CONTROLLING HYPERCHAOS AND SYNCHRONIZATION OF AN UNCERTAIN MODIFIED HYPERCHAOTIC LÜ SYSTEM

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ABSTRACT

This paper investigates the adaptive control and synchronization of an uncertain modified hyperchaotic Lü system recently discovered by Wang, Zhang, Zheng and Li (2006). This paper deploys nonlinear control for controlling the hyperchaos of the modified hyperchaotic Lü system with unknown parameters and then synchronizing two identical modified hyperchaotic Lü systems with unknown parameters. First, adaptive control laws are designed to stabilize the modified hyperchaotic Lü system to its unstable equilibrium at the origin based on the adaptive control theory and Lyapunov stability theory. Then adaptive control laws are derived to achieve global chaos synchronization of identical modified hyperchaotic Lü systems with unknown parameters. Numerical simulations are presented to demonstrate the effectiveness of the proposed adaptive control and synchronization schemes.

KEYWORDS

Adaptive Control, Hyperchaos, Synchronization, Nonlinear Control, Hyperchaotic Lü System.

1. INTRODUCTION

A chaotic system is a nonlinear dynamical system with the following characteristics: extreme sensitivity to changes in initial conditions, random-like behaviour, deterministic motion, trajectories of chaotic systems pass through any point an infinite number of times. Experimentally, chaos was first discovered by Lorenz ([1], 1963) while he was simulating weather models.

Hyperchaotic attractor is a chaotic system having more than one positive Lyapunov exponent. Historically, hyperchaos was first reported by Rössler, which is the famous hyperchaotic Rössler system ([2], 1979). The hyperchaotic attractors are found to be very useful in many engineering applications like secure communication, data encryption, etc. because the trajectories of the hyperchaotic system are expanded in more than one direction giving rise to a more complex and increasing randomness and unpredictability.

The control of chaotic systems is to design state feedback control laws that stabilizes the chaotic systems around the unstable equilibrium points. Active control technique is used when the system parameters are known and adaptive control technique is used when the system parameters are unknown [3-4].

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Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic attractors are coupled or when a hyperchaotic attractor drives another hyperchaotic attractor. In the last two decades, there has been significant interest in the literature on the synchronization of chaotic and hyperchaotic systems [5-16].

In 1990, Pecora and Carroll [5] introduced a method to synchronize two identical chaotic systems and showed that it was possible for some chaotic systems to be completely synchronized. From then on, chaos synchronization has been widely explored in a variety of fields including physical systems [6], chemical systems [7], ecological systems [8], secure communications [9-10], etc.

In most of the chaos synchronization approaches, the *master-slave* or *drive-response* formalism has been used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

The seminal work by Pecora and Carroll (1990) has been followed by a variety of impressive approaches in the literature such as the sampled-data feedback method [11], OGY method [12], time-delay feedback method [13], backstepping method [14], active control method [15-20], adaptive control method [21-25], sliding mode control method [26-28], etc.

This paper is organized as follows. In Section 2, we derive results for the adaptive control of modified hyperchaotic Lü system (Wang, Zhang, Zheng and Li, [29], 2006) with unknown parameters. In Section 3, we derive results for the adaptive synchronization of modified hyperchaotic Lü systems with unknown parameters. Section 4 contains a summary of the main results derived in this paper.

2. ADAPTIVE CONTROL OF THE MODIFIED HYPERCHAOTIC LÜ SYSTEM

2.1 Theoretical Results

The modified hyperchaotic Lü system (Wang, Zhang, Zheng and Li, [29], 2006) is one of the recently discovered four-dimensional hyperchaotic systems.
The modified hyperchaotic Lü system is described by the dynamics

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1 + x_2 x_3)\\ \dot{x}_2 &= -x_1 x_3 + b x_2 + x_4\\ \dot{x}_3 &= x_1 x_2 - c x_3\\ \dot{x}_4 &= -d x_1\end{aligned} \tag{1}$$

where $x_i \ (i = 1, 2, 3, 4)$ are the state variables and a, b, c, d are positive constants.

The system (1) is hyperchaotic when the parameter values are taken as

$$a = 35, \quad b = 14, \quad c = 3 \ \text{and} \ d = 5. \tag{2}$$

The strange attractor of the hyperchaotic system (1) is described in Figure 1.

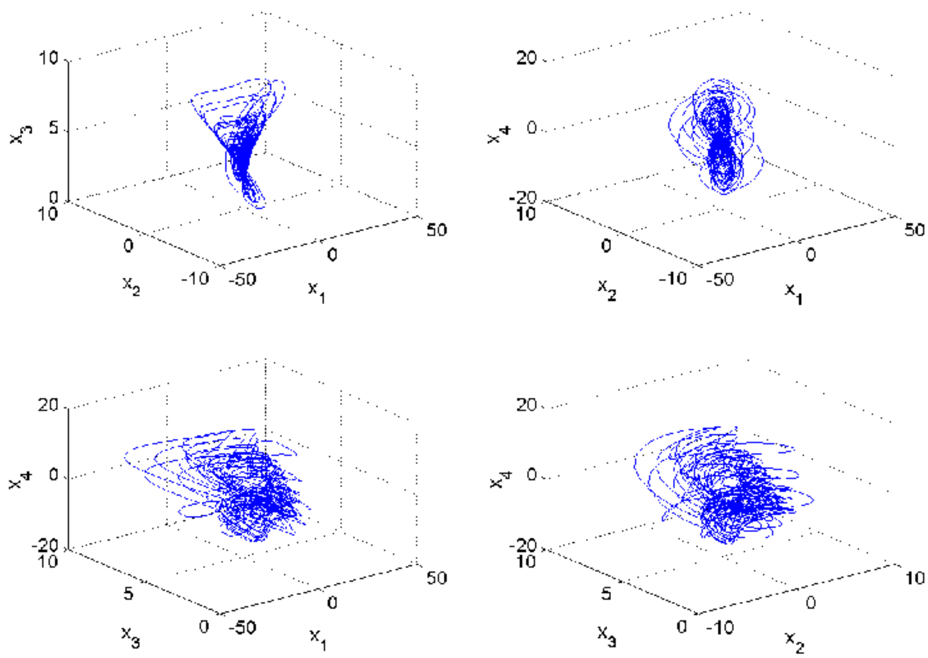

Figure 1. Strange Attractor of the Modified Hyperchaotic Lü System

When the parameter values are taken as in (2), the system (1) is hyperchaotic and the system linearization matrix at the equilibrium point $E_0 = (0, 0, 0, 0)$ is given by

$$A = \begin{bmatrix} -35 & 35 & 0 & 0 \\ 0 & 14 & 0 & 1 \\ 0 & 0 & -3 & 0 \\ -5 & 0 & 0 & 0 \end{bmatrix}$$

which has the eigenvalues

$$\lambda_1 = -35.1015, \; \lambda_2 = -3, \; \lambda_3 = 0 \quad \text{and} \quad \lambda_4 = 13.7387.$$

Since λ_4 is a positive eigenvalue, it is immediate from Lyapunov stability theory [30] that the hyperchaotic system (1) is unstable at the equilibrium point $E_0 = (0, 0, 0, 0)$.

In this section, we design adaptive control law for globally stabilizing the hyperchaotic system (1) when the parameter values are unknown.

Thus, we consider the controlled modified hyperchaotic Lü system (MHL system) as follows.

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1 + y_2 y_3) + u_1 \\
\dot{y}_2 &= -y_1 y_3 + b y_2 + y_4 + u_2 \\
\dot{y}_3 &= y_1 y_2 - c y_3 + u_3 \\
\dot{y}_4 &= -d y_1 + u_4
\end{aligned} \tag{3}$$

where u_1, u_2, u_3 and u_4 are feedback controllers to be designed using the states and estimates of the unknown parameters of the system.

In order to ensure that the controlled system (3) globally converges to the origin asymptotically, we consider the following adaptive control functions

$$\begin{aligned}
u_1 &= -\hat{a}(x_2 - x_1 + x_2 x_3) - k_1 x_1 \\
u_2 &= x_1 x_3 - \hat{b} x_2 - x_4 - k_2 x_2 \\
u_3 &= -x_1 x_2 + \hat{c} x_3 - k_3 x_3 \\
u_4 &= \hat{d} x_1 - k_4 x_4
\end{aligned} \tag{4}$$

where $\hat{a}, \hat{b}, \hat{c}$ and $\hat{d}$ are estimates of the parameters a, b, c and d, respectively, and $k_i, (i = 1, 2, 3, 4)$ are positive constants.

Substituting the control law (4) into the modified hyperchaotic Lü dynamics (1), we obtain

$$\begin{aligned}
\dot{x}_1 &= (a - \hat{a})\ (x_2 - x_1 + x_2 x_3) - k_1\ x_1 \\
\dot{x}_2 &= (b - \hat{b}) x_2 - k_2\ x_2 \\
\dot{x}_3 &= (c - \hat{c})\ x_3 - k_3\ x_3 \\
\dot{x}_4 &= (d - \hat{d})\ x_1 - k_4\ x_4
\end{aligned} \tag{5}$$

Let us now define the parameter errors as

$$e_a = a - \hat{a}, \quad e_b = b - \hat{b}, \quad e_c = c - \hat{c} \ \text{ and } \ e_d = d - \hat{d}. \tag{6}$$

Using (6), the closed-loop dynamics (5) can be written compactly as

$$\begin{aligned}
\dot{x}_1 &= e_a\ (x_2 - x_1 + x_2 x_3) - k_1\ x_1 \\
\dot{x}_2 &= e_b x_2 - k_2\ x_2 \\
\dot{x}_3 &= e_c x_3 - k_3\ x_3 \\
\dot{x}_4 &= e_d\ x_1 - k_4\ x_4
\end{aligned} \tag{7}$$

For the derivation of the update law for adjusting the parameter estimates $\hat{a}, \hat{b}, \hat{c}$ and $\hat{d}$, the Lyapunov approach is used.

Consider the quadratic Lyapunov function

$$V(x_1, x_2, x_3, x_4, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + x_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right) \tag{8}$$

which is a positive definite function on R^8.

Note also that

$$\dot{e}_a = -\dot{\hat{a}}, \;\; \dot{e}_b = -\dot{\hat{b}}, \;\; \dot{e}_c = -\dot{\hat{c}}, \;\; \dot{e}_d = -\dot{\hat{d}}. \tag{9}$$

Differentiating V along the trajectories of (7) and using (9), we obtain

$$\begin{aligned}\dot{V} = &-k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 - k_4 x_4^2 \; + e_a\left[x_1(x_2 - x_1 + x_2 x_3) - \dot{\hat{a}}\right] \\ &+ e_b\left(x_2^2 - \dot{\hat{b}}\right) + e_c\left(x_3^2 - \dot{\hat{c}}\right) + e_d\left(x_1 x_4 - \dot{\hat{d}}\right)\end{aligned} \tag{10}$$

In view of Eq. (10), the estimated parameters are updated by the following law:

$$\begin{aligned}\dot{\hat{a}} &= x_1(x_2 - x_1 + x_2 x_3) + k_5 e_a \\ \dot{\hat{b}} &= x_2^2 + k_6 e_b \\ \dot{\hat{c}} &= x_3^2 + k_7 e_c \\ \dot{\hat{d}} &= x_1 x_4 + k_8 e_d\end{aligned} \tag{11}$$

where k_5, k_6, k_7 and k_8 are positive constants.

Substituting (11) into (10), we get

$$\dot{V} = -k_1\, x_1^2 - k_2\, x_2^2 - k_3\, x_3^2 - k_4\, x_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 \tag{12}$$

which is a negative definite function on R^8.

Thus, by Lyapunov stability theory [30], we obtain the following result.

Theorem 1. The modified hyperchaotic Lü system (1) with unknown parameters is globally and exponentially stabilized for all initial conditions $x(0) \in R^4$ by the adaptive control law (4), where the update law for the parameters is given by (11) and $k_i, \; (i = 1, \ldots, 8)$ are positive constants. ■

2.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the hyperchaotic system (1) with the adaptive control law (4) and the parameter update law (11).

The parameters of the modified hyperchaotic Lü system (1) are selected as $a = 35, \;\; b = 14, \;\; c = 3$ and $d = 5.$

For the adaptive and update laws, we take $k_i = 4, \quad (i = 1, 2, \ldots, 8).$

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 6, \quad \hat{b}(0) = 8, \quad \hat{c}(0) = 7, \quad \hat{d}(0) = 4.$$

The initial state of the modified hyperchaotic Lü system (1) is taken as

$$x_1(0) = 1, \quad x_2(0) = 5, \quad x_3(0) = 2, \quad x_4(0) = 3.$$

When the adaptive control law (4) and the parameter update law (11) are used, the controlled modified hyperchaotic Lü system converges to the equilibrium $E_0 = (0, 0, 0, 0)$ exponentially as shown in Figure 2. The parameter estimates are shown in Figure 3.

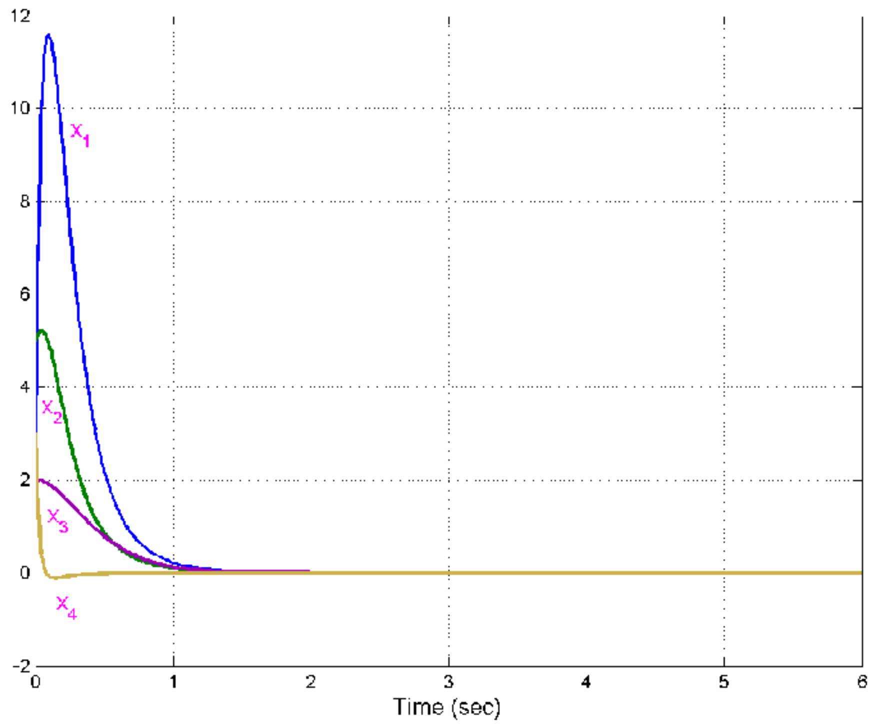

Figure 2. Time Responses of the Controlled Modified Hyperchaotic Lü System

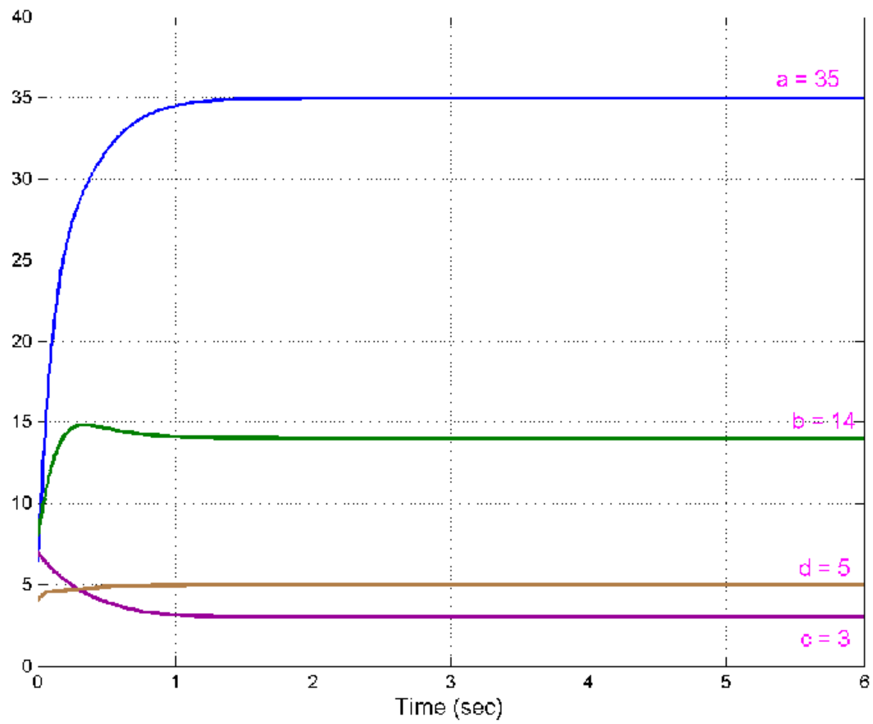

Figure 3. Parameter Estimates $\hat{a}(t),\ \hat{b}(t),\ \hat{c}(t),\ \hat{d}(t)$

3. ADAPTIVE SYNCHRONIZATION OF IDENTICAL MODIFIED HYPERCHAOTIC LÜ SYSTEMS

3.1 Theoretical Results

In this section, we discuss the adaptive synchronization of identical modified hyperchaotic Lü systems (Wang, Zhang, Zheng and Li, [29], 2006) with unknown parameters.

As the master system, we consider the modified hyperchaotic Lü dynamics described by

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1 + x_2 x_3)\\ \dot{x}_2 &= -x_1 x_3 + b x_2 + x_4\\ \dot{x}_3 &= x_1 x_2 - c x_3\\ \dot{x}_4 &= -d x_1\end{aligned} \tag{13}$$

where $x_i,\ (i = 1, 2, 3, 4)$ are the state variables and a, b, c, d are unknown system parameters.

The system (13) is hyperchaotic when the parameter values are taken as

$a = 35,\ \ b = 14,\ \ c = 3$ and $d = 5.$

As the slave system, we consider the modified hyperchaotic Lü dynamics described by

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1 + y_2 y_3) + u_1 \\
\dot{y}_2 &= -y_1 y_3 + b y_2 + y_4 + u_2 \\
\dot{y}_3 &= y_1 y_2 - c y_3 + u_3 \\
\dot{y}_4 &= -d y_1 + u_4
\end{aligned} \tag{14}$$

where $y_i,\ (i = 1,2,3,4)$ are the state variables and $u_i,\ (i = 1,2,3,4)$ are the nonlinear controllers to be designed.

The synchronization error is defined by

$$\begin{aligned}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 - x_2 \\
e_3 &= y_3 - x_3 \\
e_4 &= y_4 - x_4
\end{aligned} \tag{15}$$

Then the error dynamics is obtained as

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1 + y_2 y_3 - x_2 x_3) + u_1 \\
\dot{e}_2 &= b e_2 + e_4 - y_1 y_3 + x_1 x_3 + u_2 \\
\dot{e}_3 &= -c e_3 + y_1 y_2 - x_1 x_2 + u_3 \\
\dot{e}_4 &= -d e_1 + u_4
\end{aligned} \tag{16}$$

Let us now define the adaptive control functions $u_1(t), u_2(t), u_3(t), u_4(t)$ as

$$\begin{aligned}
u_1 &= -\hat{a}(e_2 - e_1 + y_2 y_3 - x_2 x_3) - k_1 e_1 \\
u_2 &= -\hat{b} e_2 - e_4 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\
u_3 &= \hat{c} e_3 - y_1 y_2 + x_1 x_2 - k_3 e_3 \\
u_4 &= \hat{d} e_1 - k_4 e_4
\end{aligned} \tag{17}$$

where $\hat{a}, \hat{b}, \hat{c}$ and $\hat{d}$ are estimates of the parameters a, b, c and d respectively, and $k_i, (i = 1,2,3,4)$ are positive constants.

Substituting the control law (17) into (16), we obtain the error dynamics as

$$\begin{aligned}
\dot{e}_1 &= (a - \hat{a})(e_2 - e_1 + y_2 y_3 - x_2 x_3) - k_1 e_1 \\
\dot{e}_2 &= (b - \hat{b}) e_2 - k_2 e_2 \\
\dot{e}_3 &= -(c - \hat{c}) e_3 - k_3 e_3 \\
\dot{e}_4 &= -(d - \hat{d}) e_1 - k_4 e_4
\end{aligned} \tag{18}$$

Let us now define the parameter errors as

$$\begin{aligned} e_a &= a - \hat{a} \\ e_b &= b - \hat{b} \\ e_c &= c - \hat{c} \\ e_d &= d - \hat{d} \end{aligned} \tag{19}$$

Substituting (19) into (18), the error dynamics simplifies to

$$\begin{aligned} \dot{e}_1 &= e_a(e_2 - e_1 + y_2 y_3 - x_2 x_3) - k_1 e_1 \\ \dot{e}_2 &= e_b e_2 - k_2 e_2 \\ \dot{e}_3 &= -e_c e_3 - k_3 e_3 \\ \dot{e}_4 &= -e_d e_1 - k_4 e_4 \end{aligned} \tag{20}$$

For the derivation of the update law for adjusting the estimates of the parameters, the Lyapunov approach is used.

Consider the quadratic Lyapunov function

$$V(e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_d) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2\right) \tag{21}$$

which is a positive definite function on R^8.

Note also that

$$\dot{e}_a = -\dot{\hat{a}}, \quad \dot{e}_b = -\dot{\hat{b}}, \quad \dot{e}_c = -\dot{\hat{c}}, \quad \dot{e}_d = -\dot{\hat{d}} \tag{22}$$

Differentiating V along the trajectories of (20) and using (22), we obtain

$$\begin{aligned} \dot{V} = &-k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 + e_a\left[e_1(e_2 - e_1 + y_2 y_3 - x_2 x_3) - \dot{\hat{a}}\right] \\ &+ e_b\left[e_2^2 - \dot{\hat{b}}\right] + e_c\left[-e_3^2 - \dot{\hat{c}}\right] + e_d\left[-e_1 e_4 - \dot{\hat{d}}\right] \end{aligned} \tag{23}$$

In view of Eq. (23), the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= e_1(e_2 - e_1 + y_2 y_3 - x_2 x_3) + k_5 e_a \\ \dot{\hat{b}} &= e_2^2 + k_6 e_b \\ \dot{\hat{c}} &= -e_3^2 + k_7 e_c \\ \dot{\hat{d}} &= -e_1 e_4 + k_8 e_d \end{aligned} \tag{24}$$

where k_5, k_6, k_7 and k_8 are positive constants.

Substituting (24) into (23), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2, \tag{25}$$

From (25), we find that $\dot{V}$ is a negative definite function on R^8.

Thus, by Lyapunov stability theory [30], it is immediate that the synchronization error and the parameter error decay to zero exponentially with time for all initial conditions.

Hence, we have proved the following result.

Theorem 2. The identical modified hyperchaotic Lü systems (13) and (14) with unknown parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law (17), where the update law for parameters is given by (24) and $k_i, (i = 1, \ldots, 8)$ are positive constants. ■

3.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (13) and (14) with the adaptive control law (17) and the parameter update law (24).

We take the parameter values as in the hyperchaotic case, viz.

$$a = 35, \quad b = 14, \quad c = 3, \quad d = 5.$$

We take the positive constants k_i, $(i = 1, \ldots, 8)$ as

$$k_i = 4 \quad \text{for} \quad i = 1, 2, \ldots, 8.$$

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 15, \quad \hat{b}(0) = 9, \quad \hat{c}(0) = 6, \quad \hat{d}(0) = 8$$

We take the initial values of the master system (13) as

$$x_1(0) = 10, \quad x_2(0) = 5, \quad x_3(0) = 2, \quad x_4(0) = 7$$

We take the initial values of the slave system (14) as

$$y_1(0) = 3, \quad y_2(0) = 8, \quad y_3(0) = 11, \quad y_4(0) = 6$$

Figure 4 shows the adaptive chaos synchronization of the identical modified hyperchaotic Lü systems.

Figure 5 shows that the estimated values of the parameters $\hat{a}, \hat{b}, \hat{c}$ and $\hat{d}$ converge to the system parameters

$$a = 35, \quad b = 14, \quad c = 3 \text{ and } d = 5.$$

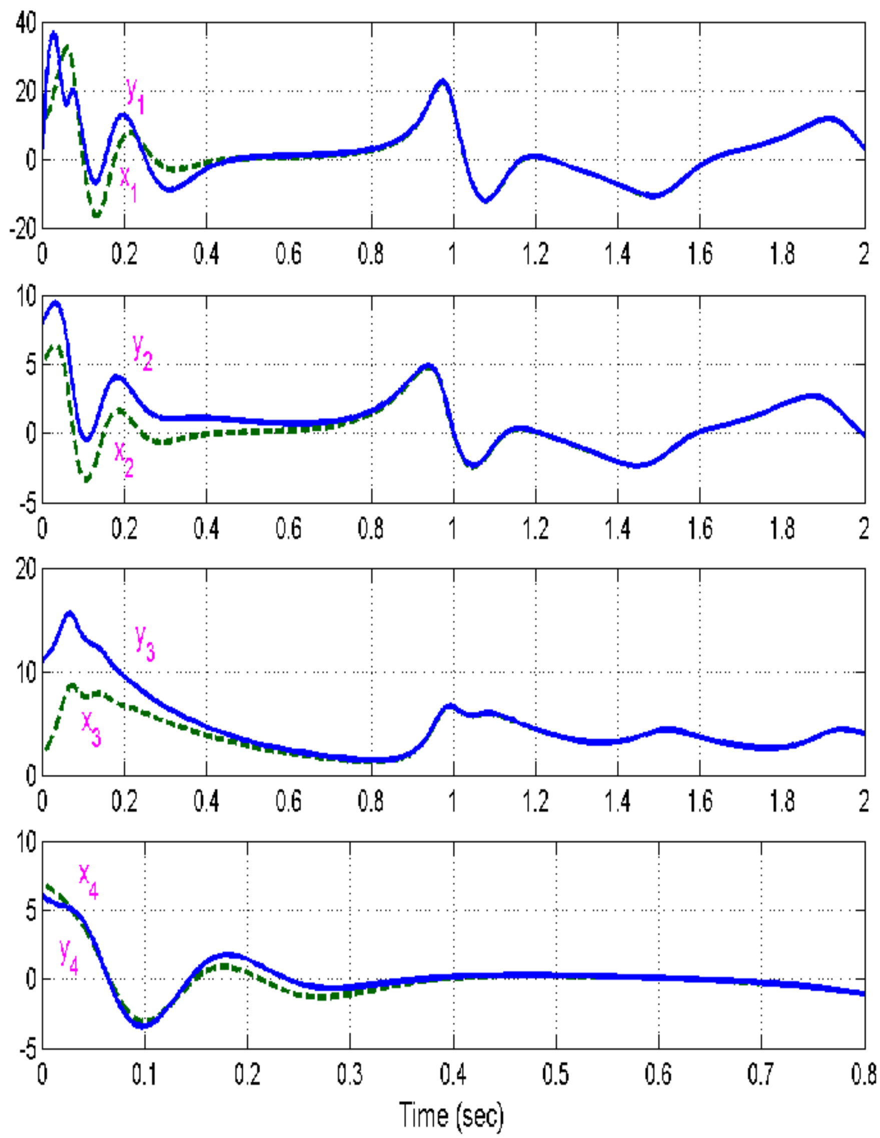

Figure 4. Adaptive Synchronization of the Modified Hyperchaotic Lü Systems

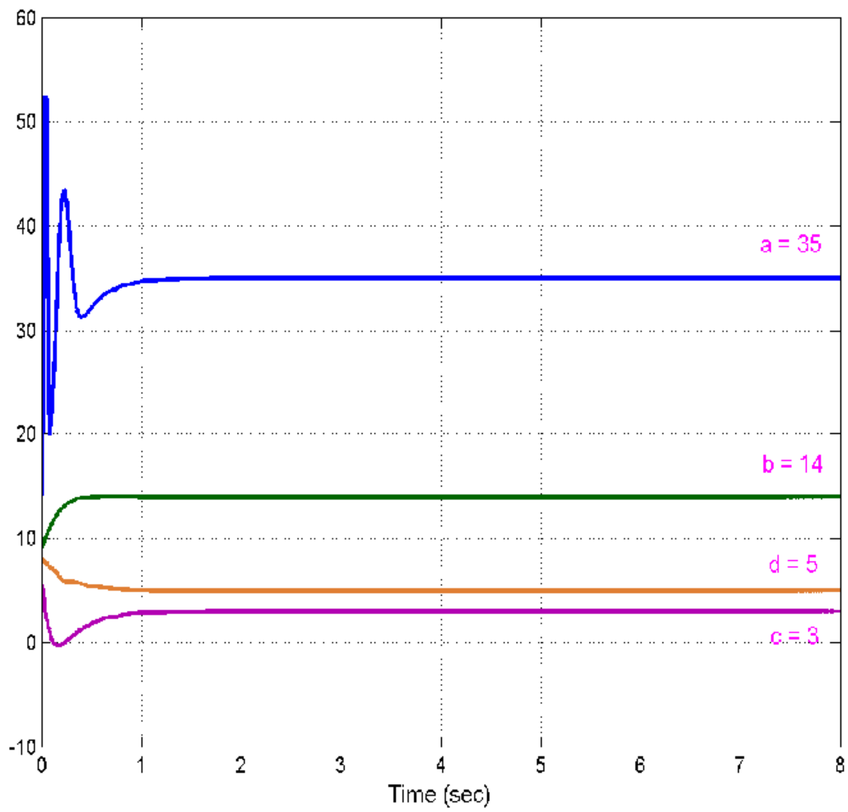

Figure 5. Parameter Estimates $\hat{a}(t),\ \hat{b}(t),\ \hat{c}(t),\ \hat{d}(t)$

4. CONCLUSIONS

Controlling hyperchaos and synchronization of identical hyperchaotic systems is an important research problem in the chaos literature. Hyperchaotic systems have important applications in electronics and communications engineering like secure communications, data encryption etc. In this paper, we applied adaptive control theory for the stabilization and synchronization of the modified hyperchaotic Lü system (Wang, Zhang, Zheng and Li, 2006) with unknown system parameters. First, we designed adaptive control laws to stabilize the modified hyperchaotic Lü system to its unstable equilibrium point at the origin based on the adaptive control theory and Lyapunov stability theory. Then we derived adaptive synchronization scheme and update law for the estimation of system parameters for the identical modified hyperchaotic Lü systems with unknown parameters. Our synchronization schemes were established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the proposed adaptive control method is very effective and convenient to achieve chaos control and synchronization of the modified hyperchaotic Lü system. Numerical simulations are shown to demonstrate the effectiveness of the proposed adaptive stabilization and synchronization schemes.

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