On the calculation of the hues that polarization develops in crystalline plates (& postscript)

by Augustin Fresnel

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with notes and analytical table of contents
by the editors of Fresnel's *Oeuvres complètes*
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Translated by Gavin R. Putland *

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English translation of A. Fresnel, "Note sur le calcul des teintes que la polarisation développe dans les lames cristallisées", *Annales de Chimie et de Physique*, Ser. 2, vol. 17, pp. 102–11 (May 1821), "II^{e} note sur la coloration des lames cristallisées", *vol. cit.*, pp. 167–96 (June 1821), and "Addition à la II^{e} note insérée dans le cahier précédent", *vol. cit.*, pp. 312–15 (July 1821), as reprinted in *Oeuvres complètes d'Augustin Fresnel* [9], vol. 1 (1866), pp. 609–48, with the corresponding extract from the "Table Analytique" in *Oeuvres complètes...* [9], vol. 3 (1870), at pp. 581–6.

The first part of this paper cites empirical results to show that polarization is analogous to the resolution of forces in a plane, proposes that unpolarized light is a "rapid succession of an infinitude of wavetrains polarized in all azimuths", and proceeds to construct the first essentially correct explanation of *chromatic polarization*—that is, the wavelength-dependent modification of polarized light by birefringent "crystalline plates", causing colors to appear when the emergent light is viewed through an analyzer.

The second part extends the analysis to two stacked plates, delivering the fatal blow to the "mobile polarization" theory of J.-B. Biot, who had tried to explain the same phenomena in corpuscular terms. Then, under the subheading "Mechanical considerations on the polarization of light", it proposes that light waves are *purely* transverse, for both polarized and unpolarized light; introduces the concept of transverse elastic waves; redefines the axis and the ordinary and extraordinary modes of propagation in a uniaxial birefringent crystal, in terms of reactions to transverse displacements, concluding that the vibrations of the ordinary waves are *perpendicular* to what has been called their *plane of polarization*; explains Malus's law as vector decomposition; explains the non-interference of perpendicularly polarized beams; trivializes the empirical rule on whether to add a half-cycle to the path difference when calculating the hue seen through the analyzer in chromatic polarization; derives a formula for the reflectivity for what we now call the 's' component of light reflected obliquely from a transparent body; and claims empirical confirmation of the formula for two angles of incidence, namely those for which D. F. J. Arago has found that as much light is transmitted as reflected, by one glass plate or two stacked glass plates (in each case, the reflectivity for the other component is calculated from the observed azimuth of the reflected beam when that of the incident beam is 45°).

The third part—a brief postscript—announces a reflectivity formula for what we now call the 'p' component (omitting details of the derivation), and presents experimental confirmation of the ratio of the two predicted reflection coefficients, for a range of angles of incidence on glass and water.

— *Translator*.

* Melbourne, Australia. Gmail address: grputland. This paper—the first of a series in which Fresnel reconstructed physical optics on the transverse-wave hypothesis between May 1821 and January 1823—has been translated in time for its bicentenary, without institutional sponsorship. My opportunity to translate more papers in the series may depend on such sponsorship.

Translator's preface

In the reprint of this paper in Fresnel's *Oeuvres complètes*, the footnote to the main title [9, vol. 1, p. 609] acknowledges where the three installments were first published (see the above abstract), and explains:

> These three parts were then combined with Mr. Arago's report in a separate printing; the deletion of a few words restores the transitions.
>
> We reproduce the text of the separate printing.
>
> This note has been placed after the various pieces from the controversy with Mr. Biot, although it had been published previously. In fact it is almost solely to Arago's report that the controversy relates.

Concerning the second installment, a footnote to the footnote (!) says in smaller type:

> (*) Note. From page 113 to page 176 of volume 17 of the *Annales*, the page numbers have been erroneously increased by one hundred.

Footnotes to the text of the present translation (but not footnotes to footnotes) are numbered sequentially. Immediately after their sequential numbers, footnotes by Fresnel are further identified by their *original numbers* in their original parentheses, and footnotes by the editors of the *Oeuvres complètes* are further identified by their *original letters* in their original parentheses. Section/article numbers, in the text and in the analytical table of contents, are from the *Oeuvres complètes*. Footnotes identified by sequential numbers alone, together with all items in *square brackets* (in the analytical table or the main text or the footnotes, and including citations such as "[3, pp. 218–9]"), have been inserted by the translator. For added clarity, some notes by the translator are signed accordingly, although their origin is already implied. In the two places where William Whewell is quoted in footnotes from the *Oeuvres complètes*, the original English text has been substituted for the editors' French translation.

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The oscillatory motion of the *ordinary rays* is executed perpendicular to the plane through the axis of the crystal, i.e. perpendicular to their *plane of polarization*, and the oscillatory motion of the *extraordinary rays* parallel to the same plane, and always *perpendicular to the rays*. — It [the latter motion] will be *parallel to the axis* when the rays are perpendicular to it, and *perpendicular to the axis* when the rays are parallel to it; so, in the last case, *the speed of propagation of the extraordinary rays will be the same as that of the ordinary rays*. — For all intermediate directions, the difference grows, and it attains its *maximum* when the oscillatory movement becomes *parallel to the axis* . . . 19

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15. It is perceived, from this new idea on the vibrations of light, why rays *polarized at right angles* can no longer interfere with each other, i.e. why they *always produce by their union the same intensity of light*. — From this is likewise deduced the [half-cycle] rule concerning the calculation of the hues produced by the crystalline plates . . . 20

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20. According to an empirical hypothesis, the intensity of the reflected light polarized perpendicularly to the plane of reflection would be represented by $\left(\frac{\sin 2i - \sin 2i'}{\sin 2i + \sin 2i'}\right)^2$. — This formula, together with that which has been given for light polarized in the plane of reflection, gives for the intensity of reflected light which has not suffered any prior polarization: $\left(\frac{\sin(i-i')}{\sin(i+i')}\right)^2 + \left(\frac{\sin 2i - \sin 2i'}{\sin 2i + \sin 2i'}\right)^2$; which agrees with the observations of Arago . . . 24

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[First part] [1]

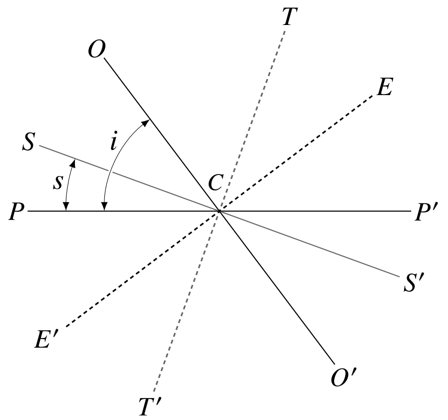
[**Context**: Polarized light is passed through a birefringent "crystalline plate" (which we would now call a waveplate), whose "principal section" (which may contain the fast axis or the slow axis) makes an angle i with the initial plane of polarization (that plane being normal to the vibrations, as Fresnel will show). As the "ordinary" and "extraordinary" waves travel at different speeds through the plate, the emergent light has a state of polarization that varies with wavelength ("chromatic polarization"), causing colors to appear when the emergent light is viewed through an analyzer ("colors of crystalline plates"). Here the analyzer takes the form of a "rhomb" of calcite, which splits the image into an ordinary image and

[1] First published as (when translated) "Note on the calculation of the hues that polarization develops in crystalline plates" in *Annales de Chimie et de Physique*, Ser. 2, vol. 17, pp. 102–11 (May 1821).

an extraordinary image, polarized perpendicularly to each other. Hence, if the light is initially white, the two images must be of complementary colors. Fresnel's first paragraph refers to a correspondence between hue and path difference, this correspondence having been established by Newton's rings in transmitted light. The problem is to explain how the colors of a crystalline plate may show the same correspondence, or the complementary one, with varying degrees of color saturation. —*Translator*.]

1. We have seen, in the report of Mr. Arago [9, vol. 1, pp. 553–68], that the nature of these hues [French: *teintes*] is determined by the path difference between the two wavetrains [*systèmes d'ondes*] into which the light is divided in traversing a crystal that shows double refraction; but, because the two images produced by the rhomb of calcite though which we pass the emergent light are always complementary [in color], it necessarily follows that, if one corresponds to the path difference of the two wavetrains in the crystalline plate, the other corresponds to that difference augmented or diminished by a half-cycle. Here is the general rule which indicates for which of the two images we must add a half-cycle to the difference of the two paths traveled: *the image whose color corresponds precisely to the difference of the traveled paths is the one of which the planes of polarization of the two constituent beams, after being separated from each other, come back together by a contrary motion to meet again; whereas the planes of polarization of the two constituent beams of the complementary image continue to move away from each other (considered from one side of their common intersection), until they are placed on the extension of each other.*

This rule becomes easier to perceive with the aid of the following figure, in which PP' represents the initial plane of polarization of the incident rays, OO' the principal section of the crystalline plate, and SS' that of the rhomb through which we view it.



We see that the incident light, initially polarized per [*suivant*] CP, divides itself, in crossing the crystalline plate, into two parts, the one which suffers the ordinary refraction and receives a new polarization per CO, and the other which suffers the extraordinary refraction and finds itself polarized in [*dans*] a plane CE' perpendicular to CO.[2] Let us denote the first by F_o and the second by F_e. The passage through the rhomb divides F_o, polarized per CO, into two other wavetrains, the one polarized in the principal section CS, which I denote by $F_{o+o'}$, and the second polarized in the perpendicular plane CT, which I shall call $F_{o+e'}$. Likewise F_e, polarized per CE', divides itself in the rhomb into two wavetrains, the first $F_{e+o'}$, polarized per CS, and the second $F_{e+e'}$, polarized per CT'. If we follow the movement of the planes of polarization of the two beams $F_{o+o'}$ and $F_{e+o'}$, which concur in the formation of the ordinary image (considering them from one side of their common intersection projected on C), we see that, departing initially from CP, they move away from each other to take the directions CO and CE', and then, coming back together again, reunite in CS. Now in this case the ordinary image corresponds precisely to the difference of the paths traveled, at the same instant, by the ordinary and extraordinary rays that have come out from the crystalline plate. If we similarly follow the course of the planes of polarization of the two beams $F_{o+e'}$ and $F_{e+e'}$ of the extraordinary image, we see that, having both departed from CP, and having taken the directions CO and CE' in the crystalline plate, instead of coming closer they are placed on the extension of each other in the directions CT and CT'; so, according to the rule that we have just given, we must add a half-cycle to the difference of the paths traveled by

[2] To specify the plane of polarization, Fresnel uses the prepositions *suivant* (per, according to) and *dans* (in) synonymously, but seems to prefer the former, presumably because he has deduced that the vibrations are not "in" that plane. —*Translator*.

the two wavetrains—or, equivalently, change the signs of the oscillatory movements in one of them—in order to calculate, by the formula of interference, the wavetrain that results from the reunion of the two beams. We see that things happen just as if it were a matter of the combination of forces in the plane of the figure, perpendicular to the rays, in their planes of polarization or perpendicular to those planes; for the two force components CO and CE', which would reunite in CS, would have the same sign, like the two beams $F_{o+o'}$ and $F_{e+o'}$ which have been reunited there, and the two other components CT and CT', acting in opposite senses, must be preceded by opposite signs.

The principle of conservation of energy [*conservation des forces vives*] indicated in advance that the two images must be complementary to each other; but it did not specify which of the two corresponds to the difference of the paths traveled, and which corresponds to that difference augmented by a half-cycle; this is why I have resorted to facts, and deduced from the observations of Mr. Biot the rule that I have just stated.

The rule explains why two beams of direct light, which have been polarized at right angles [without a common prior polarization], do not present any appearance of mutual influence when we bring them back to a common plane of polarization by the action of a stack of glass plates or of a rhomb of calcite. It is not because they do not then exercise any influence on each other—for, independently of mechanical considerations, this supposition would be too contrary to analogy—but because the effects produced by the different wavetrains of direct light compensate and neutralize each other. Indeed we may consider direct light [i.e., light not yet polarized] as the assemblage or, more precisely, the rapid succession of an infinitude of wavetrains polarized in all azimuths, so that there is always as much of the light polarized in any one plane as in the perpendicular plane: but then it follows from the rule just stated that if, for example, we must add a half-cycle to the difference of the traveled paths in order to calculate the extraordinary image produced by the light polarized per the first plane, we must not add it for the extraordinary image that results from the light polarized per the second; hence the two colors that they bring together or successively in the extraordinary image are complementary [and cancel out]. The compensation established in this way, and in the same manner for all azimuths, precludes the perception of the effects of interference.

2. Let us return to the case represented in the figure, where the incident light has undergone a prior polarization per the plane PP' before the traversal of the crystalline plate, whose principal section OO' makes an angle i with that plane. For a particular species of homogeneous [i.e., monochromatic] light of wavelength λ, let us ask what must be the intensities of the ordinary and extraordinary images given by the rhomb of calcite, whose principal section SS' makes an angle s with the initial plane PP'. In this calculation I shall neglect the loss of light due to the partial reflections at the two surfaces of the crystalline plate and of the rhomb, because this affects only the absolute intensities of the images, and not their relative intensities, which alone concern us here. I denote by F the amplitude[3] of the velocities of the aether molecules in their oscillations,[4] for the polarized incident beam; its *intensity of light* will be represented by F^2, or the intensity of the energy, according to the same sense that we attach to the first expression and the manner in which we evaluate the intensities of light in all optical experiments; for it is the sum of the energies, and not that of the velocities of oscillation, that remains constant, like the total intensities in the various subdivisions that the light can undergo. That being said, the incident beam, in crossing the crystalline plate, is divided into two others whose luminous intensities, according to Malus's law, must be equal to $F^2 \cos^2 i$ for that which suffers the ordinary refraction, and $F^2 \sin^2 i$ for that which suffers the extraordinary refraction; the amplitude of the velocities of oscillation will then be $F \cos i$ in the first and $F \sin i$ in the second.[5] So the incident light, in crossing the crystalline plate, is

[3] French: *intensité*.

[4, (1)] Henceforth I call these velocities, for brevity, *velocities of oscillation*. They must not be confused with the period of oscillation, which remains always constant for the same species of rays, whatever be the intensity of the light.

[5, (2)] If the oscillations of light, as I am strongly led to believe, are executed solely in the plane of the wave, perpendicularly to the plane of polarization, then Malus's law becomes a simple and rigorous consequence of the composition and decomposition of small movements. [This footnote, by Fresnel, is numbered (1) in the original publication and (2) in the *Oeuvres complètes*, due to different page breaks.]

divided into two wavetrains, which we can represent as follows:

$$\begin{array}{cc} F_o \cos i & F_e \sin i \\ P.O & P.E'. \end{array}$$

The small letters o and e, subfixed to F, do not at all change the value of this quantity; they only indicate the length of the path traveled at the same instant by the ordinary and extraordinary rays after they have come out of the crystalline plate, and thereby determine, by their difference $o{-}e$, the spacing between the corresponding points of the two wavetrains. The upper-case $P.O$ and $P.E'$ show the successive positions of the plane of polarization of each beam, to facilitate the application of the rule previously stated.

Each of these two wavetrains is divided into two others by the action of the calcite rhomb, which will produce in total the four following beams, of which the first two are produced by the first wavetrain, and the other two by the second:

$$\begin{array}{cc} F_{o+o'} \cos i \cos(i{-}s) & F_{o+e'} \cos i \sin(i{-}s) \\ P.O.S & P.O.T \\[6pt] F_{e+o'} \sin i \sin(i{-}s) & F_{e+e'} \sin i \cos(i{-}s) \\ P.E'.S & P.E'.T'. \end{array}$$

The first with the third [left column] constitute the ordinary image, and the second with the fourth [right column] the extraordinary image. Let us first calculate the intensity of the latter.

3. We see, from the progress of the planes of polarization indicated by the upper-case letters under each beam, that the second and the fourth, when brought back to a common plane of polarization, must differ by a half-cycle that is independent of the difference $o{-}e$ between the paths traveled; we must therefore add a half-cycle to $o{-}e$ or, equivalently, change the sign of one of the expressions that represent the amplitude or the common factor of the velocities of oscillation.[6] So the problem is to find the resultant of two wavetrains, of which the path difference is $o{-}e$ and the amplitudes of the velocities of oscillation are respectively

$$F \cos i \sin(i{-}s) \qquad \text{and} \qquad -F \sin i \cos(i{-}s)\,.$$

By applying here the general formula that I have given in the extract of my Memoir on diffraction [7], page 258 of volume 11 of the *Annales de Chimie et de Physique*,[7]

$$A^2 = a^2 + a'^2 + 2aa' \cos \tfrac{2\pi c}{\lambda}\,,$$

in which a and a' denote the amplitudes of the velocities of oscillation of the two wavetrains, 2π the circumference of which the radius is 1, c the difference of paths traveled, and λ the wavelength, we find for the intensity of the homogeneous light in the extraordinary image:[8]

$$F^2 \left\{ \cos^2 i \sin^2(i{-}s) + \sin^2 i \cos^2(i{-}s) - 2 \sin i \cos i \sin(i{-}s) \cos(i{-}s) \cos \tfrac{2\pi(o-e)}{\lambda} \right\}, \qquad [*]$$

or[9]

$$F^2 \left\{ \Big(-\cos i \sin(i{-}s) + \sin i \cos(i{-}s) \Big)^2 + 2 \sin i \cos i \sin(i{-}s) \cos(i{-}s) \Big(1 - \cos \tfrac{2\pi(o-e)}{\lambda} \Big) \right\},$$

or finally[10]

$$F^2 \left\{ \sin^2 s + \sin 2i \sin 2(i{-}s) \sin^2\!\Big(\pi \tfrac{o-e}{\lambda} \Big) \right\}.$$

[6] In other words, the coefficients $F_{o+e'}$ and $F_{e+e'}$ have opposite signs (so that their product is $-F^2$) because the final planes T and T', although coincident, differ by 180°. —*Translator*.

[7, (a)] Page 291 of the present volume [9, vol. 1]. [Or p. 106 of the translation in [4].]

[8] The minus sign before $2 \sin i$ in [*] arises from the different signs of $F_{o+e'}$ and $F_{e+e'}$.

[9] Completing the square on the first two terms.

[10] Recognizing the expression to be squared as the sine of a difference, and applying the identities $\sin\theta \cos\theta = \frac{1}{2} \sin 2\theta$ (twice) and $1 - \cos 2\theta = 2 \sin^2\theta$.

Making a similar calculation on the two constituent beams of the ordinary image, and observing that the two expressions $F\cos i\,\cos(i{-}s)$ and $F\sin i\,\sin(i{-}s)$ must have the same [leading] sign, due to the movement of the planes of polarization,[11] we find for the intensity of the light in the ordinary image:

$$F^2\left\{\cos^2 s \;-\; \sin 2i\,\sin 2(i{-}s)\,\sin^2\!\left(\pi\,\tfrac{o-e}{\lambda}\right)\right\}.$$

Here are the general formulae that give the intensity of each species of homogeneous light in the ordinary and extraordinary images in terms of its wavelength and the difference $o{-}e$ in the paths traveled by the rays that have crossed the crystalline plate. Knowing its thickness and the velocities of the ordinary and extraordinary rays in the crystal, it will be easy to determine $o{-}e$. In calcium sulfate, quartz, and most other crystals showing double refraction, $o{-}e$ suffers only very slight variations due to the difference in nature of the rays of light, so that we can regard it as a constant quantity, at least for the crystals that we consider here, in which the *dispersion of double refraction* is very small relative to the double refraction. If, having calculated the path difference $o{-}e$, we divide it successively by the mean wavelength of each of the seven principal species of colored rays, and substitute the different quotients successively into the above expressions, we will have the intensities of each species of colored rays in the ordinary and extraordinary images, and we will then be able to determine their hues with the aid of the empirical formula that Newton has given, for finding the hue resulting from any mixture of diverse rays of which we know the relative intensities. This is why we must consider the general formulae, which give the intensity of each species of homogeneous light in terms of its wavelength, as the true expression of the color [*teinte*] produced by white light. At least that is all that can now be deduced from the theory, and for the rest we must resort to Newton's empirical construction founded on experiment; for theoretically explaining and calculating the effect produced on the eye by a mixture of heterogeneous rays is a double problem of physics and of physiology, which we are undoubtedly still far from solving.

4. Let us repeat the above formulae, removing the common factor F^2, which we can take for the unit of light:

$$\text{Ordinary image:}\qquad \cos^2 s \;-\; \sin 2i\,\sin 2(i{-}s)\,\sin^2\!\left(\pi\,\tfrac{o-e}{\lambda}\right);$$

$$\text{Extraordinary image:}\qquad \sin^2 s \;+\; \sin 2i\,\sin 2(i{-}s)\,\sin^2\!\left(\pi\,\tfrac{o-e}{\lambda}\right).$$

We see, on inspection of these formulae, that the two images must become white when the term containing

$$\sin^2\!\left(\pi\,\tfrac{o-e}{\lambda}\right)$$

vanishes, as this is the only one that varies with the wavelength, giving different intensities for the different colored rays. So the images will become white when we have

$$\sin 2i\,\sin 2(i{-}s) = 0,$$

which is satisfied by setting to zero

$$\sin 2i \qquad\text{or}\qquad \sin 2(i{-}s),$$

[giving in the first case

$$i = 0,\ 90^\circ,\ 180^\circ,\ \text{or } 270^\circ,$$

and in the second

$$i = s,\ s+90^\circ,\ s+180^\circ,\ \text{or } s+270^\circ$$

].[12]

[11] That is, $F_{o+o'}$ and $F_{e+o'}$ have the same sign, because the final planes are the same (namely S), with no 180° difference. This calculation differs from the first in that $\sin(i{-}s)$ is replaced by $\cos(i{-}s)$ and vice versa, and *one* of the replacements is accompanied by a change of sign, so that the minus sign before $2\sin i$ in [∗] becomes a plus, and the expression to be squared in the next equation becomes (plus or minus) the *cosine* of a difference.

[12] The bracketed passage has been corrected so as to follow from what precedes it and imply what follows it. I do not know what Fresnel's manuscript had in its place; but both printed French editions have "which gives for i the four values $i=0$, $i=90^\circ$, $i=180^\circ$, $i=360^\circ$; and for s: $s=i$, $s=90^\circ-i$, $s=180^\circ-i$, $s=360^\circ-i$", which raises multiple issues. —*Translator*.

So, for the images to become white, it suffices that one of these eight conditions be satisfied—that is, that the principal section of the crystalline plate be parallel or perpendicular to the initial plane of polarization [as in the first case] or to the principal section of the rhomb [as in the second]. This could easily be deduced from the theory without recourse to the formula; for, when the principal section of the plate is parallel or perpendicular to the initial plane, the incident light suffers only one species of refraction of the crystal; and when this principal section is parallel or perpendicular to that of the rhomboid, each image contains only rays that have suffered the same refraction in the crystalline plate: so, in one case as in the other, each image contains but a single wavetrain, no longer with any colors, since there is no longer any interference.

The two images are, on the contrary, both colored with the greatest possible saturation [*vivacité*] when the coefficient of the variable term is equal to unity, which happens when $s = 0$ and $i = 45°$; the two expressions then become

$$\text{Ordinary image:} \qquad 1 - \sin^2\!\left(\pi\,\frac{o-e}{\lambda}\right), \quad \text{i.e.} \quad \cos^2\!\left(\pi\,\frac{o-e}{\lambda}\right);$$

$$\text{Extraordinary image:} \qquad \sin^2\!\left(\pi\,\frac{o-e}{\lambda}\right).$$

It should be noted that the second expression is similar to that which gives, for the colored rings [Newton's rings], the resultant of two wavetrains reflected under perpendicular incidence at the first and at the second surface of the layer of air,[13] when its thickness is equal to $\frac{1}{2}(o-e)$, which makes the difference of the traveled paths equal to $o{-}e$. Indeed, representing the amplitude of oscillation of each wavetrain by 1/2, we note that their velocities of oscillation must be taken with opposite signs, because the one is reflected inside the denser medium and the other outside—which implies an opposition of the sign, as follows from the calculation of Dr. Young and Mr. Poisson[14] on the reflection of waves at the surface of contact of two elastic media with different densities. On that basis, we find for the intensity of the resultant light, by the formula that we have already employed:[15]

$$\frac{1}{4} + \frac{1}{4} - 2\,.\,\frac{1}{2}\,.\,\frac{1}{2}\cos\frac{2\pi(o-e)}{\lambda}\,,$$

or

$$\frac{1}{2} - \frac{1}{2}\cos\frac{2\pi(o-e)}{\lambda}\,,$$

or finally

$$\sin^2\!\left(\pi\,\frac{o-e}{\lambda}\right).$$

So the hues of the extraordinary image produced by crystalline plates must be similar to those of the reflected colored rings, at least if the path difference $o{-}e$ produced by the crystal does not sensibly vary with the nature of the rays; for in the colored rings, the path difference, being twice the thickness of the layer of air, is rigorously the same for all species of rays.

5. The above expressions

$$\cos^2\!\left(\pi\,\frac{o-e}{\lambda}\right) \qquad \text{and} \qquad \sin^2\!\left(\pi\,\frac{o-e}{\lambda}\right),$$

which respectively give the intensities of the ordinary and extraordinary images in a homogeneous light whose wavelength is λ, provided that the axis of the crystalline plate makes an angle of 45° with the initial plane of polarization, and that the principal section of the rhomb is parallel to that plane, show that the *totality* [*ensemble*] of the two wavetrains that come out of the crystalline plate must be polarized

[13] As the second expression gives the hue of the reflection ring, the *first* gives the hue (but not the saturation) of the *transmission* ring for the same thickness of air.

[14] In this paper, Fresnel (or perhaps the original publisher) does not distinguish between honorifics, but uses "M." (*Monsieur*) for all. The notes by the editors of the *Oeuvres complètes* are more specific, with the exception of the Rev. Dr. Whewell, who becomes plain "M." For better or worse, I have followed the editors' example throughout. —*Translator*.

[15] That is, from p. 7 above, $A^2 = a^2 + a'^2 + 2aa'\cos\frac{2\pi c}{\lambda}$, with $a = \frac{1}{2}$, $a' = -\frac{1}{2}$, and $c = o{-}e$.

per the initial plane of polarization whenever $o{-}e$ is equal to zero or a whole number of wavelengths [*ondulations*], because, as

$$\sin^2\left(\pi\,\tfrac{o-e}{\lambda}\right)$$

then becomes equal to zero, the extraordinary image vanishes. On the contrary, whenever $o{-}e$ is equal to an odd number of half-wavelengths, it is

$$\cos^2\left(\pi\,\tfrac{o-e}{\lambda}\right)$$

that becomes zero, and consequently the ordinary image that vanishes, whence we must conclude that *all* of the light is polarized in the plane perpendicular to the principal section, which is precisely what Mr. Biot calls azimuth $2i$. But, for all intermediate values of λ, the *totality* of the two wavetrains can only present a partial polarization; and indeed it must appear completely depolarized when $o{-}e$ is equal to an odd number of quarter-wavelengths, because, as both

$$\cos^2\left(\pi\,\tfrac{o-e}{\lambda}\right) \qquad \text{and} \qquad \sin^2\left(\pi\,\tfrac{o-e}{\lambda}\right)$$

then become equal to $1/2$, the two images are of the same intensity; and this happens regardless of the azimuth in which we turn the principal section of the rhomb, as we can convince ourselves by the general formulae presented above, by setting

$$i = 45^\circ \qquad \text{and} \qquad \sin^2\left(\pi\,\tfrac{o-e}{\lambda}\right) = \tfrac{1}{2}\ ;$$

for then they become

$$\begin{aligned} &\text{Ordinary image:} && \cos^2 s - \tfrac{1}{2}\cos 2s \;=\; \tfrac{1}{2}\ ; \\ &\text{Extraordinary image:} && \sin^2 s + \tfrac{1}{2}\cos 2s \;=\; \tfrac{1}{2}\ . \end{aligned}$$

It is easy to see similarly in the general formulae, regardless of the value of i, that when $o{-}e$ is equal to zero or an even number of half-wavelengths, the extraordinary image vanishes for $s = 0$, and that when $o{-}e$ is equal to an odd number of half-wavelengths, the same expression becomes zero if we set $s = 2i$, and that consequently all of the light is polarized per the initial plane in the first case, and in the second case per the azimuth $2i$, whereas for all intermediate values of $o{-}e$, there cannot be complete disappearance of either image, whichever way we turn the principal section of the rhomb. All these consequence of the theory are confirmed by experiment.

In a second note, I shall indicate the manner of calculation of the hues produced by an arbitrary number of superimposed plates, and give the general formulae for the case of two plates whose principal sections make between them an arbitrary angle. I shall also add some mechanical considerations on polarization and double refraction, and on the cause of the remarkable properties that Mr. Arago and I have discovered in polarized light [1].

Second note on the coloration of crystalline plates [16]

6. I have just given[17] the general formulae for the hues of a single crystalline plate; I shall now calculate the effects that result from the assembly of several plates. I shall always suppose these crystals to have parallel faces, perpendicular to the incident ray, in order not to be obliged to enter into the calculation of the deviations of the planes of polarization produced by the inclination of the surfaces, for which we have no rigorous formula,[18] and which should be taken into account at least for large obliquities.

[16] First published in *Annales de Chimie et de Physique*, Ser. 2, vol. 17, pp. 167–96 (June 1821). In the original edition, up to p. 176, the printed page numbers were incorrectly increased by 100; but the starting page number in the table of contents at the back of the volume is correct.

[17] Or, from the original edition, "In the first Note, I have given..."

[18] This problem was solved by MacCullagh and Neumann in the 1830s; see e.g. [5, pp. 148–50] or [6, pp. 230–31].

It matters little, however, whether the faces of the plates are parallel or oblique to their [optic] axes, or whether they have one or two [axes], provided that the position of the planes of polarization of the ordinary and extraordinary rays is known in each plate, together with their path difference, which we can always calculate if we know their respective velocities; the arguments that we are about to make will apply equally in all cases.

If we superimpose any number of crystalline plates by placing their principal sections[19] in the same direction, the ordinary and extraordinary rays that come out of the first plate proceed to undergo in the second the same type of refraction that they have suffered in the first; thus only two wavetrains finally result, as in the case of a single plate. We can therefore apply to such an assembly of crystalline plates the formulae that we have given for a single one, substituting therein the total path difference produced by the passage of the light through all these plates. This difference will be equal to the sum of those that result from each plate, if it is the rays of the same name (for example, the ordinary rays) that cross them all with the greater speed. Otherwise we must add the path differences produced by the plates in which the speed of propagation of the ordinary rays is greater than that of the extraordinary rays, then make the sum of the path differences given by the plates in which the ordinary rays travel less fast than the extraordinary rays, and subtract the two sums one from the other; we shall then have the final difference of the paths traveled at the same instant by the two wavetrains that have come out of this assembly of crystalline plates.

If the principal sections of one subset of the plates are perpendicular to those of the others, which I suppose parallel among themselves, it is clear that again only two wavetrains result, as in the preceding case; only the rays that have been refracted ordinarily by the former would be refracted extraordinarily by the others, and the extraordinary rays of the former would become ordinary in the latter. Hence we see that, to obtain the final path difference of the two wavetrains, we must make the sum of the differences produced by all the *attractive* crystals (to employ the usual expression),[20] whose principal sections are parallel to the first direction, and subtract the sum of the differences produced by the repulsive crystals whose principal sections have the same direction, make a similar calculation for the plates whose principal sections are perpendicular to the first direction, and subtract the two results one from the other; or, equivalently, we add the path differences due to crystals of the same kind that have their principal sections parallel to each other [attractive crystals with sections in one direction and repulsive crystals with sections in the perpendicular direction], with the path differences due to the crystals of the opposite kind whose principal sections are perpendicular to them [attractive crystals with sections in the latter direction and repulsive crystals with sections in the former], and subtract, one from the other, the two sums thus obtained.

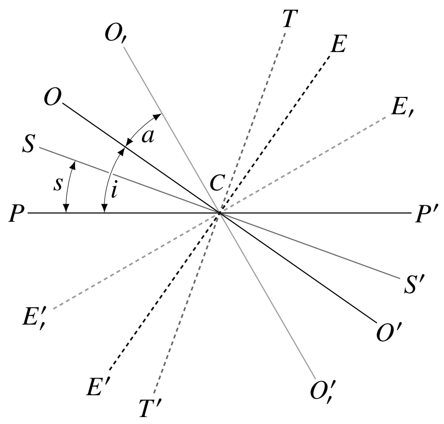
7. Having considered the particular cases where we can apply to the assembly of any number of crystalline plates the formulae that we have given for a single one, let us now deal with the general case of two superimposed plates whose principal sections make between them any angle, and are arranged

[19, (1)] By *principal section*, I [Fresnel] mean here the plane of polarization of the ordinary rays, whether the crystal has two axes or only one. [In a biaxial crystal, as Fresnel would discover later that year [3, 260–67], neither ray is ordinary in the sense of having a speed independent of direction. But one of the rays has a plane of polarization that divides the acute angle between the axes; and this plane, in the degenerate case of a uniaxial crystal, becomes the plane of the ray and the axis, which is the plane of polarization of the ordinary ray. Hence, in a biaxial crystal, the ray whose plane of polarization divides the acute angle between the axes may be taken as the generalization of the "ordinary" ray; *cf.* Fresnel's "Second Memoir" on double refraction [8], at pp. 325–8. —*Translator*.]

[20] An "attractive" crystal, now called a *positive* crystal, is one for which the extraordinary waves are slower (i.e., the extraordinary refractive index is higher), whereas a "repulsive" crystal, now called a *negative* crystal, is one for which the extraordinary waves are faster (i.e., the extraordinary refractive index is lower—as in calcite). The older terms refer to the attempt by Malus and Laplace to construct a corpuscular theory of extraordinary refraction. On the shortcomings of this attempt, see (e.g.) the first few pages of Fresnel's "Second Memoir" [8]. —*Translator*.

in any manner with respect to the initial plane of polarization, and to the principal section of the calcite rhomb which serves to analyze the emergent light.

If PP' is the initial plane of polarization, OO' the principal section of the first plate, $O_{,}O'_{,}$ that of the second, SS' the principal section of the rhomb, and EE', $E_{,}E'_{,}$, TT' the planes respectively perpendicular to the preceding three: I denote by i the angle OCP that the first plane makes with the initial plane of polarization, by a the angle $OCO_{,}$, that the principal section of the second plate makes with that of the first, and by s the angle PCS of the principal section of the rhomb with the initial plane.



In the first plate the incident light will divide itself into two polarized wavetrains, the one per CO and the other per CE'; in the second plate each of these will divide itself into two polarized wavetrains, the one per $O_{,}O'_{,}$ and the other per $E_{,}E'_{,}$; and finally, in crossing the rhomb, each of these four beams will divide itself into two others, the one polarized in the principal section SS', and the other in the perpendicular plane TT'. The four beams finally polarized per SS' will form the ordinary image, and the four others polarized per TT' will constitute the extraordinary image. We shall only occupy ourselves with the ordinary image, the other image being always complementary to this one. We find, for the amplitudes of the velocities of oscillation of the four constituent beams of the ordinary image, the following expressions:

$$\begin{array}{cc} F_{o+o'} \cos i \cos a \cos(a+i-s) & F_{o+e'} \cos i \sin a \sin(a+i-s) \\ P.O.O_{,}.S & P.O.E'_{,}.S \\ -F_{e+o'} \sin i \sin a \cos(a+i-s) & F_{e+e'} \sin i \cos a \sin(a+i-s) \\ P.E'.O'_{,}.S' & P.E'.E'_{,}.S\,, \end{array}$$

in which F denotes always the amplitude of the velocities of oscillation of the incident light, or, more precisely, of this light diminished by all that it loses in crossing the three crystals. We have marked [with subscripts on F] only the paths traveled at the same instant by the different wavetrains after they have crossed the two crystalline plates, without dealing with their path in the rhomb, since they have all suffered the ordinary refraction there. We have given the third expression a minus sign, due to the course of the plane of polarization of this wavetrain compared with those of the planes of polarization of the other three: by following the successive changes in these planes of polarization, indicated by the capital letters placed under each expression, we know indeed that for the third beam, the extremity P of the initial plane has come to sit finally at S', while for the other three it has gone to S; whence follows the opposition of sense which implies the opposition of sign, as in the composition of forces.

To find the resultant of these four wavetrains, it is necessary to follow the rule that I have given in my Memoir on diffraction already cited, page 256.[21] It consists in decomposing each wavetrain into two others, whose positions are the same for all, and different from each other by a quarter-cycle; we then make the sum of the components in the first position, then that of the components in the second, and, by adding the squares of the two sums, we have the intensity of the total light that results from the interference of the different wavetrains. I choose for the position of the former components that which corresponds to the traveled path $o+e$, for example; the position of the others will differ from this by a

[21,(a)] Page 289 of the present volume [9, vol. 1]. [Or p. 104 of the translation in [4]. Fresnel's method of superposing sinewaves of the same frequency, although first published in the context of diffraction [4, pp. 103–6], was first *submitted* in the context of chromatic polarization [9, vol. 1, pp. 489–93]. In the earlier version (submitted January 1818), the wavelength was called d; in the later, better-known version (submitted July 1818), it was called λ.]

quarter-cycle. We will have, for the sum of the first: [22]

$$\cos i \cos a \cos(a+i-s) \cos\frac{2\pi(e-o')}{\lambda} + \cos i \sin a \sin(a+i-s) \cos\frac{2\pi(e-e')}{\lambda}$$
$$- \sin i \sin a \cos(a+i-s) \cos\frac{2\pi(o-o')}{\lambda} + \sin i \cos a \sin(a+i-s) \cos\frac{2\pi(o-e')}{\lambda} ;$$

and for the sum of the second:

$$\cos i \cos a \cos(a+i-s) \sin\frac{2\pi(e-o')}{\lambda} + \cos i \sin a \sin(a+i-s) \sin\frac{2\pi(e-e')}{\lambda}$$
$$- \sin i \sin a \cos(a+i-s) \sin\frac{2\pi(o-o')}{\lambda} + \sin i \cos a \sin(a+i-s) \sin\frac{2\pi(o-e')}{\lambda} .$$

Here I have removed the common factor F, which would uselessly complicate the calculation, and which we can take as unity.

Squaring the two sums and adding the two squares, we find, after several simplifications,[23]

$$\cos^2 s + \sin 2a \sin 2i \cos 2(a+i-s) \sin^2\Big(\pi\,\frac{o-e}{\lambda}\Big) - \sin 2a \cos 2i \sin 2(a+i-s) \sin^2\Big(\pi\,\frac{o'-e'}{\lambda}\Big)$$
$$- \cos^2 a \sin 2i \sin 2(a+i-s) \sin^2\Big(\pi\,\frac{o-e+o'-e'}{\lambda}\Big) + \sin^2 a \sin 2i \sin 2(a+i-s) \sin^2\Big(\pi\,\frac{o-e-(o'-e')}{\lambda}\Big) .$$

This is the general expression for the intensity of one simple [i.e., monochromatic] light in the ordinary image. At the same time, we can consider it as representing the color [*teinte*] produced by white light, since this formula gives the relative intensity of each species of colored rays as a function of its wavelength.

8. We see that this expression contains four variable terms with the length λ of the wave of light, multiplied by coefficients that depend only on the angles a, i, and s. The first function of λ is

$$\sin^2\Big(\pi\,\frac{o-e}{\lambda}\Big),$$

and the second

$$\sin^2\Big(\pi\,\frac{o'-e'}{\lambda}\Big),$$

and the third

$$\sin^2\Big(\pi\,\frac{o-e+o'-e'}{\lambda}\Big),$$

and the fourth

$$\sin^2\Big(\pi\,\frac{o-e-(o'-e')}{\lambda}\Big).$$

These are precisely those that would form the variable term for a single crystalline plate, whose thickness were supposed equal to, respectively, that of the first plate, that of the second, that of the sum of their thicknesses, and that of the difference, if the two plates are of the same nature. And in fact the system of two crossed plates can present the same effects as a single plate having successively the thicknesses that we have just indicated: first, when the principal section of the rhomb is parallel or perpendicular to that of the second plate, since then the image given by the rhomb contains only the rays that have suffered the same refraction in that plate, and between which it [the plate] has not established any new difference of paths traveled; second, when the principal section of the first plate is parallel or perpendicular to the initial plane of polarization, since then the incident rays suffer only one mode of refraction in this plate; third, when the principal sections of the two plates are parallel to each other; and fourth, when they are perpendicular. These last two cases fall among those that we had already mentioned before calculating the formula. The experiments of Mr. Biot had demonstrated in advance these consequences of the theory, which one can deduce from the formula by successively putting therein:

$$a+i-s=0\,, \quad a+i-s=90^\circ, \quad i=0\,, \quad i=90^\circ, \quad a=0\,, \quad a=90^\circ .$$

[22] The subscripts on F are subtracted from $o+e$ to obtain the corresponding final path differences. —*Translator*.

[23] Misprint: In the original edition and the *Oeuvres complètes*, the last numerator in this long expression ends in $-(o-e')$, but the ensuing discussion indicates that it should end in $-(o'-e')$, as given here. Moreover, the former option would admit a trivial simplification, which would surely not have been left undone. —*Translator*.

By a process similar to that which we have just indicated for two plates, one could also calculate the general formulae for the intensities of diverse species of colored rays in the ordinary and extraordinary images, for three, four, five, etc. superimposed plates, whose principal sections made any angles between them. The application of the theory to these more complicated cases would be equally easy; only the calculations would take longer.

9. We see what advantage this theory has over that of *mobile polarization*, which becomes so embarrassing when we want to know how *the oscillations of the axes of the molecules of light* reconnect in the passage from one plate to another whose principal section makes an arbitrary angle with that of the first. Moreover the theory of mobile polarization has furnished Mr. Biot with the means of determining all the coefficients of his formulae, for two superimposed plates, only in some very particular cases; and there is even one where the facts are not exactly represented by his formulae, as I have been warned by my own: this is the case in which the principal section of the rhomb is parallel or perpendicular to the initial plane, the two plates being of the same nature and the same thickness, with their axes crossed at an angle of 45°. Mr. Biot has concluded from his formulae that when we rotate this assembly of two crossed plates in its plane, the hues of the images must remain constant.[24] The general expression that we have just found for the intensity of each species of simple light in the ordinary image leads to a different consequence. Indeed, in the case in question, $o'-e' = o-e$ since the two plates are of the same nature and the same thickness, and $a = 45°$, and $s = 0$ or $90°$. Supposing that $s = 0$ and substituting these results into the formula, we will have, after all simplifications,[25]

$$\cos^2\!\left(\pi\,\frac{o-e}{\lambda}\right) - \frac{1}{4}\sin 4i\,\sin^2\!\left(2\pi\,\frac{o-e}{\lambda}\right).$$

This expression, not being independent of i, which is the angle that the principal section of the first plate makes with the initial plane of polarization, must change in value when we rotate the assembly of two plates in its plane. When $\sin 4i = 0$, it becomes

$$\cos^2\!\left(\pi\,\frac{o-e}{\lambda}\right),$$

which is precisely the formula that we have found for a single plate of the same thickness as one of the two in question, when its principal section makes an angle of 45° with the principal plane; and indeed in all positions of the assembly of two crossed plates where $\sin 4i = 0$, i.e. when i is equal to 45°, 90°, 135°, 180°, etc., the hue of the image is perfectly similar to that given by one of the two plates taken separately, and turned so that its principal section is in the azimuth of 45°, just as Mr. Biot had stated, and as one can easily verify by experiment. But, for all intermediate values of i, the formula differs more or less from

$$\cos^2\!\left(\pi\,\frac{o-e}{\lambda}\right);$$

and this difference attains its *maximum* when $\sin 4i$ becomes equal [in magnitude] to 1, i.e. when i is equal to an odd number of quarters of a quadrant.

It should be noted that, even in the last case, the coefficient of $\sin 4i$ cannot exceed $1/4$, whatever be the value of

$$\sin^2\!\left(2\pi\,\frac{o-e}{\lambda}\right),$$

since this is multiplied by $1/4$. Moreover, it vanishes for the two species of rays whose wavelength makes

$$\frac{o-e}{\lambda}$$

equal to a whole number, or a whole number plus $1/2$, since

$$\sin^2\!\left(2\pi\,\frac{o-e}{\lambda}\right)$$

[24] In a note published two months later, Fresnel acknowledged Biot's defense that Biot's formulae as applied in this case were not a necessary consequence of the mobile-polarization theory (see [9], vol. 1, pp. 606–7, quoted in translation by Buchwald [3], p. 444, note 11). That being conceded, Fresnel's "advantage" is not that his theory predicts the facts while Biot's predicts something counterfactual, but merely that Fresnel's theory predicts the facts while Biot's does not. —*Translator*.

[25] Notice that the argument of $\sin^2(\,)$ is twice that of $\cos^2(\,)$; this pattern persists for the remainder of the section.

is then equal to zero: now the first species of rays is that which dominates in the ordinary image, since

$$\cos^2\left(\pi\,\frac{o-e}{\lambda}\right)$$

becomes equal to 1 when

$$\frac{o-e}{\lambda}$$

is a whole number, and the second species is that which is entirely excluded, since

$$\cos^2\left(\pi\,\frac{o-e}{\lambda}\right)$$

becomes null when

$$\frac{o-e}{\lambda}$$

is equal to a whole number plus 1/2. Thus the variations of i should only bring fairly slight changes in the tint of the ordinary image when white light is employed, first because the term that contains i is multiplied by 1/4, and second because it is null for the rays that dominate in the image and for those that are entirely excluded from it, so that consequently these two kinds of rays, which especially determine the nature of the tint [i.e., the hue], do not suffer any change of intensity as we rotate the assembly of two crossed plates in its plane. It is therefore only the other kinds of rays whose intensities vary; but as these variations are multiplied by a quarter, we see that, in general, they can hardly change the color of the image in a very perceptible way, and that their usual effect must be to make it only more or less dark. This is undoubtedly why these slight variations could have escaped the attention of an observer so skilled and experienced as Mr. Biot, or could appear to him as mere anomalies independent of the principal phenomenon.

When i is equal to a quarter of a quadrant or, in general, to a whole number plus 1/4 of a quadrant, $\sin 4i = 1$, and all the rays that mix with the dominant rays are reduced to *minimum* intensity, because the variable term reaches its negative extreme; thus the hue of the ordinary image must then become purer and darker, since it contains less of the heterogeneous light. When, on the contrary, i is equal to 3/4 of a quadrant, or to a whole number plus 3/4 of a quadrant, $\sin 4i = -1$, and all the heterogeneous rays that we have just mentioned attain their *maximum* intensity; then the ordinary image must be both brighter and of a less pure color than in the first case. This is what will be easily recognized by performing the experiment with attention.

The variations in intensity of these rays become much more perceptible when, instead of white light, one employs a roughly homogeneous light, choosing light for which $o-e$ is an odd number of quarter-cycles, or the thickness of plate that satisfies this condition. It is easy to recognize when it is fulfilled; for, as we have seen, the homogeneous light must in this case be completely depolarized[26] by passing through only one of the two plates, whose principal section has been directed at 45° from the initial plane. Then, if we rotate the assembly of two crossed plates in its plane, we shall see the intensity of the ordinary image change considerably, as the formula indicates; for when $e-o$ is an odd number of quarter-cycles,

$$\sin^2\left(2\pi\,\frac{o-e}{\lambda}\right)$$

attains its *maximum* and is equal to 1, whereas

$$\cos^2\left(\pi\,\frac{o-e}{\lambda}\right)$$

is equal to 1/2, and the formula becomes $\frac{1}{2}-\frac{1}{4}\sin 4i$, which gives 1/4 when i is equal to a quarter of a quadrant or to a whole number plus 1/4 of a quadrant, or 3/4 when i is equal to three quarters of a quadrant or to a whole number plus 3/4 of a quadrant; whence, in the second case, the intensity of the ordinary image is triple that which it is in the first. It is seen that this difference must in general be diminished by the departure from homogeneity of the light employed, and the more diminished as the plates are thicker.

[26] That is, *circularly polarized*, but Fresnel had not yet coined that term; *cf.* his footnote 35 on p. 20, below.

Mechanical considerations on the polarization of light

10. While I was occupied with the drafting of my first Memoir on the coloration of crystalline plates (in September 1816), I noted that the polarized light waves act on each other like forces perpendicular to the rays and directed in their planes of polarization, since they neither enfeeble nor strengthen each other when their planes are at right angles, and that two wavetrains show an opposition of sign independent of the difference of traveled paths when their planes of polarization, at first united, separate and then return to a common plane by placing themselves on the extension of each other. Mr. Ampère, to whom I had communicated these experimental results, had the same thought with respect to the opposition of sign resulting from the course of the planes of polarization. We both felt that these phenomena would be explained with the greatest simplicity if the oscillatory movements of polarized waves occurred only in the very plane of the waves. But then what would become of the longitudinal oscillations along the rays? How would they be destroyed by the act of polarization, and how would they not reappear when polarized light was reflected or refracted obliquely by a glass plate?[27]

These difficulties seemed to me so embarrassing that I abandoned our first idea, and continued to suppose longitudinal oscillations in polarized rays, admitting at the same time transverse movements therein, without which it always seemed to me impossible to conceive of polarization and of mutual non-influence of rays polarized at right angles. It has only been for a few months that in meditating more attentively on this subject, I have realized that it was very probable that the oscillatory movements of light waves were executed solely along the plane of these waves, for direct light as well as for polarized light. I cannot enter here into the detail of the calculations on the various combinations of longitudinal and transverse movements that have led me to this conclusion. I shall only pause to make it clear that the hypothesis which I present contains nothing physically impossible, and that it can already serve to explain the principal properties of polarized light by means of very simple mechanical considerations.

11. The geometers who have applied themselves to the vibrations of elastic fluids have, I believe, considered as an accelerating force only the difference in condensation or dilatation between consecutive layers. At least in their equations I do not see anything that indicates, for example, that an infinite layer, by sliding between two others, must communicate movement to them, and it is evident that in this respect their equations do not tell all that really occurs. This is consistent with the fact that they mathematically represent elastic fluids as a union of small differential elements susceptible to condensation or dilation, and *juxtaposed*, whereas in nature elastic fluids are presumably composed of material points separated by intervals that are more or less considerable relative to the dimensions of these molecules. Now let us imagine, in a fluid, three infinite parallel and consecutive rows of material points thus arranged: if we suppose a certain law of repulsion between these molecules, they will assume, in the state of equilibrium and of absolute rest, a regular arrangement whereby they will be equally spaced in the three rows; and those of the intermediate row will, I suppose, correspond to the midpoints of the intervals between the molecules of the other two. I indicate this particular arrangement only to fix ideas, for clearly it could not take place in all directions. But whatever be the direction of the rows that we consider in the elastic medium, their material points will always tend to place themselves in the relative positions that admit stable equilibrium. Suppose that this condition is satisfied; then, if we slightly displace the intermediate row by sliding it along itself, but only by a distance that is very small compared with the spacing of two consecutive molecules, and then set it free, each of its material points will come back towards its first position (neglecting what happens at the ends of the row, since we assume it infinite)[28], and will oscillate from side to side like a pendulum that has been pushed aside from the vertical. But if we

[27] Fresnel had discovered, in 1817, that initially-polarized light remains polarized after reflection at any angle by a transparent body, although its azimuth usually changes; see Buchwald [3, pp. 218–9], quoting (& translating) Fresnel [9, vol. 1, pp. 441–2]. Mechanically, it is hard to imagine how an oblique reflection or refraction would not partly convert a transverse wave into a longitudinal one, thereby partly depolarizing the light, if longitudinal waves were possible at all.

[28,(1)] As it never happens that the waves of light have this indefinite extent, in the direction perpendicular to the rays, which we have considered here to simplify the argument, one might wonder how these transverse motions do not propagate sensibly beyond the edge of the waves. Presumably they cannot abruptly annihilate themselves at their extremity; but it is easy to see that at a distance somewhat large compared with the wavelength of light, the contrary oscillations sent there by the different parts of the wavetrain must cancel each other.

had displaced these molecules far enough from their points of departure to place them exactly opposite the molecules of the two other rows (supposed immobile), a new equilibrium would have ensued. Let us further slide the intermediate file until its material points again correspond to the midpoints of the gaps of the other two, and it will enter a third state of equilibrium similar to the first. We see that by continuing to slide in the same direction, it would be in equilibrium at each half-interval of molecules, and therefore at intermediate positions would only suffer the action of retarding forces whose effect would be compensated, after each very short period, by the accelerating forces that succeeded them.

It is quite possible that the fluidity of a body comes from the fact that, by virtue of a great randomness [*dissémination*] of its molecules, these different positions of equilibrium are much closer together than in solids, so that the retarding force which tends to restore the system to its first state, as it can grow only in too small an interval, never acquires a great intensity; but it is conceivable that, when it is only a question of displacements that are very small compared with the intervals separating two consecutive molecules, the retarding force could have as much intensity, or even more, in a liquid as in a solid. Now it is only very small displacements of this kind, in the layers of aether and transparent bodies, that constitute the vibration of light, according to the hypothesis that I have newly adopted.[29]

I have supposed, in order to simplify the ideas and explain more clearly the nature of the forces of equilibrium that I wanted to discuss, that the two slices neighboring the intermediate slice remained at rest while the latter slid along itself. It is clear that things do not happen in this manner, and that one slice cannot be displaced without setting the neighboring slices in motion. The greater or lesser rapidity with which the motion is propagated depends on the energy of the accelerating force that tends to restore the contiguous slices to the same relative positions, and on the masses of these slices, as the velocity of propagation of sound waves in air (as we usually think of them) depends on the relation between its density and the resistance that it offers to compression. It is evident that we can apply to these new oscillations, perpendicular to the rays, the same arguments and the same calculations as to those where the oscillatory movement is executed along the direction of propagation. The principle of interference and all the consequences that Dr. Young has deduced therefrom to explain various phenomena of light, as well as the formulae by means of which I have represented the laws of diffraction, agree as well with this new hypothesis on light as with the one that I had initially adopted.

12. Having brought to attention the possibility of such vibrations in a fluid, it remains for me to explain how it can happen that its molecules suffer sensible oscillations only along the very surface of the waves, perpendicularly to the rays. For this it suffices to suppose a law of repulsion between the molecules such that the force opposing the approach of two slices of fluid is much greater that that which opposes the sliding of one of them relative to another, and then to admit that the oscillations of small solid bodies, which set the fluid in vibration, have absolute speeds infinitely smaller than the speeds with which the condensations and dilatations are transmitted in the fluid. And indeed, if we suppose that the equality of stress is re-established with extreme rapidity, due to the great resistance offered to compression, we perceive that during the much slower movement of the small oscillating body, the equilibrium of pressure is re-established around this body at each instant, between the contiguous part of the fluid which it tends to compress by approaching it, and the part situated on the other side, which it tends to dilate by moving away from it; whence we see that the main movements of the molecules will consist in a sort of oscillatory circulation about the oscillating small solid. This movement will be communicated step-by-step to all the concentric layers, weakening and regularizing as it moves away from the center of agitation, and at a short distance there will no longer be any sensible displacement of the aether molecules except in the very direction of the surface of the waves. Such is, in my view, the

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[29, (1)] If, as seems probable to me, the molecules of diaphanous bodies participate in the vibrations of the aether that surrounds them on all sides, the forces developed by the relative displacements of the slices of the medium parallel to the waves must be far superior in intensity to those that propagate sound waves in the same media, relative to the masses of the slices that the former and the latter [forces] set in motion, since the velocity of propagation of light is incomparably greater than that of sound. But that may be because the displacements that constitute the oscillations of sound take place between particles of a much more composite order, between slices much thicker, than those that constitute the vibrations of light, and because the former displacements do not give rise to accelerating forces so energetic, relative to the masses of the slices that they set in motion.

idea that must be formed of the nature of light waves, to account for the different phenomena that they present, particularly in polarization and double refraction.

13. I must state here that an item in a letter from Dr. Young, dated 29 April 1818, which had been communicated to me by Mr. Arago, has contributed to making me doubt the existence of longitudinal oscillations. Dr. Young concluded from the optical properties of crystals of two axes, discovered by Dr. Brewster, that the undulations of the aether might well resemble those of a stretched rope of indefinite length, and propagate in the same manner. There is undoubtedly a great analogy between this definition of light waves and the one I have just given; but I do not believe that Dr. Young has shown how one could reconcile such a mutual dependence of the molecules of aether with its fluidity, and conceive of the production of these undulations to the exclusion of the oscillations directed along the line of propagation. This was the same difficulty that had embarrassed me until now, and had prevented me from holding to my first idea. I must admit nevertheless that, if he has not explained it, Dr. Young is the first who has positively stated the possibility of such a property in an elastic fluid.[30] I do not know if this learned physicist has published his views on this subject, or even if they are well settled in his mind; but I thought that the publicity I am giving to them here could not be disagreeable to him.[31]

If the polarization of a light ray consists in the fact that all its vibrations are executed in the same direction, it follows from my hypothesis on the generation of light waves that a ray emanating from a single center of agitation is always found to be polarized according to a certain plane at a given moment. But, a moment later, the direction of the movement changes, and with it the plane of polarization; and these changes follow each other as rapidly as the perturbations of the vibrations of the illuminating particle, so that, even if the light emanating from it could be separated from that of the other luminous points, we would surely recognize no appearance of polarization. If we now consider the effect produced

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[30, (a)] CHROMATICS, from the *Supplement to the Encyclopedia Britannica*, sect. XVI, art. 5 (*Miscellaneous Works*, vol. I [12], from p. 332). CORRESPONDENCE RELATING TO OPTICAL SUBJECTS—*From* Dr. YOUNG *to* M. ARAGO (12 January 1817), *Miscellaneous Works*, vol. I [12], from p. 380 [esp. p. 383]. Note, appended to the Memoir of Dr. Brewster "On the laws of polarisation and double refraction in regularly crystallized bodies" [2], *Philosophical Transactions* for 1818, from p. 272.

[31, (a)] Dr. Young's ideas probably had not in fact taken a very precise form in his mind. Admittedly we do not know his letter to Arago of 29 April 1818; but in that of 12 January 1817 [in English] and in his article CHROMATICS, written at the same time for the *Supplement to the Encyclopedia Britannica*, he regards it as mechanically impossible to find in the constitution of elastic media some force analogous to gravity, which, on the surface of liquids, determines the propagation of waves perpendicularly to the direction of the oscillations.

Transverse vibrations therefore do not have, according to him, any probability as a physical explanation of the phenomena, but only a usefulness for the mathematical representation thereof; they are a mechanical *postulatum* of the undulatory theory.

Later, in 1827 [sic], Dr. Young analyzes the ideas of Fresnel in an article from the same work, titled "Theoretical investigations intended to illustrate the phenomena of polarisation: Being an addition made by Dr. Young to M. Arago's 'Treatise on the Polarisation of Light' " (*Miscellaneous Works*, vol. I [12], from p. 412). He reiterates the same observations and comes to the conclusion that the aether should be not only very elastic, but solid. [This "addition" was written in January 1823 and first published in the *Supplement to the Fourth, Fifth, and Sixth Editions of the Encyclopædia Britannica*, vol. 6, part 2 (issued April 1824), pp. 860–63. The title has been corrected to match the reprint in [12]. —*Translator*.]

These objections of Young let us judge the resistance that other mathematicians must have offered to concepts which came to overturn all the received ideas on the constitution of elastic fluids. Arago had recoiled at such bold novelties.

"And M. Arago was afterwards wont to relate," said Mr. Whewell, "that when he and Fresnel had obtained their joint experimental results, of the non-interference of oppositely [i.e., perpendicularly] polarized pencils [1], and when Fresnel pointed out that transverse vibrations were the only possible translation of this fact into the undulatory theory, he himself protested that he had not courage to publish such a conception; and accordingly, the second part of the Memoir was published in Fresnel's name alone." — *History of the Inductive Sciences*, by Wil. Whewell, vol. II [11], p. 454 (2nd Ed., London, 1847). See also *Oeuvres complètes de F. Arago*, vol. VII, p. 428. — DE SÉNARMONT.

To be completely fair to Arago, it is well to supplement the story just read with another quote which we also borrow from Mr. Whewell's work and which, like the first, is probably based on personal communications of the learned historian of experimental sciences, with Arago. We read in the same work, in the chapter which describes the reception given by the contemporaries of Young and Fresnel to their theories, "M. Arago would perhaps have at once adopted the conception of transverse vibrations, when it was suggested by his fellow-labourer, Fresnel, if it had not been that he was a member of the Institute, and had to bear the brunt of the war, in the frequent discussions on the undulatory theory; to which theory Laplace, and other leading members, were so vehemently opposed, that they would not even listen with toleration to the arguments in its favour" (*op. cit.*, vol. II [11], p. 473). It has seemed useful to include Young's letter of 12 January 1817 in the present edition (see no. LVI [9, vol. 2, at pp. 742–4; in English]). — E. VERDET.

by the union of all the waves that emanate from the different points of an illuminating body, we shall perceive that at every instant, for a given point of the aether, the general resultant of all the movements that cross it will have a definite direction; but this direction will vary from one instant to another. Thus direct light can be considered as the meeting, or more exactly as the rapid succession, of wavetrains polarized in all directions. According to this way of looking at things, the act of polarization consists not in creating these transverse movements, but in decomposing them into two fixed perpendicular directions and in separating the two components from each other; for then, in each of them, the oscillatory movements will always operate according to the same plane.

14. Applying these ideas to double refraction, let us consider a uniaxial crystal as an elastic medium in which the accelerating force due to a row of molecules being displaced perpendicular to the axis, relative to the adjoining rows, is the same all around the axis, while displacements parallel to the axis produce accelerating forces of a different intensity, stronger if the crystal is repulsive (to employ the usual expression), and weaker if it is attractive.[32] As the distinguishing character of the rays that undergo ordinary refraction is to propagate with the same speed in all directions, one must admit that their oscillatory movements are executed perpendicular to the plane defined by these rays and the axis of the crystal; for then the displacements that they cause, being effected always in directions perpendicular to the axis, will always develop, by hypothesis, the same accelerating forces. But, according to the meaning attached to the term *plane of polarization*, the plane of which we have just spoken is precisely the plane of polarization of ordinary rays; thus, in a polarized beam, the oscillatory movement is executed perpendicularly to what is called the *plane of polarization*.

The oscillations of the ordinary rays being perpendicular to the plane led by the axis, the oscillations of the extraordinary rays will be parallel to this plane and, of course, always perpendicular to the rays.[33] We see then that as they [the extraordinary rays] change their inclination relative to the axis, the direction of the oscillatory movement will also change: it will be parallel to the axis when the rays are perpendicular to it, and perpendicular to the axis when the rays are parallel to it; thus, in the latter case, the speed of propagation of the extraordinary rays will be the same as that of the ordinary rays. But for all the other directions of the former, as the small disturbances of the lines of molecules are no longer executed perpendicular to the axis, the resulting accelerating forces, and therefore the speed of propagation, can no longer be the same. This difference gradually increases until the oscillatory movement is parallel to the axis: then the difference reaches its *maximum*.

Let us consider this last case, to simplify the ideas, and suppose that we present perpendicularly to the incident beam a crystalline plate [whose faces are] parallel to the axis, so that the rays passing through it are perpendicular to the latter. Suppose further that the incident beam is polarized per a certain plane making an angle i with the principal section of the crystal; its oscillations will be perpendicular to this plane. Then, by the principle of the composition and decomposition of small movements, we can consider each of the oscillation velocities of the incident waves to be decomposed into two others, one perpendicular and the other parallel to the principal section; the former components will produce ordinary waves, and the latter those [waves] that suffer extraordinary refraction. However, if we take as unity the common factor that multiplies all the oscillation velocities of the various strata of the wave entering the crystal, then $\cos i$ will be the common factor of the former components or their amplitudes[34], and $\sin i$ that of the latter components; and as the intensities of light are proportional to the energies, the

[32, (1)] I suppose the crystal particles and the intervals that separate them to be infinitesimal in relation to the wavelength of light, and here I consider these particles and the aether surrounding them as together forming a homogeneous medium. This mathematical conception, which is not applicable to opaque or imperfectly transparent bodies, may nevertheless, in many cases, represent the mechanical effects of diaphanous media on light to a sufficient approximation.

[33] Fresnel would later correct himself: if the oscillations are tangential to the wavefronts, they must be always perpendicular to the *wave-normals*, which, for *extraordinary* waves, do not generally have the same direction as the corresponding rays; see (e.g.) his "Second Memoir" on double refraction [8], at pp. 318–20. In electromagnetic terms, Fresnel's oscillations are parallel to those of the electric displacement $\mathbf{D}$, which is tangential to the wavefront, whereas the electric field $\mathbf{E}$ is perpendicular to the corresponding ray (as I have tried to show in the most elementary possible manner in [10], chapter 2). —*Translator*.

[34] Literally "or their intensity of absolute velocity", where the "absolute" velocity is the velocity of a particle of the aether, as distinct from the velocity of propagation of the wave.

light intensities of the ordinary and extraordinary rays will be to each other as $\cos^2 i$ to $\sin^2 i$. Behold a very simple mechanical explanation of Malus's law. The oscillations of these two wavetrains, being perpendicular, will be executed in the crystal independently; and, because of the energy difference of the accelerating forces that result from the small displacements of the molecules of the medium parallel or perpendicular to the axis, the two wavetrains will propagate with different speeds, and the distance between their corresponding points will grow the more considerable as they have traversed a greater thickness of crystal.

If it is direct [unpolarized] light that falls on the crystal, we can apply to the various polarized wavetrains of which it is composed what we have just said for a single one: each one will be likewise divided into ordinary and extraordinary waves, whose intensities will generally differ; but since, by reason of the multitude of chances, there must be in the aggregate as much light polarized in any plane as in the perpendicular plane, the ordinary and extraordinary rays will have the same intensity.

15. I shall not pause to explain in detail, according to this new idea of luminous vibrations, the properties that Mr. Arago and I have discovered in the polarized rays [1]. But we can see why rays polarized at right angles can no longer interfere [4, p. 155, rule (1)]—that is, why they always produce by their union the same intensity of light, whatever the difference of the paths traversed—since, by virtue of the perpendicularity of their oscillations, the square of the resultant of the two absolute velocities imposed on each point of the aether is always equal to the sum of the squares of its two components, and thus the sum of the energies of the resultant wavetrain is always equal to the sum of the energies of the two components, whatever the difference of the paths traveled. It is equally easy to see the reason for the rule that I have given in the calculation of the hues produced by crystalline plates, for knowing when we must add a half-cycle to the difference of paths traveled, due to the changes of the planes of polarization [*cf.* [4], pp. 154–5, including rule (5)].

16. I would have desired to show in some detail, by the composition of the oscillatory movements at each point, how the two wavetrains of simple light that come out of a crystalline plate really give, *by their reunion*, a wavetrain polarized per the initial plane of polarization when the difference of paths traveled is *zero* or an even number of half-cycles, and polarized with azimuth $2i$ when this difference is equal to an odd number of half-cycles; why the total light only presents partial polarization in the intermediate cases, and even appears completely depolarized if, while the difference of paths traveled is equal to an odd whole number of quarter-cycles, the principal section of the plate is at 45° to the initial plane.[35] But it seems to me more necessary to employ the little space left to me to make a note about the formulae for the intensity of the light reflected obliquely on transparent bodies, to which I have just been led by the same theoretical ideas.[36]

17. The incident direct light that has fallen on a reflective surface can always be decomposed into two beams of equal intensity, the one polarized according to the plane of the reflection, and the other according to a perpendicular plane.[37] So far I have found the general formula only for the first. But it is easy to determine the intensity ratio between the two beams from the simple deviation of the plane of polarization of a ray initially polarized in the azimuth of 45°, and reflected under the same incidence as the beam of direct light; for the wavetrain polarized in the azimuth of 45° can be divided into two other wavetrains of equal intensity, the one polarized according to the plane of reflection and the other according to a perpendicular plane, which will be reflected in unequal proportions by the transparent body; and these proportions are precisely the same as for the two beams that constitute the ordinary light. Now, if we represent by 1 the intensity of the wavetrain after its reflection, the intensities of light of its two components will in general be represented by $\sin^2 s$ and $\cos^2 s$, and it is easy to see that the angle s will be precisely the azimuth of the plane of polarization of the reflected wavetrain. So if we have

[35, (1)] A remarkable consequence of the composition of oscillations in the last case is that in the resultant wavetrain, the aether molecules, instead of oscillating, revolve around their equilibrium positions with a constant speed.

[36] Thus Fresnel postpones the explanation of what he would later call linear polarization, elliptical polarization, and circular polarization, in order to get started on what we now call the **Fresnel equations**.

[37] Literally "and the other perpendicularly to that plane"—but implying that the vibrations of the "other" are *in* that plane, since the "plane of polarization" is perpendicular to the vibrations.

determined the angle s by experiment for the particular incidence in question, knowing the quantity of reflected light in the beam polarized per the plane of incidence, it will suffice to multiply this by $\tan^2 s$ in order to have [the quantity in] the other reflected beam. I shall now show how we can calculate the intensity of the reflected light for arbitrary incidence of the beam polarized per the plane of reflection.[38]

18. What makes this calculation easy is that the oscillations, being therefore perpendicular to the plane of reflection, have the same direction in the incident beam, the reflected beam, and the refracted beam. Let m be the mass of a differential element of the first medium, which, by sliding along itself, sets in motion a contiguous differential element m' of the reflective medium, which I suppose of the same elasticity.[39] At the first instant, m' was at rest, and m had velocity v; an instant later, the two elements have the same velocity, and that is when the displacement of the first with respect to the second is stationary; but, due to the displacement thus effected, the first must receive afterwards, in the opposite direction, all of that part of the initial velocity that it has lost.[40] At the instant that we have just mentioned, the common velocity of the two elements was

$$\frac{mv}{m+m'} \;;$$

so the velocity lost by m is

$$v - \frac{mv}{m+m'}\,, \qquad \text{i.e.} \qquad \frac{m'v}{m+m'}\;;$$

and consequently the velocity of m will be finally[41]

$$v\left(\frac{m-m'}{m+m'}\right).$$

So if we take as unity the amplitude of the oscillation velocity[42] of the incident wave, then

$$\frac{m-m'}{m+m'}$$

will represent the amplitude of the oscillation of the reflected wave, and

$$\left(\frac{m-m'}{m+m'}\right)^2$$

its intensity of light.[43] Solving the problem is then only a matter of determining the proportions of the masses m and m', of the differential elements of the incident and refracted waves, that shake each other in the two media.

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[38] That is, the component whose vibration is normal (*s*quare; German *senkrecht*) to the plane of reflection and tangential to the reflective *s*urface; that is, the 's' component.

[39] In §14 above, Fresnel has characterized birefringence as a variation of *elasticity* with direction. But below, in order to explain partial reflection, he needs to suppose that the two media have equal elasticities but different *densities*—whereas in real life, variations in optical properties of transparent media, both within media and between media, are almost entirely due to variations in electrical properties alone, not magnetic properties. This difficulty arises because electromagnetic waves are not, in fact, fully analogous to transverse elastic waves with **D** as the displacement; see e.g. [5], or [6, pp. 225–44].

[40, (1)] This abridged reasoning, which I borrow from Dr. Young,[(a)] and which presents only an equivalent of what happens, has been verified in its consequences for an analogous case by the rigorous analysis of Mr. Poisson.[(b)]

[(a)] Chromatics, from the *Supplement to the Encyclopedia Britannica*, sect. XVI, art. 6 (*Misc. Works*, vol. I [12], from p. 336).

[(b)] *Mémoires de l'Académie royale des sciences de l'Institut* for 1817, from p. 305. [Notes (a) and (b) are from the *Oeuvres* [9].]

[41] That is, when it has overshot the instantaneous common velocity by 100%.

[42] Literally "take for unity the intensity of the absolute velocities".

[43, (1)] It should be noted that when m' is greater than m, i.e. when the second medium is more refractive than the first, this expression for the velocity of oscillation of the reflected rays is of opposite sign to that of the incident rays—so that at the point of departure the oscillations of the former are made from left to right, for example, if those of the incident rays are made from right to left, this being equivalent to the difference of a half-cycle that experiment had presented to me. Thus the difficulty that followed when we supposed the direction of the luminous vibrations to be parallel to the rays no longer exists with the new hypothesis; and we can now consider the reflection as arising from the difference in density of the two media, composed of their own molecules and those of the aether, without being led to counterfactual conclusions. It is possible that things do not happen strictly this way; and yet that this mechanical model represents most of the optical properties of *transparent bodies* with sufficient accuracy. The phenomenon of dispersion can likewise be explained without abandoning this mechanical model, by supposing only that the mutual dependence of the molecules of the medium extends to sensible distances relative to the wavelength of light; for it then follows that the velocity of propagation must diminish somewhat with the wavelength.

For this, one must take care that as each refracted wave is produced by each incident wave, if we consider them divided into the same number of infinitely thin layers, each elementary layer of the refracted wave will be the part of the second medium disturbed by the corresponding slice of the incident wave; thus the thicknesses of the elements of the two media that communicate the disturbance, measured in the direction of the rays, are in the same ratio as the wavelengths—that is, in the ratio of $\sin i$ to $\sin i'$, where we denote by i and i' the angles of incidence and refraction. Hence, to obtain the ratios of their volumes, it remains only to determine their relative widths. Let us consider two parallel incident rays and the same rays refracted; the waves included between these incident rays occupy, after the refraction, all the space between the refracted rays; thus the width of the element of the first medium, which communicates the disturbance to the element of the second, will be to the width of the latter as the distance between the two incident rays is to the distance between the two refracted rays, or as $\cos i$ is to $\cos i'$. Multiplying the new ratio by the first, we shall have

$$\frac{\sin i \cos i}{\sin i' \cos i'}\,,$$

which will be the ratio between the volumes of the two elements. Here I ignore the dimension perpendicular to the plane of reflection, which is the same in the incident and refracted rays. Hence, to obtain the masses m and m', one must multiply the volumes by the densities of the media: now, since we consider the difference in velocity of propagation in the two media as resulting from their difference in density, their densities must be inversely proportional to the squares of their velocities; so the density of the first is to the density of the second as $\sin^2 i'$ to $\sin^2 i$. Multiplying this ratio of densities by that of the volumes, we shall have the ratio of the masses m and m', which will be

$$\frac{\sin i' \cos i}{\sin i \cos i'}\,, \qquad \text{i.e.} \qquad \frac{\tan i'}{\tan i}\,;$$

thus, if m is represented by $\tan i'$, then $\tan i$ will represent m'. If we substitute these values into the formula

$$\left(\frac{m-m'}{m+m'}\right)^2,$$

we shall have for the expression of the intensity of the reflected light

$$\left(\frac{\tan i - \tan i'}{\tan i + \tan i'}\right)^2,$$

whereby we can calculate *a priori*, for any incidence, the proportion of light reflected by a diaphanous medium whose refractive power is known, when the incident light is all polarized per the plane of reflection.

[Multiplying the numerator and denominator in the last expression by $\cos i \cos i'$, we obtain the alternative form

$$\left(\frac{\sin(i-i')}{\sin(i+i')}\right)^2,$$

which may be more familiar to modern readers, and which is used in the "Postscript". —*Translator*.]

19. I have not yet verified this formula directly by intensity measurements made in the same case, knowing only some results obtained with ordinary light. Fortunately, using the deviation of the plane of polarization observed for the same incidence, one can calculate the intensity ratio between the light reflected from the beam polarized according to the plane of reflection, and the light reflected from the beam polarized according to the perpendicular plane, as we have seen previously, and thus deduce the second intensity from the first. This is the indirect process that I have followed in order to verify my formula on two valuable results of the observations of Mr. Arago, which he had the kindness to communicate to me.[44] He found that an unsilvered glass plate with parallel faces reflected as much light as it let pass when it was inclined to the rays at $11°\,23'$; this is the mean of four observations made with

[44, (a)] See Arago, *Oeuvres complètes*, vol. X, art. xxv, p. 468 et seq.

much care, and of which the greatest variations were barely a third of a degree, despite the difference in procedures. He has likewise found that two similar glass plates let pass as much light as they reflect when they are inclined at $16^\circ\,58'$. This is also the mean of four observations, but between two of them there was nearly one degree of difference. By measuring, under the same incidences, the deviation of the plane of polarization of a ray polarized in the azimuth of 45°, I have found the new azimuth s to be $31^\circ\,45'$ in the first case, and $24^\circ\,30'$ in the second. I have supposed that the ratio of refraction of the glass plates employed by Mr. Arago was 1.51, which is that of most of the glasses of Saint-Gobain. According to this assumption, which should hardly deviate from reality, I have calculated the value of the angle of refraction i' for each of the two incidences and, substituting the value of $\tan i$ and $\tan i'$ into the formula, found in the first case 0.4994, and in the second 0.3604, for the proportion of light reflected by a single surface when the incident beam is polarized per the plane of reflection. Let us first consider the former case, where the light is reflected by the two surfaces of a single plate. If we represent by 2 the intensity of all the direct light that has fallen on the plate, then that of each of the two perpendicularly polarized beams into which we divide it is equal to 1, and the sum of the rays reflected at the first surface is 0.4994 for the beam polarized per the plane of incidence: multiplying this number by $\tan^2 31^\circ\,45'$, we shall have, for the portion of light reflected from the second beam, 0.1912. Hence, for each of the two beams we find, by adding a geometric progression, that if n represents the light reflected at the first surface and m the light transmitted, so that $m+n=1$, the sum total of the infinitely many reflections that the second surface of the plate adds to that of the first is equal to[45]

$$\frac{mn}{1+n}\ .$$

Applying this formula to the first beam, for which $n=0.4994$ and $m=0.5006$, we find 0.1667, which, added to 0.4994, gives 0.6661. We obtain in the same manner, for the totality of light reflected from the second beam, 0.3211; now these two numbers combined give 0.9872, which only differs by the order of a hundredth from half of the incident light which I have supposed equal to 2.

In the second case, by multiplying by $\tan^2 24^\circ\,30'$ the number 0.3604, which is the portion of light reflected from the beam polarized per the plane of incidence of the first surface, we have, for the second beam, 0.0748. With these two given, we can easily calculate by the simple geometric progressions the sum of light resulting from all the reflections produced by the four surfaces of the two plates,[46] and we find in this manner, for the totality of the light reflected from the first beam, 0.6926, and for the second 0.2444, which when added together make 0.9370. We see that this number differs by little more than 6 percent from half of the incident light.

The table of Bouguer offered me some simpler cases and some more varied incidences: but, Mr. Arago having warned me that it was very inaccurate, I have deemed it unnecessary to compare it to the theory.[47]

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[45] To derive this formula, one must invoke the premise that the fraction of the light reflected from the back surface is the same as that reflected from the front. For the 's' component, this premise is predicted by the preceding theory: the formula for the reflectivity gives the same result if i and i' are interchanged. But, at this stage of the argument, the reflectivity formula for the other component has not yet been derived. [*This footnote and all subsequent ones are by the translator.*]

[46] As the addition of a third surface, let alone a fourth, creates infinitely many sequences of reflections and refractions by which a ray entering through the first surface may find its way back to that surface, to be either refracted out or reflected back in to repeat the process, it is not obvious (at least to the translator) how to perform the summation, and therefore not obvious that the shortfall in the calculated net reflected light is not due to omission of some terms.

[47] Young, in the passage last cited [12, at p. 338–9], quoted two measurements by Bouguer, of which modern readers will recognize the first as impressively accurate and the second as useless.

Postscript [48]

20. As the printing of this note was finishing, I found, by a mechanical argument, supported however by an empirical hypothesis,[49] a formula for the intensity of the reflected light polarized perpendicular to the plane of reflection;[50] this formula, which I propose to derive anew following some more rigorous considerations,[51] appears to be exact, at least when judged by its agreement with the several experimental results with which I have compared it. If we always represent by i and i' the angles of incidence and refraction, and by 1 the intensity of the incident beam polarized perpendicular to the plane of reflection, the intensity of the reflected portion of the light is equal to

$$\left(\frac{\sin 2i - \sin 2i'}{\sin 2i + \sin 2i'}\right)^2 .$$

This formula,[52] together with that which I have already given for the light polarized per the plane of reflection, must give the intensity of the reflected light when the incident light has not suffered any prior polarization; representing the intensity of the latter by 2, that of the reflected light will be equal to

$$\frac{\sin^2(i-i')}{\sin^2(i+i')} + \left(\frac{\sin 2i - \sin 2i'}{\sin 2i + \sin 2i'}\right)^2 .$$

This formula, applied to the two already cited observations of Mr. Arago, agrees with the first to nigh one percent,[53] and for the second gives six percent of difference.[54]

21. Having long ago measured several deviations of the plane of polarization in the reflection on glass and on water, I can put these formulae to a new test, by deducing from them the general expression for the azimuth of the plane of polarization of the reflected beam, and applying it to the observed cases. When the plane of polarization of the incident light is inclined at 45° to the plane of reflection, the two beams polarized parallel and perpendicular to the plane of incidence, into which we can decompose it, are equal; and if a and b denote the amplitudes of the velocities of oscillation of the respective reflected beams, then b/a is the tangent of the angle that the plane of polarization of the total reflected light makes with the plane of incidence. But we have, for the values of b and of a: [55]

$$a = \frac{\sin(i-i')}{\sin(i+i')} \quad \text{and} \quad b = \frac{\sin 2i - \sin 2i'}{\sin 2i + \sin 2i'} ;$$

so the tangent of the azimuth of the plane of polarization of the reflected light is equal to

$$\frac{(\sin 2i - \sin 2i')\,\sin(i+i')}{(\sin 2i + \sin 2i')\,\sin(i-i')} .$$

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48 First published as "Addition to the Second Note inserted in the previous issue", *Annales de Chimie et de Physique*, Ser. 2, vol. 17, pp. 312–15 (July 1821). In the reprint in the *Oeuvres complètes* [9, vol. 1, pp. 646–8], there is no subtitle, but the label "Post-Scriptum" is inserted after the number "20." at the beginning of the first paragraph. The same label appears in the corresponding place in the editors' analytical table of contents [9, vol. 3, p.585]. Logically, this label has the scope of a subtitle in both places, and the present translation promotes it accordingly.

49 French: *par une solution mécanique, mais fondée sur une hypothèse empirique*.

50 That is, the component whose vibration is ***p***arallel to the ***p***lane of reflection; that is, the 'p' component.

51 A derivation was included in a later memoir read on 7 January 1823 [9, vol. 1, pp. 767–99], in which Fresnel interpreted complex values of the reflection coefficients as predicting the phase shifts in total internal reflection.

52 The fraction in parentheses is equivalent to $\frac{\tan(i-i')}{\tan(i+i')}$, as can be verified by working back from the latter. At Brewster's angle, i and i' are complementary, so $2i$ and $2i'$ are supplementary, whence their sines are equal, so that the numerator of the fraction in the text is zero. *N.B.*: At Brewster's angle, while it is true that (i) the reflected ray is perpendicular to the refracted ray and (ii) the electric vibration of the reflected ray is perpendicular to the refracted ray, *the first fact does not directly imply the second*; to see this, consider the magnetic vibration instead of the electric one! Moreover, condition (i) depends on the two media having equal magnetic permeabilities.

53 As the reflectivity formula for the 'p' component has now been found, the equality of the reflectivities at the front and back surfaces, for each component, now becomes a prediction from theory.

54 Again it is not obvious that all reflections between the four surfaces have been accounted for. But, in view of what comes next, that might no longer seem so important.

55 Nowadays we would put a minus sign in front of the expression for a, as Fresnel effectively did in the subsequent memoir of 7 January 1823 [9, vol. 1, p. 773, eq. (1)].

The following table offers the comparison of several angles deduced from this formula with those that had been given to me by observation.

Reflection of light polarized with an azimuth of 45°
relative to the plane of reflection

INCIDENCES measured from the perpendicular	AZIMUTH of the plane of polarization of the reflected light with the sign relative to the image of the initial plane — By the formula	By observation	DIFFERENCES
ON GLASS			
24°	37° 54′	38° 55′	−1° 01′
39°	24° 38′	24° 35′	+0° 03′
49°	10° 52′	11° 45′	−0° 53′
The light was polarized in the plane of reflection at the incidence of 56° $\frac{1}{2}$, in conformity with Brewster's law, with which the formula agrees.			
60°	− 5° 29′	− 5° 15′	−0° 14′
70°	−20° 24′	−19° 52′	−0° 32′
80°	−33° 25′	−32° 45′	−0° 40′
85°	−39° 19′	−38° 55′	−0° 24′
87°	−41° 36′	−40° 55′	−0° 41′
88°	−42° 44′	−41° 15′	−1° 29′
89°	−43° 52′	−44° 35′	+0° 43′
ON WATER			
The reflected light was polarized in the plane of reflection at the incidence of 53°, in conformity with the calculation.			
60°	−10° 51′	−10° 20′	−0° 31′
70°	−24° 48′	−25° 20′	+0° 32′
80°	−35° 49′	−36° 20′	+0° 31′
85°	−40° 32′	−40° 50′	+0° 18′

We see that the most considerable difference between calculation and observation is a degree and a half, for the reflection on glass at the incidence of 88°, and that this somewhat strong discrepancy probably arises from the inaccuracy of the observation, at least if one judges by the contrary signs of the difference which follows it and that which precedes it. It is difficult to determine the plane of polarization of a beam of light with great precision by observing it through a rhomb of calcite, because the extraordinary image is invisible a little before and after the moment when the principal section of the rhomb coincides with the plane of polarization. I hope however to obtain the results more accurately by using the light of the sun, whose great brightness [*vivacité*] allows one to follow the extraordinary image closer to the plane of polarization. Pending these new verifications, one can consider the exactness of the formula as very probable, by its fairly satisfactory agreement with the observations made, and its still more certain coincidence with experiment in the three principal cases: first, when the incident rays are perpendicular to the reflective surface; second, when they make therewith the angle of complete polarization [Brewster's angle]; third, when they are parallel thereto. For it indicates that in the first case, the plane of polarization does not change; that in the second, it merges with the plane of reflection; and that in the third, it is 45° away therefrom, on the side opposite to the image of the initial plane of polarization, so that it is on the extension of that plane.[56] And all these consequences of the formula agree with observation.

22. The two intensity formulae that I have just given can also be used to calculate the proportion of light polarized by reflection; for this it suffices to subtract

$$\left(\frac{\sin 2i - \sin 2i'}{\sin 2i + \sin 2i'}\right)^2 \qquad \text{from} \qquad \frac{\sin^2(i-i')}{\sin^2(i+i')}\,,$$

and divide their difference by their sum.

Acknowledgment

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Revision history

Version 2 adds reference [5] and mentions the plate in the 1826 printing of reference [7]. In the footnotes, it adds five citations (including [9, vol. 1, pp. 489–93] in footnote 21) and indicates that the cited surviving letter from Young to Arago [12, pp. 380–84] is in English. In the analytical table of contents, at § 14, it removes an ambiguity by changing one instance of "latter" to "last". In the text, it gains slightly from an additional proof-reading.

[56] In the first case, as we approach normal incidence, we can use the approximation $\sin\theta = \theta$ and find that the tangent of the azimuth is 1. In the second (Brewster's angle), the numerator in the formula is zero. In the third, the formula reduces to 1; and indeed, near grazing incidence, the reflections of the 's' and 'p' components are both inverting and near-complete, so that the direction of vibration hardly changes.

References

[1] D.F.J. Arago & A. Fresnel, “Mémoire sur l'action que les rayons de lumière polarisés exercent les uns sur les autres”, *Annales de Chimie et de Physique*, Ser. 2, vol. 10, pp. 288–305, March 1819; reprinted in [9], vol. 1, pp. 509–22; translated as “On the action of rays of polarized light upon each other” in Crew [4], pp. 145–55. (The title in [9] changes “polarisés” to “polarisée”, which is followed in [4].)

[2] D. Brewster, “On the laws of polarisation and double refraction in regularly crystallized bodies” (read 15 Jan. 1818), *Philosophical Transactions of the Royal Society*, vol. 108 (1818), pp. 199–273, rstl.royalsocietypublishing.org/content/108/199.

[3] J.Z. Buchwald, *The Rise of the Wave Theory of Light: Optical Theory and Experiment in the Early Nineteenth Century*, University of Chicago Press, 1989.

[4] H. Crew (ed.), *The Wave Theory of Light: Memoirs by Huygens, Young and Fresnel* [Vol. X in J.S. Ames (ed.), Scientific Memoirs], American Book Co., 1900; archive.org/details/wavetheoryofligh00crewrich.

[5] O. Darrigol, “James MacCullagh's ether: An optical route to Maxwell's equations?” *European Physical Journal H*, vol. 35, no. 2 (November 2010), pp. 133–72; doi.org/10.1140/epjh/e2010-00009-3.

[6] O. Darrigol, *A History of Optics: From Greek Antiquity to the Nineteenth Century*, Oxford, 2012.

[7] A. Fresnel, “Mémoire sur la diffraction de la lumière”, submitted 29 July 1818, “crowned” 15 March 1819; partly published in *Annales de Chimie et de Physique*, Ser. 2, vol. 11, pp. 246–96, 337–78 (1819); fully published in *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, vol. V (for 1821 & 1822, printed 1826), pp. 339–455 & plate after p. 474; reprinted in [9], vol. 1, pp. 247–364; partly translated as “Fresnel's prize memoir on the diffraction of light” in Crew [4], pp. 81–144.

[8] A. Fresnel, “Mémoire sur la double réfraction”, *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, vol. VII (nominally for 1824, printed 1827), pp. 45–176; reprinted as “Second mémoire...” in [9], vol. 2, pp. 479–596; translated by A.W. Hobson as “Memoir on double refraction” in R. Taylor (ed.), *Scientific Memoirs*, vol. V (London: Taylor & Francis, 1852), pp. 238–333, archive.org/details/scientificmemoir05memo/page/238. Cited page numbers are from the translation. For notable errata in the original printing, and consequently in the translation, see [9], vol. 2, p. 596*n*.

[9] A. Fresnel (ed. H. de Sénarmont, E. Verdet, & L. Fresnel), *Oeuvres complètes d'Augustin Fresnel* (3 vols.), Paris: Imprimerie Impériale, 1866, 1868, 1870.

[10] G.R. Putland, *Wave Foundations of Ray Optics*, doi.org/10.5281/zenodo.3901935, 2020 (in progress).

[11] W. Whewell, *History of the Inductive Sciences*, 2nd Ed., vol. II, London: J.W. Parker, 1847; books.google.com/books?id=JcrIfVHljaEC.

[12] T. Young (ed. G. Peacock), *Miscellaneous Works of the late Thomas Young, M.D., F.R.S., &c.*, vol. I, London: J. Murray, 1855; archive.org/details/miscellaneouswo01youngoog.