North American Academic Research

Journal homepage: http://twasp.info/journal/home

Research

An Approach for Solving Minimum Spanning Tree Problem and Transportation Problem Using Modified Ant Colony Algorithm

Uthpala Ekanayake[1]*,Wasantha Daundasekara[2] ,Pantalian Perera[3]

[1]Department of Physical Sciences, Faculty of Applied Sciences, Rajarata University of Sri Lanka, Mihinthale, Sri Lanka

[2]Departments of Mathematics, Faculty of Science, University of Peradeniya, Sri Lanka

[3]Departments of Eng. Mathematics, Faculty of Engineering, University of Peradeniya, Sri Lanka

**Corresponding author*

***Accepted**: 30 September , 2020;**Online**: 07 October, 2020*

***DOI** : https://doi.org/10.5281/zenodo.4072472*

***Abstract:** The minimum spanning tree problem consists of finding a minimum cost spanning tree in an undirected graph in various types of networks. Minimum weight spanning tree visits all vertices that are in the similar associated part as the beginning node. In this investigation, we examine the various techniques for solving a Generalized Minimum Weight Spanning Tree Problem. Also, we present the An Approach for Solving Minimum Spanning Tree Problem and Transportation Problem Using Modified Ant Colony Algorithm. Different methodologies have been made in the composition for dealing with transportation on finding an initial basic feasible solution and the rest to find the optimal solution to the TP. Northwest, Least Cost, and Vogel's Approximation techniques are created to find an initial basic feasible solution whereas the Modified Distribution (MODI) Method and Stepping Stone Method is designed to find an optimal solution to the TP. In this examination, we propose a heuristic method known as the Modified Ant Colony Optimization Algorithm, which is based on Ant Colony Algorithm (ACA) procedure has demonstrated to provide near-optimal solutions to a reasonable degree of satisfaction to large scale TPs. In this novel approach, the degree of satisfaction of the optimal solution has been improved by modifying ACA with the incorporation of the transition Rule and Pheromone Update Rule. The algorithmic approach proposed by this study is less complicated compared to the well-known meta-heuristic algorithms in the literature. In the end, we represent the conclusion we illustrate the proposed method using a case to study.*

***Keywords**: Transportation problem, Ant Colony Algorithm, A Minimum Spanning Tree, Optimal solution.*

Introduction

A Minimum Spanning Tree in an undirected associated weighted graph is a spanning tree of minimum weight. Minimum spanning tree is generally applied in different zones at least cost electrical wiring associations, least expense interfacing transportation networks, telephone organize structure business with a few workplaces, Civil Network Planning, algorithms for tackling mobile sales rep problems, and network flows. Ongoing methodologies in breaking down different biomedical problems like clinical imaging, bio-psychological warfare, and so on have utilized the ideas of minimal spanning tree (MST). The problem is to find a spanning subtree of a given associated network which has a minimum total length.

Various arranged arrangement procedures exist for comprehending the MSTP. One of the principal realized arrangements was given by Czech researcher Otakar Borůvka developed the primary known algorithm for finding a minimum spanning tree, in 1926. He needed to tackle the problem of finding an efficient coverage of Moravia with electricity. This algorithm is called Borůvka's algorithm. Thereafter Prim and Kruskal proposed two different algorithms that had all the earmarks of being more proficient. One of them was created by Vojtěch Jarník in 1930 and put in practice by Robert C. Prim in 1957. Edsger W Dijkstra rediscovered it in 1959 and called it Prim's algorithm. The other algorithm is called Kruskal's algorithm and was published by Joseph Kruskal in 1956.

Ant Colony Algorithm (ACA) is a class of algorithms, whose initial segment, called the Ant System, was from the outset proposed by Colorni, Dorigo and Maniezzo. The rule concealed idea, inexactly motivated by the conduct of real ants is that of an equal inquiry more than a few productive computational strings dependent on neighborhood issue information and a powerful memory structure containing information on the nature of the recently acquired outcome. The collective behavior emerging from the interaction of the different search threads has demonstrated compelling in unraveling combinatorial improvement (CO) problems. The improvement of this algorithm was inspired by the observation of ant colonies. Ants are social insects and wander randomly. They are strolling between their colony and the food source, ants store pheromones along with the ways they move. The pheromone level on the paths increases (sum and the idea of the food) with the number of ants going through and food source is drained with pheromones level gradually rots. Shorter ways attract more pheromone and pheromone power helps ants to distinguish shorter paths to the food source.

Problem Definition.

An undirected connected graph is usually denoted by consist set of vertices $V(G)$ and set of edges $E(G)$.

Definition 1.A tree is an undirected graph that is connected and acyclic. It is easy to show that if a graph that satisfies any two of the following properties also satisfies the third, and is, therefore, a tree:

• is connected

• is acyclic

•$|E| = |V| - 1$

Definition 2. Weighted Graphs**.** A weighted graph G(V, E, W) consists of vertices set V and edge set E, weighted set W and each edges connects a pair of vertices.

Definition 3. Weight of a Graph**:** The sum of the weights of all edges.

Definition 4. A tree is an undirected graph in which any two vertices are connected by exactly one path. In other words, any acyclic connected graph is a tree.

Definition 5. General Properties of Spanning Tree

A spanning tree, in the graph, is a tree with that has all vertices covered, That is the. Following are a few properties of the spanning tree connected to graph G.

- A connected graph G can have more than one spanning tree.
- All conceivable spanning trees of graph G, have a similar number of edges and vertices.
- The spanning-tree does not have any cycle (loops).
- Eliminating one edge from the spanning tree will make the graph disconnected, i.e. the spanning tree is minimally connected.
- Adding one edge to the spanning tree will make a circuit or loop, i.e. the spanning tree is maximally acyclic.

Definition 6.Minimum Weight Spanning Tree (MWST)

In a weighted graph, a minimum spanning tree (MST) or minimum weight spanning tree consisting of all the vertices of the same graph. The weight of a spanning tree is the sum of weights given to each edge of the spanning tree.

An edge-weighted graph is a pair where is a graph and is a weight function and that assigns a real weight to each edge. Our task is to find the minimum spanning tree of, that is, the spanning tree that minimizes the function

$$w(T) = \sum_{e \in T} w(e)$$

Definition 7 . Modified Ant Colony Algorithm (H. Hingrajiya)

An ant k currently at city *i* (i.e., the nest) choose to move to city *j* (i.e., the food) by applying the following probabilistic transition rule Transition rule ;

$$P_{ij}(c) = \frac{{\tau_{ij}}^{\alpha} {\mu_{ij}}^{\beta}}{\sum_{\Omega_k(l)} {\tau_{il}}^{\alpha} {\mu_{il}}^{\beta}} \text{ if } j \in \Omega_k(l)$$

$$=0 \qquad \text{otherwise}$$

Here,τ_{ij} denotes the amount of pheromone on a component between states i and j, and μ_{ij}denotes it's heuristic value. α and β are both parameters(0 ≤α,β≤ 1) used to control the importance of the pheromone trail and heuristic information during component selection.$\Omega_k(l)$ is a set of cities which remain to be visited when the ant is at city i.

For our development we assumed $\beta = 1$ in the transition rule,

With;

$$P_{ij} = \frac{{C_{ij}}^* - C_{ij}}{\sum_{s \in Allownodes(} {C_{ij}}^* - C_{ij})}; \quad i^{\text{th}} \text{ ant visits the } j^{\text{th}} city$$

$$0 \qquad ; \text{ Otherwise}$$

$\tau_{ij} = 1$; Quantity on pheromone between the source *i* to destination *j*.

c_{ij}; is cost between node *i* and node *j* and ${C_{ij}}^*$=max[c_{ij}]

P_{ij}; Probability to branch from node *i* to node *j*.

Pheromone Update Rule..

After all ants complete their tours, the local update rule of the pheromone trails is applied for each route according to [1],

$$Q_{ij} = (1-\rho)Q_{ij}(t) + \sum_{k=1}^{m} \Delta Q^{k}{}_{ij}$$

After that, apply the global pheromone update rule in which the amount of pheromone is added to the best route which has the lowest cost. This rule is defined in (4). First let's look at it mathematically:

$\Delta Q^{k}{}_{ij} = \frac{\zeta}{Y^{k}}$;   if component $(i, j)$was used by ant (best route)

= 0 ;Otherwise

Here, $Y^{k}$is the distance of the best route. $\zeta$is simply a parameter to adjust the amount of pheromone deposited, typically it would be set to 1. We sum $\frac{\zeta}{Y^{k}}$for every solution which used component (i, j), then that value becomes the amount of pheromone to be deposited on component(i, j). In our case ,

$$Q_{ij}(t+1) = (1-\rho)Q_{ij}(t)$$

Where $Q_{ij}(t)$=*1 and* $0 < \rho \leq 1$.

Definition 8.Transportation Problems

Transportation problems are characterized by problems that are trying to distribute commodities from any supply center, known as sources, to any group of receiving centers, known as destinations. Two major assumptions are needed in these types of problems:

i.The Requirement Assumption

Each source has a fixed supply which must be distributed to destinations, while each destination has a fixed demand that must be received from the sources

ii.The Cost Assumption

The cost of distributing commodities from the source to the destination is directly proportional to the number of units distributed

3.Mathematical formulation of the Transportation problem

Let X_{ij} be the amount of commodity shipped from source i to destination j

Minimize$\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}C_{ij}$

Subject to the constraints

(i) Supply at each source must be used:

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1,2,\dots,m$$

(ii) Demand at each destination must be met:

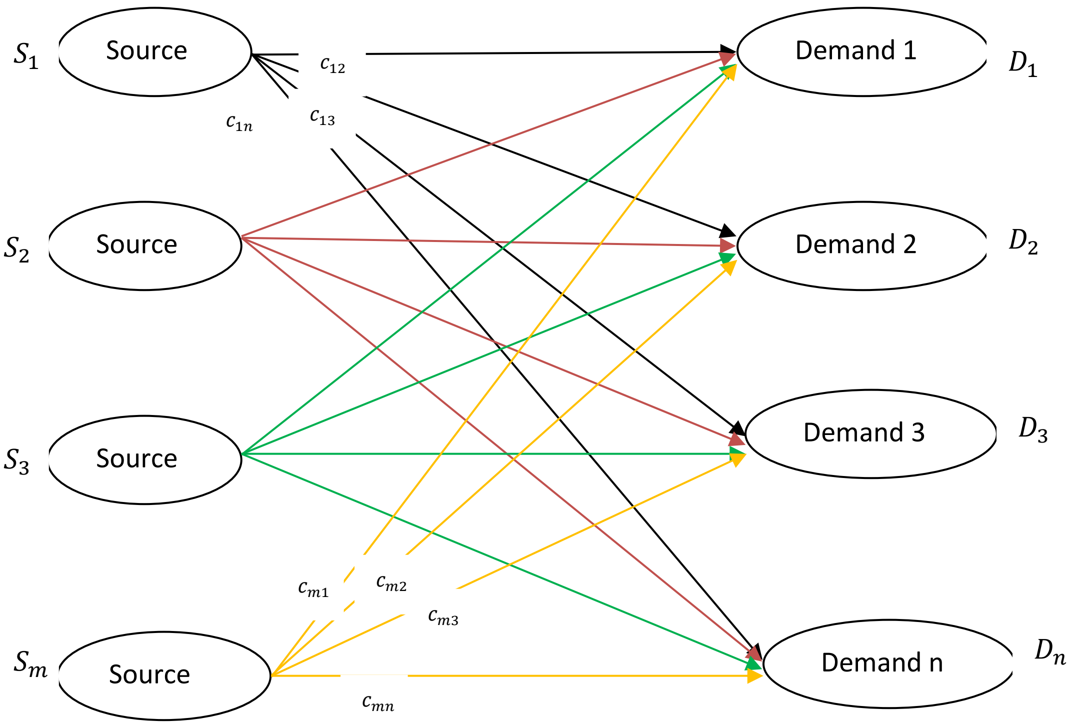
$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1,2,\dots,n$$ and

(iii) Flows must be nonnegative:

$X_{ij} \geq 0$for all $i = 1,2,\dots,m$ and $j = 1,2,\dots,n$

Note that here the sum of the supplies equals the sum of the demands. i.e. $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Such problems are called balanced transportation problems and otherwise, i.e. $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$,known as unbalanced transportation problems

Network Diagram



Proposed Method.

In this section, a proposed method, Improved Ant Colony Algorithm, for finding an optimal solution. Following are the steps for solving minimum spanning Problem.

Proposed New Algorithm For minimum spanning Tree

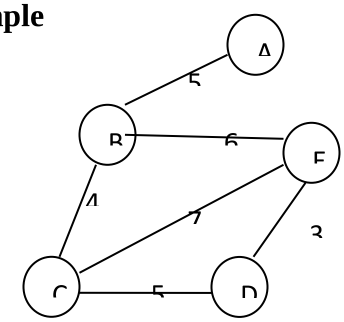
Step1.Sort all the edges in the cost table of their weight.

Step2.Calculate the edge probability using modified Ant colony Algorithm

Step 3. Start adding edges to the MST from the edge maximum probability weight and Remove particular edge from the cost table and the graph

Step4.Repeat step 3until the whole of what vertices have been related..

Example



.

i.

Edge	AB	BC	CD	DE	EB	EC
Weight	5	4	5	-	6	7
$C_{ij}{}^{*} - C_{ij}$	2	3	2	-	1	0
Probability	0.166	0.25	0.17	-	0.0833	0

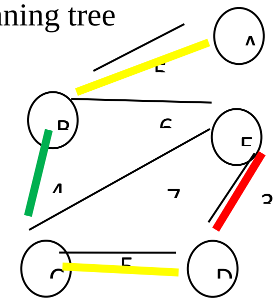
ii.

Edge	AB	BC	CD	DE	EB	EC
Weight	5	-	5	-	6	7
$C_{ij}{}^{*} - C_{ij}$	2	-	2	-	1	0
Probability	0.166	-	0.17	-	0.0833	0

iii.

Edge	AB	BC	CD	DE	EB	EC
Weight	5	-	5	-	6	7
$C_{ij}{}^{*} - C_{ij}$	2	-	2	-	1	0
Probability	0.166	-	0.166	-	0.0833	0

Spanning tree



Proposed New Algorithm For TP

Step 1: Construct the transportation cost table from the given problem

Step 2: Next compute the path according to the probability Table using modified ***Transition Rule***.

Step 3: Identify the cell for allocation which has the maximum probability cell for each row or each column.

Step 4: Starting nodes with the Zero cell column or row in the probability matrix to make the first allocation. Allocate $X_{ij} = \min(a_i, b_j)$ in the i-j^{th} with respect to the selected cell in Step – 3.

Step 5: Allocate this minimum value to the selected cell in Step – 3 and subtract this minimum value from the supply and demand values with respect to the selected cell.

Step 6: If the demand in the column (or supply in the row) is satisfied, delete that column (or row) then move to the next maximum value in the Demand row or Supply column.

Step 7**:** Continue this process until satisfaction of all the supply and demand is met go to Step - 8 ,Otherwise go to Step – 4.

Step 8**:** Stop and calculate the initial feasible solution.

Comparative Assessment

This section provides performance comparisons across the various well-known methods – Kruskal Algorithm, and some other popular methods by the solutions obtained from disparate problems. Comparative assessments are performed and illustrated in the immediately following sections.

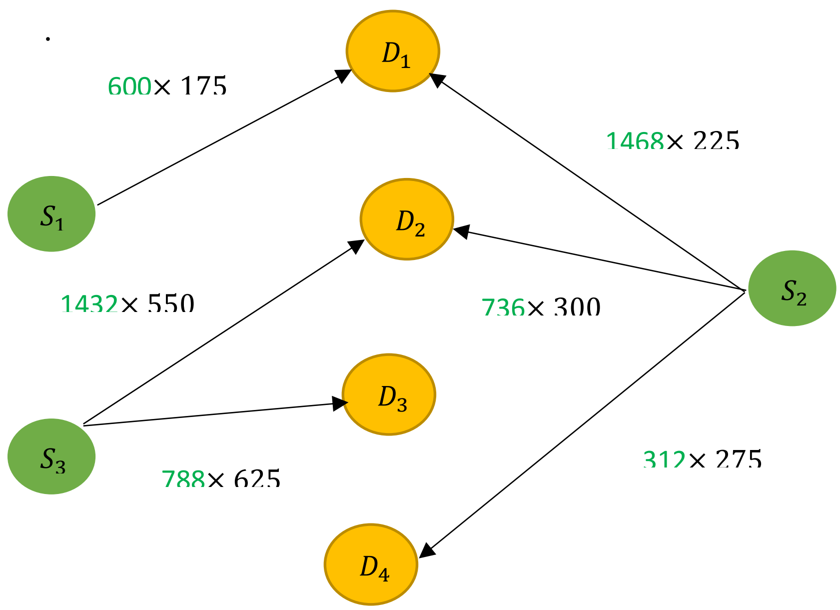
This is demonstrated by comparative study AKPAN, N. P. & IWOK, I. A and NEW METHOD .

Cable troughs were shipped from three different suppliers:

Supplier 1, Supplier 2, and Supplier 3. They are collected at four different locations at the construction site viz: Sec1, Sec2, Sec3, and Sec4. Our aim here, using the Kruskal algorithm for minimum spanning tree, is to figure out which plan for assigning the shipments would minimize the total shipping cost, subject to the restrictions imposed by the fixed output from each supplier and the fixed allocation in each location at the site. For this problem, the data that needed to be gathered included three categories: the output from each source (Supply), the allocation at each destination (Demand), the cost per unit shipped from each source to each destination (In AED). The procurement department at the company involved provided the data needed for the first and third categories above. Engineers supervising the work in each of the four locations at the site provided data of the second category. The cost from each supply end to each location depends on the distance, the nature of road, and the time of shipment. Since each trip can load four troughs, the shipping cost per trough can be found by dividing the shipping cost per trip by 4. This information of supply and demand (in units of cable troughs), along with the shipping cost per cable trough for each supplier-location combination is given in the Table below.

	LOCA	**TION**		**AT**	**THE SITES**
SUPPLIERS	SEC 1	SEC 2	SEC 3	SEC 4	CAPACITY
AL MERAKNY	175	250	325	525	600
UPC	225	300	375	275	2,516
EPC	500	550	625	525	2,220
ALLOCATION	2,068	2,168	788	312	

Solution of **AKPAN & IWOK** method



MIN Z= (175x600)+(225x1468)+(275x312)+(300x736)+(550x1432)+(625x788)=2022000

Next Apply our new method

	D_1	D_2	D_3	D_4	Supply
S_1	175	350	325	225	600
S_2	225	300	375	275	2516
S_3	500	500	625	525	2220
Demand	2068	2168	788	312	

Using modified***Transition Rule***.

	D_1	D_2	D_3	D_4	Supply

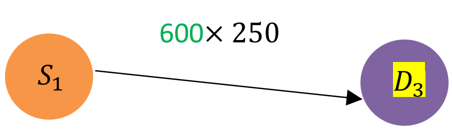
S_1	450	275	300	400	600
S_2	400	325	250	350	2516
S_3	125	125	0	100	2220
Demand	2068	2168	788	312	

Probability table

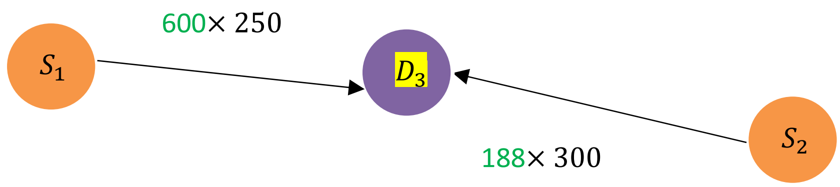
	D_1	D_2	D_3	D_4	Supply
S_1	0.142	0.087	0.103	0.126	600
S_2	0.126	0,103	0.079	0.111	2516
S_3	0.039	0.039	0	0.031	2220
Demand	2068	2168	788	312	

	D_1	D_2	D_3	D_4	Supply
S_1	~~0.142~~	~~0.087~~	0.103*600	~~0.126~~	0
S_2	0.126*2068	0,103	0.079*188	0.111*260	0
S_3	~~0.039~~	0.039*2168	0	0.031*52	0
Demand	0	0	0	0	

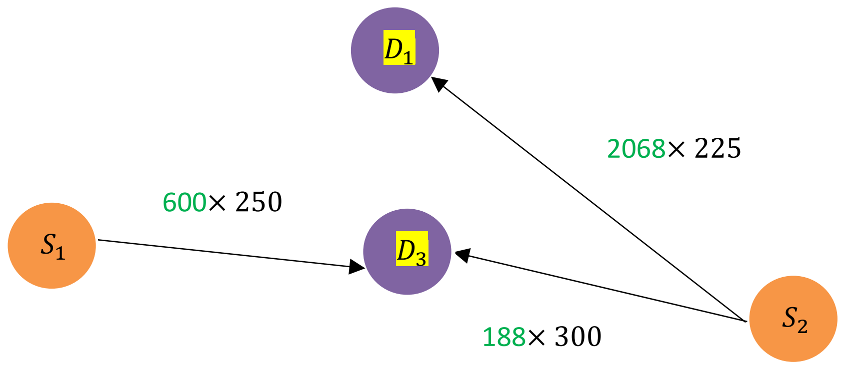
i.



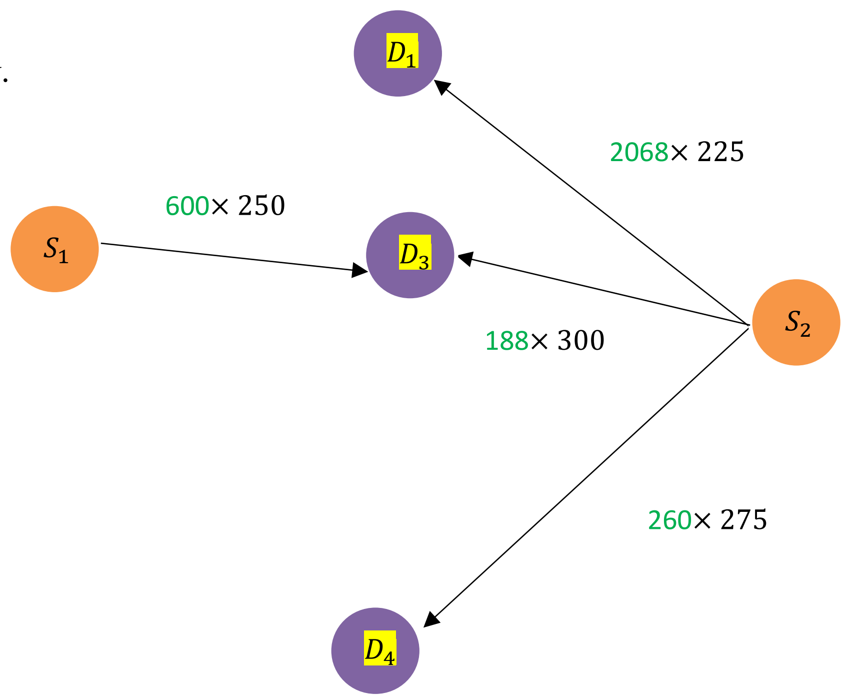
ii.


600× 250
S_1
D_3
S_2
188× 300

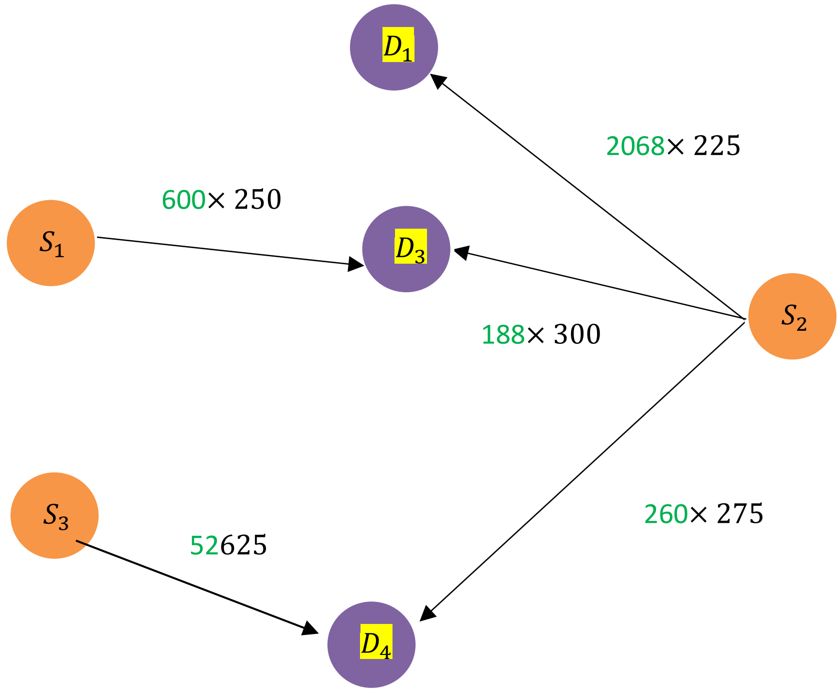
iii.


D_1
2068× 225
600× 250
S_1
D_3
S_2
188× 300

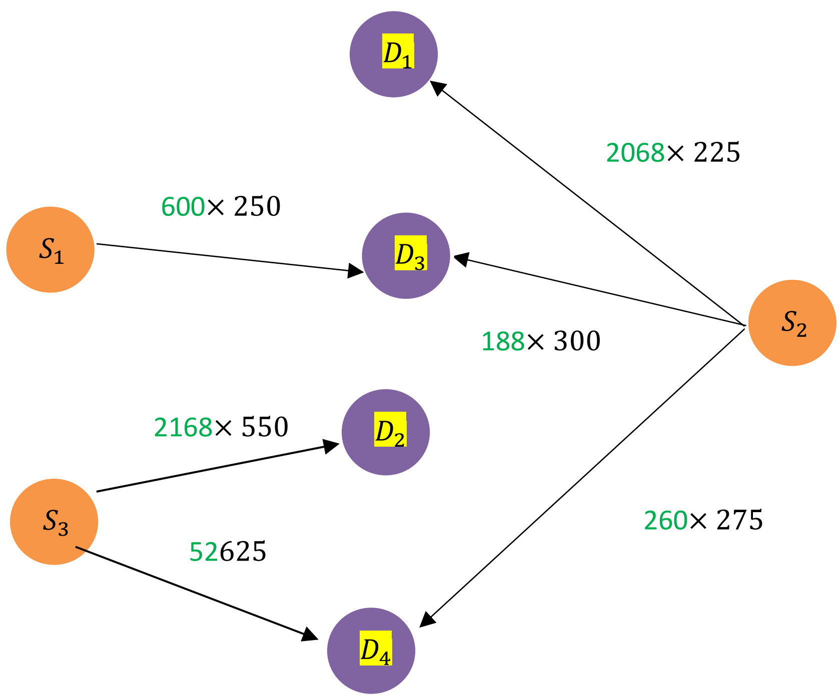
iv.


D_1
2068× 225
600× 250
S_1
D_3
S_2
188× 300
260× 275
D_4

v.



vi.



Minimumcost=600x325+188x375+2068x225+260x275+52x525+2168x500=1,913,600

It can be easily observed from above Problem that our NEW METHOD provides the better solutions.

Comparative outcomes obtained by Modified Kruskal's Algorithm, and the proposed method for the four benchmark models are showed up in the going with Table 1. Detailed data depiction of these four problems is given in **Appendix A**:

Table 1. A comparative results obtained by Modified Kruskal's Algorithm and New method for the four benchmark models

Problem chosen from Kadhim & Anwar **(2015)**			**Optimal cost(O_c)**	**From cost**	
	MKA	NEW		MKA	NEW
Problem 1.	600	600	600	0.00	0.00
Problem 2.	2225	2375	2225	0.00	6.74
Problem 3.	293200	291600	91600	0.005	0.00
Problem 4.	447	311	311	43.72	0.00

The comparative results got in Table 1 are moreover depicted using bar graphs and the results are given in Fig.1.

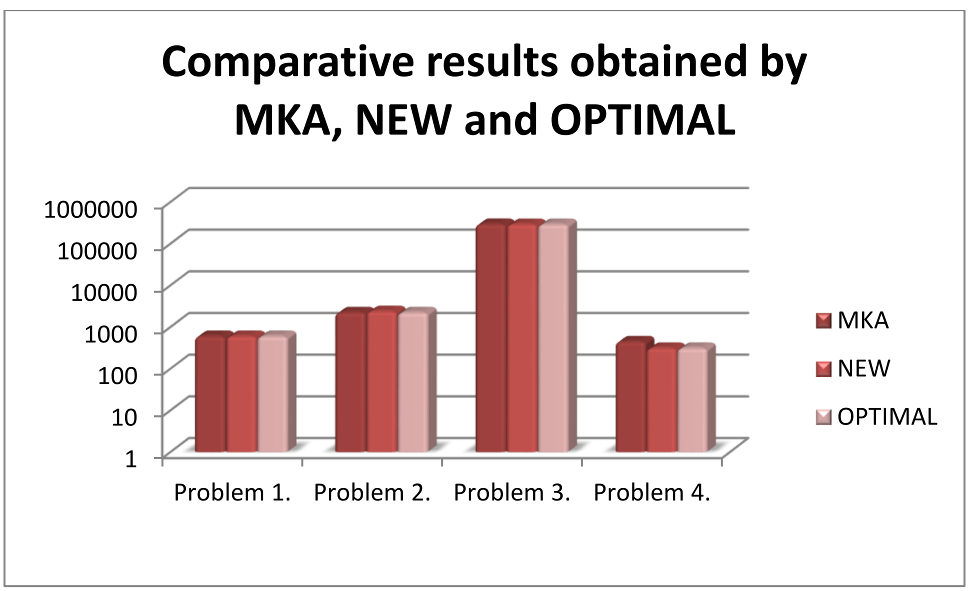


Fig.1. Comparative Study of the Result obtained by MKA , NEW Method and OPTIMAL solution

Line graphs for the percentage deviation (of the MKA and New method) from minimal complete cost solution obtained in Table 1 are depicted in Fig.2:

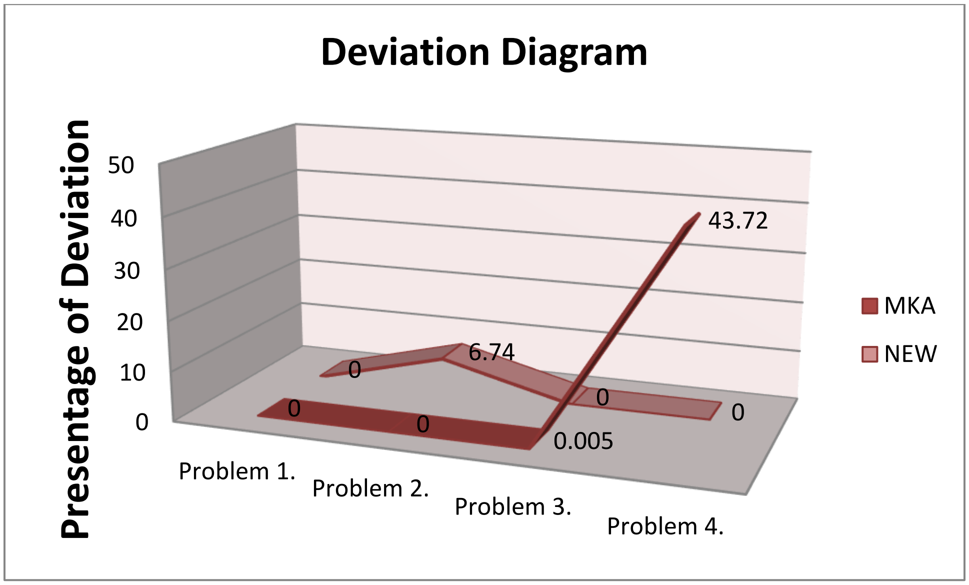


Fig.2. Percentage of Deviation of the Results obtained by MKA and the proposed NEW METHOD

As observed from the above results (Table 1, Fig. 2 Fig. 3), the proposed is more efficient than MKA for each situation where an improvement in effectiveness was possible (3 / 4 case). Nonetheless, this cannot deny the value of our proposed method, modified ant colony algorithm. Note that, here, the formula is utilized to obtain the percentage deviation from the optimal result.

Conclusion

A minimum spanning tree of an edge-weighted graph G is a spanning tree of G such that minimizes the maximum weight of an edge in the spanning tree. We have built up another definite algorithm for finding the MST of graph G, utilizing the Ant colony algorithm to obtain an MST from a given connected graph.

However, in this research paper, we discuss a new elective method, a modified ant colony optimization algorithm, and a modified minimum spanning tree which gives frequently an optimal solution to the transportation Optimum Utilization of Transportation System. Subsequently, Charnes and Cooper built up the developed stepping stone method and Dantzig built up the Transportation Simplex Method. problem. At first, the TP was first developed by Hitchcock and then Koopmans developed

A few heuristic solution approaches can be found in the literature such as Northwest Corner Method, Minimum Cost Method, VAM -Vogel's Approximation Method, MODI Method, and Stepping Stone Method. This investigation presents an overview of the concept of ACA and provides a review of its applications for solving TP. The proposed algorithm is heuristic in nature and less complicated in the implementation compared to many existing heuristic algorithms. Several modifications have been incorporated into the ACA and utilizing a minimum spanning tree to upgrade the combination rate to arrive at an acceptable optimal solution to the TP. This is accomplished by modifying ACA with the consolidation of the Transition Rule and Pheromone Update Rule.

References

Acierno LD and Gallo M et al(2011). Ant Colony Optimisation approaches for the transportation assignment problem. WIT Transactions on The Built Environment.2011, Vol 111.

Adlakhaand V and Kowalski K(2009).Alternate Solutions Analysis For Transportation Problems.*J.Bus.Econ.Res,7(11)*,41-49.

Ahmed MM, Khan AR,Uddin MS and Ahmed F(2016). Incessant Allocation Method for Solving Transportation Problems.American Journal of Operations Research. 6, 236-244.

AKPAN NP and IWOK IA(2017).A Minimum Spanning Tree Approach of Solving a Transportation Problem ; International Journal of Mathematics and Statistics Invention (IJMSI) ,Volume 5 Issue 3 , PP-08-17 Bertsimas DJ(1990) .The Probabilistic Minimum Spanning Tree Problem, NETWORKS.1990,Vol. 20 , 245-275 , John Wiley & Sons, Inc.,

Biswasa Pet a(2016)l. An Efficient Greedy Minimum Spanning Tree Algorithm Based on Vertex Associative Cycle Detection ,Procedia Computer Science, 92 , 513 – 519

Boria N and Paschos VT(2010). Fast reoptimization for the minimum spanning tree problem, Journal of Discrete Algorithms ,8 ,(296–310).

Blum C(2005).Ant colony optimization: Introduction and recent trends. Physics of Life Reviews.2 , 353–373.

Charnes A and Cooper WW(1954). The stepping stone method of explaining linear programming calculations in transportation problems. *Management Science*. 1 ,49-69.

Dantzig GB(1951). Application of the Simplex Method to a Transportation Problem, Activity Analysis of Production and Allocation. In: Koopmans, T.C., Ed., John Wiley and Sons, New York, 359-373.

Dorigo ACM, Maniezzo V and Coloni A(1996). The Ant system; Optimization by a colony of cooperating agents.IEEE Trance action of system ,Man, and cyberneties part B. 26(1):29-41

Dorigo M and Gambardella LM(1997).Ant colony system: a cooperative learning approach to the traveling. IEEE Transactions on Evolutionary Computation. 1: 53-66

Ekanayake EMUSB, Perera SPC., Daundasekara WB and Juman ZAMS. (2020). A Modified Ant Colony Optimization Algorithm for Solving a Transportation Problem. *Journal of Advances in Mathematics and Computer Science, 35*(5), 83-101

Hingrajiya H, Gupta RK and GS Chande (2012).An Ant Colony Optimization Algorithm for Solving Travelling Salesman Problem Krishna, International Journal of Scientific and Research Publications. Volume 2, Issue 8.

Ghodsia M et al.(2017). Spanning trees with minimum weighted degrees Information Processing Letters. 104 ,113–116

Kadhim B. S. Aljanabi & Anwar Nsaif Jasim(2015), An Approach for Solving Transportation Problem Using Modified Kruskal's Algorithm: International Journal of Science and Research (IJSR), Volume 4 Issue 7

Kenneth S and Gerrit K (2005). Janssens, An algorithm to generate all spanning trees of a graph in order of increasing cost, Pesquisa operational journal. vol.25 no.2 Rio de Janeiro May/Aug.

Koopmans C(1949) .A model of transportation. *Activity Analysis of Production and Allocation*. Proceedings of the Linear Programming Conference held at the University of Chicago, June 1949. Wiley, New York.1 222–259.

MatteucciM and Mussone L(2006).Ant colony optimization technique for equilibrium assignment in congested transportation networks. Conference: Genetic and Evolutionary Computation Conference, GECCO .

Mistuo GEN and Yinzhen M(1999). Solving Multi-Objective Transportation Problem by Spanning Tree-Based Genetic Algorithm, IEICE TRANS. FUNDAMENTALS. VOL.E82–A, NO.12.

Nicolas Boria and Vangelis Paschos (2008). Fast reoptimization for the minimum spanning tree problem. hal-00906970

Acknowledgments

This work was partially supported by the International grant of Rajarata University of SriLanka.

Dedication

This study is dedicated to my Gold mother, my lovely wife, son and my two daughters.

Conflicts of Interest

The authors declare that they have no conflict of interest.

Appendix A

Problem chosen from.	Data of the problem

Problem 1	$c_{ij} = [4,2; 7,5; 3,10], s_i = [60,40,70], d_j = [105,65\ ]$
Problem 2	$c_{ij} = [15,30,20; 30,40,35], s_i = [50,30], d_j = [25,45,10]$
Problem 3	$c_{ij} = [80,215; 100,108; 102,68], s_i = [1000,1300,1200], d_j = [2300,1400]$
Problem 4	$c_{ij} = [3,16,9,2; 1,9,3,8; 4,5,2,5], s_i = [40,20,50], d_j = [\ 25,25,42,8]$