**International Journal of Electrical and Computer Engineering (IJECE)**
Vol. 7, No. 4, August 2017, pp. 2109~2115
ISSN: 2088-8708, DOI: 10.11591/ijece.v7i4.pp2109-2115 

# A Novel Algorithm to Estimate Closely Spaced Source DOA

**Sidi Mohamed Hadj Irid[1] , Samir Kameche[2], Said Assous[3]**
[1,2]Lab STIC, University of Tlemcen, Tlemcen University, Algeria
[3]Ultrasound Research Labs, University of Leicester, United Kingdom

**Article Info**

***Article history:***

Received Jun 28, 2016
Revised Mar 27, 2017
Accepted Apr 10, 2017

***Keyword:***

Array geometry
Closely spaced sources
Music method
Resolution
Spatial processing

**ABSTRACT**

In order to improve resolution and direction of arrival (DOA) estimation of two closely spaced sources, in context of array processing, a new algorithm is presented. However, the proposed algorithm combines both spatial sampling technic to widen the resolution and a high resolution method which is the Multiple Signal Classification (MUSIC) to estimate the DOA of two closely spaced sources impinging on the far-field of Uniform Linear Array (ULA). Simulations examples are discussed to demonstrate the performance and the effectiveness of the proposed approach (referred as Spatial sampling MUSIC SS-MUSIC) compared to the classical MUSIC method when it's used alone in this context.



***Corresponding Author:***

S. Mohamed Hadj Irid,
Lab STIC, University of Tlemcen,
Algeria.
Email: smh_irid@mail.univ-tlemcen.dz

## 1. INTRODUCTION

When two sources are in the ambiguity range (very closed in space), the radar detects them like one target. The spatial resolution limits for two closely spaced sources in the context of array processing still an active research [1], [2]. In fact, there has been a tremendous involvement in the investigation of how many source (target) can be detected. Most of them, [2, 3, 4], exploit high resolution methods like Multiple Signal Classification (MUSIC) or Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT), and detect sources using eigenvalues obtained from covariance of samples. However, first a predetermination of model order is imperative for these techniques to know the number of uncorrelated sources. This estimation is based on information theoretic criteria like AIC (AKAIKE) and Rissanen's minimum description length criterion (MDL) algorithms to estimate the number of source [1], [5], [6]. In other hand, the performance of these techniques, when DOA's become closer, stays very poor for low SNR, short sample size and presence of impulsive white noise. To reduce this hurtful effect and improve the robustness of the covariance estimator, many authors proposed estimators combining high resolution, statistic techniques and time signal processing for different geometry design [3], [5], [7], [8].

In this work, we propose an improved algorithm which combines spatial sampling and high resolution method (MUSIC) to estimate closely spaced number of source and their direction of arrivals (DOA). The spatial sampling consist to curve up the array network of antenna, into L subarrays; in each subarray, it's applied MUSIC algorithm to estimate the number of closely spaced source and their Directions of Arrival (DOA). Numerical simulations are given to assess the performance of the technique.

The paper is organised as follows. The data, array model and MUSIC method description are introduced in section 2, followed by spatial sampling model description in section 3. The proposed algorithm described in section 4. Simulation results are given in section 5, before discussion and conclusion in section 6

## 2. SYSTEM MODELS

Let assume a ULA composed of M sensors, with equip-spacing d=λ/2 as shown in fig.1; where λ is the wavelength of the source signal. Consider a K narrowband far-field uncorrelated source impinging on the array with (M > K), such that sources have a direction of arrival (DOA) $\theta_k$, with k=1… K. The received snapshots at this array, at instance t are given by [2], [9], [10]:

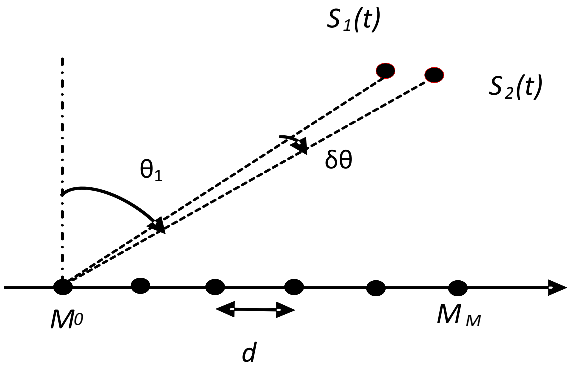


Figure 1. Localization of two closely spaced sources using ULA

$$y(t)=As(t)+n(t) \tag{1}$$

The matrices and vectors in Equation (1) have the following forms:

$$y(t)=[y_1(t) \ \dots \ y_M(t)]^T \tag{2}$$

$$S(t)=[S_1(t) \ \dots \ S_M(t)]^T \tag{3}$$

$$n(t)=[n_1(t) \ \dots \ n_M(t)]^T \tag{4}$$

$$A=[a_1 \ \dots \ a_k]^T \tag{5}$$

$$a_k=[1 \ a_k \ a_k^2 \ \dots \ a_k^{(M-1)}]^T$$
(6)

$$a_k=e^{(-j\,2\pi(d/\lambda)\sin\theta_k)} \tag{7}$$

Superscript $(.)^T$ presents the transpose operation.

Where $y_k(t)$ denotes the output of $k^{th}$ sensors, and $n_k(t)$ is a stationary model, temporally white, zero-mean Gaussian random process independent of the source signals.

$$E\{n(t)n^H(t)\}=\sigma^2 I \tag{8}$$

Where the superscript $(.)^H$ stands for the conjugate transposition, $\sigma^2$ is variance and I indicate the identity matrix.

A is steering matrix, it's (M x K) complex matrix, it's assumed to be full rank.

The covariance of receiving data is

$$R_{yy}=E[Y.Y^H]=AR_S A^H+\sigma^2 I \tag{9}$$

Where

$$R_S = E\{S(t)S^H(t)\}$$

Furthermore, the covariance matrix is estimated by [1- 3], [9-12]:

$$R_{yy} = \frac{1}{N} Y.Y^H \quad (10)$$

where N is the number of snapshots, and $R_{yy}$ is a Hermitian positive definite matrix, so its eigenvalues are real and positive, it can be decomposed [2], [13], [14] :

$$R_{yy} = U\Lambda U^H \quad (11)$$

U and Λ denotes eigenvectors matrix and diagonal eigenvalue matrix, respectively. Λ contains on its diagonal eigenvalues of signal space and noise space. The eigen-decomposition (EVD) of $R_{yy}$ [2], [4]:

$$R_{yy} = E_S \Lambda E_S^H + E_n \Lambda E_n^H \quad (12)$$

$E_s$ and $E_n$ are orthogonal [2], [11], and are the eigenvectors matrix that spans the signal and noise subspaces of $R_{yy}$, respectively.

The eigenvalues are given as follows

$$\rho_1 \geq \rho_2 \geq \ldots \geq \rho_k \geq \rho_{k+1} = \rho_M = \sigma^2$$

where the first K eigenvalues belong to the source signal, and the last (k-M) to the noise.

MUSIC plots the pseudo-spectrum:

$$V_{Music} = \frac{1}{a^H(\theta) E_n E_n^H a(\theta)} \quad (13)$$

Note, the estimated signal directions are the K largest peaks in the pseudo-spectrum [2], [5], [10].

## 3. THE SPATIAL SUBSAMPLING

In order to determine the DOA of impinging source on array antenna, spatial subsampling consists to divide the whole network array antenna into L subarrays as shown in Figure.2, and use just a one subarray at each sample time [4].

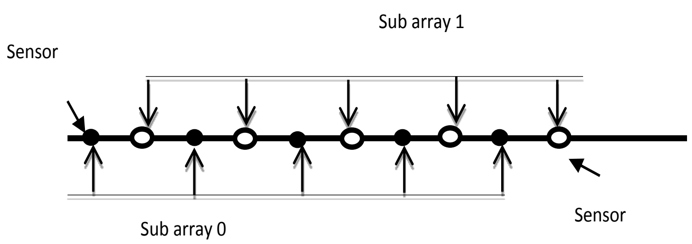


Figure 2. A ULA antenna is divided into L=2 sub array, each sub array contain M/L sensors

Let Consider L=2, in this case we use two sub networks. The signal will be received on sub network 0, first, then on second sub network 1.

The receiving signal is:

$$Y = A(\theta)S + N_0 \quad (14)$$

In case of using sub network 0

$$Y_0 = \begin{bmatrix} y_1(0) & .... & y_1(T) \\ y_{L+1}(0) & .... & y_{L+1}(T) \\ . & & . \\ . & & . \\ y_{((N_L-1)L+1)}(0) & .... & y_{((N_L-1)L+1)}(T) \end{bmatrix} \quad (15)$$

with T is snapshot number, and $N_L = M/L$

In this case:

$$A_0(\theta) = \begin{bmatrix} 1 & .... & 1 \\ e^{j\pi \sin\theta_1} & .... & e^{j\pi \sin\theta_q} \\ e^{j\pi(L+1)\sin\theta_1} & .... & e^{j\pi(L+1)\sin\theta_q} \\ . & & . \\ . & & . \\ e^{j\pi(N_L-1)(L+1)\sin\theta_1} & .... & e^{j\pi(N_L-1)(L+1)\sin\theta_q} \end{bmatrix} \quad (16)$$

The same for the output for sub network sign 1

$$Y_1 = \begin{bmatrix} y_2(0) & .... & y_2(T) \\ y_{L+2}(0) & .... & y_{L+2}(T) \\ . & & . \\ . & & . \\ y_{((N_L-1)L+2)}(0) & .... & y_{((N_L-1)L+2)}(T) \end{bmatrix} \quad (17)$$

$$A_1(\theta) = \begin{bmatrix} 1 & .... & 1 \\ e^{j\pi \sin\theta_1} & .... & e^{j\pi \sin\theta_q} \\ e^{j\pi(L+2)\sin\theta_1} & .... & e^{j\pi(L+2)\sin\theta_q} \\ . & & . \\ . & & . \\ e^{j\pi(N_L-1)(L+2)\sin\theta_1} & .... & e^{j\pi(N_L-1)(L+2)\sin\theta_q} \end{bmatrix} \quad (18)$$

Let note, that spatial subsampling, improve angular resolution because of the increasing of the imaginary part in elements matrix output in Equations (16) and (18). If we assume two closely spaced source where their DOA are $\theta_1$ and $\theta_2$ such as :

$$\theta_1 = \theta_2 + \delta\theta \text{ with } \delta\theta \prec 1 \; radian,$$

## 4. THE PROPOSED ALGORITHM:

Many authors in the literatures have tried to reproduce pseudo-spectrum in Equation (13) in order to improve the resolution algorithm MUSIC [10], [14], [15]. However, most of the proposed solutions require an exhaustive calculation and searching which is wasteful and inefficient when it comes to the real time implementation. In this section, we propose a new algorithm which improves the resolution with less computational cost. This algorithm combines a high resolution method MUSIC and spatial subsampling; we summarize the operational process in following steps:

- Step1: Apply first rough estimation using MDL algorithm, in order to detect first DOA sources.
- Step2: Determine an interval around each angle detected where it should be another close source, the interval can be the wide of main lobe of radiation pattern of antenna network.
- Step 3: apply a spatial subsampling (interleaving method) wich lead to L subarrays.
- Step 4: Applying MUSIC algorithm on each subarray and searching for the largest peaks in order to detect the DOA.
- Step 5: On each interval we choose subarray given a maximum of sources, and then we calculate the mean of angles to obtain final angle sources.
- Step 6: computing the final DOA, after sorting and calculate the average from each interval and selected subarray presenting the maximum peaks

## 5. RESULTS OF SIMULATION

In this section, some numerical results are presented to analyze and compare the estimation of behavior of the new proposed algorithm. A Uniform Linear Array (ULA) with 12 inter sensor spacing of half-length wavelength is employed. Assume that there are two closely spaced uncorrelated narrowband signal sources with the same wavelength $\lambda$. Simulation result were obtained based on 100 snapshot and 100 Monte Carlo simulation runs. The method proposed in this paper is denoted SS-MUSIC (spatial sampling MUSIC).

Figure 3 shows an array of 12 sensors and three sub networks (L=3) with an SNR=7dB. The figure 3 illustrates the detection right rate versus the angle difference of our proposed algorithm and the standard MUSIC alone. We can see that for low angular separation $\delta\theta$ standard MUSIC cannot detect the both sources, while the proposed algorithm detects low degree angular separation. In this scenario, Standard MUSIC detects an angular separation of $2.5^0$ , when our method SS-MUSIC (combining MUSIC and spatial sampling) detects the angular separation at $0.5^0$. Thus, the proposed approach can improve the angular resolution.

In Figure 4 we plot detection probability rate versus SNR for $\delta\theta = 2^0$ . We can see a considerable improvement of detection probability for low SNR when our algorithm is used compared with MUSIC standard. For example, at SNR=10, the probability detection for MUSIC standard is less than 10%, on the other side, it’s more than 65% when we use SS-MUSIC.

In figure 5, we illustrate the mean square error (MSE) for directions of arrivals when $\delta\theta = 6^0$ , it indicates that SS-MUSIC follows the same principle as MUSIC. Meanwhile, it indicates that the estimation angle is better for weak SNR, in the other hand, for higher SNR the new method doesn’t give any better improvement.

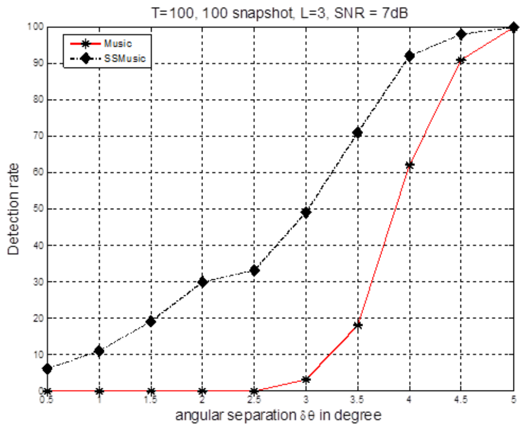


Figure 3. Angular resolution versus detection rate

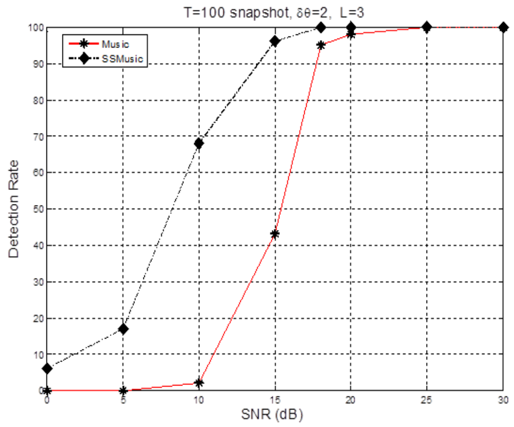


Figure 4. SNR versus detection rate

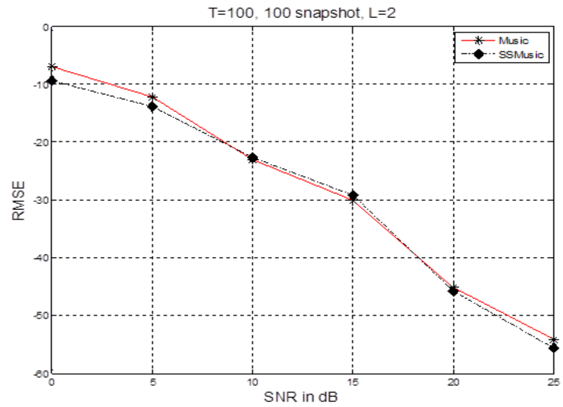


Figure 5. SNR versus RMSE

## 6. CONCLUSION

Estimation closed space source number problem can be met in many fields as radar, sonar and communication. In this paper, a new technique combining MUSIC and spatial sampling approach is being demonstrated with simulated cases to outperform the conventional MUSIC method in separating closely spaced sources. We compared our new technique to standard MUSIC. Under the assumption of number of sensors must be larger than the number of sources, the proposed algorithm improves the detection of two closely sources at low SNR and improves the resolution with less computational cost, that's why; it fits with real time implementation. However, the SS-MUSIC improves the resolution but it doesn't improve the estimation accuracy.

## REFERENCES

[1] T. Bao, Mohammed El Korso, H. H. Ouslimani, "Cramér–Rao Bound And Statistical Resolution Limit Investigation For Near-Field Source Localization", *Elsevier Digital Signal Processing*, (2015), pp12-17.

[2] S. Marcos, "Les méthodes à Haute Résolution, traitement d'antennes et analyse spectrale", (Éditions Hermès 1998) 2nd Ed.

[3] N. Nemri, H.W. Badri, R.Ghayoula, H. Trabelsi, A.Gharsallah, "Synthesis and Implementation (In STM8S) of Phased Circular Antenna Arrays Using Taguchi Method", *International Journal of Electrical and Computer Engineering (IJECE), June* 2016, DOI: 10.11591/ijece.v6i3.9193

[4] R. Chavannes, "Application des Méthodes d'égalisation au Radar Transhorizon Nostradamus", PhD thesis Telecom Paris, 2011.

[5] A. Zeeshan, A. Iftikhar, "Three Decades of Development in DOA Estimation Technology", *TELKOMNIKA Indonesian Journal of Electrical Engineering*, Vol 12 No 8, 2014, pages 6297-6312.

[6] S.K. Bodhe, B.G. Hogade, Shailesh D. Nandgaonkar, "Beamforming Techniques for Smart Antenna using Rectangular Array Structure", *International Journal of Electrical and Computer Engineering (IJECE)*, Vol. 4, No. 2, April 2014, pp. 257~264.

[7] T. D. Wang, L. Gan, P. Wei, H. S. Liao, "DOA Estimation of Coherently Distributed Sources based on block Spares Constraint with Measurement Matrix Uncertainty", *IEICE Electron. Express* **10** (2013) 20120863.

[8] H. Akaike, "A new look at the statistical model identification", *IEEE Tran, on aut control* AC-19. **6**, (1974).

[9] W. Jiang, S. Pennock, and P. Shepherd, "A Novel W-MUSIC Algorithm for GPR Target Detection in Noisy and Distorted Signals", *IEEE Radar Conference*. (2009) IEEE, pp. 1-6.

[10] Q. Yuan, Q. Chen, K. Sawaya., "Experimental Study on MUSIC Based DOA Estimation by Using Universal Steering Vector", *IEICE Trans. Commun.* Vol.E91-B No.5 pp.1575-1580, May2008.

[11] Kusay F, "A New improved-Music Algorithm for High resolution direction of arrival", *Journal of Theoretical and Applied Information Technology*, 2015, Vol 72 N°1.

[12] JC. Pascal, "*Formation de voie*", PhD thesis Telecom Paris, (2009).

[13] D. Ruiyan, J. Wang, F. Liu, B. Wang, "A Robust Algorithm for Source Number Detection based on s Space Time Averaging Techniques ", *ICP* (2010).

[14] F. Liu, D. Ruiyan, J. Wang, "A Robust Source Number Detection method via Rotational Invariance", *ICSP* (2008), pp. 124-130.

[15] Elhafiz A. B.Yagoup, Z. Liu, Y. Xu, "DOA Estimation by FourthOrder Cumulants without Source Enu meration and Eigendecomposition", *TELKOMNIKA Indonesian Journal of Electrical Engineering*, Vol 12 No 6, 2014, pages 4237-4242.