Quasinormal-mode perturbation theory for dissipative and dispersive optomechanics



André G. Primo*, Natália C. Carvalho, Cauê M. Kersul, Newton C. Frateschi, Gustavo S. Wiederhecker, Thiago P. M. Alegre**

*agprimo@ifi.unicamp.br
**alegre@unicamp.br





Available files

- Ring resonator coupled to absorber medium (Comsol, Ο
- Split-beam nanocavity (Comsol, Python) Ο
- Nanoparticle-on-a-mirror (Comsol, Mathematica, Pyth Ο
- Infinite nanocylinder (Comsol, Mathematica, Python) Ο
- Nanoparticle-on-a-mirror Dielectric nanosphere (Comsol, Python) Ο

Python)	Comsol version 5.4 is required. The accompanying
	simulation files were not tested on other versions. Details
	in the structure of the simulations can be found in the
non)	repository for: "Brillouin Optomechanics in Nanophotonic
	Structures" - DOI: 10.1063/1.5088169.







Ring resonator coupled to absorber medium

File "comsol_simulation_file_fig_2.mph" calculates optical and mechanical modes of a GaAs ring resonator coupled to a heavily doped GaAs ring. Implementations of the normalmode and quasi-normal mode perturbation theory are found in the simulations.



Python:

1. Open file "fig_2_plot_data.ipynb". Load "Fig_2_MB_mod.txt" and "Fig_2_PE_mod.txt" to generate plots.

Generating data from Comsol:

- 4.

All variables with subscript "c" are defined to compute results using the normal-mode perturbation theory.

Moving boundary or Photoelastic calculations are chosen by setting the variable *mmesh = 0* or *mmesh* = 1 (*Global definitions -> Parameters 1*), respectively.

If *mmesh = 0*, probes gOMB (var1) and gOMB_c(var2) must be set to compute gOMB and gOMB_c, respectively.

If *mmesh = 1*, probes gOMB (var1) and gOMB_c(var2) must be set to compute gOPE and gOPE_c, respectively.

mponent 1 <i>(comp1)</i> Definitions ^{a=} basic - global ^{a=} basic - optical	Label: g0MB Variable name: var1		
a= basic - mechanical	 Expression 	🕂 💌 🎽 🕶	gOMB
a= MB - mechanical	Expression:		gOPE
a= MB - mixed	g0MB		gOMB_c
a= PE - mechanical	Table and plot unit:		ØOPF c
a= PE - mixed			
a= Total - mixed	1/s	¥	
🔎 gap (var7)			
🔎 freq (<i>var</i> 6)			
🔎 g0MB (var1)			

Run the simulation on the mechanics node. This one is set to solve for the mechanical breathing mode of the disk.

Run the simulation on the optics node. This one will sweep over the gap distance between the intrinsic and doped GaAs rings. For each gap, the optical modes for the deformed and undeformed geometries are computed. This follows from the usage of the Moving Mesh module. Data is generated automatically in a table.

6. Exported data can be found in the "fig_2_MB_mod.txt" and "fig_2_PE_mod.txt" files.







^{🔺 🕋} Con **A** =

Split-beam nanocavity

File "comsol_simulation_file_fig_3.mph" calculates optical and mechanical modes of a split-beam nanocavity. Implementations of quasi-normal mode perturbation theory are found in the simulations.



Ge	enera
1.	Set th
	comsol_sim Global I Compor Study 1
	Para
	im Ste im Ste ▷ în Solv ▷ 🛃 Job
2. 3. 4.	Run S Data Expor
	Pytho

1.

ting data from Comsol:

he desired z-gap in the *Parametric Sweep* node, under *Study 1:*

nulation_file_fig_3 (1).mph <i>(root)</i> Definitions nent 1 <i>(comp1)</i>	Settings Parametric Sweep = Compute	↓]
ametric Sweep ep 1: eigen_opt ep 2: eigen_mech ver Configurations	Label: Parametric Sweep Study Settings	
Configurations	Sweep type: Specified combinations Parameter name Parameter value list P	▼ eter unit

Study 1.

- is automatically generated in a table.
- rted data can be found in "data_fig3.csv".

on:

Open file "fig_3_plot_data.ipynb". Load "data_fig3.csv" to generate plots.







Nanoparticle-on-a-mirror

File "comsol_simulation_file_fig_4.mph" and "comsol_simulation_file_fig_4_no_PML.mph" calculate plasmonic modes of a gold nanoparticle placed on top of a gold mirror – in the presence and absence of PMLs, respectively. Gold is modeled through a Drude-Lorentz-type permittivity. Implementations of quasi-normal mode perturbation theory are found in the simulations, assuming the vibrational modes in question are of molecules treated as point-dipoles.



Python:

1. Figure S2 c) and d) are generated from the same Comsol file, although the radius of the gold nanosphere is different than the one used on the main text. The single difference being that a sweep over the gap between nanoparticle and PML is performed. This is found in file "comsol_simulation_file_S2_c_d.mph".

Output data ("fig_S2_c_d_data.txt") is loaded in 2.

"fig_S2_plot_data.ipynb"

Run Study 1. 🔺 \infty Study 1 🔺 💻 Results Data Sets 🕨 🎶 Views Derived Values 3. 🔺 \infty Study 1 🔰 Param Step Ster Step 🔺 🐂 Solver 🔺 📑 So Þ 🖬 þцv;w Þ 📊

Mathematica:





Under the *Results -> Derived values* node, compute *Global Evaluation 1*. The output is the value for the generalized optomechanical coupling and the Q-factor of the cavity. Those are used as input for the Mathematica file used to generate Figs. 4 c) and d2).

Step 1: Eigenfrequency (p) Step 2: Eigenfrequency (s) Solver Configurations

(8) Global Evaluation 1

Be sure that *transform point* is on and set to *freq._guess. Transform point* can be found in *Solver* configurations -> Eigenvalue Solver 1 (or 2).

	 Values of I 	Linearization Point	
etric Sweep			
netric Sweep 2	Prescribed by:	Solution 🗸	
o 1: Stationary			
o 2: Eigenfrequency (p)	Solution:	Solution 6 (sol6)	
o 3: Eigenfrequency (s)	Use:	Solution Store 1 (sol7)	
Configurations			
lution 6 (sol6)	Selection:	Automatic 🗸 🗸	
Solution Store 1 (sol7)	Store linearization point and deviation in output		
Compile Equations: Eigenfrequency (p)	Value of eigenvalue linearization point		
Dependent Variables 2	Transform point		
Eigenvalue Solver 1		om	
Solution Store 2 (sol8)	Point:	freq_guess	
Compile Equations: Eigenfrequency (s)			
Dependent Variables 3			
Eigenvalue Solver 2			

Results from the Comsol simulation are inputs for variables f0, $g\kappa$ and $g\omega$, kabs, and $g\kappa$ abs. The force spectrum and damping rate modifications are calculated from these results.





Infinite nanocylinder

This model supports both analytic and numerical solutions. The analytic solution is found in "fig_S2_a_b_mathematica.nb" or "fig_S1_b_mathematica.nb", whereas the numerical solution is found in "comsol_simulation_file_S2_a_b.mph". Both compute optical and mechanical modes of the structure. In this exemple, only moving boundary contributions are considered in the optomechanical coupling.



Generating data from Comsol:

- structure.

Mathematica:

Python:

- 2.

Run the *Optomechanics* node. This is set to calculate optical and mechanical modes of the

2. A sweep over the gap distances between cylinder and PML will be performed. The output is automatically generated in a table.

1. Analytic results for the optical and mechanical modes are available. In

"fig_S2_a_b_mathematica.nb", the exact and perturbation theory moving boundary optomechanical couplings are evaluated. This data is then used as benchmark for our Comsol simulations.

2. In "fig_S1_b_mathematica.nb" we use the same calculations to analyse the convergence of the perturbation series. For that purpose, we gradually increase the boundary deformations on the cylinder and compare exact and perturbation theory predictions.

1. Using "fig_S2_plot_data.ipynb" we import the Comsol data found in "fig_S2_a_b_data.txt". Results from the analytic calculations were already incorporated in the .ipynb file. Run the code to generate plots.





NPoM – Dielectric nanosphere

File "comsol_simulation_file_figS3_sphere" and "comsol_simulation_file_figS3_sphere_no_PML.mph" follow the same implementantions used in Fig. 4. A dielectric subnanometer sphere is placed at the center of the gap between the sphere and mirror. Its radial mechanical breathing mode is computed and used as input for Moving-Mesh calculations. A collection cone is defined in the geometry builder and is used to compute radiative decay rates in its inside and outside.



Generating data from Comsol:

- 2.
- 3.
- 4.
- 5.

Python:

Under Global Definitions -> Parameters 1 -> NA: set the desired numerical aperture desired. Note that for the purposes of the simulation absent of PML implementation, the numerical aperture plays no role and thus this step may be ignored.

Run *Study 2.* This will compute the mechanical mode used as input for the Moving-Mesh module. Run Study 1. A table will be automatically generated and should be exported.

Exported data can be found in "sphere_FigS3_NA_XX.txt" or "sphere_FigS3_noPML.txt", depending on which of the Comsol files were run. Here, XX stands for the chosen numerical aperture. Be sure that *transform point* is on and set to *freq._guess. Transform point* can be found in *Solver* configurations -> Eigenvalue Solver 1 (or 2).

Files generated in Comsol are loaded on file "fig_S3_plot_data.ipynb" and plots are generated.





