Neutrosophic Sets and Systems, Vol. 36, 2020
University of New Mexico

Fuzzy Neutrosophic Strongly Alpha Generalized Closed Sets in Fuzzy Neutrosophic Topological spaces

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Abstract: In this paper, we will define a new set called fuzzy neutrosophic strongly alpha generalized closed set, so we will prove some theorems related to this concept. After that, we will give some interesting properties were investigated and referred to some results related to the new definitions by theorems, propositions to get some relationships among fuzzy neutrosophic strongly alpha generalized closed sets, fuzzy neutrosophic closed sets, fuzzy neutrosophic regular closed sets, fuzzy neutrosophic alpha closed sets, fuzzy neutrosophic alpha generalized closed sets and fuzzy neutrosophic pre closed sets which are compared with necessary examples based of fuzzy neutrosophic topological spaces .

Keywords: Fuzzy neutrosophic set, fuzzy neutrosophic topological space, fuzzy neutrosophic strongly alpha generalized closed set.

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1. Introduction

The concept of fuzzy set "FS"was introduced by Lotfi Zadeh in 1965 [1], then Chang depended the fuzzy set to introduce the concept of fuzzy topological space "FTS" in 1968 [7]. After that the concept of fuzzy set was developed into the concept of intutionistic fuzzy set "IFS" by Atanassov in 1983 [4-6], the intutionistic fuzzy set gives a degree of membership and a degree of non- membership functions. Cokor in 1997 [7] relied on intutionistic fuzzy set to introduced the concept of intutionistic fuzzy topological space."IFTS". In 2005 Smaradache [23] study the concept of neutrosophic set. "NS". After that and as developed the term of neutrosophic set, Salama has studied neutrosophic topological space "NTS"and many of its applications [18-21]. In 2013 Arockiarani Sumathi and Martina Jency [2] introduced the concept of fuzzy neutrosophic set as generalizes the concept of fuzzy set and intutionistic fuzzy set. where each element had three associated defining functions on the universe of discourse X, namely the membership function (T), indeterminacy function (I), the non-membership function (F) that is added an indeterminacy degree between the degree of membership and the degree of non- membership. In 2012 Salama and Alblowi defined fuzzy neutrosophic topological space [18].

In the present work, we will generalized the concept of strongly alpha generalized closed set in fuzzy neutrosophic topological spaces which was studied by Santhi and Sakthivel in 2011 [22] via intuitinistic topological spaces and generalizing our works in 2018 [9,10], the new set will called fuzzy neutrosophic strongly alpha generalized closed set in fuzzy neutrosophic topological spaces.

Finally, there are many application of neutrosophic sets in many fields so we can enhance our work, we will try in the future to applied this work in different fields such as many authors applications see [11] and [13-17]

2. Preliminaries:

In this section, we will define some basic definitions and some operations which are useful in our present study.

Definition 2.1 [18]: Let X be a non-empty fixed set. The fuzzy neutrosophic set (FNS, for short), η_N is an object having the form $\eta_N = \{< x, \mu_{\eta N}(x), \sigma_{\eta N}(x), \nu_{\eta N}(x) >: x \in X\}$ where the functions $\mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}: X \rightarrow [0, 1]$ denote the degree of membership function (namely $\mu_{\eta N}(x)$), the degree of indeterminacy function (namely $\sigma_{\eta N}(x)$) and the degree of non-membership (namely $\nu_{\eta N}(x)$) respectively of each element $x \in X$ to the set η_N and $0 \le \mu_{\eta N}(x) + \sigma_{\eta N}(x) + \nu_{\eta N}(x) \le 3$, for each $x \in X$.

Remark 2.2 [18]: FNS $\eta_N = \{< x, \mu_{\eta N}(x), \sigma_{\eta N}(x), \nu_{\eta N}(x) >: x \in X\}$ can be identified to an ordered triple $< x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N} >$ in [0, 1] on X.

Definition 2.3 [18]: Let X be a non-empty set and the FNSs η_N and γ_N be in the form:
$\eta_N = \{< x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N} >: x \in X\}$ and $\gamma_N = \{< x, \mu_{\gamma N}, \sigma_{\gamma N}, \nu_{\gamma N} >: x \in X\}$ on X then:

i. $\eta_N \subseteq \gamma_N$ iff $\mu_{\eta N} \le \mu_{\gamma N}$, $\sigma_{\eta N} \le \sigma_{\gamma N}$ and $\nu_{\eta N} \ge \nu_{\gamma N}$.

ii. $\eta_N = \gamma_N$ iff $\eta_N \subseteq \gamma_N$ and $\gamma_N \subseteq \eta_N$,

iii. $1_N - \eta_N = \{< x, \nu_{\eta N}, 1 - \sigma_{\eta N}, \mu_{\eta N} >: x \in X\}$,

iv. $\eta_N \cup \gamma_N = \{< x, \mathrm{Max}(\mu_{\eta N}, \mu_{\gamma N}), \mathrm{Max}(\sigma_{\eta N}, \sigma_{\gamma N}), \mathrm{Min}(\nu_{\eta N}, \nu_{\gamma N}) >: x \in X\}$,

v. $\eta_N \cap \gamma_N = \{< x, \mathrm{Min}(\mu_{\eta N}, \mu_{\gamma N}), \mathrm{Min}(\sigma_{\eta N}, \sigma_{\gamma N}), \mathrm{Max}(\nu_{\eta N}, \nu_{\gamma N}) >: x \in X\}$,

vi. $0_N = < x, 0, 0, 1>$ and $1_N = <x, 1, 1, 0 >$.

Definition 2.4 [18]: "Fuzzy neutrosophic topology (FNT, for short) on a non-empty set X is a family τ_N of fuzzy neutrosophic subsets in X satisfying the following axioms.

i. $0_N, 1_N \in \tau_N$,

ii. $\eta_{N1} \cap \eta_{N2} \in \tau_N$ for any $\eta_{N1}, \eta_{N2} \in \tau_N$,

iii. $\cup \eta_{Ni} \in \tau_N$, $\forall \{\eta_{Ni}: i \in J\} \subseteq \tau_N$."

In this case the pair (X, τ_N) is called fuzzy neutrosophic topological space (FNTS, for short). The elements of τ_N are called fuzzy neutrosophic open set (FNOS, for short). The complement of FNOS in the FNTS (X, τ_N) is called fuzzy neutrosophic closed set (FNCS, for short).

Definition 2.5 [18]: Let (X, τ_N) be FNTS and $\eta_N = < x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N} >$ be FNS in X. Then the fuzzy neutrosophic closure of η_N (FNCL, for short) and fuzzy neutrosophic interior of η_N (FNIn, for short) are defined by:

$$FNCL(\eta_N) = \cap \{C_N : C_N \text{ is FNCS in X and } \eta_N \subseteq C_N \},$$
$$FNIn(\eta_N) = \cup \{O_N : O_N \text{ is FNOS in X and } O_N \subseteq \eta_N \}.$$

We know, FNCL(η_N) is FNCS and FNIn (η_N) is FNOS in X. Further,

i. η_N is FNCS in X iff FNCL (η_N) = η_N,

ii. η_N is FNOS in X iff FNIn (η_N) = η_N.

Proposition 2.6 [25]: Let (X, τ_N) is FNTS and η_N, γ_N are FNSs in X. Then the following properties hold:

i. FNIn (η_N) $\subseteq \eta_N$ and $\eta_N \subseteq$ FNCL(η_N),

ii. $\eta_N \subseteq \gamma_N \Longrightarrow$ FNIn (η_N) $\subseteq$ FNIn (γ_N) and $\eta_N \subseteq \gamma_N \Longrightarrow$ FNCL(η_N) $\subseteq$ FNCL(γ_N),

iii. FNIn (FNIn (η_N)) = FNIn (η_N) and FNCL(FNCL(η_N)) = FNCL(η_N),

iv. FNIn ($\eta_N \cap \gamma_N$) = FNIn (η_N) $\cap$ FNIn (γ_N) and FNCL($\eta_N \cup \gamma_N$) = FNCL(η_N) $\cup$FNCL(γ_N),

v. FNIn (1_N) =1_N and FNCL(1_N) = 1_N,

vi. FNIn (0_N) = 0_N and FNCL(0_N) = 0_N.

Definition 2.7 [9]: FNS η_N in FNTS (X, τ_N) is called:

i. Fuzzy neutrosophic regular closed set (FNRCS, for short) if η_N = FNCL(FNIn (η_N)).

ii. Fuzzy neutrosophic pre closed set (FNPCS, for short) if FNCL(FNIn (η_N)) $\subseteq \eta_N$.

iii. Fuzzy neutrosophic α closed set (FNαCS, for short) if FNCL(FNIn(FNCL(η_N))) $\subseteq \eta_N$.

Definition 2.8 [10]: Let (X, τ_N) be FNTS and $\eta_N = < x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N} >$ be FNS in X. Then the fuzzy neutrosophic alpha closure of η_N (FNαCL, for short) and fuzzy neutrosophic alpha interior of η_N (FNαIn,

for short) are defined by:

$$FN\alpha CL(\eta_N) = \cap \{C_N : C_N \text{ is FN}\alpha\text{CS in X and } \eta_N \subseteq C_N \},$$
$$FN\alpha In(\eta_N) = \cup \{O_N : O_N \text{ is FN}\alpha\text{OS in X and } O_N \subseteq \eta_N\}.$$

We know, FNαCL(η_N) is FNαCLS and FNαIn (η_N) is FNαOS in X. Further,

i. η_N is FNαCS in X iff FNαCL(η_N) = η_N,

ii. η_N is FNαOS in X iff FNαIn (η_N) = η_N.

Definition 2.9 [9,10]: Fuzzy neutrosophic sub set η_N of FNTS (X, τ_N) is called:

i. fuzzy neutrosophic generalized closed set (FNGCS, for short) if FNCL(η_N) $\subseteq U_N$ wherever, $\eta_N \subseteq U_N$ and U_N is FNOS in X. And η_N is said to be fuzzy neutrosophic generalized open set (FNGOS, for short) if the complement 1_N- η_N is FNGCS set in (X, τ_N).

ii. fuzzy neutrosophic alpha generalized closed set (FNαGCS, for short) if FNαCL(η_N) $\subseteq U_N$ wherever, $\eta_N \subseteq U_N$ and U_N is FNOS in X. And η_N is said to be fuzzy neutrosophic alpha generalized open set (FNαGOS, for short) if the complement 1_N- η_N is FNαGCS set in (X, τ_N).

3. Fuzzy Neutrosophic Strongly Alpha Generalized Closed Sets in Fuzzy Neutrosophic Topological Spaces.

Now, we will introduce the concept of fuzzy neutrosophic strongly alpha generalized closed set in fuzzy neutrosophic topological spaces.

Definition 3.1: Fuzzy neutrosophic subset η_N of FNTS (X, τ_N) is called fuzzy neutrosophic strongly alpha generalized closed set (FNSαGCS, for short) if FNαCL (η_N) $\subseteq U_N$ wherever, $\eta_N \subseteq U_N$ and U_N is FNGOS in X.

Example 3.2: Let X= {a, b} define FNS η_N in X as follows:
η_N =<x, (0.2$_{(a)}$, 0.3$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.8$_{(a)}$,0.7$_{(b)}$) >, where the family τ_N ={0_N, 1_N, η_N }.
If we take, ψ_N = <x, (0.8$_{(a)}$, 0.7$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.1$_{(a)}$,0$_{(b)}$) >.

And, $U_N = 1_N$ where U_N is FNGOS such that, $\psi_N \subseteq U_N$. Then, FNαCL(ψ_N) = 1_N. So, FNαCL(ψ_N) $\subseteq$ U_N.
Hence, ψ_N is FNSαGCS.

Theorem 3.3: For any FNSs, the following statements are true in general:
i. Every FNOS is FNGOS.
ii. Every FNCS is FNαCS.
iii. Every FNCS is FNSαGCS.
iv. Every FNRCS is FNSαGCS.
v. Every FNαCS is FNSαGCS.
vi. Every FNαGCS is FNSαGCS.
vii. Every FNRCS is FNCS.
viii. Every FNαCS is FNαGCS.

Proof:.
i. Let η_N = <x, $\mu_{\eta N}$, $\sigma_{\eta N}$, $\nu_{\eta N}$ > be FNOS in the FNTS (X,τ_N).
Then **by Definition 2.5 ii** we get, FNIn (η_N) = η_N.

Now, let U_N is FNCS such that, $U_N \subseteq \eta_N$. Therefore, FNIn (η_N) = $\eta_N \supseteq U_N$.

Hence, η_N is FNGOS in (X,τ_N) .

ii. Let η_N = <x, $\mu_{\eta N}$, $\sigma_{\eta N}$, $\nu_{\eta N}$ > be FNCLS in the FNTS (X, τ_N).
Then by **Definition 2.5 (i)** we get, FNC(η_N) = η_N……(1).
And **by Proposition 2.6 i** we get, FNIn (η_N) $\subseteq \eta_N$

So **by 1** we get, FNIn (FNCl(η_N)) $\subseteq \eta_N$
This implies FNC(FNIn (FNCL(η_N))) $\subseteq$ FNCL(η_N).
So **by (1)** we get, FNCL(FNIn (FNCL(η_N))) $\subseteq \eta_N$.
Hence, η_N is FNαCS in (X, τ_N).

iii. Let $\eta_N = <x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}>$ be FNCS in FNTS (X,τ_N).

Then by **Definition 2.5 (i)** we get, FNCl(η_N) = η_N. Now, let U_N be FNGOS such that, $\eta_N \subseteq U_N$.
Since, FNαCL(η_N) $\subseteq$FNCL(η_N) **by Definition 2.5 and Definition 2.8.**
So we get, FNαCL(η_N) $\subseteq$ FNCL(η_N) = $\eta_N \subseteq U_N$.
Hence, η_N is FNSαGCS in (X,τ_N) .

iv. Let $\eta_N = <x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}>$ be FNRCS in the FNTS (X, τ_N).
Then, FNCL(FNIn (η_N)) = η_N…….(1).
This implies, FNCL(FNIn (η_N)) = FNCL(η_N)…….(2).

Now, let U_N be FNGOS such that, $\eta_N \subseteq U_N$.

From **(1) and (2)** we get, FNCL(η_N) = η_N.
That η_N is FNCS in X.
So **by iii** we get, FNαCL(η_N) $\subseteq$ FNCL(η_N) = $\eta_N \subseteq U_N$.
Hence, η_N is FNSαGCS in (X,τ_N) .
v. Let $\eta_N = <x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}>$ be FNαCLOS in the FNTS (X, τ_N).
Then **by Definition 2.8 i** we get, FNαCL (η_N) = η_N.

Now, let U_N be FNGOS such that, $\eta_N \subseteq U_N$. So, FNαCL (η_N) = $\eta_N \subseteq U_N$.

Hence, η_N is FNSαGCS in (X, τ_N).
vi. Let $\eta_N = <x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}>$ be FNαGCS in the FNTS (X, τ_N).
Then, FNαCL (η_N) $\subseteq U_N$, $\eta_N \subseteq U_N$ and U_N be FNOS, so **by i** we get , FNOS be FNGOS in (X, τ_N).
Therefore, FNαCL (η_N) $\subseteq U_N$, $\eta_N \subseteq U_N$ and U_N be FNGOS. Hence, η_N is FNSαGCS in (X, τ_N).

vii. Let $\eta_N = <x, \mu_{\eta N}, \sigma_{\eta N}, \nu_{\eta N}>$ be FNαCS in the FNTS (X, τ_N). Then, FNαCL (η_N) = η_N.

Now, let U_N be FNOS such that, $\eta_N \subseteq U_N$, so, FNαCL (η_N) = $\eta_N \subseteq U_N$.

Hence, η_N is FNαGCS in (X, τ_N).

Remark 3.4: The convers of **Theorem 3.3** is not true and this can be clarified in the following examples.
Example 3.5:

i. Let X= {a, b} define FNS η_N in X as follows:

η_N = <x, (0.5(a), 0.7(b)), (0.5(a), 0.5(b)), (0.5(a),0.2(b)) >.

The family τ_N = {0_N, 1_N, η_N } be FNT.

If we take, ψ_N = <x, (0.1(a), 0.6(b)), (0.5(a), 0.5(b)), (0.9(a),0.3(b)) >.

And let, U_N = 0_N, where U_N be FNCS such that, $U_N \subseteq \psi_N$.

Then, FNIn (ψ_N) = <x, (0(a), 0(b)), (0(a), 0(b)), (1(a),1(b)) > $\subseteq$ <x, (0.1(a), 0.6(b)), (0.5(a), 0.5(b)),

(0.9(a),0.3(b)) > such that, (0(a), 0(b)) $\leq$ (0.1(a), 0.6(b)), (0(a), 0(b)) $\leq$ (0.5(a), 0.5(b)) and (1(a),1(b))

$\geq$ (0.9(a),0.3(b)) = 0_N. So, FNIn (ψ_N) $\supseteq$ U_N. Hence, ψ_N is FNGOS but, not FNOS.

Since $\psi_N \notin \tau_N$.

ii. Let X={a } define the FNSs η_N and γ_N in X as follows:

η_N = <x, (0.5(a)), (0.4(a)), (0.7(a)) >, γ_N =<x, (0.4(a)), (0.1(a)), (0.8(a)) >.

The family τ_N ={0_N, 1_N, η_N, γ_N } be FNT.

If we take, ψ_N = <x, (0.8(a)), (0.6(a)), (0.5(a)) >.

Then, FNCL(ψ_N) = <x, (0.8(a)), (0.9(a)), (0.4(a)) >. And, FNIn (FNCL(ψ_N)) = <x, (0.5(a)), (0.4(a)), (0.7(a)) >. So, FNCL(FNIn (FNCL(ψ_N))) = <x, (0.7(a)), (0.6(a)), (0.5(a)) >.

Therefore, <x, (0.7(a)), (0.6(a)), (0.5(a)) > $\subseteq$ ψ_N .

Hence, ψ_N is FNαCS but not FNCS. Since $\psi_N \notin 1_N$-τ_N

iii. Take **Example 3.2.** Then, ψ_N is FNSαGCS but, not FNCS.

Since, $\psi_N \notin 1_N$- τ_N.

iv. Take **Example 3.2**. Then ψ_N is FNSαGCS but, not FNRCS.

Since, FNIn (ψ_N) = <x, (0.2(a), 0.3(b)), (0.5(a), 0.5(b)), (0.8(a),0.7(b)) > and

FNCL(FNIn (ψ_N)) = <x, (0.8(a), 0.7(b)), (0.5(a), 0.5(b)), (0.2(a),0.3(b)) > $\neq$ ψ_N.

v. Let X={a, b } define the FNSs η_N and γ_N in X as follows:

η_N = <x, (0.4(a), 0.2(b)), (0.5(a), 0.5(b)), (0.6(a),0.7(b)) >,

γ_N =<x, (0.8(a), 0.8(b)), (0.5(a), 0.5(b)), (0.2(a),0.2(b)) >.

The family τ_N ={0_N, 1_N, η_N, γ_N } be FNT.

Now if, ψ_N = <x, (0.6(a), 0.7(b)), (0.5(a), 0.5(b)), (0.4(a),0.3(b)) >.

By **Theorem 3.3** **i.** If U_N is FNOS then is FNGOS.

So, U_N = γ_N where, U_N be FNGOS such that, $\psi_N \subseteq U_N$.

By **Theorem 3.3** **ii**. Every FNCS is FNαCS.

Then, $FN\alpha CL(\psi_N) = 1_N - \eta_N$. Therefore $FN\alpha CL(\psi_N) \subseteq U_N$.

Hence, ψ_N is FNSαGCS but, not FNαCS.

Since, $FNCL(\psi_N) = 1_N - \eta_N$, $FNIn(FNCL(\psi_N)) = \eta_N$ and

$FNCL(FNIn(FNCL(\psi_N))) = 1_N - \eta_N \nsubseteq \psi_N$.

vi. Let X={a} define the FNSs η_N and γ_N in X as follows:

$\eta_N = \langle x, (0.5_{(a)}), (0.5_{(a)}), (0.5_{(a)}) \rangle$, $\gamma_N = \langle x, (0.5_{(a)}), (0_{(a)}), (1_{(a)}) \rangle$.

The family $\tau_N = \{0_N, 1_N, \eta_N, \gamma_N\}$ be FNT.

Now if, $\psi_N = \langle x, (0.6_{(a)}), (0.6_{(a)}), (0.6_{(a)}) \rangle$.

Let $U_N = \langle x, (1_{(a)}), (1_{(a)}), (0.4_{(a)}) \rangle$ be FNGOS such that, $\psi_N \subseteq U_N$.

Then, $FN\alpha Cl(\psi_N) = 1_N - \gamma_N$. So $FN\alpha Cl(\psi_N) \subseteq U_N$.

Hence, ψ_N is FNSαGCS but, not FNαGCS.

Since, U_N is FNGOS but not FNOS.

vii. Let X={a } define the FNSs η_N and γ_N in X as follows:

$\eta_N = \langle x, (0.5_{(a)}), (0.5_{(a)}), (0.7_{(a)}) \rangle$, $\gamma_N = \langle x, (0.4_{(a)}), (0_{(a)}), (1_{(a)}) \rangle$.

The family $\tau_N = \{0_N, 1_N, \eta_N, \gamma_N\}$ be FNT.

Now if, $\psi_N = \langle x, (1_{(a)}), (1_{(a)}), (0.4_{(a)}) \rangle$.

Then, ψ_N is FNCS. Since $\psi_N \in 1_N - \tau_N$ but, not FNRCS.

Since $FNIn(\psi_N) = \langle x, (0.5_{(a)}), (0.5_{(a)}), (0.7_{(a)}) \rangle$ and

$FNCL(FNIn(\psi_N)) = \langle x, (0.7_{(a)}), (0.5_{(a)}), (0.5_{(a)}) \rangle \neq \psi_N$.

viii. Let X={a} define the FNSs η_N and γ_N in X as follows:

$\eta_N = \langle x, (0.5_{(a)}), (0.5_{(a)}), (0.6_{(a)}) \rangle$, $\gamma_N = \langle x, (0.5_{(a)}), (0_{(a)}), (1_{(a)}) \rangle$.

The family $\tau_N = \{0_N, 1_N, \eta_N, \gamma_N\}$ be FNT.

Now if, $\psi_N = \langle x, (0.6_{(a)}), (0.6_{(a)}), (0.6_{(a)}) \rangle$.

Let, $U_N = 1_N$ be FNOS such that, $\psi_N \subseteq U_N$.

Then, $FNCL(\psi_N) = \langle x, (1_{(a)}), (1_{(a)}), (0.5_{(a)}) \rangle$ and $FNCL(\psi_N) \subseteq U_N$.

Hence, ψ_N is FNαGCS but, not FNαCS.

Since, $FNCL(\psi_N) = \langle x, (1_{(a)}), (1_{(a)}), (0.5_{(a)}) \rangle$, $FNIn(FNCL(\psi_N)) = \langle x, (0.5_{(a)}), (0.5_{(a)}), (0.6_{(a)}) \rangle$ and

$FNCL(FNIn(FNCL(\psi_N))) = \langle x, (0.6_{(a)}), (0.5_{(a)}), (0.5_{(a)}) \rangle \nsubseteq \psi_N$.

Remark 3.6: i. The relation between FNPCS and FNSαGCS is independent and this can be clarified in the next example.

ii. The intersection of two FNSαGCS is not FNSαGCS in general and we explained it in the next example.

Example 3.7:

i. (1) Let X= {a, b} define FNS η_N in X as follows:

η_N = <x, (0.5$_{(a)}$, 0.5$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.4$_{(a)}$,0.5$_{(b)}$) >.

The family τ_N = {0_N, 1_N, η_N } be FNT.

Now if, ψ_N = <x, (0.5$_{(a)}$, 0.4$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.6$_{(a)}$,0.5$_{(b)}$) >.

Then, FNIn (ψ_N) = 0_N and FNCL(FNIn (ψ_N)) = 0_N. So, FNCL(FNIn (ψ_N)) $\subseteq$ ψ_N.

Hence, ψ_N is FNPCS but, not FNSαGCS. Since

Let, U_N = η_N, where U_N be FNGOS such that, $\psi_N \subseteq U_N$. Then, FNαCL(ψ_N) = 1_N. So FNαCL(ψ_N) $\not\subseteq$ U_N.

(2) Let X={a, b} define the FNSs η_N and γ_N in X as follows:

η_N = <x, (0.5$_{(a)}$, 0.2$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.5$_{(a)}$,0.7$_{(b)}$) >,

γ_N =<x, (0.8$_{(a)}$, 0.8$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.2$_{(a)}$,0.2$_{(b)}$) >.

The family τ_N ={0_N, 1_N, η_N, γ_N }be FNT.

Now if, ψ_N = <x, (0.5$_{(a)}$, 0.7$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.5$_{(a)}$,0.3$_{(b)}$) >.

Let, U_N = γ_N, where U_N be FNGOS such that, $\psi_N \subseteq U_N$.

Then, FNαCL(ψ_N) = <x, (0.5$_{(a)}$, 0.7$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.5$_{(a)}$,0.2$_{(b)}$) > $\subseteq$ U_N.

Hence, ψ_N is FNSαGCS but, not FNPCS.

Since, FNIn (ψ_N) = η_N and FNCL(FNIn (ψ_N)) = <x, (0.5$_{(a)}$, 0.7$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.5$_{(a)}$,0.2$_{(b)}$) >.

So, FNCL(FNIn (ψ_N)) $\not\subseteq$ ψ_N.

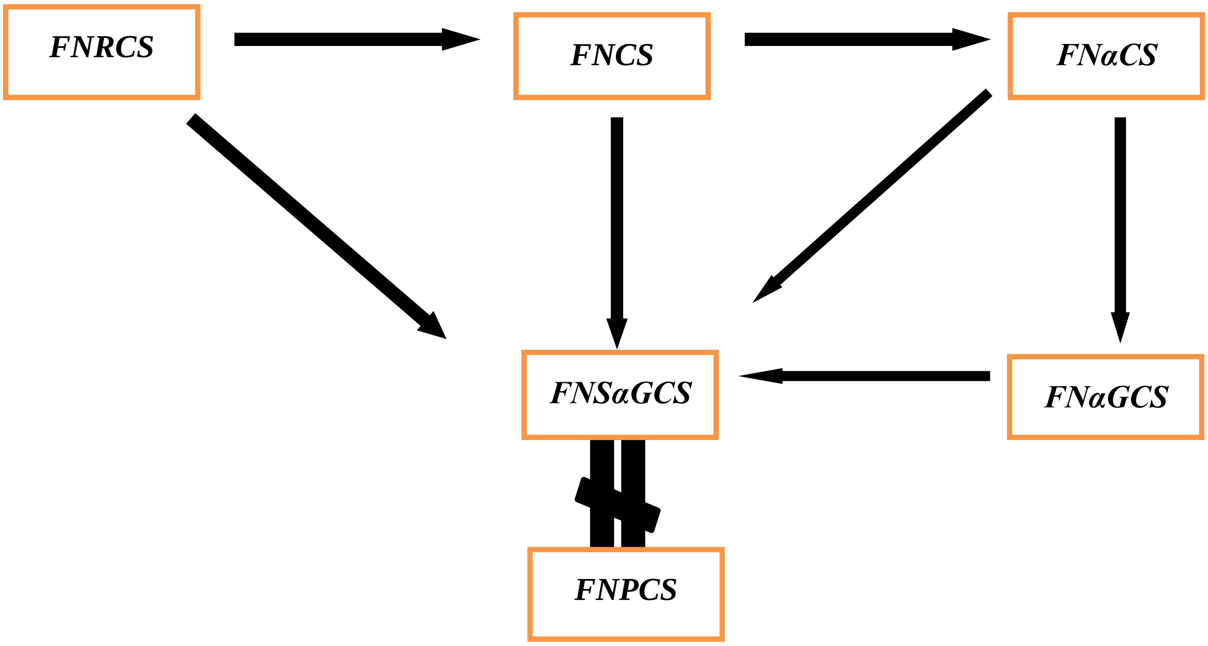
ii. Let X= {a, b} define FNS η_N in X as follows: η_N = <x, (0.5$_{(a)}$, 0$_{(b)}$), (0.5$_{(a)}$, 0.5$_{(b)}$), (0.1$_{(a)}$,1$_{(b)}$) >. The family τ_N = {0_N, 1_N, η_N } be FNT.

Now if, ψ_{N1} = <x, (0.2$_{(a)}$, 1$_{(b)}$), (1$_{(a)}$, 1$_{(b)}$), (0.7$_{(a)}$,0$_{(b)}$) > and ψ_{N2} = <x, (0.6$_{(a)}$, 0$_{(b)}$), (1$_{(a)}$, 1$_{(b)}$), (0.3$_{(a)}$,1$_{(b)}$) > are FNSαGCS. But, $\psi_{N1} \cap \psi_{N2}$ = <x, (0.2$_{(a)}$, 0$_{(b)}$), (1$_{(a)}$, 1$_{(b)}$), (0.7$_{(a)}$,1$_{(b)}$)>.

Now let, U_N = η_N, where U_N be FNGOS such that, $\psi_{N1} \cap \psi_{N2} \subseteq U_N$. Then, FN$\alpha$CL($\psi_{N1} \cap \psi_{N2}$) = 1_N $\not\subseteq$ U_N.

Hence, $\psi_{N1} \cap \psi_{N2}$ is not FNSαGCS.

Remark 3.7: The next diagram explains the relationships among different sets in the FNTS and the convers is not true in general.



5. Conclusions

In this present paper, we have defined new class of neutrosophic generalized closed sets called, fuzzy neutrosophic strongly alpha generalized closed set in fuzzy neutrosophic topological spaces. Many results have been discussed with some properties. Further, we giving some theorems, propositions and provided some useful examples where such properties failed to be preserved in order to get the relations between fuzzy neutrosophic strongly alpha generalized closed set and existing fuzzy neutrosophic closed sets in fuzzy neutrosophic topological spaces . We think, our studied class of sets belongs to the new class of fuzzy neutrosophic sets which is useful not only in the deepening of our understanding of some special features of the well-known notions of fuzzy neutrosophic topology but also useful in neutrosophic control theory.

Acknowledgments:

In this section the authors would like to thank the referees for their valuable suggestions to improve the paper.

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Received: May 2, 2020. Accepted: September 22, 2020