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Nonlinear elastic behaviour of aero-engine materials for microstructure and stress state characterization

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Abstract
In this paper the overview of third order elastic constants measurements on sample surface with 2 approaches are presented. One of the methods is the acousto-elastic constants (AEC) determination. Results of AEC measurements with velocity dispersion for Inconel 718 are shown. In the other part utilization of surface acoustic waves (SAW) mixing for TOEC determination is examined. Preliminary results of acoustic nonlinear parameter measurement for Ti6246 are presented.

1 Introduction

Elements of jet engine are exposed to severe environmental conditions like high temperatures and mechanical stresses. This framework requires the advanced means of design, including usage of superalloys and additional mechanical surface treatment. One of these processes is shot peening, a procedure in which small metal beads are bombarding the engine part. This process introduces residual stresses which are held in the specimen by dislocations. In the course of the fatigue lifetime these dislocations will form slip bands, which will move on to the surface and generate micro-cracks there. During lifetime RS is aimed to increase the fatigue characteristics by suppressing the crack propagation (1). At the same time, operating in hot temperature environment includes the possibility of metal annealing and recovery in surface layer stress field (2, 3, 4). This contradiction, as well as lacking of commercial method for non-destructive evaluation (NDE) for surface RS measurement, enforces manufacturers to increase safety margin and therefore limit the benefits of shot peening application. Existing RS measurement techniques, like hole drilling, eddy current or X-ray testing, has its own limitations and restrictions for regular maintenance operations (1).
In this work the authors examined the possible options for third order elastic constants (TOEC) measurements in surface layer with Rayleigh waves for subsequent residual stress determination. Along with theoretical review, existing experimental data on Inconel 718 and Ti6246 measurements is presented.

2 Theoretical description of problem

Acoustic NDE techniques utilize particular characteristics of the propagating stress waves. One of fundamental features is the wavespeed, that could be measured directly when the propagation path is known (5). The Method of AEC measurements is based on the acoustoelastic effect, i.e. the change in the speed of elastic wave propagation in a body undergoing static elastic deformation.
Another approach is the utilization of wave scattering effect, i.e. producing of a secondary wave with mixed frequency. This problem is similar to quantum mechanics anharmonic phonon-phonon scattering (6). Along with the complexness of theoretical description, experimental detection of scattered wave is arduous, due to weakness of the scattered wave amplitude.

2.1 Acoustoelastic and third order elastic constants

To establish a relationship between the relative velocity change of acoustic waves to the state of stress, we start with the equation of motion in the pre-strained state (initial state)

$$\rho^{(\mathrm{I})}\frac{\partial^2 u_I}{\partial t^2}=\frac{\partial}{\partial X_J}\left(C_{IJKL}+\delta_{IK}\sigma^{(\mathrm{I})}_{JL}\right) \tag{1}$$

where $\rho^{(\mathrm{I})}$ and $\left(\sigma^{(\mathrm{I})}_{JL}\right)$ are the density and the Cauchy stress tensor in the initial state and u^I the small displacement caused by the acoustic wave, measured with respect to the initial state. The stiffness C_{IJKL} refers to the stiffness in the initial state, which in turn is related to the elastic constants $(C_{\alpha\beta\gamma\delta})$ and $(C_{\alpha\beta\gamma\delta\epsilon\mu})$ in the undeformed state (natural state) as (7)

$$\begin{aligned}C_{IJKL}=\delta_{I\alpha}\delta_{J\beta}\delta_{K\gamma}\delta_{L\delta}\Big[&C_{\alpha\beta\gamma\delta}\left(1-\epsilon^{(\mathrm{I})}_{\lambda\lambda}\right)+C_{\alpha\beta\gamma\delta\epsilon\nu}\epsilon^{(\mathrm{I})}_{\epsilon\nu}\\&+C_{\mu\beta\gamma\delta}\frac{\partial u^{(\mathrm{I})}_{\alpha}}{\partial\xi_\mu}+C_{\alpha\mu\gamma\delta}\frac{\partial u^{(\mathrm{I})}_{\beta}}{\partial\xi_\mu}+C_{\alpha\beta\mu\delta}\frac{\partial u^{(\mathrm{I})}_{\gamma}}{\partial\xi_\mu}+C_{\alpha\beta\gamma\mu}\frac{\partial u^{(\mathrm{I})}_{\delta}}{\partial\xi_\mu}\Big],\end{aligned} \tag{2}$$

where $\epsilon^{(\mathrm{I})}_{\epsilon\nu}$ denotes the infinitesimal strain tensor in the initial state. Here and in the following, summation of repeated indices is implied. In an isotropic material, the elastic strain energy has to be invariant with respect to arbitrary rotations. This symmetry requirement greatly simplifies the elastic constants leaving only two independent second order parameters and three third order constants.
Considering a plane wave, propagating through the pre-stressed medium, one is led to a generalized Christoffel equation, the derivative of which with respect to some free strain parameter yields the desired acoustoelastic equations (5)

$$\begin{aligned}L_{22}&=\frac{\mathrm{d}c_{22}/c^0_{22}}{\mathrm{d}\epsilon}=-2\nu(1+\frac{m-\mu l/\lambda}{\lambda+2\mu}),\\L_{21}&=\frac{\mathrm{d}c_{21}/c^0_{21}}{\mathrm{d}\epsilon}=\frac{\nu n}{4\mu}+\frac{\lambda+2\mu+m}{2(\lambda+\mu)},\\L_{23}&=\frac{\mathrm{d}c_{23}/c^0_{23}}{\mathrm{d}\epsilon}=\frac{m-2\lambda}{2(\lambda+\mu)}-\frac{n}{4\mu},\end{aligned} \tag{3}$$

where c_{IJ} denotes the phase velocity of a wave propagating along the X_I-axis with particle displacement along the X_J-axis. The static load is applied in X_1-direction. The constants λ and μ are the usual second order Lamé parameters and l, m, and n are the third order elastic constants in the notation of Murnaghan (8).
Measurements of the phase velocities while changing the static stress then yield the desired acoustoelastic constants. These stress induced changes in the phase velocity of bulk waves can now be used to obtain the corresponding change of the Rayleigh wave velocity. One way to achieve this goal is to put an ansatz for a plane surface wave into the equation of motion in the initial state with boundary conditions and solve the resulting equation for the phase velocity (9). Another way is to use the so called Grüneisen constants, describing the change of phonon frequency with respect to an applied strain. Originally introduced for volume waves (10) these can be generalized to surface waves (11) giving the relative change of Rayleigh wave velocity

$$\frac{\delta c}{c} = \left(-\gamma_{KL} - \hat{k}_K \hat{k}_L\right) E_{KL} = A^{(1)}_{R13}\sigma_{11} + A^{(2)}_{R13}\sigma_{22} \tag{4}$$

where $\hat{k}_K$ denotes the unit vector in the direction of the wave vector k_K, (E_{IK}) the Green-Lagrange strain tensor and γ_{KL} are the Grüneisen-constants defined as (11)

$$\gamma_{KL} = [-Nc]^{-1} \frac{1}{2} \left(\delta_{aK}\delta_{BL} + \delta_{aL}\delta_{BK}\right) S_{aB\,mM\,nN} D^*_{Mr} b^*_{mr} \, D_{Nr'} b_{nr'} \left[\alpha^*_r + \alpha_{r'}\right]^{-1}, \tag{5}$$

where the tensor $S_{aB\,mM\,nN}$ is a combination of second and third order elastic constants. Here, we assume the Rayleigh wave to propagate in the X_1-direction with its displacement components lying parallel to the X_1-X_3-plane, indicated by the lower indices of the AEC $A^{(\cdot)}_{R13}$. The upper index in the latter coefficients denotes the direction of the normal stress component, the influence of which onto the RW velocity change the coefficient describes. In the isotropic case

$$D_{Lr} = \delta_{L1}\mathrm{i} + \delta_{L3}\alpha_r, \tag{6}$$

$$b_{mr} = \begin{pmatrix} 1 & -\sqrt{\alpha_1\alpha_2} \\ 0 & 0 \\ -\mathrm{i}\alpha_1 & -\mathrm{i}\sqrt{\alpha_1/\alpha_2} \end{pmatrix}_{mr} \tag{7}$$

where $\alpha_r = \sqrt{1 - c/c_r}$ with the longitudinal and transversal wave velocities c_1 and c_2. The scaling constant

$$Nc = \rho c^2 \frac{(\alpha_1 - \alpha_2)\left(\alpha_1 - \alpha_2 + 2\alpha_1\alpha_2^2\right)}{\alpha_1\alpha_2^2} \tag{8}$$

has the dimension of an elastic constants and does not dependent on the frequency.
In Table 1 some values for Rayleigh wave AEC are listed. These are partly taken directly from the works cited and partly computed with the parameters given there. The values given in (9) are the earliest ones in the table. The sign of the parallel AEC for this material is negative, the other one positive. For a long time, this was supposed to be the usual behavior for all alloys. However, independent measurements in (12, 13, 2, 3) consistently show that this is not true in the case of Inconel 718. All measurements show a positive constant $A^{(1)}_{R13}$ and a negative $A^{(2)}_{R13}$.

Table 1: Acoustoelastic constants for different alloys. The values for Inconel 718 are obtained by virtue of Grüneisen constants and the second and third order elastic constants given in the corresponding references.

material	$A^{(1)}_{R13}$ [10^{-6}/MPa]	$A^{(3)}_{R13}$ [10^{-6}/MPa]
Inconel 718 (12)	0.68	−2.58
Inconel 718 (13)	1.30	
Inconel 718 (2)	2.36	−1.20
Inconel 718-1 (3)	3.01	−1.48
Inconel 718-2 (3)	1.69	−1.24
Inconel 718-3a (3)	1.57	−1.22
Inconel 718-3b (3)	1.44	−1.20
Inconel 718-4a (3)	2.43	−1.37
Inconel 718-4b (3)	1.90	−1.28
Inconel 718-4 (3)	2.55	−1.40
Inconel 718-5 (3)	1.11	−1.14
Inconel 718-6a (3)	2.19	−1.33
Inconel 718-6b (3)	3.52	−1.57
Ti6246 (12)	−3.25	3.57
Ti6246 (2)	2.79	1.62
Aluminum alloy 2214 (9)	−14.40	7.60

In (13) the wave field of a Rayleigh wave propagating on a strained specimen made of Inconel 718 was measured. The acoustoelastic constant $A^{(1)}_{R13}$ determined in these experiments is 1.3×10^{-6} MPa^{-1}, which is in good agreement with some results obtained using the values of Hubel et al. (3) (see Tab. 1). Unfortunately, no result for $A^{(2)}_{R13}$ was given in (13). Besides the consistency of the sign in the case of Inconel 718 there is no further consistency as to the absolute values. One reason for this is that it's difficult to obtain these constants in tensile tests. The relative changes in phase velocity are fairly low and even small changes in the measurement setup can lead to large deviations in the results. Moreover, the AEC and thereby the TOEC are highly dependent on the state of the material itself. Different treatments will strongly affect the material and alter the second order as well as the third order elastic constants, where the latter ones are much more sensitive to those changes than the SOEC. In case of Ti6246 even the sign of both AEC changes. As was already pointed out in (12), this can be attributed to the strong heterogeneity of such titanium base alloys.

2.2 Nonlinear wave mixing

Another approach for TOEC determination is the mixing of elastic waves. Waves with frequencies ω_1 and ω_2 may interact in nonlinear (anharmonic) solid, producing the secondary wave. Resonance conditions for this type of interactions are(14, 15, 6):

$$\begin{aligned} \omega_{\text{scattered}} &= \omega_1 \pm \omega_2 \\ \vec{k}_{\text{scattered}} &= \vec{k}_1 \pm \vec{k}_2 \end{aligned} \tag{9}$$

where $\vec{k}_i$ is the wave vector. Due to existence of two propagation velocities for bulk elastic wave in a solid, multiple resonance interactions becomes possible. According to (6), only 10 out of 54 combination cases are possible for bulk waves.

Surface acoustic waves (SAW), or Rayleigh waves scattering theory has been developed primarily for anisotropic materials (11). Based on slowness surfaces construction, introduced by Herring (16, 17), next scattering combinations are allowed by resonance conditions for SAW in isotropic material (Eq. (9)):

Table 2: Scattering processes for elastic waves in isotropic solids.

Interacting waves		Scattered wave		
ω_1	ω_2	RW	L	T
RW	RW	‖	x	x
RW	L		x	x
RW	T		x	x
L	L	x	‖	x
L	T	x	x	x
T	T	x	x	

In Table 2 RW is the Rayleigh wave, L is longitudinal and T is transverse wave. Sign "x" stands for scattering possibility, ‖ for additional collinearity condition for waves interaction. Polarization restrictions have not been accounted. Information on bulk wave interactions was taken from (15, 6).

Scattered waves amplitude measurements for bulk wave interaction allows to define the third order elastic constants (18) in the volume.

After numerical simulation on scattered wave amplitudes on the surface and introducing the inversion procedure the determination of surface layer stress state would become feasible. This work would be described in the following papers.

3 ANP evaluation for high-strength alloys

As a first step towards surface residual stress quantification through wave mixing, measurements of acoustic nonlinear parameter (ANP) was performed for Ti6246 sample without peening and with 4 A and 8 A peening intensity. For longitudinal wave, propagating in pre-stressed solid, ANP could be defined as (19)

$$\beta = \frac{\overline{C}_{111111} + 3\overline{C}_{1111}}{\overline{C}_{1111} + \sigma_{11}^0}, \tag{10}$$

where σ_{11}^0 is residual stress and $\overline{C}$ are modified second- and third- order elastic constants. For Rayleigh wave experimental characterization, one could express the ANP in the following way (19)

$$\beta = \frac{8u_{2\omega}}{\omega^2 X_1 u_\omega^2} \frac{c_L\sqrt{c_L^2 - c_R^2}}{2(c_T/c_R)^2 - 1}, \tag{11}$$

where X_1 is the distance of RW propagation, c_L, c_T, c_R are longitudinal, shear and Rayleigh wave speeds, ω is the fundamental frequency and u_ω, $u_{2\omega}$ are the amplitudes

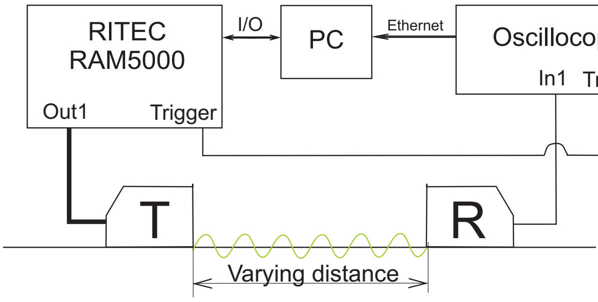

Figure 1: Experimental setup layout.

of fundamental and second harmonic, respectively. Therefore, by measuring the second harmonic variation through wave propagation path, one could obtain information on the sample stress state.
With the experimental setup depicted on Fig. 1 measurements on ANP change over distance have been conducted. During experiment 2 commercial PZT transducers with fundamental frequencies 4 MHz (T) and 10 MHz (R) have been used, for each measurement sine burst signal on the transmitter fundamental frequency have been utilized. Time data was recorded with the LeCroy digital scope and processed on PC. First results are displayed on Fig. 2, left. Unlike the (19) results, Rayleigh wave ANP measurements on Ti6246 sample surface have not provided the sufficient information for peening intensity determination. Also the check of results' repeatability was made with series of independent measurements under equal conditions (Fig. 2 right) within the simplified setup: RITEC GA2500 gated amplifier and Agilent 33220A signal generator as a replacement of RAM5000. 3 measurements on ANP changes with the distance on non-peened surface have shown satisfactory repeatability. For $\overline{\beta}$ estimation linear regression of measured $u_{2\omega}(u_{\omega}^2)$ dependency for each amplitude step was used.

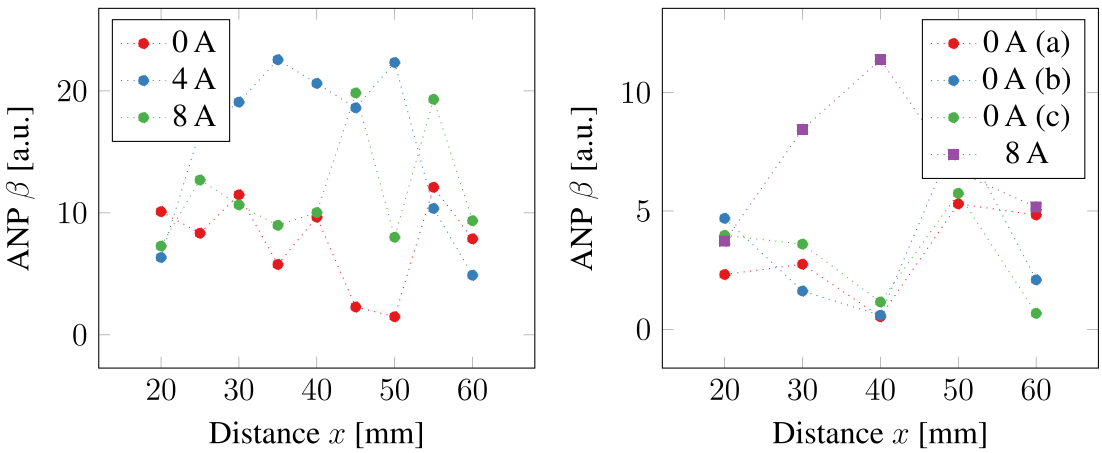

Figure 2: Variation of ANP with distance (left) and repeatability test with GA2500 (right), conducted on a Ti6246 specimen.

For 24 out of 27 points on Fig. 2 left, R^2-coefficient of determination was greater than 0.99. In 16 points out of 20 for Fig. 2 right, R^2 was greater than 0.97. ANP oscillations over distance could be related with inhomogeneous coupling conditions, which altered from point to point due to manual coupling.

4 Conclusion and future work

In this work, an overview of two methods for the determination of third order elastic constants for isotropic materials was given. Measurements of the Rayleigh wave acoustoelastic constants for Inconel 718, made by two independent groups, have shown to be in good agreement. Another approach could be the use of wave mixing. In this work the authors have made first assumptions on the allowed interactions for surface waves. Measurements of the Rayleigh wave acoustic nonlinearity parameter in Titanium sample were conducted. The obtained results are supposed to be used as a benchmark tool for a SAW mixing experimental setup due to similar amplitude and frequency range. Despite satisfactory repeatability and good correlation for each individual point, this data are not in accordance with previous works on the topic.
The next steps include the optimization of the experimental procedure by numerical simulation of surface acoustic waves mixing and second harmonic generation where the limitations of the instruments have to be taken into account.

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