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Fermat's Last theorem Algebraic Proof

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Abstract:

In 1995, A, Wiles of the Princeton Institute for Advanced Study, announced, using cyclic groups (a subject area which was not available at the time of Fermat), a proof of Fermat's Last Theorem, which is stated as follows: If π is an odd prime and x, y, z are relatively prime positive integers, then $z^\pi \neq x^\pi + y^\pi$. In this paper, a proof of this theorem is given using only elementary Algebra. It is shown that if π is an odd prime and x, y, z are positive inyegera satisfying $z^\pi = x^\pi + y^\pi$, then $x, y,$ and z are each divisible by π.

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Introduction

For other theorems named after Pierre de Fermat, see the book by H. Edwards [1]. The 1670 edition of Diophantus' Arithmetica includes Fermat's commentary, particularly his "Last Theorem" (Observatio Domini Petri de Fermat). In number theory, Fermat's Last Theorem (sometimes called Fermat's conjecture, especially in older texts) states that no three positive integers x, y, and z satisfy the equation $z^\pi = x^\pi + y^\pi$ for any integer value of π greater than two. The cases n = 1 and π = 2 were known to have infinitely many solutions. This theorem was first conjectured by Pierre de Fermat in 1637 in the margin of a copy of Arithmetica where he claimed he had a proof that was too large to fit in the margin. The first proof agreed upon as successful was released in 1994 by Andrew Wiles, using cyclic groups, and formally published in 1995, after 358 years of effort by mathematicians. The unsolved problem stimulated the development of algebraic number theory in the 19th century and the proof of the modularity theorem in the 20th century. It is among the most notable theorems in the history of mathematics It is known that if x; y; z are relatively prime positive integers, $z^4 \neq x^4 + y^4$ [1]. In view of this fact, it is only necessary to prove if x; y; z; are relatively prime positive integers, π is an odd prime, $z^\pi = x^\pi + y^\pi$, then x, y, z are each divisible by π. Before and since Wiles paper, many papers and books have been written trying to solve this problem in an elegant algebraic way, but none have succeeded. (See [1], and go to a search engine on the computer and search Fermat's Last Theorem). In the remainder of this paper, π will represent an odd prime.

Theorem 1. *If x, y, z are positive integers and $z^\pi = x^\pi + y^\pi$, then $z^\pi = x^\pi + y^\pi$,* then x, y, z are each divisible by π.

Theorem 1 is arrived at through lemmas, all but one part of which are proved in [2],, and are given here for completeness.

$x \equiv 0 \ (mod\ \pi),\ y \equiv 0\ (mod\ \pi), z \equiv 0\ (mod\ \pi).$

Theorem 1 is arrived at as a result of two Lemmas.

Lemma 1. *If* $z^{\pi} = x^{\pi} + y^{\pi}$ *, then*

(1) $x + y - z \equiv 0\ (mod\ \pi);$

(2) $(x+y)^{\pi} - z^{\pi} \equiv 0\ (mod\ \pi^{2});$

(3) $z \not\equiv 0\ (mod\ \pi), then\ \ \pi = 2.$

Proof. It is obvious that $(x+y)^{\pi} - z^{\pi} \equiv 0\ (mod\ \pi);$ also,

$$(x+y)^{\pi} - z^{\pi} = (x+y+z-z)^{\pi} - z^{\pi} = \sum_{1}^{\pi-1} C(\pi.k)(x+y-z)^{\pi-k} z^{k}.$$

From this equation, it follows that

$$(1)\ \ x+y-z \equiv 0(mod\,\pi),\ \ (2)\ \ (x+y)^{\pi} - z^{\pi} \equiv 0(mod\,\pi^{2})$$

As for (3), (1) and (2) and the Prime Factorization Theorem lead to

$$\pi^{\pi} = \pi^{2}, \pi = 2$$

□

Lemma 1(3) contradicts π odd.

Lemma 2. *If* x, y, z *are positive integers such that* $z^{\pi} = x^{\pi} + y^{\pi}$ *, then* $y \equiv 0(mod\,\pi)$.

Proof. It follows from Lemma 1(1) and Lemma 1(3), that

$$x + y \equiv 0\ (mod\ \pi).$$

So

$$z^{\pi} \equiv 0(mod\,\pi^{\pi}) \equiv (x+y)\sum_{0}^{\pi-1}(x)^{p-1-k} y^{k};$$

$$y^{\pi-1} \equiv 0(mod\,\pi^{\pi-1})$$

Fermat's Last Theorem. If x, y, z are relatively prime positive integers, then $z^{\pi} \neq x^{\pi} + y^{\pi}$

Proof. From Lemma 1. $x + y - z \equiv 0(mod\,\pi),\ \ z \equiv 0(mod\,\pi), y \equiv 0(mod\,\pi)$ by Lemma 2. So x, y, z are not relatively prime. □

References

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[4] A. Wiles and R. Taylor, Ring-theoretic properties of certain Heche algebras, Ann. Math. 141 (1995), 553-573.