Financial Sensitivity

Summary

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Financial Sensitivity Definition

- Financial sensitivity is the measure of the value reaction of a financial instrument to changes in underlying factors.
- The value of a financial instrument is impacted by many factors, such as interest rate, stock price, implied volatility, time, etc.
- Financial sensitivities are also called Greeks, such as Delta, Gamma, Vega and Theta.
- Financial sensitivities are risk measures that are more important than fair values.
- They are vital for risk management: isolating risk, hedging risk, explaining profit and loss, etc.

Sensitivity

Delta Definition

- Delta is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying asset price.
- Interest rate Delta:

$$IrDelta = \frac{\partial V}{\partial r} = \frac{V(r + 0.0001) - V(r)}{0.0001}$$

  where *V*(*r*) is the instrument value and *r* is the underlying interest rate.
- PV01, or dollar duration, is analogous to interest rate Delta but has the change value of a one-dollar annuity given by

$$PV01 = V(r + 0.0001) - V(r)$$

Delta Definition (Cont)

- Credit Delta applicable to fixed income and credit product is given by

$$CreditDelta = \frac{\partial V}{\partial c} = \frac{V(c + 0.0001) - V(c)}{0.0001}$$

  where c is the underlying credit spread.

- CR01 is analogous to credit Delta but has the change value of a one-dollar annuity given by

$$PV01 = V(r + 0.0001) - V(r)$$

- Equity/FX/Commodity Delta

$$Delta = \frac{\partial V}{\partial S} = \frac{V(1.01S) - V(S)}{0.01 * S}$$

  where S is the underlying equity price or FX rate or commodity price

Vega Definition

- Vega is a first-order Greek that measures the value change of a financial instrument with respect to changes in the underlying implied volatility.

$$Vega = \frac{\partial V}{\partial \sigma} = \frac{V(\sigma + \Delta\sigma) - V(\sigma)}{\Delta\sigma}$$

where σ is the implied volatility.

- Only non-linear products, such as options, have Vegas.

Gamma Definition

- Gamma is a second order Greek that measures the value change of a financial instrument with respect to changes in the underlying price.

$$Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{V(S + 0.5 * \Delta S) + V(S - 0.5 * \Delta S) - 2V(S)}{\Delta S^2}$$

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Theta Definition

- Theta is a first order Greek that measures the value change of a financial instrument with respect to time.

$$Theta = \frac{\partial V}{\partial t} = \frac{V(t + \Delta t) - V(t)}{\Delta t}$$

Curvature Definition

- Curvature is a new risk measure for options introduced by Basel FRTB.
- It is a risk measure that captures the incremental risk not captured by the delta risk of price changes in the value of an option.

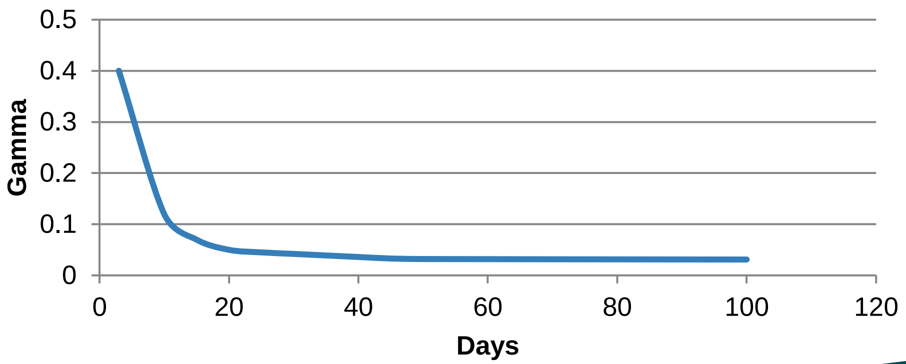
$$Curvature = min\{V(S + \Delta W) - V(S) - \Delta W * Delta, V(S - \Delta W) - V(S) - \Delta W * Delta\}$$

where ΔW is the risk weight.

Sensitivity

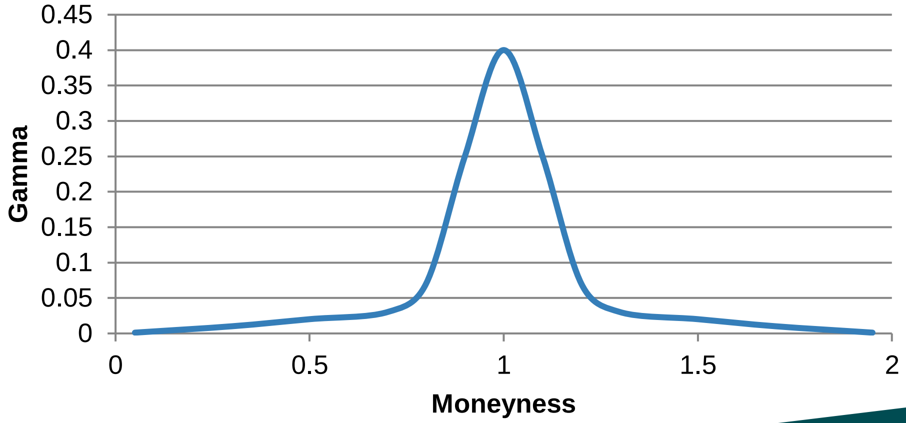
Option Sensitivity Pattern

- Sensitivity behaviors are critical for managing risk.
- Gamma
  - Gamma behavior in relation to time to maturity shown below.
  - Gamma has a greater effect on shorter dated options.



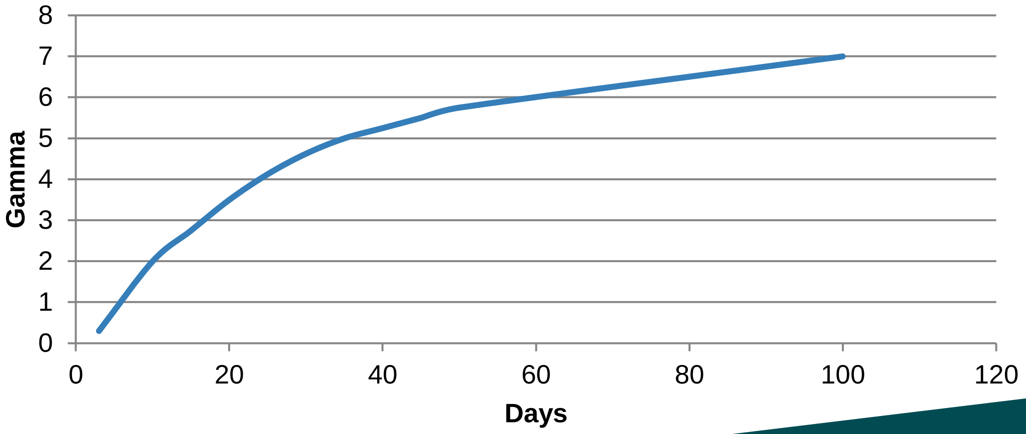
Option Sensitivity Pattern (Cont)

- Gamma behavior in relation to moneyness shown below.
- Gamma has the greatest impact on at-the-money options.



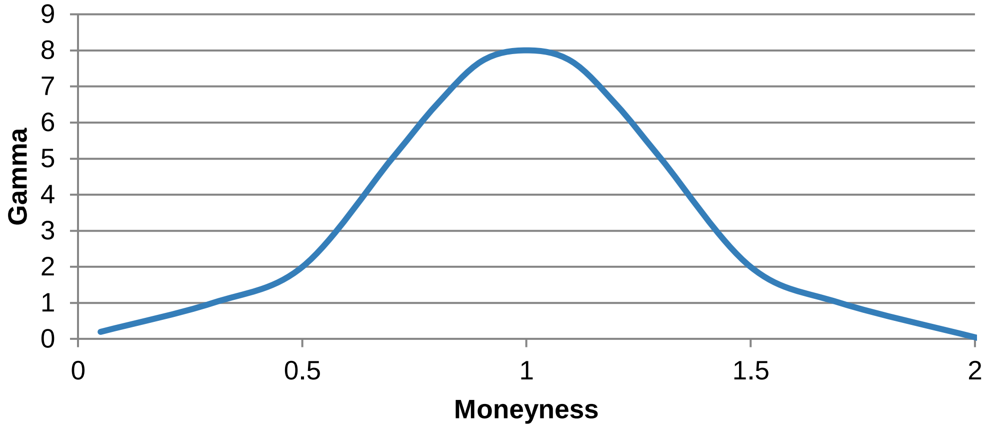
Option Sensitivity Pattern (Cont)

- Vega
  - Vega behavior in relation to time to maturity shown below.
  - Vega has a greater effect on longer dated options.



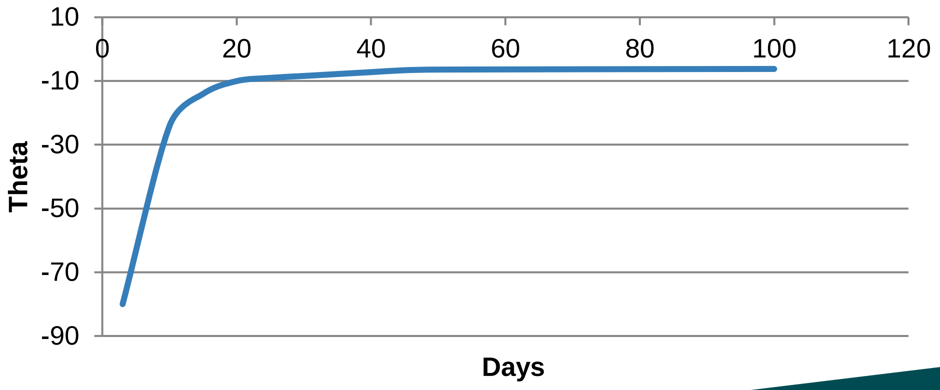
Option Sensitivity Pattern (Cont)

- Vega behavior in relation to moneyness shown below.
- Vega has the greatest impact on at-the-money options.



Option Sensitivity Pattern (Cont)

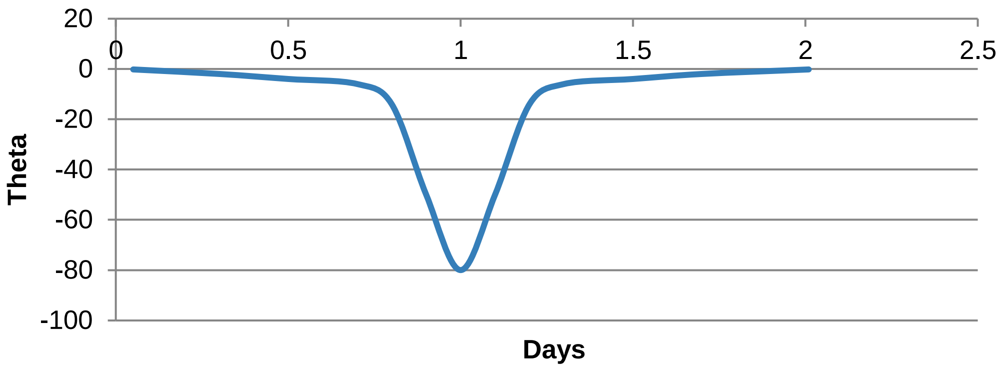
- Theta or time decay
  - Theta is normally negative except some deeply in-the-money deals.
  - Theta behavior in relation to time to maturity shown below.
  - Theta has a greater effect on shorter dated options.



Sensitivity

Option Sensitivity Pattern (Cont)

- Theta behavior in relation to moneyness shown below.
- Theta has the biggest impact on at-the-money options.



Sensitivity Hedging

- The objective of hedging is to have a lower price volatility that eliminates both downside risk (loss) and upside profit.
- Hedging is a double-edged sword.
- The profit of a broker or an investment bank comes from spread rather than market movement. Thus it is better to hedge all risks.
- Delta is normally hedged.
- Vega can be hedged by using options.
- Gamma is hardly hedged in real world.

Sensitivity Profit & Loss (P&L)

- Hypothetic P&L is the P&L that is purely driven by market movement.
- Hypothetic P&L is calculated by revaluing a position held at the end of the previous day using the market data at the end of the current day, i.e.,

$$HypotheticalP\&L = V(t-1, P_{t-1}, M_t) - V(t-1, P_{t-1}, M_{t-1})$$

where *t-1* is yesterday; *t* is today; P_{t-1} is the position at yesterday; M_{t-1} is yesterday's market and M_t is today's market.

- Sensitivity P&L is the sum of Delta P&L, Vega P&L and Gamma P&L.
- Unexplained P&L = HypotheticalP&L – SensitivityP&L.

Sensitivity Profit & Loss (Cont)

- Delta P&L:

$$DeltaP\&L = Delta * (S_t - S_{t-1})$$

where S_t is today's underlying price and S_{t-1} is yesterday's underlying price.

- Vega P&L:

$$VegaP\&L = Vega * (\sigma_t - \sigma_{t-1})$$

where σ_t is today's implied volatility and σ_{t-1} is yesterday's implied volatility.

- Gamma P&L:

$$GammaP\&L = 0.5 * Gamma * (S_t - S_{t-1})^2$$

Backbone Adjustment

- Backbone adjustment is an advanced topic in sensitivity P&L.
- It can be best explained mathematically.
- Assume the value of an option is a function of the underlying price S and implied volatility σ, i.e., $V = F(S, \sigma)$.
- If the implied volatility is a function of the ATM volatility and strike (sticky strike assumption), i.e., $\sigma = \sigma_A + f(K)$, the first order approximation of the option value is

$$\Delta V = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial \sigma_A} d\sigma_A = DeltaP\&L + VegaP\&L$$

where $DeltaP\&L = \frac{\partial F}{\partial S} dS$ and $VegaP\&L = \frac{\partial F}{\partial \sigma_A} d\sigma_A$

Backbone Adjustment (Cont)

- If the implied volatility is a function of the ATM volatility and moneyness K/S (sticky moneyness or stricky Delta assumption), i.e., $\sigma = \sigma_A + f(S, K)$, the first order approximation of the option value is

$$\Delta V = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial \sigma_A} d\sigma_A + \frac{\partial F}{\partial \sigma}\frac{\partial \sigma}{\partial S} dS = DeltaP\&L + VegaP\&L$$

  where $DeltaP\&L = (\frac{\partial F}{\partial S} + \frac{\partial F}{\partial \sigma}\frac{\partial \sigma}{\partial S})dS$ and $VegaP\&L = \frac{\partial F}{\partial \sigma_A} d\sigma_A$

- Under sticky moneyness/Delta assumption, the DeltaP&L above has one more item, i.e., $\frac{\partial F}{\partial \sigma}\frac{\partial \sigma}{\partial S} dS$ that is the backbone adjustment.

Thanks!

You can find more details at
https://finpricing.com/lib/IrCurveIntroduction.html