Threshold resummation and pion structure

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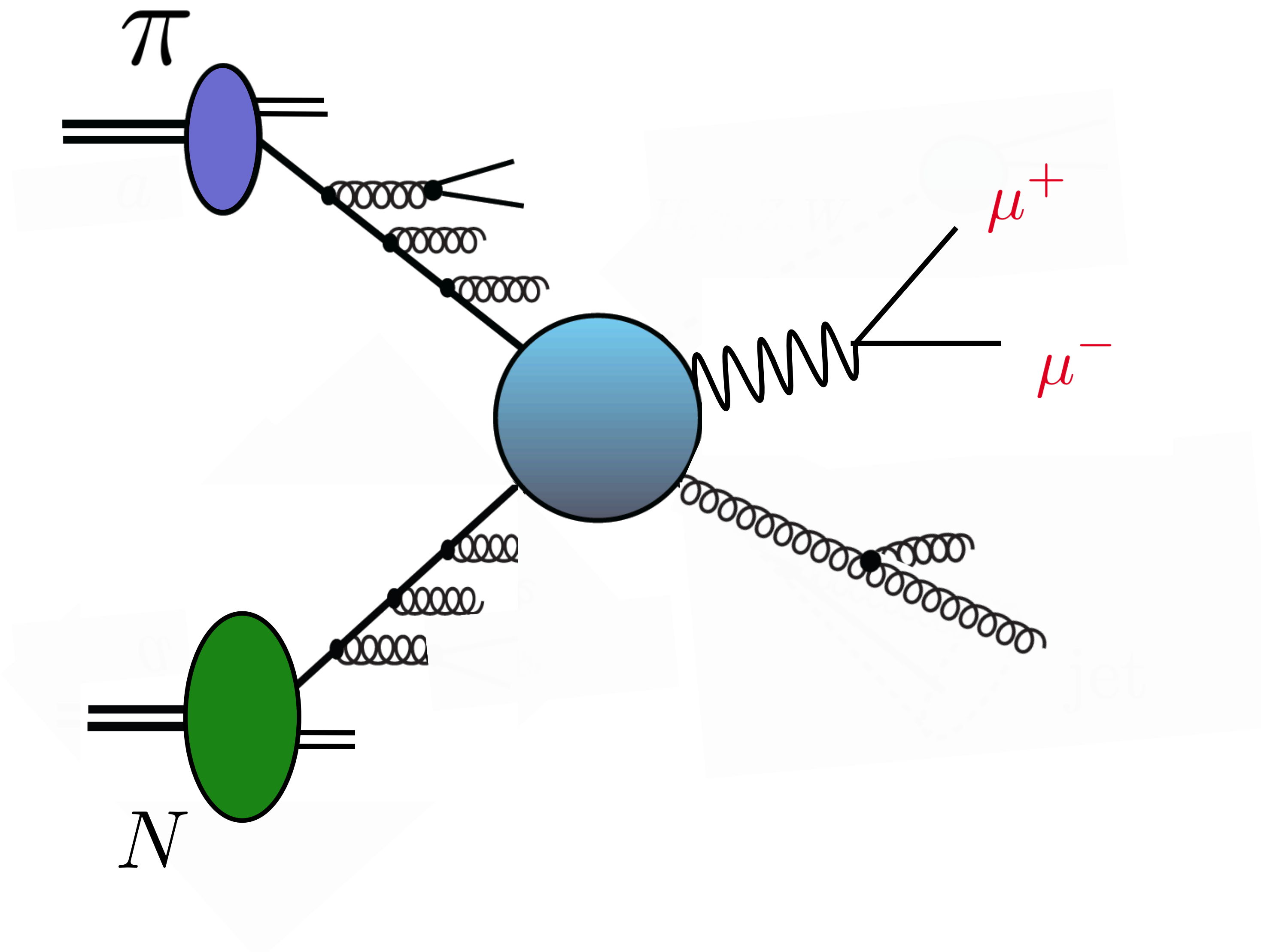
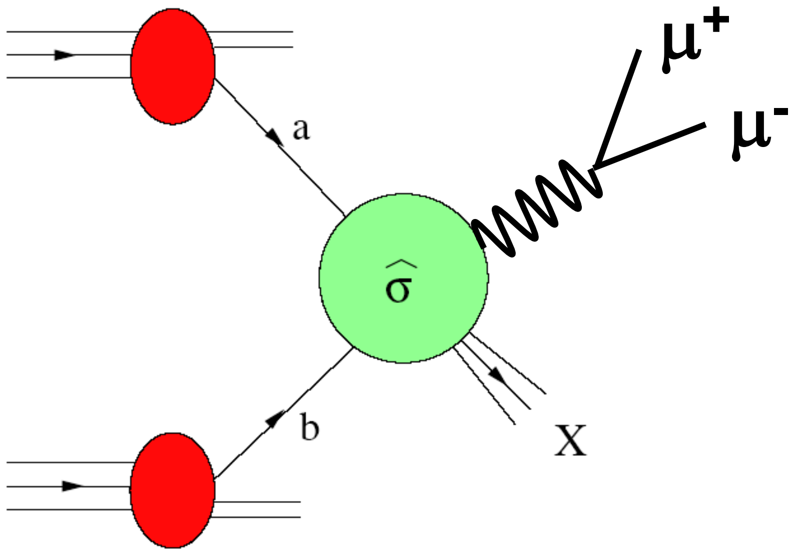


π
μ^+
μ^-
N

Outline:

- Basics of threshold resummation for Drell-Yan
- Drell-Yan in πN scattering
- Possible improvements
- Conclusions

Basics of threshold resummation for Drell-Yan



hard scale Q

$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a,\mu) \, f_b(x_b,\mu) \, \omega_{ab}\left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu}\right) + \ldots$$

universal pdfs

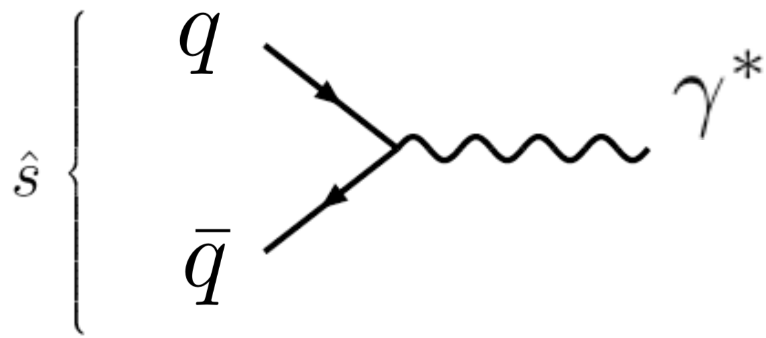
partonic hard scatt.
perturbative QCD

$\mu \sim Q$ fact./ren. scale

$$\omega_{ab} = \omega_{ab}^{(\mathrm{LO})} + \frac{\alpha_s}{2\pi} \omega_{ab}^{(\mathrm{NLO})} + \ldots$$

(up to power corrections $1/Q^2$)

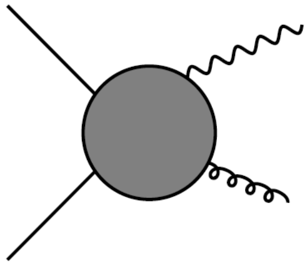
LO :



$$z = \frac{Q^2}{\hat{s}}$$

$$\omega_{q\bar{q}}^{(\mathrm{LO})} \propto \delta(1-z)$$

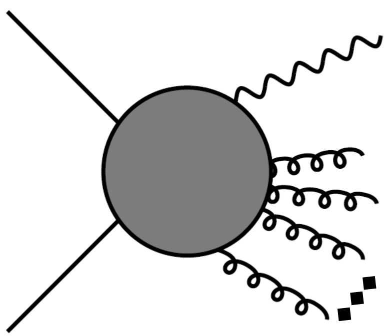
- NLO correction:



$$z \longrightarrow 1:$$

$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \propto \alpha_s \left(\frac{\log(1-z)}{1-z}\right)_+ + \ldots$$

- higher orders:



$$\omega_{q\bar{q}}^{(\mathrm{N^kLO})} \propto \alpha_s^k \left(\frac{\log^{2k-1}(1-z)}{1-z}\right)_+ + \ldots$$

"threshold logarithms"

- for z → 1 real radiation inhibited

- logs emphasized by parton distributions:

$$d\sigma \sim \int_\tau^1 \frac{dz}{z}\, \mathcal{L}^{\mathrm{PDF}}_{q\bar{q}}\left(\frac{\tau}{z}\right) \omega_{q\bar{q}}(z) \qquad \tau = \frac{Q^2}{S}$$

z = 1 relevant,
in particular as $\tau \to 1$

- logs more relevant at lower hadronic energies

- large logs may spoil perturbative series, unless taken into account to all orders

  = (threshold) resummation !

- particularly relevant for (lower-energy) fixed-target

- work began in the ‘80s with Drell-Yan process

  Sterman; Catani, Trentadue

  various new techniques: Laenen, Sterman, WV
  Vogt; Bonvini, Forte, Ridolfi;
  Becher, Neubert;
  van Neerven, Smith, Ravindran;
  Laenen, Magnea; Tackmann et al.
  …

- Mellin moments:

$$\int_0^1 dz\, z^{N}\, \alpha_s^k \left(\frac{\log^{2k-1}(1-z)}{1-z} \right)_+ \propto \alpha_s^k \log^{2k}(N) \equiv \alpha_s^k L^{2k}$$

Fixed Order

Resummation

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	...
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	$\alpha_s^k L^{2k-5}$

LL NLL NNLL

- NLL resummation in Mellin-moment space : ($\overline{\text{MS}}$ scheme)

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) = e_q^2\,\mathcal{C}(\alpha_s(Q^2))\,\exp\left[2\int_0^1 dy\,\frac{y^N-1}{1-y}\int_{\mu_F^2}^{Q^2(1-y)^2}\frac{dk_\perp^2}{k_\perp^2}A_q(\alpha_s(k_\perp^2))\right]$$

N independent,
perturbative

$$A_q(\alpha_s) = C_F\left\{\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{C_A}{2}\left(\frac{67}{18}-\zeta(2)\right)-\frac{5}{9}T_R n_f\right]+\ldots\right\}$$

Catani,Mangano,
Nason,Trentadue

$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \propto \exp\left\{2\ln\bar{N}\,h^{(1)}(\lambda) + 2h^{(2)}\left(\lambda,\frac{Q^2}{\mu^2}\right)\right\}$$

LL NLL

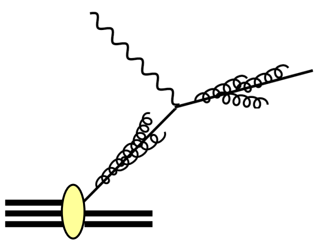
$$\lambda = \alpha_s(\mu^2)\,b_0\,\log(N e^{\gamma_E})$$

$$h^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0\lambda}\left[2\lambda+(1-2\lambda)\ln(1-2\lambda)\right] \qquad h^{(2)} = \ldots$$

- resummation enhances cross section:

LL :
$$\tilde{\omega}_{q\bar{q}}^{(\mathrm{res})}(N) \propto \exp\left[+ \frac{2C_F}{\pi} \alpha_s \ln^2 N + \ldots \right]$$

compare DIS:

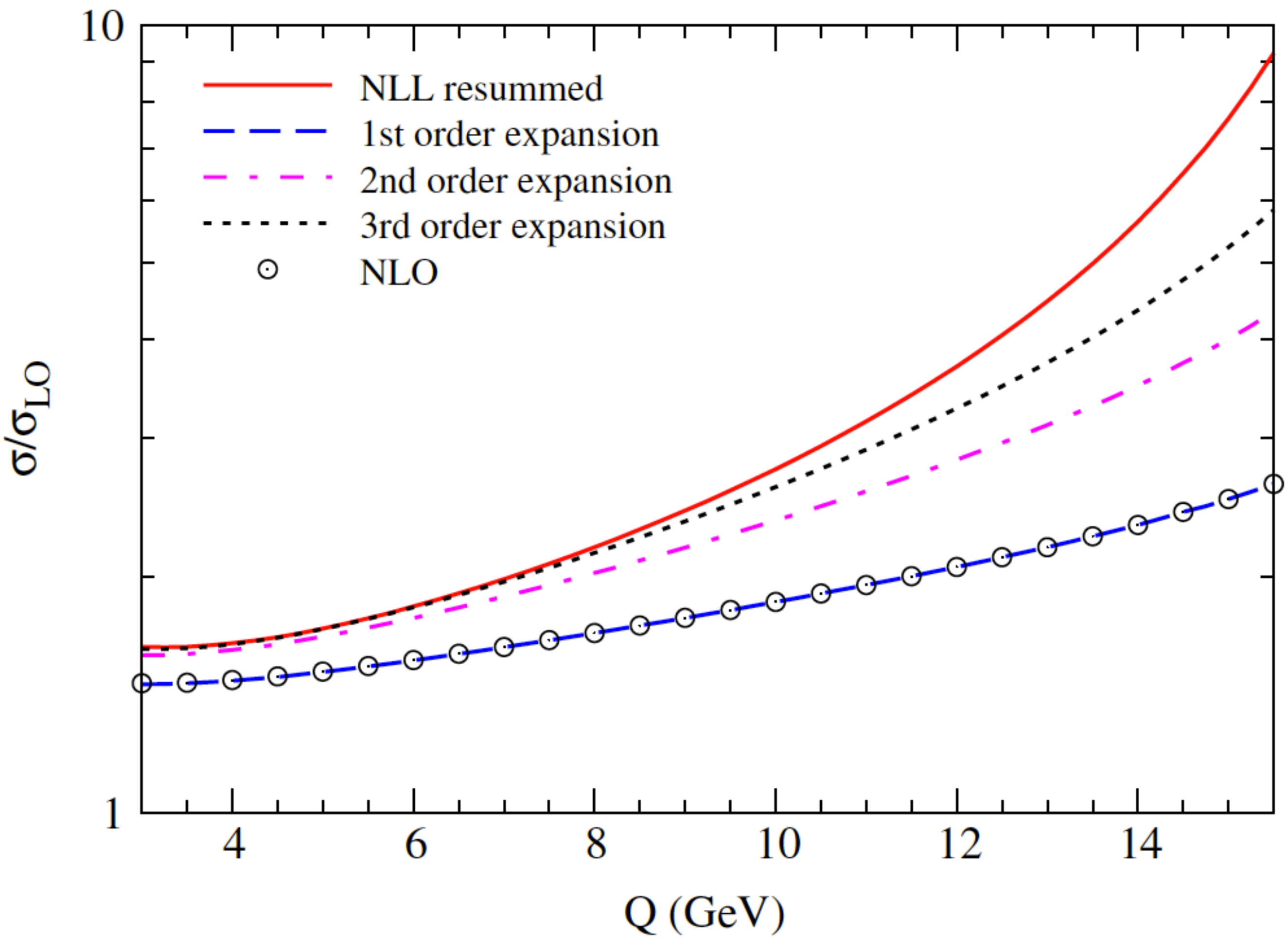


$$\tilde{\omega}_{\mathrm{DIS}}^{(\mathrm{res})} \propto \exp\left[\frac{1}{2} \frac{C_F \alpha_s}{\pi} \ln^2(N) \right]$$

$$\pi N \to \mu^+ \mu^- X$$

$$\sqrt{S} = 19\,\mathrm{GeV}$$

(COMPASS)

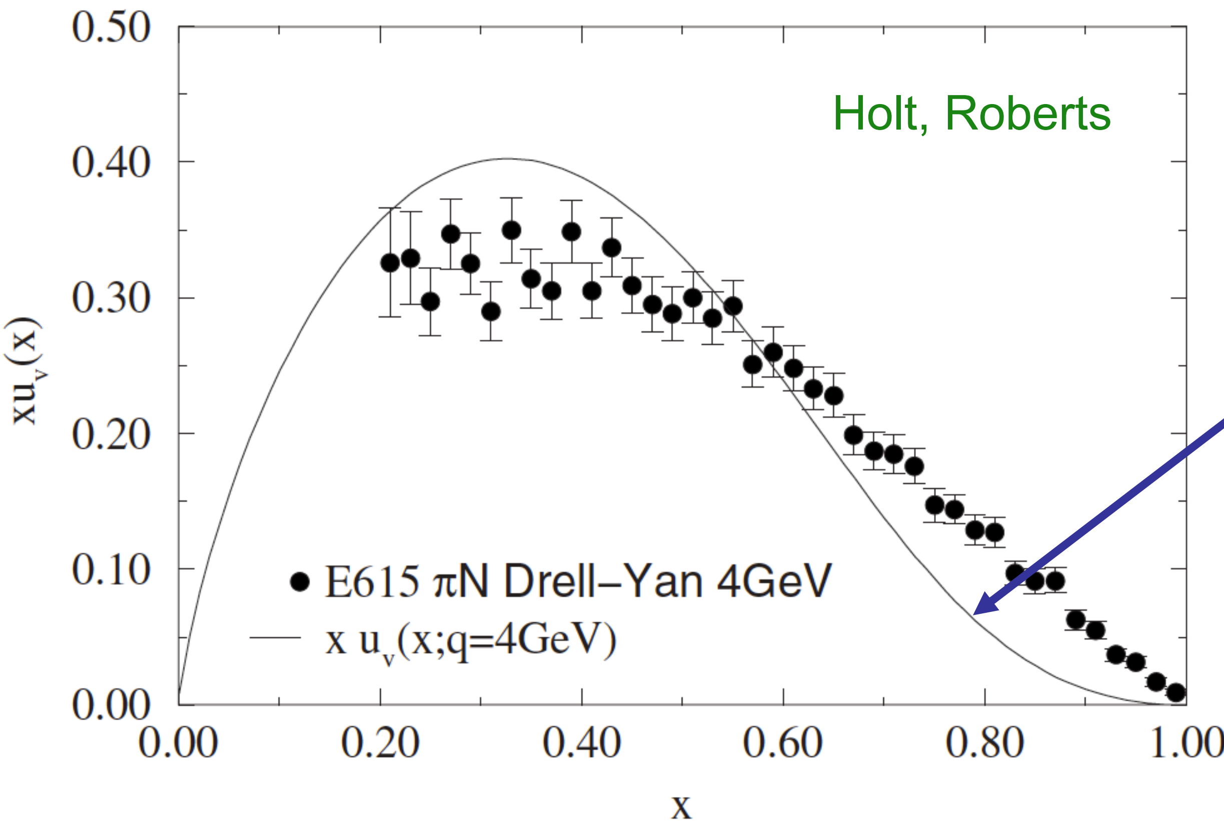


Aicher, Schäfer, WV
(earlier studies: Shimizu, Sterman, WV, Yokoya)

Drell-Yan process in πN scattering

M. Aicher, A.Schäfer, WV (2010)

- LO extraction of u_v from E615 data: $\sqrt{S} = 21.75\,\mathrm{GeV}$



$$\sim (1-x)^2$$

pQCD counting rules

Farrar, Jackson;
Berger, Brodsky; Yuan
Blankenbecler, Gunion, Nason

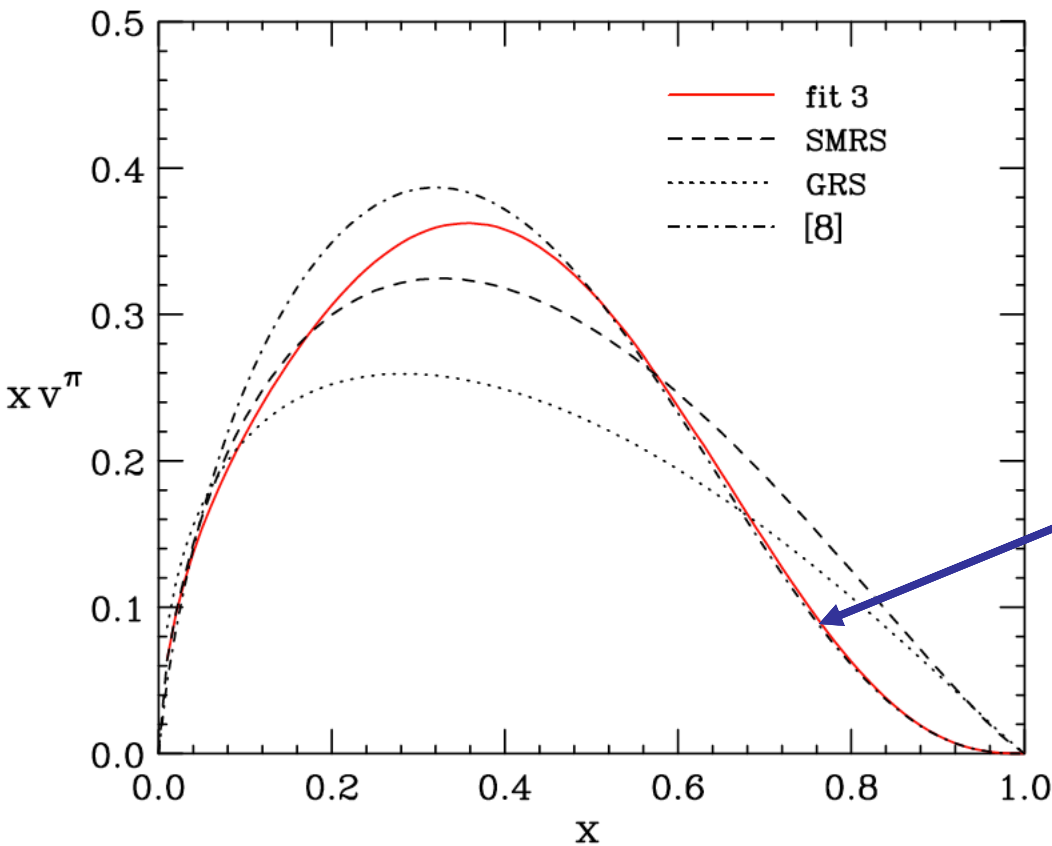
Dyson-Schwinger

Hecht et al.

→ LO adequate?

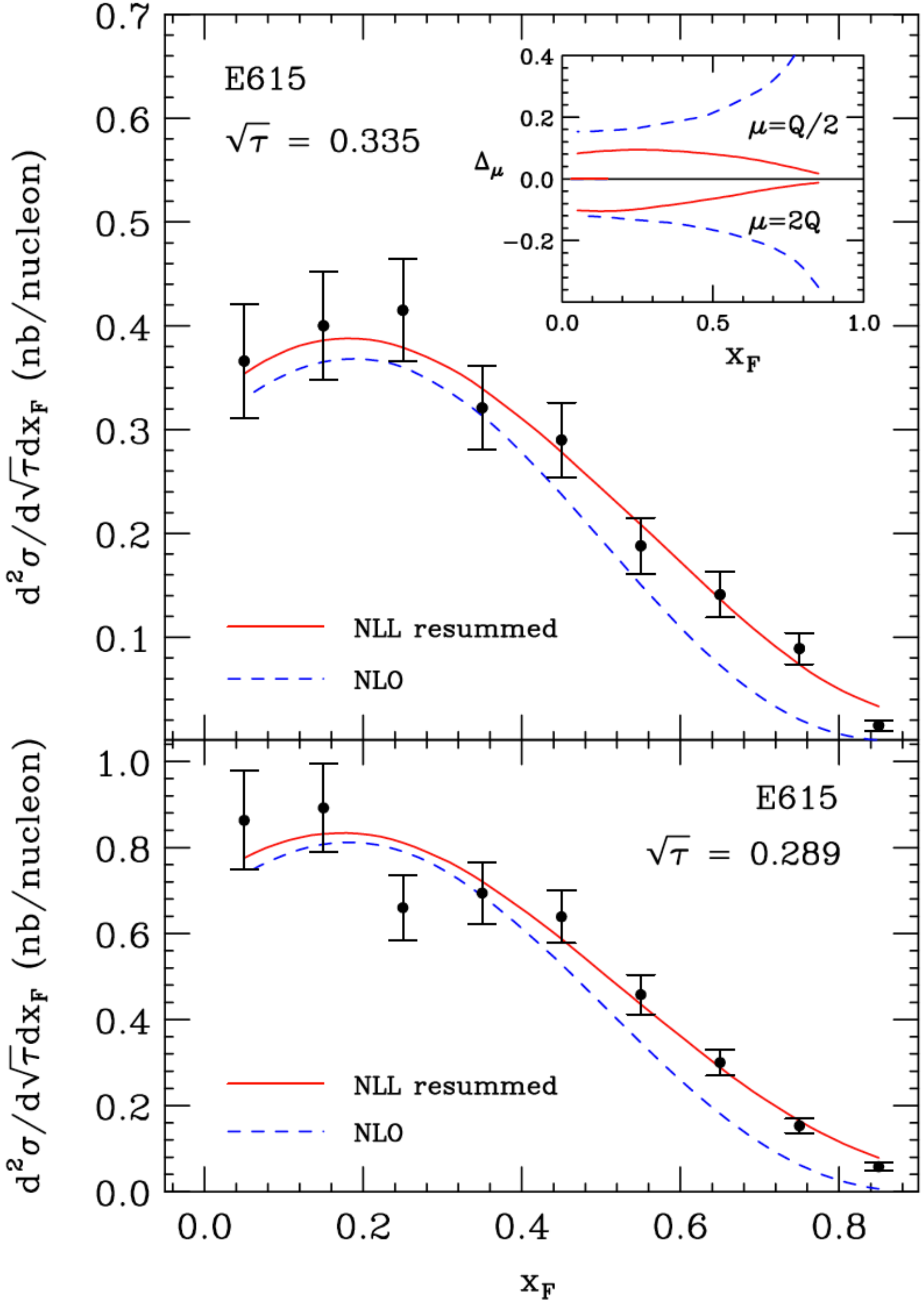
$$xv^{\pi}(x, Q_0^2) = N_v x^{\alpha}(1-x)^{\beta}(1+\gamma x^{\delta})$$ $Q_0 = 0.63\,\mathrm{GeV}$

Fit	$2\langle xv^{\pi}\rangle$	α	β	γ	K	χ^2 (no. of points)
1	0.55	0.15 ± 0.04	1.75 ± 0.04	89.4	0.999 ± 0.011	82.8 (70)
2	0.60	0.44 ± 0.07	1.93 ± 0.03	25.5	0.968 ± 0.011	80.9 (70)
3	0.65	0.70 ± 0.07	2.03 ± 0.06	13.8	0.919 ± 0.009	80.1 (70)
4	0.7	1.06 ± 0.05	2.12 ± 0.06	6.7	0.868 ± 0.009	81.0 (70)



$Q = 4\ \mathrm{GeV}$

$\sim (1-x)^{2.34}$


E615
$\sqrt{\tau}$ = 0.335
Δ_μ
μ=Q/2
μ=2Q
x_F
NLL resummed
NLO
E615
$\sqrt{\tau}$ = 0.289
NLL resummed
NLO
$d^2\sigma/d\sqrt{\tau}dx_F$ (nb/nucleon)
x_F

Possible improvements

- extension of previous work to NNLL (or even beyond) would be straightforward:

Fixed Order →

Resummation ↓

LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2	
...	...	...	...	...	...	...
N^{k}LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	$\alpha_s^k L^{2k-5}$

LL NLL NNLL

$$\tilde{\omega}^{(\mathrm{res})}_{q\bar{q}}(N) \;=\; e_q^2\,\mathcal{C}\big(\alpha_s(Q^2)\big)\;\exp\left[2\int_0^1 dy\,\frac{y^N-1}{1-y}\int_{\mu_F^2}^{Q^2(1-y)^2}\frac{dk_\perp^2}{k_\perp^2}A_q\big(\alpha_s(k_\perp^2)\big)\right.$$

$$\left.+\int_0^1 dy\,\frac{y^N-1}{1-y}\,D_q\big(\alpha_s(Q^2(1-y)^2)\big)\right]$$

A. Vogt

$$D_q(\alpha_s) \;=\; \left(\frac{\alpha_s}{\pi}\right)^2\left\{C_F\left[C_A\left(-\frac{101}{27}+\frac{11}{3}\zeta(2)+\frac{7}{2}\zeta(3)\right)+N_f\left(\frac{14}{27}-\frac{2}{3}\zeta(2)\right)\right]\right\}+\mathcal{O}(\alpha_s^3)$$

- also, may consider subleading-power contributions, esp. $\log(N)/N$

Bonocore, Laenen, Magnea, Vernazza, White;
Beneke, Broggio et al.

- πN data require resummation at fixed rapidity (or x_F)

$$\frac{d\sigma}{dQ^2 dy} = \sigma_0 \sum_{a,b} \int_{x_1^0}^1 \frac{dx_1}{x_1} f_a(x_1) \int_{x_2^0}^1 \frac{dx_2}{x_2} f_b(x_2)\, \Omega_{ab}\left(\xi_1 = \frac{x_1^0}{x_1}, \xi_2 = \frac{x_2^0}{x_2}, \ldots\right)$$

where

$$x_1^0 = \sqrt{\frac{Q^2}{S}}\, \mathrm{e}^{y} \qquad x_2^0 = \sqrt{\frac{Q^2}{S}}\, \mathrm{e}^{-y}$$

- LO:

$$\Omega_{q\bar{q}}^{(\mathrm{LO})} \propto \delta(1-\xi_1)\,\delta(1-\xi_2)$$

- NLO:

$$\omega_{q\bar{q}}^{(\mathrm{NLO})} \propto \frac{\alpha_s}{2\pi} C_F \left[\delta(1-\xi_1) \left(\frac{\ln(1-\xi_2)}{1-\xi_2}\right)_+ + \delta(1-\xi_2) \left(\frac{\ln(1-\xi_1)}{1-\xi_1}\right)_+ \right.$$

$$\left. + \frac{1}{(1-\xi_1)_+(1-\xi_2)_+} + \ldots \right]$$

$$\frac{d\sigma}{dQ^2 dy} = \sigma_0 \sum_{a,b} \int_{x_1^0}^1 \frac{dx_1}{x_1} f_a(x_1) \int_{x_2^0}^1 \frac{dx_2}{x_2} f_b(x_2)\, \Omega_{ab}\left(\xi_1 = \frac{x_1^0}{x_1}, \xi_2 = \frac{x_2^0}{x_2}, \ldots\right)$$

- take *double* Mellin moments:

$$\int_0^1 dx_1^0\, \left(x_1^0\right)^{N_1-1} \int_0^1 dx_2^0\, \left(x_2^0\right)^{N_2-1} \frac{d\sigma}{dQ^2 dy}$$

$$= \sigma_0 \sum_{a,b} \left(\int_0^1 dx_1\, x_1^{N_1-1} f_a(x_1)\right) \times \left(\int_0^1 dx_2\, x_2^{N_2-1} f_b(x_2)\right) \times \left(\int_0^1 dy_1 \int_0^1 dy_2\, y_1^{N_1-1}\, y_2^{N_2-1}\, \Omega_{ab}(y_1, y_2)\right)$$

$$\equiv \sigma_0 \sum_{a,b} f_a^{N_1} \times f_b^{N_2} \times \tilde{\Omega}_{ab}(N_1, N_2)$$

- NLL resummed expression:

Cacciari, Catani; Owens, Westmark; Ravindran et al.

$$\tilde{\Omega}_{q\bar{q}}(N_1,N_2) \;=\; e_q^2\,\mathcal{C}\big(\alpha_s(Q^2)\big)\;\exp\Bigg[\int_0^1 dy_1\,\frac{y_1^{N_1-1}-1}{1-y_1}\int_{\mu_F^2}^{(1-y_1)Q^2}\frac{dk_\perp^2}{k_\perp^2}\,A_q\big(\alpha_s(k_\perp^2)\big)$$

$$+\int_0^1 dy_2\,\frac{y_2^{N_2-1}-1}{1-y_2}\int_{\mu_F^2}^{(1-y_2)Q^2}\frac{dk_\perp^2}{k_\perp^2}\,A_q\big(\alpha_s(k_\perp^2)\big)$$

$$+\int_0^1 dy_1\,\frac{y_1^{N_1-1}-1}{1-y_1}\int_0^1 dy_2\,\frac{y_2^{N_2-1}-1}{1-y_2}\,A_q\big(\alpha_s(Q^2(1-y_1)(1-y_2))\big)\Bigg]$$

- for $N_1=N_2=N$: reduces to resummation for total Drell-Yan:

$$\tilde{\Omega}_{q\bar{q}}(N,N) \;=\; \tilde{\omega}_{q\bar{q}}(N)$$

- this approximation introduced by Laenen, Sterman
  (see also Bonvini, Forte, Ridolfi) and used in our pion paper:

$$\int_0^1 dx_1^0\,\left(x_1^0\right)^{N_1-1}\int_0^1 dx_2^0\,\left(x_2^0\right)^{N_2-1}\frac{d\sigma^{(\mathrm{res})}}{dQ^2dy}\approx\sigma_0\sum_{a,b}f_a^{N_1}\times f_b^{N_2}\times\tilde{\Omega}_{ab}^{(\mathrm{res})}(N,N)$$

where $N=\frac{1}{2}(N_1+N_2)$

- full dependence on N_1, N_2 becomes relevant at large rapidity or x_F, where it moderates the enhancement due to resummation

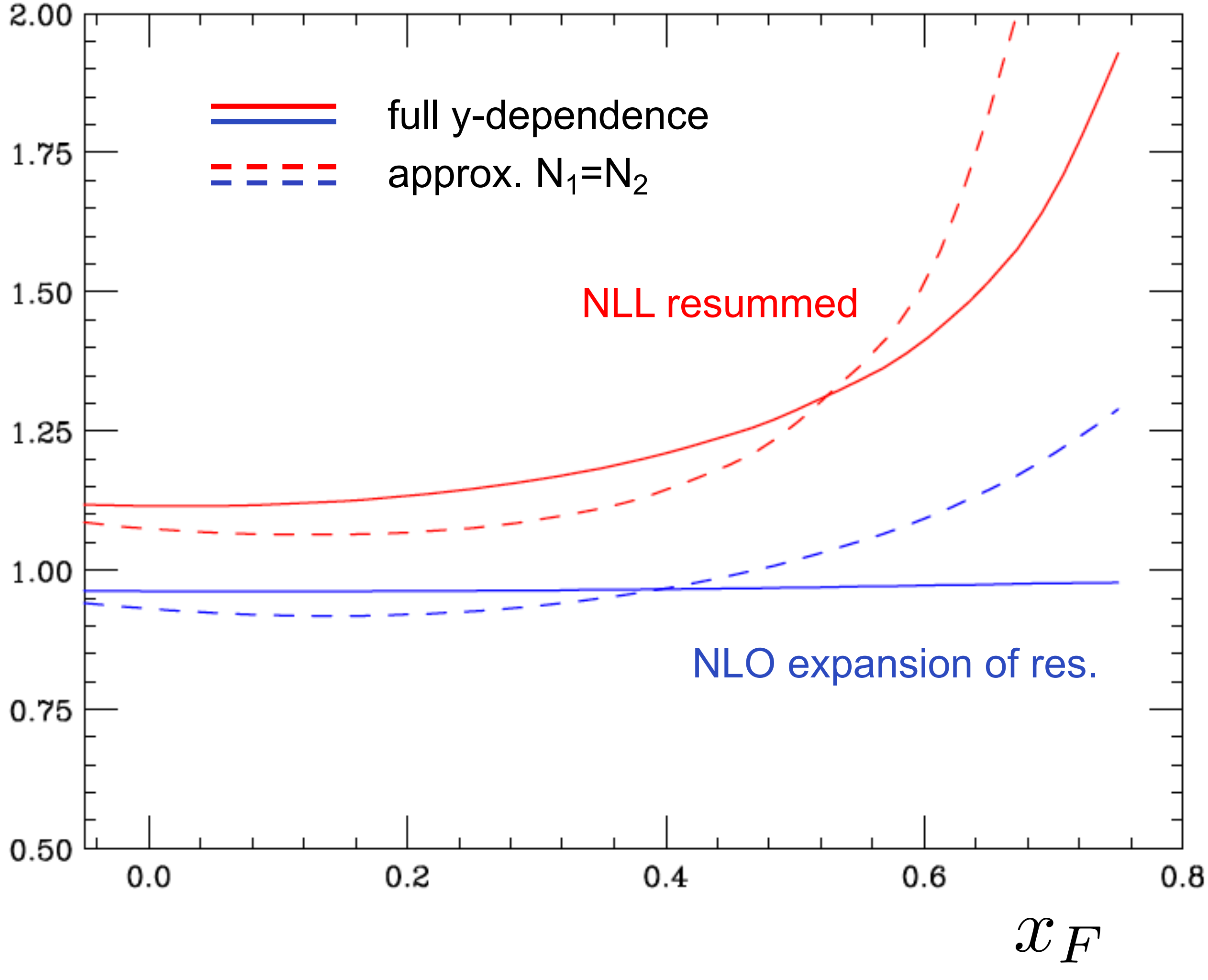
Owens, Westmark

- assumption that only PDFs carry rapidity dependence not warranted in general

Lustermans, Michel, Tackmann
Banerjee, Das, Dhani, Ravindran

$$\pi^- N \qquad \sqrt{s} = 21.7 \text{ GeV} \qquad \sqrt{\tau} = 0.335$$

$$K \equiv \frac{\sigma}{\sigma^{\rm NLO}}$$



- nevertheless, close relation between resummations for total cross section and rapidity dependence:

Exponentiation of Drell-Yan "web" diagrams near threshold gives rise to resummed expression of the form (prior to $\overline{\text{MS}}$ factn.)

$$\tilde{\Omega}^{(\text{eik})}_{q\bar{q}}(N_1, N_2, \epsilon) \propto \exp\left[\int \frac{d^{2-2\epsilon}k_T}{\Omega_{1-2\epsilon}} \int_0^{Q^2-k_T^2} dk^2\, w_{ab}\left(k^2, k_T^2+k^2, \mu^2, \alpha_s(\mu), \epsilon\right) \right.$$

Laenen, Sterman, WV

$$\left. \times \int_{\frac{k_T^2+k^2}{Q^2}}^{Q} \frac{dk^+}{k^+} \left(\mathrm{e}^{-\left(N_1 \frac{k^+}{Q} + N_2 \frac{k_T^2+k^2}{Q k^+}\right)} - 1 \right) \right]$$

- at large N_1,N_2:

$$\tilde{\Omega}^{(\text{eik})}_{q\bar{q}}(N_1, N_2, \epsilon) \propto \exp\left[2 \int \frac{d^{2-2\epsilon}k_T}{\Omega_{1-2\epsilon}} \int_0^{Q^2-k_T^2} dk^2\, w_{ab}\left(k^2, k_T^2+k^2, \mu^2, \alpha_s(\mu), \epsilon\right) \right.$$

$$\left. \times \left\{ K_0\left(2\sqrt{N_1 N_2} \sqrt{\frac{k_T^2+k^2}{Q^2}} \right) - \ln \sqrt{\frac{Q^2}{k_T^2+k^2}} \right\} \right]$$

$$\tilde{\Omega}^{(\mathrm{eik})}_{q\bar{q}}(N_1, N_2, \epsilon) \propto \exp\left[2 \int \frac{d^{2-2\epsilon} k_T}{\Omega_{1-2\epsilon}} \int_0^{Q^2-k_T^2} dk^2 \; w_{ab}\left(k^2, k_T^2+k^2, \mu^2, \alpha_s(\mu), \epsilon\right) \right.$$

$$\left. \times \left\{ K_0\left(2\sqrt{N_1 N_2} \sqrt{\frac{k_T^2+k^2}{Q^2}} \right) - \ln \sqrt{\frac{Q^2}{k_T^2+k^2}} \right\} \right]$$

- compare *total* Drell-Yan:

$$\tilde{\omega}^{(\mathrm{eik})}_{q\bar{q}}(N, \epsilon) \propto \exp\left[2 \int \frac{d^{2-2\epsilon} k_T}{\Omega_{1-2\epsilon}} \int_0^{Q^2-k_T^2} dk^2 \; w_{ab}\left(k^2, k_T^2+k^2, \mu^2, \alpha_s(\mu), \epsilon\right) \right.$$

$$\left. \times \left\{ K_0\left(2N \sqrt{\frac{k_T^2+k^2}{Q^2}} \right) - \ln \sqrt{\frac{Q^2}{k_T^2+k^2}} \right\} \right]$$

- correspondence $N \leftrightarrow \sqrt{N_1 N_2}$ to all logs (cf. Anderle, Ringer, WV)
- consistent with explicit NNLL results Banerjee, Das, Dhani, Ravindran
- N.B., full collinear kinematics N_1 large, N_2 arbitrary:

Lustermans, Michel, Tackmann

Conclusions:

- QCD radiative corrections important for extracting information on pion structure

- a lot known for higher-order corrections in Drell-Yan

- note: besides Drell-Yan, also data for $\pi N \to \gamma X$

  For global analysis, need resummation as well !

  Recent NNLL progress on single-inclusive high-p_T:

Hinderer, Ringer, Sterman, WV