Coordination of Power and Natural Gas Markets via Financial Instruments

Anna Schwele, Christos Ordoudis, Jalal Kazempour, Pierre Pinson

This document serves as an electronic companion for the paper "Coordination of Power and Natural Gas Markets via Financial Instruments". It contains seven sections: The input data for the case study is shown in Section 1. Section 2 analyses the impact of our assumption of perfect knowledge of virtual bidders. Section 3 presents the detailed formulation of all optimization problems from the original manuscript including the equivalent equilibrium problems following Remark 1. The Karush Kuhn Tucker (KKT) conditions of all optimization and equilibrium problems are provided in Section 4. Sections 5 and 6 show the proofs of Propositions 1 and 2, respectively. Section 7 contains an overview over computational performance.

Nomenclature

Sets

- $\mathcal{I}$ Set of dispatchable power production units i.
- $\mathcal{C}$ Subset of non-gas power plants ($\mathcal{C} \subset \mathcal{I}$).
- $\mathcal{G}$ Subset of natural gas-fired power plants ($\mathcal{G} \subset \mathcal{I}$).
- $\mathcal{S}$ Subset of slow-start power plants ($\mathcal{S} \subset \mathcal{I}$).
- F Subset of fast-start power plants ($\mathcal{F} \subset \mathcal{I}$).
- $\mathcal{SS}$ Subset of self-scheduling power plants ($\mathcal{SS} \subset \mathcal{I}$).
- $\mathcal{J}$ Set of wind power units j.
- $\mathcal{K}$ Set of natural gas supply units k.
- $\mathcal{R}$ Set of electricity virtual bidders r.
- $\mathcal{Q}$ Set of natural gas virtual bidders q.
- Ω Set of wind power scenarios ω.
- $\mathcal{T}$ Set of time periods t.

Note that $\mathcal{C} \cup \mathcal{G} = \mathcal{I}$, $\mathcal{F} \cap \mathcal{S} = \emptyset$, $\mathcal{F} \cap \mathcal{SS} = \emptyset$ and $\mathcal{S} \cap \mathcal{SS} = \emptyset$.

Variables

- $p_{i,t}^{\mathrm{DA}}, w_{j,t}^{\mathrm{DA}}$ Day-ahead dispatch of units i and j in period t, respectively [MW].
- $p_{i,t,\omega}^{\mathrm{RT}}$ Power production adjustment of unit i in scenario ω, period t [MW].
- $w_{j,t,\omega}^{\mathrm{RT}}$ Wind power production adjustment of unit j in scenario ω, period t [MW].
- $l_{t,\omega}^{\mathrm{sh,E}}, l_{t,\omega}^{\mathrm{sh,G}}$ Electricity and natural gas load shedding under scenario ω, period t [MW, kcf/h].
- $g_{k,t}^{\mathrm{DA}}$ Day-ahead dispatch of unit k in period t [kcf/h].

$g^{\mathrm{RT}}_{k,t,\omega}$ Natural gas adjustment by unit k in scenario ω, period t [kcf/h].
$\hat{\lambda}^{\mathrm{E}}_t$ Day-ahead electricity price in period t [\$/MWh].
$\tilde{\lambda}^{\mathrm{E}}_{t,\omega}$ Probability-weighted real-time electricity price in period t, scenario ω [\$/MWh].
$\hat{\lambda}^{\mathrm{G}}_t$ Day-ahead natural gas price in period t [\$/kcf].
$\tilde{\lambda}^{\mathrm{G}}_{t,\omega}$ Probability-weighted real-time natural gas price in period t, scenario ω [\$/kcf].
μ, ν Set of dual variables in day-ahead and real-time markets, respectively.
$c^{\mathrm{DA}}_{i,t}$ Start-up cost of dispatchable unit i in period t [\$].
$c^{\mathrm{RT}}_{i,t,\omega}$ Start-up cost adjustment of dispatchable fast-start unit i in period t under scenario s [\$].
$u^{\mathrm{DA}}_{i,t}$ Relaxed unit commitment status of dispatchable unit i in period t.
$u^{\mathrm{RT}}_{i,t,\omega}$ Relaxed unit commitment adjustment of fast-start unit i in period t, scenario ω.
$v^{\mathrm{DA,E}}_{r,t}$ Day-ahead trade of electricity virtual bidder r in period t [MW].
$v^{\mathrm{RT,E}}_{r,t}$ Real-time trade of electricity virtual bidder r in period t [MW].
$v^{\mathrm{DA,G}}_{q,t}$ Day-ahead trade of natural gas virtual bidder q in period t [kcf/h].
$v^{\mathrm{RT,G}}_{q,t}$ Real-time trade of natural gas virtual bidder q in period t [kcf/h].

Parameters

D^{E}_t Electricity demand in period t [MWh].
D^{G}_t Natural gas demand in period t [kcf/h].
C^{E}_i Production cost of unit i [\$/MWh].
$C^{\mathrm{sh,E}}$ Value of electricity lost load [\$/MWh].
C^{G}_k Day-ahead offer price of unit k [\$/kcf].
$C^{\mathrm{sh,G}}$ Value of natural gas lost load [\$/kcf].
$P^{\max}_i$ Capacity of dispatchable unit i [MW].
$P^{\min}_i$ Minimum production level of dispatchable unit i [MW].
ϕ_i Power conversion factor of natural gas unit $i \in G$ [kcf/MWh].
$W_{j,t,\omega}$ Wind power realization of unit j in period t, scenario ω [MW].
$W^{\mathrm{DA}}_{j,t}$ Day-ahead wind power forecast for unit j in period t [MW].
$\overline{W}_j$ Capacity of wind power unit j [MW].
$G^{\max}_k$ Capacity of natural gas unit k [kcf].
G^{adj}_k Adjustment limit of natural gas unit k [kcf/h].
π_ω Probability of scenario ω.
C^{SU}_i Start-up cost of dispatchable unit i [\$].
U^{ini}_i Initial commitment status of dispatchable unit i [0/1].
P^{ini}_i Initial dispatch of unit i [MW].
R_i Up/down ramping limit of dispatchable unit i [MW/h].

1. Input Data

Table 3 gives the technical characteristics of power generators, whose columns one to ten show the unit name, minimum power production ($P^{\min}_i$), capacity ($P^{\max}_i$), ramp rate (R_i), start-up cost

Unit	$P_i^{\min}$ [MW]	$P_i^{\max}$ [MW]	R_i [MW/h]	C_i^{SU} [$]	U^{ini} [0/1]	P_i^{ini} [MWh]	Type	C_i^{E} [$/MWh]	ϕ_i [kcf/MWh]
$\mathcal{C}^1$	0	40	20	17,462	1	40	non gas-fired	22.18	-
$\mathcal{C}^2$	0	152	50	13,207	1	100	non gas-fired	33.2	-
$\mathcal{C}^3$	0	300	195	22,313	0	0	non gas-fired	37.14	-
$\mathcal{C}^4$	100	591	230	28,272	0	0	non gas-fired	38.2	-
$\mathcal{C}^5$	400	400	400	50,000	1	400	non gas-fired	22.34	-
$\mathcal{C}^6$	0	350	80	33,921	0	0	non gas-fired	20.92	-
$\mathcal{G}^1$	0	155	100	21,450	1	100	gas-fired	-	15.23
$\mathcal{G}^2$	0	60	60	10,721	0	0	gas-fired	-	16.98
$\mathcal{G}^3$	0	310	200	42,900	0	0	gas-fired	-	12.65
$\mathcal{G}^4$	0	300	150	10,000	0	0	gas-fired	-	14.88

Table 1 Technical characteristics of power generators

Supplier	$G_k^{\min}$ [kcf]	$G_k^{\max}$ [kcf]	G_k^{adj} [kcf/h]	C_k^{G} [$/kcf]
$\mathcal{K}^1$	0	9,000	1,000	2
$\mathcal{K}^2$	0	6,000	1,000	2.2
$\mathcal{K}^3$	0	15,000	1,000	2.5
$\mathcal{K}^4$	0	15,000	1,000	3.3

Table 2 Technical characteristics of gas suppliers

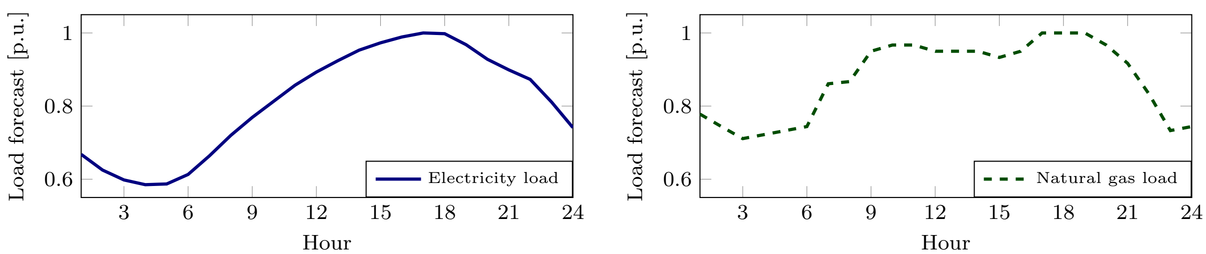

Figure 1 Electricity and natural gas demand. The plots on the left- and right-hand sides show the total hourly demand for power and natural gas, respectively.

(C_i^{SU}), initial commitment status at the beginning of time horizon (U^{ini}), initial dispatch (P^{ini}), type, production cost for non gas-fired generators (C_i^{E}), and gas-to-power conversion ratio for gas-fired generators (ϕ_i), respectively. In addition, Table 4 provides the technical characteristics of four gas suppliers, including minimum and maximum gas capacity ($G_k^{\min}$ and $G_k^{\max}$), ramp rate (G_k^{adj}), and supply cost (C_k^{G}). The total hourly demand in both power and natural gas sectors is shown in Fig. 1. The profile of deterministic wind power forecast (in per-unit) in day-ahead is illustrated by a solid curve in the left-hand side plot of Fig. 2, while the right-hand side plot provides the five equiprobable wind scenarios that may realize in real-time. Due to potential forecast error in day-ahead, observe that the day-ahead deterministic forecast (solid curve in the left-hand side plot) is not necessarily identical to the expected wind power realization in real-time (dashed curve in the same plot). In this case, the day-ahead wind forecast underestimates the available wind power production during hours 1 to 6 and 19 to 23, while overestimates it from hour 7 to 18.

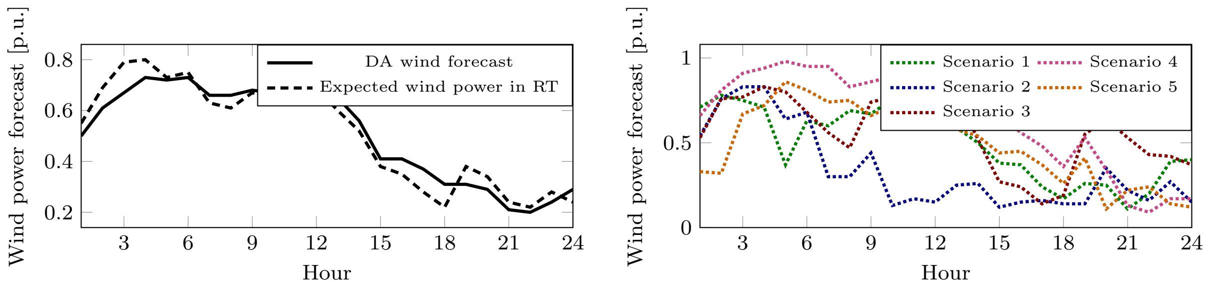

Figure 2 Wind power forecast in day-ahead (DA) and potential scenarios in real-time (RT): The upper plot shows the deterministic wind power forecast in DA and the expected value of five wind power scenarios in RT. These five equiprobable scenarios in RT are depicted in the lower plot.

		Seq	*Seq+eVB*	*Seq+iVB*	*Seq+VB*	*Ideal*
In-sample		\$1,464,320	-6.83%	-6.37%	-6.94%	-7.06%
Out-of-sample	Same distribution	\$1,360,886	-4.49%	-3.36%	-4.29%	-5.33%
	Higher first moments	\$1.344.285	-3.91%	-3.06%	-3.73%	-4.76%
	Lower first moments	\$1,410,223	-5.24%	-3.45%	-4.85%	-6.12%
	Different distribution	\$1,252.643	-3.30%	-3.99%	-3.79%	-4.60%

Table 3 Total expected cost of the electricity and natural gas systems under different market setups for in-sample and out-of-sample scenarios. The percentages show the differences in the total expected system cost compared to that cost in the fully uncoordinated sequential setup Seq.

2. Out-of-sample Analysis

This section presents an ex-post out-of-sample analysis to evaluate the performance of the proposed market setups against our assumption of perfect knowledge of virtual bidders and against estimations of natural gas prices. For this purpose, we test the impact of unseen scenarios on the market setups. To assess the impact of the assumption of perfect knowledge of virtual bidders on market outcomes, we generate a set of 100 new scenarios from the same distribution and 100 scenarios from a distribution with different first and second moments. Fixing the day-ahead schedules to those obtained with the original in-sample simulations, we solve a real-time electricity market and then a real-time gas market for each out-of-sample scenario. The expected total system costs achieved with the sequential setup *Seq* decrease compared to those in the in-sample simulation, due to the updated expected electricity and gas adjustment costs under the previously unseen scenarios, as exhibited in Table 3. The fully coordinated ideal model *Ideal* still provides a lower bound for the expected total system cost. The setups including soft coordination via financial instruments consistently achieve lower expected system costs compared to the fully sequential setup *Seq*. The effectiveness of virtual bidders to improve sectoral and temporal coordination and make day-ahead schedules more efficient is not overly sensitive to the quality of information of these agents.

3. Optimization Problems

3.1. Explicit Electricity Virtual Bidder

The profit maximization problem of each virtual bidder r participating in the electricity market given the day-ahead and expectation of real-time prices $\hat{\lambda}_t^{\mathrm{E}}$ and $\tilde{\lambda}_{t,\omega}^{\mathrm{E}}$, respectively, is given below:

$$\left\{\begin{aligned} &\max_{v_{r,t}^{\mathrm{DA,E}},v_{r,t}^{\mathrm{RT,E}}} \sum_{t\in\mathcal{T}} \left(\hat{\lambda}_t^{\mathrm{E}}\ v_{r,t}^{\mathrm{DA,E}} + \sum_{\omega\in\Omega}\tilde{\lambda}_{t,\omega}^{\mathrm{E}}\right)\ v_{r,t}^{\mathrm{RT,E}} && \text{(1a)}\\ &\text{subject to } v_{r,t}^{\mathrm{DA,E}} + v_{r,t}^{\mathrm{RT,E}} = 0 : \rho_{r,t},\ \ \forall t, \end{aligned}\right\},\ \ \forall r\in\mathcal{R}, \qquad \text{(1b)}$$

where $\Theta^{\mathrm{VBE}} = \{v_{r,t}^{\mathrm{DA,E}}, v_{r,t}^{\mathrm{RT,E}}, \forall r,t\}$ is the set of primal optimization variables. Objective function (1a) maximizes the expected profit of arbitraging in the day-ahead and real-time electricity markets. Equation (1b) ensures that each virtual bidder sells (buys) the same amount back in the real-time market that was bought (sold) in the day-ahead market. The market operators treat the virtual bidders' dispatch decision as fixed input into the market clearing in the following.

3.2. Day-Ahead Electricity Market

The day-ahead electricity market clears with given day-ahead positions of virtual trade as:

$$\min_{\Theta^{\mathrm{EDA}}} \sum_{t\in\mathcal{T}} \Big(\sum_{i\in\mathcal{C}} C_i^{\mathrm{E}}\ p_{i,t}^{\mathrm{DA}} + \sum_{i\in\mathcal{G}} \hat{\lambda}_t^{\mathrm{G}}\ \phi_i\ p_{i,t}^{\mathrm{DA}} + \sum_{i\in\mathcal{I}} c_{i,t}^{\mathrm{DA}}\Big) \tag{2a}$$

$$\text{subject to } \sum_{i\in\mathcal{I}} p_{i,t}^{\mathrm{DA}} + \sum_{j\in\mathcal{J}} w_{j,t}^{\mathrm{DA}} - D_t^{\mathrm{E}} + \sum_{r\in\mathcal{R}} v_{r,t}^{\mathrm{DA,E}} = 0 : \hat{\lambda}_t^{\mathrm{E}},\ \forall t, \tag{2b}$$

$$0 \le w_{j,t}^{\mathrm{DA}} \le W_{j,t}^{\mathrm{DA}} : \underline{\mu}_{j,t}^{\mathrm{W}}, \overline{\mu}_{j,t}^{\mathrm{W}},\ \ \forall j,t, \tag{2c}$$

$$u_{i,t}^{\mathrm{DA}}\ P_i^{\min} \le p_{i,t}^{\mathrm{DA}} \le u_{i,t}^{\mathrm{DA}}\ P_i^{\max} : \underline{\mu}_{i,t}^{\mathrm{P}}, \overline{\mu}_{i,t}^{\mathrm{P}},\ \ \forall i\in\mathcal{I},t, \tag{2d}$$

$$-u_{i,(t-1)}^{\mathrm{DA}}\ R_i \le \left(p_{i,t}^{\mathrm{DA}} - p_{i,(t-1)}^{\mathrm{DA}}\right) \le u_{i,t}^{\mathrm{DA}}\ R_i : \underline{\mu}_{i,t}^{\mathrm{R}}, \overline{\mu}_{i,t}^{\mathrm{R}},\ \ \forall i\in\mathcal{I}, t>1, \tag{2e}$$

$$-U_i^{\mathrm{ini}}\ R_i \le \left(p_{i,t}^{\mathrm{DA}} - P_i^{\mathrm{ini}}\right) \le u_{i,t}^{\mathrm{DA}}\ R_i : \underline{\mu}_{i,t}^{\mathrm{R}}, \overline{\mu}_{i,t}^{\mathrm{R}},\ \ \forall i\in\mathcal{I}, t=1, \tag{2f}$$

$$0 \le u_{i,t}^{\mathrm{DA}} \le 1 : \underline{\mu}_{i,t}^{\mathrm{B}}, \overline{\mu}_{i,t}^{\mathrm{B}},\ \ \forall i\in\mathcal{I},t, \tag{2g}$$

$$C_i^{\mathrm{SU}}\ \left(u_{i,t}^{\mathrm{DA}} - u_{i,(t-1)}^{\mathrm{DA}}\right) \le c_{i,t}^{\mathrm{DA}} : \overline{\mu}_{i,t}^{\mathrm{SU}},\ \ \forall i\in\mathcal{I}, t>1, \tag{2h}$$

$$C_i^{\mathrm{SU}}\left(u_{i,t}^{\mathrm{DA}} - U_i^{\mathrm{ini}}\right) \le c_{i,t}^{\mathrm{DA}} : \overline{\mu}_{i,t}^{\mathrm{SU}},\ \ \forall i\in\mathcal{I}, t=1, \tag{2i}$$

$$0 \le c_{i,t}^{\mathrm{DA}} : \underline{\mu}_{i,t}^{\mathrm{SU}},\ \ \forall i\in\mathcal{I},t, \tag{2j}$$

where $\Theta^{\mathrm{EDA}} = \{p_{i,t}^{\mathrm{DA}}, c_{i,t}^{\mathrm{DA}}, u_{i,t}^{\mathrm{DA}}, \forall i\in\mathcal{I},t; w_{j,t}^{\mathrm{DA}}, \forall j,t\}$ is the set of primal optimization variables. The objective (2a) of the deterministic day-ahead market-clearing problem is to minimize the day-ahead generation cost. The total cost stems from the cost of non-gas and gas-fired power plants. These units are assumed to bid in the market truthfully, i.e., offer at prices equal to their marginal cost of production. For the case of gas-fired units, we assume that the marginal cost of production is described by a linear function of the estimated natural gas price, i.e., $C_i = \hat{\lambda}_t^{\mathrm{G}}\phi_i$, $\forall i\in\mathcal{G}$. Constraint

(2b) is the day-ahead power balance with inelastic demand treating the virtual day-ahead positions $\sum_{r\in\mathcal{R}} v_{r,t}^{\mathrm{DA,E}}$ as given inputs. Constraints (2c) and (2d) enforce lower and upper bounds on the day-ahead dispatch of wind and conventional generation. Constraints (2e), (2f) ensure the ramping limits of conventional generators and represent in combination with (2g) the tight relaxation of unit commitment. Constraints (2h), (2i), and (2j) enforce the start-up cost of each generator.

The optimization problem for day-ahead electricity market clearing can be equivalently formulated as the following equilibrium model with each unit maximizing her profit and a price-setting agent according to Remark 1. Each non gas-fired generator $\mathcal{C}$ maximizes her day-ahead profit with respect to her operational constraints according to

$$\left\{\begin{aligned} &\max_{p_{i,t}^{\mathrm{DA}},c_{i,t}^{\mathrm{DA}},u_{i,t}^{\mathrm{DA}}} \sum_{t\in\mathcal{T}} \left[\left(\hat{\lambda}_t^{\mathrm{E}} - C_i^{\mathrm{E}}\right) p_{i,t}^{\mathrm{DA}} - c_{i,t}^{\mathrm{DA}}\right] && \text{(3a)} \\ &\text{subject to } \ (2\mathrm{d}) - (2\mathrm{j}) && \text{(3b)} \end{aligned}\right\} \ \forall (i \in \mathcal{C}).$$

Similarly, gas-fired generator $\mathcal{G}$ decides her day-ahead dispatch based on estimated marginal cost via natural gas price forecast $\hat{\lambda}_t^{\mathrm{G}}$:

$$\left\{\begin{aligned} &\max_{p_{i,t}^{\mathrm{DA}},c_{i,t}^{\mathrm{DA}},u_{i,t}^{\mathrm{DA}}} \sum_{t\in\mathcal{T}} \left[\left(\hat{\lambda}_t^{\mathrm{E}} - \hat{\lambda}_t^{\mathrm{G}}\ \phi_i\right) p_{i,t}^{\mathrm{DA}} - c_{i,t}^{\mathrm{DA}}\right] && \text{(4a)} \\ &\text{subject to } \ (2\mathrm{d}) - (2\mathrm{j}) && \text{(4b)} \end{aligned}\right\} \ \forall (i \in \mathcal{G}).$$

Wind farm $\mathcal{J}$ maximizes her profit according to the day-ahead wind forecast $W_{j,t}^{\mathrm{DA}}$ as

$$\left\{\begin{aligned} &\max_{w_{j,t}^{\mathrm{DA}}} \sum_{t\in\mathcal{T}} \hat{\lambda}_t^{\mathrm{E}}\ w_{j,t}^{\mathrm{DA}} && \text{(5a)} \\ &\text{subject to } \ (2\mathrm{c}) && \text{(5b)} \end{aligned}\right\} \ \forall j \in \mathcal{J}.$$

A price setting agent decides the day-ahead electricity price $\hat{\lambda}_t^{\mathrm{E}}$ according to

$$\min_{\hat{\lambda}_t^{\mathrm{E}}} \ \sum_{t\in\mathcal{T}} \hat{\lambda}_t^{\mathrm{E}} \Big(\sum_{i\in\mathcal{I}} p_{i,t}^{\mathrm{DA}} + \sum_{j\in\mathcal{J}} w_{j,t}^{\mathrm{DA}} - D_t^{\mathrm{E}} + \sum_{r\in\mathcal{R}} v_{r,t}^{\mathrm{DA,E}}. \Big) \tag{6a}$$

The equilibrium problem (3)-(6) is equivalent to the day-ahead market optimization problem (2), since the Karush-Kuhn-Tucker (KKT) conditions are identical, see Section 4.

3.3. Real-Time Electricity Market

In real-time operation, wind power production $W_{j,t,\omega}$ is realized and the real-time markets are cleared to adjust for imbalances. The day-ahead schedule is treated as fixed parameters in the following formulation. The RT market-clearing problem for wind generation scenario ω is formulated as

$$\left\{ \min_{\Theta^{\mathrm{ERT}}} \ \pi_\omega \sum_{t\in\mathcal{T}} \Big(\sum_{i\in\mathcal{C}} C_i^{\mathrm{E}} \ p_{i,t,\omega}^{\mathrm{RT}} + \sum_{i\in\mathcal{G}} \tilde{\lambda}_{t,\omega}^{\mathrm{G}} \ \phi_i \ p_{i,t,\omega}^{\mathrm{RT}} + C^{\mathrm{sh,E}} \ l_{t,\omega}^{\mathrm{sh,E}} + \sum_{i\in\mathcal{F}} c_{i,t,\omega}^{\mathrm{RT}} \Big) \right. \tag{7a}$$

$$\text{subject to} \ \sum_{i\in\mathcal{I}} p_{i,t,\omega}^{\mathrm{RT}} + l_{t,\omega}^{\mathrm{sh,E}} + \sum_{r\in\mathcal{R}} v_{r,t}^{\mathrm{RT,E}} + \sum_{j\in\mathcal{J}} w_{j,t,\omega}^{\mathrm{RT}} = 0 : \tilde{\lambda}_{t,\omega}^{\mathrm{E}}, \ \forall t, \tag{7b}$$

$$0 \le l_{t,\omega}^{\mathrm{sh,E}} \le D_t^{\mathrm{E}} : \underline{\nu}_{t,\omega}^{\mathrm{DE}}, \overline{\nu}_{t,\omega}^{\mathrm{DE}}, \ \ \forall t, \tag{7c}$$

$$0 \le \left(w_{j,t}^{\mathrm{DA}} + w_{j,t,\omega}^{\mathrm{RT}} \right) \le W_{j,t,\omega} : \underline{\nu}_{j,t,\omega}^{\mathrm{W}}, \overline{\nu}_{j,t,\omega}^{\mathrm{W}}, \ \ \forall j,t, \tag{7d}$$

$$u_{i,t}^{\mathrm{DA}} \ P_i^{\min} \le \left(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} \right) \le u_{i,t}^{\mathrm{DA}} \ P_i^{\max} : \underline{\nu}_{i,t,\omega}^{\mathrm{P}}, \overline{\nu}_{i,t,\omega}^{\mathrm{P}}, \ \ \forall i \in \mathcal{S}, t, \tag{7e}$$

$$-u_{i,(t-1)}^{\mathrm{DA}} \ R_i \le \left(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - p_{i,(t-1)}^{\mathrm{DA}} - p_{i,(t-1),\omega}^{\mathrm{RT}} \right) \le u_{i,t}^{\mathrm{DA}} \ R_i : \underline{\nu}_{i,t,\omega}^{\mathrm{R}}, \overline{\nu}_{i,t,\omega}^{\mathrm{R}}, \ \ \forall i \in \mathcal{S}, t > 1, \tag{7f}$$

$$-U_i^{\mathrm{ini}} \ R_i \le \left(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - P_i^{\mathrm{ini}} \right) \le u_{i,t}^{\mathrm{DA}} \ R_i : \underline{\nu}_{i,t,\omega}^{\mathrm{R}}, \overline{\nu}_{i,t,\omega}^{\mathrm{R}}, \ \ \forall i \in \mathcal{S}, t=1, \tag{7g}$$

$$\left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} \right) P_i^{\min} \le \left(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} \right) \le \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} \right) P_i^{\max} : \underline{\nu}_{i,t,\omega}^{\mathrm{P}}, \overline{\nu}_{i,t,\omega}^{\mathrm{P}}, \ \ \forall i \in \mathcal{F}, t, \tag{7h}$$

$$\begin{aligned} -\left(u_{i,(t-1)}^{\mathrm{DA}} + u_{i,(t-1),\omega}^{\mathrm{RT}} \right) R_i \le \left(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - p_{i,(t-1)}^{\mathrm{DA}} - p_{i,(t-1),\omega}^{\mathrm{RT}} \right) \le \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} \right) R_i \\ : \underline{\nu}_{i,t,\omega}^{\mathrm{R}}, \overline{\nu}_{i,t,\omega}^{\mathrm{R}}, \ \ \forall i \in \mathcal{F}, t > 1, \end{aligned} \tag{7i}$$

$$-U_i^{\mathrm{ini}} \ R_i \le \left(p_{i,t}^{\mathrm{DA}} + p_{i,t,\omega}^{\mathrm{RT}} - P_i^{\mathrm{ini}} \right) \le \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} \right) \ R_i : \underline{\nu}_{i,t,\omega}^{\mathrm{R}}, \overline{\nu}_{i,t,\omega}^{\mathrm{R}}, \ \ \forall i \in \mathcal{F}, t=1, \tag{7j}$$

$$0 \le \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} \right) \le 1 : \underline{\nu}_{i,t,\omega}^{\mathrm{B}}, \overline{\nu}_{i,t,\omega}^{\mathrm{B}}, \ \ \forall i \in \mathcal{F}, t, \tag{7k}$$

$$C_i^{\mathrm{SU}} \ \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} - u_{i,(t-1)}^{\mathrm{DA}} - u_{i,(t-1),\omega}^{\mathrm{RT}} \right) \le \left(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}} \right) : \overline{\nu}_{i,t,\omega}^{\mathrm{SU}}, \ \ \forall i \in \mathcal{F}, t, \tag{7l}$$

$$C_i^{\mathrm{SU}} \ \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} - U_i^{\mathrm{ini}} \right) \le \left(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}} \right) : \overline{\nu}_{i,t,\omega}^{\mathrm{SU}}, \ \ \forall i \in \mathcal{F}, t = 1, \tag{7m}$$

$$\left. 0 \le \left(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}} \right) : \underline{\nu}_{i,t,\omega}^{\mathrm{SU}}, \ \ \forall i \in \mathcal{F}, t \right\} \forall \omega, \tag{7n}$$

where $\Theta^{\mathrm{ERT}} = \{ p_{i,t,\omega}^{\mathrm{RT}}, \forall i \in \mathcal{I}, t, \omega; w_{j,t,\omega}^{\mathrm{RT}}, \forall j,t,\omega; \ l_{t,\omega}^{\mathrm{sh,E}}, \ \forall t,\omega; \ u_{i,t,\omega}^{\mathrm{RT}}, \ c_{i,t,\omega}^{\mathrm{RT}} \ \forall i \in \mathcal{F}, t, \omega \}$ is the set of primal optimization variables. The objective of (7) is to minimize the probability-weighted system cost in the real-time market under scenarios ω. Objective function (7a) describes the real-time cost of power adjustments to cover excess or deficit of wind power production. Electricity load shedding cost is also taken into account. Constraint (7b) balances the deviations in real-time from the day-ahead schedule with the position of virtual bidders $\sum_{r\in\mathcal{R}} v_{r,t}^{\mathrm{RT,E}}$ as fixed input. Constraints (7c),(7d), (7e), and (7h) enforce lower and upper bounds on the real-time adjustment of load levels, wind generation, and conventional slow- and fast-starting generators, respectively. Constraints (7f),(7g),(7i), and (7j) ensure the ramp-rate limits of conventional slow- and fast-starting generators and represent in combination with (7k) the tight relaxation of unit commitment for fast-starting units. Constraints (7l), (7m), and (7n) enforce the start-up cost of fast-starting generators.

Following Remark 1, optimization problem (7) can be equivalently formulated as the following equilibrium problem. Slow-starting non gas-fired generator $\mathcal{C} \cap \mathcal{S}$ maximizes her profit in real-time with respect to her day-ahead commitment decision as

$$\left\{\begin{aligned} &\max_{p^{\mathrm{RT}}_{i,t,\omega}} \sum_{t\in\mathcal{T}} \left(\tilde{\lambda}^{\mathrm{E}}_{t,\omega} - \pi_\omega \; C^{\mathrm{E}}_i\right) p^{\mathrm{RT}}_{i,t,\omega} && \text{(8a)}\\ &\text{subject to } \; (7\mathrm{e}) - (7\mathrm{g}) \end{aligned}\right\} \forall (i \in \mathcal{C} \cap \mathcal{S}), \omega, \quad \text{(8b)}$$

while each fast-starting generator $\mathcal{C} \cap \mathcal{F}$ can update her commitment decision in real-time according to

$$\left\{\begin{aligned} &\max_{p^{\mathrm{RT}}_{i,t,\omega}, c^{\mathrm{RT}}_{i,t,\omega}, u^{\mathrm{RT}}_{i,t,\omega}} \sum_{t\in\mathcal{T}} \left[\left(\tilde{\lambda}^{\mathrm{E}}_{t,\omega} - \pi_\omega \; C^{\mathrm{E}}_i\right) p^{\mathrm{RT}}_{i,t,\omega} - \pi_\omega \; c^{\mathrm{RT}}_{i,t,\omega}\right] && \text{(9a)}\\ &\text{subject to } \; (7\mathrm{h}) - (7\mathrm{n}) \end{aligned}\right\} \forall (i \in \mathcal{C} \cap \mathcal{F}), \omega. \quad \text{(9b)}$$

Similarly, gas-fired generators optimize their dispatch decisions in real-time based on real-time natural gas price estimation $\tilde{\lambda}^{\mathrm{G}}_{t,\omega}$ as slow-starters $\mathcal{G} \cap \mathcal{S}$ according to

$$\left\{\begin{aligned} &\max_{p^{\mathrm{RT}}_{i,t,\omega}} \sum_{t\in\mathcal{T}} \left(\tilde{\lambda}^{\mathrm{E}}_{t,\omega} - \pi_\omega \; \tilde{\lambda}^{\mathrm{G}}_{t,\omega} \; \phi_i\right) p^{\mathrm{RT}}_{i,t,\omega} && \text{(10a)}\\ &\text{subject to } \; (7\mathrm{e}) - (7\mathrm{g}) \end{aligned}\right\} \forall (i \in \mathcal{G} \cap \mathcal{S}), \omega \quad \text{(10b)}$$

and as fast-starters $\mathcal{G} \cap \mathcal{F}$, as

$$\left\{\begin{aligned} &\max_{p^{\mathrm{RT}}_{i,t,\omega}, c^{\mathrm{RT}}_{i,t,\omega}, u^{\mathrm{RT}}_{i,t,\omega}} \sum_{t\in\mathcal{T}} \left[\left(\tilde{\lambda}^{\mathrm{E}}_{t,\omega} - \pi_\omega \; \tilde{\lambda}^{\mathrm{G}}_{t,\omega} \; \phi_i\right) p^{\mathrm{RT}}_{i,t,\omega} - \pi_\omega \; c^{\mathrm{RT}}_{i,t,\omega}\right] && \text{(11a)}\\ &\text{subject to } \; (7\mathrm{h}) - (7\mathrm{n}) \end{aligned}\right\} \forall (i \in \mathcal{G} \cap \mathcal{F}), \omega. \quad \text{(11b)}$$

Wind farm $\mathcal{J}$ adjusts her dispatch in real-time according to the actual wind power realization $W_{j,t,\omega}$:

$$\left\{\begin{aligned} &\max_{w^{\mathrm{RT}}_{j,t,\omega}} \sum_{t\in\mathcal{T}} \tilde{\lambda}^{\mathrm{E}}_{t,\omega} \; w^{\mathrm{RT}}_{j,t,\omega} && \text{(12a)}\\ &\text{subject to } \; (7\mathrm{d}) \end{aligned}\right\} \forall j, \omega. \quad \text{(12b)}$$

Power demand is able to shed load in real-time incurring cost as

$$\left\{\begin{aligned} &\min_{l^{\mathrm{sh,E}}_{t,\omega}} \sum_{t\in\mathcal{T}} \left(\pi_\omega\ C^{\mathrm{sh,E}} - \tilde{\lambda}^{\mathrm{E}}_{t,\omega}\right) l^{\mathrm{sh,E}}_{t,\omega} \quad (13a)\\ &\text{subject to (7c)} \end{aligned}\right\}\forall\omega. \quad (13b)$$

For each scenario, the real-time electricity price $\tilde{\lambda}^{\mathrm{E}}_{t,\omega}$ is set according to

$$\left\{\min_{\tilde{\lambda}^{\mathrm{E}}_{t,\omega}} \sum_{t\in\mathcal{T}} \tilde{\lambda}^{\mathrm{E}}_{t,\omega}\Big(\sum_{i\in\mathcal{I}} p^{\mathrm{RT}}_{i,t,\omega} + l^{\mathrm{sh,E}}_{t,\omega} + \sum_{j\in\mathcal{J}} w^{\mathrm{RT}}_{j,t,\omega} + \sum_{r\in\mathcal{R}} v^{\mathrm{RT,E}}_{r,t}\Big)\right\}\forall\omega. \quad (14a)$$

The equilibrium problem (8)-(14) is equivalent to the real-time market optimization problem (7) for each scenario ω.

3.4. Explicit Natural Gas Virtual Bidder

We also introduce virtual bidding in natural gas markets. Similarly to electricity virtual bidding, the profit maximization problem of each virtual bidder q participating in the natural gas market is given below for day-ahead and real-time distribution of natural gas spot price $\hat{\lambda}^{\mathrm{G}}_t$ and $\tilde{\lambda}^{\mathrm{G}}_{t,\omega}$, respectively:

$$\left\{\begin{aligned} &\max_{\Theta^{\mathrm{VBG}}} \sum_{t\in\mathcal{T}} \left(\hat{\lambda}^{\mathrm{G}}_t\ v^{\mathrm{DA,G}}_{q,t} + \sum_{\omega\in\Omega} \tilde{\lambda}^{\mathrm{G}}_{t,\omega}\ v^{\mathrm{RT,G}}_{q,t}\right) \quad (15a)\\ &\text{subject to } v^{\mathrm{DA,G}}_{q,t} + v^{\mathrm{RT,G}}_{q,t} = 0 : \psi_{q,t},\ \ \forall t, \end{aligned}\right\},\ \ \forall q\in\mathcal{Q}, \quad (15b)$$

where $\Theta^{\mathrm{VBG}} = \{v^{\mathrm{DA,G}}_{q,t}, v^{\mathrm{RT,G}}_{q,t}, \forall q,t\}$ is the set of primal optimization variables. Objective function (15a) maximizes the expected profit of virtual bidder participating in the day-ahead and real-time natural gas markets and equation (15b) balances the virtual bidders day-ahead and real-time trade.

3.5. Day-Ahead Natural Gas Market

Both the day-ahead dispatch of virtual bidders and gas-fired units are inputs into the natural gas day-ahead market clearing problem. The power dispatch of gas-fired units is translated into a time-varying demand for natural gas by $\sum_{i\in\mathcal{G}} \phi_i\ p^{\mathrm{DA}}_{i,t}, \forall t$. Operating cost of the natural gas system in day-ahead is minimized according to

$$\min_{\Theta^{\mathrm{GD}}} \sum_{t\in\mathcal{T}} \Big(\sum_{k\in\mathcal{K}} C^{\mathrm{G}}_k\ g^{\mathrm{DA}}_{k,t}\Big) \quad (16a)$$

$$\text{subject to } \sum_{k\in\mathcal{K}} g^{\mathrm{DA}}_{k,t} - \sum_{i\in\mathcal{G}} \phi_i\ p^{\mathrm{DA}}_{i,t} - D^{\mathrm{G}}_t + \sum_{q\in\mathcal{Q}} v^{\mathrm{DA,G}}_{q,t} = 0 : \hat{\lambda}^{\mathrm{G}}_t,\ \ \forall t, \quad (16b)$$

$$0 \le g^{\mathrm{DA}}_{k,t} \le G^{\max}_k : \underline{\mu}^{\mathrm{G}}_{k,t}, \overline{\mu}^{\mathrm{G}}_{k,t},\ \ \forall k,t, \quad (16c)$$

where $\Theta^{\text{GD}} = \{g_{k,t}^{\text{DA}}, \forall k,t\}$ is the set of primal optimization variables. Objective function (16a) gives the cost of natural gas supply. Equation (16b) represents the day-ahead gas supply balance with inelastic demand including fixed gas demand for power production $\sum_{i\in\mathcal{G}} \phi_i \; p_{i,t}^{\text{DA}}$ and amount of virtual trade $\sum_{q\in\mathcal{Q}} v_{q,t}^{\text{DA,G}}$. Constraint (16c) enforces lower and upper bounds on the gas supply.

The optimization problem for day-ahead gas market clearing can be equivalently formulated as the following equilibrium model with each supplier maximizing their profit and a price-setting agent. Each natural gas supplier or producer maximizes her day-ahead profit with respect to her operational constraints according to

$$\left\{\begin{aligned} &\max_{\Theta^{\text{GD}}} \sum_{t\in\mathcal{T}} \left(\hat{\lambda}_t^{\text{G}} - C_k^{\text{G}}\right) g_{k,t}^{\text{DA}} && (17\text{a})\\ &\text{subject to } (16\text{c}) \end{aligned}\right\} \forall k, \qquad (17\text{b})$$

with the day-ahead price for natural gas $\hat{\lambda}_t^{\text{G}}$ set by

$$\min_{\hat{\lambda}_t^{\text{G}}} \sum_{t\in\mathcal{T}} \hat{\lambda}_t^{\text{G}} \Big(\sum_{k\in\mathcal{K}} g_{k,t}^{\text{DA}} - \sum_{i\in\mathcal{G}} \phi_i \; p_{i,t}^{\text{DA}} - D_t^{\text{G}} + \sum_{q\in\mathcal{Q}} v_{q,t}^{\text{DA,G}} \Big) \qquad (18\text{a})$$

The KKT conditions of optimization problem (16) are equivalent to those of equilibrium problem (17)-(18).

3.6. Real-Time Natural Gas Market

The real-time natural gas market is cleared for adjusted fuel consumption by gas-fired units converted to a time-varying demand deviation via $\sum_{i\in\mathcal{G}} \phi_i \; p_{i,t,\omega}^{\text{RT}}, \forall t,\omega$. The day-ahead schedule of the natural gas system as well as real-time electricity adjustments of gas-fired units and dispatch decisions by virtual bidders are treated as fixed parameters in the following formulation:

$$\left\{\begin{aligned} &\min_{\Theta^{\text{GR}}} \; \pi_\omega \sum_{t\in\mathcal{T}} \Big(\sum_{k\in\mathcal{K}} C_k^{\text{G}} \; g_{k,t,\omega}^{\text{RT}} + C^{\text{sh,G}} \; l_{t,\omega}^{\text{sh,G}} \Big) && (19\text{a})\\ &\text{subject to } \sum_{k\in\mathcal{K}} g_{k,t,\omega}^{\text{RT}} - \sum_{i\in\mathcal{G}} \phi_i \; p_{i,t,\omega}^{\text{RT}} + l_{t,\omega}^{\text{sh,G}} + \sum_{q\in\mathcal{Q}} v_{q,t}^{\text{RT,G}} = 0 : \tilde{\lambda}_{t,\omega}^{\text{G}}, \; \forall t, && (19\text{b})\\ &0 \le \left(g_{k,t}^{\text{DA}} + g_{k,t,\omega}^{\text{RT}}\right) \le G_k^{\max} : \underline{\nu}_{k,t,\omega}^{\text{G}}, \overline{\nu}_{k,t,\omega}^{\text{G}}, \;\; \forall k,t, && (19\text{c})\\ &g_{k,t,\omega}^{\text{RT}} \le G_k^{\text{adj}} : \underline{\nu}_{t,\omega}^{\text{GR}}, \overline{\nu}_{t,\omega}^{\text{GR}}, \;\; \forall k,t, && (19\text{d})\\ &0 \le l_{t,\omega}^{\text{sh,G}} \le D_t^{\text{G}} : \underline{\nu}_{t,\omega}^{\text{DG}}, \overline{\nu}_{t,\omega}^{\text{DG}}, \;\; \forall t, && (19\text{e}) \end{aligned}\right\} \forall \omega$$

where $\Theta^{\text{GR}} = \{g_{k,t,\omega}^{\text{RT}}, \forall k,t,\omega; l_{t,\omega}^{\text{sh,G}}, \forall t,\omega\}$ is the set of primal optimization variables. The real-time cost of the natural gas system is given in objective function (19a). The probability-weighted cost of natural gas adjustments along with natural gas load shedding is minimized in (19a)under scenarios

ω. Constraint (19b) represents the balance of gas supply adjustments in real-time including fixed amount of virtual trade $\sum_{q\in\mathcal{Q}} v_{q,t}^{\mathrm{RT,G}}$. Constraints (19c), (19d), and (19e) enforce lower and upper bounds on gas supply, gas adjustments and gas load shedding, respectively.

Market-clearing problem (19) is equivalent to the following equilibrium problem (20)-(22). Each gas supplier updates her supply in real-time as

$$\left\{\begin{aligned} &\max_{\Theta^{\mathrm{GR}}} \sum_{t\in\mathcal{T}} \Big(\tilde{\lambda}_{t,\omega}^{\mathrm{G}} - \pi_\omega\ C_k^{\mathrm{G}}\Big) g_{k,t,\omega}^{\mathrm{RT}} && \text{(20a)}\\ &\text{subject to } \ (19\text{c}), (19\text{d}) \end{aligned}\right\} \forall k, \omega, \quad \text{(20b)}$$

and cost incurred by gas demand curtailment is minimized according to

$$\left\{\begin{aligned} &\min_{\Theta^{\mathrm{GR}}} \sum_{t\in\mathcal{T}} \Big(\pi_\omega\ C^{\mathrm{sh,G}} - \tilde{\lambda}_{t,\omega}^{\mathrm{G}}\Big) l_{t,\omega}^{\mathrm{sh,G}} && \text{(21a)}\\ &\text{subject to } \ (19\text{e}) \end{aligned}\right\} \forall \omega. \quad \text{(21b)}$$

The real-time natural gas price is determined for each scenario ω as

$$\left\{\min_{\tilde{\lambda}_{t,\omega}^{\mathrm{G}}} \sum_{t\in\mathcal{T}} \tilde{\lambda}_{t,\omega}^{\mathrm{G}} \Big(\sum_{k\in\mathcal{K}} g_{k,t,\omega}^{\mathrm{RT}} - \sum_{i\in\mathcal{G}} \phi_i\ p_{i,t,\omega}^{\mathrm{RT}} + l_{t,\omega}^{\mathrm{sh,G}} + \sum_{q\in\mathcal{Q}} v_{q,t}^{\mathrm{RT,G}}\Big)\right\} \forall \omega. \quad \text{(22a)}$$

3.7. Self-Scheduling Gas-Fired Generators

For improving the sectoral coordination, we allow natural gas-fired units to self-schedule outside the markets for optimally allocating their flexibility in the power and natural gas markets. Each gas-fired unit maximizes its expected profit given a perfect anticipation of the distribution of both electricity and natural gas real-time market prices.

3.7.1. Self-scheduling slow-starting gas-fired unit The profit maximization problem of each self-scheduling gas-fired unit $\mathcal{G}\cap\mathcal{S}$ participating in the electricity and natural gas market is given below:

$$\left\{\begin{aligned} &\max_{\Theta^{\mathrm{SSS}}} \sum_{t\in\mathcal{T}} \Big[p_{i,t}^{\mathrm{DA}} \Big(\hat{\lambda}_t^{\mathrm{E}} - \phi_i\ \hat{\lambda}_t^{\mathrm{G}}\Big) - c_{i,t}^{\mathrm{DA}} + \sum_{\omega\in\Omega} p_{i,t,\omega}^{\mathrm{RT}} \Big(\tilde{\lambda}_{t,\omega}^{\mathrm{E}} - \phi_i\ \tilde{\lambda}_{t,\omega}^{\mathrm{G}}\Big)\Big] && \text{(23a)}\\ &\text{subject to} (2\text{d}) - (2\text{j}) && \text{(23b)}\\ &(7\text{e}) - (7\text{g}) \forall\omega \end{aligned}\right\} \ \forall i \in (\mathcal{G}\cap\mathcal{S}), \quad \text{(23c)}$$

where $\Theta^{\mathrm{SSS}} = \{p_{i,t}^{\mathrm{DA}}, u_{i,t}^{\mathrm{DA}}, c_{i,t}^{\mathrm{DA}}, \forall i \in (\mathcal{G}\cap\mathcal{S}), t; p_{i,t,\omega}^{\mathrm{RT}}, \forall i\in(\mathcal{G}\cap\mathcal{S}), t, \omega\}$ is the set of primal optimization variables. Objective function (23a) maximizes the expected profit of self-scheduling gas-fired generators and simultaneously considering the day-ahead (2d)-(2j) and real-time (7e)-(7g) constraints for all scenarios $\omega \in \Omega$. Note that the self-scheduler's dispatch decisions $p^{\mathrm{DA}}_{i,t}, u^{\mathrm{DA}}_{i,t}$, $p^{\mathrm{RT}}_{i,t,\omega}$ are fixed input in the market-clearing problems (2), (7), (16), and (19).

3.7.2. Self-scheduling fast-starting gas-fired unit The profit maximization problem of each fast-start self-scheduling gas-fired unit $\mathcal{G} \cap \mathcal{F}$ participating in the electricity and natural gas market is given below:

$$\left\{\begin{aligned} &\max_{\Theta^{\mathrm{SSF}}} \sum_{t\in\mathcal{T}} \Big[p^{\mathrm{DA}}_{i,t} \left(\hat{\lambda}^{\mathrm{E}}_t - \phi_i \ \hat{\lambda}^{\mathrm{G}}_t\right) - c^{\mathrm{DA}}_{i,t} + \sum_{\omega\in\Omega} p^{\mathrm{RT}}_{i,t,\omega} \left(\tilde{\lambda}^{\mathrm{E}}_{t,\omega} - \phi_i \ \tilde{\lambda}^{\mathrm{G}}_{t,\omega}\right) - \pi_\omega \ c^{\mathrm{RT}}_{i,t,\omega} \Big] && (24a)\\ &\text{subject to } \ (2d) - (2j) && (24b)\\ &\qquad (7h) - (7n) \forall \omega \end{aligned}\right\} \ \forall i \in (\mathcal{G} \cap \mathcal{F}), \qquad (24c)$$

where $\Theta^{\mathrm{SSF}} = \{p^{\mathrm{DA}}_{i,t}, u^{\mathrm{DA}}_{i,t}, c^{\mathrm{DA}}_{i,t}, \forall i \in (\mathcal{G}\cap\mathcal{F}), t; p^{\mathrm{RT}}_{i,t,\omega}, u^{\mathrm{RT}}_{i,t,\omega}, c^{\mathrm{RT}}_{i,t,\omega}, \forall i \in (\mathcal{G}\cap\mathcal{F}), t, \omega\}$ is the set of primal optimization variables. Objective function (24a) maximizes the expected profit of self-scheduling gas-fired generators and simultaneously considering the day-ahead (2d)-(2j) and real-time (7h)-(7n) constraints for all scenarios $\omega \in \Omega$.

3.8. Ideal Benchmark: Stochastic Integrated Electricity and Natural Gas Market

The stochastic and fully-coupled dispatch model simulates the integrated power and natural system by jointly modeling the day-ahead and real-time stages. The problem is formulated as a two-stage stochastic program aiming to minimize the total expected cost and writes as follows,

$$\begin{aligned} \min_{\Theta^{\mathrm{SC}}} \ & \sum_{t\in\mathcal{T}} \Big[\sum_{i\in\mathcal{C}} \left(C^{\mathrm{E}}_i \ p^{\mathrm{DA}}_{i,t}\right) + \sum_{i\in\mathcal{I}} c^{\mathrm{DA}}_{i,t} + \sum_{k\in\mathcal{K}} C^{\mathrm{G}}_k \ g^{\mathrm{DA}}_{k,t} + \sum_{\omega\in\Omega} \pi_\omega \\ & \Big(\sum_{i\in\mathcal{C}} C^{\mathrm{E}}_i \ p^{\mathrm{RT}}_{i,t,\omega} + \sum_{i\in\mathcal{F}} c^{\mathrm{RT}}_{i,t} + \sum_{k\in\mathcal{K}} C^{\mathrm{G}}_k \ g^{\mathrm{RT}}_{k,t,\omega} + C^{\mathrm{sh,E}} \ l^{\mathrm{sh,E}}_{t,\omega} + C^{\mathrm{sh,G}} \ l^{\mathrm{sh,G}}_{t,\omega} \Big) \Big] \end{aligned} \tag{25a}$$

subject to

$$(2d) - (2g), \ \ \forall i, \ \ (16c), (16b), \tag{25b}$$

$$(7e) - (7k), \ \ \forall i, \omega, \ \ (19c) - (19b), \ \ \forall \omega, \tag{25c}$$

where $\Theta^{\mathrm{SC}} = \{\Theta^{\mathrm{ED}}, \Theta^{\mathrm{GD}}, \Theta^{\mathrm{ER}}, \Theta^{\mathrm{GR}}\}$ is the set of primal optimization variables. In this model, the temporal coordination of the two trading floors is taken into account by anticipating the real-time constraints (25c) for all scenarios $\omega \in \Omega$.

4. Karush-Kuhn-Tucker Conditions

4.1. Explicit Electricity Virtual Bidder

$$\frac{\partial L}{\partial v^{\mathrm{DA,E}}_{r,t}} = \hat{\lambda}^{\mathrm{E}}_t - \rho_{r,t} = 0, \ \ \forall r, t, \tag{26a}$$

$$\frac{\partial L}{\partial v^{\mathrm{RT,E}}_{r,t}} = \sum_{\omega\in\Omega} \tilde{\lambda}^{\mathrm{E}}_{t,\omega} - \rho_{r,t} = 0, \ \ \forall r, t, \tag{26b}$$

$$v^{\mathrm{DA,E}}_{r,t} + v^{\mathrm{RT,E}}_{r,t} = 0 : \rho_{r,t}, \ \ \forall r, t. \tag{26c}$$

4.2. Day-Ahead Electricity Market

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = C^{\mathrm{E}}_i - \hat{\lambda}^{\mathrm{E}}_t + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} = 0, \;\; \forall i \in \mathcal{C} \setminus \mathcal{SS}, t < |\mathcal{T}|, \tag{27a}$$

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = C^{\mathrm{E}}_i - \hat{\lambda}^{\mathrm{E}}_t + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \underline{\mu}^{\mathrm{R}}_{i,t} = 0, \;\; \forall i \in \mathcal{C} \setminus \mathcal{SS}, t = |\mathcal{T}|, \tag{27b}$$

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = \hat{\lambda}^{\mathrm{G}}_t \phi_i - \hat{\lambda}^{\mathrm{E}}_t + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} = 0, \;\; \forall i \in \mathcal{G} \setminus \mathcal{SS}, t < |\mathcal{T}|, \tag{27c}$$

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = \hat{\lambda}^{\mathrm{G}}_t \phi_i - \hat{\lambda}^{\mathrm{E}}_t + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \underline{\mu}^{\mathrm{R}}_{i,t} = 0, \;\; \forall i \in \mathcal{G} \setminus \mathcal{SS}, t = |\mathcal{T}|, \tag{27d}$$

$$\frac{\partial L}{\partial u^{\mathrm{DA}}_{i,t}} = -P^{\max}_i \overline{\mu}^{\mathrm{P}}_{i,t} + P^{\min}_i \underline{\mu}^{\mathrm{P}}_{i,t} - R_i \overline{\mu}^{\mathrm{R}}_{i,t} - R_i \underline{\mu}^{\mathrm{R}}_{i,(t+1)} + C^{\mathrm{SU}}_i (\overline{\mu}^{\mathrm{SU}}_{i,t} - \overline{\mu}^{\mathrm{SU}}_{i,(t+1)}) + \overline{\mu}^{\mathrm{B}}_{i,t} - \underline{\mu}^{\mathrm{B}}_{i,t} = 0,$$
$$\forall i \in \mathcal{I} \setminus \mathcal{SS}, t < |\mathcal{T}|, \tag{27e}$$

$$\frac{\partial L}{\partial u^{\mathrm{DA}}_{i,t}} = -P^{\max}_i \overline{\mu}^{\mathrm{P}}_{i,t} + P^{\min}_i \underline{\mu}^{\mathrm{P}}_{i,t} - R_i \overline{\mu}^{\mathrm{R}}_{i,t} + C^{\mathrm{SU}}_i \overline{\mu}^{\mathrm{SU}}_{i,t} + \overline{\mu}^{\mathrm{B}}_{i,t} - \underline{\mu}^{\mathrm{B}}_{i,t} = 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t = |\mathcal{T}|, \tag{27f}$$

$$\frac{\partial L}{\partial w^{\mathrm{DA}}_{j,t}} = -\hat{\lambda}^{\mathrm{E}}_t \overline{\mu}^{\mathrm{W}}_{j,t} - \underline{\mu}^{\mathrm{W}}_{j,t} = 0, \;\; \forall j, t \tag{27g}$$

$$\frac{\partial L}{\partial c^{\mathrm{DA}}_{i,t}} = 1 - \overline{\mu}^{\mathrm{SU}}_{i,t} - \underline{\mu}^{\mathrm{SU}}_{i,t} = 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t \tag{27h}$$

$$0 \le (p^{\mathrm{DA}}_{i,t} - u^{\mathrm{DA}}_{i,t} P^{\min}_i) \perp \underline{\mu}^{\mathrm{P}}_{i,t} \ge 0 \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t, \tag{27i}$$

$$0 \le (u^{\mathrm{DA}}_{i,t} P^{\max}_i - p^{\mathrm{DA}}_{i,t}) \perp \overline{\mu}^{\mathrm{P}}_{i,t} \ge 0 \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t, \tag{27j}$$

$$0 \le w^{\mathrm{DA}}_{j,t} \perp \underline{\mu}^{\mathrm{W}}_{j,t} \ge 0, \;\; \forall j, t \tag{27k}$$

$$0 \le (W^{\mathrm{DA}}_{j,t} - w^{\mathrm{DA}}_{j,t}) \perp \overline{\mu}^{\mathrm{W}}_{j,t} \ge 0, \;\; \forall j, t, \tag{27l}$$

$$0 \le \left[(p^{\mathrm{DA}}_{i,t} - p^{\mathrm{DA}}_{i,(t-1)}) + u^{\mathrm{DA}}_{i,(t-1)} R_i\right] \perp \underline{\mu}^{\mathrm{R}}_{i,t} \ge 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t > 1, \tag{27m}$$

$$0 \le \left[u^{\mathrm{DA}}_{i,t} R_i - (p^{\mathrm{DA}}_{i,t} - p^{\mathrm{DA}}_{i,(t-1)})\right] \perp \overline{\mu}^{\mathrm{R}}_{i,t} \ge 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t > 1, \tag{27n}$$

$$0 \le \left[(p^{\mathrm{DA}}_{i,t} - P^{\mathrm{ini}}_i) + U^{\mathrm{ini}}_i R_i\right] \perp \underline{\mu}^{\mathrm{R}}_{i,t} \ge 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t = 1, \tag{27o}$$

$$0 \le \left[u^{\mathrm{DA}}_{i,t} R_i - (p^{\mathrm{DA}}_{i,t} - P^{\mathrm{ini}}_i)\right] \perp \overline{\mu}^{\mathrm{R}}_{i,t} \ge 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t = 1, \tag{27p}$$

$$0 \le \left[c^{\mathrm{DA}}_{i,t} - C^{\mathrm{SU}}_i (u^{\mathrm{DA}}_{i,t} - u^{\mathrm{DA}}_{i,(t-1)})\right] \perp \overline{\mu}^{\mathrm{SU}}_{i,t} \ge 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t > 1, \tag{27q}$$

$$0 \le \left[c^{\mathrm{DA}}_{i,t} - C^{\mathrm{SU}}_i (u^{\mathrm{DA}}_{i,t} - U^{\mathrm{ini}}_i)\right] \perp \overline{\mu}^{\mathrm{SU}}_{i,t} \ge 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t = 1, \tag{27r}$$

$$0 \le c^{\mathrm{DA}}_{i,t} \perp \underline{\mu}^{\mathrm{SU}}_{i,t} \ge 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t, \tag{27s}$$

$$0 \le u^{\mathrm{DA}}_{i,t} \perp \underline{\mu}^{\mathrm{B}}_{i,t} \ge 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t, \tag{27t}$$

$$0 \le (1 - u^{\mathrm{DA}}_{i,t}) \perp \overline{\mu}^{\mathrm{B}}_{i,t} \ge 0, \;\; \forall i \in \mathcal{I} \setminus \mathcal{SS}, t, \tag{27u}$$

$$\sum_{i \in \mathcal{I}} p^{\mathrm{DA}}_{i,t} + \sum_{j \in \mathcal{J}} w^{\mathrm{DA}}_{j,t} - D^{\mathrm{E}}_t + \sum_{r \in \mathcal{R}} v^{\mathrm{DA,E}}_{r,t} = 0 : \hat{\lambda}^{\mathrm{E}}_t, \forall t. \tag{27v}$$

$$\tag{27w}$$

4.3. Real-Time Electricity Market

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = \pi_\omega C^{\mathrm{E}}_i - \tilde{\lambda}^{\mathrm{E}}_{t,\omega} + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} = 0,$$

$$\forall i \in \mathcal{C} \setminus \mathcal{SS}, t < |\mathcal{T}|, \omega, \tag{28a}$$

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = \pi_\omega C^{\mathrm{E}}_i - \tilde{\lambda}^{\mathrm{E}}_{t,\omega} + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} = 0, \;\; \forall i \in \mathcal{C} \setminus \mathcal{SS}, t = |\mathcal{T}|, \omega, \tag{28b}$$

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = \tilde{\lambda}^{\mathrm{G}}_t \phi_i - \tilde{\lambda}^{\mathrm{E}}_{t,\omega} + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} = 0,$$

$$\forall i \in \mathcal{G} \setminus \mathcal{SS}, t < |\mathcal{T}|, \omega, \tag{28c}$$

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = \tilde{\lambda}^{\mathrm{G}}_t \phi_i - \tilde{\lambda}^{\mathrm{E}}_{t,\omega} + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} = 0, \;\; \forall i \in \mathcal{G} \setminus \mathcal{SS}, t = |\mathcal{T}|, \omega, \tag{28d}$$

$$\frac{\partial L}{\partial w^{\mathrm{RT}}_{j,t,\omega}} = -\tilde{\lambda}^{\mathrm{E}}_{t,\omega} \overline{\nu}^{\mathrm{W}}_{j,t,\omega} - \underline{\nu}^{\mathrm{W}}_{j,t,\omega} = 0, \;\; \forall j, t, \omega, \tag{28e}$$

$$\frac{\partial L}{\partial l^{\mathrm{sh,E}}_{t,\omega}} = \pi_\omega \; C^{\mathrm{sh,E}} - \tilde{\lambda}^{\mathrm{E}}_{t,\omega} + \overline{\nu}^{\mathrm{DE}}_{t,\omega} - \underline{\nu}^{\mathrm{DE}}_{t,\omega} = 0, \;\; \forall t, \omega, \tag{28f}$$

$$\frac{\partial L}{\partial u^{\mathrm{RT}}_{i,t,\omega}} = -P^{\max}_i \overline{\nu}^{\mathrm{P}}_{i,t,\omega} + P^{\min}_i \underline{\nu}^{\mathrm{P}}_{i,t,\omega} - R_i \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - R_i \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} + C^{\mathrm{SU}}_i (\overline{\nu}^{\mathrm{SU}}_{i,t,\omega} - \overline{\nu}^{\mathrm{SU}}_{i,(t+1),\omega})$$

$$+ \overline{\nu}^{\mathrm{B}}_{i,t,\omega} - \underline{\nu}^{\mathrm{B}}_{i,t,\omega} = 0, \;\; \forall i \in \mathcal{F}, t < \mathcal{T}, \omega, \tag{28g}$$

$$\frac{\partial L}{\partial u^{\mathrm{RT}}_{i,t,\omega}} = -P^{\max}_i \overline{\nu}^{\mathrm{P}}_{i,t,\omega} + P^{\min}_i \underline{\nu}^{\mathrm{P}}_{i,t,\omega} - R_i \overline{\nu}^{\mathrm{R}}_{i,t,\omega} + C^{\mathrm{SU}}_i \overline{\nu}^{\mathrm{SU}}_{i,t,\omega} + \overline{\nu}^{\mathrm{B}}_{i,t,\omega} - \underline{\nu}^{\mathrm{B}}_{i,t,\omega} = 0, \;\; \forall i \in \mathcal{F}, t = |\mathcal{T}|, \omega, \tag{28h}$$

$$\frac{\partial L}{\partial c^{\mathrm{RT}}_{i,t,\omega}} = \pi_\omega - \overline{\nu}^{\mathrm{SU}}_{i,t,\omega} - \underline{\nu}^{\mathrm{SU}}_{i,t,\omega} = 0, \;\; \forall i \in \mathcal{F}, t, \omega, \tag{28i}$$

$$0 \leq \left[(p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega}) - u^{\mathrm{DA}}_{i,t} P^{\min}_i\right] \perp \underline{\nu}^{\mathrm{P}}_{i,t,\omega} \geq 0, \;\; \forall i \in \mathcal{S}, t, \omega, \tag{28j}$$

$$0 \leq \left[u^{\mathrm{DA}}_{i,t} P^{\max}_i - (p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega})\right] \perp \overline{\nu}^{\mathrm{P}}_{i,t,\omega} \geq 0, \;\; \forall i \in \mathcal{S}, t, \omega, \tag{28k}$$

$$0 \leq \left[(p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega}) - (u^{\mathrm{DA}}_{i,t} + u^{\mathrm{RT}}_{i,t,\omega}) P^{\min}_i\right] \perp \underline{\nu}^{\mathrm{P}}_{i,t,\omega} \geq 0, \forall i \in \mathcal{F}, t, \omega, \tag{28l}$$

$$0 \leq \left[(u^{\mathrm{DA}}_{i,t} + u^{\mathrm{RT}}_{i,t,\omega}) P^{\max}_i - (p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega})\right] \perp \overline{\nu}^{\mathrm{P}}_{i,t,\omega} \geq 0, \forall i \in \mathcal{F}, t, \omega, \tag{28m}$$

$$0 \leq (w^{\mathrm{DA}}_{j,t} + w^{\mathrm{RT}}_{j,t,\omega}) \perp \underline{\nu}^{\mathrm{W}}_{j,t,\omega} \geq 0, \;\; \forall j, t, \omega, \tag{28n}$$

$$0 \leq \left[W_{j,t,\omega} - (w^{\mathrm{DA}}_{j,t} + w^{\mathrm{RT}}_{j,t,\omega})\right] \perp \overline{\nu}^{\mathrm{W}}_{j,t,\omega} \geq 0, \;\; \forall j, t, \omega, \tag{28o}$$

$$0 \leq l^{\mathrm{sh,E}}_{t,\omega} \perp \underline{\nu}^{\mathrm{DE}}_{t,\omega} \geq 0, \;\; \forall t, \omega, \tag{28p}$$

$$0 \leq D^{\mathrm{E}}_t - l^{\mathrm{sh,E}}_{t,\omega} \perp \overline{\nu}^{\mathrm{DE}}_{t,\omega} \geq 0, \;\; \forall t, \omega, \tag{28q}$$

$$0 \leq \left[(p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - p^{\mathrm{DA}}_{i,(t-1)} - p^{\mathrm{RT}}_{i,(t-1),\omega}) + u^{\mathrm{DA}}_{i,(t-1)} R_i\right] \perp \underline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \;\; \forall i \in \mathcal{S}, t > 1, \omega, \tag{28r}$$

$$0 \leq \left[u^{\mathrm{DA}}_{i,t} R_i - (p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - p^{\mathrm{DA}}_{i,(t-1)} - p^{\mathrm{RT}}_{i,(t-1),\omega})\right] \perp \overline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \forall i \in \mathcal{S}, t > 1, \omega, \tag{28s}$$

$$0 \leq \left[(p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - P^{\mathrm{ini}}_i) + U^{\mathrm{ini}}_i R_i\right] \perp \underline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \forall i \in \mathcal{S}, t{=}1, \omega, \tag{28t}$$

$$0 \leq \left[u^{\mathrm{DA}}_{i,t} R_i - (p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - P^{\mathrm{ini}}_i)\right] \perp \overline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \forall i \in \mathcal{S}, t{=}1, \omega, \tag{28u}$$

$$0 \leq \left[(p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - p^{\mathrm{DA}}_{i,(t-1)} - p^{\mathrm{RT}}_{i,(t-1),\omega}) + (u^{\mathrm{DA}}_{i,(t-1)} + u^{\mathrm{RT}}_{i,(t-1)}) R_i\right] \perp \underline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \;\; \forall i \in \mathcal{F}, t > 1, \omega, \tag{28v}$$

$$0 \leq \left[(u^{\mathrm{DA}}_{i,t} + u^{\mathrm{RT}}_{i,t}) R_i - (p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - p^{\mathrm{DA}}_{i,(t-1)} - p^{\mathrm{RT}}_{i,(t-1),\omega})\right] \perp \overline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \;\; \forall i \in \mathcal{F}, t > 1, \omega, \tag{28w}$$

$$0 \leq \left[p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - P^{\mathrm{ini}}_i + U^{\mathrm{ini}}_i R_i\right] \perp \underline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \forall i \in \mathcal{F}, t{=}1, \omega, \tag{28x}$$

$$0 \leq \left[(u^{\mathrm{DA}}_{i,t} + u^{\mathrm{RT}}_{i,t}) R_i - (p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - P^{\mathrm{ini}}_i)\right] \perp \overline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \forall i \in \mathcal{F}, t{=}1, \omega, \tag{28y}$$

$$0 \leq \left[(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}}) - C_i^{\mathrm{SU}}(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} - u_{i,(t-1)}^{\mathrm{DA}} - u_{i,(t-1),\omega}^{\mathrm{RT}})\right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{SU}} \geq 0, \;\; \forall i \in \mathcal{F}, t > 1, \omega, \tag{28z}$$

$$0 \leq \left[(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}}) - C_i^{\mathrm{SU}}(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}} - U_i^{\mathrm{ini}})\right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{SU}} \geq 0, \;\; \forall i \in \mathcal{F}, t{=}1, \omega, \tag{28aa}$$

$$0 \leq \left(c_{i,t}^{\mathrm{DA}} + c_{i,t,\omega}^{\mathrm{RT}}\right) \perp \underline{\nu}_{i,t,\omega}^{\mathrm{SU}} \geq 0, \;\; \forall i \in \mathcal{F}, t, \omega, \tag{28ab}$$

$$0 \leq \left(u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}}\right) \perp \underline{\nu}_{i,t,\omega}^{\mathrm{B}} \geq 0, \;\; \forall i \in \mathcal{F}, t, \omega, \tag{28ac}$$

$$0 \leq \left[1 - (u_{i,t}^{\mathrm{DA}} + u_{i,t,\omega}^{\mathrm{RT}})\right] \perp \overline{\nu}_{i,t,\omega}^{\mathrm{B}} \geq 0, \;\; \forall i \in \mathcal{F}, t, \omega, \tag{28ad}$$

$$\sum_{i \in \mathcal{I}} p_{i,t,\omega}^{\mathrm{RT}} + l_{t,\omega}^{\mathrm{sh,E}} + \sum_{r \in \mathcal{R}} v_{r,t}^{\mathrm{RT,E}} + \sum_{j \in \mathcal{J}} w_{j,t,\omega}^{\mathrm{RT}} = 0 \!:\! \tilde{\lambda}_{t,\omega}^{\mathrm{E}}, \forall t. \tag{28ae}$$

4.4. Explicit Natural Gas Virtual Bidder

$$\frac{\partial L}{\partial v_{q,t}^{\mathrm{DA,G}}} = \hat{\lambda}_t^{\mathrm{G}} - \psi_{q,t} = 0, \;\; \forall q, t, \tag{29a}$$

$$\frac{\partial L}{\partial v_{q,t}^{\mathrm{RT,G}}} = \sum_{\omega \in \Omega} \tilde{\lambda}_{t,\omega}^{\mathrm{G}} - \psi_{q,t} = 0, \;\; \forall q, t, \tag{29b}$$

$$v_{q,t}^{\mathrm{DA,G}} + v_{q,t}^{\mathrm{RT,G}} = 0 : \psi_{q,t}, \;\; \forall q, t. \tag{29c}$$

4.5. Day-Ahead Natural Gas Market

$$\frac{\partial L}{\partial g_{k,t}^{\mathrm{DA}}} = C_k^{\mathrm{G}} - \hat{\lambda}_t^{\mathrm{G}} + \overline{\mu}_{k,t}^{\mathrm{G}} - \underline{\mu}_{k,t}^{\mathrm{G}} = 0 \;\; \forall k, t, \tag{30a}$$

$$0 \leq g_{k,t}^{\mathrm{DA}} \perp \underline{\mu}_{k,t}^{\mathrm{G}} \geq 0 \;\; \forall k, t, \tag{30b}$$

$$0 \leq (G_k^{\max} - g_{k,t}^{\mathrm{DA}}) \perp \overline{\mu}_{k,t}^{\mathrm{G}} \geq 0 \;\; \forall k, t, \tag{30c}$$

$$\sum_{k \in \mathcal{K}} g_{k,t}^{\mathrm{DA}} - \sum_{i \in \mathcal{G}} \phi_i \, p_{i,t}^{\mathrm{DA}} - D_t^{\mathrm{G}} + \sum_{q \in \mathcal{Q}} v_{q,t}^{\mathrm{DA,G}} = 0 : \hat{\lambda}_t^{\mathrm{G}}, \;\; \forall t. \tag{30d}$$

4.6. Real-Time Natural Gas Market

$$\frac{\partial L}{\partial g_{k,t,\omega}^{\mathrm{RT}}} = \pi_\omega C_k^{\mathrm{G}} - \tilde{\lambda}_{t,\omega}^{\mathrm{G}} + \overline{\nu}_{k,t,\omega}^{\mathrm{G}} - \underline{\nu}_{k,t,\omega}^{\mathrm{G}} + \overline{\nu}_{k,t,\omega}^{\mathrm{GR}} = 0, \;\; \forall k, t, \omega, \tag{31a}$$

$$\frac{\partial L}{\partial l_{t,\omega}^{\mathrm{sh,G}}} = \pi_\omega C^{\mathrm{sh,G}} - \tilde{\lambda}_{t,\omega}^{\mathrm{G}} + \overline{\nu}_{t,\omega}^{\mathrm{DG}} - \underline{\nu}_{t,\omega}^{\mathrm{DG}} = 0, \;\; \forall t, \omega, \tag{31b}$$

$$0 \leq (g_{k,t}^{\mathrm{DA}} + g_{k,t,\omega}^{\mathrm{RT}}) \perp \underline{\nu}_{k,t,\omega}^{\mathrm{G}} \geq 0, \;\; \forall k, t, \omega, \tag{31c}$$

$$0 \leq \left[G_k^{\max} - (g_{k,t}^{\mathrm{DA}} + g_{k,t,\omega}^{\mathrm{RT}})\right] \perp \overline{\nu}_{k,t,\omega}^{\mathrm{G}} \geq 0, \;\; \forall k, t, \omega, \tag{31d}$$

$$0 \leq (G_k^{\mathrm{adj}} - g_{k,t,\omega}^{\mathrm{RT}}) \perp \overline{\nu}_{t,\omega}^{\mathrm{GR}} \geq 0, \;\; \forall k, t, \omega, \tag{31e}$$

$$0 \leq l_{t,\omega}^{\mathrm{sh,G}} \perp \underline{\nu}_{t,\omega}^{\mathrm{DG}} \geq 0, \;\; \forall t, \omega, \tag{31f}$$

$$0 \leq (D_t^{\mathrm{G}} - l_{t,\omega}^{\mathrm{sh,G}}) \perp \overline{\nu}_{t,\omega}^{\mathrm{DG}} \geq 0, \;\; \forall t, \omega, \tag{31g}$$

$$\sum_{k \in \mathcal{K}} g_{k,t,\omega}^{\mathrm{RT}} - \sum_{i \in \mathcal{G}} \phi_i \, p_{i,t,\omega}^{\mathrm{RT}} + l_{t,\omega}^{\mathrm{sh,G}} + \sum_{q \in \mathcal{Q}} v_{q,t}^{\mathrm{RT,G}} = 0 : \tilde{\lambda}_{t,\omega}^{\mathrm{G}}, \forall t, \omega. \tag{31h}$$

4.7. Self-Scheduling Slow-Starting Gas-Fired Generator

$$\begin{cases} \frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = -\hat{\lambda}^{\mathrm{E}}_t + \hat{\lambda}^{\mathrm{G}}_t \phi_i + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} \end{cases}$$

$$+ \sum_{\omega \in \Omega} \Big[\overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} \Big] = 0, \;\; \forall t < |\mathcal{T}|, \tag{32a}$$

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = -\hat{\lambda}^{\mathrm{E}}_t + \hat{\lambda}^{\mathrm{G}}_t \phi_i + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \underline{\mu}^{\mathrm{R}}_{i,t} + \sum_{\omega \in \Omega} \Big[\overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} \Big] = 0, t = |\mathcal{T}|, \tag{32b}$$

$$\frac{\partial L}{\partial u^{\mathrm{DA}}_{i,t}} = -P^{\max}_i \overline{\mu}^{\mathrm{P}}_{i,t} + P^{\min}_i \underline{\mu}^{\mathrm{P}}_{i,t} - R_i \overline{\mu}^{\mathrm{R}}_{i,t} - R_i \underline{\mu}^{\mathrm{R}}_{i,(t+1)} + C^{\mathrm{SU}}_i (\overline{\mu}^{\mathrm{SU}}_{i,t} - \overline{\mu}^{\mathrm{SU}}_{i,(t+1)}) + \overline{\mu}^{\mathrm{B}}_{i,t} - \underline{\mu}^{\mathrm{B}}_{i,t}$$

$$+ \sum_{\omega \in \Omega} \big(-P^{\max}_i \overline{\nu}^{\mathrm{P}}_{i,t,\omega} + P^{\min}_i \underline{\nu}^{\mathrm{P}}_{i,t,\omega} - R_i \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - R_i \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} \big) = 0, \;\; \forall t < |\mathcal{T}|, \tag{32c}$$

$$\frac{\partial L}{\partial u^{\mathrm{DA}}_{i,t}} = -P^{\max}_i \overline{\mu}^{\mathrm{P}}_{i,t} + P^{\min}_i \underline{\mu}^{\mathrm{P}}_{i,t} - R_i \overline{\mu}^{\mathrm{R}}_{i,t} + C^{\mathrm{SU}}_i \overline{\mu}^{\mathrm{SU}}_{i,t} + \overline{\mu}^{\mathrm{B}}_{i,t} - \underline{\mu}^{\mathrm{B}}_{i,t}$$

$$+ \sum_{\omega \in \Omega} \big(-P^{\max}_i \overline{\nu}^{\mathrm{P}}_{i,t,\omega} + P^{\min}_i \underline{\nu}^{\mathrm{P}}_{i,t,\omega} - R_i \overline{\nu}^{\mathrm{R}}_{i,t,\omega} \big) = 0, \;\; t = |\mathcal{T}|, \tag{32d}$$

$$\frac{\partial L}{\partial c^{\mathrm{DA}}_{i,t}} = 1 - \overline{\mu}^{\mathrm{SU}}_{i,t} - \underline{\mu}^{\mathrm{SU}}_{i,t} = 0, \;\; \forall t, \tag{32e}$$

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = -\tilde{\lambda}^{\mathrm{E}}_{t,\omega} + \phi_i \tilde{\lambda}^{\mathrm{G}}_{t,\omega} + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} = 0,$$

$$\forall t < |\mathcal{T}|, \omega, \tag{32f}$$

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = -\tilde{\lambda}^{\mathrm{E}}_{t,\omega} + \phi_i \tilde{\lambda}^{\mathrm{G}}_{t,\omega} + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} = 0, \;\; t = |\mathcal{T}|, \omega, \tag{32g}$$

$$0 \le (p^{\mathrm{DA}}_{i,t} - u^{\mathrm{DA}}_{i,t} P^{\min}_i) \perp \underline{\mu}^{\mathrm{P}}_{i,t} \ge 0 \;\; \forall t, \tag{32h}$$

$$0 \le (u^{\mathrm{DA}}_{i,t} P^{\max}_i - p^{\mathrm{DA}}_{i,t}) \perp \overline{\mu}^{\mathrm{P}}_{i,t} \ge 0 \;\; \forall t, \tag{32i}$$

$$0 \le \big[(p^{\mathrm{DA}}_{i,t} - p^{\mathrm{DA}}_{i,(t-1)}) + u^{\mathrm{DA}}_{i,(t-1)} R_i \big] \perp \underline{\mu}^{\mathrm{R}}_{i,t} \ge 0, \;\; \forall t > 1, \tag{32j}$$

$$0 \le \big[u^{\mathrm{DA}}_{i,t} R_i - (p^{\mathrm{DA}}_{i,t} - p^{\mathrm{DA}}_{i,(t-1)}) \big] \perp \overline{\mu}^{\mathrm{R}}_{i,t} \ge 0, \;\; \forall t > 1, \tag{32k}$$

$$0 \le \big[(p^{\mathrm{DA}}_{i,t} - P^{\mathrm{ini}}_i) + U^{\mathrm{ini}}_i R_i \big] \perp \underline{\mu}^{\mathrm{R}}_{i,t} \ge 0, \;\; \forall t = 1, \tag{32l}$$

$$0 \le \big[u^{\mathrm{DA}}_{i,t} R_i - (p^{\mathrm{DA}}_{i,t} - P^{\mathrm{ini}}_i) \big] \perp \overline{\mu}^{\mathrm{R}}_{i,t} \ge 0, \;\; \forall t = 1, \tag{32m}$$

$$0 \le \big[c^{\mathrm{DA}}_{i,t} - C^{\mathrm{SU}}_i (u^{\mathrm{DA}}_{i,t} - u^{\mathrm{DA}}_{i,(t-1)}) \big] \perp \overline{\mu}^{\mathrm{SU}}_{i,t} \ge 0, \;\; \forall t > 1, \tag{32n}$$

$$0 \le \big[c^{\mathrm{DA}}_{i,t} - C^{\mathrm{SU}}_i (u^{\mathrm{DA}}_{i,t} - U^{\mathrm{ini}}_i) \big] \perp \overline{\mu}^{\mathrm{SU}}_{i,t} \ge 0, \;\; \forall t = 1, \tag{32o}$$

$$0 \le c^{\mathrm{DA}}_{i,t} \perp \underline{\mu}^{\mathrm{SU}}_{i,t} \ge 0, \;\; \forall t, \tag{32p}$$

$$0 \le u^{\mathrm{DA}}_{i,t} \perp \underline{\mu}^{\mathrm{B}}_{i,t} \ge 0, \;\; \forall t, \tag{32q}$$

$$0 \le (1 - u^{\mathrm{DA}}_{i,t}) \perp \overline{\mu}^{\mathrm{B}}_{i,t} \ge 0, \;\; \forall t, \tag{32r}$$

$$0 \le \big[(p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega}) - u^{\mathrm{DA}}_{i,t} P^{\min}_i \big] \perp \underline{\nu}^{\mathrm{P}}_{i,t,\omega} \ge 0, \;\; \forall t, \omega, \tag{32s}$$

$$0 \le \big[u^{\mathrm{DA}}_{i,t} P^{\max}_i - (p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega}) \big] \perp \overline{\nu}^{\mathrm{P}}_{i,t,\omega} \ge 0, \;\; \forall t, \omega, \tag{32t}$$

$$0 \le \big[(p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - p^{\mathrm{DA}}_{i,(t-1)} - p^{\mathrm{RT}}_{i,(t-1),\omega}) + u^{\mathrm{DA}}_{i,(t-1)} R_i \big] \perp \underline{\nu}^{\mathrm{R}}_{i,t,\omega} \ge 0, \;\; \forall t > 1, \omega, \tag{32u}$$

$$0 \leq \left[u^{\mathrm{DA}}_{i,t} R_i - (p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - p^{\mathrm{DA}}_{i,(t-1)} - p^{\mathrm{RT}}_{i,(t-1),\omega})\right] \perp \overline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \forall t > 1, \omega, \tag{32v}$$

$$\left.\begin{aligned} &0 \leq \left[(p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - P^{\mathrm{ini}}_i) + U^{\mathrm{ini}}_i R_i\right] \perp \underline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \forall t{=}1, \omega, \quad (32\mathrm{w})\\ &0 \leq \left[u^{\mathrm{DA}}_{i,t} R_i - (p^{\mathrm{DA}}_{i,t} + p^{\mathrm{RT}}_{i,t,\omega} - P^{\mathrm{ini}}_i)\right] \perp \overline{\nu}^{\mathrm{R}}_{i,t,\omega} \geq 0, \forall t{=}1, \omega \end{aligned}\right\} \ \forall i \in (\mathcal{G} \cap \mathcal{SS}) \tag{32x}$$

5. Proof of Proposition 1

The KKT optimality conditions of each self-scheduling gas-fired generator, whose day-ahead dispatch is restricted by operational bounds, enforce

$$\begin{cases} \dfrac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = -\hat{\lambda}^{\mathrm{E}}_t + \hat{\lambda}^{\mathrm{G}}_t \phi_i + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} \\ \qquad + \sum_{\omega \in \Omega} \left[\overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega}\right] = 0, \ \ \forall t < T, \end{cases} \tag{33a}$$

and

$$\left.\begin{aligned} &\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = -\pi_\omega \Big(\frac{\tilde{\lambda}^{\mathrm{E}}_{t,\omega}}{\pi_\omega} - \phi_i \frac{\tilde{\lambda}^{\mathrm{G}}_{t,\omega}}{\pi_\omega}\Big) + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} = 0,\\ &\qquad \forall t < T, \omega, \end{aligned}\right\} \ \forall i \in (\mathcal{G} \cap \mathcal{SS}). \tag{33b}$$

The summation of condition (33b) over all scenarios, i.e., $\sum_\omega$(33b), shows that when virtual bidders in electricity and natural gas markets enforce price convergence in expectation, i.e., $\hat{\lambda}^{\mathrm{E}}_t = \sum_\omega \tilde{\lambda}^{\mathrm{E}}_{t,\omega}$ and $\hat{\lambda}^{\mathrm{G}}_t = \sum_\omega \tilde{\lambda}^{\mathrm{G}}_{t,\omega}$, the problem is feasible if only if $\overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} = 0, \ \forall i, t$, e.g., for the case when day-ahead operational bounds are non-binding.

6. Proof of Proposition 2

The KKT optimality conditions of the stochastic two-stage optimization problem (25) and those of the equilibrium problem (1), (2), (7), (15), (16), (19), (23), (24) with all gas-fired units as implicit virtual bidders are identical under the conditions that day-ahead operational bounds on $p^{\mathrm{DA}}_{i,t}$, $w^{\mathrm{DA}}_{j,t}$, $c^{\mathrm{DA}}_{i,t}$, $u^{\mathrm{DA}}_{i,t}$, and $g^{\mathrm{DA}}_{k,t}$ are non-binding (e.g., $\overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} = 0, \ \forall i, t$ and $\overline{\mu}^{\mathrm{G}}_{k,t} - \underline{\mu}^{\mathrm{G}}_{k,t} = 0, \ \forall k, t$) so that day-ahead and real-time prices converge in expectation (i.e., $\hat{\lambda}^{\mathrm{E}}_t = \sum_\omega \tilde{\lambda}^{\mathrm{E}}_{t,\omega}$ and $\hat{\lambda}^{\mathrm{G}}_t = \sum_\omega \tilde{\lambda}^{\mathrm{G}}_{t,\omega}$), see (26)-(32) .

Generator $i \in \mathcal{I}$ in sequential setup:

$$\frac{\partial L}{\partial p^{\mathrm{DA}}_{i,t}} = C^{\mathrm{E}}_i - \hat{\lambda}^{\mathrm{E}}_t + \overline{\mu}^{\mathrm{P}}_{i,t} - \underline{\mu}^{\mathrm{P}}_{i,t} + \overline{\mu}^{\mathrm{R}}_{i,t} - \overline{\mu}^{\mathrm{R}}_{i,(t+1)} - \underline{\mu}^{\mathrm{R}}_{i,t} + \underline{\mu}^{\mathrm{R}}_{i,(t+1)} = 0, \ \ \forall t, \tag{34a}$$

$$\frac{\partial L}{\partial p^{\mathrm{RT}}_{i,t,\omega}} = \pi_\omega C^{\mathrm{E}}_i - \tilde{\lambda}^{\mathrm{E}}_{t,\omega} + \overline{\nu}^{\mathrm{P}}_{i,t,\omega} - \underline{\nu}^{\mathrm{P}}_{i,t,\omega} + \overline{\nu}^{\mathrm{R}}_{i,t,\omega} - \overline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} - \underline{\nu}^{\mathrm{R}}_{i,t,\omega} + \underline{\nu}^{\mathrm{R}}_{i,(t+1),\omega} = 0, \ \ \forall t. \tag{34b}$$

Generator $i \in \mathcal{I}$ in two-stage stochastic setup:

$$\frac{\partial L}{\partial p_{i,t}^{\mathrm{DA}}} = C_i^{\mathrm{E}} - \hat{\lambda}_t^{\mathrm{E}} + \overline{\mu}_{i,t}^{\mathrm{P}} - \underline{\mu}_{i,t}^{\mathrm{P}} + \overline{\mu}_{i,t}^{\mathrm{R}} - \overline{\mu}_{i,(t+1)}^{\mathrm{R}} - \underline{\mu}_{i,t}^{\mathrm{R}} + \underline{\mu}_{i,(t+1)}^{\mathrm{R}}$$

$$+ \sum_{\omega \in \Omega} \left[\overline{\nu}_{i,t,\omega}^{\mathrm{P}} - \underline{\nu}_{i,t,\omega}^{\mathrm{P}} + \overline{\nu}_{i,t,\omega}^{\mathrm{R}} - \overline{\nu}_{i,(t+1),\omega}^{\mathrm{R}} - \underline{\nu}_{i,t,\omega}^{\mathrm{R}} + \underline{\nu}_{i,(t+1),\omega}^{\mathrm{R}} \right] = 0, \;\; \forall t, \tag{34c}$$

$$\frac{\partial L}{\partial p_{i,t,\omega}^{\mathrm{RT}}} = \pi_\omega \; C_i^{\mathrm{E}} - \tilde{\lambda}_{t,\omega}^{\mathrm{E}} + \overline{\nu}_{i,t,\omega}^{\mathrm{P}} - \underline{\nu}_{i,t,\omega}^{\mathrm{P}} + \overline{\nu}_{i,t,\omega}^{\mathrm{R}} - \overline{\nu}_{i,(t+1),\omega}^{\mathrm{R}} - \underline{\nu}_{i,t,\omega}^{\mathrm{R}} + \underline{\nu}_{i,(t+1),\omega}^{\mathrm{R}} = 0, \;\; \forall t, \omega. \tag{34d}$$

Gas supplier $k \in \mathcal{K}$ in sequential setup:

$$\frac{\partial L}{\partial g_{k,t}^{\mathrm{DA}}} = C_k^{\mathrm{G}} - \hat{\lambda}_t^{\mathrm{G}} + \overline{\mu}_{k,t}^{\mathrm{G}} - \underline{\mu}_{k,t}^{\mathrm{G}} = 0 \;\; \forall t, \tag{34e}$$

$$\frac{\partial L}{\partial g_{k,t,\omega}^{\mathrm{RT}}} = \pi_\omega C_k^{\mathrm{G}} - \tilde{\lambda}_{t,\omega}^{\mathrm{G}} + \overline{\nu}_{k,t,\omega}^{\mathrm{G}} - \underline{\nu}_{k,t,\omega}^{\mathrm{G}} + \overline{\nu}_{k,t,\omega}^{\mathrm{GR}} = 0, \;\; \forall t, \omega. \tag{34f}$$

Gas supplier $k \in \mathcal{K}$ in two-stage stochastic setup:

$$\frac{\partial L}{\partial g_{k,t}^{\mathrm{DA}}} = C_k^{\mathrm{G}} - \hat{\lambda}_t^{\mathrm{G}} + \overline{\mu}_{k,t}^{\mathrm{G}} - \underline{\mu}_{k,t}^{\mathrm{G}} + \sum_{\omega \in \Omega} \left[\overline{\nu}_{k,t,\omega}^{\mathrm{G}} - \underline{\nu}_{k,t,\omega}^{\mathrm{G}} + \overline{\nu}_{k,t,\omega}^{\mathrm{GR}} \right] = 0 \;\; \forall t, \tag{34g}$$

$$\frac{\partial L}{\partial g_{k,t,\omega}^{\mathrm{RT}}} = \pi_\omega C_k^{\mathrm{G}} - \tilde{\lambda}_{t,\omega}^{\mathrm{G}} + \overline{\nu}_{k,t,\omega}^{\mathrm{G}} - \underline{\nu}_{k,t,\omega}^{\mathrm{G}} + \overline{\nu}_{k,t,\omega}^{\mathrm{GR}} = 0, \;\; \forall t, \omega. \tag{34h}$$

(34i)

7. Computational Performance

Model	**Postsolved residual**	**Computational time [s]**
Seq	-	0.142
Seq+eVB	2.46E-09 & 5.03E-8	13.99 + 0.20
Seq+iVB	0.64	863.97
Seq+VB	3.80E-09	251.7
Ideal	-	0.19

Table 4 **Computational performance**