

Endless Fun

By the Experimental Quantum Optics Group

Players: 2-7

Ages: 12 and up

Duration: approx. 30min

The Idea of the game

Build your own high-dimensional quantum computer. By applying quantum logic operations to high-dimensional quantum bits – so-called *qudits* -, you can change their values. Try to reach the highest possible value on your qudit, while keeping the other player's qudit as low as possible. The evaluation software calculates the final state after each round, in which a certain number of measurements will project possible complex superposition states into a final value. The player with the highest qudit values in three rounds wins the game.

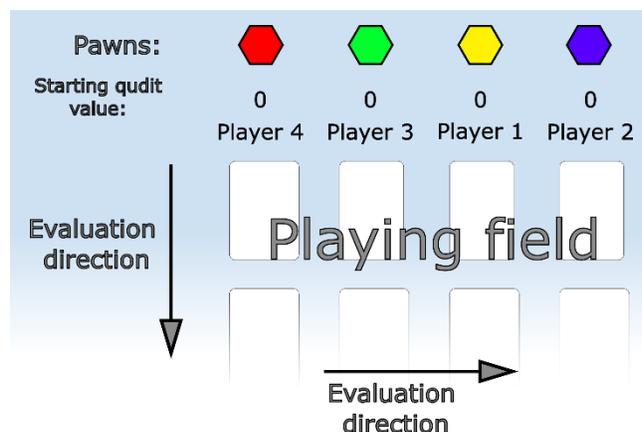
What you will need

- 55 Cards
- As many pawns as players
- 1 die
- Paper and pen to write down the players points
- The evaluation software

Setup

- The players agree on playing in 2D or 3D. In 2D, qudit values of 0 and 1 are possible. In 3D, the qudit values can be 0, 1 and 2. In 2D the H3 cards are removed from the card deck.
- Every player gets 7 random cards and a pawn.
- Decide on a starting player by throwing a dice. The player with the highest number starts.
- The starting player is the first to choose his or her qudit position. The player marks the qudit's position by placing his or her pawn.
- Then, the players take turns going to the left. The players can place their pawns only on positions neighboring already set qudits.

The layout at the start of a 4-player game after the starting positions are decided.



Playing the game

1st Round:

- The starting qudit value for all players is 0.
- The starting player starts the round by playing one card from his or her hand.
- Going to the left, the other players take turn in playing their operations.
- The round is finished after every player has played three cards.
- Then, the played quantum operations are entered into the evaluation software to determine the winning state.
- The players earn as many points as the value of their qudit in the round's winning state. Write the points down on a paper.

2nd Round:

Round 2 is played as the previous round with the following changes:

- The starting state of all players is their winning value of the last round.
- The starting player is the player to the left side of the previous starting player.
- Every player plays only 2 cards.

3rd Round:

Round 3 is played as the previous round with the following changes:

- Only 1 card is played per player.

End of the game:

- The player with the most points summed up over all three rounds wins the game.

How to play cards

There are three different types of quantum operations. For all quantum operations the cards must be placed on the highest possible position beneath a qudit, but spaces to the left or right are allowed. Take a look at the truth tables at the end of the instructions to learn how they act.

Note:

- The players can put the card on any position, i.e. also apply logic operations on qudits of other players.
- The way a computation on this quantum computer works: every unitary is applied from left to right sequentially and top to bottom

1. Single qudit operations

Single qudit cards act only on one qudit and are placed beneath the qudit on which they should act on.

The cards:

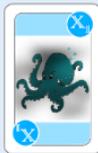
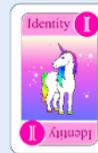
| | | |
|-----|----------|---|
| 4x | I | Identity operation. It does nothing. |
| 4x | X | X_1 and X_2 operation in 3D; X operation in 2D |
| 4x | Y | Y_1 and Y_2 operation in 3D; Y operation in 2D |
| 4x | Z | Z_1 and Z_2 operation in 3D; Z operation in 2D |
| 15x | Hadamard | H_1, H_2 and H_3 operation in 3D; H_1 and H_2 operation in 2D |

A possible way to place cards and the respective evaluation in 2D could be:

| | | |
|--|---|--|
| 1 | 0 | 1 |
| Player 1 | Player 2 | Player 3 |
|  |  |  |
| |  | |



It is not allowed to leave empty spaces beneath a qudit.

| | | |
|--|---|---|
| 1 | 0 | 1 |
| Player 1 | Player 2 | Player 3 |
|  | |  |
| |  | |

2. Two qudit operations

These cards act on two quantum states. The quantum operations can only be played on neighboring qudits. Tilting the card signifies on which neighboring state the quantum operation acts on.

The cards:

- 9x CNOT CNOT operation. The chosen control and target are placed next to each other on the field. The card is placed on the control qudit and tilted to the target qudit. In the software a CNOT with control on the left and target on the right is entered as CNOTr (and vice versa).
- 8x Swap Swaps the state with the chosen neighboring state. A tilt of the card signifies which states are swapped. In the software, swapping a state with the neighboring state on the right is entered as SWAPr (and vice versa).

Here, the state of player 1 acts as the control and the state of player 2 as the target of the CNOT gate.

1 0

Player 1 Player 2



Endless Fun

Starting state: 2D 3D

Quantum operations:

Winning state:

Number of measurements:

Reset Calculate

Complete end state:
1.*exp(i*0.)*|11>

The swap card is tilted into the direction of the state, with which the qudit states should be swapped.

0 0

Player 1 Player 2



Endless Fun

Starting state: 2D 3D

Quantum operations:

Winning state:

Number of measurements:

Reset Calculate

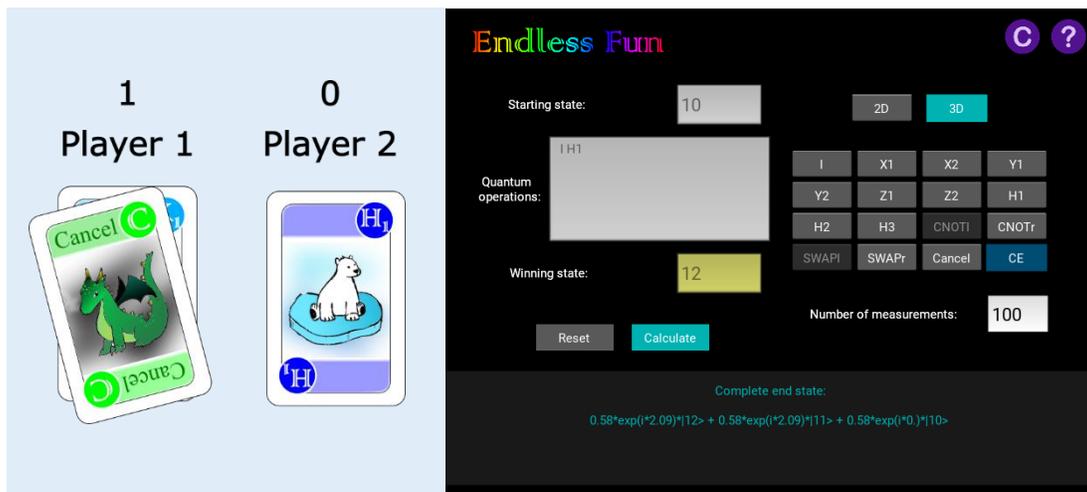
Complete end state:
1.*exp(i*0.)*|01>

3. Other cards

The cards:

- | | | |
|----|--------|--|
| 2x | Steal | Draw a random card from someone else and play it immediately. |
| 5x | Cancel | Cancel the operation of any card already played. They subsequently turn into identities and are entered as identities in the evaluation. |

The cancel card is placed on top of the card that should be nullified. It is then entered as an identity operation in the software.



How to use the evaluation software

- Run the evaluation application. Running the software in python is more powerful. The executable might crash for many players depending on the computer.
- Choose if you want to play in 2 or 3 dimensions by clicking on the buttons in the top right.
- Input the starting state
 - In 2D qubit values of 0 and 1 are allowed. In 3D the possible qudit values are 0, 1 and 2.
- Input the played quantum logic gates, by clicking on the buttons in the right panel.
 - Remember that the quantum computer applies the operations sequentially from left to right and top to bottom. The gates must also be entered in that order.
 - Use the identity operation as a placeholder for empty spaces.
- Define the number of measurements or use the default value of 100.
- Click the 'Calculate'-button to compute the winning state. The winning state is shown in the winning state text field, which is highlighted in yellow once the state is calculated.
 - If the calculate button is not enabled, some inputs are incorrect. Click the reset button next to it and enter the parameters again.
 - The winning state is the state, which includes the highest achieved qudit-value for at least one qudit and which was statistically measured the most often in the measurements.
- Click on the 'Reset'-button to evaluate the next round.

Note:

- The more measurements, the smaller the deviation of the outcome from the theoretical probability is, i.e. less luck is involved. In other words: one measurement leads to a pretty random result.
- The complete state is displayed at the bottom. It is the quantum state before its projection onto a single state through a measurement.

A first round played with 4 players in 3D in which player 1 starts with 0, player 2 with 2, player 3 with 1 and player 4 with 0 could for example be:

0 2 1 0
Player 1 Player 2 Player 3 Player 4

Starting state: 0210 2D 3D

Quantum operations: SWAPr X1 I H3 Z2 I I CNOTI I I I Y1

Winning state: 2122

Number of measurements: 100

Complete end state:
 $0.58 \cdot \exp(i \cdot 0) \cdot |2122\rangle + 0.58 \cdot \exp(i \cdot -2.09) \cdot |2111\rangle + 0.58 \cdot \exp(i \cdot 2.09) \cdot |2100\rangle$

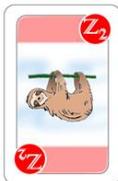
Since the winning state is 2122, player 2 earns 1 and the other players 2 points in this round.

Rookie version: "Finite Fun in Flatland"

- This simplified version helps to understand the game and to get ready for having endless fun in high dimensions.
- The differences during the full game described above are the following:
 - The game is played in 2D. The H3 cards are removed.
 - In every round, the players starting with the starting state 0.
 - The players are only allowed to put cards on their own qubit.

Truth tables of the quantum gates

| | | | | |
|-----------|---|-----------|---|---|
| Identity |  | X-gate |  |  |
| 3D | $ 0\rangle \rightarrow 0\rangle$ $ 1\rangle \rightarrow 1\rangle$ $ 2\rangle \rightarrow 2\rangle$ | 3D | $ 0\rangle \rightarrow 1\rangle$ $ 1\rangle \rightarrow 2\rangle$ $ 2\rangle \rightarrow 0\rangle$ | $ 0\rangle \rightarrow 2\rangle$ $ 1\rangle \rightarrow 0\rangle$ $ 2\rangle \rightarrow 1\rangle$ |
| 2D | $ 0\rangle \rightarrow 0\rangle$ $ 1\rangle \rightarrow 1\rangle$ | 2D | $ 0\rangle \rightarrow 1\rangle$ $ 1\rangle \rightarrow 0\rangle$ | |

| | | | | | | | |
|-----------|--|-----------------|--|-----------------|-----------|--|--|
| Y-gate |  | $Y_1 = Z_1 X_1$ |  | $Y_2 = Z_2 X_2$ | Z-gate |  |  |
| 3D | $ 0\rangle \rightarrow e^{i\frac{2\pi}{3}} 1\rangle$ $ 1\rangle \rightarrow e^{-i\frac{2\pi}{3}} 2\rangle$ $ 2\rangle \rightarrow 0\rangle$ | | $ 0\rangle \rightarrow e^{i\frac{2\pi}{3}} 2\rangle$ $ 1\rangle \rightarrow 0\rangle$ $ 2\rangle \rightarrow e^{-i\frac{2\pi}{3}} 1\rangle$ | | 3D | $ 0\rangle \rightarrow 0\rangle$ $ 1\rangle \rightarrow e^{i\frac{2\pi}{3}} 1\rangle$ $ 2\rangle \rightarrow e^{-i\frac{2\pi}{3}} 2\rangle$ | $ 0\rangle \rightarrow 0\rangle$ $ 1\rangle \rightarrow e^{-i\frac{2\pi}{3}} 1\rangle$ $ 2\rangle \rightarrow e^{i\frac{2\pi}{3}} 2\rangle$ |
| 2D | $ 0\rangle \rightarrow i 1\rangle$ $ 1\rangle \rightarrow i 0\rangle$ | | | | 2D | $ 0\rangle \rightarrow 0\rangle$ $ 1\rangle \rightarrow - 1\rangle$ | |

| | | | |
|--------------------|--|--|--|
| H-gate Hadamard |  |  |  |
| 3D | $ 0\rangle \rightarrow \frac{1}{\sqrt{3}} (0\rangle + e^{i\frac{2\pi}{3}} 1\rangle + e^{i\frac{2\pi}{3}} 2\rangle)$ $ 1\rangle \rightarrow \frac{1}{\sqrt{3}} (0\rangle + e^{-i\frac{2\pi}{3}} 1\rangle + 2\rangle)$ $ 2\rangle \rightarrow \frac{1}{\sqrt{3}} (0\rangle + 1\rangle + e^{-i\frac{2\pi}{3}} 2\rangle)$ | $ 0\rangle \rightarrow \frac{1}{\sqrt{3}} (0\rangle + e^{-i\frac{2\pi}{3}} 1\rangle + e^{-i\frac{2\pi}{3}} 2\rangle)$ $ 1\rangle \rightarrow \frac{1}{\sqrt{3}} (0\rangle + 1\rangle + e^{i\frac{2\pi}{3}} 2\rangle)$ $ 2\rangle \rightarrow \frac{1}{\sqrt{3}} (0\rangle + e^{i\frac{2\pi}{3}} 1\rangle + 2\rangle)$ | $ 0\rangle \rightarrow \frac{1}{\sqrt{3}} (0\rangle + 1\rangle + 2\rangle)$ $ 1\rangle \rightarrow \frac{1}{\sqrt{3}} (0\rangle + e^{i\frac{2\pi}{3}} 1\rangle + e^{-i\frac{2\pi}{3}} 2\rangle)$ $ 2\rangle \rightarrow \frac{1}{\sqrt{3}} (0\rangle + e^{-i\frac{2\pi}{3}} 1\rangle + e^{i\frac{2\pi}{3}} 2\rangle)$ |
| 2D | $ 0\rangle \rightarrow 0\rangle + 1\rangle$ $ 1\rangle \rightarrow 0\rangle - 1\rangle$ | | $ 0\rangle \rightarrow 0\rangle + i 1\rangle$ $ 1\rangle \rightarrow 0\rangle - i 1\rangle$ |

CNOT



SWAP



3D

$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$
 $|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$
 $|0\rangle|2\rangle \rightarrow |0\rangle|2\rangle$
 $|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$
 $|1\rangle|1\rangle \rightarrow |1\rangle|2\rangle$
 $|1\rangle|2\rangle \rightarrow |1\rangle|0\rangle$
 $|2\rangle|0\rangle \rightarrow |2\rangle|2\rangle$
 $|2\rangle|1\rangle \rightarrow |2\rangle|0\rangle$
 $|2\rangle|2\rangle \rightarrow |2\rangle|1\rangle$

3D

$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$
 $|0\rangle|1\rangle \rightarrow |1\rangle|0\rangle$
 $|0\rangle|2\rangle \rightarrow |2\rangle|0\rangle$
 $|1\rangle|0\rangle \rightarrow |0\rangle|1\rangle$
 $|1\rangle|1\rangle \rightarrow |1\rangle|1\rangle$
 $|1\rangle|2\rangle \rightarrow |2\rangle|1\rangle$
 $|2\rangle|0\rangle \rightarrow |0\rangle|2\rangle$
 $|2\rangle|1\rangle \rightarrow |1\rangle|2\rangle$
 $|2\rangle|2\rangle \rightarrow |2\rangle|2\rangle$

2D

$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$
 $|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$
 $|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$
 $|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$

2D

$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$
 $|0\rangle|1\rangle \rightarrow |1\rangle|0\rangle$
 $|1\rangle|0\rangle \rightarrow |0\rangle|1\rangle$
 $|1\rangle|1\rangle \rightarrow |1\rangle|1\rangle$

Credits

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NOTE: The general idea is strongly based on the great card game *Q/Cards* by Oskari Kerppo, Jorden Senior, Sabrina Maniscallo, Guillermo Garcia-Perez, Samuli Jääskeläinen, Sylvia Smatanova, Krista Erkkilä, Elie Abraham, which is a very similar game using only qubits. It can be downloaded here: <https://zhamul.itch.io/qcards>

