Amortizing and Accreting Caps and Floors Vaulation

FinPricing

Summary

- Interest Rate Amortizing and Accreting Cap and Floor Introduction
- The Use of Amortizing or Accreting Caps and Floors
- Caplet and Floorlet Payoffs
- Valuation
- Practical Notes
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Amortizing /Accreting Caps and Floors Introduction

◆ An interest rate cap is a financial contract between two parties that provides an interest rate ceiling or cap on the floating rate payments.

◆ An interest rate cap actually consists of a series of European call options (caplets) on interest rates.

◆ An amortizing cap is an interest rate cap whose notional principal amount declines during the life of the contract.

◆ An accreting cap is an interest rate cap whose notional principal amount increases during the life of the contract.

Amortizing /Accreting Caps and Floors Introduction (Cont)

◆ An interest rate floor is a financial contract between two parties that provides an interest rate "floor" on the floating rate payments.

◆ An interest rate floor consists of a series of European put options (floorlets) on interest rates.

◆ An amortizing floor is an interest rate floor whose notional principal amount declines during the life of the contract.

◆ An accreting floor is an interest rate floor whose notional principal amount increases during the life of the contract.

The Use of Amortizing /Accreting Caps and Floors

◆ Amortizing caps and floors are primarily used to hedge loans whose principal declines on a scheduled basis.

◆ An accreting cap or floor is primarily used to hedge construction loans whose principal increases on a scheduled basis to meet the expanding working capital requirements.

◆ Amortizing caps are frequently purchased by issuers of floating rate debt where the loan principal declines during the life.

◆ Accreting caps are frequently purchased by issuers of floating rate debt where the loan principal increases during the life.

The Use of Amortizing /Accreting Caps and Floors

◆ Amortizing floors are frequently purchased by purchasers of floating rate debt where the loan principal declines during the life.

◆ Amortizing floors are frequently purchased by purchasers of floating rate debt where the loan principal increases during the life.

◆ The amortizing/accreting cap holders wish to protect themselves from the increased financing costs that would result from fluctuation in interest rates.

◆ The amortizing/accreting floor holders wish to protect themselves from the loss of income that would result from a decrease in interest rates.
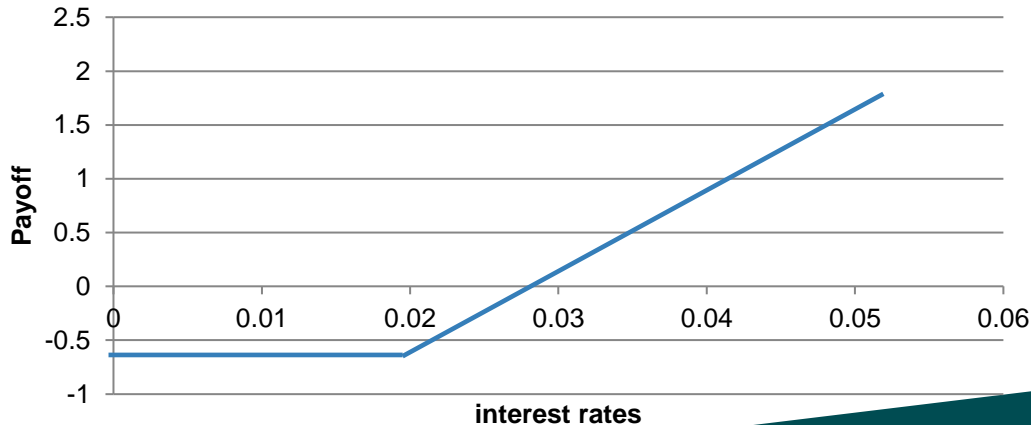
Caplet Payoff

◆ The payoff of a caplet

$$Payoff = N * \tau * max(R - K, 0)$$

where N − notional; R − realized interest rate; K − strike; τ − day count fraction.
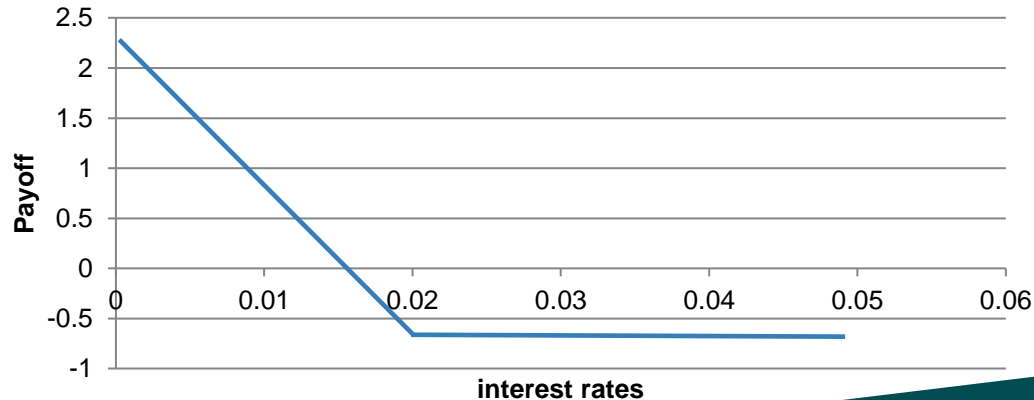
◆ Payoff diagram

Floorlet Payoff

◆ The payoff of a floorlet is given by

$$Payoff = N * \tau * max(K - R, 0)$$

where N − notional; R − realized interest rate; K − strike; τ − day count fraction.

◆ Payoff diagram

Valuation

◆ The analytics is similar to a vanilla cap and floor except the principal amount used by each period may be different.

◆ The present value of a cap is given by

$$PV(0) = \sum_{i=1}^{n} N_i \tau_i D_i (F_i \Phi(d_1) - K\Phi(d_2))$$

where

$D_i = D(0, T_i)$ – the discount factor;

$F_i = F(t; T_{i-1}, T_i) = \left(\frac{D_{i-1}}{D_i} - 1\right) / \tau_i$ – the forward rate for period (T_{i-1}, T_i).

Φ – the accumulative normal distribution function

$$d_{1,2} = \frac{\ln(\frac{F_i}{K}) \pm 0.5\sigma_i^2 T_i}{\sigma_i \sqrt{T_i}}$$

Valuation

◆ The present value of a floor is given by

$$PV(0) = \sum_{i=1}^{n} N_i \tau_i D_i \left(K\Phi(-d_2) - F_i\Phi(-d_1) \right)$$

where

$D_i = D(0, T_i) -$ the discount factor;

$F_i = F(t; T_{i-1}, T_i) = \left(\frac{D_{i-1}}{D_i} - 1 \right) / \tau_i -$ the forward rate for period (T_{i-1}, T_i).

$\Phi -$ the accumulative normal distribution function

$$d_{1,2} = \frac{\ln(\frac{F_i}{K}) \pm 0.5\sigma_i^2 T_i}{\sigma_i\sqrt{T_i}}$$

Practical Notes

◆ Amortizing and accreting caps are valued via the Black model in the market.

◆ The forward rate is simply compounded.

◆ The first key to value a cap is to generate the cash flows. The cash flow generation is based on the start time, end time and payment frequency, plus calendar (holidays), business convention (e.g., modified following, following, etc.) and whether sticky month end.

◆ Then you need to construct interest zero rate curve by bootstrapping the most liquid interest rate instruments in the market. The most common used yield curve is continuously compounded.

Practical Notes

◆ Another key for accurately pricing an outstanding cap/floor is to construct an arbitrage-free volatility surface.

◆ The accrual period is calculated according to the start date and end date of a cash flow plus day count convention

◆ The formula above doesn't contain the last live reset cash flow whose reset date is less than valuation date but payment date is greater than valuation date. The reset value is

$$PV_{reset} = N_0 * \tau * max(R - K, 0) \text{ for cap}$$
$$PV_{reset} = N * \tau * max(K - R, 0) \text{ for floor}$$

which should be added into the above present value.

Amortizing Cap/Floor

A Real World Example

Cap Terms and Conditions		Notional Schedule	
Buy Sell	Sell	9000000	2/6/2015
Cap or Floor	Cap	8785714.29	3/31/2015
Strike	0.025	8464285.72	6/30/2015
Trade Date	2/6/2015	8142857.15	9/30/2015
Start Date	2/6/2015	7821428.58	12/31/2015
Maturity Date	2/4/2019	7500000.01	3/31/2016
Currency	USD	7178571.44	6/30/2016
Day Count	dcAct360	6857142.87	9/30/2016
Rate type	Float	6535714.3	12/30/2016
Notional	9000000	6214285.73	3/31/2017
Pay Receive	Pay	5892857.16	6/30/2017
Payment Frequency	1M	5571428.59	9/29/2017
Index Tenor	1M	5250000.02	12/29/2017
Index Type	LIBOR	4928571.45	3/30/2018

Thanks!

You can find more details at
https://finpricing.com/lib/IrSwap.html