# Effects of nonlinear electrodynamics on slowly rotating BH spacetimes and neutrino propagation

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## Abstract

*Huge electromagnetic fields are known to be present during the late stages of the dynamics of SNe. Thus, when dealing with electrodynamics in this context, the possibility may arise to probe nonlinear theories (generalizations of the Maxwellian electromagnetism). We solve Einstein field equations minimally coupled to an arbitrary nonlinear Lagrangian of electrodynamics (NLED) in the slow rotation regime $a \ll M$ (black hole's mass), up to first order in $a/M$. We use Born-Infeld Lagrangian in order to compare and contrast the physical properties of such NLED spacetime with its Maxwellian counterpart. We focus on the astrophysics of both neutrino flavor oscillations ($\nu_e \to \nu_{\mu,\tau}$) and spin-flip ($\nu_L \to \nu_R$), the equivalent to gyroscopic precessions. Such analysis proves that in the spacetime of a slowly rotating nonlinear charged black hole, intrinsically associated with the assumption the electromagnetism is nonlinear, the neutrino dynamics in core-collapse SNe could be significantly changed. In such an astrophysical environment, the electron fraction $Y_e$, hence the r-processes, may significantly differ with respect to the standard electrodynamics.*

## 1. Field equation for slowly rotating black holes

We wish study neutrino spin-flip and neutrino oscillations for the Lagrangian density put forward by Born and Infeld [1]

$$L_{BI} = b^2\left[1-\sqrt{1-\frac{F}{2b^2}-\frac{G}{16b^6}}\right]\,,\quad F = F_{\mu\nu}F^{\mu\nu}\,,\, G = F^{\mu\nu}F^*_{\mu\nu}\,,$$

where $b$ is a parameter. The field equations are [2]

$$G_{\mu\nu} = 8\pi T_{\mu\nu}\,,\quad \partial_\mu\left[\sqrt{-g}(L_{BI,F}F^{\mu\nu}+L_{BI,G}F^{*\mu\nu})\right]=0\,. \qquad (1)$$

Assuming the metric of a slowly rotating BH

$$ds^2 = g_{00}(r)dt^2 - \frac{1}{g_{00}(r)}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 - 2a\sin^2\theta A(r)dt\,d\phi,$$

an exact solution to Einstein's equations in the spherically symmetric case is

$$g_{00} = 1-2u+\frac{2(bM)^2}{3u^2}\left(1-\sqrt{1+\frac{\alpha u^4}{(bM)^2}}\right)+\frac{2\alpha^2 u}{3}\sqrt{\frac{bM}{\alpha}}\mathcal{F}[z,\frac{1}{\sqrt{2}}]\,.$$

where $u = GM/r$, $\alpha = Q/M$, $z = \cos^{-1}\frac{bM-\alpha u^2}{bM+\alpha u^2}$, and $\mathcal{F}[z,q]$ is the elliptic function of first species.

## 2. Neutrino oscillations and spin-flip in BI Lagrangian

*Neutrino flavor oscillation* - Neutrino flavor oscillations are related to the mass eigenstate by the transformation $|\nu_\alpha\rangle = U_{\alpha j}\exp[-i\Phi_j]|\nu_j\rangle$ where $\alpha$ and $j$ stand for the neutrino flavor and mass eigenstates, $U$ the mixing matrix, and $\Phi_j$ is the phase associated with the $j$th mass eigenstate. In curved spacetimes, $\Phi_j$ reads $\Phi_j = \int p_{(j)\mu}dx^\mu$ [3, 4], being $p_{(j)\mu}$ the four momentum of the mass eigenstate $j$. The transition probability and the oscillation length reads

$$P(\nu_\alpha\to\nu_\beta) = \sin^2(2\Theta)\sin^2\left(\frac{\Phi_{jk}}{2}\right),\quad \Phi_{jk} \doteq \Phi_j - \Phi_k\,.$$

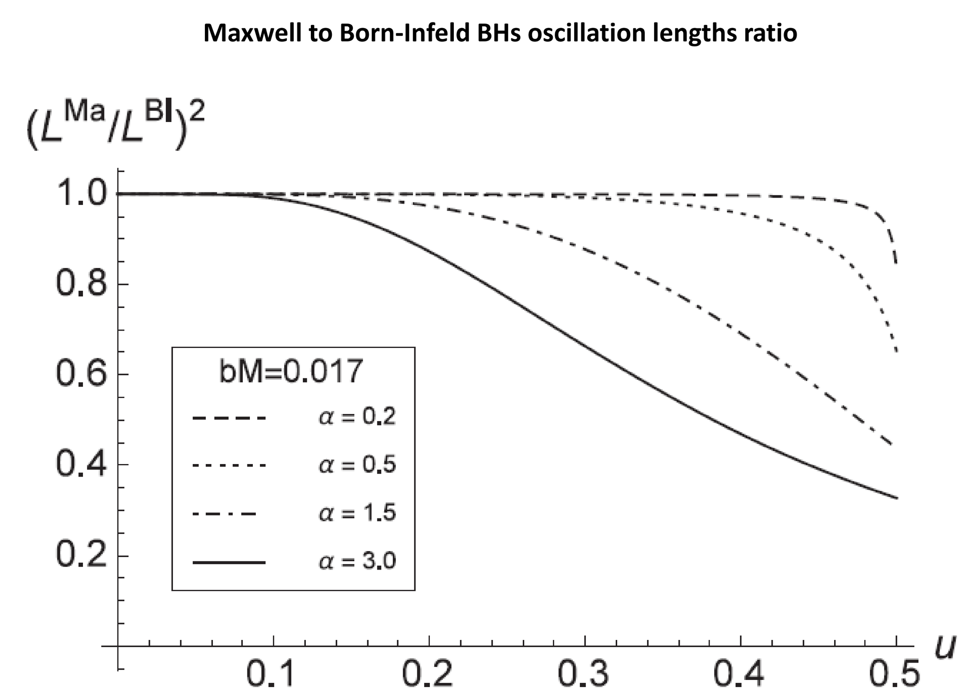
$$L_{osc} \doteq \frac{dl_{pr}}{d\Phi_{jk}/(2\pi)} = \frac{2\pi E}{\sqrt{g_{00}}(m_j^2-m_k^2)},\quad dl_{pr}^2 = \left(-g_{ij}+\frac{g_{0i}g_{0j}}{g_{00}}\right)dx^i dx^j\,.$$

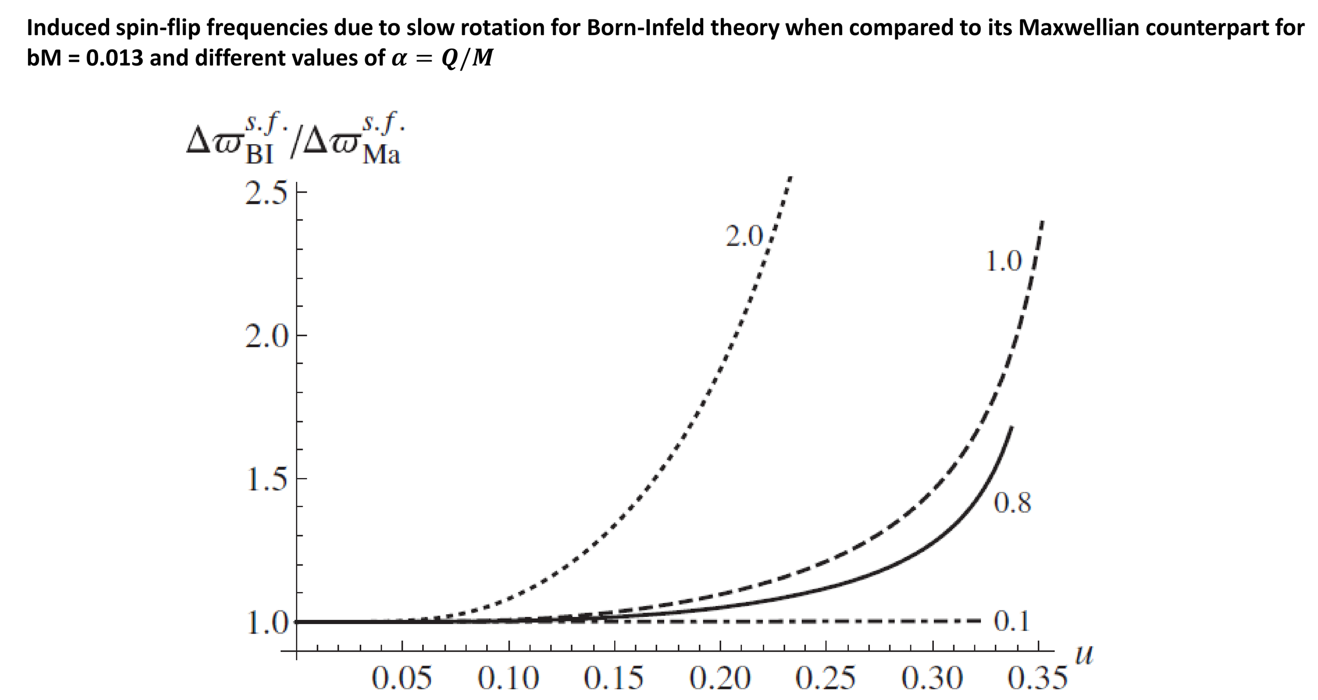
$$\Phi_j = \int dr\frac{m_j}{\dot r} = m_j^2\int\frac{dr}{\sqrt{E^2-g_{00}(r)\left[\frac{l^2}{\sin^2\theta_0 r^2}-\frac{2EaA(r)}{g_{00}(r)r^2}+m_j^2\right]}}\,.$$

*Neutrino spin precession* - The equations governing the spin $S^\mu$ coupling of test particles with the gravitational field are (neutrino flavor spin precession) [4, 2]

$$\frac{DS^\mu}{d\lambda}=0,\ \frac{Dv^\mu}{d\lambda}=0\ \to\ \frac{d\mathbf{S}}{d\tau} = \boldsymbol{\omega}\times\mathbf{S}\,,\ \boldsymbol{\omega} = \mathbf{B}+\frac{\mathbf{E}\times\mathbf{v}}{1+v^0}\,,$$

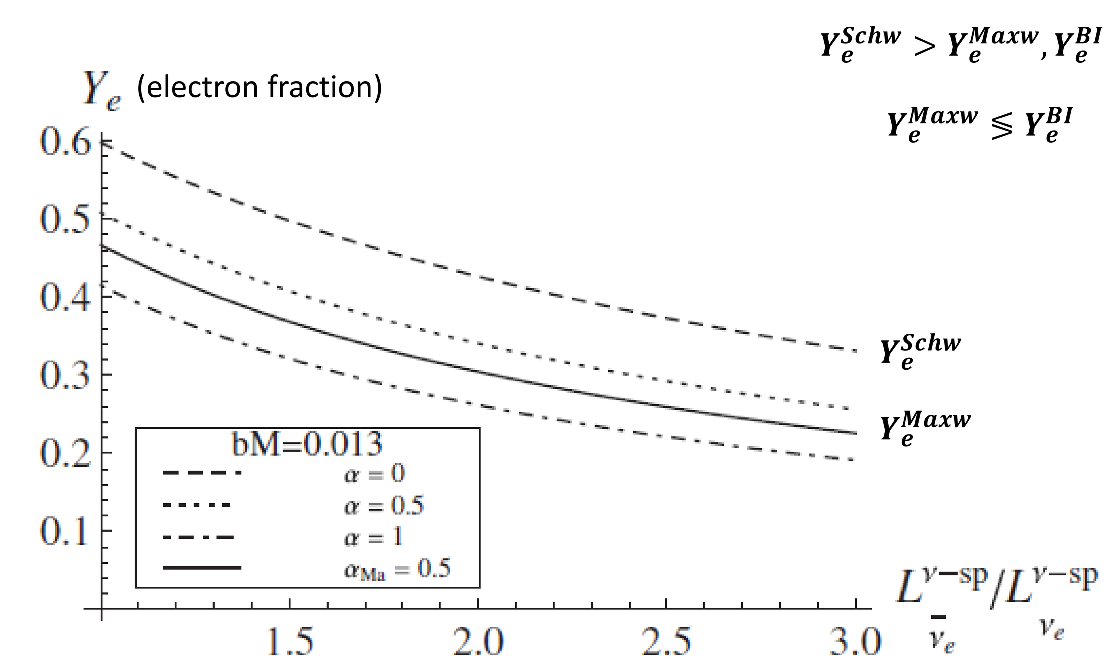
while the oscillation spin-flip probability is $P_{s.f.} = \sin^2\omega\tau$. Results of the neutrino and spin-flip oscillation are reported in the following Figures [2].




## 3. Estimates of electron fraction $Y_e$

The presence of magnetic field changes the neutron-to-proton ratio in SN ejecta, which already change the r-processes. This would mean that, in principle, nonlinear charged BH could indeed influence more SN events and the formation of heavier elements than Schwarzschild ones. In Figure we report $Y_e$ for different cases.




## 4. Conclusion

We have solved Einstein's equations for slowly rotating black holes minimally coupled to nonlinear Lagrangians of the electromagnetism dependent upon its two local invariants. Neutrinos could probe some of the aspects of these spacetimes, which may discern charged and uncharged black holes, as well as Maxwellian from NLED. In this study we mainly focalised in the analysis of the spin-flip, the neutrino flavor oscillation, and the r-processes.

## References

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