Vibration Protection of the Robotic Arm from External Effects on the Base

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Abstract—**The paper considers the topical problem of robotic arms protection from small base vibrations. To derive dynamic equations, the matrix method and the matrix Lagrange equations are used. A mathematical model of the movement of the manipulator with small vibrations of the base is presented. The solution was obtained by the method of polynomial transformations. A motion model of a manipulator mounted on shock absorbers consisting of elastic elements and dampers is presented. The shock absorber allows you to completely absorb small vibrations of the base of the manipulator. Dependences of the depreciation coefficient on the vibration frequency of the base and the coefficient of vibration protection on the damping coefficient are presented. In this case, the depreciation coefficient is significantly less than unity and the shock absorber effectively dampens small vibrations of the base. The method of model development and the method of transformations presented in this work for a qualitative and quantitative analysis of the effectiveness of vibration protection can be used to study the vibration protection of a wide class of manipulators with many degrees of freedom.**

I. INTRODUCTION

The problem of vibration protection of robotic arms from external influences on the base is considered. Small vibrations can be caused by fast-rotating elements such as flywheels, rotor systems, and electric motors. The main reason for small fluctuations is the residual imbalance of the elements and their wear. This leads to the effect of periodic inertia forces proportional to the square of the angular velocity of rotation. For robotic manipulators, this effect leads to stresses in the elements of robots and inaccuracy in the performance of production tasks.

A lot of modern scientific works are devoted to the problems of robotics and vibration protection of robotic manipulators.

In [1], the effect of joint flexibility on the vibrational characteristics of a composite box-shaped manipulator is researched. The theoretical results obtained using finite element analysis are compared with experimental results.

The study [2] considers the problem of dynamic modelling and the task of developing active vibration control for a flexible robot manipulator Tymoshenko. For a practical robot manipulator, the dynamic characteristic is studied taking into account the shear strain and the elastic beam deflection. A flexible system is described using a coupled partial differential equation and a model of ordinary differential equations.

In [3], modelling and analysis of vibration of a parallel manipulator with two degrees of freedom in a hybrid machine with five degrees of freedom are performed. The model is obtained using the connection graph, and the elastic beam deflection with the lowest stiffness is taken into account in the simulation. An approach is proposed for studying the interaction between the mechanical subsystem and the control subsystem in the frequency domain. Based on the interaction between the mechanical subsystem and the control subsystem, forced oscillations are investigated.

In [4], dynamic modelling is performed to control vibration in a three-dimensional flexible manipulator. A flexible manipulator is described by a system of distributed parameters with partial differential equations and ordinary differential equations.

The study [5] considers the vibrational characteristics of a space manipulator with flexible connections. The vibration of the manipulator, caused by flexibility, not only reduces the efficiency of the manipulator but also affects the accuracy. The flexibility of the space manipulator is due to the structural flexibility of the links and the flexibility of the transmission of the harmonic gearbox in the joints. The vibrations created by these two types of flexibility are interconnected, which complicates the dynamic characteristics of the space manipulator system. The article defines the dynamic equations of a multi-link flexible manipulator with several degrees of freedom. In this work, the vibrational characteristics of the tip were studied for various moduli of elasticity, damping, and joint stiffness.

A hybrid control for a system of elastic electro-hydraulic manipulators with a variable stiffness drive was presented in [6]. The system includes a sequential elastic manipulator, an adjustable stiffness mechanism and an electro-hydraulic system. This system can provide quick stiffness control over a wide range. The study examines the new design of the adaptive fuzzy sliding control mode.

The article [7] presents experimental studies on the control of active vibrations of a two-link flexible manipulator using a generalized self-tuning scheme with minimal dispersion and fuzzy neural network control schemes based on the Takagi-

Sugeno model. Experimental results show that the developed controller can damp the vibrations of both large and small amplitudes of a two-link flexible manipulator faster than a traditional linear controller.

To improve the control accuracy of industrial robotic manipulators, an adaptive control method based on a system of recurrent fuzzy wavelet-neural networks was proposed in [8]. The parameters of the neural network system are configured online using an adaptive learning algorithm. Online adaptive control laws are determined by the Lyapunov stability theorem.

In the study [9], the problem of vibration during the operation of industrial robotic manipulators is considered and iterative learning control is proposed to suppress vibration. A simulation study is performed and the method is applied to a system with two degrees of freedom.

A new manipulation system is being developed for industrial robotic manipulators based on wireless communications and a tablet PC [10]. The developed graphical interface allows the operator to perform manipulator control tasks more efficiently.

In [11], an adaptive controller based on neural networks with a radial basis function is proposed to increase the accuracy of control of industrial robotic manipulators in uncertain dynamic environments. The parameters of the control system are determined by the Lyapunov stability theorem and are tuned online using an adaptive learning algorithm.

In the above works on the vibration protection of manipulators, all studies were carried out by numerical methods, and quantitative results were obtained. The use of numerical methods for the analysis of nonlinear vibration protection systems does not fully allow obtaining qualitative characteristics of vibration protection. In our work, for the study of nonlinear models of manipulators, the analytical method of polynomial transformations is applied, which allows one to obtain qualitative and quantitative characteristics of the vibration-proof system.

In Section 2, we obtain a mathematical model of a manipulator with six degrees of freedom under the action of small vibrations of the base. The model is represented by a nonlinear system of six second-order differential equations. Non-linearity for the system under study is presented to the fourth degree, relative to phase variables. To solve the system of nonlinear differential equations, the method of polynomial transformations is applied.

Section 3 presents the polynomial transformation method for solving a nonlinear system of m differential equations. Theorems on finding a stationary solution to a system of m differential equations by the transformation method are presented. In the case of nonlinearities in the system to the fourth degree, the system of linear m algebraic equations is obtained by the method of transformations, which determines the stationary motion of the system. In the case of sixth-degree nonlinearities, a system of m quadratic algebraic equations is obtained by the method of transformations.

Section 4 presents a vibration-proof model for a manipulator with six degrees of freedom under the influence of small vibrations of the base. The model is represented by a nonlinear system of six second-order differential equations. Non-linear shock absorbers and dampers are used in the vibration protection system. Using the method of transformations, nonlinear characteristics of the vibration protection system, the coefficient of vibration protection, and the dependence of the coefficient of vibration protection on the damping coefficient are obtained. The use of a vibration protection system allows you to completely damp out small vibrations of the manipulator.

II. Research of Robot-Manipulator with External Periodic Influence on the Basis

Consider a mathematical model of a manipulator with six degrees of freedom with small vibrations of the base without a shock absorber and with a shock absorber.

Degrees of freedom provide a unique orientation of the manipulator in three-dimensional space. The kinematic diagram of the manipulator has the form of an open spatial lever mechanism, the links of which form rotational and translational pairs. The links of the manipulator are modelled by rod elements connected by the scheme. In the model, we assume that low friction in the links of the links does not affect the movement of the manipulator.

Electric motors for manipulator drives are located on the base and can be a source of small base vibrations. This can lead to significant errors when positioning the grip of the manipulator.

Suppose that the base of the manipulator, the circuit of which is shown in Fig. 1, experiences vertical vibrations according to the law $Z_A = A\,Sin\left(wt\right)$.

In this case, the portable inertia force can be applied in the centre of mass of each link of the manipulator

$$F_{iz} = -m_i A w^2\, Sin\left(wt\right).$$

After that, the reference system associated with the base of the manipulator O_0 can be considered motionless.

To obtain the dynamic equations of the manipulator, we apply the matrix method [12] and the matrix dynamic Lagrange equations.

Define the coordinate system relative to the links of the manipulator at points $O_1, O_2, O_3, O_4, O_5, O_6$. The initial coordinate system is connected with the base of the manipulator at a point O_0.

We take as generalized q_i the coordinates of the manipulator with six degrees of freedom: the angle of rotation around the rack, the angle of inclination of the rack, the length of the arm, the angle of rotation of the arm, the angle of rotation and inclination of the grip. Here the angles are measured in radians and lengths in centimetres.

We introduce the notation of periodic functions:

$$C_i = \mathrm{Cos}\left[q_i\left(t\right)\right], S_i = \mathrm{Sin}\left[q_i\left(t\right)\right],$$

$$\mathrm{Sin}\left[2q_2\right] = S_7, \mathrm{Cos}\left[2q_2\right] = C_7, \mathrm{Sin}\left[2\left(q_2+q_4\right)\right] = S_8,$$

$$\mathrm{Cos}\left[2\left(q_2+q_4\right)\right]=C_8,\mathrm{Sin}\left[2q_2+q_4\right]=S_9,$$

$$\mathrm{Cos}\left[2q_2+q_4\right]=C_9,\mathrm{Sin}\left[q_2+q_4\right]=S_{10},\mathrm{Cos}\left[q_2+q_4\right]=C_{10},$$

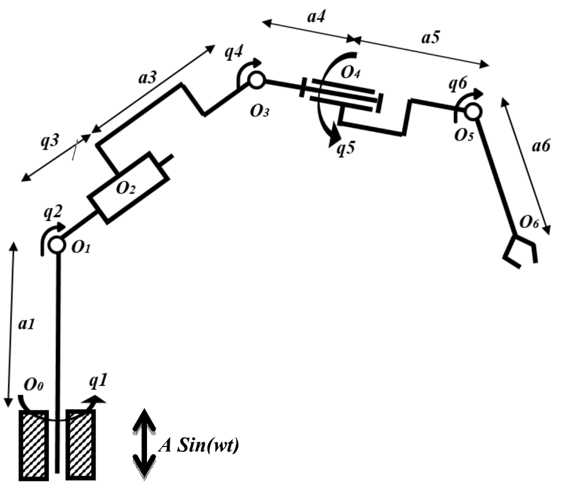

Fig. 1. Scheme of the manipulator

Grip coordinates in the coordinate system O_0 have the form:

$$x_6=-a_3C_2S_1-a_4C_2S_1-q_3S_1+a_5\left(-C_2C_4S_1+S_1S_2S_4\right)+ a_6\left(-C_2C_4S_1+S_1S_2S_4\right),$$

$$y_6=a_3C_1C_2+a_4C_1C_2+C_1q_3+a_5\left(C_1C_2C_4-C_1S_2S_4\right)+ a_6\left(C_1C_2C_4-C_1S_2S_4\right),$$

$$z_6=a_1+a_3S_2+a_4S_2+a_5\left(C_4S_2+C_2S_4\right)+ a_6\left(C_4S_2+C_2S_4\right).$$

We determine the total kinetic energy of all links:

$$T=\sum_{i=1}^{6}T_i$$

We determine the total potential energy of all links:

$$P=\left(-g-A w^2\,Sin\left[wt\right]\right)(-a_1\left(m_1+m_2+m_3+m_4+m_5+m_6\right)- (a_4\left(m_4+m_5+m_6\right)+a_3\left(m_3+m_4+m_5+m_6\right)+ C_4\left(a_5m_5+\left(a_5+a_6\right)m_6\right))S_2-C_2\left(a_6m_6+a_5\left(m_5+m_6\right)\right)S_4)$$

We compose a system of dynamic equations of motion of an industrial manipulator using the Lagrange equation in matrix form:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial P}{\partial q_i}-Q_i=0$$

where Q_i – generalized forces created by the electric link.

We substitute the kinetic, potential energy and generalized forces into the Lagrange matrix equations, we obtain the system of equations of motion of the industrial manipulator with six degrees of freedom.

$$0.5\left(k_{20}+m_1i_1^2+m_2i_2^2+m_3i_3^2+m_4i_4^2+m_5i_5^2+m_6i_6^2\right)q_1''+k_{31}q_3q_1''+ k_{12}q_1'q_2'+k_{13}q_1'q_3'+k_{14}q_1'q_4'+k_{312}q_3q_1'q_2'+k_{313}q_3q_1'q_3'=Q_1$$

$$0.5\left(k_{22}+m_2i_2^2+m_3i_3^2+m_4i_4^2+m_5i_5^2+m_6i_6^2\right)q_2''+k_{24}q_2'q_4'+ k_{21}\left(q_1'\right)^2+p_1\mathrm{Sin}\left[tw\right]+p_2\mathrm{Cos}\left[tw\right]+p_0=Q_2$$

$$(m_3+m_4+m_5+m_6)q_3''+k_{33}\left(q_1'\right)^2- (m_2+m_3+m_4+m_5+m_6)q_3\left(q_1'\right)^2=Q_3$$

$$0.5\left(k_{40}+m_4i_4^2+m_5i_5^2+m_6i_6^2\right)q_4''+k_{41}\left(q_1'\right)^2+k_{42}\left(q_2'\right)^2+ p_{41}\mathrm{Sin}\left[tw\right]+p_{42}\mathrm{Cos}\left[tw\right]+p_{40}=Q_4$$

$$0.5\left(i_5^2m_5+i_6^2m_6\right)q_5''=Q_5\ ,\ 0.5i_6^2m_6q_6''=Q_6$$

We integrate the fifth and sixth equations of the system:

$$q_5\left(t\right)=\frac{t^2Q_5}{0.5i_5^2m_5+0.5i_6^2m_6};\ \ q_6\left(t\right)=\frac{t^2Q_6}{0.5i_6^2m_6};$$

To solve the remaining four differential equations of system (4), we apply the polynomial transform method [13-16] with the following parameters:

$$m_1=200kg,m_2=60kg,m_3=30kg,$$
$$m_4=20kg,m_5=20kg,m_6=20kg,$$

$$Q_1=6000,Q_2=60000,Q_3=0.01,Q_4=20000,$$

$$Q_5=0.01,Q_6=0.01,$$

$$a_1=30cm,a_2=20cm,a_3=20cm,a_4=30cm,$$

$$a_5=30cm,a_6=20cm,$$

The base makes small vertical vibrations $0.1Sin\left[30t\right]$.

The transformation method allows you to build a solution taking into account all the nonlinear components of the original system. The solution of the system of three differential equations is obtained by the method of transformations:

$$q_1\left(t\right)=0.0005+\left(-0.0009+0.008t\right)t+ \left(-0.0002+\left(0.0002-0.00003t\right)t\right)\mathrm{Cos}\left[30t\right]+ \left(-0.0004+\left(0.0005-0.0001t\right)t\right)\mathrm{Sin}\left[30t\right],$$

$$q_2\left(t\right)=0.0004+\left(-0.04+0.012t\right)t+ \left(-0.0002+\left(0.0001-0.00002t\right)t\right)\mathrm{Cos}\left[30t\right]+ \left(0.001+\left(0.0004-0.00008t\right)t\right)\mathrm{Sin}\left[30t\right],$$

$$q_3\left(t\right)=0.06+\left(-0.13257+0.0498t\right)t+ \left(-0.0186+\left(0.01455-0.0024t\right)t\right)\mathrm{Cos}\left[30t\right]+ \left(-0.029+\left(0.031-0.0065t\right)t\right)\mathrm{Sin}\left[30t\right],$$

$$q_4(t) = 0.0009 + (-0.072 + 0.032t)t + (-0.0004 + (0.0003 - 0.00005t)t)\mathrm{Cos}[30t] + (0.0019 + (0.001 - 0.0002t)t)\mathrm{Sin}[30t]$$

$$q_5(t) = 0.025t^2,\ q_6(t) = 0.05t^2\ ,$$

Fig. 2 shows the calculation of the generalized coordinates for the manipulator.

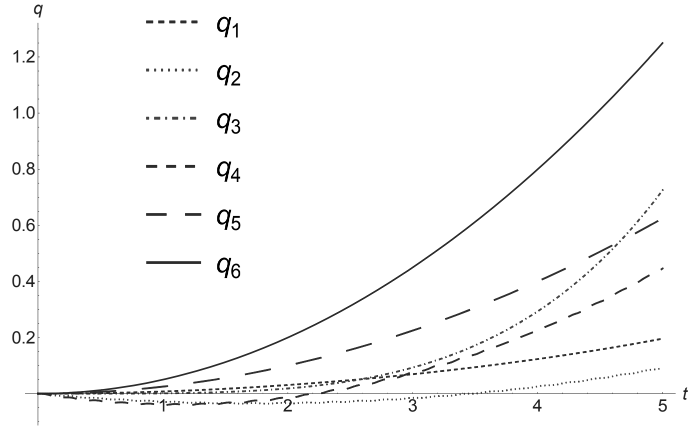

Fig. 2. The coordinates of the links of the manipulator

Fig. 3 shows the trajectory of vertical movements of the grip of the manipulator relative to the z-axis.

The figure shows the vertical oscillations of the grip of the manipulator with small vibrations of the base of the robot.

Fig. 3. The trajectory of the vertical movement of the capture of the manipulator

Consider the transformation method [15,16] used to construct an analytical stationary solution to the system of nonlinear differential equations of motion of the manipulator.

III. Transformation method for solving differential equations of manipulator motion

The transformation method [15,16] allows us to build a stationary analytical solution and includes an exponential change, linear and polynomial transformation.

By the method of polynomial transformations, the system is reduced to an autonomous form. For autonomous differential equations, the solutions asymptotically approach the stationary state. For the stability of stationary systems, all the roots of the characteristic equation must have negative real parts.

As a result of polynomial transformations, we obtain a system of algebraic equations, the solution of which determines a stationary solution. We consider theorems on determining the stationary state of a system by the method of transformations.

Theorem 1 Given the existence of a stable stationary state for a system *m* of second-order differential equations in the non-resonant case with small nonlinear parts in the form of polynomials of fourth degrees, there exists a system *m* of linear algebraic equations with real coefficients, obtained as a result of polynomial transformations, which determines the stationary state of the system.

The proof.

Assuming the existence of a stable stationary state in the non-resonant case, for a system of *m* nonlinear second-order differential equations, we determine the solution by the polynomial transformation method. We assume that for the system of differential equations the conditions of Picard's theorem on the existence and uniqueness of a solution to the Cauchy problem are satisfied, the characteristic matrix equation for the system has complex conjugate roots with small negative real parts. The right-hand side of the system is defined, continuous, and satisfies the Lipschitz condition.

Consider a system *m* of second-order differential equations with small nonlinear parts in the form of polynomials of fourth degrees.

In the absence of resonances, the autonomous transformed system has the following form:

$$\dot{\rho}_S = \rho_S \lambda_S + \sum_{|\nu|=2}^{4} \left(q_\nu^S\right) \rho_3^{\nu_3+\nu_4} \ldots \rho_{2m+1}^{\nu_{2m+1}+\nu_{2m+2}},$$

$$\rho_S \dot{\theta}_S = \sum_{|\nu|=2}^{4} \rho_3^{\nu_3+\nu_4} \ldots \rho_{2m+1}^{\nu_{2m+1}+\nu_{2m+2}} \left(p_\nu^S\right).$$

To determine the special indices, we have the matrix forms for $s = 3, \ldots, 2m+1$

$$M_{I3} = M_I + I_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & \ldots & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & \ldots & 1 & 1 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 2 & 1 & \ldots & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_{I(2m+1)} = M_I + I_{2m+1} = \begin{bmatrix} 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & \ldots & 1 & 1 & 1 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 1 & 1 & \ldots & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Each row of the matrix represents a special vector index.

For the sum of special indices, the equalities are satisfied:

$$\sum_{i=3}^{2m+2} \nu_i = 3, \quad \nu_1 = \nu_2 = 0\,, \; \sum_{i=3}^{2m+2} \nu_i = 1\,, \; \nu_1 = \nu_2 = 1\,.$$

Given that the sum of the special indices determines the degrees of monomials in the transformed system of differential equations and the equalities $\rho_1 = \rho_2 = \rho_1^{\nu_1} = \rho_2^{\nu_2} = 1$, in each equation of the transformed system, there are only monomials of the first and third degree of the desired variables.

To determine the stationary solution, we divide by ρ_S the first equation of the transformed system, we equate the right-hand sides to zero, we obtain a system of m algebraic equations:

$$\lambda_S + \sum_{|\nu|=2}^{4} \left(q_\nu^S\right) \rho_3^{\nu_3+\nu_4} \ldots \rho_{2m+1}^{\nu_{2m+1}+\nu_{2m+2}} \rho_S^{-1} = 0 \;, \quad s = 3,\ldots,2m+1\,.$$

When divided by ρ_S degrees of the monomials have decreased by one and in the equation, there are only monomials of zero (free terms) and second degree of the desired variables.

After a quadratic change of variables $r_S = \rho_S^2$, we obtain a linear system m of algebraic equations $s = 3,\ldots,2m+1$ with the following real coefficients:

$$\lambda_S + \sum_{|\nu|=3} \left(q_\nu^S\right) r_3^{\nu_3} r_5^{\nu_5} \ldots r_S^{\nu_S - 1} \ldots r_{2m+1}^{\nu_{2m+1}} = 0 \;, \quad s = 3,\ldots,2m+1$$

In the linear system of algebraic equations, only monomials of degree zero and first degree will be present, in accordance with the equalities for special indices:

$$\nu_3 + \nu_5 + \ldots + \nu_S - 1 + \ldots + \nu_{2m+1} = 1, \text{ for } \nu_1 = \nu_2 = 0\,,$$

$$\nu_3 + \nu_5 + \ldots + \nu_S - 1 + \ldots + \nu_{2m+1} = 0, \text{ for } \nu_1 = \nu_2 = 1\,.$$

Thus, a system of linear algebraic equations is obtained, and the theorem is proved for the system m second order differential equations.

Thus, a stationary solution is determined for a system of m differential equations of the second order with nonlinear parts in the form of polynomials of fourth degrees in the nonresonant case.

We give a theorem for the case of a polynomial of sixth degrees.

Theorem 2: Given the existence of a stable stationary state for a system of m second-order differential equations in the non-resonant case with small nonlinear parts in the form of polynomials of sixth-degree, there exists a system of m algebraic equations in the form of polynomials of the second degree with real coefficients, obtained as a result of polynomial transformations, which determines the stationary state of the system.

The proof.

Assuming the existence of a stable stationary state in the non-resonant case, for a system of m nonlinear second-order differential equations, we determine stationary solutions by the method of polynomial transformations. We assume that for the system of differential equations the conditions of Picard's theorem on the existence and uniqueness of a solution to the Cauchy problem are satisfied, the characteristic matrix equation for the system has complex conjugate roots with small negative real parts. The right-hand side of the system is defined, continuous, and satisfies the Lipschitz condition.

Consider a system m of second-order differential equations with small nonlinear parts in the form of polynomials of fourth degrees.

In the absence of resonances, the autonomous transformed system has the following form:

$$\dot{\rho}_S = \rho_S \lambda_S + \sum_{|\nu|=2}^{6} \left(q_\nu^S\right) \rho_3^{\nu_3+\nu_4} \ldots \rho_{2m+1}^{\nu_{2m+1}+\nu_{2m+2}}\,,$$

$$\rho_S \dot{\theta}_S = \sum_{|\nu|=2}^{6} \rho_3^{\nu_3+\nu_4} \ldots \rho_{2m+1}^{\nu_{2m+1}+\nu_{2m+2}} \left(p_\nu^S\right).$$

To define special indices $|\nu| = 2,3,4$ we write matrix forms for $s = 3,\ldots,2m+1$

$$M_{I3} = M_I + I_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & \ldots & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & \ldots & 1 & 1 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 2 & 1 & \ldots & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_{I(2m+1)} = M_I + I_{2m+1} = \begin{bmatrix} 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & \ldots & 1 & 1 & 1 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 1 & 1 & \ldots & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Each row of the matrix represents a special vector index.

For the sum of special indices, the equalities are satisfied:

$$\sum_{i=3}^{2m+2} \nu_i = 3, \quad \nu_1 = \nu_2 = 0\,, \; \sum_{i=3}^{2m+2} \nu_i = 1, \quad \nu_1 = \nu_2 = 1\,.$$

Given that the sum of the special indices determines the degrees of monomials in the transformed system of differential equations and the equalities $\rho_1 = \rho_2 = \rho_1^{\nu_1} = \rho_2^{\nu_2} = 1$, in each equation of the transformed system, there are only monomials of the first and third degree of the desired variables.

To define special indices $|\nu| = 5,6$ we write the matrix form s $= 3,\ldots,2m+1$.

For $s = 3$ matrices for determining special indices:

$$MJ_{1,2} = \begin{bmatrix} 2 & 2 & 1 \ldots 0 \end{bmatrix}, \; k = 1\,,$$

$$MJ_{2,4} = \begin{bmatrix} 0 & 0 & 3 & 2 & 0\ldots0 \\ 1 & 1 & 2 & 1 & 0\ldots0 \end{bmatrix},\ k = 2,$$

$$MJ_{3,6} = \begin{bmatrix} 0 & 0 & 1 & 0 & 2 & 2 & 0\ldots0 \\ 0 & 0 & 2 & 1 & 1 & 1 & 0\ldots0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0\ldots0 \end{bmatrix},\ k = 3,$$

$$MJ_{4,8} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 2 & 2 & 0\ldots0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0\ldots0 \\ 0 & 0 & 2 & 1 & 0 & 0 & 1 & 1 & 0\ldots0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0\ldots0 \end{bmatrix},\ k = 4,$$

$$MJ_{k,2m+2} = J_{k,2k+2} + I_3 =$$
$$\begin{bmatrix} 0 & 0 & 1 & 0 & \ldots & 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 & \ldots & 1 & 1 & 1 & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 2 & 1 & \ldots & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & \ldots & 0 & 0 & 1 & 1 \end{bmatrix},\ k = m+1.$$

For $s = 2m+1$ we obtain matrices for determining special indices:

$$MJ_{1,2} = \begin{bmatrix} 2 & 2 & 0\ldots0 & 1 & 0 \end{bmatrix},\ k = 1,$$

$$MJ_{2,4} = \begin{bmatrix} 0 & 0 & 2 & 2 & 0\ldots0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0\ldots0 & 1 & 0 \end{bmatrix},\ k = 2,$$

$$MJ_{3,6} = \begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 2 & 0\ldots0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0\ldots0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0\ldots0 & 1 & 0 \end{bmatrix},\ k = 3,$$

$$MJ_{4,8} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0\ldots0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0\ldots0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0\ldots0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0\ldots0 & 1 & 0 \end{bmatrix},$$
$$k = 4,$$

$$MJ_{k,2m+2} = J_{k,2k+2} + I_{2m+2} =$$
$$\begin{bmatrix} 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 & \ldots & 1 & 1 & 2 & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 1 & 1 & \ldots & 0 & 0 & 2 & 1 \\ 1 & 1 & 0 & 0 & \ldots & 0 & 0 & 2 & 1 \end{bmatrix},\ k = m+1.$$

Each row of the matrix $MJ_{k,2k}$ represents a special vector index. For special indices $s = 3,5,\ldots,2m+1$ the equalities are satisfied:

$$\sum_{i=3}^{2m+2} \nu_i = 5,\quad \nu_1 = \nu_2 = 0,\ \sum_{i=3}^{2m+2} \nu_i = 3,\quad \nu_1 = \nu_2 = 1,$$

$$\sum_{i=3}^{2m+2} \nu_i = 1,\quad \nu_1 = \nu_2 = 2,$$

Given that the sum of special indices $\sum_{i=3}^{2m+2} \nu_i$ determines the degree of monomials in the transformed system of differential equations and equality $\rho_1 = \rho_2 = \rho_1^{\nu_1} = \rho_2^{\nu_2} = 1$, each differential equation of the transformed system contains only monomials of the first, third, and fifth-degree of the desired variables.

To determine the stationary solution, we divide by ρ_S the first equation of the transformed system, equate the right-hand sides to zero, and obtain the system m of algebraic equations:

$$\lambda_S + \sum_{|\nu|=2}^{6} \left(q_\nu^S\right) \rho_3^{\nu_3+\nu_4} \ldots \rho_{2m+1}^{\nu_{2m+1}+\nu_{2m+2}} \rho_S^{-1} = 0\ ,\ s = 3,\ldots,2m+1.$$

When dividing by ρ_S the degrees of the monomials decreased by one and in the equation, there are only monomials of zero (free terms), second and fourth degrees of the desired variables.

After a quadratic change of variables, $r_S = \rho_S^2$ is done, we get a system m of algebraic equations in the form of polynomials of the second degree $s = 3,\ldots,2m+1$ with the following real coefficients:

$$\begin{matrix} \lambda_S + \sum_{|\nu|=3} \left(q_\nu^S\right) r_3^{\nu_3} r_5^{\nu_5} \ldots r_S^{\nu_S - 1} \ldots r_{2m+1}^{\nu_{2m+1}} + \\ \sum_{|\nu|=5} \left(q_\nu^S\right) r_3^{\nu_3} r_5^{\nu_5} \ldots r_S^{\nu_S - 1} \ldots r_{2m+1}^{\nu_{2m+1}} = 0 \end{matrix}\ ,\ s = 3,\ldots,2m+1$$

The system of algebraic equations will contain only monomials of degree zero, first and second degree, in accordance with the equalities for special indices:

$$\nu_3 + \ldots + \nu_S - 1 + \ldots + \nu_{2m+1} = 1, \nu_1 = \nu_2 = 0, |\nu| = 3,$$

$$\nu_3 + \ldots + \nu_S - 1 + \ldots + \nu_{2m+1} = 0, \nu_1 = \nu_2 = 1,\ |\nu| = 3,$$

$$\nu_3 + \ldots + \nu_S - 1 + \ldots + \nu_{2m+1} = 2, \nu_1 = \nu_2 = 0,\ |\nu| = 5,$$

$$\nu_3 + \ldots + \nu_S - 1 + \ldots + \nu_{2m+1} = 1, \nu_1 = \nu_2 = 1,\ |\nu| = 5,$$

$$\nu_3 + \ldots + \nu_S - 1 + \ldots + \nu_{2m+1} = 0, \nu_1 = \nu_2 = 2,\ |\nu| = 5.$$

Thus, a system of algebraic equations in the form of polynomials of the second degree is obtained, and the theorem is proved for a system of m differential equations of the second order. The solution to the system of algebraic equations is obtained by the Newton method.

Thus, a stationary solution to the system of m differential equations of the second order with non-linear parts in the form of polynomials of six degrees in the non-resonant case is determined.

The transformation method is applicable to solve the practical problem of vibration protection of manipulators.

IV. Vibration Protection of the Manipulator

To reduce vibrations, shock absorbers are installed between the base and the arm of the manipulator, consisting of an elastic element, stiffness c_A and a damping device that creates a drag force proportional to the speed of action. The damping device creates a resistance force, consumes energy of oscillations and leads to their damping.

The vertical force acts on the base: $Z_A = A\,Sin(wt)$.

As a result of the installation of the shock absorber, the resistance force acting on the base consists of the force transmitted by the elastic element and the damper.

$$R_z = c_A Z + b_A Z' = A\left(c_A\,Sin(wt) + b_A w Cos(wt)\right),$$

where b_A – is the damping coefficient of the shock absorber.

We apply the matrix equations of Lagrange, we obtain a system of equations of motion of the manipulator with a shock absorber.

$$0.5\left(k_{20} + m_1 i_1^2 + m_2 i_2^2 + m_3 i_3^2 + m_4 i_4^2 + m_5 i_5^2 + m_6 i_6^2\right) q_1'' + k_{31} q_3 q_1'' + k_{12} q_1' q_2' + k_{13} q_1' q_3' + k_{14} q_1' q_4' + k_{312} q_3 q_1' q_2' + k_{313} q_3 q_1' q_3' = Q_1$$

$$0.5\left(k_{22} + m_2 i_2^2 + m_3 i_3^2 + m_4 i_4^2 + m_5 i_5^2 + m_6 i_6^2\right) q_2'' + k_{24} q_2' q_4' + k_{21}\left(q_1'\right)^2 + h_1 \operatorname{Sin}[tw] + h_2 \operatorname{Cos}[tw] = Q_2$$

$$(m_3 + m_4 + m_5 + m_6) q_3'' + k_{33}\left(q_1'\right)^2 - (m_2 + m_3 + m_4 + m_5 + m_6) q_3 \left(q_1'\right)^2 = Q_3$$

$$0.5\left(k_{40} + m_4 i_4^2 + m_5 i_5^2 + m_6 i_6^2\right) q_4'' + k_{41}\left(q_1'\right)^2 + k_{42}\left(q_2'\right)^2 + h_{41} \operatorname{Sin}[tw] + h_{42} \operatorname{Cos}[tw] = Q_4$$

$$0.5\left(i_5^2 m_5 + i_6^2 m_6\right) q_5'' = Q_5\ ,\ 0.5 i_6^2 m_6 q_6'' = Q_6$$

To solve the system of differential equations, we apply the polynomial transform method with the following parameters:

$$m_1 = 200kg, m_2 = 60kg, m_3 = 30kg,$$
$$m_4 = 20kg, m_5 = 20kg, m_6 = 20kg,$$
$$Q_1 = 6000, Q_2 = 60000, Q_3 = 0.01, Q_4 = 20000,$$
$$Q_5 = 0.01, Q_6 = 0.01,$$
$$a_1 = 30cm, a_2 = 20cm, a_3 = 20cm, a_4 = 30cm,$$
$$a_5 = 30cm, a_6 = 20cm,$$

The base makes small vertical vibrations $0.1 Sin(30t)$.

For the installed shock absorber, the stiffness coefficient c = 0.1, the damping coefficient b = 0.01.

The transformation method allows to build a solution taking into account all the nonlinear components of the original system. The solution of the system of three differential equations by the transformation method is obtained:

$$q_1(t) = 0.005 - 0.0096t + 0.011t^2\ ,$$

$$q_2(t) = 0.0043 - 0.0085t + 0.0142t^2\ ,$$

$$q_3(t) = 0.0776 - 0.1665t + 0.0598t^2\ ,$$

$$q_4(t) = 0.01 - 0.021t + 0.038t^2$$

$$q_5(t) = 0.025t^2,\ q_6(t) = 0.05t^2\ ,$$

Fig. 4 shows the generalized coordinates for the manipulator.

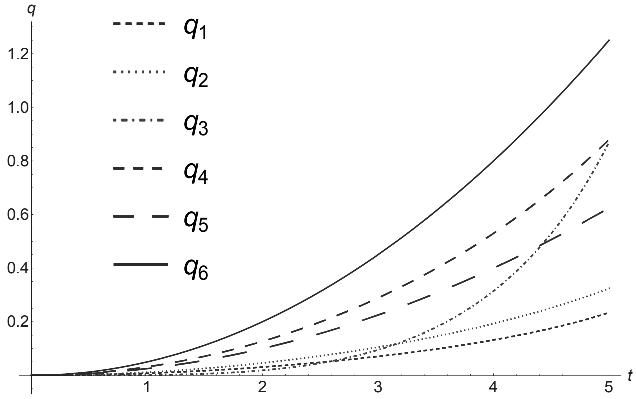

Fig. 4. Coordinates for the manipulator

Fig. 5 shows the trajectory of the spatial movements of the capture of the manipulator with a shock absorber in the conditions of small base vibrations.

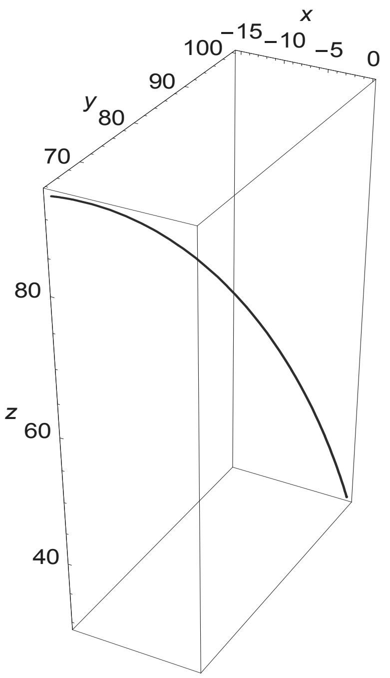

Fig. 5. Manipulator trajectory

As a result of installing the shock absorber, the vertical vibrations of the gripper are effectively suppressed.

To determine the characteristics of the damping device, a logarithmic attenuation decrement is used $\delta = \frac{2\pi\mu}{\sqrt{1+\mu^2}}$.

Here μ –is a damping coefficient.

Fig. 6 shows the dependence of the logarithmic damping decrement on the damping coefficient.

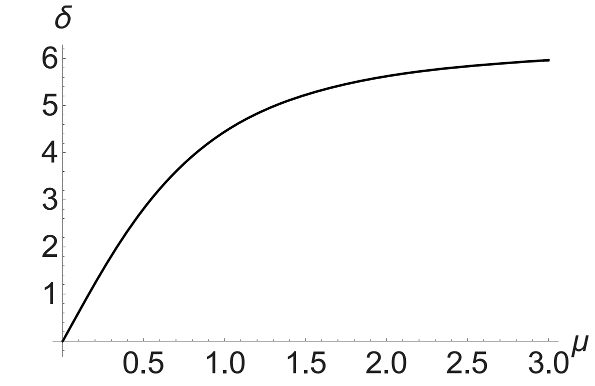

Fig. 6. Dependence of the logarithmic attenuation decrement on the damping coefficient.

To assess the degree of vibration protection, we consider the coefficient of vibration protection γ. The vibration protection coefficient is equal to the ratio of the amplitude of the oscillations of the links of the manipulator to the amplitude of the vibrations of the base.

$$\gamma = \sqrt{\frac{\sigma^2\mu^2+1}{\left(\sigma^2\mu^2+\sigma^2-1\right)^2}} ,$$

Here $\sigma = \frac{\omega_0}{\omega}$ is the ratio of the vibration frequencies of the base and the vibration frequencies of the links of the manipulator.

Fig. 7 shows the dependence of the vibration protection coefficient on the damping coefficient at $0 \le \mu \le 0.8$.

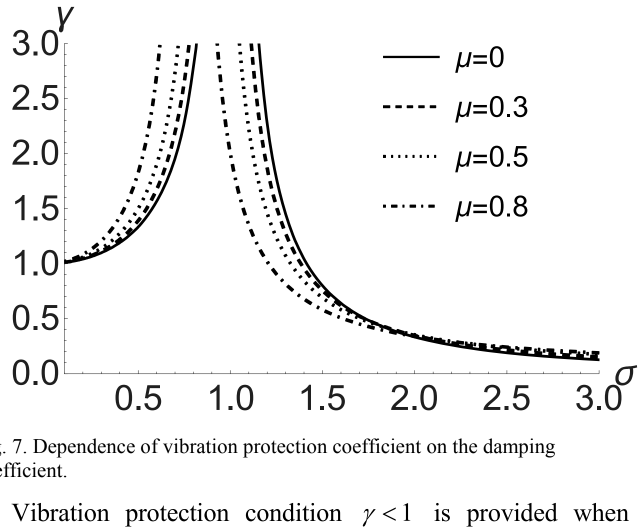

Fig. 7. Dependence of vibration protection coefficient on the damping coefficient.

Vibration protection condition $\gamma < 1$ is provided when $\sigma > \sqrt{2}$.

To assess the effectiveness of the shock absorber, consider the depreciation rate τ. The depreciation coefficient is equal to the ratio of the maximum acceleration of the system with a shock absorber to the maximum acceleration of the system without a shock absorber.

To effectively reduce vibrations, it is necessary to have damping factor value be significantly less than one $\tau < 1$.

For a robot manipulator, Fig. 8 shows the dependence of the depreciation coefficient on the vibration frequency.

For the manipulator's shock absorber to work effectively for given characteristics, the depreciation coefficient must be significantly less than one. In this case, the depreciation rate $\tau = 0.01$ and shock absorber effectively reduce vibration.

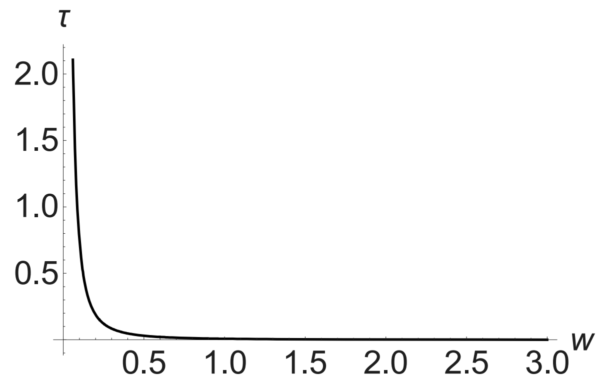

Fig. 8. Dependence of the depreciation coefficient on the vibration frequency.

V. Conclusion

The work considers the actual problem of vibration protection of the manipulator from small vibrations of the base. A mathematical model of the movement of the manipulator with small vibrations of the base, as well as a model of vibration protection of the manipulator is presented. The model of vibration protection of the manipulator is presented in the form of a nonlinear system of differential equations of the second order. The solution is determined by the polynomial transform method.

The method of transformations allows one to obtain an analytical solution of a nonlinear system of differential equations and to conduct a qualitative and quantitative study of the vibration-proof model. Theorems are presented on the method of transforming a system of nonlinear equations to a system of algebraic equations that determines stationary motion.

Vibration protection is performed using elastic elements and dampers. Dependences of the depreciation coefficient on the vibration frequency of the base and the coefficient of vibration protection on the damping coefficient are presented. The dependence of the depreciation coefficient on the frequency of external influence is presented. As a result, the task of vibration protection of the manipulator against external periodic exposure is solved, and the shock absorber effectively damps small vibrations of the base.

Thus, the presented method of constructing a nonlinear model of vibration protection of the manipulator and the method of analysis of the vibration protection model allows us to study the qualitative and quantitative characteristics of vibration protection and allows the development of vibration protection systems for various manipulators.

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