Ice-shelf vibrations modeled by a full 3-D elastic model: field equations

Y. V. Konovalov[1,2]

[1]Mathematical Department, Financial University under the Government of the Russian Federation, Leningradsky Prospekt 49, Moscow, Russian Federation, 125993, GSP-3.

[2]Department of Mathematics, National University of Science and Technology MISIS, Leninskiy Prospekt 4, Moscow, Russian Federation, 119049.

Correspondence to: Y.V. Konovalov (yu-v-k@yandex.ru)

Abstract

Forced ice shelf vibration modeling is performed using a full 3D finite-difference elastic model, which also takes into account sub-ice seawater flow. The sea water flow is described by the wave equation. Ice shelf flexure therefore results from hydrostatic pressure perturbations in the sub-ice seawater layer. Numerical experiments were undertaken for idealized rectangular ice-shelf geometry. The ice-plate vibrations were modeled for harmonic incoming pressure perturbations and for a wide range of incoming wave frequencies. The spectra showed distinct resonant peaks, which demonstrate the ability of the model to simulate a resonant-like motion.

1. Model description and field equations

2.1 Basic equations

The 3D elastic model is based on the well-known momentum equations (e.g. [1], [2]):

$$\begin{cases} \dfrac{\partial \sigma_{xx}}{\partial x}+\dfrac{\partial \sigma_{xy}}{\partial y}+\dfrac{\partial \sigma_{xz}}{\partial z}=\rho\,\dfrac{\partial^2 U}{\partial\, t^2}; \\ \dfrac{\partial \sigma_{yx}}{\partial x}+\dfrac{\partial \sigma_{yy}}{\partial y}+\dfrac{\partial \sigma_{yz}}{\partial z}=\rho\,\dfrac{\partial^2 V}{\partial\, t^2}; \\ \dfrac{\partial \sigma_{zx}}{\partial x}+\dfrac{\partial \sigma_{zy}}{\partial y}+\dfrac{\partial \sigma_{zz}}{\partial z}=\rho\,\dfrac{\partial^2 W}{\partial t^2}+\rho g; \\ 0<x<L; y_1(x)<y<y_2(x);\ h_b(x,y)<z<h_s(x,y), \end{cases} \tag{1}$$

where (XYZ) is a rectangular coordinate system with X axis along the central line, and Z axis is pointing vertically upward; U, V and W are two horizontal and one vertical ice displacements, respectively; $\boldsymbol{\sigma}$ is the stress tensor; and ρ is ice density. The ice shelf is of length L along the central line. The geometry of the ice shelf is assumed to be given by lateral boundary functions $y_{1,2}(x)$ at sides labeled 1 and 2 and functions for the surface and base elevation, $h_{s,b}(x,y)$, denoted by subscripts s and b, respectively. Thus, the domain on which Eqs. (1) are solved is $\Omega=\{0<x<L,\ y_1(x)<y<y_2(x),\ h_b(x,y)<z<h_s(x,y)\}$.

Sub-ice water is assumed to be an incompressible inviscid fluid of uniform density. Another assumption is that water depth in the cavity below the ice shelf changes gradually in the horizontal directions. Thus, the ice-front and other such features are not considered here. Moreover, the ice is considered to be a continuous solid elastic plate. Under these three assumptions, sub-ice water flow is independent of z in a vertical column. Manipulating the governing equations of the shallow sub-ice water layer yields the wave equation [3]:

$$\frac{\partial^2 W_b}{\partial\, t^2}=\frac{1}{\rho_w}\frac{\partial}{\partial\, x}\left(d_0\frac{\partial\, P\prime}{\partial\, x}\right)+\frac{1}{\rho_w}\frac{\partial}{\partial\, y}\left(d_0\frac{\partial\, P\prime}{\partial\, y}\right), \tag{2}$$

where ρ_w is sea water density; $d_0(x, y)$ is the depth of the sub-ice water layer; $W_b(x, y, t)$ is the vertical deflection of the ice-shelf base, and $W_b(x, y, t) = W(x, y, h_b(x, y), t)$; and $P'(x, y, t)$ is the deviation of the sub-ice water pressure from the hydrostatic value.

2.2 Boundary conditions

The boundary conditions are: (i) a stress free ice surface; (ii) the normal stress exerted by seawater at the ice-shelf free edges and at the ice-shelf base; and (iii) rigidly fixed edges at the grounding line of the ice-shelf.

In this model, the boundary conditions are considered in the form of the linear combination

$$\alpha_1 F_i(U, V, W) + \alpha_2 \Phi_i(U, V, W) = 0, \; i = 1,2,3, \quad (3)$$

where:

(i) $F_i(U, V, W) = 0$ is the typical and well-known form of the boundary conditions where, for example, the condition on the ice-shelf surface is expressed as $\sigma_{ik} n_k = 0$ ($\vec{n}$ is the unit vector normal to the surface);

(ii) $\Phi_i(U, V, W) = 0$ is the approximation based on the integration of the typical boundary conditions into the momentum equations (1);

and

(iii) the coefficients α_1 and α_2 satisfy the condition $\alpha_1 + \alpha_2 = 1$.

Thus, these boundary conditions (3) are the superposition of the typical boundary conditions (see **Appendix A**) and those based on the integration of the basic/typical boundary conditions into the momentum equations (see **Appendix B**). Thus, the boundary conditions formulated here are notable because they are "mixed".

The boundary conditions to the sea water layer correspond to the frontal incident wave. They are

(i) at $x = 0$: $\frac{\partial P\prime}{\partial x} = 0$;

(ii) at $y = y_1, y = y_2$: $\frac{\partial P\prime}{\partial y} = 0$;

(iii) at $x = L$: $P' = A_0\, \rho_w g\, e^{i\omega t}$, where A_0 is the amplitude of the incident wave.

2.3 Discretization of the model

The numerical solutions were obtained by a finite-difference method, which is based on the standard coordinate transformation $x, y, z \rightarrow x, \eta = \frac{y-y_1}{y_2-y_1}, \xi = (h_s - z)/H$, where H is the ice thickness ($H = h_s - h_b$). The coordinate transformation maps the ice domain Ω into the rectangular parallelepiped $\Pi = \{0 \leq x \leq L; 0 \leq \eta \leq 1; 0 \leq \xi \leq 1\}$, which presents simplification to the numerical discretization.

2.4 Equations for ice-shelf displacements

Constitutive relationships between stress tensor components and displacements correspond to Hooke's law (e.g., [2], [4]):

$$\sigma_{ij} = \frac{E}{1+\nu}\left(u_{ij} + \frac{\nu}{1-2\nu}\, u_{ll}\delta_{ij}\right) \quad , \qquad (4)$$

where u_{ij} are the strain components, E - Young's modulus, ν - Poisson's ratio.

Substitution of these relationships into Eq. (1) gives final equations of the model:

$$\begin{cases} \frac{2(1-\nu)}{1-2\nu}\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}+\frac{1}{1-2\nu}\left(\frac{\partial^2 V}{\partial x\,\partial y}+\frac{\partial^2 W}{\partial x\,dz}\right)=\frac{2(1+\nu)}{E}\rho\,\frac{\partial^2 U}{\partial t^2}; \\ \frac{\partial^2 V}{\partial x^2}+\frac{2(1-\nu)}{1-2\nu}\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}+\frac{1}{1-2\nu}\left(\frac{\partial^2 U}{\partial y\,\partial x}+\frac{\partial^2 W}{\partial y\,\partial z}\right)=\frac{2(1+\nu)}{E}\rho\,\frac{\partial^2 V}{\partial t^2}; \\ \frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+\frac{2(1-\nu)}{1-2\nu}\frac{\partial^2 W}{\partial z^2}+\frac{1}{1-2\nu}\left(\frac{\partial^2 U}{\partial z\,\partial x}+\frac{\partial^2 V}{\partial z\,\partial y}\right)-\frac{2(1+\nu)}{E}\rho g=\frac{2(1+\nu)}{E}\rho\,\frac{\partial^2 W}{\partial t^2}; \end{cases} \quad (5)$$

2.5 Ice-shelf harmonic vibrations. The eigenvalue problem.

It is assumed that for harmonic vibrations all variables are periodic in time, with the periodicity of the incident wave (of the forcing) given by the frequency ω, i.e.,

$$\tilde{\varsigma}(x,y,z,t)=\varsigma(x,y,z)\,e^{i\omega t}, \quad (6)$$

where $\tilde{\varsigma}=\{U,V,W,\ \sigma_{ij}\}$,

where we are interested in the real part of the variables expressed in complex form.

This assumption also implies that the full solution of the linear partial differential Eqs. (2), (5) is a sum of the solution for the steady-state flexure of the ice shelf and solution (6) for the time-dependent problem. In other words, solution (6) implies that the deformation due to the gravitational forcing can be separated from the vibration problem, i.e. the term ρg as well as the appropriate terms in the boundary conditions (3) are absent from the final equations formulated for the vibration problem, because a time-independent solution accounting for them applies and is not of interest in this study.

The separation of variables in Eq. (6) and its substitution into Eqs. (2), (5) yields the same equations, but with the operator $\frac{\partial^2}{\partial t^2}$ replaced with the constant $-\omega^2$, i.e. we obtain equation for $\varsigma(x,y,z)$:

$$\mathcal{L}\,\varsigma=-\omega^2\varsigma, \quad (7)$$

where $\mathcal{L}$ is a linear partial differential operator.

The numerical solution of Eq. (7) at different values of ω yields the dependence of ς on the frequency of the forcing ω. When the frequency of the forcing converges to the eigenfrequency of the system, we observe the typical rapid increase of deformation/stresses in the spectra in the form of the resonant peaks.

Note that here, the term "eigenvalue" refers to the eigenfrequency (ω_n) of the ice/water system or corresponding periodicity ($T_n = \frac{2\pi}{\omega_n}$). As mentioned previously, the term "eigenvalue" is employed in the same meaning like in a Sturm-Liouville Eigenvalue Problem (e.g. [5]). Eigenvalues (where resonant peaks would be observed) are denoted by the letters ω_n or T_n with the subscript n (or other), which is integer, because the array of the eigenvalues is a countable set.

Letters ω or T without the subscript denote the non-resonant values of frequency or periodicity of the ice/water system. They are defined by the frequency of the incident wave (of the forcing).

The eigenvalues can be derived from the equation $D(\omega) = 0$, where D is the determinant of the matrix, which results from the discretization of Eq. (7) and of the corresponding boundary conditions. However, the probability of the appearance of the forcing at any specific frequency is practically zero. This can be seen when we consider only events within the frequency range $(\omega_i - \Delta\omega,\ \omega_i + \Delta\omega)$. The probability of a forcing that is within the frequency range, is non-zero:

$$p\{\omega \in (\omega_i - \Delta\omega,\ \omega_i + \Delta\omega)\} = \frac{2\Delta\omega}{\Omega}, \qquad (8)$$

where Ω is the width of the range in omega space, which includes all possible frequencies of the forcing. Eq. (8) also assumes that the events have equal probabilities in different parts of Ω.

Thus, the probability of the resonant-like motion is higher when the value $\Delta\omega$, which is defined by the width of the resonant peak, is higher too. Therefore, the width of the resonant peaks is an important parameter, from a practical standpoint, because it defines the probability of the suitable resonant-like motion.

Computation of the spectra, such as provided below, thus provides important information about the width of resonant peaks within the likely range of forcing frequencies found in nature. By assessing the widths of such peaks, a better understanding of the probability that any one specific forcing event, at a specific ω can be assessed.

2. Numerical results & Discussion

The numerical experiments with ice shelf/tongue forced vibrations were carried out for a physically idealized ice shelf with the geometry of a rectangular parallelepiped as described above. In the undeformed ice shelf, the four edges had coordinates $x = 0, x = L,\ y_1 = 0,\ y_2 = B$, where L is the plate length along the X axis and B is the plate width along the Y axis ($B = y_2 - y_1$, see Eq. (1)). Furthermore, the ice shelf thickness $H = h_s(x, y) - h_b(x, y)$ was held constant in these experiments.

The numerical results and the discussion were presented in [6].

Appendix A: Basic boundary conditions in x, η, ξ variables

1) The **<u>boundary conditions on the ice-shelf surface</u>** (stress-free surface) are expressed as

$$\begin{cases} -\sigma_{xx}\dfrac{\partial h_s}{\partial x} - \sigma_{xy}\dfrac{\partial h_s}{\partial y} + \sigma_{xz} = 0; \\ -\sigma_{yx}\dfrac{\partial h_s}{\partial x} - \sigma_{yy}\dfrac{\partial h_s}{\partial y} + \sigma_{yz} = 0; \\ -\sigma_{zx}\dfrac{\partial h_s}{\partial x} - \sigma_{zy}\dfrac{\partial h_s}{\partial y} + \sigma_{zz} = 0. \end{cases} \tag{9}$$

Respectively, using Eq. (4), we obtain the follow boundary conditions on the ice-shelf surface for the displacements in x, y, z variables

$$\begin{cases} -\frac{2}{1-2\nu}\left\{(1-\nu)\frac{\partial U}{\partial x}+\nu\left(\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}\right)\right\}\cdot\frac{\partial h_s}{\partial x}-\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)\cdot\frac{\partial h_s}{\partial y}+\left(\frac{\partial U}{\partial z}+\frac{\partial W}{\partial x}\right)=0; \\ -\left(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x}\right)\frac{\partial h_s}{\partial x}-\frac{2}{1-2\nu}\left\{(1-\nu)\frac{\partial V}{\partial y}+\nu\left(\frac{\partial U}{\partial x}+\frac{\partial W}{\partial z}\right)\right\}\frac{\partial h_s}{\partial y}+\left(\frac{\partial V}{\partial z}+\frac{\partial W}{\partial y}\right)=0; \\ -\left(\frac{\partial U}{\partial z}+\frac{\partial W}{\partial x}\right)\cdot\frac{\partial h_s}{\partial x}-\left(\frac{\partial V}{\partial z}+\frac{\partial W}{\partial y}\right)\cdot\frac{\partial h_s}{\partial y}+\frac{2}{1-2\nu}\left\{(1-\nu)\frac{\partial W}{\partial z}+\nu\left(\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}\right)\right\}=0. \end{cases} \quad (10)$$

Therefore, in x, η, ξ variables we can write follow equations that express the stress-free conditions on the ice surface:

a) **the first equation** (**lines 11055-11336 in the program code**) is

$$-\frac{2(1-\nu)}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta'_x\frac{\partial U}{\partial \eta}+\xi'_x\frac{\partial U}{\partial \xi}\right\}_1^{i.j}\cdot\left(\frac{\partial h_s}{\partial x}\right)^{i,j}-\frac{2\nu}{1-2\nu}\left\{\eta'_y\frac{\partial V}{\partial \eta}+\xi'_y\frac{\partial V}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_s}{\partial x}\right)^{i,j}-$$
$$\frac{2\nu}{1-2\nu}\left\{\xi'_z\frac{\partial W}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_s}{\partial x}\right)^{i,j}-\left\{\eta'_y\frac{\partial U}{\partial \eta}+\xi'_y\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta'_x\frac{\partial V}{\partial \eta}+\xi'_x\frac{\partial V}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_s}{\partial y}\right)^{i,j}+$$
$$\left\{\xi'_z\frac{\partial U}{\partial \xi}+\frac{\partial W}{\partial x}+\eta'_x\frac{\partial W}{\partial \eta}+\xi'_x\frac{\partial W}{\partial \xi}\right\}_1^{i,j}=0; \quad (11)$$

b) **the second equation** (**lines 11341-11604 in the program code**) is

$$-\left\{\eta'_y\frac{\partial U}{\partial \eta}+\xi'_y\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta'_x\frac{\partial V}{\partial \eta}+\xi'_x\frac{\partial V}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_s}{\partial x}\right)^{i,j}-\frac{2(1-\nu)}{1-2\nu}\left\{\eta'_y\frac{\partial V}{\partial \eta}+\right.$$
$$\left.\xi'_y\frac{\partial V}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_s}{\partial y}\right)^{i,j}-\frac{2\nu}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta'_x\frac{\partial U}{\partial \eta}+\xi'_x\frac{\partial U}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_s}{\partial y}\right)^{i,j}-\frac{2\nu}{1-2\nu}\left\{\xi'_z\frac{\partial W}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_s}{\partial y}\right)^{i,j}+$$
$$\left\{\xi'_z\frac{\partial V}{\partial \xi}+\eta'_y\frac{\partial W}{\partial \eta}+\xi'_y\frac{\partial W}{\partial \xi}\right\}_1^{i,j}=0; \quad (12)$$

c) **the third equation** (**lines 11609-11845 in the program code**) is

$$-\left\{\xi'_z\frac{\partial U}{\partial \xi}+\frac{\partial W}{\partial x}+\eta'_x\frac{\partial W}{\partial \eta}+\xi'_x\frac{\partial W}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_s}{\partial x}\right)^{i,j}-\left\{\xi'_z\frac{\partial V}{\partial \xi}+\eta'_y\frac{\partial W}{\partial \eta}+\xi'_y\frac{\partial W}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_s}{\partial y}\right)^{i,j}+$$
$$\frac{2(1-\nu)}{1-2\nu}\left\{\xi'_z\frac{\partial W}{\partial \xi}\right\}_1^{i,j}+\frac{2\nu}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta'_x\frac{\partial U}{\partial \eta}+\xi'_x\frac{\partial U}{\partial \xi}\right\}_1^{i,j}+\frac{2\nu}{1-2\nu}\left\{\eta'_y\frac{\partial V}{\partial \eta}+\xi'_y\frac{\partial V}{\partial \xi}\right\}_1^{i,j}=0; \quad (13)$$

where the bottom index "1" corresponds to the grid layer located at the ice shelf surface.

2) The **boundary conditions on the ice-shelf base** (the surface under the water pressure forcing) are expressed as

$$\begin{cases} \sigma_{xx}\frac{\partial h_b}{\partial x}+\sigma_{xy}\frac{\partial h_b}{\partial y}-\sigma_{xz}=-P\frac{\partial h_b}{\partial x}; \\ \sigma_{yx}\frac{\partial h_b}{\partial x}+\sigma_{yy}\frac{\partial h_b}{\partial y}-\sigma_{yz}=-P\frac{\partial h_b}{\partial y}; \\ \sigma_{zx}\frac{\partial h_b}{\partial x}+\sigma_{zy}\frac{\partial h_b}{\partial y}-\sigma_{zz}=P; \end{cases} \quad (14)$$

where P is the sum of the hydrostatic pressure and the pressure perturbations result from ocean swell:

$$P=\rho gH+P'. \quad (15)$$

Respectively, in x,η,ξ variables we can write follow equations that express the boundary conditions on the ice base in terms of the displacements (and that look likewise Eq. (11)-(13)):

a) **the first equation** (**lines 16295-16583 in the program code**) is

$$\frac{2(1-\nu)}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta'_x\frac{\partial U}{\partial \eta}+\xi'_x\frac{\partial U}{\partial \xi}\right\}_{N_\xi}^{i.j}\cdot\left(\frac{\partial h_b}{\partial x}\right)^{i,j}+\frac{2\nu}{1-2\nu}\left\{\eta'_y\frac{\partial V}{\partial \eta}+\xi'_y\frac{\partial V}{\partial \xi}\right\}_{N_\xi}^{i,j}\left(\frac{\partial h_b}{\partial x}\right)^{i,j}+$$
$$\frac{2\nu}{1-2\nu}\left\{\xi'_z\frac{\partial W}{\partial \xi}\right\}_{N_\xi}^{i,j}\left(\frac{\partial h_b}{\partial x}\right)^{i,j}+\left\{\eta'_y\frac{\partial U}{\partial \eta}+\xi'_y\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta'_x\frac{\partial V}{\partial \eta}+\xi'_x\frac{\partial V}{\partial \xi}\right\}_{N_\xi}^{i,j}\left(\frac{\partial h_b}{\partial y}\right)^{i,j}-$$
$$\left\{\xi'_z\frac{\partial U}{\partial \xi}+\frac{\partial W}{\partial x}+\eta'_x\frac{\partial W}{\partial \eta}+\xi'_x\frac{\partial W}{\partial \xi}\right\}_{N_\xi}^{i,j}+P'\left(\frac{\partial h_b}{\partial x}\right)^{i,j}\cdot\frac{2(1+\nu)}{E}=-\rho gH\left(\frac{\partial h_b}{\partial x}\right)^{i,j}\cdot\frac{2(1+\nu)}{E}; \quad (16)$$

b) **the second equation** (**lines 16588-16859 in the program code**) is

$$\left\{\eta_y'\frac{\partial U}{\partial \eta}+\xi_y'\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta_x'\frac{\partial V}{\partial \eta}+\xi_x'\frac{\partial V}{\partial \xi}\right\}_{N_\xi}^{i,j}\left(\frac{\partial h_b}{\partial x}\right)^{i,j}+\frac{2(1-\nu)}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+\xi_y'\frac{\partial V}{\partial \xi}\right\}_{N_\xi}^{i,j}\left(\frac{\partial h_b}{\partial y}\right)^{i,j}+\frac{2\nu}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_{N_\xi}^{i,j}\left(\frac{\partial h_b}{\partial y}\right)^{i,j}+\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_{N_\xi}^{i,j}\left(\frac{\partial h_b}{\partial y}\right)^{i,j}-\left\{\xi_z'\frac{\partial V}{\partial \xi}+\eta_y'\frac{\partial W}{\partial \eta}+\xi_y'\frac{\partial W}{\partial \xi}\right\}_{N_\xi}^{i,j}+P'\left(\frac{\partial h_b}{\partial y}\right)^{i,j}\cdot\frac{2(1+\nu)}{E}=-\rho g H\left(\frac{\partial h_b}{\partial y}\right)^{i,j}\cdot\frac{2(1+\nu)}{E}; \quad (17)$$

c) **the third equation** (**lines 16864-17109 in the program code**) is

$$\left\{\xi_z'\frac{\partial U}{\partial \xi}+\frac{\partial W}{\partial x}+\eta_x'\frac{\partial W}{\partial \eta}+\xi_x'\frac{\partial W}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_b}{\partial x}\right)^{i,j}+\left\{\xi_z'\frac{\partial V}{\partial \xi}+\eta_y'\frac{\partial W}{\partial \eta}+\xi_y'\frac{\partial W}{\partial \xi}\right\}_1^{i,j}\left(\frac{\partial h_b}{\partial y}\right)^{i,j}-\frac{2(1-\nu)}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_1^{i,j}-\frac{2\nu}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_1^{i,j}-\frac{2\nu}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+\xi_y'\frac{\partial V}{\partial \xi}\right\}_1^{i,j}-P'\cdot\frac{2(1+\nu)}{E}=\rho g H\cdot\frac{2(1+\nu)}{E}; \quad (18)$$

3) The **boundary conditions on the ice-shelf front ($x = L$)** are expressed as

$$\begin{cases}\sigma_{xx}=f(\xi);\\ \sigma_{yx}=0;\\ \sigma_{zx}=0;\end{cases} \quad (19)$$

where

$$f(\xi)=\begin{cases}0, \xi<\frac{h_s}{H};\\ \rho_w g(h_s-\xi H), \xi\geq\frac{h_s}{H}.\end{cases} \quad (20)$$

In x, η, ξ variables equations (19) in terms of the displacements are expressed as

a) **the first equation** (**lines 1570-1636 in the program code**) is

$$\frac{2(1-\nu)}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_k^{N_x,j}+\frac{2\nu}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+\xi_y'\frac{\partial V}{\partial \xi}\right\}_k^{N_x,j}+\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_k^{N_x,j}=\frac{2(1+\nu)}{E}\cdot f(\xi); \quad (21)$$

b) **the second equation** (**lines 1640-1693 in the program code**) is

$$\left\{\eta_y'\frac{\partial U}{\partial \eta}+\xi_y'\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta_x'\frac{\partial V}{\partial \eta}+\xi_x'\frac{\partial V}{\partial \xi}\right\}_k^{N_x,j}=0; \tag{22}$$

c) **the third equation** (**lines 1697-1740 in the program code**) is

$$\left\{\xi_z'\frac{\partial U}{\partial \xi}+\frac{\partial W}{\partial x}+\eta_x'\frac{\partial W}{\partial \eta}+\xi_x'\frac{\partial W}{\partial \xi}\right\}_k^{N_x,j}=0. \tag{23}$$

4) The **boundary conditions on the ice-shelf lateral edge $y = y_1(x)$** are expressed as

$$\begin{cases} \sigma_{xx}\frac{dy_1}{dx}-\sigma_{xy}=f_x^1(\xi); \\ \sigma_{yx}\frac{dy_1}{dx}-\sigma_{yy}=f_y^1(\xi); \\ \quad \sigma_{zx}\frac{dy_1}{dx}-\sigma_{zy}=0; \end{cases} \tag{24}$$

where

$$f_x^1(\xi)=\begin{cases} 0, \ \xi<\frac{h_s}{H}; \\ \rho_w g(h_s-\xi H)\ \frac{dy_1}{dx}, \xi\geq\frac{h_s}{H}; \end{cases} \tag{25}$$

and

$$f_y^1(\xi)=\begin{cases} 0, \ \xi<\frac{h_s}{H}; \\ -\rho_w g(h_s-\xi H), \xi\geq\frac{h_s}{H}. \end{cases} \tag{26}$$

Respectively, in x,η,ξ variables equations (24) in terms of the displacements are expressed as

a) **the first equation** (**lines 3847-3978 in the program code**) is

$$\frac{2(1-\nu)}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_k^{i.1}\cdot\left(\frac{dy_1}{dx}\right)^i+\frac{2\nu}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+\xi_y'\frac{\partial V}{\partial \xi}\right\}_k^{i,1}\cdot\left(\frac{dy_1}{dx}\right)^i+$$
$$\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_k^{i,1}\cdot\left(\frac{dy_1}{dx}\right)^i-\left\{\eta_y'\frac{\partial U}{\partial \eta}+\xi_y'\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta_x'\frac{\partial V}{\partial \eta}+\xi_x'\frac{\partial V}{\partial \xi}\right\}_k^{i,1}=\frac{2(1+\nu)}{E}\cdot f_x^1(\xi);$$

(27)

b) **the second equation** (**lines 3982-4112 in the program code**) is

$$\left\{\eta_y'\frac{\partial U}{\partial \eta}+\xi_y'\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta_x'\frac{\partial V}{\partial \eta}+\xi_x'\frac{\partial V}{\partial \xi}\right\}_k^{i,1}\cdot\left(\frac{dy_1}{dx}\right)^i-\frac{2(1-\nu)}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+\xi_y'\frac{\partial V}{\partial \xi}\right\}_k^{i,1}-$$

$$\frac{2\nu}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_k^{i,1}-\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_k^{i,1}=\frac{2(1+\nu)}{E}\cdot f_y^1(\xi); \qquad (28)$$

c) **the third equation** (**lines 4116-4195 in the program code**) is

$$\left\{\xi_z'\frac{\partial U}{\partial \xi}+\frac{\partial W}{\partial x}+\eta_x'\frac{\partial W}{\partial \eta}+\xi_x'\frac{\partial W}{\partial \xi}\right\}_k^{i,1}\cdot\left(\frac{dy_1}{dx}\right)^i-\left\{\xi_z'\frac{\partial V}{\partial \xi}+\eta_y'\frac{\partial W}{\partial \eta}+\xi_y'\frac{\partial W}{\partial \xi}\right\}_k^{i,1}=0. \qquad (29)$$

5) The **boundary conditions on the ice-shelf lateral edge $y = y_2(x)$** look like Eq. (24) and are expressed as

$$\begin{cases} -\sigma_{xx}\frac{dy_2}{dx}+\sigma_{xy}=f_x^2(\xi); \\ -\sigma_{yx}\frac{dy_2}{dx}+\sigma_{yy}=f_y^2(\xi); \\ -\sigma_{zx}\frac{dy_2}{dx}+\sigma_{zy}=0; \end{cases} \qquad (30)$$

where

$$f_x^2(\xi)=\begin{cases} 0, \ \xi<\frac{h_s}{H}; \\ -\rho_w g(h_s-\xi H)\ \frac{dy_2}{dx}, \xi\geq\frac{h_s}{H}; \end{cases} \qquad (31)$$

and

$$f_y^2(\xi)=\begin{cases} 0, \ \xi<\frac{h_s}{H}; \\ \rho_w g(h_s-\xi H), \xi\geq\frac{h_s}{H}. \end{cases} \qquad (32)$$

Respectively, in x, η, ξ variables equations (30) in terms of the displacements look like Eqs (27)-(29) and are expressed as

a) **the first equation** (**lines 6295-6425 in the program code**) is

$$-\frac{2(1-\nu)}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_k^{i.N_\eta}\cdot\left(\frac{dy_2}{dx}\right)^i-\frac{2\nu}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+\xi_y'\frac{\partial V}{\partial \xi}\right\}_k^{i,N_\eta}\cdot\left(\frac{dy_2}{dx}\right)^i-$$

$$\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_k^{i,N_\eta}\cdot\left(\frac{dy_2}{dx}\right)^i+\left\{\eta_y'\frac{\partial U}{\partial \eta}+\xi_y'\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta_x'\frac{\partial V}{\partial \eta}+\xi_x'\frac{\partial V}{\partial \xi}\right\}_k^{i,N_\eta}=\frac{2(1+\nu)}{E}\cdot f_x^2(\xi);$$

(33)

b) **the second equation** (**lines 6430-6560 in the program code**) is

$$-\left\{\eta_y'\frac{\partial U}{\partial \eta}+\xi_y'\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta_x'\frac{\partial V}{\partial \eta}+\xi_x'\frac{\partial V}{\partial \xi}\right\}_k^{i,N_\eta}\cdot\left(\frac{dy_2}{dx}\right)^i+\frac{2(1-\nu)}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+\xi_y'\frac{\partial V}{\partial \xi}\right\}_k^{i,N_\eta}+$$

$$\frac{2\nu}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_k^{i,N_\eta}+\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_k^{i,N_\eta}=\frac{2(1+\nu)}{E}\cdot f_y^2(\xi); \quad (34)$$

c) **the third equation** (**lines 6564-6647 in the program code**) is

$$-\left\{\xi_z'\frac{\partial U}{\partial \xi}+\frac{\partial W}{\partial x}+\eta_x'\frac{\partial W}{\partial \eta}+\xi_x'\frac{\partial W}{\partial \xi}\right\}_k^{i,N_\eta}\cdot\left(\frac{dy_2}{dx}\right)^i+\left\{\xi_z'\frac{\partial V}{\partial \xi}+\eta_y'\frac{\partial W}{\partial \eta}+\xi_y'\frac{\partial W}{\partial \xi}\right\}_k^{i,N_\eta}=0. \quad (35)$$

Appendix B: The approximation of the boundary conditions based on the integration of the basic boundary conditions into the momentum equations (1)

In this study, the technique, in which the basic boundary conditions (Appendix A) are included in the momentum equations (1), is considered.

The procedure for this inclusion consists following successive steps.

1) We rewrite, for instance, the first equation from (1) using the new variables:

$$\frac{\partial \sigma_{xx}}{\partial x}+\eta'_x\frac{\partial \sigma_{xx}}{\partial \eta}+\xi'_x\frac{\partial \sigma_{xx}}{\partial \xi}+\eta'_y\frac{\partial \sigma_{xy}}{\partial \eta}+\xi'_y\frac{\partial \sigma_{xy}}{\partial \xi}+\xi'_z\frac{\partial \sigma_{xz}}{\partial \xi}=\rho\frac{\partial^2 U}{\partial t^2}; \quad (36)$$

2) We write the approximation of the derivative $\frac{\partial \sigma_{xz}}{\partial \xi}$ at the ice-shelf base towards the substance (glacier):

$$\left(\xi'_z \frac{\partial \sigma_{xz}}{\partial \xi}\right)_{N_\xi}^{i,j} = -\frac{1}{H}\left(\frac{\partial \sigma_{xz}}{\partial \xi}\right)_{N_\xi}^{i,j} \approx -\frac{1}{H}\frac{1}{2\,\Delta\xi}(\sigma_{xz})_{N_\xi-2}^{i,j} + \frac{1}{H}\frac{4}{2\,\Delta\xi}(\sigma_{xz})_{N_\xi-1}^{i,j} - \frac{1}{H}\frac{3}{2\,\Delta\xi}(\sigma_{xz})_{N_\xi}^{i,j}; \qquad (37)$$

where index "N_ξ" corresponds to the grid layer located at the ice shelf base.

3) The standard (typical) boundary condition at the ice-shelf base (first equation from (14)) requires that $\sigma_{xz} = \sigma_{xx}\frac{\partial h_b}{\partial x} + \sigma_{xy}\frac{\partial h_b}{\partial y} + P\frac{\partial h_b}{\partial x}$. Thus, we should replace $(\sigma_{xz})_{N_\xi}^{i,j}$ in agreement with the standard boundary conditions, with $\left\{\sigma_{xx}\frac{\partial h_b}{\partial x} + \sigma_{xy}\frac{\partial h_b}{\partial y}\right\}_{N_\xi}^{i,j} + \left(P\frac{\partial h_b}{\partial x}\right)_{N_\xi}^{i,j}$. Finally, we obtain the following approximation of the derivative $\left(\xi'_z \frac{\partial \sigma_{xz}}{\partial \xi}\right)_{N_\xi}^{i,j}$ at the ice-shelf base:

$$\left(\xi'_z \frac{\partial \sigma_{xz}}{\partial \xi}\right)_{N_\xi}^{i,j} \approx -\frac{1}{H}\frac{1}{2\,\Delta\xi}(\sigma_{xz})_{N_\xi-2}^{i,j} + \frac{1}{H}\frac{4}{2\,\Delta\xi}(\sigma_{xz})_{N_\xi-1}^{i,j} - \frac{1}{H}\frac{3}{2\,\Delta\xi}\left\{\sigma_{xx}\frac{\partial h_b}{\partial x} + \sigma_{xy}\frac{\partial h_b}{\partial y}\right\}_{N_\xi}^{i,j} - \frac{1}{H}\frac{3}{2\,\Delta\xi}\left(P'\frac{\partial h_b}{\partial x}\right)^{i,j} - \frac{3}{2\,\Delta\xi}\rho g\left(\frac{\partial h_b}{\partial x}\right)^{i,j}; \qquad (38)$$

where $P = \rho g H + P'$

4) Thus, at the ice shelf base, we apply the equation

$$\left(\frac{\partial \sigma_{xx}}{\partial x}\right)_{N_\xi}^{i,j} + \left(\eta'_x \frac{\partial \sigma_{xx}}{\partial \eta}\right)_{N_\xi}^{i,j} + \left(\xi'_x \frac{\partial \sigma_{xx}}{\partial \xi}\right)_{N_\xi}^{i,j} + \left(\eta'_y \frac{\partial \sigma_{xy}}{\partial \eta}\right)_{N_\xi}^{i,j} + \left(\xi'_y \frac{\partial \sigma_{xy}}{\partial \xi}\right)_{N_\xi}^{i,j} -$$

$$\frac{1}{H}\frac{1}{2\,\Delta\xi}(\sigma_{xz})_{N_\xi-2}^{i,j} + + \frac{1}{H}\frac{4}{2\,\Delta\xi}(\sigma_{xz})_{N_\xi-1}^{i,j} - \frac{1}{H}\frac{3}{2\,\Delta\xi}\left\{\sigma_{xx}\frac{\partial h_b}{\partial x} + \sigma_{xy}\frac{\partial h_b}{\partial y}\right\}_{N_\xi}^{i,j} -$$

$$\frac{1}{H}\frac{3}{2\,\Delta\xi}\left(P'\frac{\partial h_b}{\partial x}\right)_{N_\xi}^{i,j} \approx \frac{3}{2\,\Delta\xi}\rho g\left(\frac{\partial h_b}{\partial x}\right)^{i,j} + \rho\left(\frac{\partial^2 U}{\partial t^2}\right)_{N_\xi}^{i,j}; \qquad (39)$$

which is the first equation at the ice-shelf base, instead of the standard equation

$\sigma_{xz} = \sigma_{xx}\frac{\partial h_b}{\partial x} + \sigma_{xy}\frac{\partial h_b}{\partial y} + P\frac{\partial h_b}{\partial x}$.

Therefore, after the coordinate transformation, the applicable **equations at the ice-shelf base** can be written as

$$
\begin{cases}
\left(\frac{\partial \sigma_{xx}}{\partial x}\right)_{N_\xi}^{i,j} + \left(\eta'_x \frac{\partial \sigma_{xx}}{\partial \eta}\right)_{N_\xi}^{i,j} + \left(\xi'_x \frac{\partial \sigma_{xx}}{\partial \xi}\right)_{N_\xi}^{i,j} + \left(\eta'_y \frac{\partial \sigma_{xy}}{\partial \eta}\right)_{N_\xi}^{i,j} + \left(\xi'_y \frac{\partial \sigma_{xy}}{\partial \xi}\right)_{N_\xi}^{i,j} - \\
\quad \frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} (\sigma_{xz})_{N_\xi-2}^{i,j} + \frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} (\sigma_{xz})_{N_\xi-1}^{i,j} - \\
\quad \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left\{\sigma_{xx} \frac{\partial h_b}{\partial x} + \sigma_{xy} \frac{\partial h_b}{\partial y}\right\}_{N_\xi}^{i,j} - \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(P' \frac{\partial h_b}{\partial x}\right)^{i,j} \approx \\
\quad \approx \frac{3}{2\,\Delta\xi} \rho g \left(\frac{\partial h_b}{\partial x}\right)^{i,j} + \rho \left(\frac{\partial^2 U}{\partial t^2}\right)_{N_\xi}^{i,j}; \\
\left(\frac{\partial \sigma_{yx}}{\partial x}\right)_{N_\xi}^{i,j} + \left(\eta'_x \frac{\partial \sigma_{yx}}{\partial \eta}\right)_{N_\xi}^{i,j} + \left(\xi'_x \frac{\partial \sigma_{yx}}{\partial \xi}\right)_{N_\xi}^{i,j} + \left(\eta'_y \frac{\partial \sigma_{yy}}{\partial \eta}\right)_{N_\xi}^{i,j} + \left(\xi'_y \frac{\partial \sigma_{yy}}{\partial \xi}\right)_{N_\xi}^{i,j} - \\
\quad \frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} (\sigma_{yz})_{N_\xi-2}^{i,j} + \frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} (\sigma_{yz})_{N_\xi-1}^{i,j} - \\
\quad \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left\{\sigma_{yx} \frac{\partial h_b}{\partial x} + \sigma_{yy} \frac{\partial h_b}{\partial y}\right\}_{N_\xi}^{i,j} - \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(P' \frac{\partial h_b}{\partial y}\right)^{i,j} \approx \\
\quad \approx \frac{3}{2\,\Delta\xi} \rho g \left(\frac{\partial h_b}{\partial y}\right)^{i,j} + \rho \left(\frac{\partial^2 V}{\partial t^2}\right)_{N_\xi}^{i,j}; \\
\left(\frac{\partial \sigma_{zx}}{\partial x}\right)_{N_\xi}^{i,j} + \left(\eta'_x \frac{\partial \sigma_{zx}}{\partial \eta}\right)_{N_\xi}^{i,j} + \left(\xi'_x \frac{\partial \sigma_{zx}}{\partial \xi}\right)_{N_\xi}^{i,j} + \left(\eta'_y \frac{\partial \sigma_{zy}}{\partial \eta}\right)_{N_\xi}^{i,j} + \left(\xi'_y \frac{\partial \sigma_{zy}}{\partial \xi}\right)_{N_\xi}^{i,j} - \\
\quad \frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} (\sigma_{zz})_{N_\xi-2}^{i,j} + \frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} (\sigma_{zz})_{N_\xi-1}^{i,j} - \\
\quad \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left\{\sigma_{zx} \frac{\partial h_b}{\partial x} + \sigma_{zy} \frac{\partial h_b}{\partial y}\right\}_{N_\xi}^{i,j} + \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} (P')^{i,j} \approx \\
\quad \approx -\frac{3}{2\,\Delta\xi} \rho g + \rho g + \rho \left(\frac{\partial^2 W}{\partial t^2}\right)_{N_\xi}^{i,j}.
\end{cases}
\tag{40}
$$

In terms of the displacements equations (40) are expressed as

a) **the first equation at the ice-shelf base** (**lines 11857-13535 in the program code**) is

$$
\frac{2(1-\nu)}{1-2\nu} \left\{\frac{\partial^2 U}{\partial x^2} + \frac{\partial}{\partial x}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_x \frac{\partial U}{\partial \xi}\right) + \eta'_x \frac{\partial^2 U}{\partial \eta \partial x} + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\xi'_x \frac{\partial U}{\partial \xi}\right) + \xi'_x \frac{\partial^2 U}{\partial \xi \partial x} + \xi'_x \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial U}{\partial \xi}\right)\right\}_{N_\xi}^{i.j} + \frac{2\nu}{1-2\nu} \left\{\frac{\partial}{\partial x}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_y \frac{\partial V}{\partial \xi}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) + \right.
$$

$$\eta'_x \frac{\partial}{\partial \eta}\left(\xi'_y \frac{\partial V}{\partial \xi}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial V}{\partial \xi}\right)\Big\}_{N_\xi}^{i,j} + \frac{2\nu}{1-2\nu}\Big\{\frac{\partial}{\partial x}\left(\xi'_z \frac{\partial W}{\partial \xi}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\xi'_z \frac{\partial W}{\partial \xi}\right) +$$

$$\xi'_x \frac{\partial}{\partial \xi}\left(\xi'_z \frac{\partial W}{\partial \xi}\right)\Big\}_{N_\xi}^{i,j} + \Big\{\eta'_y \frac{\partial}{\partial \eta}\left(\eta'_y \frac{\partial U}{\partial \eta}\right) + \eta'_y \frac{\partial}{\partial \eta}\left(\xi'_y \frac{\partial U}{\partial \xi}\right) + \eta'_y \frac{\partial^2 V}{\partial \eta \partial x} + \eta'_y \frac{\partial}{\partial \eta}\left(\eta'_x \frac{\partial V}{\partial \eta}\right) + \eta'_y \frac{\partial}{\partial \eta}\left(\xi'_x \frac{\partial V}{\partial \xi}\right) +$$

$$\xi'_y \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial U}{\partial \eta}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial U}{\partial \xi}\right) + \xi'_y \frac{\partial^2 V}{\partial \xi \partial x} + \xi'_y \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial V}{\partial \eta}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial V}{\partial \xi}\right)\Big\}_{N_\xi}^{i,j} - \frac{1}{H^{i,j}}\frac{1}{2\,\Delta\xi}\Big\{\xi'_z \frac{\partial U}{\partial \xi} +$$

$$\frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi}\Big\}_{N_\xi-2}^{i,j} + \frac{1}{H^{i,j}}\frac{4}{2\,\Delta\xi}\Big\{\xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi}\Big\}_{N_\xi-1}^{i,j} -$$

$$\frac{1}{H^{i,j}}\frac{3}{2\,\Delta\xi}\left(\frac{\partial h_b}{\partial x}\right)^{i,j}\frac{2(1-\nu)}{1-2\nu}\Big\{\frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi}\Big\}_{N_\xi}^{i.j} - \frac{1}{H^{i,j}}\frac{3}{2\,\Delta\xi}\left(\frac{\partial h_b}{\partial x}\right)^{i,j}\frac{2\nu}{1-2\nu}\Big\{\eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi}\Big\}_{N_\xi}^{i,j} -$$

$$\frac{1}{H^{i,j}}\frac{3}{2\,\Delta\xi}\left(\frac{\partial h_b}{\partial x}\right)^{i,j}\frac{2\nu}{1-2\nu}\Big\{\xi'_z \frac{\partial W}{\partial \xi}\Big\}_{N_\xi}^{i,j} - \frac{1}{H^{i,j}}\frac{3}{2\,\Delta\xi}\left(\frac{\partial h_b}{\partial y}\right)^{i,j}\Big\{\eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi}\Big\}_{N_\xi}^{i,j} -$$

$$\frac{2(1+\nu)}{E}\cdot\frac{1}{H^{i,j}}\frac{3}{2\,\Delta\xi}\left(P'\frac{\partial h_b}{\partial x}\right)^{i,j} \approx \frac{2(1+\nu)}{E}\cdot\frac{3}{2\,\Delta\xi}\rho g\left(\frac{\partial h_b}{\partial x}\right)^{i,j} + \frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 U}{\partial t^2}\right)_{N_\xi}^{i,j}; \qquad (41)$$

b) **the second equation at the ice-shelf base** (**lines 13540-15145 in the program code**) is

$$\Big\{\frac{\partial}{\partial x}\left(\eta'_y \frac{\partial U}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_y \frac{\partial U}{\partial \xi}\right) + \frac{\partial^2 V}{\partial x^2} + \frac{\partial}{\partial x}\left(\eta'_x \frac{\partial V}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_x \frac{\partial V}{\partial \xi}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_y \frac{\partial U}{\partial \eta}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\xi'_y \frac{\partial U}{\partial \xi}\right) +$$

$$\eta'_x \frac{\partial^2 V}{\partial \eta \partial x} + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_x \frac{\partial V}{\partial \eta}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\xi'_x \frac{\partial V}{\partial \xi}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial U}{\partial \eta}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial U}{\partial \xi}\right) + \xi'_x \frac{\partial^2 V}{\partial \xi \partial x} +$$

$$\xi'_x \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial V}{\partial \eta}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial V}{\partial \xi}\right)\Big\}_{N_\xi}^{i,j} + \frac{2(1-\nu)}{1-2\nu}\Big\{\eta'_y \frac{\partial}{\partial \eta}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) + \eta'_y \frac{\partial}{\partial \eta}\left(\xi'_y \frac{\partial V}{\partial \xi}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) +$$

$$\xi'_y \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial V}{\partial \xi}\right)\Big\}_{N_\xi}^{i,j} + \frac{2\nu}{1-2\nu}\Big\{\eta'_y \frac{\partial^2 U}{\partial \eta \partial x} + \eta'_y \frac{\partial}{\partial \eta}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) + \eta'_y \frac{\partial}{\partial \eta}\left(\xi'_x \frac{\partial U}{\partial \xi}\right) + \xi'_y \frac{\partial^2 U}{\partial \xi \partial x} + \xi'_y \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) +$$

$$\xi'_y \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial U}{\partial \xi}\right)\Big\}_{N_\xi}^{i,j} + \frac{2\nu}{1-2\nu}\Big\{\eta'_y \frac{\partial}{\partial \eta}\left(\xi'_z \frac{\partial W}{\partial \xi}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\xi'_z \frac{\partial W}{\partial \xi}\right)\Big\}_{N_\xi}^{i,j} - \frac{1}{H^{i,j}}\frac{1}{2\,\Delta\xi}\Big\{\xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} +$$

$$\xi'_y \frac{\partial W}{\partial \xi}\Big\}_{N_\xi-2}^{i,j} + \frac{1}{H^{i,j}}\frac{4}{2\,\Delta\xi}\Big\{\xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi}\Big\}_{N_\xi-1}^{i,j} - \frac{1}{H^{i,j}}\frac{3}{2\,\Delta\xi}\left(\frac{\partial h_b}{\partial x}\right)^{i,j}\Big\{\eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} +$$

$$\eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi}\Big\}_{N_\xi}^{i,j} - \frac{1}{H^{i,j}}\frac{3}{2\,\Delta\xi}\left(\frac{\partial h_b}{\partial y}\right)^{i,j}\frac{2(1-\nu)}{1-2\nu}\Big\{\eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi}\Big\}_{N_\xi}^{i,j} - \frac{1}{H^{i,j}}\frac{3}{2\,\Delta\xi}\left(\frac{\partial h_b}{\partial y}\right)^{i,j}\frac{2\nu}{1-2\nu}\Big\{\frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} +$$

$$\left.\xi'_x \frac{\partial U}{\partial \xi}\right\}_{N_\xi}^{i,j} - \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_b}{\partial y}\right)^{i,j} \frac{2\nu}{1-2\nu} \left\{\xi'_z \frac{\partial W}{\partial \xi}\right\}_{N_\xi}^{i,j} - \frac{2(1+\nu)}{E} \cdot \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(P' \frac{\partial h_b}{\partial y}\right)^{i,j} \approx \frac{2(1+\nu)}{E} \cdot$$

$$\frac{3}{2\,\Delta\xi} \rho g \left(\frac{\partial h_b}{\partial y}\right)^{i,j} + \frac{2(1+\nu)}{E} \cdot \rho \left(\frac{\partial^2 V}{\partial t^2}\right)_{N_\xi}^{i,j}; \qquad (42)$$

c) **the third equation at the ice-shelf base** (**lines 15150-16268 in the program code**) is

$$\left\{\frac{\partial}{\partial x}\left(\xi'_z \frac{\partial U}{\partial \xi}\right) + \frac{\partial^2 W}{\partial x^2} + \frac{\partial}{\partial x}\left(\eta'_x \frac{\partial W}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_x \frac{\partial W}{\partial \xi}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\xi'_z \frac{\partial U}{\partial \xi}\right) + \eta'_x \frac{\partial^2 W}{\partial \eta \partial x} + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_x \frac{\partial W}{\partial \eta}\right) + \right.$$

$$\left.\eta'_x \frac{\partial}{\partial \eta}\left(\xi'_x \frac{\partial W}{\partial \xi}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_z \frac{\partial U}{\partial \xi}\right) + \xi'_x \frac{\partial^2 W}{\partial \xi \partial x} + \xi'_x \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial W}{\partial \eta}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial W}{\partial \xi}\right)\right\}_{N_\xi}^{i,j} + \left\{\eta'_y \frac{\partial}{\partial \eta}\left(\xi'_z \frac{\partial V}{\partial \xi}\right) + \right.$$

$$\left.\eta'_y \frac{\partial}{\partial \eta}\left(\eta'_y \frac{\partial W}{\partial \eta}\right) + \eta'_y \frac{\partial}{\partial \eta}\left(\xi'_y \frac{\partial W}{\partial \xi}\right) + \quad \xi'_y \frac{\partial}{\partial \xi}\left(\xi'_z \frac{\partial V}{\partial \xi}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial W}{\partial \eta}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial W}{\partial \xi}\right)\right\}_{N_\xi}^{i,j} -$$

$$\frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} \frac{2(1-\nu)}{1-2\nu} \left\{\xi'_z \frac{\partial W}{\partial \xi}\right\}_{N_\xi-2}^{i,j} - \frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} \frac{2\nu}{1-2\nu} \left\{\frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi}\right\}_{N_\xi-2}^{i,j} - \frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} \frac{2\nu}{1-2\nu} \left\{\eta'_y \frac{\partial V}{\partial \eta} + \right.$$

$$\left.\xi'_y \frac{\partial V}{\partial \xi}\right\}_{N_\xi-2}^{i,j} + \frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} \frac{2(1-\nu)}{1-2\nu} \left\{\xi'_z \frac{\partial W}{\partial \xi}\right\}_{N_\xi-1}^{i,j} + \frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} \frac{2\nu}{1-2\nu} \left\{\frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi}\right\}_{N_\xi-1}^{i,j} +$$

$$\frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} \frac{2\nu}{1-2\nu} \left\{\eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi}\right\}_{N_\xi-1}^{i,j} - \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_b}{\partial x}\right)^{i,j} \left\{\xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi}\right\}_{N_\xi}^{i,j} -$$

$$\frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_b}{\partial y}\right)^{i,j} \left\{\xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi}\right\}_{N_\xi}^{i,j} + \frac{2(1+\nu)}{E} \cdot \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} (P')^{i,j} \approx -\frac{2(1+\nu)}{E} \cdot \frac{3}{2\,\Delta\xi} \rho g + \frac{2(1+\nu)}{E} \cdot$$

$$\rho g + \frac{2(1+\nu)}{E} \cdot \rho \left(\frac{\partial^2 W}{\partial t^2}\right)_{N_\xi}^{i,j}; \qquad (43)$$

The same technique yields the similar **equations at the ice surface**

$$
\begin{cases}
\left(\frac{\partial \sigma_{xx}}{\partial x}\right)_1^{i,j} + \left(\eta'_x \frac{\partial \sigma_{xx}}{\partial \eta}\right)_1^{i,j} + \left(\xi'_x \frac{\partial \sigma_{xx}}{\partial \xi}\right)_1^{i,j} + \left(\eta'_y \frac{\partial \sigma_{xy}}{\partial \eta}\right)_1^{i,j} + \left(\xi'_y \frac{\partial \sigma_{xy}}{\partial \xi}\right)_1^{i,j} + \\
\frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left\{\sigma_{xx} \frac{\partial h_s}{\partial x} + \sigma_{xy} \frac{\partial h_s}{\partial y}\right\}_1^{i,j} - \frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} (\sigma_{xz})_2^{i,j} + \frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} (\sigma_{xz})_3^{i,j} \approx \rho \left(\frac{\partial^2 U}{\partial t^2}\right)_1^{i,j} ; \\
\left(\frac{\partial \sigma_{yx}}{\partial x}\right)_1^{i,j} + \left(\eta'_x \frac{\partial \sigma_{yx}}{\partial \eta}\right)_1^{i,j} + \left(\xi'_x \frac{\partial \sigma_{yx}}{\partial \xi}\right)_1^{i,j} + \left(\eta'_y \frac{\partial \sigma_{yy}}{\partial \eta}\right)_1^{i,j} + \left(\xi'_y \frac{\partial \sigma_{yy}}{\partial \xi}\right)_1^{i,j} + \\
\frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left\{\sigma_{yx} \frac{\partial h_s}{\partial x} + \sigma_{yy} \frac{\partial h_s}{\partial y}\right\}_1^{i,j} - \frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} (\sigma_{yz})_2^{i,j} + \frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} (\sigma_{yz})_3^{i,j} \approx \rho \left(\frac{\partial^2 V}{\partial t^2}\right)_1^{i,j} ; \\
\left(\frac{\partial \sigma_{zx}}{\partial x}\right)_1^{i,j} + \left(\eta'_x \frac{\partial \sigma_{zx}}{\partial \eta}\right)_1^{i,j} + \left(\xi'_x \frac{\partial \sigma_{zx}}{\partial \xi}\right)_1^{i,j} + \left(\eta'_y \frac{\partial \sigma_{zy}}{\partial \eta}\right)_1^{i,j} + \left(\xi'_y \frac{\partial \sigma_{zy}}{\partial \xi}\right)_1^{i,j} + \\
\frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left\{\sigma_{zx} \frac{\partial h_s}{\partial x} + \sigma_{zy} \frac{\partial h_s}{\partial y}\right\}_1^{i,j} - \frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} (\sigma_{zz})_2^{i,j} + \frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} (\sigma_{zz})_3^{i,j} \approx \rho g + \rho \left(\frac{\partial^2 W}{\partial t^2}\right)_1^{i,j} ;
\end{cases}
\tag{44}
$$

where the lower indices 1-3 in Eq (44) denote respectively the numbers of the grid layers starting from the ice surface to moving downward in the vertical direction.

In terms of the displacements equations (44) are expressed as

a) **the first equation at the ice-shelf surface** (**lines 6658-8319 in the program code**) is

$$
\frac{2(1-\nu)}{1-2\nu} \left\{\frac{\partial^2 U}{\partial x^2} + \frac{\partial}{\partial x}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_x \frac{\partial U}{\partial \xi}\right) + \eta'_x \frac{\partial^2 U}{\partial \eta \partial x} + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\xi'_x \frac{\partial U}{\partial \xi}\right) + \xi'_x \frac{\partial^2 U}{\partial \xi \partial x} + \right.
$$

$$
\left. \xi'_x \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial U}{\partial \xi}\right)\right\}_1^{i.j} + \frac{2\nu}{1-2\nu} \left\{\frac{\partial}{\partial x}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_y \frac{\partial V}{\partial \xi}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) + \right.
$$

$$
\left. \eta'_x \frac{\partial}{\partial \eta}\left(\xi'_y \frac{\partial V}{\partial \xi}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial V}{\partial \xi}\right)\right\}_1^{i,j} + \frac{2\nu}{1-2\nu} \left\{\frac{\partial}{\partial x}\left(\xi'_z \frac{\partial W}{\partial \xi}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\xi'_z \frac{\partial W}{\partial \xi}\right) + \right.
$$

$$
\left. \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_z \frac{\partial W}{\partial \xi}\right)\right\}_1^{i,j} + \left\{\eta'_y \frac{\partial}{\partial \eta}\left(\eta'_y \frac{\partial U}{\partial \eta}\right) + \eta'_y \frac{\partial}{\partial \eta}\left(\xi'_y \frac{\partial U}{\partial \xi}\right) + \eta'_y \frac{\partial^2 V}{\partial \eta \partial x} + \eta'_y \frac{\partial}{\partial \eta}\left(\eta'_x \frac{\partial V}{\partial \eta}\right) + \eta'_y \frac{\partial}{\partial \eta}\left(\xi'_x \frac{\partial V}{\partial \xi}\right) + \right.
$$

$$\xi'_y \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial U}{\partial \eta}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial U}{\partial \xi}\right) + \xi'_y \frac{\partial^2 V}{\partial \xi \partial x} + \xi'_y \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial V}{\partial \eta}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial V}{\partial \xi}\right)\Big\}_1^{i,j} + \frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} \Big\{\xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi}\Big\}_3^{i,j} - \frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} \Big\{\xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi}\Big\}_2^{i,j} + \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_s}{\partial x}\right)^{i,j} \frac{2(1-\nu)}{1-2\nu} \Big\{\frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi}\Big\}_1^{i.j} + \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_s}{\partial x}\right)^{i,j} \frac{2\nu}{1-2\nu} \Big\{\eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi}\Big\}_1^{i,j} + \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_s}{\partial x}\right)^{i,j} \frac{2\nu}{1-2\nu} \Big\{\xi'_z \frac{\partial W}{\partial \xi}\Big\}_1^{i,j} + \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_s}{\partial y}\right)^{i,j} \Big\{\eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi}\Big\}_1^{i,j} \approx \frac{2(1+\nu)}{E} \cdot \rho \left(\frac{\partial^2 U}{\partial t^2}\right)_{N_\xi}^{i,j}; \qquad (45)$$

b) **the second equation at the ice-shelf surface** (**lines 8324-9909 in the program code**) is

$$\Big\{\frac{\partial}{\partial x}\left(\eta'_y \frac{\partial U}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_y \frac{\partial U}{\partial \xi}\right) + \frac{\partial^2 V}{\partial x^2} + \frac{\partial}{\partial x}\left(\eta'_x \frac{\partial V}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_x \frac{\partial V}{\partial \xi}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_y \frac{\partial U}{\partial \eta}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\xi'_y \frac{\partial U}{\partial \xi}\right) + \eta'_x \frac{\partial^2 V}{\partial \eta \partial x} + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_x \frac{\partial V}{\partial \eta}\right) + \eta'_x \frac{\partial}{\partial \eta}\left(\xi'_x \frac{\partial V}{\partial \xi}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial U}{\partial \eta}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial U}{\partial \xi}\right) + \xi'_x \frac{\partial^2 V}{\partial \xi \partial x} + \xi'_x \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial V}{\partial \eta}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial V}{\partial \xi}\right)\Big\}_1^{i,j} + \frac{2(1-\nu)}{1-2\nu} \Big\{\eta'_y \frac{\partial}{\partial \eta}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) + \eta'_y \frac{\partial}{\partial \eta}\left(\xi'_y \frac{\partial V}{\partial \xi}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial V}{\partial \xi}\right)\Big\}_1^{i,j} + \frac{2\nu}{1-2\nu} \Big\{\eta'_y \frac{\partial^2 U}{\partial \eta \partial x} + \eta'_y \frac{\partial}{\partial \eta}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) + \eta'_y \frac{\partial}{\partial \eta}\left(\xi'_x \frac{\partial U}{\partial \xi}\right) + \xi'_y \frac{\partial^2 U}{\partial \xi \partial x} + \xi'_y \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial U}{\partial \xi}\right)\Big\}_1^{i,j} + \frac{2\nu}{1-2\nu} \Big\{\eta'_y \frac{\partial}{\partial \eta}\left(\xi'_z \frac{\partial W}{\partial \xi}\right) + \xi'_y \frac{\partial}{\partial \xi}\left(\xi'_z \frac{\partial W}{\partial \xi}\right)\Big\}_1^{i,j} + \frac{1}{H^{i,j}} \frac{1}{2\,\Delta\xi} \Big\{\xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi}\Big\}_3^{i,j} - \frac{1}{H^{i,j}} \frac{4}{2\,\Delta\xi} \Big\{\xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi}\Big\}_2^{i,j} + \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_s}{\partial x}\right)^{i,j} \Big\{\eta'_y \frac{\partial U}{\partial \eta} + \xi'_y \frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x \frac{\partial V}{\partial \eta} + \xi'_x \frac{\partial V}{\partial \xi}\Big\}_1^{i,j} + \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_s}{\partial y}\right)^{i,j} \frac{2(1-\nu)}{1-2\nu} \Big\{\eta'_y \frac{\partial V}{\partial \eta} + \xi'_y \frac{\partial V}{\partial \xi}\Big\}_1^{i,j} + \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_s}{\partial y}\right)^{i,j} \frac{2\nu}{1-2\nu} \Big\{\frac{\partial U}{\partial x} + \eta'_x \frac{\partial U}{\partial \eta} + \xi'_x \frac{\partial U}{\partial \xi}\Big\}_1^{i,j} + \frac{1}{H^{i,j}} \frac{3}{2\,\Delta\xi} \left(\frac{\partial h_s}{\partial y}\right)^{i,j} \frac{2\nu}{1-2\nu} \Big\{\xi'_z \frac{\partial W}{\partial \xi}\Big\}_1^{i,j} \approx \frac{2(1+\nu)}{E} \cdot \rho \left(\frac{\partial^2 V}{\partial t^2}\right)_1^{i,j}; \qquad (46)$$

c) **the third equation at the ice-shelf surface** (**lines 9914-11047 in the program code**) is

$$\left\{\frac{\partial}{\partial x}\left(\xi_z'\frac{\partial U}{\partial \xi}\right)+\frac{\partial^2 W}{\partial x^2}+\frac{\partial}{\partial x}\left(\eta_x'\frac{\partial W}{\partial \eta}\right)+\frac{\partial}{\partial x}\left(\xi_x'\frac{\partial W}{\partial \xi}\right)+\eta_x'\frac{\partial}{\partial \eta}\left(\xi_z'\frac{\partial U}{\partial \xi}\right)+\eta_x'\frac{\partial^2 W}{\partial \eta\partial x}+\eta_x'\frac{\partial}{\partial \eta}\left(\eta_x'\frac{\partial W}{\partial \eta}\right)+\right.$$

$$\left.\eta_x'\frac{\partial}{\partial \eta}\left(\xi_x'\frac{\partial W}{\partial \xi}\right)+\xi_x'\frac{\partial}{\partial \xi}\left(\xi_z'\frac{\partial U}{\partial \xi}\right)+\xi_x'\frac{\partial^2 W}{\partial \xi\partial x}+\xi_x'\frac{\partial}{\partial \xi}\left(\eta_x'\frac{\partial W}{\partial \eta}\right)+\xi_x'\frac{\partial}{\partial \xi}\left(\xi_x'\frac{\partial W}{\partial \xi}\right)\right\}_1^{i,j}+\left\{\eta_y'\frac{\partial}{\partial \eta}\left(\xi_z'\frac{\partial V}{\partial \xi}\right)+\right.$$

$$\left.\eta_y'\frac{\partial}{\partial \eta}\left(\eta_y'\frac{\partial W}{\partial \eta}\right)+\eta_y'\frac{\partial}{\partial \eta}\left(\xi_y'\frac{\partial W}{\partial \xi}\right)+\quad \xi_y'\frac{\partial}{\partial \xi}\left(\xi_z'\frac{\partial V}{\partial \xi}\right)+\xi_y'\frac{\partial}{\partial \xi}\left(\eta_y'\frac{\partial W}{\partial \eta}\right)+\xi_y'\frac{\partial}{\partial \xi}\left(\xi_y'\frac{\partial W}{\partial \xi}\right)\right\}_1^{i,j}+$$

$$\frac{1}{H^{i,j}}\frac{1}{2\,\Delta\xi}\frac{2(1-\nu)}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_3^{i,j}+\frac{1}{H^{i,j}}\frac{1}{2\,\Delta\xi}\frac{2\nu}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_3^{i,j}+\frac{1}{H^{i,j}}\frac{1}{2\,\Delta\xi}\frac{2\nu}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+\right.$$

$$\left.\xi_y'\frac{\partial V}{\partial \xi}\right\}_3^{i,j}-\frac{1}{H^{i,j}}\frac{4}{2\,\Delta\xi}\frac{2(1-\nu)}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_2^{i,j}-\frac{1}{H^{i,j}}\frac{4}{2\,\Delta\xi}\frac{2\nu}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_2^{i,j}-\frac{1}{H^{i,j}}\frac{4}{2\,\Delta\xi}\frac{2\nu}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+\right.$$

$$\left.\xi_y'\frac{\partial V}{\partial \xi}\right\}_2^{i,j}+\frac{1}{H^{i,j}}\frac{3}{2\,\Delta\xi}\left(\frac{\partial h_s}{\partial x}\right)^{i,j}\left\{\xi_z'\frac{\partial U}{\partial \xi}+\frac{\partial W}{\partial x}+\eta_x'\frac{\partial W}{\partial \eta}+\xi_x'\frac{\partial W}{\partial \xi}\right\}_1^{i,j}+\frac{1}{H^{i,j}}\frac{3}{2\,\Delta\xi}\left(\frac{\partial h_s}{\partial y}\right)^{i,j}\left\{\xi_z'\frac{\partial V}{\partial \xi}+\eta_y'\frac{\partial W}{\partial \eta}+\right.$$

$$\left.\xi_y'\frac{\partial W}{\partial \xi}\right\}_1^{i,j}\approx\frac{2(1+\nu)}{E}\cdot\rho g+\frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 W}{\partial t^2}\right)_1^{i,j}; \qquad (47)$$

Accounting for Eqs (19)-(20), the same manipulations lead to the following **equations at the ice-shelf front ($x = L$)**:

$$\begin{cases} \frac{1}{2\,\Delta x}(\sigma_{xx})_k^{N_x-2,j}-\frac{4}{2\,\Delta x}(\sigma_{xx})_k^{N_x-1,j}\approx-\frac{3}{2\,\Delta x}f(\xi)-\left(\xi'_x\right)_k^{N_x,j}\frac{\partial f(\xi)}{\partial \xi}+\rho\left(\frac{\partial^2 U}{\partial t^2}\right)_k^{N_x,j}; \\ \frac{1}{2\,\Delta x}\left(\sigma_{yx}\right)_k^{N_x-2,j}-\frac{4}{2\,\Delta x}\left(\sigma_{yx}\right)_k^{N_x-1,j}+\left(\eta'_y\frac{\partial\sigma_{yy}}{\partial \eta}\right)_k^{N_x,j}+\left(\xi'_y\frac{\partial\sigma_{yy}}{\partial \xi}\right)_k^{N_x,j}+\left(\xi'_z\frac{\partial\sigma_{yz}}{\partial \xi}\right)_k^{N_x,j}\approx \\ \approx\rho\left(\frac{\partial^2 V}{\partial t^2}\right)_k^{N_x,j}; \\ \frac{1}{2\,\Delta x}(\sigma_{zx})_k^{N_x-2,j}-\frac{4}{2\,\Delta x}\,(\sigma_{zx})_k^{N_x-1,j}+\left(\eta'_y\frac{\partial\sigma_{zy}}{\partial \eta}\right)_k^{N_x,j}+\left(\xi'_y\frac{\partial\sigma_{zy}}{\partial \xi}\right)_k^{N_x,j}+\left(\xi'_z\frac{\partial\sigma_{zz}}{\partial \xi}\right)_k^{N_x,j}\approx \\ \approx\rho g+\rho\left(\frac{\partial^2 W}{\partial t^2}\right)_k^{N_x,j}; \end{cases} \qquad (48)$$

where $f(\xi)$ is defined by Eq (20).

In terms of the displacements equations (48) are expressed as

a) **the first equation at the ice-shelf front** (**lines 877-1016 in the program code**) is

$$\frac{1}{2\,\Delta x}\cdot\frac{2(1-\nu)}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_k^{N_x-2.j}+\frac{1}{2\,\Delta x}\cdot\frac{2\nu}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+\xi_y'\frac{\partial V}{\partial \xi}\right\}_k^{N_x-2,j}+\frac{1}{2\,\Delta x}\cdot \frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_k^{N_x-2,j}-\frac{4}{2\,\Delta x}\cdot\frac{2(1-\nu)}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta_x'\frac{\partial U}{\partial \eta}+\xi_x'\frac{\partial U}{\partial \xi}\right\}_k^{N_x-1.j}-\frac{4}{2\,\Delta x}\cdot\frac{2\nu}{1-2\nu}\left\{\eta_y'\frac{\partial V}{\partial \eta}+ \xi_y'\frac{\partial V}{\partial \xi}\right\}_k^{N_x-1,j}-\frac{4}{2\,\Delta x}\cdot\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial W}{\partial \xi}\right\}_k^{N_x-1,j}\approx-\frac{2(1+\nu)}{E}\cdot\frac{3}{2\,\Delta x}f(\xi)-\frac{2(1+\nu)}{E}\cdot\left({\xi'}_x\right)_k^{N_x,j}\frac{\partial f(\xi)}{\partial \xi}+\frac{2(1+\nu)}{E}\cdot \rho\left(\frac{\partial^2 U}{\partial t^2}\right)_k^{N_x,j}; \qquad (49)$$

b) **the second equation at the ice-shelf front** (**lines 1020-1319 in the program code**) is

$$\frac{1}{2\,\Delta x}\cdot\left\{\eta_y'\frac{\partial U}{\partial \eta}+\xi_y'\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta_x'\frac{\partial V}{\partial \eta}+\xi_x'\frac{\partial V}{\partial \xi}\right\}_k^{N_x-2,j}-\frac{4}{2\,\Delta x}\cdot\left\{\eta_y'\frac{\partial U}{\partial \eta}+\xi_y'\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta_x'\frac{\partial V}{\partial \eta}+ \xi_x'\frac{\partial V}{\partial \xi}\right\}_k^{N_x-1,j}+\frac{2(1-\nu)}{1-2\nu}\left\{\eta_y'\frac{\partial}{\partial \eta}\left(\eta_y'\frac{\partial V}{\partial \eta}\right)+\eta_y'\frac{\partial}{\partial \eta}\left(\xi_y'\frac{\partial V}{\partial \xi}\right)\right\}_k^{N_x,j}+\frac{2\nu}{1-2\nu}\left\{\eta_y'\frac{\partial^2 U}{\partial \eta\,\partial x}+\eta_y'\frac{\partial}{\partial \eta}\left(\eta_x'\frac{\partial U}{\partial \eta}\right)+ \eta_y'\frac{\partial}{\partial \eta}\left(\xi_x'\frac{\partial U}{\partial \xi}\right)\right\}_k^{N_x,j}+\frac{2\nu}{1-2\nu}\left\{\eta_y'\frac{\partial}{\partial \eta}\left(\xi_z'\frac{\partial W}{\partial \xi}\right)\right\}_k^{N_x,j}+\frac{2(1-\nu)}{1-2\nu}\left\{\xi_y'\frac{\partial}{\partial \xi}\left(\eta_y'\frac{\partial V}{\partial \eta}\right)+\xi_y'\frac{\partial}{\partial \xi}\left(\xi_y'\frac{\partial V}{\partial \xi}\right)\right\}_k^{N_x,j}+ \frac{2\nu}{1-2\nu}\left\{\xi_y'\frac{\partial^2 U}{\partial \xi\,\partial x}+\xi_y'\frac{\partial}{\partial \xi}\left(\eta_x'\frac{\partial U}{\partial \eta}\right)+\xi_y'\frac{\partial}{\partial \xi}\left(\xi_x'\frac{\partial U}{\partial \xi}\right)\right\}_k^{N_x,j}+\frac{2\nu}{1-2\nu}\left\{\xi_y'\frac{\partial}{\partial \xi}\left(\xi_z'\frac{\partial W}{\partial \xi}\right)\right\}_k^{N_x,j}+\left\{\xi_z'\frac{\partial}{\partial \xi}\left(\xi_z'\frac{\partial V}{\partial \xi}\right)+ \xi_z'\frac{\partial}{\partial \xi}\left(\eta_y'\frac{\partial W}{\partial \eta}\right)+\xi_z'\frac{\partial}{\partial \xi}\left(\xi_y'\frac{\partial W}{\partial \xi}\right)\right\}_k^{N_x,j}\approx\frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 V}{\partial t^2}\right)_k^{N_x,j}; \qquad (50)$$

c) **the third equation at the ice-shelf front (lines 1323-1562 in the program code)** is

$$\frac{1}{2\,\Delta x}\cdot\left\{\xi_z'\frac{\partial U}{\partial \xi}+\frac{\partial W}{\partial x}+\eta_x'\frac{\partial W}{\partial \eta}+\xi_x'\frac{\partial W}{\partial \xi}\right\}_k^{N_x-2,j}-\frac{4}{2\,\Delta x}\cdot\left\{\xi_z'\frac{\partial U}{\partial \xi}+\frac{\partial W}{\partial x}+\eta_x'\frac{\partial W}{\partial \eta}+\xi_x'\frac{\partial W}{\partial \xi}\right\}_k^{N_x-1,j}+$$

$$\left\{\eta_y'\frac{\partial}{\partial \eta}\left(\xi_z'\frac{\partial V}{\partial \xi}\right)+\eta_y'\frac{\partial}{\partial \eta}\left(\eta_y'\frac{\partial W}{\partial \eta}\right)+\eta_y'\frac{\partial}{\partial \eta}\left(\xi_y'\frac{\partial W}{\partial \xi}\right)\right\}_k^{N_x,j}+\left\{\xi_y'\frac{\partial}{\partial \xi}\left(\xi_z'\frac{\partial V}{\partial \xi}\right)+\xi_y'\frac{\partial}{\partial \xi}\left(\eta_y'\frac{\partial W}{\partial \eta}\right)+\right.$$

$$\left.\xi_y'\frac{\partial}{\partial \xi}\left(\xi_y'\frac{\partial W}{\partial \xi}\right)\right\}_k^{N_x,j}+\frac{2(1-\nu)}{1-2\nu}\left\{\xi_z'\frac{\partial}{\partial \xi}\left(\xi_z'\frac{\partial W}{\partial \xi}\right)\right\}_k^{N_x,j}+\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial^2 U}{\partial \xi\partial x}+\xi_z'\frac{\partial}{\partial \xi}\left(\eta_x'\frac{\partial U}{\partial \eta}\right)+\right.$$

$$\left.\xi_z'\frac{\partial}{\partial \xi}\left(\xi_x'\frac{\partial U}{\partial \xi}\right)\right\}_k^{N_x,j}+\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial}{\partial \xi}\left(\eta_y'\frac{\partial V}{\partial \eta}\right)+\xi_z'\frac{\partial}{\partial \xi}\left(\xi_y'\frac{\partial V}{\partial \xi}\right)\right\}_k^{N_x,j}\approx\frac{2(1+\nu)}{E}\cdot\rho g+\frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 W}{\partial t^2}\right)_k^{N_x,j};$$

(51)

Accounting for Eqs (24)-(26), the considered technique leads to the following **equations at the ice-shelf lateral edge $y = y_1(x)$**:

$$\begin{cases} \left(\frac{\partial\sigma_{xx}}{\partial x}\right)_k^{i,1} + \left(\eta'_x \frac{\partial\sigma_{xx}}{\partial\eta}\right)_k^{i,1} + \left(\xi'_x \frac{\partial\sigma_{xx}}{\partial\xi}\right)_k^{i,1} - \left(\eta'_y\right)^i \frac{1}{2\,\Delta\eta}\left(\sigma_{xy}\right)_k^{i,3} + \\ \quad \left(\eta'_y\right)^i \frac{4}{2\,\Delta\eta}\left(\sigma_{xy}\right)_k^{i,2} - \left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta}\left(\sigma_{xx}\right)_k^{i,1}\left(\frac{dy_1}{dx}\right)^i + \\ \quad\quad \left(\xi'_y \frac{\partial\sigma_{xx}}{\partial\xi}\right)_k^{i,1}\left(\frac{dy_1}{dx}\right)^i + \left(\xi'_z \frac{\partial\sigma_{xz}}{\partial\xi}\right)_k^{i,1} \approx \\ \quad \approx -\left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta} f_x^1(\xi) + \left(\xi'_y\right)_k^{i,1}\frac{df_x^1}{d\xi} + \rho\left(\frac{\partial^2 U}{\partial t^2}\right)_k^{i,1}; \\ \\ \left(\frac{\partial\sigma_{yx}}{\partial x}\right)_k^{i,1} + \left(\eta'_x \frac{\partial\sigma_{yx}}{\partial\eta}\right)_k^{i,1} + \left(\xi'_x \frac{\partial\sigma_{yx}}{\partial\xi}\right)_k^{i,1} - \left(\eta'_y\right)^i \frac{1}{2\,\Delta\eta}\left(\sigma_{yy}\right)_k^{i,3} + \\ \quad \left(\eta'_y\right)^i \frac{4}{2\,\Delta\eta}\left(\sigma_{yy}\right)_k^{i,2} - \left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta}\left(\sigma_{yx}\right)_k^{i,1}\left(\frac{dy_1}{dx}\right)^i + \\ \quad\quad \left(\xi'_y \frac{\partial\sigma_{yx}}{\partial\xi}\right)_k^{i,1}\left(\frac{dy_1}{dx}\right)^i + \left(\xi'_z \frac{\partial\sigma_{yz}}{\partial\xi}\right)_k^{i,1} \approx \\ \quad \approx -\left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta} f_y^1(\xi) + \left(\xi'_y\right)_k^{i,1}\frac{df_y^1}{d\xi} + \rho\left(\frac{\partial^2 V}{\partial t^2}\right)_k^{i,1}; \\ \\ \left(\frac{\partial\sigma_{zx}}{\partial x}\right)_k^{i,1} + \left(\eta'_x \frac{\partial\sigma_{zx}}{\partial\eta}\right)_k^{i,1} + \left(\xi'_x \frac{\partial\sigma_{zx}}{\partial\xi}\right)_k^{i,1} - \left(\eta'_y\right)^i \frac{1}{2\,\Delta\eta}\left(\sigma_{zy}\right)_k^{i,3} + \\ \quad \left(\eta'_y\right)^i \frac{4}{2\,\Delta\eta}\left(\sigma_{zy}\right)_k^{i,2} - \left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta}\left(\sigma_{zx}\right)_k^{i,1}\left(\frac{dy_1}{dx}\right)^i + \\ \quad \left(\xi'_y \frac{\partial\sigma_{zx}}{\partial\xi}\right)_k^{i,1}\left(\frac{dy_1}{dx}\right)^i + \left(\xi'_z \frac{\partial\sigma_{zz}}{\partial\xi}\right)_k^{i,1} \approx \rho g + \rho\left(\frac{\partial^2 W}{\partial t^2}\right)_k^{i,1}; \end{cases} \tag{52}$$

where the upper indices 1-3 in Eq (52) denote respectively the numbers of the grid layers starting from the ice lateral edge $y = y_1(x)$, moving in the horizontal transverse direction in the glacier.

In terms of the displacements equations (52) are expressed as

a) **the first equation at the ice-shelf lateral edge $y = y_1(x)$ (lines 1751-2555 in the program code)** is

$$\frac{2(1-\nu)}{1-2\nu}\Big\{\frac{\partial^2 U}{\partial x^2} + \frac{\partial}{\partial x}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_x \frac{\partial U}{\partial \xi}\right) + \eta'_x \frac{\partial^2 U}{\partial\eta\partial x} + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_x\frac{\partial U}{\partial\eta}\right) + \eta'_x \frac{\partial}{\partial\eta}\left(\xi'_x \frac{\partial U}{\partial \xi}\right) + \xi'_x \frac{\partial^2 U}{\partial\xi\partial x} +$$

$$\xi'_x \frac{\partial}{\partial\xi}\left(\eta'_x \frac{\partial U}{\partial\eta}\right) + \xi'_x \frac{\partial}{\partial\xi}\left(\xi'_x\frac{\partial U}{\partial\xi}\right)\Big\}_k^{i.1} + \frac{2\nu}{1-2\nu}\Big\{\frac{\partial}{\partial x}\left(\eta'_y \frac{\partial V}{\partial\eta}\right) + \frac{\partial}{\partial x}\left(\xi'_y\frac{\partial V}{\partial\xi}\right) + \eta'_x\frac{\partial}{\partial\eta}\left(\eta'_y\frac{\partial V}{\partial\eta}\right) +$$

$$\eta'_x \frac{\partial}{\partial\eta}\left(\xi'_y \frac{\partial V}{\partial\xi}\right) + \xi'_x\frac{\partial}{\partial\xi}\left(\eta'_y\frac{\partial V}{\partial\eta}\right) + \xi'_x\frac{\partial}{\partial\xi}\left(\xi'_y\frac{\partial V}{\partial\xi}\right)\Big\}_k^{i,1} + \frac{2\nu}{1-2\nu}\Big\{\frac{\partial}{\partial x}\left(\xi'_z\frac{\partial W}{\partial\xi}\right) + \eta'_x\frac{\partial}{\partial\eta}\left(\xi'_z\frac{\partial W}{\partial\xi}\right) +$$

$$\xi'_x\frac{\partial}{\partial\xi}\left(\xi'_z\frac{\partial W}{\partial\xi}\right)\Big\}_k^{i,1} - \left(\eta'_y\right)^i \frac{1}{2\,\Delta\eta}\Big\{\eta'_y\frac{\partial U}{\partial\eta} + \xi'_y\frac{\partial U}{\partial\xi} + \frac{\partial V}{\partial x} + \eta'_x\frac{\partial V}{\partial\eta} + \xi'_x\frac{\partial V}{\partial\xi}\Big\}_k^{i,3} + \left(\eta'_y\right)^i\frac{4}{2\,\Delta\eta}\Big\{\eta'_y\frac{\partial U}{\partial\eta} +$$

$$\xi'_y\frac{\partial U}{\partial\xi} + \frac{\partial V}{\partial x} + \eta'_x\frac{\partial V}{\partial\eta} + \xi'_x\frac{\partial V}{\partial\xi}\Big\}_k^{i,2} - \left(\eta'_y\right)^i\frac{3}{2\,\Delta\eta}\frac{2(1-\nu)}{1-2\nu}\Big\{\frac{\partial U}{\partial x} + \eta'_x\frac{\partial U}{\partial\eta} + \xi'_x\frac{\partial U}{\partial\xi}\Big\}_k^{i,1}\left(\frac{dy_1}{dx}\right)^i -$$

$$\left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\Big\{\eta'_y\frac{\partial V}{\partial\eta} + \xi'_y\frac{\partial V}{\partial\xi}\Big\}_k^{i,1}\left(\frac{dy_1}{dx}\right)^i - \left(\eta'_y\right)^i\frac{3}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\Big\{\xi'_z\frac{\partial W}{\partial\xi}\Big\}_k^{i,1}\left(\frac{dy_1}{dx}\right)^i + \frac{2(1-\nu)}{1-2\nu}\Big\{\xi'_y\frac{\partial^2 U}{\partial\xi\partial x} +$$

$$\xi'_y\frac{\partial}{\partial\xi}\left(\eta'_x\frac{\partial U}{\partial\eta}\right) + \xi'_y\frac{\partial}{\partial\xi}\left(\xi'_x\frac{\partial U}{\partial\xi}\right)\Big\}_k^{i.1}\cdot\left(\frac{dy_1}{dx}\right)^i + \frac{2\nu}{1-2\nu}\Big\{\xi'_y\frac{\partial}{\partial\xi}\left(\eta'_y\frac{\partial V}{\partial\eta}\right) + \xi'_y\frac{\partial}{\partial\xi}\left(\xi'_y\frac{\partial V}{\partial\xi}\right)\Big\}_k^{i,1}\cdot\left(\frac{dy_1}{dx}\right)^i +$$

$$\frac{2\nu}{1-2\nu}\Big\{\xi'_y\frac{\partial}{\partial\xi}\left(\xi'_z\frac{\partial W}{\partial\xi}\right)\Big\}_k^{i,1}\cdot\left(\frac{dy_1}{dx}\right)^i + \Big\{\xi'_z\frac{\partial}{\partial\xi}\left(\xi'_z\frac{\partial U}{\partial\xi}\right) + \xi'_z\frac{\partial^2 W}{\partial\xi\partial x} + \xi'_z\frac{\partial}{\partial\xi}\left(\eta'_x\frac{\partial W}{\partial\eta}\right) + \xi'_z\frac{\partial}{\partial\xi}\left(\xi'_x\frac{\partial W}{\partial\xi}\right)\Big\}_k^{i,1} \approx$$

$$-\frac{2(1+\nu)}{E}\cdot\left(\eta'_y\right)^i\frac{3}{2\,\Delta\eta}f_x^1(\xi) + \frac{2(1+\nu)}{E}\cdot\left(\xi'_y\right)_k^{i,1}\frac{df_x^1}{d\xi} + \frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 U}{\partial t^2}\right)_k^{i,1}; \qquad (53)$$

b) **the second equation at the ice-shelf lateral edge $y = y_1(x)$ (lines 2562-3256 in the program code)** is

$$\Big\{\frac{\partial}{\partial x}\left(\eta'_y\frac{\partial U}{\partial\eta}\right) + \frac{\partial}{\partial x}\left(\xi'_y\frac{\partial U}{\partial\xi}\right) + \frac{\partial^2 V}{\partial x^2} + \frac{\partial}{\partial x}\left(\eta'_x\frac{\partial V}{\partial\eta}\right) + \frac{\partial}{\partial x}\left(\xi'_x\frac{\partial V}{\partial\xi}\right)\Big\}_k^{i,1} + \Big\{\eta'_x\frac{\partial}{\partial\eta}\left(\eta'_y\frac{\partial U}{\partial\eta}\right) + \eta'_x\frac{\partial}{\partial\eta}\left(\xi'_y\frac{\partial U}{\partial\xi}\right) +$$

$$\eta'_x\frac{\partial^2 V}{\partial\eta\partial x} + \eta'_x\frac{\partial}{\partial\eta}\left(\eta'_x\frac{\partial V}{\partial\eta}\right) + \eta'_x\frac{\partial}{\partial\eta}\left(\xi'_x\frac{\partial V}{\partial\xi}\right)\Big\}_k^{i,1} + \Big\{\xi'_x\frac{\partial}{\partial\xi}\left(\eta'_y\frac{\partial U}{\partial\eta}\right) + \xi'_x\frac{\partial}{\partial\xi}\left(\xi'_y\frac{\partial U}{\partial\xi}\right) + \xi'_x\frac{\partial^2 V}{\partial\xi\partial x} +$$

$$\xi'_x\frac{\partial}{\partial\xi}\left(\eta'_x\frac{\partial V}{\partial\eta}\right) + \xi'_x\frac{\partial}{\partial\xi}\left(\xi'_x\frac{\partial V}{\partial\xi}\right)\Big\}_k^{i,1} - \left(\eta'_y\right)^i\frac{1}{2\,\Delta\eta}\frac{2(1-\nu)}{1-2\nu}\Big\{\eta'_y\frac{\partial V}{\partial\eta} + \xi'_y\frac{\partial V}{\partial\xi}\Big\}_k^{i,3} - \left(\eta'_y\right)^i\frac{1}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\Big\{\frac{\partial U}{\partial x} +$$

$$\eta'_x\frac{\partial U}{\partial\eta} + \xi'_x\frac{\partial U}{\partial\xi}\Big\}_k^{i,3} - \left(\eta'_y\right)^i\frac{1}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\Big\{\xi'_z\frac{\partial W}{\partial\xi}\Big\}_k^{i,3} + \left(\eta'_y\right)^i\frac{4}{2\,\Delta\eta}\frac{2(1-\nu)}{1-2\nu}\Big\{\eta'_y\frac{\partial V}{\partial\eta} + \xi'_y\frac{\partial V}{\partial\xi}\Big\}_k^{i,2} +$$

$$\left(\eta'_y\right)^i\frac{4}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\Big\{\frac{\partial U}{\partial x} + \eta'_x\frac{\partial U}{\partial\eta} + \xi'_x\frac{\partial U}{\partial\xi}\Big\}_k^{i,2} + \left(\eta'_y\right)^i\frac{4}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\Big\{\xi'_z\frac{\partial W}{\partial\xi}\Big\}_k^{i,2} - \left(\eta'_y\right)^i\frac{3}{2\,\Delta\eta}\Big\{\eta'_y\frac{\partial U}{\partial\eta} +$$

$$\xi'_y\frac{\partial U}{\partial\xi} + \frac{\partial V}{\partial x} + \eta'_x\frac{\partial V}{\partial\eta} + \xi'_x\frac{\partial V}{\partial\xi}\Big\}_k^{i,1}\left(\frac{dy_1}{dx}\right)^i + \Big\{\xi'_y\frac{\partial}{\partial\xi}\left(\eta'_y\frac{\partial U}{\partial\eta}\right) + \xi'_y\frac{\partial}{\partial\xi}\left(\xi'_y\frac{\partial U}{\partial\xi}\right) + \frac{\partial^2 V}{\partial\xi\partial x} + \xi'_y\frac{\partial}{\partial\xi}\left(\eta'_x\frac{\partial V}{\partial\eta}\right) +$$

$$\xi'_y \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial V}{\partial \xi}\right)\Big\}_k^{i,1} \left(\frac{dy_1}{dx}\right)^i + \left\{\xi'_z \frac{\partial}{\partial \xi}\left(\xi'_z \frac{\partial V}{\partial \xi}\right) + \xi'_z \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial W}{\partial \eta}\right) + \xi'_z \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial W}{\partial \xi}\right)\right\}_k^{i,1} \approx -\frac{2(1+\nu)}{E} \cdot$$

$$\left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta} f_y^1(\xi) + \frac{2(1+\nu)}{E} \cdot \left(\xi'_y\right)_k^{i,1} \frac{df_y^1}{d\xi} + \frac{2(1+\nu)}{E} \cdot \rho \left(\frac{\partial^2 V}{\partial t^2}\right)_k^{i,1}; \qquad (54)$$

c) **the third equation at the ice-shelf lateral edge $y = y_1(x)$** (**lines 3261-3836 in the program code**) is

$$\left\{\frac{\partial}{\partial x}\left(\xi'_z \frac{\partial U}{\partial \xi}\right) + \frac{\partial^2 W}{\partial x^2} + \frac{\partial}{\partial x}\left(\eta'_x \frac{\partial W}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_x \frac{\partial W}{\partial \xi}\right)\right\}_k^{i,1} + \left\{\eta'_x \frac{\partial}{\partial \eta}\left(\xi'_z \frac{\partial U}{\partial \xi}\right) + \eta'_x \frac{\partial^2 W}{\partial \eta \partial x} + \eta'_x \frac{\partial}{\partial \eta}\left(\eta'_x \frac{\partial W}{\partial \eta}\right) +\right.$$

$$\left.\eta'_x \frac{\partial}{\partial \eta}\left(\xi'_x \frac{\partial W}{\partial \xi}\right)\right\}_k^{i,1} + \left\{\xi'_x \frac{\partial}{\partial \xi}\left(\xi'_z \frac{\partial U}{\partial \xi}\right) + \xi'_x \frac{\partial^2 W}{\partial \xi \partial x} + \xi'_x \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial W}{\partial \eta}\right) + \xi'_x \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial W}{\partial \xi}\right)\right\}_k^{i,1} -$$

$$\left(\eta'_y\right)^i \frac{1}{2\,\Delta\eta}\left\{\xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi}\right\}_k^{i,3} + \left(\eta'_y\right)^i \frac{4}{2\,\Delta\eta}\left\{\xi'_z \frac{\partial V}{\partial \xi} + \eta'_y \frac{\partial W}{\partial \eta} + \xi'_y \frac{\partial W}{\partial \xi}\right\}_k^{i,2} -$$

$$\left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta}\left\{\xi'_z \frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x \frac{\partial W}{\partial \eta} + \xi'_x \frac{\partial W}{\partial \xi}\right\}_k^{i,1} \left(\frac{dy_1}{dx}\right)^i + \left\{\xi'_y \frac{\partial}{\partial \xi}\left(\xi'_z \frac{\partial U}{\partial \xi}\right) + \xi'_y \frac{\partial^2 W}{\partial \xi \partial x} + \xi'_y \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial W}{\partial \eta}\right) +\right.$$

$$\left.\xi'_y \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial W}{\partial \xi}\right)\right\}_k^{i,1} \cdot \left(\frac{dy_1}{dx}\right)^i + \frac{2(1-\nu)}{1-2\nu}\left\{\xi'_z \frac{\partial}{\partial \xi}\left(\xi'_z \frac{\partial W}{\partial \xi}\right)\right\}_k^{i,1} + \frac{2\nu}{1-2\nu}\left\{\xi'_z \frac{\partial^2 U}{\partial \xi \partial x} + \xi'_z \frac{\partial}{\partial \xi}\left(\eta'_x \frac{\partial U}{\partial \eta}\right) +\right.$$

$$\left.\xi'_z \frac{\partial}{\partial \xi}\left(\xi'_x \frac{\partial U}{\partial \xi}\right)\right\}_k^{i,1} + \frac{2\nu}{1-2\nu}\left\{\xi'_z \frac{\partial}{\partial \xi}\left(\eta'_y \frac{\partial V}{\partial \eta}\right) + \xi'_z \frac{\partial}{\partial \xi}\left(\xi'_y \frac{\partial V}{\partial \xi}\right)\right\}_k^{i,1} \approx \frac{2(1+\nu)}{E} \cdot \rho g + \frac{2(1+\nu)}{E} \cdot \rho \left(\frac{\partial^2 W}{\partial t^2}\right)_k^{i,1};$$

(55)

Like in Eqs(52), accounting for Eqs (30)-(32), we obtain the following **equations at the ice-shelf lateral edge $y = y_2(x)$**:

$$\begin{cases} \left(\frac{\partial\sigma_{xx}}{\partial x}\right)_k^{i,N_\eta} + \left(\eta'_x \frac{\partial\sigma_{xx}}{\partial\eta}\right)_k^{i,N_\eta} + \left(\xi'_x \frac{\partial\sigma_{xx}}{\partial\xi}\right)_k^{i,N_\eta} + \left(\eta'_y\right)^i \frac{1}{2\,\Delta\eta}\left(\sigma_{xy}\right)_k^{i,N_\eta-2} - \\ \quad \left(\eta'_y\right)^i \frac{4}{2\,\Delta\eta}\left(\sigma_{xy}\right)_k^{i,N_\eta-1} + \left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta}\left(\sigma_{xx}\right)_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i + \\ \quad \left(\xi'_y \frac{\partial\sigma_{xx}}{\partial\xi}\right)_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i + \left(\xi'_z \frac{\partial\sigma_{xz}}{\partial\xi}\right)_k^{i,N_\eta} \approx \\ \quad \approx -\left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta} f_x^2(\xi) - \left(\xi'_y\right)_k^{i,N_\eta}\frac{df_x^2}{d\xi} + \rho\left(\frac{\partial^2 U}{\partial t^2}\right)_k^{i,N_\eta}; \\ \\ \left(\frac{\partial\sigma_{yx}}{\partial x}\right)_k^{i,N_\eta} + \left(\eta'_x \frac{\partial\sigma_{yx}}{\partial\eta}\right)_k^{i,N_\eta} + \left(\xi'_x \frac{\partial\sigma_{yx}}{\partial\xi}\right)_k^{i,N_\eta} + \left(\eta'_y\right)^i \frac{1}{2\,\Delta\eta}\left(\sigma_{yy}\right)_k^{i,N_\eta-2} - \\ \quad \left(\eta'_y\right)^i \frac{4}{2\,\Delta\eta}\left(\sigma_{yy}\right)_k^{i,N_\eta-1} + \left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta}\left(\sigma_{yx}\right)_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i + \\ \quad \left(\xi'_y \frac{\partial\sigma_{yx}}{\partial\xi}\right)_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i + \left(\xi'_z \frac{\partial\sigma_{yz}}{\partial\xi}\right)_k^{i,N_\eta} \approx \\ \quad \approx -\left(\eta'_y\right)^{N_\eta} \frac{3}{2\,\Delta\eta} f_y^2(\xi) - \left(\xi'_y\right)_k^{i,N_\eta}\frac{df_y^2}{d\xi} + \rho\left(\frac{\partial^2 V}{\partial t^2}\right)_k^{i,N_\eta}; \\ \\ \left(\frac{\partial\sigma_{zx}}{\partial x}\right)_k^{i,N_\eta} + \left(\eta'_x \frac{\partial\sigma_{zx}}{\partial\eta}\right)_k^{i,N_\eta} + \left(\xi'_x \frac{\partial\sigma_{zx}}{\partial\xi}\right)_k^{i,N_\eta} + \left(\eta'_y\right)^i \frac{1}{2\,\Delta\eta}\left(\sigma_{zy}\right)_k^{i,N_\eta-2} - \\ \quad \left(\eta'_y\right)^i \frac{4}{2\,\Delta\eta}\left(\sigma_{zy}\right)_k^{i,N_\eta-1} + \left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta}\left(\sigma_{zx}\right)_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i + \\ \quad \left(\xi'_y \frac{\partial\sigma_{zx}}{\partial\xi}\right)_k^{i,N_\eta}\frac{dy_2}{dx} + \left(\xi'_z \frac{\partial\sigma_{zz}}{\partial\xi}\right)_k^{i,N_\eta} \approx \\ \quad \approx \rho g + \rho\left(\frac{\partial^2 W}{\partial t^2}\right)_k^{i,N_\eta}; \end{cases} \tag{56}$$

where indices, "$N_\eta - 2$", "$N_\eta - 1$", and "N_η", denote respectively the numbers of the grid layers, moving from "$N_\eta - 2$" in the horizontal transverse direction and ending at the ice lateral edge $y = y_2(x)$.

In terms of the displacements equations (56) are expressed as

a) **the first equation at the ice-shelf lateral edge $y = y_2(x)$ (lines 4206-5003 in the program code)** is

$$\frac{2(1-\nu)}{1-2\nu}\left\{\frac{\partial^2 U}{\partial x^2}+\frac{\partial}{\partial x}\left(\eta'_x\frac{\partial U}{\partial \eta}\right)+\frac{\partial}{\partial x}\left(\xi'_x\frac{\partial U}{\partial \xi}\right)+\eta'_x\frac{\partial^2 U}{\partial\eta\partial x}+\eta'_x\frac{\partial}{\partial \eta}\left(\eta'_x\frac{\partial U}{\partial \eta}\right)+\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_x\frac{\partial U}{\partial \xi}\right)+\xi'_x\frac{\partial^2 U}{\partial\xi\partial x}+\right.$$

$$\left.\xi'_x\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial U}{\partial \eta}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial U}{\partial \xi}\right)\right\}_k^{i.N_\eta}+\frac{2\nu}{1-2\nu}\left\{\frac{\partial}{\partial x}\left(\eta'_y\frac{\partial V}{\partial \eta}\right)+\frac{\partial}{\partial x}\left(\xi'_y\frac{\partial V}{\partial \xi}\right)+\eta'_x\frac{\partial}{\partial \eta}\left(\eta'_y\frac{\partial V}{\partial \eta}\right)+\right.$$

$$\left.\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_y\frac{\partial V}{\partial \xi}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial V}{\partial \eta}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,N_\eta}+\frac{2\nu}{1-2\nu}\left\{\frac{\partial}{\partial x}\left(\xi'_z\frac{\partial W}{\partial \xi}\right)+\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_z\frac{\partial W}{\partial \xi}\right)+\right.$$

$$\left.\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,N_\eta}+\left(\eta'_y\right)^i\frac{1}{2\,\Delta\eta}\left\{\eta'_y\frac{\partial U}{\partial \eta}+\xi'_y\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta'_x\frac{\partial V}{\partial \eta}+\xi'_x\frac{\partial V}{\partial \xi}\right\}_k^{i,N_\eta-2}-$$

$$\left(\eta'_y\right)^i\frac{4}{2\,\Delta\eta}\left\{\eta'_y\frac{\partial U}{\partial \eta}+\xi'_y\frac{\partial U}{\partial \xi}+\frac{\partial V}{\partial x}+\eta'_x\frac{\partial V}{\partial \eta}+\xi'_x\frac{\partial V}{\partial \xi}\right\}_k^{i,N_\eta-1}+\left(\eta'_y\right)^i\frac{3}{2\,\Delta\eta}\frac{2(1-\nu)}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta'_x\frac{\partial U}{\partial \eta}+\right.$$

$$\left.\xi'_x\frac{\partial U}{\partial \xi}\right\}_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i+\left(\eta'_y\right)^i\frac{3}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\left\{\eta'_y\frac{\partial V}{\partial \eta}+\xi'_y\frac{\partial V}{\partial \xi}\right\}_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i+$$

$$\left(\eta'_y\right)^i\frac{3}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\left\{\xi'_z\frac{\partial W}{\partial \xi}\right\}_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i+\frac{2(1-\nu)}{1-2\nu}\left\{\xi'_y\frac{\partial^2 U}{\partial\xi\partial x}+\xi'_y\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial U}{\partial \eta}\right)+\xi'_y\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial U}{\partial \xi}\right)\right\}_k^{i.N_\eta}\cdot$$

$$\left(\frac{dy_2}{dx}\right)^i+\frac{2\nu}{1-2\nu}\left\{\xi'_y\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial V}{\partial \eta}\right)+\xi'_y\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,N_\eta}\cdot\left(\frac{dy_2}{dx}\right)^i+\frac{2\nu}{1-2\nu}\left\{\xi'_y\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,N_\eta}\cdot\left(\frac{dy_2}{dx}\right)^i+$$

$$\left\{\xi'_z\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial U}{\partial \xi}\right)+\xi'_z\frac{\partial^2 W}{\partial\xi\partial x}+\xi'_z\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial W}{\partial \eta}\right)+\xi'_z\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,N_\eta}\approx-\frac{2(1+\nu)}{E}\cdot\left(\eta'_y\right)^i\frac{3}{2\,\Delta\eta}f_x^2(\xi)-$$

$$\frac{2(1+\nu)}{E}\cdot\left(\xi'_y\right)_k^{i,N_\eta}\frac{df_x^2}{d\xi}+\frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 U}{\partial t^2}\right)_k^{i,N_\eta};\qquad(57)$$

b) **the second equation at the ice-shelf lateral edge $y = y_2(x)$ (lines 5008-5703 in the program code)** is

$$\left\{\frac{\partial}{\partial x}\left(\eta'_y\frac{\partial U}{\partial \eta}\right)+\frac{\partial}{\partial x}\left(\xi'_y\frac{\partial U}{\partial \xi}\right)+\frac{\partial^2 V}{\partial x^2}+\frac{\partial}{\partial x}\left(\eta'_x\frac{\partial V}{\partial \eta}\right)+\frac{\partial}{\partial x}\left(\xi'_x\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,N_\eta}+\left\{\eta'_x\frac{\partial}{\partial \eta}\left(\eta'_y\frac{\partial U}{\partial \eta}\right)+\right.$$

$$\left.\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_y\frac{\partial U}{\partial \xi}\right)+\eta'_x\frac{\partial^2 V}{\partial\eta\partial x}+\eta'_x\frac{\partial}{\partial \eta}\left(\eta'_x\frac{\partial V}{\partial \eta}\right)+\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_x\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,N_\eta}+\left\{\xi'_x\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial U}{\partial \eta}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial U}{\partial \xi}\right)+\right.$$

$$\left.\xi'_x\frac{\partial^2 V}{\partial\xi\partial x}+\xi'_x\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial V}{\partial \eta}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,N_\eta}+\left(\eta'_y\right)^i\frac{1}{2\,\Delta\eta}\frac{2(1-\nu)}{1-2\nu}\left\{\eta'_y\frac{\partial V}{\partial \eta}+\xi'_y\frac{\partial V}{\partial \xi}\right\}_k^{i,N_\eta-2}+$$

$$\left(\eta'_y\right)^i\frac{1}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta'_x\frac{\partial U}{\partial \eta}+\xi'_x\frac{\partial U}{\partial \xi}\right\}_k^{i,N_\eta-2}+\left(\eta'_y\right)^i\frac{1}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\left\{\xi'_z\frac{\partial W}{\partial \xi}\right\}_k^{i,N_\eta-2}-$$

$$\left(\eta'_y\right)^i\frac{4}{2\,\Delta\eta}\frac{2(1-\nu)}{1-2\nu}\left\{\eta'_y\frac{\partial V}{\partial \eta}+\xi'_y\frac{\partial V}{\partial \xi}\right\}_k^{i,N_\eta-1}-\left(\eta'_y\right)^i\frac{4}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\left\{\frac{\partial U}{\partial x}+\eta'_x\frac{\partial U}{\partial \eta}+\xi'_x\frac{\partial U}{\partial \xi}\right\}_k^{i,N_\eta-1}-$$

$$\left(\eta'_y\right)^i \frac{4}{2\,\Delta\eta}\frac{2\nu}{1-2\nu}\left\{\xi'_z\frac{\partial W}{\partial \xi}\right\}_k^{i,N_\eta-1} + \left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta}\left\{\eta'_y\frac{\partial U}{\partial \eta} + \xi'_y\frac{\partial U}{\partial \xi} + \frac{\partial V}{\partial x} + \eta'_x\frac{\partial V}{\partial \eta} + \xi'_x\frac{\partial V}{\partial \xi}\right\}_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i +$$

$$\left\{\xi'_y\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial U}{\partial \eta}\right) + \xi'_y\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial U}{\partial \xi}\right) + \frac{\partial^2 V}{\partial \xi \partial x} + \xi'_y\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial V}{\partial \eta}\right) + \xi'_y\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i +$$

$$\left\{\xi'_z\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial V}{\partial \xi}\right) + \xi'_z\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial W}{\partial \eta}\right) + \xi'_z\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,N_\eta} \approx -\frac{2(1+\nu)}{E}\cdot\left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta} f_y^2(\xi) - \frac{2(1+\nu)}{E}\cdot$$

$$\left(\xi'_y\right)_k^{i,1}\frac{df_y^2}{d\xi} + \frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 V}{\partial t^2}\right)_k^{i,1}; \qquad (58)$$

c) **the third equation at the ice-shelf lateral edge $y = y_2(x)$** (**lines 5708-6286 in the program code**) is

$$\left\{\frac{\partial}{\partial x}\left(\xi'_z\frac{\partial U}{\partial \xi}\right) + \frac{\partial^2 W}{\partial x^2} + \frac{\partial}{\partial x}\left(\eta'_x\frac{\partial W}{\partial \eta}\right) + \frac{\partial}{\partial x}\left(\xi'_x\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,N_\eta} + \left\{\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_z\frac{\partial U}{\partial \xi}\right) + \eta'_x\frac{\partial^2 W}{\partial \eta \partial x} + \eta'_x\frac{\partial}{\partial \eta}\left(\eta'_x\frac{\partial W}{\partial \eta}\right) +$$

$$\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_x\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,N_\eta} + \left\{\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial U}{\partial \xi}\right) + \xi'_x\frac{\partial^2 W}{\partial \xi \partial x} + \xi'_x\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial W}{\partial \eta}\right) + \xi'_x\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,N_\eta} +$$

$$\left(\eta'_y\right)^i \frac{1}{2\,\Delta\eta}\left\{\xi'_z\frac{\partial V}{\partial \xi} + \eta'_y\frac{\partial W}{\partial \eta} + \xi'_y\frac{\partial W}{\partial \xi}\right\}_k^{i,N_\eta-2} - \left(\eta'_y\right)^i \frac{4}{2\,\Delta\eta}\left\{\xi'_z\frac{\partial V}{\partial \xi} + \eta'_y\frac{\partial W}{\partial \eta} + \xi'_y\frac{\partial W}{\partial \xi}\right\}_k^{i,N_\eta-1} +$$

$$\left(\eta'_y\right)^i \frac{3}{2\,\Delta\eta}\left\{\xi'_z\frac{\partial U}{\partial \xi} + \frac{\partial W}{\partial x} + \eta'_x\frac{\partial W}{\partial \eta} + \xi'_x\frac{\partial W}{\partial \xi}\right\}_k^{i,N_\eta}\left(\frac{dy_2}{dx}\right)^i + \left\{\xi'_y\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial U}{\partial \xi}\right) + \xi'_y\frac{\partial^2 W}{\partial \xi \partial x} +$$

$$\xi'_y\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial W}{\partial \eta}\right) + \xi'_y\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,N_\eta}\cdot\left(\frac{dy_2}{dx}\right)^i + \frac{2(1-\nu)}{1-2\nu}\left\{\xi'_z\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,N_\eta} + \frac{2\nu}{1-2\nu}\left\{\xi'_z\frac{\partial^2 U}{\partial \xi \partial x} +$$

$$\xi'_z\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial U}{\partial \eta}\right) + \xi'_z\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial U}{\partial \xi}\right)\right\}_k^{i,N_\eta} + \frac{2\nu}{1-2\nu}\left\{\xi'_z\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial V}{\partial \eta}\right) + \xi'_z\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,N_\eta} \approx \frac{2(1+\nu)}{E}\cdot\rho g +$$

$$\frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 W}{\partial t^2}\right)_k^{i,N_\eta}; \qquad (59)$$

<u>Appendix C</u>: Equations (5) in x, η, ξ variables

In x, η, ξ variables equations (5) are expressed as

a) **the first equation** (**lines 17121-17573 in the program code**) is

$$\frac{2(1-\nu)}{1-2\nu}\left\{\frac{\partial^2 U}{\partial x^2}+\frac{\partial}{\partial x}\left(\eta'_x\frac{\partial U}{\partial \eta}\right)+\frac{\partial}{\partial x}\left(\xi'_x\frac{\partial U}{\partial \xi}\right)+\eta'_x\frac{\partial^2 U}{\partial \eta\partial x}+\eta'_x\frac{\partial}{\partial \eta}\left(\eta'_x\frac{\partial U}{\partial \eta}\right)+\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_x\frac{\partial U}{\partial \xi}\right)+\xi'_x\frac{\partial^2 U}{\partial \xi\partial x}+\right.$$

$$\left.\xi'_x\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial U}{\partial \eta}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial U}{\partial \xi}\right)\right\}_k^{i.j}+\frac{2\nu}{1-2\nu}\left\{\frac{\partial}{\partial x}\left(\eta'_y\frac{\partial V}{\partial \eta}\right)+\frac{\partial}{\partial x}\left(\xi'_y\frac{\partial V}{\partial \xi}\right)+\eta'_x\frac{\partial}{\partial \eta}\left(\eta'_y\frac{\partial V}{\partial \eta}\right)+\right.$$

$$\left.\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_y\frac{\partial V}{\partial \xi}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial V}{\partial \eta}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,j}+\frac{2\nu}{1-2\nu}\left\{\frac{\partial}{\partial x}\left(\xi'_z\frac{\partial W}{\partial \xi}\right)+\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_z\frac{\partial W}{\partial \xi}\right)+\right.$$

$$\left.\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,j}+\left\{\eta'_y\frac{\partial}{\partial \eta}\left(\eta'_y\frac{\partial U}{\partial \eta}\right)+\eta'_y\frac{\partial}{\partial \eta}\left(\xi'_y\frac{\partial U}{\partial \xi}\right)+\eta'_y\frac{\partial^2 V}{\partial \eta\partial x}+\eta'_y\frac{\partial}{\partial \eta}\left(\eta'_x\frac{\partial V}{\partial \eta}\right)+\eta'_y\frac{\partial}{\partial \eta}\left(\xi'_x\frac{\partial V}{\partial \xi}\right)+\right.$$

$$\left.\xi'_y\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial U}{\partial \eta}\right)+\xi'_y\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial U}{\partial \xi}\right)+\xi'_y\frac{\partial^2 V}{\partial \xi\partial x}+\xi'_y\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial V}{\partial \eta}\right)+\xi'_y\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,j}+\left\{\xi'_z\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial U}{\partial \xi}\right)+\right.$$

$$\left.\xi'_z\frac{\partial^2 W}{\partial \xi\partial x}+\xi'_z\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial W}{\partial \eta}\right)+\xi'_z\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,j}=\frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 U}{\partial t^2}\right)_k^{i,j}; \tag{60}$$

b) **the second equation** (**lines 17579-17980 in the program code**) is

$$\left\{\frac{\partial}{\partial x}\left(\eta'_y\frac{\partial U}{\partial \eta}\right)+\frac{\partial}{\partial x}\left(\xi'_y\frac{\partial U}{\partial \xi}\right)+\frac{\partial^2 V}{\partial x^2}+\frac{\partial}{\partial x}\left(\eta'_x\frac{\partial V}{\partial \eta}\right)+\frac{\partial}{\partial x}\left(\xi'_x\frac{\partial V}{\partial \xi}\right)+\eta'_x\frac{\partial}{\partial \eta}\left(\eta'_y\frac{\partial U}{\partial \eta}\right)+\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_y\frac{\partial U}{\partial \xi}\right)+\right.$$

$$\eta'_x\frac{\partial^2 V}{\partial \eta\partial x}+\eta'_x\frac{\partial}{\partial \eta}\left(\eta'_x\frac{\partial V}{\partial \eta}\right)+\eta'_x\frac{\partial}{\partial \eta}\left(\xi'_x\frac{\partial V}{\partial \xi}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial U}{\partial \eta}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial U}{\partial \xi}\right)+\xi'_x\frac{\partial^2 V}{\partial \xi\partial x}+$$

$$\left.\xi'_x\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial V}{\partial \eta}\right)+\xi'_x\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,j}+\frac{2(1-\nu)}{1-2\nu}\left\{\eta'_y\frac{\partial}{\partial \eta}\left(\eta'_y\frac{\partial V}{\partial \eta}\right)+\eta'_y\frac{\partial}{\partial \eta}\left(\xi'_y\frac{\partial V}{\partial \xi}\right)+\xi'_y\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial V}{\partial \eta}\right)+\right.$$

$$\left.\xi'_y\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,j}+\frac{2\nu}{1-2\nu}\left\{\eta'_y\frac{\partial^2 U}{\partial \eta\partial x}+\eta'_y\frac{\partial}{\partial \eta}\left(\eta'_x\frac{\partial U}{\partial \eta}\right)+\eta'_y\frac{\partial}{\partial \eta}\left(\xi'_x\frac{\partial U}{\partial \xi}\right)+\ \xi'_y\frac{\partial^2 U}{\partial \xi\partial x}+\xi'_y\frac{\partial}{\partial \xi}\left(\eta'_x\frac{\partial U}{\partial \eta}\right)+\right.$$

$$\left.\xi'_y\frac{\partial}{\partial \xi}\left(\xi'_x\frac{\partial U}{\partial \xi}\right)\right\}_k^{i,j}+\frac{2\nu}{1-2\nu}\left\{\eta'_y\frac{\partial}{\partial \eta}\left(\xi'_z\frac{\partial W}{\partial \xi}\right)+\xi'_y\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,j}+\left\{\xi'_z\frac{\partial}{\partial \xi}\left(\xi'_z\frac{\partial V}{\partial \xi}\right)+\xi'_z\frac{\partial}{\partial \xi}\left(\eta'_y\frac{\partial W}{\partial \eta}\right)+\right.$$

$$\left.\xi'_z\frac{\partial}{\partial \xi}\left(\xi'_y\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,j}=\frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 V}{\partial t^2}\right)_k^{i,j}; \tag{61}$$

c) **the third equation** (**lines 17985-18313 in the program code**) is

$$\left\{\frac{\partial}{\partial x}\left(\xi_z'\frac{\partial U}{\partial \xi}\right)+\frac{\partial^2 W}{\partial x^2}+\frac{\partial}{\partial x}\left(\eta_x'\frac{\partial W}{\partial \eta}\right)+\frac{\partial}{\partial x}\left(\xi_x'\frac{\partial W}{\partial \xi}\right)+\eta_x'\frac{\partial}{\partial \eta}\left(\xi_z'\frac{\partial U}{\partial \xi}\right)+\eta_x'\frac{\partial^2 W}{\partial \eta\partial x}+\eta_x'\frac{\partial}{\partial \eta}\left(\eta_x'\frac{\partial W}{\partial \eta}\right)+\right.$$

$$\left.\eta_x'\frac{\partial}{\partial \eta}\left(\xi_x'\frac{\partial W}{\partial \xi}\right)+\xi_x'\frac{\partial}{\partial \xi}\left(\xi_z'\frac{\partial U}{\partial \xi}\right)+\xi_x'\frac{\partial^2 W}{\partial \xi\partial x}+\xi_x'\frac{\partial}{\partial \xi}\left(\eta_x'\frac{\partial W}{\partial \eta}\right)+\xi_x'\frac{\partial}{\partial \xi}\left(\xi_x'\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,j}+\left\{\eta_y'\frac{\partial}{\partial \eta}\left(\xi_z'\frac{\partial V}{\partial \xi}\right)+\right.$$

$$\left.\eta_y'\frac{\partial}{\partial \eta}\left(\eta_y'\frac{\partial W}{\partial \eta}\right)+\eta_y'\frac{\partial}{\partial \eta}\left(\xi_y'\frac{\partial W}{\partial \xi}\right)+\quad\xi_y'\frac{\partial}{\partial \xi}\left(\xi_z'\frac{\partial V}{\partial \xi}\right)+\xi_y'\frac{\partial}{\partial \xi}\left(\eta_y'\frac{\partial W}{\partial \eta}\right)+\xi_y'\frac{\partial}{\partial \xi}\left(\xi_y'\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,j}+$$

$$\frac{2(1-\nu)}{1-2\nu}\left\{\xi_z'\frac{\partial}{\partial \xi}\left(\xi_z'\frac{\partial W}{\partial \xi}\right)\right\}_k^{i,j}+\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial^2 U}{\partial \xi\partial x}+\xi_z'\frac{\partial}{\partial \xi}\left(\eta_x'\frac{\partial U}{\partial \eta}\right)+\xi_z'\frac{\partial}{\partial \xi}\left(\xi_x'\frac{\partial U}{\partial \xi}\right)\right\}_k^{i,j}+$$

$$\frac{2\nu}{1-2\nu}\left\{\xi_z'\frac{\partial}{\partial \xi}\left(\eta_y'\frac{\partial V}{\partial \eta}\right)+\xi_z'\frac{\partial}{\partial \xi}\left(\xi_y'\frac{\partial V}{\partial \xi}\right)\right\}_k^{i,j}=\frac{2(1+\nu)}{E}\cdot\rho g+\frac{2(1+\nu)}{E}\cdot\rho\left(\frac{\partial^2 W}{\partial t^2}\right)_k^{i,j}; \qquad (62)$$

References

1. Lamb, H.: Hydrodynamics. (6th ed.). Cambridge: Cambridge University Press, 1994.
2. Landau, L.D., E.M. Lifshitz: Theory of Elasticity. (3rd ed.). Oxford: Butterworth-Heinemann, (Vol. 7), 1986.
3. Holdsworth, G. & J. Glynn: Iceberg calving from floating glaciers by a vibrating mechanism. Nature, 274, 464-466, 1978.
4. Lurie, A.I.: Theory of Elasticity. Berlin: Springer, (Foundations of Engineering Mechanics), 2005.
5. Tikhonov, A.N., Samarskii, A.A.: Equations of Mathematical Physics. Pergamon Press Ltd., USA, 1963
6. Konovalov Y.V. (2019) Ice-shelf vibrations modeled by a full 3-D elastic model. *Annals of Glaciology,* 60(79), 68-74. doi:10.1017/aog.2019.9 (http://dx.doi.org/10.1017/aog.2019.9)
7. Ashcroft, N.W., & Mermin, N.D.: Solid state physics. Books Cole, Belmont, CA, 1976
8. Balmforth, N.J., & Craster, R.V.: Ocean waves and ice sheets. J. Fluid Mech., 395, 89-124, doi: 10.1017/S0022112099005145, 1999
9. Bassis, J.N., Fricker, H.A., Coleman, R., Minster, J.-B.: An investigation into the forces that drive ice-shelf rift propagation on the Amery Ice Shelf, East Antarctica. J. Glaciol., 54 (184), 17-27, doi: 10.3189/002214308784409116, 2008
10. Bennetts, L.G., Biggs, N.R.T., Porter, D.: The interaction of flexural-gravity waves with periodic geometries. Wave Motion, 46 (1), 57-73, doi.org/10.1016/j.wavemoti.2008.08.002, 2009

11. Bromirski, P.D., Sergienko, O.V., MacAyeal, D.R.: Transoceanic infragravity waves impacting Antarctic ice shelves. Geophys. Res. Lett., 37, L02502. doi:10.1029/2009GL041488, 2009
12. Bromirski, P. D., Diez, A., Gerstoft, P., Stephen, R. A., Bolmer, T., Wiens, D. A., Aster, R. C., and Nyblade, A.: Ross ice shelf vibrations. Geophys. Res. Lett., 42, 7589–7597, doi:10.1002/2015GL065284, 2015
13. Gerstoft P., Bromirski P., Chen Z., Stephen R.A, Aster R.C., Wiens D.A., Nyblade, A.: Tsunami excitation of the Ross Ice Shelf, Antarctica. The Journal of the Acoustical Society of America, 141(5), 3526, doi:10.1121/1.4987434, 2017
14. Chen, Z., Bromirski, P., Gerstoft, P., Stephen, R., Wiens, D., Aster, R., & Nyblade, A.: Ocean-excited plate waves in the Ross and Pine Island Glacier ice shelves. J. Glaciol., 64(247), 730-744, doi:10.1017/jog.2018.66, 2018
15. Chou, T.: Band structure of surface flexural-gravity waves along periodic interfaces. J. Fluid Mech., 369, 333-350, 1998.
16. Goodman, D.J., Wadhams, P., & Squire, V.A.: The flexural response of a tabular ice island to ocean swell. Ann. Glaciol., 1, 23–27, 1980
17. Freed-Brown, J., Amundson J., MacAyeal, D., & Zhang, W.: Blocking a wave: Frequency band gaps in ice shelves with periodic crevasses. Ann. Glaciol., 53(60), 85-89, doi: 10.3189/2012A0G60A120, 2012
18. Holdsworth, G.: Tidal interaction with ice shelves. Ann. Geophys., 33, 133-146, 1977
19. Holdsworth, G., & Glynn, J.: Iceberg calving from floating glaciers by a vibrating mechanism. Nature, 274, 464-466, 1978
20. Hughes, T. J.: West Antarctic ice streams. Reviews of Geophysics and Space Physics, 15(1), 1-46, 1977
21. Lingle, C. S., Hughes, T. J., Kollmeyer, R. C.: Tidal flexure of Jakobshavns Glacier, West Greenland. J. Geophys. Res., 86(B5), 3960-3968, 1981

22. MacAyeal, D.R., Okal, E.A., Aster, R.C., Bassis, J.N., Brunt, K.M., Cathles, L.M., Drucker, R., Fricker, H.A., Kim, Y.-J., Martin, S., Okal, M.H., Sergienko, O.V., Sponsler, M.P., & Thom, J.E.: Transoceanic wave propagation links iceberg calving margins of Antarctica with storms in tropics and Northern Hemisphere. Geophys. Res. Lett., 33, L17502. doi:10.1029/2006GL027235, 2006
23. MacAyeal, D., Sergienko, O., Banwell, A.: A model of viscoelastic ice-shelf flexure. J. Glaciol., 61(228), 635-645, doi:10.3189/2015JoG14J169, 2015
24. Mei, C.C.: Resonant reflection of surface water waves by periodic sandbars. J. Fluid Mech., 152, 315-335, doi: S0022112085000714, 1985
25. Meylan, M., Squire, V.A., & Fox, C.: Towards realism in modelling ocean wave behavior in marginal ice zones. J. Geophys. Res., 102(C10), 22981–22991, 1997
26. Reeh, N., Christensen, E.L., Mayer, C., Olesen, O.B.: Tidal bending of glaciers: a linear viscoelastic approach. Ann. Glaciol., 37, 83–89, 2003
27. Robin, G. de Q.: Seismic shooting and related investigations. In Norwegian-British-Swedish Antarctic Expedition, Sci. Results 5, Glaciology 3, Norsk Poarinstitutt (pp. 1949-1952). Oslo: University Press, 1958
28. Rosier, S.H.R., Gudmundsson, G.H., & Green, J.A.M.: Insights into ice stream dynamics through modeling their response to tidal forcing. The Cryosphere, 8, 1763–1775, 2014
29. Scambos, T.A., Hulbe, C., Fahnestock, M., Bohlander, J.: The link between climate warming and breack-up of ice shelves in the Atlantic Peninsula. J. Glaciol., 46(154), 516-530, doi:10.3189/172756500781833043, 2000
30. Shearman, E.D.R.: Radio science and oceanography, Radio Sci., 18(3), 299–320, doi:10.1029/RS018i003p00299, 1983
31. Sheng, P.: Introduction to wave scattering, localization and mesoscopic phenomena. Springer, Berlin, 2006

32. Schmeltz, M., Rignot, E., & MacAyeal, D.R.: Tidal flexure along ice-sheets margins: Comparison of InSAR with an elastic plate model. Ann. Glaciol., 34, 202-208, 2001
33. Schulson, E.M.: The Structure and Mechanical Behavior of Ice. JOM, 51 (2), 21-27, 1999
34. Sergienko, O.V.: Elastic response of floating glacier ice to impact of long-period ocean waves. J. Geophys. Res., 115, F04028. doi:10.1029/2010JF001721, 2010
35. Smith, A.M.: The use of tiltmeters to study the dynamics of Antarctic ice shelf grounding lines. J. Glaciol., 37, 51–58, 1991
36. Squire, V.A., Dugan, J.P., Wadhams, P., Rottier, P.J., & Liu, A.K.: Of ocean waves and sea ice. Annu. Rev. Fluid Mech., 27, 115–168, 1995
37. Stephenson, S.N.: Glacier flexure and the position of grounding lines: measurements by tiltmeter on Rutford Ice Stream, Antarctica. Ann. Glaciol., 5, 165-169, 1984
38. Turcotte, D.L., Schubert, G.: Geodynamics. (3rd ed.). Cambridge: Cambridge University Press, 2002
39. Van der Veen C.J.: Fracture mechanics approach to penetration of bottom crevasses on glaciers. Cold Reg. Sci. Technol., 27(3), 213-223, 1998
40. Vaughan, D.G.: Tidal flexure at ice shelf margins. J. Geophys. Res., 100(B4), 6213–6224, doi:10.1029/94JB02467, 1995
41. Wadhams, P.: The seasonal ice zone. In Untersteiner, N. (Ed.), Geophysics of sea ice (pp. 825–991), London: Plenum Press, 1986
42. Walker, R.T., Parizek, B.R., Alley, R.B., Anandakrishnan, S., Riverman, K.L., Christianson, K.: Ice-shelf tidal flexure and subglacial pressure variations. Earth and Planetary Science Letters, 361, 422–428, doi: 10.1016/j.epsl.2012.11.008, 2013