Corona, mathematics, and the limits of democratic control

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Abstract

Maturity and citizenship in a democracy require that laypersons are able to critically reflect on experts' use of mathematics. Learning to critically reflect on the use of mathematics, including the learning of mathematics required to that end, has been repeatedly postulated as an indispensable goal of compulsory education in mathematics. However, it remained unclear in how far control by reflection is possible, even for the well-educated layperson in mathematics. We use different discourses on the SARS-CoV-2 crisis in 2020 as examples of discourses with far-reaching individual and social consequences. The selected discourses build heavily on mathematical concepts such as mortality rates, casualty numbers, reproduction numbers and exponential growth. We identify the concepts and discuss how far they can be understood by laypersons. On the one hand, we found that some mathematical models are inappropriate, which can also be determined by laypersons. On the other hand, we found uses of mathematics where ideal concepts are intermingled with statistical concepts in the public discourse. While only the ideal concepts can be understood by laypersons, only the statistical concepts lead to actual data. The identification of both types of concepts leads to a situation where the use of mathematics evades social control and opens spaces for misconceptions and manipulation. We conclude that the control of experts' use of mathematics by laypersons is not possible in all relevant cases, even if they had enjoyed critical mathematics education.

Keywords

Critical reflection, democratic control, expert-layperson-communication, coronavirus.

1 Mathematics in the crisis

In 2020, the world experienced a globalcrisis as many countries invoked extreme political measures in reaction to the pandemic spread of diseases associated to the "severe acute respiratory syndrome coronavirus 2" (SARS-CoV-2), which is publicly also referred to as "the

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coronavirus", especially of the coronavirus disease 2019 (COVID-19). The shock caused by media reports of overburdened medical facilities and human casualties associated to the virus as well as warnings by virologists, epidemiologists and other experts led a majority of countries worldwide to react forcefully with a social lockdown including the closing of national borders and the suspension of immutable human rights such as the right to freedom of movement, the right to work, the right to freedom of assembly, and the right to asylum. Executive measures were met with debates of their adequacy, while the official legitimisation of the measures mainly rested on scientific analyses.

As such analyses often relied heavily on mathematics, the understanding of mathematics had assumed a central role in governing the crisis. As controversies, some of which we will present later, show, mathematical understanding was often limited and caused misunderstanding or confusion among politicians, journalists and the wider public. But instead of arguing for a more extensive investment in the mathematical education of the general public or specific social agents, we critically address the question in how far mathematical laypersons can be prepared for such understanding at all. This discussion will relate to theories in mathematics education that propose that preparing for critical communication between experts and laypersons should be a central goal in secondary education.

As we were composing the first draft of this article in May 2020, the numbers of people tested positively for SARS-CoV-2 was already decreasing in Europe and executive measures were slowly cancelled. However, some scholars and politicians warned that revoking the lockdown too fast might result in a second wave of the pandemic spread of COVID-19.

2 Mathematics, education and communication

2.1 Mathematics and understanding

When addressing mathematics and understanding, mathematics education research usually discusses questions of understanding mathematics. In the course of the current global crisis, it might be straightforward to ask for the mathematical knowledge and skills that should be understood in order to understand and react to the crisis. The underlying assumption is that we need to understand mathematics so that mathematics can help us understand the crisis. Here, the social relevance of mathematics is derived from its role as a tool for understanding. The relationship between reality and mathematics is often conceptualised through a positivist paradigm that assumes that reality exists independent of human understanding and that human understanding resembles that reality. Hacking (2014) discussed different forms of essentialist philosophies of mathematics. These philosophies postulate that reality is inherently mathematical and that mathematics is the obvious way to understand our world. On the basis of such assumptions, mathematics appears to be objective, eternal, infallible and value-free. A positivist understanding of reality and mathematics is also widespread in mathematics education research. For example, in research on mathematical modelling in education, the modelling circle assumes that reality is mathematised in a one-directional process in which the mathematics used is defined by reality but not vice versa (Kaiser, 2014).

Positivist philosophies of mathematics have been attacked mainly because they cannot explain how we come to know the mathematics that is supposed to lie beyond human existence, because they lead to philosophical contradictions, and because its assumptions have problematic political effects for the use of mathematics. The modern crisis in the foundations of mathematics has made it clear that mathematical knowledge cannot be totally safe from human error and that objectivity, therefore, is not absolute but rather an ideal that mathematicians aspire to (Davis & Hersh, 1980). Applications of mathematics in ever more arenas of social life have furthermore documented that mathematics is not the only and most obvious form of describing reality but a very specific perspective with certain strengths and clear epistemological limitations (Davis & Hersh, 1986). Above that, applications of mathematics have proved to be anything but a mere resemblance of reality, they are actually participating in the formation of reality. The most famous example for such a formation may be Galilei's law of gravity, which failed validation in every experiment conducted under Galilei's conditions but made a nice and easy mathematical formula (François & Sutter, 2005). In *How the Laws of Physics Lie*, Cartwright (1983) discussed more generally how theory-building in modern physics trusts in easy mathematical solutions but often does not work out in practice. Most recently, the argument has been promoted again in Lindley's (2020) *The Dream Universe: How Fundamental Physics Lost Its Way*. This use of mathematics proved not to be restricted to physics but to potentially apply to all areas where mathematics is used (Porter, 1996; Skovsmose, 1994). In ever more areas of understanding, up to the computation of love, mathematics is formatting our understanding of reality while non-mathematical forms of understanding are devalued. This means that when we are attempting to describe reality with mathematics, we are always-already constituting reality in specifically biased and alterable ways.

A way out of this philosophical mess is to understand mathematics as a social process (Ernest, 1998). From this perspective, mathematicians negotiate the rules of their practice in a never finished process. They provide abstract theories, whose strength lies in their wide applicability and in the fact that they allow for computational use and for the wide avoidance of dissent (Fischer, 2006). Through application, mathematics co-determines how the problems are conceptualised. The creation and application of different mathematics or the decision that no mathematics should be used for a specific problem, can have severe impacts on how we understand the problems in our world. In this sense, mathematical models of problems in reality are productive in constituting these problems and reality in particular ways. As different understandings of problems in the social realm quickly lead to different policies from which specific people benefit in different ways than others, mathematics becomes political. Mathematics education can then no longer reduce itself to the production of competences for the positivistic description of reality. Instead, it has to prepare students to critically reflect on the uses of mathematics. We will shortly present two educational philosophies in mathematics education that attempt to theorise this position.

2.2 Critical reflection as an educational task

The insight that mathematics is not a one-to-one representation of reality but allows many different perspectives which each can serve distinct interests raised the question how mathematics education should address this problem. Would it be ethical to introduce students to mathematics without teaching them to critically question uses of mathematics? If not, how could such critical mathematics education be organised? Without claiming to cover all approaches that have been presented in this direction, we will restrict our further discussion to the work of Skovsmose, who worked in the tradition of critical mathematics education, and Fischer, who worked on educational theory in the German-speaking community.

Fischer (2001) worked out an educational theory in which he stressed that, in a society with a highly specialised workforce, people as laypersons will have to engage with experts. This happens in the private sphere, for example when comparing different offers for a loan or when seeking medical counsel, and also in the public sphere, especially when citizens evaluate decisions which were reached by politicians. This asymmetry of knowledge means that laypersons have to trust and to control experts. Here, we will focus on control, which the Cambridge Dictionary understands as the act of ordering, limiting, or ruling something, or someone's actions ("Control", 2020). For Fischer, higher secondary education, especially in mathematics, should prepare for the critical communication of the student with such experts:

> One will usually rely on the subject-specific *correctness* of the expertise, on the fact that the expertise is up to date and that in this respect mutual control between the experts of a discipline works. In the question of *significance*, i.e. how important one considers a certain expert judgement to be, how one weights it, one is dependent on one's own judgement. In the end, you have to judge experts, even though you understand less than they do. (p. 152, original emphasis, our translation)

Fischer (2001) differentiated between three fields of knowledge in a discipline such as mathematics: *basic knowledge* about concepts and notations, *operative skills* in typical procedures, and *reflective knowledge* about the meaning, the potentials and the limits of the concepts and procedures. Laypersons, and that includes students in higher secondary education, should acquire operative skills only as far as they are needed for reflective knowledge. Especially for mathematics education, Fischer demanded that teaching switched its focus from learning mathematical skills to learning to reflect on mathematics.

Skovsmose (1985) reflected on the use of mathematics in modern societies and argued that some kind of Critical Mathematics Education was necessary if mathematics education was not to be reduced to "socializing students into the technological society and at the same time destroying the possibilities of developing a critical attitude towards precisely this technological society" (p. 338). Anticipating Fischer's call for reflection, he already proposed to address the applicability of mathematics in class, together with the interests behind it, assumptions it rests on, its functions and its limitations.

Some years later, Skovsmose (1994) also addressed the problem that arises for democratic participation "if a knowledge elite develops the conditions and the arguments for the decisions to be taken by the politicians" (p. 39). He warned "that the ground for decisions taken by the authorities may be inaccessible to people other than the technicians and the people in charge" (ibid.) and asked if it was "possible to secure a critical citizenship in a highly technological society" (p. 40). It might be added that even politicians would usually and especially in the current crises be laypersons and no knowledge elite at all. Skovsmose (1994) then turns to the idea of reflective knowing as a solution to the problem, whereat he differentiates between technological and reflective knowledge in a similar if yet less differentiated manner as Fischer.

Eventually, Skovsmose (2005) referred to the philosophy of mathematics to argue that positivist images of mathematics create the illusion that mathematical models depict how things really are and that there would be nothing to call into question. Only on the basis of an understanding of mathematics as a social process would it be possible to acknowledge the uncertainty that comes with mathematics and to take the responsibility of reflecting mathematics critically.

2.3 Potentials and limits of critical reflection

Both Fischer and Skovsmose see education in and critical reflection of mathematics as an adequate and possible solution for the private evaluation and democratic control of decisions based on mathematical expertise. But, although there are several projects illustrating such reflection in mathematics education (e.g., by Skovsmose, 1994), it is unclear in how far students can be prepared to reflect on all or at least a majority of socially relevant applications of mathematics. Obviously, there are boundaries set by the mathematical contents covered in school education and by the complexity of the mathematical theories involved. A possible answer to this objection might be to assume that people educated in various mathematical areas, including their critical reflection, are able to transfer their skills in learning and reflecting mathematics to new mathematical topics, even to complex ones.

In this article, we want to use selected discourses in the current crisis to put the idea of control through critical reflection under a reality check. Our main question is *whether it will be possible to evaluate the different discourses on the corona crisis from a mathematical perspective as a mathematical layperson*. More specifically, we are asking *if typical school contents serve as a good basis for reflection of these discourses*, *if otherwise the basis for reflection can realistically be built up*, *if positivistic images of mathematics stand in the way of reflection and control* and *if political problems based on a lack of reflection can be identified*.

The methodology of our study might be understood as an abridged discourse analysis (Jørgensen & Phillips, 2010), in which each of the discourses we selected is organised around a mathematical concept. We will show that these mathematical concepts have influenced the public communication of the crisis and possibly also political decision-making. We will then attempt to understand the mathematics involved in these concepts in order to later evaluate in how far laypersons can understand the mathematical basis of the concepts in question. We called our discourse analysis "abridged" as we have neither the intent, nor the space to present a full analysis of the discourses we address. Instead, we will as far as possible constrain our remarks on mathematical aspects, and will we comment on non-mathematical aspects only as far as necessary for the understanding of the mathematical concepts and their use.

3 Attempts to understand the crisis

3.1 How dangerous is SARS-CoV-2?

The amount to which we are personally worried and to which political stakeholders see a need for special executive measures depends on the perception of the dangerousness of the virus. Initially, the restrictions on public life were not legitimised on the basis of the number of COVID-19 casualties but on the basis of the number of people who tested positive. However, this is a problematic number, for the more people we test, the more infected we find. There are various viruses whose rate of infection of the population is one hundred percent. So, the infection rate alone is not the problem.

The key question is: Is it really problematic if a lot of people are infected with SARS-CoV-2? A high number of infected people is problematic when it results in serious illness, in an overload of the medical infrastructure and in human casualties. So, it is crucial to ask for the rate of infected people who need serious medical attention and for the mortality rate. We discuss these questions here only on the basis of the mortality rate, which is the proportion of people who died from COVID-19 among those who were infected with SARS-CoV-2. If we knew this rate, it would be possible to determine the expected institutional stress and the expected casualties on the basis of the number of infections and vice versa. So far, we only needed mathematics from early secondary school, which leaves the impression of facing an easy mathematical problem, but different than in the classroom, the numerator and denominator that constitute our ratio are not given.

According to an early report by the World Health Organisation (2020), 3.5% of those infected in China had died during the outbreak there. The mortality rate was 5.8% in Wuhan, the city in China where the virus was first identified, but only 0.7% in the rest of China (p. 12). At that time, it was already clear that the high rate in Wuhan was associated to low test capacities which resulted in testing to be confined to severe cases. Nevertheless, the figure of 3.5% became the focus of public debate. For example, the *Süddeutsche Zeitung*, one of the largest daily newspapers in Germany, printed a warning of the high mortality with an explicit reference to the 3.5% in China (Endt, Hosse, Mainka, & Witzenberger, 2020). Also *Der Spiegel*, Europe's largest weekly news magazine, explained that "the authorities estimate the mortality rate at around two to four percent" (Dandan et al., 2020, p. 16, our translation). On the basis of such figures, Karl Lauterbach, Member of the German parliament, health economist and the most prominent health expert of the Social Democrats, stated that "more than one million people could die in Germany alone" (Hammerstein & Feldenkirchen, 2020, p. 42, our translation).

The general problem here is that two widely unrelated concepts are mixed up in public communication. On the one hand, there is the positivistic assumption that a fixed proportion of infected people will die from the virus. This proportion is called the lethality of the virus. However, we neither know how many people are infected, nor do we know how many people die from the virus, as many cases of SARS-CoV-2 infections and probably also some cases of COVID-19 casualties go unnoticed. So, although lethality is an easy and very general concept, it is nearly impossible to determine directly and can only be estimated by complex models. On the other hand, it is possible to statically determine the case fatality rate, that is the ratio of people who died from COVID-19 among those who tested positive for SARS-CoV-2. However, the case fatality rate in a certain environment depends heavily on the numbers of tests administered, the ratio of infections that are noticed or go unnoticed by the test, and on the medical capacities to prevent severe cases from dying. Consequently, the case fatality rate varies strongly between different environments and its relation to the lethality of the virus remains opaque. However, the case fatality rates of different hot spots of the crisis were the information that was easily available, and confusion with lethality seemed to have contributed to the public panic.

Around the time when major events were banned in Germany, the first studies were published which estimated the lethality of SARS-CoV-2. In these studies, rates of 0.12% to 0.5% were reported (Mizumoto, Kagaya, & Chowell, 2020; Russell et al., 2020). It was thus established that, compared to the case fatality rates reported earlier, lethality was much lower than the value initially communicated.[1]

When we look at the mathematics used in these two studies, we find that they build on disputable assumptions and that they use mathematical techniques that require advanced tertiary studies in statistics. In order to arrive at estimations of the lethality, such approaches seem to be necessary, as for a representative study that truly represents a normal population, you would need a huge number of subjects. Taking Germany as an example: In April 2020, only 0.2% of Germans tested positive and 0.005% of Germans died with COVID-19. So even with 1 million test persons, you would end up with only 50 assumed COVID-19 deaths. Truly representative studies of lethality are therefore practically impossible, at least for the time being.

3.2 The number of COVID-19 casualties

A naïve understanding of the concept of a COVID-19 casualty might be the following: Someone comes to the hospital with symptoms of a cold, tests positive for SARS-CoV-2, is treated for a few days, and then dies and counts as a COVID-19 casualty. The statistical practice, at least in Germany, indeed works like that (Schilling, Diercke, Altmann, Haas, & Buda, 2020). Also, someone who visits a hospital for a different reason and gets infected with SARS-CoV-2 there, eventually counts as a COVID-19 casualty. And it is also possible that a SARS-CoV-2 test is administered on a dead person, and even then, this person might come to count as a COVID-19 casualty.

It is a complex question in how far COVID-19 is responsible for the death of these casualties. Often, COVID-19 casualties had not only a SARS-CoV-2 infection but also underlying medical conditions, without which these people might not have died. Are these people actually casualties that have to be counted for their underlying conditions instead of for COVID-19? That's splitting hairs, of course. But at the moment when, within political decisions, the number of COVID-19 casualties decides on the freedom, health and economic existence of millions, this hair-splitting becomes a vital distinction.

More, though not total clarity can be obtained by an autopsy. In Germany there is only one federal state, Hamburg, where everyone who died with SARS-CoV-2 was examined for their cause of death. Wichmann et al. (2020) reported on the cause of death of the first twelve consecutive deaths with identified SARS-CoV-2 infections and found that all these patients had severe pre-existing conditions which in most cases were the main cause of death. In a press conference, the involved researchers reported of yet unpublished data from now 192 autopsies,

[1] This lethality is in the range of conventional influenza viruses. However, this lethality does not suffice to estimate the dangerousness of SARS-CoV-2, as it does not take into account, for example, which medical measures are required to save a patient or the speed of the spread of the virus.

which again were *all* reported to have had serious medical preconditions, sometimes without awareness of them (Betzholz, 2020). At least one of the researchers involved proposed that COVID-19 is harmless for the majority of people and only critically affects people who are already in a weakened condition. From such a perspective, the number of people who died not only *with* but *because of* COVID-19 becomes impossible to determine.

The use of different models might be a solution to the problem. A simple idea might be to count one fifth COVID-19 casualty if somebody died with five diseases that contributed to that person's death. The total number of COVID-19 casualties would then be a cumulation of many different fractions. This might give a better estimate of the danger of COVID-19. However, such a model would lead to new difficulties, for example concerning the identification of the appropriate fraction in each case or concerning the interpretation of the resulting numbers.

3.3 R

When evaluating the current trend of the epidemic, experts rely on biometric numbers. The *Berliner Morgenpost*, one of Berlin's largest daily newspapers, reported on the use of R, the reproduction number, which they presented as the number specifying "how many people a coronavirus-infected person infects on average" ("Die Bedeutung der Reproduktionszahl", 2020, our translation). The newspaper article further explained:

> The R-value provides orientation for political decisions. When virologists measure the 'fever' of the pandemic, they do so according to the R-value. 'Even if we assume that everyone contaminates 1.1 persons, we would have reached the performance limit of our health care system with the assumed intensive care beds in October', explained Federal Chancellor Angela Merkel […] in mid-April. (our translation)

As the concept of R, as defined by the newspaper cited above, only requires an understanding of the arithmetic mean, it appears as an easy concept at first sight. However, we again face serious problems determining R. Not only do we not know how many people are indeed infected. We also get the data of identified cases with a delay of some days. The Robert Koch Institute (RKI), a German governmental agency for research on infectious diseases, therefore used a complex procedure for the daily up-to-date estimation of R.

The RKI employees Heiden and Hamouda (2020) published information on the assumptions and procedures of the calculation of R. Every infection identified in Germany had to be reported to the RKI, together with the information when the first symptoms were experienced in each case. In order to use all reported cases in the further calculation, the RKI "assigned an artificial onset of disease" if no information about first symptoms was given in the reported data (p. 11, our translation). To achieve that assignment, "a so-called multiple imputation was carried out, in which the missing data values are estimated on the basis of the statistical relationships of the known data" (ibid., our translation). For further detail, the authors merely referred to Little and Rubin (2020), a textbook that provides technical explanations and relies on knowledge that is well beyond introductory lectures in statistics in tertiary education. On top of that, RKI used a procedure called "nowcasting" to estimate the number of cases that had not yet been but would be diagnosed and reported to the present day. Here, the authors referenced a biometric study published in 2014 and a publication in statistics describing a general procedure for dealing with reporting delays. As in the case of the assignment of artificial onset of disease, the concrete adaptation is not reported, neither by Heiden and Hamouda (2020) nor elsewhere. Eventually, R is computed by dividing the cumulated numbers of new infections in the last four days by the cumulated numbers of new infections in the four days before that period.

As in the case of the mortality rate, the resulting values for R face a serious problem. In the public discourse, R was expected to change with the implementation of anti-spreading measures and was seen as an indicator of the success of such measures and of a de-escalating social behaviour in general. However, as the estimation of R rests not on the number of *all infected* but on the number of the *positively tested* only, the number of tests administered impacts R heavily. If the number of infections increased and the number of tests stayed constant, then this increase would be underestimated by R. If the number of tests was increased and the number of infections stayed constant, R would rise, indicating a faster spread of the virus. Heiden and Hamouda (2020) were aware of this effect:

> This structural effect and the resulting increase in the number of reports can lead to the current R-value slightly overestimating the real events. An adjustment for the higher test rates is not possible without further ado, since no sufficiently differentiated test data is available. (p. 15)

Eventually, it was unclear in how far the communicated values for R were generally over- or underestimated, and how the interpretation of R should have changed with the increase of test capacity during the crisis.

3.4 Exponential growth

German media, experts and politicians repeatedly warned that infection numbers in Germany would rise exponentially if no special measures were taken (e.g., Heiden & Hamouda, 2020; Müller-Jung, 2020). This idea relies on a mathematical model that is taught in German secondary schools and assumes that every infected person infects a fixed number of other persons. Exponential growth was often used to legitimise warnings against the threat posed by SARS-CoV-2. For example, the German popular mathematician Beutelspacher explained that "this growth is usually so that you don't notice anything at all in the beginning and you are tempted to underestimate it all [but] once the momentum starts, it's almost unstoppable", only to add that "This is a growth that people do not understand" (Welty, 2020). But was it sound to expect that mysterious growth?

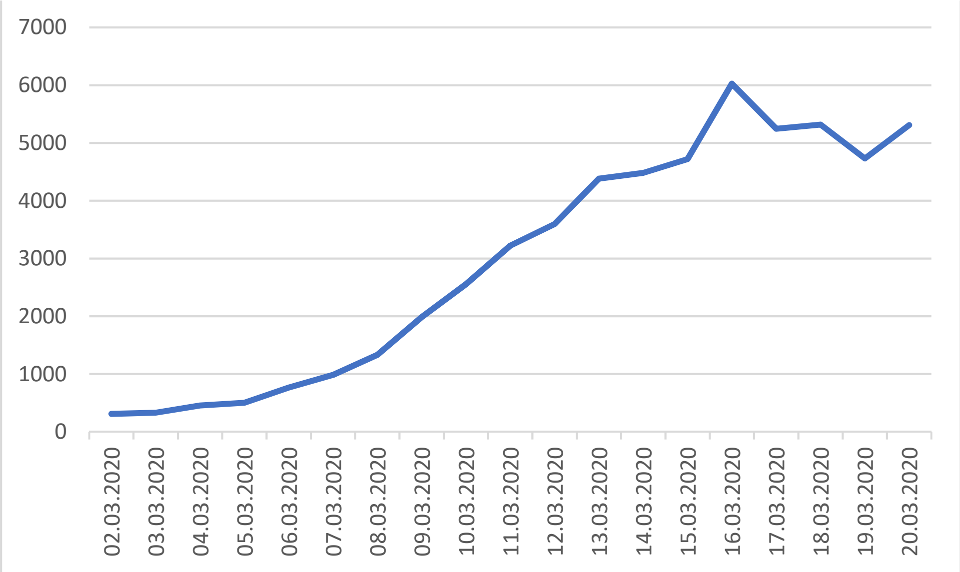

Fig. 1. Number of reported and estimated daily onsets of COVID-19 in Germany (Robert-Koch-Institut, 2020)

Figure 1 shows the number of reported and estimated daily onsets of COVID-19 for the period before the lockdown started in Germany.[2] Following the exponential-growth assumption, the number of daily onsets should have grown proportionally to the number of overall infections, which appears not to be the case. Indeed, regression analysis proposes that a linear model ($R^2 \approx .960$) is much more appropriate than an exponential model ($R^2 \approx .417$).[3]

Also mathematically, the exponential-growth assumption was problematic. One problem is that infection numbers cannot grow without limit. As the number of human beings is limited, so are infection numbers. The more people are infected with SARS-CoV-2, the more difficult it gets for an infected person to find a non-infected person to contaminate. This phenomenon of saturation can be modelled mathematically and leads to growth scenarios which are exponential only in the very beginning. Thereby, we do not even know if the saturation rate of SARS-CoV-2 would lie at 100% or lower. For example, it could be that previous infections with SARS-CoV-2 or a sufficiently similar coronavirus caused an immunity against SARS-CoV-2 among a considerable proportion of the population. Furthermore, increasing test capacities might lead to an increase in case numbers without saying anything about the increase of the number of

[2] It should be noted that from 9 March 2020 on, events with more than 1,000 participants were forbidden, and from 16 March 2020 on, further regulations were implemented. Assuming an incubation period of four days, it should be reasonable to present data up to 20 March 2020. After that date, the graph was more or less constant.

[3] The sceptical reader may check a selection of graphs from his or her environment, if necessary by a regression analysis, to convince him- and herself.

infections. Another problem is that exponential growth assumes constant doubling times while we witnessed strong fluctuations in the doubling times of reported new cases even before the lockdown. It appears that people already became more careful when they consumed media reports on the danger of COVID-19. Obviously, constant doubling times could only have been expected in a society that was totally ignorant to the growing health crises and did not change its behavioural patterns at all.

4 Reflections of our attempts

We raised the research questions whether it would be possible to evaluate the different discourses on the corona crisis from a mathematical perspective as a mathematical layperson. This included the more specific questions whether school mathematics would be sufficient for such understanding or at least a realistic basis to build up the necessary mathematical understanding, whether positivistic images of mathematics stand in the way of reflection and control, and whether political problems based on a lack of reflection can be identified.

In the four discourses on the corona crises we chose for our analysis, we faced concepts whose definitions rested on mathematics that was well within usual secondary school curricula, at least in Germany. Proportions allowed to understand the idea of lethality, the number of COVID-19 casualties appeared to be a case of mere addition, the reproduction number can be understood as the arithmetic mean of a factor, the expected growth rates could be understood on the grounds of exponential functions. Despite the fact that these mathematical models were presented in the media, they all turned out to be problematic. In the case of the exponential function, which we will address later, the model used appears to be simply misleading. In the cases of lethality, the number of COVID-19 casualties and the reproduction number, we faced ideal concepts which are easy to define and possible to understand as mathematical laypersons but utterly impractical or even impossible to actually determine. In each case, the numbers that were determined and used in the public discourse were something rather different than the ideal concepts originally laid out. In the case of lethality, we saw that our limited data yields inappropriately high rates and allows for estimations only. In the case of the reproduction number, we face the same problem and estimations are used as well. When we looked at the mathematics behind the estimations used to approximate the reproduction number, we encountered mathematics that lies beyond the contents of a usual mathematics and statistics degree course and is certainly out of reach for mathematical laypersons. The same goes for the estimations of lethality. In the case of the number of COVID-19 casualties, we came to understand that the definition, as easy as it might sound, is highly problematic as it is unclear how to identify a COVID-19 casualty.

A first result from this analysis is that initial mathematical understanding of the presented discourses is possible but misleading, as this initial understanding concerns only ideal concepts, which cannot be determined empirically. In contrast to that, a profound understanding of what is happening mathematically seems to be out of reach for mathematical laypersons, even if they were to invest private resources to increase their mathematical knowledge.

Obviously, it is problematic that discourses present ideal concepts but then use numbers whose production and meaning remains cryptic. One explanation for the use of these ideal concepts might be the will to communicate complex mathematical concepts in a way that mathematical laypersons can understand. Such a communication seems necessary and it is clear that any simplification will have to leave blind spots. Another explanation for the use of these ideal concepts might be a positivistic attitude. Experts might have the assumption that SARS-CoV-2, its lethality, its reproduction rate, its exponential spread pattern and the number of COVID-19 casualties are a part of reality irrespective of human understanding, and that it is the task of research to estimate these figures ever more accurately. Understanding mathematics as a social practice, we would rather argue that all these numbers are concepts that were made up. This might be rather obvious for the number of COVID-19 casualties but less obvious for the lethality of SARS-CoV-2 or its spread pattern.

Eventually, both explanations go hand in hand: If you share the positivistic assumption that numbers such as the virus's lethality exist in an unalterable form and that researchers are successively finding better estimations of these numbers, then it is only fair to talk about these concepts in an ideal form. However, if you agree that numbers are arbitrary inventions, then the ideal numbers discussed here are nothing but fantasies and the mathematical procedures used to actually produce numbers must be discussed critically. But, as we explained, these procedures are not open for the understanding of laypersons. In a very pointed interpretation, that might mean that laypersons are presented concepts that have little empirical meaning while experts work with different, empirical concepts, whose underlying mathematics is out of reach of democratic control.

The issue of evaded democratic control of the use of mathematics in public discourse might be less problematic if a critical analysis of the mathematics motivated no criticism. The case of exponential spread models for SARS-CoV-2 shows that such serenity would be far from appropriate. As we could see, the publicly communicated danger of an exponential increase of SARS-CoV-2 infections is neither mathematically sound nor empirically valid. From a mathematical perspective, the narrative of an exponential increase might again be understood as a simplification for the sake of public communication. Indeed, mathematical models for the spread of infectious deceases assume a spread behaviour which is almost exponential in the early phase of an epidemic. However, empirical data of the number of SARS-CoV-2 infections did not provide any reason to assume an exponential increase, as even laypersons could conclude from a view on the respective graphs.[4] We assume that we witness mathematics as a formatting power here. One the one hand, the mathematical models allowing a prediction are too complex for laypersons and of yet unverified applicability. On the other hand, the public just as most journalists, politicians, virologists, epidemiologists and other mathematical laypersons know the exponential function, which allows an easy explanation of the spread of the virus. We propose to consider whether it was the availability alone of the exponential function as a description of increasing numbers of infections that caused its use. If this was the case, then we would face a case of the formatting of reality on the basis of the mathematics available.

As we could see that the mathematical concepts used in the discourses on the current crisis are of debatable significance, would it be reasonable to refrain from using mathematics in our discussions on the crisis? We find that to be an interesting thought experiment. We assume that political decision making relies so heavily on numbers that the process of political decision making requires each problem to be quantified in order to approach it at all. Ironically, this even holds true if the concepts used for such a quantification do not make much sense. We found examples of such a paradox use of numbers in public media. One example is Müller-Jung's (2020) article in the *Frankfurter Allgemeine*, Germany's third-largest daily newspaper. Müller-Jung first stated:

> The lethality or mortality rate is also not easy to determine because a large proportion of infected persons have not yet been identified. If the mortality rate is low and the infection is mild or asymptomatic, many virus carriers do not appear at all in clinics, at doctors and thus in any statistics. (our translation)

Thereafter, the only numbers he presented to answer the question "How deadly is the virus?" are those of the "case-related lethality" which would be estimated "between 2 and 4 percent" (our translation). Another example is Weimer's (2020) article in *The European*, a German-English political magazine, where Weimer started with an estimation of 1.9 million casualties in Germany based on an expected lethality of 3.4%, only to discuss afterwards that such a high number was rather unlikely and due to insufficient data. It should be noted that both articles did not present alternative data, on the basis of which less alarming predictions could be made. Both articles clearly explicated why using the current numbers does not make much sense. In spite of this insight, they could not refrain from using these numbers for their arguments.

[4] One of the authors of this article was contacted by a layperson who had made the observation that the graphs are rather linear in most states and countries.

5 Conclusions

On the basis of the reflections of our attempts to understand the mathematics behind the selected discourses on the SARS-CoV-2 crisis, we notice that we face discourses that are politically highly relevant and controversial and whose mathematical components cannot be understood by mathematical laypersons. This does not mean that the idea of control by reflection failed in general. Of course, in negotiations of a private loan or when discussing which taxes to raise how drastically for the funding of a certain welfare policy, the underlying mathematics might already be familiar from secondary school or at least approachable on the basis of one's abilities. Therefore, we still support Fischer's (2001) and Skovsmose's (1994) ideas of critical reflection of mathematics as a central goal of mathematics education. However, we discern that critical reflection of mathematics cannot be the sole solution to the problem it stood up against, namely the control of experts by laymen both in the political and in the private sphere.

The observation that relevant discourses rely on mathematics that is out of reach of the understanding of laypersons raises the question how individual maturity and democratic control can be possible and can be secured by mathematics education. The discourses analysed here make extreme cases, as they accompanied one of the most extreme interventions in social life by governmental authorities. We assume that there are many more, more subtle and everyday-cases of relevant decisions with underlying mathematics that is out of reach of a layperson's understanding. It is unclear how to solve the problem of maturity and control. Even with an increased investment in mathematics education, it will be impossible to prepare the layperson with enough understanding to allow for the reflection of each and every relevant use of mathematics. One idea might be to politically secure independent positions for experts who are not involved in the interest-guided production of mathematical solutions but reserve their talents for a multi-faceted critique and public problematisation of the uses of mathematics (Greer & Mukhopadhyay, 2012).

Ironically, one might ask if mathematics education actually lies at the heart of the problem. As we pointed out, the public discourse features idealised mathematical concepts on the basis of mathematical theories that are covered in secondary education, whereas experts work with alternative concepts that are too complex for a layperson's understanding but can be approached empirically. We suggested that the mingling of ideal and empirical concepts is at least misleading if not even hindering a critical discussion of the empirical concepts. However, if mathematics education for the general public had not existed, there would have been no way to simplify the mathematical ideas and the sole mathematical discourse could have been performed under experts. Obviously, abolishing mathematics education is not an option as it would cause other problems, such as rendering control by reflection impossible. Still, it is important to note that mathematics education is responsible for the social availability of comparatively simple mathematical concepts which might then serve the need for simplification and result in misunderstanding or even in deception.[5]

We want to conclude with general concerns of the role of journalists and politicians. Both Fischer (2001) and Skovsmose (1994) argued that critical reflection on mathematics and academic theories in general is important to decide within a field of competing explanations. From journalists and politicians, one would expect that they reached conclusions on the basis of the consideration of such competing explanations. Instead, at least in Germany and Austria, the corona crisis was accompanied with the establishment of the hegemonic discourse of a dangerous decease that legitimised a lockdown. Merkel went as far as criticising "opening discussion orgies" (Chornley, 2020). In a few cases, even mainstream journalists complained that alternative positions, even those of decorated researchers, were officially discarded as conspiracy theories (Martenstein, 2020). As alternative positions were barely discussed in mainstream media and especially not communicated and assessed by politicians, they were eventually presented in alternative media with the effect that accounts by researchers were published side by side with absurd explanations that lack any academic verification. We believe that it is dangerous for a democracy to create an antagonism between the official and the obscure. For the sake of democratic control and well-informed decision making, the critical reflection on concepts underlying central social discourses, especially the mathematical ones, must be assigned a central role in our public discourses.

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[5] Accusing politicians of deception might be a strong claim, but there is evidence for interest-driven choices of mathematical models. For example, Russia used a definition of COVID-19 casualties that allowed for very low numbers of casualties, numbers then used to document the healthy condition of the country (Vasilyeva, 2020). Leaked protocols of meetings of Sebastian Kurz, the Chancellor of Austria, with his corona taskforce revealed that Kurz intended to achieve "obedience through fear" and wanted estimations of "100,000 deaths" to be communicated – a number that exceeded the estimations of the experts in the taskforce by far and for which a mathematical legitimisation would have been useful for the government (Tóth, 2020, our translation).

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