



# A Study of the Integration of Neutrosophic Thick Function

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## Abstract

In this paper, the definition of neutrosophic thick function and its integral are introduced. The main objective is defining a differential linear equation based on the thick function and finding solutions for this equation.

**Keywords:** Neutrosophic Thick Function, Neutrosophic Integration.

## 1. Introduction

Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probability and alike, are recent creations of F. Smarandache, being characterized by having the indeterminacy, as component of their framework and a notable feature of neutrosophic logic is that can be considered a generalization of fuzzy logic, encompassing the classical logic as well[1]. Also, in 2015, F. Smarandache has defined the concept of continuity of a neutrosophic function in [1], and neutrosophic mereo-limit [1], mereo-continuity. Moreover, in 2014, F. Smarandache has defined the concept of a neutro-oscillator differential in [3], and mereo-derivative. Finally, in 2013 F. Smarandache introduced neutrosophic integration in [2], and mereo-integral, besides the classical definitions of limit, continuity, derivative, and integral respectively.

Among the recent applications there are: neutrosophic crisp set theory in image processing[4][5], neutrosophic sets medical field [6][7][8][9][10], geographic information systems [11] and possible applications to database [12]. Also, neutrosophic triplet group application to physics[13]. Moreover, several types of research have made multiple contributions to neutrosophic topology [14][15] [16] [17] [18] [19] [20] Also more researches have made multiple contributions to the neutrosophic analysis[21]. Finally, the neutrosophic integration may be applied in calculating the area between two neutrosophic functions.

## 2. Preliminaries

In this paper  $m(x) = [m_1(x), m_2(x)]$  is called a neutrosophic thick function. Now, we recall some definitions which are useful in this paper.

**Definition 2.1. [1]**

A neutrosophic crisp function:

$f: A \rightarrow B$  is called a crisp relation if there exists an element  $a \in A$  with  $f(a) = b$  and  $f(a) = c$  where  $b, c \in B$  then  $b \equiv c$ . (this is the classical vertical line test).

**Definition 2.2. [1]**

Neutrosophic subset or crisp function:

A neutrosophic (subset or crisp) function in general is a function that has some indeterminacy.

**Definition 2.3. [2]**

The neutrosophic constant thick function:

$$l: R \rightarrow P(R) ; \text{for example: } l(x) = [2,3]; \text{ for any } x \in R$$

Where  $P(R)$  is the set of all subsets of  $R$ .

For example,  $l(7)$ , is the vertical segment of line  $[2,3]$  shown in figure 2.1.



Figure 2.1.

**Definition 2.4. [2]**

A neutrosophic non-constant thick function:

$$k: R \rightarrow P(R)$$

For example,  $k(x) = [2x, 2x + 1]$ ;

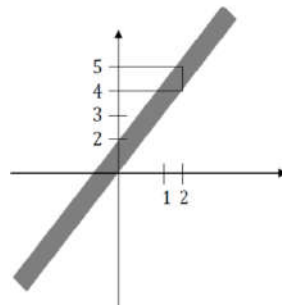


Figure 2.2

Let  $k(x) = [2x, 2x + 1]$  then  $k(2) = [2(2), 2(2) + 1] = [4, 5]$ . The respective graph is the Figure 2.2

**Definition 2.5.** [2]

The general neutrosophic thick function is defined as:

There is something confusing in the use of notations for intervals. For example, it is used  $(m_1(x), m_2(x))$  for open interval and  $]m_1(x), m_2(x)[$  and  $[m_1(x), m_2(x)[$  for half closed (or semi closed) intervals.

$$m: R \rightarrow P(R) ; m(x) = [m_1(x), m_2(x)]$$

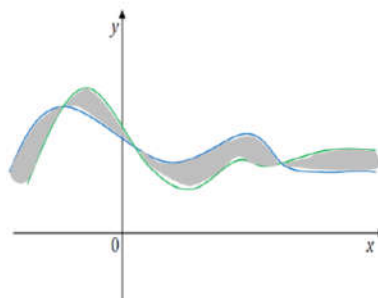


Figure 2.3

**3. A Neutrosophic Integration**

As in Euclidean integration, integration is the opposite of differentiation.

In other words, the anti-thesis of the derivative of the neutrosophic function  $f(x)$  is also a neutrosophic function  $F(x)$ . That the definition of neutrosophic integral is:

$$F(x) = \int f(x, I) dx$$

Where I represents the indeterminacy and constant integration is  $a + bI$ .

**Example 3.1.**

$$f : R \rightarrow R \cup \{I\} ; f(x) = 5x^2 + (3x + 1)I$$

Then:

$$F(x) = \int [5x^2 + (3x + 1)I]dx = \int 5x^2 dx + \int (3x + 1)I dx = 5\frac{x^3}{3} + \left(3\frac{x^2}{2} + x\right)I + a + bI.$$

Where  $a + bI$  is the neutrosophic integration constant.

#### 4. Neutrosophic integration thick function

In this section is given the definition of the neutrosophic thick function, as well the operation of integration over it.

**Definition 4.1.** Let  $m(x) = [m_1(x), m_2(x)]$  be a neutrosophic thick function. Then we define the integration of this function as:

$$\int m(x)dx = \int [m_1(x), m_2(x)]dx = \left[ \int m_1(x) dx + c_1, \int m_2(x) dx + c_2 \right] = [A, B]$$

Where  $c_1 = a_1 + b_1I_1$ ,  $c_2 = a_2 + b_2I_2$ .

**Example 4.1.** Let  $m(x) = [m_1(x), m_2(x)] = [xe^x, xe^{x^2}]$  then:

$$\int m(x)dx = \int [xe^x, xe^{x^2}]dx = \left[ \int xe^x dx + c_1, \int xe^{x^2} dx + c_2 \right] = [A, B]$$

$$A = \int xe^x dx + c_1 = x \cdot e^x - e^x + c_1$$

$$B = \int xe^{x^2} dx + c_2 = \frac{1}{2} \cdot e^{x^2} + c_2$$

$$\int m(x)dx = \left[ x \cdot e^x - e^x + c_1, \frac{1}{2} \cdot e^{x^2} + c_2 \right]$$

Where  $c_1 = a_1 + b_1I_1$ ,  $c_2 = a_2 + b_2I_2$ .

**Example 4.2.** let  $m(x) = [m_1(x), m_2(x)] = \left[ \frac{1}{1+x^2}, \frac{x^2}{1+x^2} \right]$  then:

$$\int m(x)dx = \int \left[ \frac{1}{1+x^2}, \frac{x^2}{1+x^2} \right] dx = \left[ \int \frac{1}{1+x^2} dx + c_1, \int \frac{x^2}{1+x^2} dx + c_2 \right] = [A, B]$$

$$A = \int \frac{1}{1+x^2} dx + c_1 = \text{arc tg}(x) + c_1$$

$$B = \int \frac{x^2}{1+x^2} dx + c_2 = x - \text{arc tg}(x) + c_2$$

$$\int m(x)dx = [\text{arc tg}(x) + c_1, x - \text{arc tg}(x) + c_2]$$

Where  $c_1 = a_1 + b_1I_1$ ,  $c_2 = a_2 + b_2I_2$ .

#### 5. Neutrosophic linear differential equation

In this section we define a linear differential equation based on the thick function and find solutions of this equation.

**Neutrosophic identical linear differential equation**

**Definition.** We define the Neutrosophic identical linear differential equation by a neutrosophic thick function form:

$$\dot{y} + m(x)y = 0 ; m(x) = [m_1(x), m_2(x)] \quad (1)$$

**Method of solution.**

$$\dot{y} + m(x)y = 0$$

$$\dot{y} + [m_1(x), m_2(x)]y = 0$$

$$\dot{y} = -[m_1(x), m_2(x)]y$$

$$\frac{\dot{y}}{y} = -[m_1(x), m_2(x)]$$

$$\frac{\ln y}{c} = - \int [m_1(x), m_2(x)] dx = - \left[ \int m_1(x) dx, \int m_2(x) dx \right]$$

$$y = c[e^{-\int m_1(x) dx}, e^{-\int m_2(x) dx}] \quad (2)$$

**Example 5.1.** Find a solution to the equation:

$$\dot{y} + \left[ \frac{1}{x}, 2x \right] y = 0$$

**Solution.**

$$y = (a + bl) \left[ e^{-\int \frac{1}{x} dx}, e^{-\int 2x dx} \right] = (a + bl) [e^{-\ln x}, e^{-x^2}] = (a + bl) \left[ \frac{1}{x}, e^{-x^2} \right]$$

**5.4 Neutrosophic non-homogeneous linear differential equation**

**Definition 5.5.** We define the Neutrosophic non-homogeneous linear differential equation by a neutrosophic thick function which takes one of the following forms:

$$\dot{y} + [m_1(x), m_2(x)]y = q(x) \dots \dots (2)$$

$$\dot{y} + p(x)y = [f_1(x), f_2(x)] \dots \dots (3)$$

$$\dot{y} + [m_1(x), m_2(x)]y = [f_1(x), f_2(x)] \dots \dots (4)$$

Now we will derive the solution of equation (2) and solution of equations (3), (4) can be driven in the same way:

**Method of solution.**

1- We find the complement factor of the equation (2) as follows:

$$\mu(x) = e^{\int m_1(x) dx, \int m_2(x) dx} = [e^{\int m_1(x) dx}, e^{\int m_2(x) dx}]$$

2- We multiply equation (2) by the complement factor:

$$\mu(x)\dot{y} + \mu(x)[m_1(x), m_2(x)]y = q(x)\mu(x)$$

$$\mu(x)\dot{y} + [m_1(x)e^{\int m_1(x)dx}, m_2(x)e^{\int m_2(x)dx}]y = [q(x)e^{\int m_1(x)dx}, q(x)e^{\int m_2(x)dx}]$$

3- Note that the left side is only the function derivative:  $\mu(x)y$  and therefor the equation can be sequenced as:

$$(y\mu(x))' = [q(x)e^{\int m_1(x)dx}, q(x)e^{\int m_2(x)dx}]$$

4- By integrating the latter, we obtain the general solution of equation(2):

$$y = \frac{1}{\mu(x)} \left( a + bI + \left[ \int q(x)e^{\int m_1(x)dx}, \int q(x)e^{\int m_2(x)dx} \right] \right) \dots \dots (5)$$

**Example 5.6.** Find the general solution of the following neutrosophic non- homogeneous linear differential equation:

$$\dot{y} + \left[ 2x, \frac{1}{x} \right] y = x$$

**Solution.** The equation is of the first form:

$$\dot{y} + [m_1(x), m_2(x)]y = q(x)$$

First we find the complement factor:

$$\mu(x) = e^{[\int m_1(x)dx, \int m_2(x)dx]} = [e^{\int m_1(x)dx}, e^{\int m_2(x)dx}] = e^{[\int 2xdx, \int \frac{1}{x}dx]} = [e^{x^2}, e^{\ln x}] = [e^{x^2}, x]$$

Now by multiplying both sides of the equation by the complement factor, we find that the general solution can be written as:

$$y = \frac{1}{\mu(x)} \left( a + bI + \int x\mu(x)dx \right) = \frac{1}{[e^{x^2}, x]} \left( a + bI + \left[ \int xe^{x^2}dx, \int x^2dx \right] \right)$$

where:

$$\int xe^{x^2}dx = \frac{1}{2} e^{x^2}$$

$$\int x^2dx = \frac{1}{3} x^3$$

Thus the general solution of the given equation is:

$$y = \frac{1}{[e^{x^2}, x]} \left( a + bI + \left[ \frac{1}{2}e^{x^2}, \frac{1}{3}x^3 \right] \right)$$

**Example 5.7.** Find the general solution for the following neutrosophic non- homogeneous linear differential equation:

$$\dot{y} + \cot(x) y = [\sin(x), \cos(x)]$$

**Solution.** The equation is of the form:

$$\dot{y} + p(x)y = [f_1(x), f_2(x)]$$

We find the complement factor as:

$$\mu(x) = e^{\int p(x)dx} = e^{\int \cot(x)dx} = e^{\ln \sin(x)} = \sin(x)$$

Now by multiplying both sides of the equation by the complement factor, we find that the general solution can be written as:

$$\begin{aligned} y &= \frac{1}{\mu(x)} \left( a + bI + \int \mu(x) \cdot [\sin(x), \cos(x)] dx \right) = \frac{1}{\sin(x)} \left( a + bI + \int \sin(x) \cdot [\sin(x), \cos(x)] dx \right) \\ &= \frac{1}{\sin(x)} \left( a + bI + \left[ \int \sin^2(x) dx, \int \sin(x) \cdot \cos(x) dx \right] \right) \end{aligned}$$

where:

$$\begin{aligned} \int \sin^2(x) dx &= \frac{1}{2}x - \frac{1}{4}\sin(2x) \\ \int \sin(x) \cdot \cos(x) dx &= \frac{1}{2} \int \sin(2x) dx = -\frac{1}{4}\cos(2x) \end{aligned}$$

Thus the general solution of the given equation is:

$$y = \frac{1}{\sin(x)} \left( a + bI + \left[ \frac{1}{2}x - \frac{1}{4}\sin(2x), -\frac{1}{4}\cos(2x) \right] \right).$$

**Example 5.8.** Find the general solution for the following neutrosophic non-homogeneous linear differential equation:

$$\dot{y} + \left[ \frac{1}{x}, x \right] y = [x^2, x]$$

**Solution:** the equation is of the third form:

$$\dot{y} + [m_1(x), m_2(x)]y = [f_1(x), f_2(x)]$$

We find the complement:

$$\mu(x) = e^{\int [m_1(x), m_2(x)] dx} = [e^{\int m_1(x) dx}, e^{\int m_2(x) dx}] = [e^{\ln x}, e^{\frac{1}{2}x^2}] = \left[ x, e^{\frac{1}{2}x^2} \right].$$

Now by multiplying both sides of the equation by the complement factor, we find that the general solution is written as:

$$\begin{aligned} y &= \frac{1}{\mu(x)} \left( a + bI + \int \mu(x) [f_1(x), f_2(x)] dx \right) = \frac{1}{\left[ x, e^{\frac{1}{2}x^2} \right]} \left( a + bI + \int \left[ x, e^{\frac{1}{2}x^2} \right] \cdot [x^2, x] dx \right) \\ &= \frac{1}{\left[ x, e^{\frac{1}{2}x^2} \right]} \left( a + bI + \left[ \int x \cdot x^2 dx, \int x \cdot e^{\frac{1}{2}x^2} dx \right] \right) \\ &= \frac{1}{\left[ x, e^{\frac{1}{2}x^2} \right]} \left( a + bI + \left[ \int x^3 dx, \int x \cdot e^{\frac{1}{2}x^2} dx \right] \right) \end{aligned}$$

where:

$$\int x^3 dx = \frac{1}{4}x^4$$

$$\int x \cdot e^{\frac{1}{2}x^2} dx = e^{\frac{1}{2}x^2}$$

Thus the general solution of the given equation is:

$$y = \frac{1}{\left[ x, e^{\frac{1}{2}x^2} \right]} \left( a + bI + \left[ \frac{1}{4}x^4, e^{\frac{1}{2}x^2} \right] \right)$$

## 6. Conclusion

In this paper, a new type of neutrosophic integration has been defined by using the thick function, Moreover, we studied a linear differential equation based on the thick function and found solutions to this equation. In addition, solutions to other types of neutrosophic differential equations can be found depending on the thick function such as Bernoulli's equation. We will work on this in the future.

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