Deep Learning Specialization Formula Sheet

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Chapter 1 Neural Networks and Deep Learning

1 Standard Notation for Deep Learning

1.1 General Comments

Superscript (i) denotes the i^{th} training example while superscript $[l]$ denotes the l^{th} layer.
Vectors are represented by bold small letters (example: $\mathbf{x}$) and matrices are represented by bold capital letters (example: $\boldsymbol{X}$).

1.2 Sizes

m: Number of examples in the dataset.
n_x: Input size.
n_y: Output size (or number of classes).
$n_h^{[l]}$: number of hidden units of the l^{th} layer.
L: Number of layers in the network.

1.3 Objects

$\boldsymbol{X} \in \mathbb{R}^{n_x \times m}$: The input matrix.
$\mathbf{x}^{(i)} \in \mathbb{R}^{n_x}$: Is the i^{th} example represented as a column vector.
$\boldsymbol{Y} \in \mathbb{R}^{n_y \times m}$: Is the *label* matrix.
$\mathbf{y}^{(i)} \in \mathbb{R}^{n_y}$: Is the *output label* for the i^{th} example represented as a column vector.
$\boldsymbol{W}^{[l]} \in \mathbb{R}^{n_h^{[l]} \times n_h^{[l-1]}}$: is the *weight* matrix, superscript $[l]$ indicates the layer.
$\mathbf{b}^{[l]} \in \mathbb{R}^{n_h^{[l]}}$: Is the *bias* vector in the l^{th} layer.
$\hat{\mathbf{y}} \in \mathbb{R}^{n_y}$: Is the *predicted output* vector. It can also be denoted $\mathbf{a}^{[L]}$, where L is the number of layers in the network.

2 Logistic Regression

For one example $\mathbf{x}^{(i)} \in \mathbb{R}^n$:

$$\mathbf{z}^{(i)} = \mathbf{w}^\mathsf{T}\mathbf{x}^{(i)} + b$$

$$\hat{\mathbf{y}}^{(i)} = \mathbf{a}^{(i)} = \sigma(\mathbf{z}^{(i)})$$

Cross-entropy loss function (for one training example):

$$\mathcal{L}(\mathbf{a}^{(i)}, \mathbf{y}^{(i)}) = -\mathbf{y}^{(i)}\log(\mathbf{a}^{(i)}) - (1 - \mathbf{y}^{(i)})\log(1 - \mathbf{a}^{(i)})$$

The cost function (for all training examples) is then computed by summing over the loss for all training examples:

$$\mathcal{J}(\mathbf{w}, b) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(\mathbf{a}^{(i)}, \mathbf{y}^{(i)})$$

Collecting all training examples in a matrix $\boldsymbol{X}$:

$$\boldsymbol{X} = \left[\mathbf{x}^{(1)}|\mathbf{x}^{(2)}|\dots|\mathbf{x}^{(m)}\right]$$

$$\boldsymbol{A} = \sigma(\mathbf{w}^\mathsf{T}\boldsymbol{X} + b) = \left[\mathbf{a}^{(1)}|\mathbf{a}^{(2)}|\dots|\mathbf{a}^{(m)}\right]$$

$$\frac{\partial\mathcal{J}}{\partial\mathbf{w}} = \frac{1}{m}\boldsymbol{X}(\boldsymbol{A} - \boldsymbol{Y})^\mathsf{T}$$

$$\frac{\partial\mathcal{J}}{\partial b} = \frac{1}{m}\sum_{i=1}^{m}\left(\mathbf{a}^{(i)} - \mathbf{y}^{(i)}\right)$$

3 Neural Networks

3.1 Feed-Forward Propagation

$$\boldsymbol{A}^{[l]} = g^{[l]}(\boldsymbol{Z}^{[l]})$$

$$\boldsymbol{Z}^{[l]} = \boldsymbol{W}^{[l]}\boldsymbol{A}^{[l-1]} + \mathbf{b}^{[l]}$$

Input : $\boldsymbol{A}^{[0]} = X$
Output : $\boldsymbol{A}^{[L]} = \hat{\boldsymbol{Y}}$

Activation Functions

The *activation function* $g^{[l]}$ can be one of the following :

- **Sigmoid:**

$$\sigma(\boldsymbol{Z}) = \sigma(\boldsymbol{W}\boldsymbol{A} + \mathbf{b}) = \frac{1}{1 + e^{-(\boldsymbol{W}\boldsymbol{A} + \mathbf{b})}}$$

- **Rectified Linear Unit (ReLU):**

$$\text{relu}(\boldsymbol{Z}) = \max(0, \boldsymbol{Z})$$

Cost Function

Cross-entropy cost function :

$$\begin{aligned}\mathcal{J} &= -\frac{1}{m}\sum_{i=1}^{m}\left[\mathbf{y}^{(i)}\log\left(\mathbf{a}^{[L](i)}\right) + \left(1 - \mathbf{y}^{(i)}\right)\log\left(1 - \mathbf{a}^{[L](i)}\right)\right] \\ &= -\frac{1}{m}\left[\boldsymbol{Y}\cdot\log\left(\boldsymbol{A}^{[L]\mathsf{T}}\right) + (1 - \boldsymbol{Y})\cdot\log\left(1 - \boldsymbol{A}^{[L]\mathsf{T}}\right)\right]\end{aligned}$$

3.2 Backpropagation

$$\begin{aligned}
d\boldsymbol{A}^{[L]} &= \frac{\partial\mathcal{J}}{\partial\boldsymbol{A}^{[L]}} &&= -\frac{\boldsymbol{Y}}{\boldsymbol{A}^{[L]}} + \frac{1 - \boldsymbol{Y}}{1 - \boldsymbol{A}^{[L]}} \\
d\boldsymbol{Z}^{[l]} &= \frac{\partial\mathcal{J}}{\partial\boldsymbol{Z}^{[l]}} &&= d\boldsymbol{A}^{[l]} \odot g^{[l]\prime}\left(\boldsymbol{Z}^{[l]}\right) \\
d\boldsymbol{A}^{[l-1]} &= \frac{\partial\mathcal{J}}{\partial\boldsymbol{A}^{[l-1]}} &&= \boldsymbol{W}^{[l]\mathsf{T}}d\boldsymbol{Z}^{[l]} \\
d\boldsymbol{W}^{[l]} &= \frac{\partial\mathcal{J}}{\partial\boldsymbol{W}^{[l]}} &&= \frac{1}{m}d\boldsymbol{Z}^{[l]}\boldsymbol{A}^{[l-1]\mathsf{T}} \\
d\mathbf{b}^{[l]} &= \frac{\partial\mathcal{J}}{\partial\mathbf{b}^{[l]}} &&= \frac{1}{m}\sum_{i=1}^{m}d\boldsymbol{Z}^{[l](i)}
\end{aligned} \tag{1.1}$$

3.3 Gradient Descent

Update the parameters:

$$\begin{aligned}\boldsymbol{W}^{[l]} &:= \boldsymbol{W}^{[l]} - \alpha\, d\boldsymbol{W}^{[l]} \\ \mathbf{b}^{[l]} &:= \mathbf{b}^{[l]} - \alpha\, d\mathbf{b}^{[l]}\end{aligned}$$

where α is the learning rate.

Chapter 2 Improving Deep Neural Networks: Hyperparameter Tuning

1 Setting up Machine Learning Application

1.1 Train/Dev/Test Sets

Splitting the data into Train/dev(validation)/test sets according to its size

- For small dataset ($m = 100 - 1,000 - 10,000$):
  A ratio of $60\%, 20\%, 20\%$ works well.
- For large datasets ($m = 1,000,000$):
  A ratio of $98\%, 1\%, 1\%$

2 Regularization

2.1 Logistic Regression

$$\mathcal{J}(\mathbf{w}, \mathbf{b}) = \frac{1}{m}\sum_{j=1}^{m}\mathcal{L}(\hat{\mathbf{y}}^{(i)}, \mathbf{y}^{(i)}) + \text{Regularization term}$$

The regularization term can be :

- L_2 Regularization : $\dfrac{\lambda}{2m}\|\mathbf{w}\|_2^2 = \dfrac{\lambda}{2m}\displaystyle\sum_{j=1}^{n_x}w_j^2 = \mathbf{w}^\mathsf{T}\mathbf{w}$
- L_1 Regularization : $\dfrac{\lambda}{2m}\|\mathbf{w}\|_1 = \dfrac{\lambda}{2m}\displaystyle\sum_{j=1}^{n_x}|w|$

2.2 Neural Network

$$\mathcal{J}(\boldsymbol{W}^{[1]}, \mathbf{b}^{[1]}, \ldots, \boldsymbol{W}^{[L]}, \mathbf{b}^{[L]}) = \frac{1}{m}\sum_{i=1}^{m}\mathcal{L}(\hat{\mathbf{y}}^{(i)}, \mathbf{y}^{(i)}) + \frac{\lambda}{2m}\sum_{l=1}^{L}\|\boldsymbol{W}^{[l]}\|_F^2$$

Where $\|\boldsymbol{W}^{[l]}\|_F^2$ is called *Frobenius norm* and

$$\|\boldsymbol{W}^{[l]}\|_F^2 = \sum_{i=1}^{n^{[l]}}\sum_{j=1}^{n^{[l-1]}}\left(W_{i,j}^{[l]}\right)^2$$

Therefore

$$\mathcal{J}_{\text{regularized}} = \underbrace{-\frac{1}{m}\sum_{i=1}^{m}\left[\mathbf{y}^{(i)}\log\left(\mathbf{a}^{[L](i)}\right) + \left(1-\mathbf{y}^{(i)}\right)\log\left(1-\mathbf{a}^{[L](i)}\right)\right]}_{\text{cross-entropy cost}} + \underbrace{\frac{1}{m}\frac{\lambda}{2}\sum_{l}\sum_{k}\sum_{j}\boldsymbol{W}_{k,j}^{[l]2}}_{\text{L2 regularization cost}}$$

Backpropagation:

$$d\boldsymbol{W}^{[l]} \overset{(1.1)}{=} \frac{1}{m}d\boldsymbol{Z}^{[l]}\boldsymbol{A}^{[l-1]\mathsf{T}} + \frac{\lambda}{m}\boldsymbol{W}^{[l]}$$

Gradient Descent :

$$\begin{aligned}\boldsymbol{W}^{[l]} &:= \alpha\, d\boldsymbol{W}^{[l]} \\ &:= \boldsymbol{W}^{[l]} - \alpha\left[\frac{1}{m}d\boldsymbol{Z}^{[l]}\boldsymbol{A}^{[l-1]\mathsf{T}} + \frac{\lambda}{m}\boldsymbol{W}^{[l]}\right] \\ &:= \boldsymbol{W}^{[l]} - \frac{\lambda\alpha}{m}\boldsymbol{W}^{[l]} - \alpha\left(\frac{1}{m}d\boldsymbol{Z}^{[l]}\boldsymbol{A}^{[l-1]\mathsf{T}}\right) \\ &:= \underbrace{\left(1-\frac{\alpha\lambda}{m}\right)}_{\substack{\text{Weight}\\ \text{Decay}}}\boldsymbol{W}^{[l]} - \alpha\left(\frac{1}{m}d\boldsymbol{Z}^{[l]}\boldsymbol{A}^{[l-1]\mathsf{T}}\right)\end{aligned}$$

2.3 Dropout

Implementing dropout ("Inverted dropout") in Python.
Illustrate with $l = 3$.

```
keep-prob = 0.8
d3 = np.random.rand(a3.shape[0], a3.shape[1]) < keep-prob
a3 = np.multiply(a3, d3)  # a3 *= d3
a3 /= keep-prob
```

3 Setting Up Optimization Problem

3.1 Normalizing Training Sets

Mean

$$\boldsymbol{\mu} = \frac{1}{m}\sum_{i=1}^{m}\mathbf{x}^{(i)}$$

Variance

$$\boldsymbol{\sigma}^2 = \frac{1}{m}\sum_{i=1}^{m}\left(\mathbf{x}^{(i)}\odot\mathbf{x}^{(i)}\right) - \boldsymbol{\mu}^2$$

Dataset Normalization:

$$\mathbf{x}^{(i)} := \frac{\mathbf{x}^{(i)} - \boldsymbol{\mu}}{\boldsymbol{\sigma}}$$

Note: We use the same $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ to normalize the test set.

3.2 Weight Initialization for Deep Networks

To solve the problem of vanishing and exploding gradients.

For *sigmoid* or *tanh* activation function we use *Xavier initialization*:

$$\boldsymbol{W}^{[l]} = \texttt{np.random.randn}(\boldsymbol{W}^{[l]}\texttt{.shape}) * \sqrt{\frac{1}{n^{[l-1]}}}$$

or

$$\boldsymbol{W}^{[l]} = \texttt{np.random.randn}(\boldsymbol{W}^{[l]}\texttt{.shape}) * \sqrt{\frac{1}{n^{[l-1]+n^{[l]}}}}$$

For *RelU* activation function:

$$\boldsymbol{W}^{[l]} = \texttt{np.random.randn}(\boldsymbol{W}^{[l]}\texttt{.shape}) * \sqrt{\frac{2}{n^{[l-1]}}}$$

3.3 Numerical Approximation of Gradients

Two Sided difference

$$f'(\theta) = \lim_{\varepsilon\to 0}\frac{f(\theta+\varepsilon) - f(\theta-\varepsilon)}{2\varepsilon}$$

Order of the error $O(\varepsilon^2)$

One sided difference

$$f'(\theta) = \lim_{\varepsilon\to 0}\frac{f(\theta+\varepsilon) - f(\theta)}{\varepsilon}$$

Order of the error $O(\varepsilon)$

Gradient Checking for a Neural Network

Take $\boldsymbol{W}^{[l]}, \mathbf{b}^{[l]}, \ldots, \boldsymbol{W}^{[L]}, \mathbf{b}^{[L]}$ and reshape into a big vector $\boldsymbol{\theta}$

$$\begin{aligned}\mathcal{J}(\boldsymbol{W}^{[l]}, \mathbf{b}^{[l]}, \ldots, \boldsymbol{W}^{[L]}, \mathbf{b}^{[L]}) &= \mathcal{J}(\boldsymbol{\theta}) \\ &= \mathcal{J}(\theta_1, \theta_2, \ldots, \theta_i, \ldots)\end{aligned}$$

Take $d\boldsymbol{W}^{[l]}, d\mathbf{b}^{[l]}, \ldots, d\boldsymbol{W}^{[L]}, d\mathbf{b}^{[L]}$ and reshape into a big vector $d\boldsymbol{\theta}$

For each i:

$$d\theta_{i\ \text{approx}} = \frac{\mathcal{J}(\theta_1, \theta_2, \ldots, \theta_i+\varepsilon, \ldots) - \mathcal{J}(\theta_1, \theta_2, \ldots, \theta_i-\varepsilon, \ldots)}{2\varepsilon}$$

$$d\theta_{i\ \text{approx}} \approx d\theta_i = \frac{\partial\mathcal{J}}{\partial\theta_i}$$

$d\boldsymbol{\theta}_{\text{approx}} \approx d\boldsymbol{\theta}$

Check $\dfrac{\|d\boldsymbol{\theta}_{\text{approx}} - d\boldsymbol{\theta}\|_2}{\|d\boldsymbol{\theta}_{\text{approx}}\|_2 + \|d\boldsymbol{\theta}\|_2} < \epsilon$

in practice we set $\epsilon = 10^{-7}$

Gradient checking implementation notes:

- Don't use in training - only to debug
- If algorithm fails grad check, look at components $(d\mathbf{b}^{[l]}, d\boldsymbol{W}^{[l]})$ to try to identify bug.
- Remember to include regularization.
- Doesn't work with dropout.
- Run at random initialization; perhaps again after some training.

4 Optimization Algorithms

Suppose that we have m total number of examples.

Batch gradient descent: Using all training examples m at once.

Mini-batch gradient descent: Using a subset ($< m$) of training examples at a time.

Stochastic gradient descent: Using a mini-batch that has just 1 example at a time.

4.1 Mini-Batch Gradient Descent

Reference : [Hin12]

Cost function may not decrease on every iteration.

Algorithm 1: Mini-Batch Gradient Descent

Result: Trained network parameters for each layer $\boldsymbol{W}^{[l]}, \mathbf{b}^{[l]}$

- **for each** *epoch*:
  - **for each** *mini-batch t*:
    - `/* Forward-Propagation on` $\boldsymbol{X}^{\{t\}}$ `*/`
    - $\boldsymbol{A}^{[0]} = \boldsymbol{X}^{\{t\}}$
    - **for** *layer* $l = 1, \ldots, L$:
      - $\boldsymbol{Z}^{[l]} = \boldsymbol{W}^{[l]} \boldsymbol{A}^{[l-1]} + \mathbf{b}^{[l]}$
      - $\boldsymbol{A}^{[l]} = g^{[l]}(\boldsymbol{Z}^{[l]})$
    - `/* ---------------------------- */`
    - Compute Cost $\mathcal{J}^{\{t\}} = \frac{1}{k}\sum_{i=1}^{l} \mathcal{L}\underbrace{\left(\hat{\mathbf{y}}^{(i)}, \mathbf{y}^{(i)}\right)}_{\text{For } \boldsymbol{X}^{\{t\}}, \boldsymbol{Y}^{\{t\}}} + \frac{\lambda}{2 \cdot k} \sum_{l} \|\boldsymbol{W}^{[l]}\|_F^2$
    - Backpropagate to compute gradients w.r.t $\mathcal{J}^{\{t\}}$ (using $(\boldsymbol{X}^{\{t\}}, \boldsymbol{Y}^{\{t\}})$)
    - **for** *layer* $l = 1, \ldots, L$:
      - $\boldsymbol{W}^{[l]} := \boldsymbol{W}^{[l]} - \alpha \, d\boldsymbol{W}^{[l]}$
      - $\mathbf{b}^{[l]} := \mathbf{b}^{[l]} - \alpha \, d\mathbf{b}^{[l]}$

Choosing Mini-Batch Size

- If small training set ($m \leq 2000$) : Use *batch gradient descent*
- Typical mini-batch sizes : $64, 128, 256, 512$ (Powers of 2)
- Make sure that the mini-batch $\boldsymbol{X}^{\{t\}}, \boldsymbol{Y}^{\{t\}}$ fits in CPU/GPU memory.

4.2 Exponentially Weighted Averages

$$V_t = \beta V_{t-1} + (1-\beta)\theta_t$$

averages over $\approx \dfrac{1}{1-\beta}$ previous values of θ

Bias Correction

$$V_t := \frac{V_t}{1-\beta}$$

4.3 Gradient Descent with Momentum

Momentum β takes past gradients into account to smooth out the steps of gradient descent. It can be applied with batch gradient descent, mini-batch gradient descent or stochastic gradient descent.

Algorithm 2: Gradient Descent with Momentum

Result: Trained network parameters for each layer $\boldsymbol{W}^{[l]}, \mathbf{b}^{[l]}$

- $\boldsymbol{V}_{d\boldsymbol{W}^{[l]}} = \mathbf{0}, \quad \boldsymbol{V}_{d\mathbf{b}^{[l]}} = \mathbf{0}$
- **for each** *epoch*:
  - **for each** *mini-batch*:
    - Forward-propagation on current mini-batch.
    - Compute cost $\mathcal{J}$ of current mini-batch.
    - Backpropagate to compute $d\boldsymbol{W}^{[l]}, d\mathbf{b}^{[l]}$ on the current mini-batch.
    - **for** *layer* $l = 1, \ldots, L$:
      - $\boldsymbol{V}_{d\boldsymbol{W}^{[l]}} := \beta_1 \boldsymbol{V}_{d\boldsymbol{W}^{[l]}} + (1-\beta_1) d\boldsymbol{W}^{[l]}$
      - $\boldsymbol{V}_{d\mathbf{b}^{[l]}} := \beta_1 \boldsymbol{V}_{d\mathbf{b}^{[l]}} + (1-\beta_1) d\mathbf{b}^{[l]}$
      - $\boldsymbol{W}^{[l]} := \boldsymbol{W}^{[l]} - \alpha \boldsymbol{V}_{d\boldsymbol{W}^{[l]}}, \quad \mathbf{b}^{[l]} = \mathbf{b}^{[l]} - \alpha \boldsymbol{V}_{d\mathbf{b}^{[l]}}$

A common practice is to set the hyperparameter $\beta = 0.9$

4.4 RMSprop

`RMSprop` stands for *root mean square prop*

Algorithm 3: RMSprop

Result: Trained network parameters for each layer $\boldsymbol{W}^{[l]}, \mathbf{b}^{[l]}$

- $\boldsymbol{S}_{d\boldsymbol{W}^{[l]}} = \mathbf{0}, \quad \boldsymbol{S}_{d\mathbf{b}^{[l]}} = \mathbf{0}$
- **for each** *epoch*:
  - **for each** *mini-batch*:
    - Forward-propagation on current mini-batch.
    - Compute cost $\mathcal{J}$ of current mini-batch.
    - Backpropagate to compute $d\boldsymbol{W}^{[l]}, d\mathbf{b}^{[l]}$ on the current mini-batch.
    - **for** *layer* $l = 1, \ldots, L$:
      - $\boldsymbol{S}_{d\boldsymbol{W}^{[l]}} := \beta_2 \boldsymbol{S}_{d\boldsymbol{W}^{[l]}} + (1-\beta) d\boldsymbol{W}^{[l]\circ 2}$ `/* small */`
      - $\boldsymbol{S}_{d\mathbf{b}^{[l]}} = \beta_2 \boldsymbol{S}_{d\mathbf{b}^{[l]}} + (1-\beta) d\mathbf{b}^{[l]\circ 2}$ `/* large */`
      - $\boldsymbol{W}^{[l]} := \boldsymbol{W}^{[l]} - \alpha \dfrac{d\boldsymbol{W}^{[l]}}{\sqrt{\boldsymbol{S}_{d\boldsymbol{W}^{[l]}}} + \varepsilon},$
      - $\mathbf{b}^{[l]} := \mathbf{b}^{[l]} - \alpha \dfrac{d\mathbf{b}^{[l]}}{\sqrt{\boldsymbol{S}_{d\mathbf{b}^{[l]}}} + \varepsilon}$

4.5 Adam Optimization Algorithm

Adam stands for *Adaptive Moment Estimation*

Paper : [KB14]

Algorithm 4: Adam Optimization Algorithm

Result: Trained network parameters for each layer $\boldsymbol{W}^{[l]}, \mathbf{b}^{[l]}$

- $\boldsymbol{V}_{d\boldsymbol{W}^{[l]}} = \mathbf{0}, \boldsymbol{S}_{d\boldsymbol{W}^{[l]}} = \mathbf{0}, \quad \boldsymbol{V}_{d\mathbf{b}^{[l]}} = \mathbf{0}, \boldsymbol{S}_{d\mathbf{b}^{[l]}} = \mathbf{0}$
- $t = 0$
- **for each** *epoch*:
  - **for each** *mini-batch*:
    - Forward-propagation on current mini-batch.
    - Compute cost $\mathcal{J}$ of current mini-batch.
    - Backpropagate to compute $d\boldsymbol{W}^{[l]}, d\mathbf{b}^{[l]}$ on the current mini-batch.
    - $t := t + 1$
    - **for** *layer* $l = 1, \ldots, L$:
      - $\boldsymbol{V}_{d\boldsymbol{W}^{[l]}} := \beta_1 \boldsymbol{V}_{d\boldsymbol{W}^{[l]}} + (1-\beta_1) d\boldsymbol{W}^{[l]}$
      - $\boldsymbol{V}_{d\mathbf{b}^{[l]}} := \beta_1 \boldsymbol{V}_{d\mathbf{b}^{[l]}} + (1-\beta_1) d\mathbf{b}^{[l]}$ `/* "moment"` β_1 `*/`
      - $\boldsymbol{S}_{d\boldsymbol{W}^{[l]}} := \beta_2 \boldsymbol{S}_{d\boldsymbol{W}^{[l]}} + (1-\beta_2) d\boldsymbol{W}^{[l]\circ 2}$
      - $\boldsymbol{S}_{d\mathbf{b}^{[l]}} := \beta_2 \boldsymbol{S}_{d\mathbf{b}^{[l]}} + (1-\beta_2) d\mathbf{b}^{[l]\circ 2}$ `/* "RMSprop"` β_2 `*/`
      - $\boldsymbol{V}^{\text{corrected}}_{d\boldsymbol{W}^{[l]}} = \dfrac{\boldsymbol{V}_{d\boldsymbol{W}^{[l]}}}{1-(\beta_1)^t}, \quad \boldsymbol{V}^{\text{corrected}}_{d\mathbf{b}^{[l]}} = \dfrac{\boldsymbol{V}_{d\mathbf{b}^{[l]}}}{1-(\beta_1)^t}$
      - $\boldsymbol{S}^{\text{corrected}}_{d\boldsymbol{W}^{[l]}} = \dfrac{\boldsymbol{S}_{d\boldsymbol{W}^{[l]}}}{1-(\beta_2)^t}, \quad \boldsymbol{S}^{\text{corrected}}_{d\mathbf{b}^{[l]}} = \dfrac{\boldsymbol{S}_{d\mathbf{b}^{[l]}}}{1-(\beta_2)^t}$
      - $\boldsymbol{W}^{[l]} := \boldsymbol{W}^{[l]} - \alpha \dfrac{\boldsymbol{V}^{\text{corrected}}_{d\boldsymbol{W}^{[l]}}}{\sqrt{\boldsymbol{S}^{\text{corrected}}_{d\boldsymbol{W}^{[l]}}} + \varepsilon}, \quad \mathbf{b}^{[l]} := \mathbf{b}^{[l]} - \alpha \dfrac{\boldsymbol{V}^{\text{corrected}}_{d\mathbf{b}^{[l]}}}{\sqrt{\boldsymbol{S}^{\text{corrected}}_{d\mathbf{b}^{[l]}}} + \varepsilon}$

4.6 Hyperparameter Choice

α : needs to be tuned.
β_1 : 0.9 (momentum of $d\boldsymbol{W}^{[l]}$)
β_2 : 0.999 (momentum of $d\boldsymbol{W}^{[l]\circ 2}$)
ε : 10^{-8}

4.7 Learning Rate Decay

Learning rate decay is to slowly reduce learning rate over time, to help speeding up the learning algorithm.

$$\alpha = \frac{1}{1 + rt}\alpha_0$$

Where r is the decay rate, t is the epoch number.

Other Learning Rate Decay Methods

- Exponential Decay $\alpha = r^t \cdot \alpha_0$
- $\alpha = \dfrac{k}{\sqrt{t}} \cdot \alpha_0$
- Discrete staircase
- Manually setting α

5 Hyperparameter Tuning

5.1 Appropriate Scale for Hyperparameters

Suppose you want to search for a parameter $\alpha = i, \dots, j$ on a logarithmic scale instead of a linear scale.

Calculate

$$a = \log_{10} i, \qquad b = \log_{10} j$$

then

$$\alpha = 10^r$$

where

$$\begin{aligned} r &\sim U(a, b) \\ &\sim a + (b - a)U(0, 1) \end{aligned}$$

5.2 Hyperparameters for exponentially weighted averages

For sampling the hyperparameter $\beta = i, \dots, j$ used to compute exponentially weighted averages.

$$1 - \beta = 1 - i, \dots, 1 - j$$

Calculate

$$a = \log_{10}(1 - i), \quad b = \log_{10}(1 - j)$$

then

$$\beta = 1 - 10^r$$

where

$$\begin{aligned} r &\sim U(b, a) \\ &\sim b + (a - b)U(0, 1) \end{aligned}$$

6 Batch Normalization

6.1 Implementing Batch Norm

Algorithm 5: Batch Norm

Data: training data $\boldsymbol{X}$, batch size $= k$

for each *Batch* $\boldsymbol{X}^{\{t\}}$ *in* $\boldsymbol{X}$**:**
- **for each** *Intermediate value* $\boldsymbol{Z}^{\{t\}[l]} = \left[\mathbf{z}^{(1)} | \dots | \mathbf{z}^{(k)}\right]$ *in Layer l in the neural network***:**
  - $\boldsymbol{\mu}^{\{t\}[l]} = \sum_{i=1}^{k} \mathbf{z}^{(i)}$
  - $\boldsymbol{\sigma}^{\{t\}[l]^2} = \dfrac{1}{m} \sum_{i=1}^{k} \left(\mathbf{z}^{(i)} - \boldsymbol{\mu}^{\{t\}[l]}\right)^2$
  - $\mathbf{z}^{(i)}_{\text{norm}} = \dfrac{\mathbf{z}^{(i)} - \boldsymbol{\mu}^{\{t\}[l]}}{\sqrt{\boldsymbol{\sigma}^{\{t\}[l]^2} + \varepsilon}}$
  - $\tilde{\mathbf{z}}^{(i)} = \boldsymbol{\gamma}^{[l]} \mathbf{z}^{(i)}_{\text{norm}} + \boldsymbol{\beta}^{[l]}$

Batch Norm Gradient Descent

Algorithm 6: Batch Norm Gradient Descent

Result: Trained network parameters for each layer $\boldsymbol{W}^{[l]}, \boldsymbol{\beta}^{[l]}, \boldsymbol{\gamma}^{[l]}$

for each *epoch***:**
- **for** *t = 1, . . . , num(mini-batches)***:**
  - /* Forward-Propagation on $\boldsymbol{X}^{\{t\}}$ */
  - $\boldsymbol{A}^{[0]} = \boldsymbol{X}^{\{t\}}$
  - **for** *layer $l = 1, \dots, L$***:**
    - $\boldsymbol{Z}^{[l]} = \boldsymbol{W}^{[l]} \boldsymbol{A}^{[l-1]} + \mathbf{b}^{[l]}$
    - Use Batch Norm (algorithm 5) to Compute $\tilde{\boldsymbol{Z}}^{[l]}$ from $\boldsymbol{Z}^{[l]}$
    - $\boldsymbol{A}^{[l]} = g^{[l]}(\tilde{\boldsymbol{Z}}^{[l]})$
  - /* ---------------------------- */
  - Compute Cost $\mathcal{J}^{\{t\}} =$
    $$\frac{1}{k} \sum_{i=1}^{l} \mathcal{L} \underbrace{\left(\hat{\mathbf{y}}^{(i)}, \mathbf{y}^{(i)}\right)}_{\text{For } \boldsymbol{X}^{\{t\}}, \boldsymbol{Y}^{\{t\}}} + \frac{\lambda}{2 \cdot k} \sum_{l} \|\boldsymbol{W}^{[l]}\|_F^2$$
  - Backpropagate to compute gradients w.r.t $\mathcal{J}^{\{t\}}$ $(d\boldsymbol{W}^{[l]}, d\boldsymbol{\beta}^{[l]}, d\boldsymbol{\gamma}^{[l]})$ (using $(\boldsymbol{X}^{\{t\}}, \boldsymbol{Y}^{\{t\}})$)
  - **for** *layer $l = 1, \dots, L$***:**
    - $\boldsymbol{W}^{[l]} := \boldsymbol{W}^{[l]} - \alpha \, d\boldsymbol{W}^{[l]}$
    - $\boldsymbol{\beta}^{[l]} := \boldsymbol{\beta}^{[l]} - \alpha d\boldsymbol{\beta}^{[l]}$
    - $\boldsymbol{\gamma}^{[l]} := \boldsymbol{\gamma}^{[l]} - \alpha d\boldsymbol{\gamma}^{[l]}$

6.2 Batch Norm as Regularization

- Each mini-batch $\boldsymbol{X}^{\{t\}}$ is scaled by the mean/variance computed on just that mini-batch.
- This adds some noise to the values $\mathbf{z}^{[l]}$ to scale them to $\tilde{\mathbf{z}}^{[l]}$ within that mini-batch. so similar to dropout, it adds some noise to each hidden layer's activations.
- This has a slight regularization effect.

6.3 Batch Norm at Test Time

Calculate the weighted average of $\boldsymbol{\mu}^{\{t\}[l]}, \boldsymbol{\sigma}^{\{t\}[l]}$ across all mini-batches $\boldsymbol{X}^{\{t\}}$

$$\begin{aligned} \mathbf{v}_{\boldsymbol{\mu}}^{\{t\}[l]} &= \beta_w \boldsymbol{\mu}^{\{t-1\}[l]} + (1 - \beta_w) \boldsymbol{\mu}^{\{t\}[l]} \\ \mathbf{v}_{\boldsymbol{\sigma}^2}^{\{t\}[l]} &= \beta_w \boldsymbol{\sigma}^{\{t-1\}[l]^2} + (1 - \beta_w) \boldsymbol{\sigma}^{\{t\}[l]^2} \end{aligned}$$

Bias correction:

$$\boldsymbol{\mu}^{[l]} = \frac{\mathbf{v}_{\boldsymbol{\mu}}^{\{t\}[l]}}{1 - \beta_w}$$

$$\boldsymbol{\sigma}^{[l]^2} = \frac{\mathbf{v}_{\boldsymbol{\sigma}^2}^{\{t\}[l]}}{1 - \beta_w}$$

Then Use them in forward-propagation:

$$\mathbf{z}^{[l](i)}_{\text{norm}} = \frac{\mathbf{z}^{[l](i)} - \boldsymbol{\mu}^{[l]}}{\sqrt{\boldsymbol{\sigma}^{[l]^2} + \varepsilon}}$$

$$\tilde{\mathbf{z}}^{[l](i)} = \boldsymbol{\gamma}^{[l]} \mathbf{z}^{[l](i)}_{\text{norm}} + \boldsymbol{\beta}^{[l]}$$

7 Multi-Class Classification

7.1 Softmax Layer

$$\mathbf{a}^{[L]} = g^{[L]}(\mathbf{z}^{[L]}) = \frac{e^{\mathbf{z}^{[L]}}}{\sum_{i=1}^{C} e^{z_i^{[L]}}}, \qquad a_i^{[L]} = g^{[L]}(z_i^{[L]}) = \frac{e^{z_i^{[L]}}}{\sum_{i=1}^{C} e^{z_i^{[L]}}}$$

If number of classes $C = 2$, then softmax reduces to logistic regression.

7.2 Loss Function

$$\mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) = -\sum_{j=1}^{C} y_j \log \hat{y}_j = -\mathbf{y}^{\mathsf{T}} \log(\hat{\mathbf{y}})$$

Cost :

$$\mathcal{J}\left(\boldsymbol{W}^{[1]}, \mathbf{b}^{[1]}, \dots, \boldsymbol{W}^{[L]}, \mathbf{b}^{[L]}\right) = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{\mathbf{y}}^{(i)}, \mathbf{y}^{(i)})$$

Chapter 3 Convolutional Neural Networks

1 Filters

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

Vertical edge detection filter

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

Horizontal edge detection filter

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Sobel filter

$$\begin{bmatrix} 3 & 0 & -3 \\ 10 & 0 & -10 \\ 3 & 0 & -3 \end{bmatrix}$$

Scharr filter

2 Notation

n: Original image dimension.
f: Filter size.
p: Padding size.
s: Stride.

3 Padding

Types of Padding:

1. Valid: no padding
2. Same padding: pad so that the output size is the same as the input size.

$$n + 2p - f + 1 = n$$
$$\therefore p = \frac{f-1}{2}$$

f is usually odd in same padding.

4 One Layer of CNN

$\mathbf{z}^{[l]} = \boldsymbol{W}^{[l]} * \mathbf{a}^{[l-1]} + \mathbf{b}^{[l]}$
$\mathbf{a}^{[l]} = g^{[l]}\left(\mathbf{z}^{[l]}\right)$

Input	$(\mathbf{a}^{[l-1]})$	size: $(n_H^{[l-1]}$	$\times n_W^{[l-1]}$	$\times n_c^{[l-1]})$		
Filter	$(\boldsymbol{W}^{[l]})$	size: $(f^{[l]}$	$\times f^{[l]}$	$\times n_c^{[l-1]}$	$\times n_c^{[l]})$
Bias	$(\mathbf{b}^{[l]})$	size: $(1$	$\times 1$	$\times 1$	$\times n_c^{[l]})$
Output	$(\mathbf{a}^{[l]})$	size: $(n_H^{[l]}$	$\times n_W^{[l]}$		$\times n_c^{[l]})$

$$\text{Number of parameters} = \text{size}\left(\boldsymbol{W}^{[l]}\right) + \text{size}\left(\mathbf{b}^{[l]}\right)$$
$$= (f^{[l]} \times f^{[l]} \times n_c^{[l-1]} + 1) \times n_c^{[l]}$$

Output size

$$= n_H^{[l]} \times n_W^{[l]} \times n_c^{[l]}$$
$$= \left\lfloor \frac{n_H^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor \times \left\lfloor \frac{n_W^{[l-1]} + 2p^{[l]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor \times n_c^{[l]}$$

Number of multiplication operations

$$= n_H^{[l]} \times n_W^{[l]} \times \left(f^{[l]} \times f^{[l]} \times n_c^{[l-1]} \times n_c^{[l]}\right)$$

Number of summation operations is the same as multiplication.

5 Pooling Layer

No parameters to learn.

Input size:	$(n_H^{[l-1]}$	$\times n_W^{[l-1]}$	$\times n_c)$
Filter size:	$(f^{[l]}$	$\times f^{[l]}$	$\times n_c)$
Output size:	$(n_H^{[l]}$	$\times n_W^{[l]}$	$\times n_c)$

$$\text{Output size} = \left(n_H^{[l]} \times n_W^{[l]} \times n_c \right)$$
$$= \left(\left\lfloor \frac{n_H^{[l-1]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor \times \left\lfloor \frac{n_W^{[l-1]} - f^{[l]}}{s^{[l]}} + 1 \right\rfloor \times n_c \right)$$

6 Residual Networks

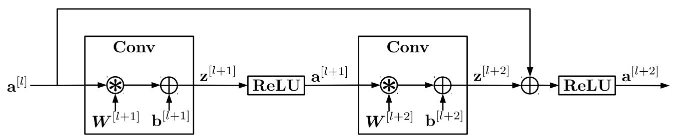
Source paper: [He+15]
Implementing "shortcut" / "skip connection" in a ResNet block:

$$\mathbf{z}^{[l+1]} = \boldsymbol{W}^{[l+1]} * \mathbf{a}^{[l]} + \mathbf{b}^{[l+1]}$$
$$\mathbf{a}^{[l+1]} = g^{[l+1]}\left(\mathbf{z}^{[l+1]}\right)$$
$$\mathbf{z}^{[l+2]} = \boldsymbol{W}^{[l+2]} * \mathbf{a}^{[l+1]} + \mathbf{b}^{[l+2]}$$

For identity block ($\mathbf{a}^{[l]}$ has the same dimensions as $\mathbf{a}^{[l+2]}$) :

$$\mathbf{a}^{[l+2]} = g^{[l+2]}\left(\mathbf{z}^{[l+2]} + \mathbf{a}^{[l]}\right)$$

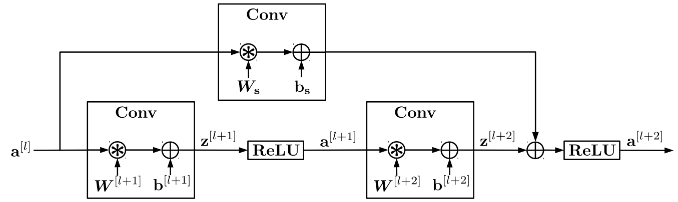
Figure 3.1: Identity block



If $\mathbf{a}^{[l]}$ has different dimensions than $\mathbf{a}^{[l+2]}$, then multiply $\mathbf{a}^{[l]}$ by an extra matrix $\boldsymbol{W_s}$

$$\mathbf{a}^{[l+2]} = g^{[l+2]}\left(\mathbf{z}^{[l+2]} + \boldsymbol{W_s} * \mathbf{a}^{[l]}\right)$$

Figure 3.2: Convolutional block



7 YOLO Object Detection

References: [Ser+13]

YOLO paper: [Red+15]

YOLO stands for "You Only Look Once"

7.1 Notation

$p_c^{[i]}$: the probability that there is an object for box number i (box i confidence probability)

$c_j^{[i]}$: the probability that the object in box i is a certain class j .

t: maximum number of boxes.

s: number of filtered(selected output boxes).

n_{grid}: Output grid size (number of grid cells in each row and column).

$b_x^{[i]}, b_y^{[i]}$: Midpoint coordinates of box i.

$b_w^{[i]}, b_h^{[i]}$: Height and width of box i.

7.2 The Algorithm

Algorithm 7: YOLO

Data: Input image of shape $(n_H, n_W, 3)$

Result:

A list of selected bounding boxes along with the recognized classes. Each bounding box is represented by 6 numbers $[p_c, b_x, b_y, b_h, b_w, c]^\mathsf{T}$. If you expand c into an n_{classes}-dimensional vector, each bounding box is then represented by $(5 + n_{\text{classes}})$ numbers. The output tensor shape is $(n_{\text{grid}}, n_{\text{grid}}, s, 6)$, where $s \le t$ and the last two dimensions can be represented by the matrix:

$$\begin{bmatrix} p_c^{[1]} & p_c^{[2]} & \dots & p_c^{[s]} \\ b_x^{[1]} & b_x^{[2]} & \dots & b_x^{[s]} \\ b_y^{[1]} & b_y^{[2]} & \dots & b_y^{[s]} \\ b_h^{[1]} & b_h^{[2]} & \dots & b_h^{[s]} \\ b_w^{[1]} & b_w^{[2]} & \dots & b_w^{[s]} \\ c^{[1]} & c^{[2]} & & c^{[s]} \end{bmatrix}$$

Steps

- The input image goes through a YOLO CNN Model, resulting in a $(n_{\text{grid}}, n_{\text{grid}}, t, 5 + n_{\text{classes}})$ dimensional output. The last two dimensions can be represented as the following matrix:

$$\begin{bmatrix} p_c^{[1]} & p_c^{[2]} & \dots & p_c^{[t]} \\ b_x^{[1]} & b_x^{[2]} & \dots & b_x^{[t]} \\ b_y^{[1]} & b_y^{[2]} & \dots & b_y^{[t]} \\ b_h^{[1]} & b_h^{[2]} & \dots & b_h^{[t]} \\ b_w^{[1]} & b_w^{[2]} & \dots & b_w^{[t]} \\ c_1^{[1]} & c_1^{[2]} & & c_1^{[t]} \\ \vdots & \vdots & \ddots & \\ c_{n_{\text{classes}}}^{[1]} & c_{n_{\text{classes}}}^{[2]} & & c_{n_{\text{classes}}}^{[t]} \end{bmatrix}$$

- From the output of the YOLO CNN model, extract the following: - `box_confidence` : tensor of shape $(n_{\text{grid}}, n_{\text{grid}}, t, 1)$. The last dimension containing p_c (confidence probability that there's some object) for each of the t boxes predicted in each of the $n_{\text{grid}} \times n_{\text{grid}}$ cells. The last two dimensions of the tensor can be represented as follows:

$$\begin{bmatrix} p_c^{[1]} & p_c^{[2]} & \dots & p_c^{[t]} \end{bmatrix}$$

- `boxes` : tensor of shape $(n_{\text{grid}}, n_{\text{grid}}, t, 4)$ containing the midpoint and dimensions $[b_x, b_y, b_h, b_w]^\mathsf{T}$ for each of the t boxes in each cell. The last two dimensions matrix is:

$$\begin{bmatrix} b_x^{[1]} & b_x^{[2]} & \dots & b_x^{[t]} \\ b_y^{[1]} & b_y^{[2]} & \dots & b_y^{[t]} \\ b_h^{[1]} & b_h^{[2]} & \dots & b_h^{[t]} \\ b_w^{[1]} & b_w^{[2]} & \dots & b_w^{[t]} \end{bmatrix}$$

- `box_class_probs` : tensor of shape $(n_{\text{grid}}, n_{\text{grid}}, t, n_{\text{classes}})$ containing the "class probabilities" $(c_1, c_2, ...c_{n_{\text{classes}}})$ for each of the n_{classes} classes for each of the t boxes per cell. The last two dimensions can be represented as:

$$\begin{bmatrix} c_1^{[1]} & c_1^{[2]} & & c_1^{[t]} \\ c_2^{[1]} & c_2^{[2]} & & c_2^{[t]} \\ \vdots & \vdots & \ddots & \\ c_{n_{\text{classes}}}^{[1]} & c_{n_{\text{classes}}}^{[2]} & & c_{n_{\text{classes}}}^{[t]} \end{bmatrix}$$

- Convert boxes to be ready for filtering functions (convert boxes from midpoint coordinates to corner coordinates):

$$\begin{bmatrix} b_x^{[1]} & b_x^{[2]} & \dots & b_x^{[t]} \\ b_y^{[1]} & b_y^{[2]} & \dots & b_y^{[t]} \\ b_h^{[1]} & b_h^{[2]} & \dots & b_h^{[t]} \\ b_w^{[1]} & b_w^{[2]} & \dots & b_w^{[t]} \end{bmatrix} \Rightarrow \begin{bmatrix} x_1^{[1]} & x_1^{[2]} & \dots & x_1^{[t]} \\ y_1^{[1]} & y_1^{[2]} & \dots & y_1^{[t]} \\ x_2^{[1]} & x_2^{[2]} & \dots & x_2^{[t]} \\ y_2^{[1]} & y_2^{[2]} & \dots & y_2^{[t]} \end{bmatrix}$$

- Calculate score and predicted class for each box:
  - Box classes: tensor of shape $(n_{\text{grid}}, n_{\text{grid}}, t, 1)$

`classes[j,k]`

$$= \begin{bmatrix} c^{[1]} & c^{[2]} & \dots & c^{[t]} \end{bmatrix}$$

$$= \operatorname{argmax} \left(\begin{bmatrix} c_1^{[1]} & c_1^{[2]} & & c_1^{[t]} \\ c_2^{[1]} & c_2^{[2]} & & c_2^{[t]} \\ \vdots & \vdots & \ddots & \\ c_{n_{\text{classes}}}^{[1]} & c_{n_{\text{classes}}}^{[2]} & & c_{n_{\text{classes}}}^{[t]} \end{bmatrix} \right)$$

  - Calculate box scores (the probability that the box contains a certain class):

The class score is $\text{scores}^{[i]} = p_c^{[i]} \times c^{[i]}$

`scores[j,k]`

$$= \begin{bmatrix} p_c^{[1]} & p_c^{[2]} & \dots & p_c^{[t]} \end{bmatrix} \odot \begin{bmatrix} c^{[1]} & c^{[2]} & \dots & c^{[t]} \end{bmatrix}$$

$$= \begin{bmatrix} p_c^{[1]} c^{[1]} & p_c^{[2]} c^{[2]} & \dots & p_c^{[t]} c^{[t]} \end{bmatrix}$$

- Select only few boxes using score-filtering and non-max suppression:
  - Perform Score-filtering with a threshold: throw away boxes that have detected a class with a $\mathit{scores}^{[i]} < \mathit{threshold}$.
  - Non-max suppression:

for each *class* c_i**:**

  Select the box that has the highest score.
  Compute the overlap of this box with all other boxes, and remove boxes that overlap significantly `(iou >= iou_threshold)` .
  Iterate until there are no more boxes with a lower score than the currently selected box.

```
/* The selected boxes count is less than the total number of boxes s ≤ t */
```

8 Face Recognition

8.1 One-Shot Learning

Learning a similarity function $d(\text{img1}, \text{img2})$ = degree of difference between images.

$$\text{If } d(\text{img1}, \text{img2}) \begin{cases} \leq \tau & \text{The two images are the same.} \\ > \tau & \text{The two images are the different.} \end{cases}$$

8.2 Siamese Network

Paper : [Tai+14]

Goal of Learning

- Parameters of the neural network define an encoding $f(\boldsymbol{X}^{(i)})$ of 128 units.
- Learn parameters so that:
  If $\boldsymbol{X}^{(i)}, \boldsymbol{X}^{(j)}$ are the same person, $d(\boldsymbol{X}^{(i)}, \boldsymbol{X}^{(j)})$ is small.
  If $\boldsymbol{X}^{(i)}, \boldsymbol{X}^{(j)}$ are different persons, $d(\boldsymbol{X}^{(i)}, \boldsymbol{X}^{(j)})$ is large.

$$d(\boldsymbol{X}^{(i)}, \boldsymbol{X}^{(j)}) = \left\| f(\boldsymbol{X}^{(i)}) - f(\boldsymbol{X}^{(j)}) \right\|_2^2$$

8.3 Triplet Loss

Paper : [SKP15]

Given three input images: an anchor image $\boldsymbol{A}$, a positive image $\boldsymbol{P}$ and a negative image $\boldsymbol{N}$,

We want

$$\|f(\boldsymbol{A}) - f(\boldsymbol{P})\|_2^2 + \alpha \leq \|f(\boldsymbol{A}) - f(\boldsymbol{N})\|_2^2$$

$$\therefore \|f(\boldsymbol{A}) - f(\boldsymbol{P})\|_2^2 + \alpha - \|f(\boldsymbol{A}) - f(\boldsymbol{N})\|_2^2 \leq 0$$

We define triplet loss function as:

$$\mathcal{L}(\boldsymbol{A}, \boldsymbol{P}, \boldsymbol{N}) = \max\left(\|f(\boldsymbol{A}) - f(\boldsymbol{P})\|_2^2 - \|f(\boldsymbol{A}) - f(\boldsymbol{N})\|_2^2 + \alpha, 0\right)$$
$$= \left[\underbrace{\|f(\boldsymbol{A}) - f(\boldsymbol{P})\|_2^2}_{(1)} - \underbrace{\|f(\boldsymbol{A}) - f(\boldsymbol{N})\|_2^2}_{(2)} + \alpha\right]_+$$

where,

- The term (1) is the squared distance between the anchor $\boldsymbol{A}$ and the positive $\boldsymbol{P}$ for a given triplet; you want this to be small.
- The term (2) is the squared distance between the anchor $\boldsymbol{A}$ and the negative $\boldsymbol{N}$ for a given triplet, you want this to be relatively large. It has a minus sign preceding it because minimizing the negative of the term is the same as maximizing that term.
- α is called the margin. It is a hyperparameter that you pick manually.

Triplet cost function can be defined as

$$\mathcal{J} = \sum_{i=1}^{m} \mathcal{L}(\boldsymbol{A}^{(i)}, \boldsymbol{P}^{(i)}, \boldsymbol{N}^{(i)})$$

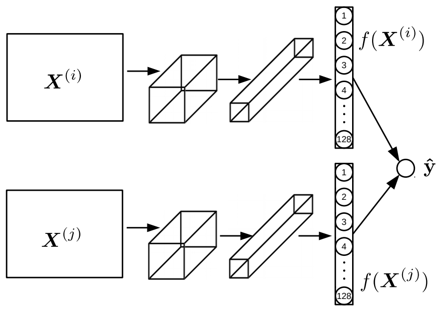
8.4 Face Verification and Binary Classification

Paper : [Tai+14]

Verification: Input is an image and name/ID. Output whether the input image is that of the claimed person.

Recognition: Has a database of K persons. Get an input image and output ID if the image is any of the K persons (or "not recognized").

Learning a Similarity Function for Face Verification



$$\hat{\mathbf{y}} = \sigma\left(\sum_{k=1}^{128} W_k \underbrace{\left| f(\boldsymbol{X}^{(i)})_k - f(\boldsymbol{X}^{(j)})_k \right|}_{(1)} + b\right)$$

Term (1) can also be the chi square ($\boldsymbol{\chi^2}$) formula:

$$\boldsymbol{\chi^2} = \frac{\left[f(\boldsymbol{X}^{(i)})_k - f(\boldsymbol{X}^{(j)})_k\right]^2}{f(\boldsymbol{X}^{(i)})_k + f(\boldsymbol{X}^{(j)})_k}$$

9 Neural Image Style Transfer

References: [ZF13], [GEB15]

The goal is to generate an image $\boldsymbol{G}$ from a content image $\boldsymbol{C}$ and a style image $\boldsymbol{S}$.

9.1 Total Cost Function

$$\mathcal{J}(\boldsymbol{G}) = \alpha \mathcal{J}_{\text{content}}(\boldsymbol{C}, \boldsymbol{G}) + \beta \mathcal{J}_{\text{style}}(\boldsymbol{S}, \boldsymbol{G})$$

Where $\mathcal{J}_{\text{content}}$ is the content cost and $\mathcal{J}_{\text{style}}$ is the style cost.

To find the generated image $\boldsymbol{G}$:

- Initiate $\boldsymbol{G}$ randomly
- Use gradient descent to minimize $\mathcal{J}(\boldsymbol{G})$:

$$\boldsymbol{G} := \boldsymbol{G} - \frac{\partial}{\partial \boldsymbol{G}} \mathcal{J}(\boldsymbol{G})$$

9.2 Content Cost

- Say you use a hidden layer l to compute content cost.
- Use pre-trained ConvNet. (E.g., VGG network).
- Let $\mathbf{a}^{[l](\boldsymbol{C})}$ and $\mathbf{a}^{[l](\boldsymbol{G})}$ be the activation of layer l on the images. If they are similar then both images have similar content. The content cost function is:

$$\mathcal{J}_{\text{content}}(\boldsymbol{C}, \boldsymbol{G}) = \frac{1}{2} \left\| \mathbf{a}^{[l](\boldsymbol{C})} - \mathbf{a}^{[l](\boldsymbol{G})} \right\|_F^2$$
$$= \frac{1}{2} \sum_{i=1}^{n_H^{[l]}} \sum_{j=1}^{n_W^{[l]}} \sum_{k=1}^{n_c^{[l]}} \left(a_{ijk}^{[l](\boldsymbol{C})} - a_{ijk}^{[l](\boldsymbol{G})} \right)^2$$

9.3 Style Cost

Gram matrix

Let $a_{i,j,k}^{[l]}$ be an element of an activation $\mathbf{a}^{[l]}$ of an input image at layer l at (i, j, k). Then the *Gram matrix* $\boldsymbol{G}_{(\text{gram})}^{[l]}$ has a shape of $n_c^{[l]} \times n_c^{[l]}$ and the matrix elements can be calculated as :

$$G_{(\text{gram})kk'}^{[l]} = \sum_{i=1}^{n_H^{[l]}} \sum_{j=1}^{n_W^{[l]}} a_{ijk}^{[l]} a_{ijk'}^{[l]}$$

Gram matrix captures the degree of correlation between a layer l channels as a measure of the style.

Style Cost Function

First calculate the gram matrix for the style image $\boldsymbol{G}_{(\text{gram})}^{[l](\boldsymbol{S})}$ and the generated image $\boldsymbol{G}_{(\text{gram})}^{[l](\boldsymbol{G})}$ for every layer l. Then the style cost function for a layer l is

$$\mathcal{J}_{\text{style}}^{[l]}(\boldsymbol{S}, \boldsymbol{G}) = \frac{1}{\left(2 n_H^{[l]} n_W^{[l]} n_c^{[l]}\right)^2} \left\| \boldsymbol{G}_{(\text{gram})}^{[l](\boldsymbol{S})} - \boldsymbol{G}_{(\text{gram})}^{[l](\boldsymbol{G})} \right\|_F^2$$
$$= \frac{1}{\left(2 n_H^{[l]} n_W^{[l]} n_c^{[l]}\right)^2} \sum_{i=1}^{n_c^{[l]}} \sum_{j=1}^{n_c^{[l]}} \left(G_{(\text{gram})ij}^{[l](\boldsymbol{S})} - G_{(\text{gram})ij}^{[l](\boldsymbol{G})} \right)^2$$

And the style cost function for all layers:

$$\mathcal{J}_{\text{style}}(\boldsymbol{S}, \boldsymbol{G}) = \sum_{l} \lambda^{[l]} \mathcal{J}_{\text{style}}^{[l]}(\boldsymbol{S}, \boldsymbol{G})$$

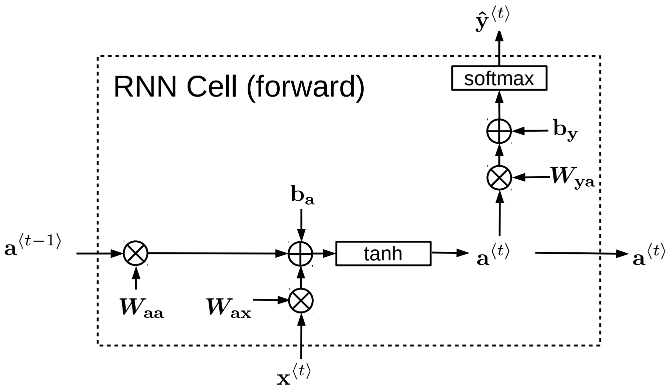
Chapter 4
Sequence Models

1 Recurrent Neural Networks

1.1 Notation

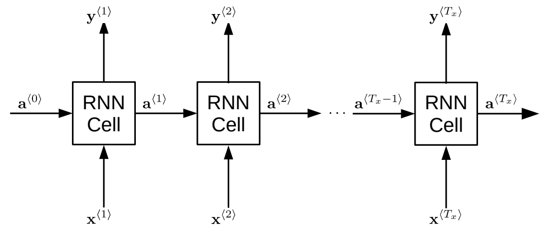
$\mathbf{x}^{\langle t\rangle}$: A one-dimensional input vector of a single example at time step t.
$\mathbf{y}^{\langle t\rangle}$: Output label at time step t.
$\hat{\mathbf{y}}^{\langle t\rangle}$: Prediction at time step t.
$\mathbf{a}^{\langle t\rangle}$: Hidden state, The activation that is passed to the RNN from one time step to another.
T_x: Length of input sequence.
T_y: Length of output sequence.
n_x: Number of units in input.
n_y: Number of units in output.
m: batch size.
$\boldsymbol{W}$: Weight matrix.
$\mathbf{b}$: Bias vector.

1.2 Recurrent Neural Networks

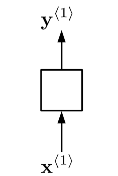


$$\begin{aligned}\mathbf{a}^{\langle t\rangle} &= \tanh\left(\boldsymbol{W}_{\mathbf{aa}}\mathbf{a}^{\langle t-1\rangle} + \boldsymbol{W}_{\mathbf{ax}}\mathbf{x}^{\langle t\rangle} + \mathbf{b_a}\right)\\ &= \tanh\left(\left[\boldsymbol{W}_{\mathbf{aa}} \mid \boldsymbol{W}_{\mathbf{ax}}\right]\begin{bmatrix}\mathbf{a}^{\langle t-1\rangle}\\ \mathbf{x}^{\langle t\rangle}\end{bmatrix} + \mathbf{b_a}\right)\\ &= \tanh\left(\boldsymbol{W}_{\mathbf{a}}\left[\mathbf{a}^{\langle t-1\rangle}, \mathbf{x}^{\langle t\rangle}\right] + \mathbf{b_a}\right)\end{aligned}$$

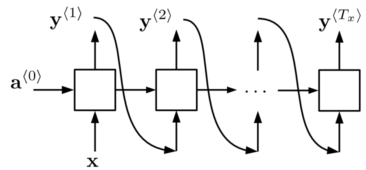
$$\begin{aligned}\hat{\mathbf{y}}^{\langle t\rangle} &= \text{softmax}\left(\boldsymbol{W}_{\mathbf{ya}}\mathbf{a}^{\langle t\rangle} + \mathbf{b_y}\right)\\ &= \text{softmax}\left(\boldsymbol{W}_{\mathbf{y}}\ \mathbf{a}^{\langle t\rangle} + \mathbf{b_y}\right)\end{aligned}$$



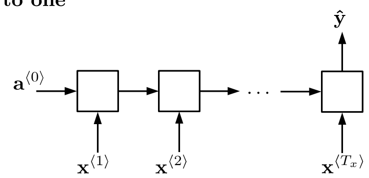
RNN Types



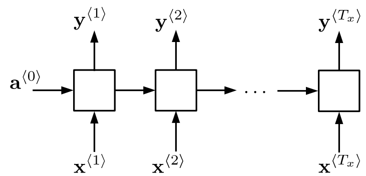
One to one



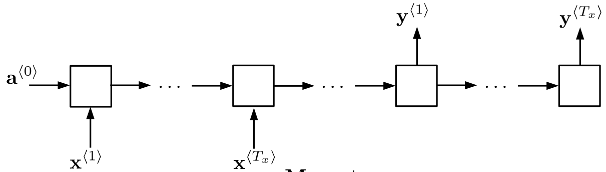
One to many



Many to one



Many to many



Many to many

Loss Function

$$\mathcal{L}^{\langle t\rangle}\left(\hat{\mathbf{y}}^{\langle t\rangle}, \mathbf{y}^{\langle t\rangle}\right) = -\mathbf{y}^{\langle t\rangle}\log\left(\hat{\mathbf{y}}^{\langle t\rangle}\right) - \left(1-\mathbf{y}^{\langle t\rangle}\right)\log\left(1-\hat{\mathbf{y}}^{\langle t\rangle}\right)$$

$$\mathcal{J}\left(\hat{\mathbf{y}}, \mathbf{y}\right) = \sum_{t=1}^{T_y}\mathcal{L}^{\langle t\rangle}\left(\hat{\mathbf{y}}^{\langle t\rangle}, \mathbf{y}^{\langle t\rangle}\right)$$

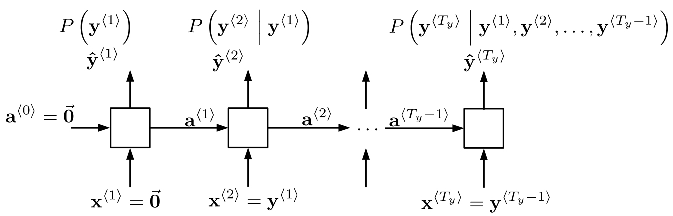
1.3 Language Model and Sequence Generation

$$P(\text{Sentence}) = P\left(\mathbf{y}^{\langle 1\rangle}, \mathbf{y}^{\langle 2\rangle}, \ldots, \mathbf{y}^{\langle T_y\rangle}\right)$$

Training

$$\begin{aligned}P\left(\mathbf{y}^{\langle 1\rangle}, \mathbf{y}^{\langle 2\rangle}, \ldots, \mathbf{y}^{\langle n\rangle}\right) =& P\left(\mathbf{y}^{\langle 1\rangle}\right) P\left(\mathbf{y}^{\langle 2\rangle} \,\middle|\, \mathbf{y}^{\langle 1\rangle}\right) P\left(\mathbf{y}^{\langle 3\rangle} \,\middle|\, \mathbf{y}^{\langle 1\rangle}, \mathbf{y}^{\langle 2\rangle}\right)\\ &\ldots P\left(\mathbf{y}^{\langle n\rangle} \,\middle|\, \mathbf{y}^{\langle 1\rangle}, \mathbf{y}^{\langle 2\rangle}, \ldots, \mathbf{y}^{\langle n-1\rangle}\right)\end{aligned}$$

$$\mathbf{x}^{\langle t+1\rangle} = \mathbf{y}^{\langle t\rangle}$$

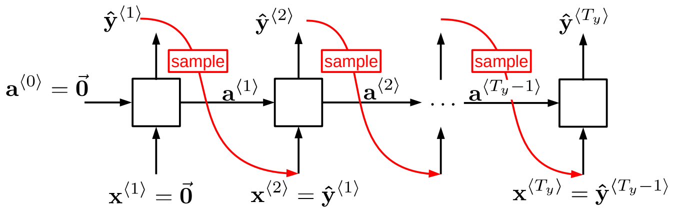


Loss Function

$$\mathcal{L}^{\langle t\rangle}\left(\hat{\mathbf{y}}^{\langle t\rangle}, \mathbf{y}^{\langle t\rangle}\right) = -\sum_i \mathbf{y}_i^{\langle t\rangle}\log\hat{\mathbf{y}}_i^{\langle t\rangle}$$

$$\mathcal{J} = \sum_t \mathcal{L}^{\langle t\rangle}\left(\hat{\mathbf{y}}^{\langle t\rangle}, \mathbf{y}^{\langle t\rangle}\right)$$

Sampling a Sequence from Trained RNN



1.4 Gated Recurrent Unit(GRU)

References: [Cho+14b], [Chu+14]

Notation

$\mathbf{c}^{\langle t\rangle}$: Memory cell state(variable) at time step t.
$\tilde{\mathbf{c}}^{\langle t\rangle}$: Candidate value for cell state.Contains information from the current time step that **may** be stored in the current cell state $\mathbf{c}^{\langle t\rangle}$. Contains values between -1 and 1.
$\boldsymbol{\Gamma}_u^{\langle t\rangle}$: Update gate. Used to decide what aspects of the candidate $\tilde{\mathbf{c}}^{\langle t\rangle}$ to add to the cell state $\mathbf{c}^{\langle t\rangle}$. It contains values that range between 0 and 1.

GRU(Full)

$$\mathbf{c}^{\langle t-1\rangle} = \mathbf{a}^{\langle t-1\rangle}$$
$$\boldsymbol{\Gamma}_u^{\langle t\rangle} = \sigma\left(\boldsymbol{W}_{\mathbf{u}}\left[\mathbf{c}^{\langle t-1\rangle}, \mathbf{x}^{\langle t\rangle}\right] + \mathbf{b}_{\mathbf{u}}\right)$$
$$\boldsymbol{\Gamma}_r^{\langle t\rangle} = \sigma\left(\boldsymbol{W}_{\mathbf{r}}\left[\mathbf{c}^{\langle t-1\rangle}, \mathbf{x}^{\langle t\rangle}\right] + \mathbf{b}_{\mathbf{r}}\right)$$
$$\tilde{\mathbf{c}}^{\langle t\rangle} = \tanh\left(\boldsymbol{W}_{\mathbf{c}}\left[\boldsymbol{\Gamma}_r^{\langle t\rangle} \odot \mathbf{c}^{\langle t-1\rangle}, \mathbf{x}^{\langle t\rangle}\right] + \mathbf{b}_{\mathbf{c}}\right)$$
$$\mathbf{c}^{\langle t\rangle} = \mathbf{a}^{\langle t\rangle} = \boldsymbol{\Gamma}_u^{\langle t\rangle} \odot \tilde{\mathbf{c}}^{\langle t\rangle} + \left(1 - \boldsymbol{\Gamma}_u^{\langle t\rangle}\right) \odot \mathbf{c}^{\langle t-1\rangle}$$
$$\hat{\mathbf{y}}^{\langle t\rangle} = \operatorname{softmax}\left(\boldsymbol{W}_{\mathbf{y}}\mathbf{a}^{\langle t\rangle} + \mathbf{b}_{\mathbf{y}}\right)$$

1.5 Long Short Term Memory(LSTM)

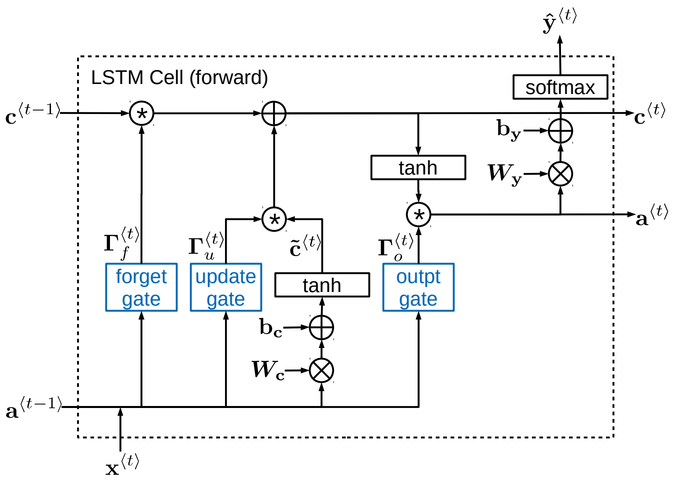
Paper: [HS97]

Notation

$\boldsymbol{\Gamma}_f^{\langle t\rangle}$: Forget gate. It contains values that range between 0 and 1.
$\boldsymbol{\Gamma}_o^{\langle t\rangle}$: Output gate. Decides what gets sent as the prediction (output) of the time step. It contains values that range between 0 and 1.
$\mathbf{a}^{\langle t\rangle}$: Hidden state. Values between -1 and 1.

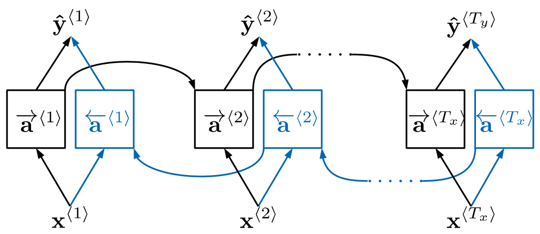
Calculations

$$\tilde{\mathbf{c}}^{\langle t\rangle} = \tanh\left(\boldsymbol{W}_{\mathbf{c}}\left[\mathbf{a}^{\langle t-1\rangle}, \mathbf{x}^{\langle t\rangle}\right] + \mathbf{b}_{\mathbf{c}}\right)$$
$$\boldsymbol{\Gamma}_u^{\langle t\rangle} = \sigma\left(\boldsymbol{W}_{\mathbf{u}}\left[\mathbf{a}^{\langle t-1\rangle}, \mathbf{x}^{\langle t\rangle}\right] + \mathbf{b}_{\mathbf{u}}\right)$$
$$\boldsymbol{\Gamma}_f^{\langle t\rangle} = \sigma\left(\boldsymbol{W}_{\mathbf{f}}\left[\mathbf{a}^{\langle t-1\rangle}, \mathbf{x}^{\langle t\rangle}\right] + \mathbf{b}_{\mathbf{f}}\right)$$
$$\boldsymbol{\Gamma}_o^{\langle t\rangle} = \sigma\left(\boldsymbol{W}_{\mathbf{o}}\left[\mathbf{a}^{\langle t-1\rangle}, \mathbf{x}^{\langle t\rangle}\right] + \mathbf{b}_{\mathbf{o}}\right)$$
$$\mathbf{c}^{\langle t\rangle} = \boldsymbol{\Gamma}_u^{\langle t\rangle} \odot \tilde{\mathbf{c}}^{\langle t\rangle} + \boldsymbol{\Gamma}_f^{\langle t\rangle} \odot \mathbf{c}^{\langle t-1\rangle}$$
$$\mathbf{a}^{\langle t\rangle} = \boldsymbol{\Gamma}_o^{\langle t\rangle} \odot \tanh\left(\mathbf{c}^{\langle t\rangle}\right)$$
$$\hat{\mathbf{y}}^{\langle t\rangle} = \operatorname{softmax}\left(\boldsymbol{W}_{\mathbf{y}}\mathbf{a}^{\langle t\rangle} + \mathbf{b}_{\mathbf{y}}\right)$$



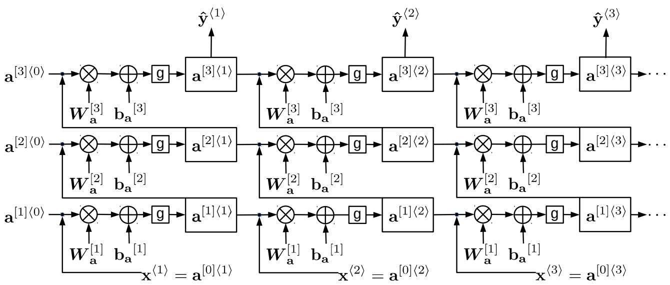
1.6 Bidirectional RNN

$$\hat{\mathbf{y}}^{\langle t\rangle} = g\left(\boldsymbol{W}_{\mathbf{y}}\left[\vec{\mathbf{a}}^{\langle t\rangle}, \overleftarrow{\mathbf{a}}^{\langle t\rangle}\right] + \mathbf{b}_{\mathbf{y}}\right)$$



1.7 Deep RNNs

$$\mathbf{a}^{[l]\langle t\rangle} = g\left(\boldsymbol{W}_{\mathbf{a}}^{[l]}\left[\mathbf{a}^{[l]\langle t-1\rangle}, \mathbf{a}^{[l-1]\langle t\rangle}\right] + \mathbf{b}_{\mathbf{a}}^{[l]}\right)$$



2 Natural Language Processing and Word Embeddings

2.1 Notation

n_v: Vocabulary size.
n_e: Embedding size, $n_e << n_v$.
$\mathbf{o}_i$: One-hot vector for a word i. Its' length is n_v.
$\mathbf{e}_i$: Feature vector (word embedding vector) for a word i.Its' length is n_e.
$\boldsymbol{O}$: One-hot matrix, of size $n_v \times n_v$.
$\boldsymbol{E}$: Embedding matrix, of size $n_e \times n_v$.

2.2 Word Representation

Reference: Visualizing word embeddings [MH08]

2.3 Transfer learning and word embeddings

1. Learn word embeddings from a large text corpus. (1 - 100B words) (Or download pre-trained embedding online.).
2. Transfer embedding to new task with smaller training set. (eg. 100k words).
3. Optional : continue to fine-tune the word embeddings with new data.

2.4 Properties of Word Embeddings

Reference : [MYZ13].

Analogies using word vectors

$$\mathbf{e}_{\text{man}} - \mathbf{e}_{\text{woman}} \approx \mathbf{e}_{\text{king}} - \mathbf{e}_w$$

Find a word w that maximizes the similarity function:

$$\arg\max_w(\operatorname{sim}(\mathbf{e}_w, \mathbf{e}_{\text{king}} - \mathbf{e}_{\text{man}} + \mathbf{e}_{\text{woman}}))$$

The similarity function can be one of the following:

- Cosine similarity (more frequently used)

$$\operatorname{sim}(\mathbf{u}, \mathbf{v}) = \frac{\mathbf{u}^{\mathsf{T}}\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \cos(\theta)$$

  Where θ is the angle between the two vectors.

- Squared distance:

$$\operatorname{sim}(\mathbf{u}, \mathbf{v}) = \|\mathbf{u} - \mathbf{v}\|^2$$

2.5 Embedding Matrix

$$\boldsymbol{E} \cdot \mathbf{o}_j = \mathbf{e}_j$$

In practice we use a specialized function to look up an embedding instead of matrix-vector multiplication.

2.6 A Simple Language Model

Reference: [Ben+03]
Given an input sequence of words for an example i , with embeddings. $\left[\mathbf{e}_1^{(i)} \quad \mathbf{e}_2^{(i)} \quad \ldots \quad \mathbf{e}_{T_x}^{(i)}\right]$
First, calculate the average of the sequence embeddings:

$$\boldsymbol{\mu}_{\mathbf{e}}^{(i)} = \mathbb{E}\left[\left[\mathbf{e}_1^{(i)} \quad \mathbf{e}_2^{(i)} \quad \ldots \quad \mathbf{e}_{T_x}^{(i)}\right]\right] = \frac{1}{T_x}\sum_{n=1}^{T_x}\mathbf{e}_n^{(i)}$$

Forward propagation:

$$\mathbf{z}^{(i)} = \boldsymbol{W}\boldsymbol{\mu}_{\mathbf{e}}^{(i)} + \mathbf{b}$$
$$\hat{\mathbf{y}}^{(i)} = \mathbf{a}^{(i)} = \operatorname{softmax}(\mathbf{z}^{(i)})$$

Loss function:

$$\mathcal{L}\left(\hat{\mathbf{y}}^{(i)}, \mathbf{y}^{(i)}\right) = \sum_{k=1}^{n_y} y_k^{(i)}\log\left(\hat{y}_k^{(i)}\right) = -\mathbf{y}^{(i)\mathsf{T}}\log\left(\hat{\mathbf{y}}^{(i)}\right)$$

Backpropagation:

$$\frac{\partial\mathcal{L}}{\partial\mathbf{z}^{(i)}} = \mathbf{a}^{(i)} - \mathbf{y}^{(i)}$$
$$\frac{\partial\mathbf{z}^{(i)}}{\partial\boldsymbol{W}} = \boldsymbol{\mu}_{\mathbf{e}}^{(i)}$$
$$\frac{\partial\mathbf{z}^{(i)}}{\partial\mathbf{b}} = \vec{\mathbf{1}}$$
$$\frac{\partial\mathcal{L}}{\partial\boldsymbol{W}} = \frac{\partial\mathcal{L}}{\partial\mathbf{z}^{(i)}}\frac{\partial\mathbf{z}^{(i)}}{\partial\boldsymbol{W}}$$
$$\frac{\partial\mathcal{L}}{\partial\mathbf{b}} = \frac{\partial\mathcal{L}}{\partial\mathbf{z}^{(i)}}\frac{\partial\mathbf{z}^{(i)}}{\partial\mathbf{b}}$$

2.7 Word2Vec

Reference: [Mik+13b]

Notation:

t: *target word*, the word we want to predict.
c: *context word*, n words before and/or after the target word.

Word2Vec Model (Skipgram model)

Vocabulary size : n_v, embedding size: n_e (for Word2Vec $n_e = 300$).

$$\mathbf{o}_c \xrightarrow{(\boldsymbol{E})} \mathbf{e}_c \xrightarrow{(\boldsymbol{\Theta})} \mathbf{z} \xrightarrow{(\text{softmax})} \hat{\mathbf{y}}$$

Where $\boldsymbol{\Theta}$ is parameter matrix, its size is $n_e \times n_v$

$$\mathbf{e}_c = \boldsymbol{E}\mathbf{o}_c$$
$$\mathbf{z} = \boldsymbol{\Theta}^\top \mathbf{e}_c$$

$$\hat{\mathbf{y}} = \text{softmax}(\mathbf{z}) = \frac{e^{\mathbf{z}}}{\sum_{i=1}^{n_v} e^{z_i}}$$

$$\hat{y}_t = P(t|c) = \frac{e^{\boldsymbol{\theta}_t^\top \mathbf{e}_c}}{\sum_{j=1}^{n_v} e^{\boldsymbol{\theta}_j^\top \mathbf{e}_c}}$$

Where $\boldsymbol{\theta}_j$ is a column vector of the parameter matrix $\boldsymbol{\Theta}$, $\boldsymbol{\theta}_t$ is the parameter vector associated with the output target word t.
The downside of the *skipgram* model is that the softmax objective function is expensive to compute.

Loss function

$$\mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) = -\sum_{i=1}^{n_v} y_i \log(\hat{y}_i) = -\mathbf{y}^\top \log(\hat{\mathbf{y}}_i)$$

2.8 Negative Sampling

Reference: [Mik+13a]
k: Number of negative examples.
y: Target label. 1 for positive example, 0 for negative example.

Model

$$P(y = 1|t, c) = \sigma\left(\boldsymbol{\theta}_t^\top \mathbf{e}_c\right)$$

On every iteration, choose k different random negative words with which to train the algorithm on. So the total number of training examples is $k + 1$ (including one positive example).

Selecting Negative Examples

Sample according to the empirical frequency of words in your corpus.

$$P(w_i) = \frac{f(w_i)^{3/4}}{\sum_{j=1}^{n_v} f(w_j)^{3/4}}$$

2.9 GloVe Word Vectors

Reference: [PSM14]
X_{ij}: Is the number of times word $\underbrace{j}_{t}$ occurs in the context of word $\underbrace{i}_{c}$.
It is a count that captures how often do words i and j appear close to each other.
If you define context to be $\pm n$ words after and before target word, then $\boldsymbol{X}$ is symmetric ($X_{ij} = X_{ji}$)

Model

$$\text{Minimize} \sum_{i=1}^{n_v} \sum_{j=1}^{n_v} \underbrace{f(X_{ij})}_{(1)} \left(\boldsymbol{\theta}_i^\top \mathbf{e}_j + b_i + b'_j - \log(X_{ij})\right)^2$$

- Term (1), $f(X_{ij})$ is a weighted sum.
- $f(X_{ij}) = 0$ if $X_{ij} = 0$, so the expression evaluates to zero ($0\log(0) = 0$).
- $\boldsymbol{\theta}_i$ and $\mathbf{e}_j$ are symmetric. They end up with the same optimization objective.
- Initialize $\boldsymbol{\theta}_w$ and $\mathbf{e}_w$ at random for every word, run gradient descent to optimize them, then take the average of $\boldsymbol{\theta}_w$ and $\mathbf{e}_w$ to calculate the final embedding:

$$\mathbf{e}_w^{(\text{final})} = \frac{\mathbf{e}_w + \boldsymbol{\theta}_w}{2}$$

2.10 Debiasing Word Embeddings

Word embeddings can reflect gender, ethnicity, age, sexual orientation and other biases of the text used to train the model, so they need to be debiased [Bol+16].
Addressing bias in word embeddings:

- Identify the bias direction (gender subspace).

  Collect n pairs of embedding vectors that differ by gender (masculine m and feminine f), subtract them, then average the result to get the bias vector $\mathbf{b}$:

$$\mathbf{b} = \frac{1}{n}\sum_{i=1}^{n}\left(\mathbf{e}_m^{(i)} - \mathbf{e}_f^{(i)}\right)$$

- Neutralize: For every word embedding that is not definitional, project to get rid of bias.

  First calculate the *bias component* $\mathbf{e}_{\boldsymbol{B}}$

$$\mathbf{e}_{\boldsymbol{B}} = \text{proj}_{\mathbf{b}}\mathbf{e} = \frac{\mathbf{e}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\mathbf{b}$$

  The debiased embedding vector $\mathbf{e}^\perp$ is the orthonormal vector to $\mathbf{e}$ it is obtained by zeroing out the component in the direction of $\mathbf{b}$:

$$\mathbf{e}^\perp = \mathbf{e} - \mathbf{e}_{\boldsymbol{B}}$$

- Equalize pairs.

  For a pair of words $w1, w2$ that differ by gender:

$$\boldsymbol{\mu} = \frac{\mathbf{e}_{w1} + \mathbf{e}_{w2}}{2}$$
$$\boldsymbol{\mu}_{\boldsymbol{B}} = \frac{\boldsymbol{\mu}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\mathbf{b}$$
$$\boldsymbol{\mu}^\perp = \boldsymbol{\mu} - \boldsymbol{\mu}_{\boldsymbol{B}}$$
$$\mathbf{e}_{w1_{\boldsymbol{B}}} = \frac{\mathbf{e}_{w1}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\mathbf{b} \qquad \mathbf{e}_{w2_{\boldsymbol{B}}} = \frac{\mathbf{e}_{w2}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\mathbf{b}$$
$$\mathbf{e}_{w1_{\boldsymbol{B}}}^{(\text{corrected})} = \sqrt{\left|1 - \|\boldsymbol{\mu}^\perp\|_2^2\right|} \odot \frac{\mathbf{e}_{w1_{\boldsymbol{B}}} - \boldsymbol{\mu}_{\boldsymbol{B}}}{\|(\mathbf{e}_{w1} - \boldsymbol{\mu}^\perp) - \boldsymbol{\mu}_{\boldsymbol{B}}\|}$$
$$\mathbf{e}_{w2_{\boldsymbol{B}}}^{(\text{corrected})} = \sqrt{\left|1 - \|\boldsymbol{\mu}^\perp\|_2^2\right|} \odot \frac{\mathbf{e}_{w2_{\boldsymbol{B}}} - \boldsymbol{\mu}_{\boldsymbol{B}}}{\|(\mathbf{e}_{w2} - \boldsymbol{\mu}^\perp) - \boldsymbol{\mu}_{\boldsymbol{B}}\|}$$
$$\mathbf{e}_1 = \mathbf{e}_{w1_{\boldsymbol{B}}}^{(\text{corrected})} + \boldsymbol{\mu}^\perp$$
$$\mathbf{e}_2 = \mathbf{e}_{w2_{\boldsymbol{B}}}^{(\text{corrected})} + \boldsymbol{\mu}^\perp$$

3 Various Sequence to Sequence Architectures

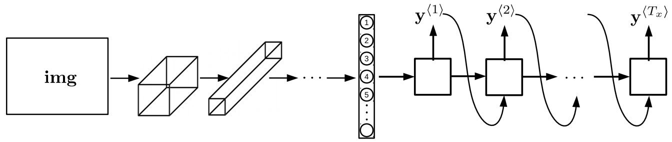
3.1 Basic Models

Sequence to sequence model

References: [SVL14], [Cho+14a]

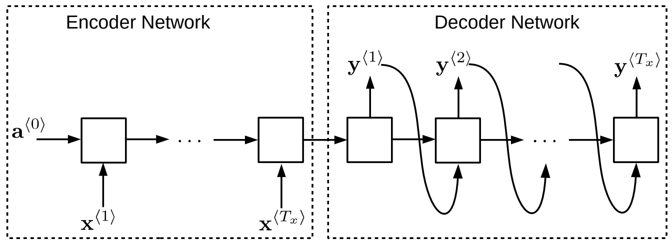
Image Captioning

References: [Mao+14], [Vin+14], [KL15]



3.2 Machine Translation

Building a Conditional Language Model



The model output the conditional probability:

$$P\left(\mathbf{y}^{\langle 1 \rangle}, \ldots, \mathbf{y}^{\langle T_y \rangle} \middle| \mathbf{x}^{\langle 1 \rangle}, \ldots, \mathbf{x}^{\langle Tx \rangle}\right)$$

In this model you don't sample words at random. Instead you find a sentence $\mathbf{y}$ that maximizes the conditional probability.
The most common algorithm to do this is called *beam search*

Beam Search

B: Beam width parameter, the number of possibilities for beam search to consider at a time.

Normalized log probability objective function (normalized log likelihood objective):

$$\begin{aligned}\frac{1}{T_y^\alpha}\log P\left(\hat{\mathbf{y}}\middle|\mathbf{x}\right) &= \frac{1}{T_y^\alpha}\log P\left(\hat{\mathbf{y}}^{\langle 1\rangle},\ldots,\hat{\mathbf{y}}^{\langle T_y\rangle}\middle|\mathbf{x}\right)\\ &= \frac{1}{T_y^\alpha}\log\prod_{t=1}^{T_y} P\left(\hat{\mathbf{y}}^{\langle t\rangle}\middle|\mathbf{x},\hat{\mathbf{y}}^{\langle 1\rangle},\ldots,\hat{\mathbf{y}}^{\langle t-1\rangle}\right)\\ &= \frac{1}{T_y^\alpha}\sum_{t=1}^{T_y}\log P\left(\hat{\mathbf{y}}^{\langle t\rangle}\middle|\mathbf{x},\hat{\mathbf{y}}^{\langle 1\rangle},\ldots,\hat{\mathbf{y}}^{\langle t-1\rangle}\right)\end{aligned}$$

Algorithm 8: Beam Search

Data: An input sequence $\mathbf{x}$, its length is T_x
Result: A sequence of predictions $\hat{\mathbf{y}}$, its length is T_y

1 Run the input sentence $\mathbf{x}$ through the encoder network.
2 Pick the the top B words from the first output of the sequence of the decoder network ($\hat{\mathbf{y}}^{\langle 1\rangle}$) with the highest probabilities as the first predicted word in the sequence.
3 **for** *sentence lengths T_y starting from 2*:
4 keep track of the top B sentences that maximize the normalized log probability objective function (normalized log likelihood objective).

$$\arg\max_{\hat{\mathbf{y}}}\left(\frac{1}{T_y^\alpha}\sum_{t=1}^{T_y}\log P\left(\hat{\mathbf{y}}^{\langle t\rangle}\middle|\mathbf{x},\hat{\mathbf{y}}^{\langle 1\rangle},\ldots,\hat{\mathbf{y}}^{\langle t-1\rangle}\right)\right)$$

5 Repeat and increment T_y until encountering an end of sentence character $\langle\text{EOS}\rangle$ for all B sentences.
6 Finally, pick up one sentence from B sentences with the highest value of normalized log likelihood objective as the final translation output.

Notes :

- To avoid numerical underflow(numerical rounding errors) that results of multiplying many small probability numbers, we maximize the log of probabilities instead.
- $\frac{1}{T_y^\alpha}$ is a length normalization term. To prevent objective function from preferring short sentences over long sentences. Reduces the penalty for outputting longer translations.
- α can range between 0 (no normalization) and 1 (full normalization), in practice it is commonly set to 0.7
- Unlike exact search algorithms, beam search runs faster but it is not guaranteed to find the exact maximum for $\arg\max_{\hat{\mathbf{y}}}\left(\frac{1}{T_y^\alpha}\log P\left(\hat{\mathbf{y}}\middle|\mathbf{x}\right)\right)$
- The larger B , the more possibilities and better results, but the algorithm becomes slower, more computationally expensive and has more memory requirements.
- For production systems $B = 10$, for research B is chosen to be up to 100.

Error Analysis in Beam Search

$\mathbf{y}^*$: Translation by a human (reference sentence).

Example:

Human: Jane visits Africa in September ($\mathbf{y}^*$).

Algorithm: Jane visited Africa last September. ($\hat{\mathbf{y}}$)

- Case 1: $P(\mathbf{y}^*|\mathbf{x}) > P(\hat{\mathbf{y}}|\mathbf{x})$

  Beam search chose $\hat{\mathbf{y}}$. But $\mathbf{y}^*$ attains higher $P(\mathbf{y}|\mathbf{x})$.

  Conclusion: Beam search is at fault.
- Case 2: $P(\mathbf{y}^*|\mathbf{x}) \le P(\hat{\mathbf{y}}|\mathbf{x})$

  $\mathbf{y}^*$ is better translation than $\hat{\mathbf{y}}$. But RNN predicted $P(\mathbf{y}^*|\mathbf{x}) \le P(\hat{\mathbf{y}}|\mathbf{x})$.

  Conclusion: RNN model is at fault.

3.3 BLEU Score

[Pap+02]

Bleu: bilingual evaluation understudy.

Modified n-gram precision (p_n) for sentences:

$$p_n = \frac{\displaystyle\sum_{n\text{-gram}\in\hat{\mathbf{y}}}\text{count}_{\text{clip}}(n\text{-gram})}{\displaystyle\sum_{n\text{-gram}\in\hat{\mathbf{y}}}\text{count}(n\text{-gram})}$$

Where $\text{count}_{\text{clip}} = \min(\text{count}, \texttt{Max\_ref\_count})$. In other words, one truncates each word's count, if necessary, to not exceed the largest count observed in any single reference for that word.

Combined **Bleu** score for n-grams up to length N:

$$\textbf{B}\text{LEU} = \text{BP}\cdot\exp\left(\frac{1}{N}\sum_{n=1}^{N}\log p_n\right)$$

Where BP: Brevity penalty.

$$\text{BP} = \begin{cases}1 & \text{if } c > r\\ e^{(1-r/c)} & \text{if } c \le r\end{cases}$$

Where c is the length of the candidate translation(machine translation) and r is the effective reference corpus length(reference output length).

3.4 Attention Model

References: [BCB15], [Xu+15]

Properties of The Model

- Pre-attention and Post-attention RNNs on both sides of the attention mechanism
  - There are two separate RNNs in this model (see figure): pre-attention and post-attention RNNs.
  - Pre-attention Bi-RNN is the one at the bottom of the picture is a Bi-directional RNN and comes before the attention mechanism.
    - The attention mechanism is shown in the middle of the left-hand diagram.
    - The pre-attention Bi-RNN goes through T_x time steps
  - Post-attention RNN: at the top of the diagram comes after the attention mechanism.

    The post-attention RNN goes through T_y time steps.
  - The post-attention RNN passes the hidden state $\mathbf{s}^{\langle t\rangle}$ from one time step to the next.
- Each time step uses predictions from the previous time step.

Notation

$\overrightarrow{\mathbf{a}}^{\langle t'\rangle}$: hidden state of the forward-direction, pre-attention RNN.

$\overleftarrow{\mathbf{a}}^{\langle t'\rangle}$: hidden state of the backward-direction, pre-attention RNN.

$\mathbf{a}^{\langle t'\rangle}$: the concatenation of the activations of both the forward-direction and backward-directions of the pre-attention Bi-RNN.

$\mathbf{e}$: is called the "energies" variable.

$\mathbf{s}^{\langle t-1\rangle}$: is the hidden state of the post-attention RNN.

$\mathbf{a}^{\langle t'\rangle}$: is the hidden state of the pre-attention RNN.

$\alpha^{\langle t,t'\rangle}$: The attention variable, amount of "attention" $\mathbf{y}^{\langle t\rangle}$ should pay to $\mathbf{a}^{\langle t'\rangle}$.

The Model

Figure 4.1: Attention Model

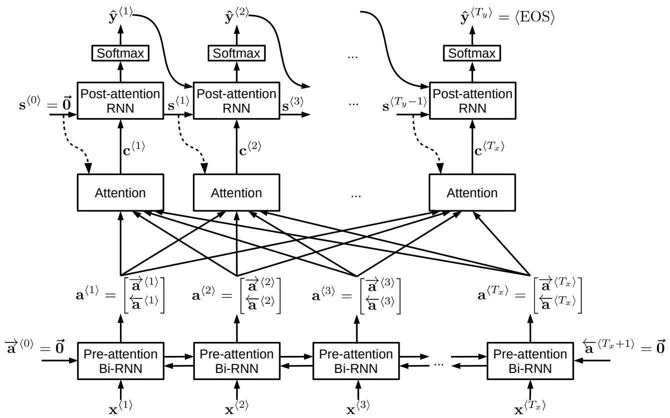
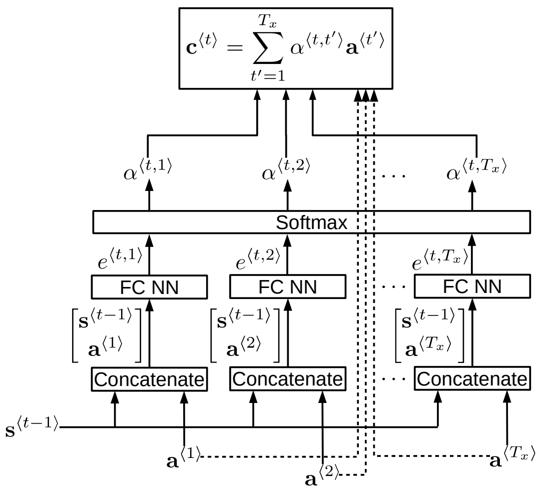


Figure 4.2: one "attention" step



Algorithm 9: Attention Model

Data: An input sequence $\mathbf{x}$, its length is T_x
Result: A sequence of predictions $\hat{\mathbf{y}}$, its length is T_y

```
/* Run the input x through the pre-attention Bi-RNN to get [a⟨1⟩, a⟨2⟩, ..., a⟨Tx⟩] */
```

for *input time steps* $t' = 1, \ldots, T_x$**:**

$$\mathbf{a}^{\langle t' \rangle} = \left[\overrightarrow{\mathbf{a}}^{\langle t' \rangle}, \overleftarrow{\mathbf{a}}^{\langle t' \rangle}\right] = \begin{bmatrix} \overrightarrow{\mathbf{a}}^{\langle t' \rangle} \\ \overleftarrow{\mathbf{a}}^{\langle t' \rangle} \end{bmatrix}$$

```
/* Pass the sequence of a⟨t'⟩ to the post-attention RNN to get the predictions ŷ⟨t⟩ */
```

for *output time steps* $t = 1, \ldots, T_y$**:**

Compute "energies" $e^{\langle t,t' \rangle}$: $\mathbf{s}^{\langle t-1 \rangle}$ and $\mathbf{a}^{\langle t' \rangle}$ are fed into a simple neural network, which learns the function to output $e^{\langle t,t' \rangle}$.

$$e^{\langle t,t' \rangle} = \mathrm{relu}\left(\mathbf{w}_e^{[2]\mathsf{T}} \cdot \tanh\left(\boldsymbol{W}_{\mathbf{e}}^{[1]}\left[\mathbf{s}^{\langle t-1 \rangle}, \mathbf{a}^{\langle t' \rangle}\right] + \mathbf{b}_{\mathbf{e}}^{[1]}\right) + b_e^{[2]}\right)$$

Calculate the attention variable $\alpha^{\langle t,t' \rangle}$

$$\alpha^{\langle t,t' \rangle} = \frac{\exp\left(e^{\langle t,t' \rangle}\right)}{\sum_{t'=1}^{T_x} \exp\left(e^{\langle t,t' \rangle}\right)}$$

Calculate the context vector $\mathbf{c}^{\langle t \rangle}$

$$\mathbf{c}^{\langle t \rangle} = \sum_{t'=1}^{T_x} \alpha^{\langle t,t' \rangle} \mathbf{a}^{\langle t' \rangle}$$

Pass the computed context vector $\mathbf{c}^{\langle t \rangle}$ to the post-attention RNN and calculate the hidden state $\mathbf{s}^{\langle t \rangle}$.

$$\mathbf{s}^{\langle t \rangle} = \tanh\left(\boldsymbol{W}_{\mathbf{s}}\left[\mathbf{s}^{\langle t-1 \rangle}, \mathbf{c}^{\langle t \rangle}, \mathbf{y}^{\langle t-1 \rangle}\right] + \mathbf{b}_{\mathbf{s}}\right)$$

Run the output of the post-attention RNN through a dense layer with softmax activation to generate a prediction $\hat{\mathbf{y}}^{\langle t \rangle}$

$$\hat{\mathbf{y}}^{\langle t \rangle} = \mathrm{softmax}\left(\boldsymbol{W}_{\mathbf{y}} \mathbf{s}^{\langle t \rangle} + \mathbf{b}_{\mathbf{y}}\right)$$

3.5 Speech Recognition

Reference: [Gra+06]

DOI: 10.5281/zenodo.3987344

References

[BCB15] Dzmitry Bahdanau, Kyunghyun Cho, and Yoshua Bengio. "Neural Machine Translation by Jointly Learning to Align and Translate". In: *3rd International Conference on Learning Representations, ICLR 2015, San Diego, CA, USA, May 7-9, 2015, Conference Track Proceedings*. Ed. by Yoshua Bengio and Yann LeCun. 2015. arXiv: 1409.0473.

[Ben+03] Yoshua Bengio et al. "A Neural Probabilistic Language Model". In: *Journal Of Machine Learning Research* 3 (Mar. 2003), pp. 1137–1155. URL: http://www.jmlr.org/papers/volume3/bengio03a/bengio03a.pdf.

[Bol+16] Tolga Bolukbasi et al. "Man is to Computer Programmer as Woman is to Homemaker? Debiasing Word Embeddings". In: *CoRR* abs/1607.06520 (July 2016). arXiv: 1607.06520.

[Cho+14a] Kyunghyun Cho et al. "Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation". In: *CoRR* abs/1406.1078 (2014). arXiv: 1406.1078.

[Cho+14b] Kyunghyun Cho et al. *On the Properties of Neural Machine Translation: Encoder-Decoder Approaches*. 2014. arXiv: 1409.1259 [cs.CL].

[Chu+14] Junyoung Chung et al. *Empirical Evaluation of Gated Recurrent Neural Networks on Sequence Modeling*. 2014. arXiv: 1412.3555 [cs.NE].

[GEB15] Leon A. Gatys, Alexander S. Ecker, and Matthias Bethge. *A Neural Algorithm of Artistic Style*. 2015. arXiv: 1508.06576 [cs.CV].

[Gra+06] Alex Graves et al. "Connectionist Temporal Classification: Labelling Unsegmented Sequence Data with Recurrent Neural Networks". In: *Proceedings of the 23rd International Conference on Machine Learning*. ICML '06. Pittsburgh, Pennsylvania, USA: ACM, 2006, pp. 369–376. ISBN: 1-59593-383-2. DOI: 10.1145/1143844.1143891. URL: https://www.cs.toronto.edu/~graves/icml_2006.pdf.

[He+15] Kaiming He et al. *Deep Residual Learning for Image Recognition*. 2015. arXiv: 1512.03385 [cs.CV].

[Hin12] Geoffrey Hinton. *Neural Networks for Machine Learning - Lecture 6a - Overview of mini-batch gradient descent*. Lecture 6 of the online course "Neural Networks for Machine Learning" on Coursera. 2012. URL: https://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf.

[HS97] Sepp Hochreiter and Jürgen Schmidhuber. "Long Short-Term Memory". In: *Neural Computation* 9.8 (1997), pp. 1735–1780. DOI: 10.1162/neco.1997.9.8.1735.

[KB14] Diederik P. Kingma and Jimmy Ba. *Adam: A Method for Stochastic Optimization*. 2014. arXiv: 1412.6980 [cs.LG].

[KL15] Andrej Karpathy and Fei-Fei Li. "Deep Visual-Semantic Alignments for Generating Image Descriptions". In: (2015). URL: https://cs.stanford.edu/people/karpathy/deepimagesent/.

[Mao+14] Junhua Mao et al. "Deep Captioning with Multimodal Recurrent Neural Networks (m-RNN)". In: (Dec. 2014). URL: http://www.cs.jhu.edu/~ayuille/Pubs15/JunhuaMaoDeepICLR2015.pdf.

[MH08] Laurens van der Maaten and Geoffrey Hinton. "Viualizing data using t-SNE". In: *Journal of Machine Learning Research* 9 (Nov. 2008), pp. 2579–2605. URL: http://www.jmlr.org/papers/v9/vandermaaten08a.html.

[Mik+13a] Tomas Mikolov et al. "Distributed Representations of Words and Phrases and their Compositionality". In: *Advances in Neural Information Processing Systems 26*. Ed. by C. J. C. Burges et al. Vol. 26. Curran Associates, Inc., Oct. 2013, pp. 3111–3119. arXiv: 1310.4546 [cs.CL].

[Mik+13b] Tomas Mikolov et al. "Efficient Estimation of Word Representations in Vector Space". In: *Proceedings of Workshop at ICLR* (Jan. 2013). Ed. by Yoshua Bengio and Yann LeCun. arXiv: 1301.3781. URL: http://arxiv.org/abs/1301.3781.

[MYZ13] Tomas Mikolov, Wen-tau Yih, and Geoffrey Zweig. "Linguistic Regularities in Continuous Space Word Representations". In: *Proceedings of the 2013 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*. Atlanta, Georgia: Association for Computational Linguistics, June 2013, pp. 746–751. URL: https://www.aclweb.org/anthology/N13-1090.

[Pap+02] Kishore Papineni et al. "Bleu: a Method for Automatic Evaluation of Machine Translation". In: *Proceedings of the 40th Annual Meeting of the Association for Computational Linguistics*. Philadelphia, Pennsylvania, USA: Association for Computational Linguistics, July 2002, pp. 311–318. DOI: 10.3115/1073083.1073135. URL: https://www.aclweb.org/anthology/P02-1040.

[PSM14] Jeffrey Pennington, Richard Socher, and Christopher D. Manning. "GloVe: Global Vectors for Word Representation". In: *Empirical Methods in Natural Language Processing (EMNLP)*. Vol. 14. Doha, Qatar: Association for Computational Linguistics, Oct. 2014, pp. 1532–1543. DOI: 10.3115/v1/D14-1162. URL: http://www.aclweb.org/anthology/D14-1162.

[Red+15] Joseph Redmon et al. *You Only Look Once: Unified, Real-Time Object Detection*. 2015. arXiv: 1506.02640 [cs.CV].

[Ser+13] Pierre Sermanet et al. *OverFeat: Integrated Recognition, Localization and Detection using Convolutional Networks*. 2013. arXiv: 1312.6229 [cs.CV].

[SKP15] Florian Schroff, Dmitry Kalenichenko, and James Philbin. "FaceNet: A unified embedding for face recognition and clustering". In: *2015 IEEE Conference on Computer Vision and Pattern Recognition (CVPR)* (June 2015). DOI: 10.1109/cvpr.2015.7298682. arXiv: 1503.03832. URL: http://dx.doi.org/10.1109/CVPR.2015.7298682.

[SVL14] Ilya Sutskever, Oriol Vinyals, and Quoc V Le. "Sequence to Sequence Learning with Neural Networks". In: *Advances in Neural Information Processing Systems 27*. Ed. by Z. Ghahramani et al. Curran Associates, Inc., 2014, pp. 3104–3112. URL: http://papers.nips.cc/paper/5346-sequence-to-sequence-learning-with-neural-networks.pdf.

[Tai+14] Y. Taigman et al. "DeepFace: Closing the Gap to Human-Level Performance in Face Verification". In: *2014 IEEE Conference on Computer Vision and Pattern Recognition*. 2014, pp. 1701–1708. URL: https://www.cs.toronto.edu/~ranzato/publications/taigman_cvpr14.pdf.

[Vin+14] Oriol Vinyals et al. "Show and Tell: A Neural Image Caption Generator". In: *CoRR* abs/1411.4555 (2014). arXiv: 1411.4555.

[Xu+15] Kelvin Xu et al. "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention". In: *CoRR* abs/1502.03044 (2015). arXiv: 1502.03044.

[ZF13] Matthew D Zeiler and Rob Fergus. *Visualizing and Understanding Convolutional Networks*. 2013. arXiv: 1311.2901 [cs.CV].