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Expansion Formulas for I-Function

*B. Satyanarayana[1], L. Prakas Rao[2], Y. Pragathi Kumar[3]

[1,2] Department of Mathematics, Acharya Nagarjuna University, Nagarjuna Nagar- 522 510
Andhra Pradesh, India
[3]Deaprtment of Mathemaitcs, College of Natural and Computational Sciences, Adigrat University, Ethiopia.

Abstract

The object of this paper is to establish some derivative formulas involving I-function of two variables and employ it to obtain expansion formulas for the I-function of two variables involving Struve's function. Some interesting special cases are pointed out.

Key words: I-function; Mellin-Barnes contour integral; Struve's function.

1. Introduction

Recently, Pragathi et.al. [8] has discussed the differentiation formulas for I-function of two variables with general class of polynomials. In the present paper we establish derivatives involving I-function of two variables and Struve's function. Also give some interesting special cases.

We use the following formulae and notations in the present investigation. The I-function of two variables defined by Shantha et. al. [13].

(1.1) $I[z_1, z_2]$

$$= I^{0,n_1:m_2,n_2;m_3,n_3}_{p_1,q_1:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1 \\ z_2 \end{matrix}\middle| \begin{matrix} (a_j;\alpha_j,A_j;\xi_j)_{1,p_1} : (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1} : (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3} \end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^2}\int_{L_s}\int_{L_t} \phi(s,t)\theta_1(s)\theta_2(t) z_1^{s} z_2^{t}\, ds\, dt$$

where

$$\phi(s,t) = \frac{\prod_{j=1}^{n_1} \Gamma^{\xi_j}\left(1-a_j+\alpha_j s + A_j t\right)}{\prod_{j=n_1+1}^{p_1} \Gamma^{\xi_j}\left(a_j-\alpha_j s - A_j t\right)\prod_{j=1}^{q_1} \Gamma^{\eta_j}\left(1-b_j+\beta_j s + B_j t\right)}$$

$$\theta_1(s)=\frac{\prod_{j=1}^{n_2}\Gamma^{U_j}(1-c_j+C_j s)\prod_{j=1}^{m_2}\Gamma^{V_j}(d_j-D_j s)}{\prod_{j=n_2+1}^{p_2}\Gamma^{U_j}(c_j-C_j s)\prod_{j=m_2+1}^{q_2}\Gamma^{V_j}(1-d_j+D_j s)}$$

$$\theta_2(t)=\frac{\prod_{j=1}^{n_3}\Gamma^{P_j}(1-e_j+E_j t)\prod_{j=1}^{m_3}\Gamma^{Q_j}(f_j-F_j t)}{\prod_{j=n_3+1}^{p_3}\Gamma^{P_j}(e_j-E_j t)\prod_{j=m_3+1}^{q_3}\Gamma^{Q_j}(1-f_j+F_j t)}$$

where $n_j, p_j, q_j (j = 1, 2, 3), m_j (j = 2, 3)$ are non negative integers such that $0 \le n_j \le p_j, q_1 \ge 0, 0 \le m_j \le q_j (j = 2, 3)$ (not all zero simultaneously), $\alpha_j, A_j (j = 1, ..., p_1)$; $\beta_j, B_j (j = 1, ..., q_1)$, $C_j (j = 1, ..., p_2)$, $D_j (j = 1, ..., q_2)$, $E_j (j = 1, ..., p_3)$, $F_j (j = 1, ..., q_3)$ are positive quantities. $a_j (j = 1, ..., p_1)$, $b_j (j = 1, ..., q_1)$, $c_j (j = 1, ..., p_2)$, $d_j (j = 1, ..., q_2)$, $e_j (j = 1, ..., p_3)$ and $f_j (j = 1, ..., q_3)$ are complex numbers. The exponents $\xi_j, \eta_j, U_j, V_j, P_j, Q_j$ may take non integer values.

L_s and L_t are suitable contours of Mellin-Barnes type. Moreover, the contour L_s is in the complex s-plane and runs from $\sigma_1 - i\infty$ to $\sigma_1 + i\infty$ (σ_1 real), so that all the poles of $\Gamma^{V_j}(d_j - D_j s)$ $(j = 1,....., m_2)$ lie to the right of L_s and all poles of $\Gamma^{U_j}(1-c_j+C_j s)$ $(j = 1,...., n_2)$, $\Gamma^{\xi_j}\left(1-a_j+\alpha_j s+A_j t\right)$ $(j = 1, ..., n_1)$ lie to the left of L_s. Similar conditions for L_t follows in complex t-plane. The detailed conditions of this function can be found in Shantha et. al.[14].

The Struve's function Pragathi et. al.. [9] (and also see [10])

(1.2) $$H^{\lambda,k}_{v,y,u}[z]=\sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{z}{2}\right)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)},$$ $Re(k) > 0, Re(\lambda) > 0, Re(y) > 0$ and $Re(v + u) > 0$.

And simple derivative notation

(1.3) $$D_x=\frac{d}{dx}$$

(1.4) $$D_x^r f(x)=\frac{d^r}{dx^r}f(x)$$

(1.5) $$(xD_x)^r f(x)=\left(x\frac{d}{dx}\right)^r f(x)$$

(1.6) $$(D_x x)^r f(x)=\left(\frac{d}{dx}x\right)^r f(x)$$

It is easy to verify that (and also see Shantha et. al. [13]).

$$(1.7)\quad \prod_{j=0}^{r-1}\left(h(v+2m+1)+h_1s+h_2t-j\right)=\frac{\Gamma\left(1+h(v+2m+1)+h_1s+h_2t\right)}{\Gamma\left(1+h(v+2m+1)+h_1s+h_2t-r\right)}$$

$$(1.8)\quad \prod_{j=1}^{r}\left(h(v+2m+1)-k_j+h_1s+h_2t\right)=\prod_{j=1}^{r}\frac{\Gamma\left(1+h(v+2m+1)-k_j+h_1s+h_2t\right)}{\Gamma\left(h(v+2m+1)-k_j+h_1s+h_2t\right)}$$

And

$$(1.9)\quad \prod_{j=1}^{r}\left(1+h(v+2m+1)-k_j+h_1s+h_2t\right)=\prod_{j=1}^{r}\frac{\Gamma\left(1+h(v+2m+1)-k_j+h_1s+h_2t+1\right)}{\Gamma\left(1+h(v+2m+1)-k_j+h_1s+h_2t\right)}$$

2. Main Results

We wish to establish the following results:

$$(2.1)\quad D_x^r\left\{H_{v,y,u}^{\lambda,k}[ax^h]I[z_1x^{h_1},z_2x^{h_2}]\right\}$$

$$=\sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{a}{2}\right)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}x^{h(v+2m+1)-r}$$

$$I_{p_1,q_1+1:p_2,q_2;p_3,q_3}^{0,n_1+1:m_2,n_2;m_3,n_3}\left[\begin{matrix}z_1x^{h_1}\\ z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(-h(v+2m+1);h_1,h_2;1),(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(r-h(v+2m+1);h_1,h_2;1):\end{matrix}\right.$$

$$\left.\begin{matrix}(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\ (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

Where h is complex number and h_1, h_2 are real and positive.

$$(2.2)\quad (xD_x-k_1)(xD_x-k_2)\ldots.(xD_x-k_r)\left\{H_{v,y,u}^{\lambda,k}[ax^h]I[z_1x^{h_1},z_2x^{h_2}]\right\}$$

$$=\sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{ax^h}{2}\right)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}$$

$$I_{p_1,q_1+r:p_2,q_2;p_3,q_3}^{0,n_1+r:m_2,n_2;m_3,n_3}\left[\begin{matrix}z_1x^{h_1}\\ z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(k_j-h(v+2m+1);h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:\\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(1+k_j-h(v+2m+1);h_1,h_2;1)_{1,r}:\end{matrix}\right.$$

$$\left.\begin{matrix}(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

Where h, k_j are complex numbers and h_1, h_2 are real and positive.

(2.3) $(D_x x-k_1)(D_x x-k_2)......(D_x x-k_r)\left\{H^{\lambda,k}_{v,y,u}[ax^h]I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$=\sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{ax^h}{2}\right)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}$$

$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix}z_1x^{h_1}\\z_2x^{h_2}\end{matrix}\middle|\begin{matrix}(k_j-h(v+2m+1)-1;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}:\\(b_j;\beta_j,B_j;\eta_j)_{1,q_1},(k_j-h(v+2m+1);h_1,h_2;1)_{1,r}:\end{matrix}\right.$$

$$\left.\begin{matrix}(c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3}\\(d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3}\end{matrix}\right]$$

Where h, k_j are complex numbers and h_1, h_2 are real and positive.

Proof of (2.1). Expressing the I-function of two variables as Mellin-Barnes type integral (1.1) and the Struve's function as series (1.2), we have

(2.4) $D_x^r\left\{H^{\lambda,k}_{v,y,u}[ax^h]\ I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$=\sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{a}{2}\right)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}\frac{1}{(2\pi i)^2}\int_{L_s}\int_{L_t}\left\{\phi(s,t)\,\theta_1(s)\,\theta_2(t)z_1^{\,s}z_2^{\,t}\right.$$

$$\left.\times\prod_{j=0}^{r-1}\left(h(v+2m+1)+h_1s+h_2t-j\right)x^{h(v+2m+1)+h_1s+h_2t-r}\right\}ds\,dt$$

Using (1.7) in (2.4) and simplifying we obtain the result (2.1).

Proof of (2.2). Using (1.1), (1.2) and simplifying we have

(2.5) $(xD_x-k_1)(xD_x-k_2)....(xD_x-k_r)\left\{H^{\lambda,k}_{v,y,u}[ax^h]I[z_1x^{h_1},z_2x^{h_2}]\right\}$

$$\sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{a}{2}\right)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)} \frac{1}{(2\pi i)^2} \int_{L_s} \int_{L_t} \Big\{ \phi(s,t)\,\theta_1(s)\,\theta_2(t) z_1^{\,s} z_2^{\,t}$$

$$\times \prod_{j=1}^{r} \left(h(v+2m+1)-k_j+h_1 s+h_2 t\right) x^{h(v+2m+1)+h_1 s+h_2 t} \Big\} ds\, dt$$

By using (1.8) in (2.5) and recalling the definition (1.1), we have the required result.

The derivation of result (2.3) runs parallel to that of result (2.1) and (2.2) as described above and we skip the details involved.

3. Special Cases

(i) By writing $k_1 = k_2 = \ldots\ldots = k_r = 0$ in (2.2), we get

(3.1) $$(xD_x)^r \left\{ H^{\lambda,k}_{v,y,u}[ax^h] I[z_1 x^{h_1}, z_2 x^{h_2}] \right\}$$

$$= \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{ax^h}{2}\right)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}$$

$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1 x^{h_1} \\ z_2 x^{h_2} \end{matrix}\middle| \begin{matrix} (-h(v+2m+1);h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}: \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(1-h(v+2m+1);h_1,h_2;1)_{1,r}: \end{matrix}\right.$$

$$\left.\begin{matrix} (c_j,C_j;U_j)_{1,p_2};(e_j,E_j;P_j)_{1,p_3} \\ (d_j,D_j;V_j)_{1,q_2};(f_j,F_j;Q_j)_{1,q_3} \end{matrix}\right]$$

Where h, k_j are complex numbers and h_1, h_2 are real and positive.

(ii) By taking $k_1 = k_2 = \ldots\ldots = k_r = 0$ in (2.3), we get

(3.2) $$(D_x x)^r \left\{ H^{\lambda,k}_{v,y,u}[ax^h] I[z_1 x^{h_1}, z_2 x^{h_2}] \right\}$$

$$= \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{ax^h}{2}\right)^{v+2m+1}}{\Gamma(km+y)\Gamma(v+\lambda m+u)}$$

$$I^{0,n_1+r:m_2,n_2;m_3,n_3}_{p_1,q_1+r:p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1 x^{h_1} \\ z_2 x^{h_2} \end{matrix}\middle| \begin{matrix} (-h(v+2m+1)-1;h_1,h_2;1)_{1,r},(a_j;\alpha_j,A_j;\xi_j)_{1,p_1}: \\ (b_j;\beta_j,B_j;\eta_j)_{1,q_1},(-h(v+2m+1);h_1,h_2;1)_{1,r}: \end{matrix}\right.$$

$$\left.\begin{matrix}(c_j, C_j; U_j)_{1,p_2}; (e_j, E_j; P_j)_{1,p_3} \\ (d_j, D_j; V_j)_{1,q_2}; (f_j, F_j; Q_j)_{1,q_3}\end{matrix}\right]$$

Where h, k_j are complex numbers and h_1, h_2 are real and positive.

(iii) By applying $h = 0, a = 1$ in (2.1), (2.2) and (2.3), we have known results given by Shantha et.al. [14].

(iv) By substitution $h = 0, a = 1, p_1 = q_1 = n_1 = 0$ and applying limit $z_2 \to 0$ in (2.1), (2.2) and (2.3) , we have the results established by Vyas et.al. [16].

(v) By writing $h = 0, a = 1, \xi_j = \eta_j = U_j = V_j = P_j = Q_j = 1$, $p_1 = q_1 = n_1 = 0$ and applying $z_2 \to 0$ in (2.1),(2.2) and (2.3), we obtain the results established by Guptha et.al. [4] and Nair [7].

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