Three examples of ancient "universal" portable sundials

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Introduction

Ancient portable sundials (see Table 1) represent only 5% to 6% of the corpus of all ancient sundials, [1] which is at present estimated to be between 500 and 600 dials. Nevertheless they must have been in rather frequent use if one is to believe Vitruvius when he writes in his *De Architectura* (between 35 and 25 BC):[2] "Again, following these types, many authors have left notices for the construction of travelers' sundials or portable sundials. If one wishes, one can find various kinds of projections in their works, so long as one is familiar with the drawings of the analemma." Unfortunately, none of these notices has reached us. For the gnomonists of antiquity, these *horologia viatoria* lent themselves particularly to the development of original models, and to the pursuit of dials intended to be usable at practically all latitudes.

One can divide ancient portable sundials into two categories:

– those that are only usable for a given latitude. This is the case with the "Portici Ham,"[3] the dials found at Mainz and Ponteilla,[4] the medallion dials,[5] and generally the cylinder dials.[6]

– those that are usable at multiple latitudes or at all latitudes. These include the Berteaucourt and Merida disks, dials of the "Crêt Chatelard" type, and lastly the most perfect of all, the armillary dial.[7]

1 The standard catalogue is now that of J. Bonnin, *La mesure du temps dans l'Antiquité*, Les Belles Lettres, Paris, 2015, accompanied by an online database at https://syrte.obspm.fr/spip/actualites/article/en-ligne-la-base-de-donnees-la-mesure-du-temps-dans-l-antiquite-temoignages/ . The well known work of S. L. Gibbs, *Greek and Roman Sundials*, New Haven, 1976, is more or less silent about portable sundials. For a succinct discussion of ancient portable sundials, see K. Schaldach, *Römische Sonnenuhren*, Verlag Harri Deutsch, Frankfurt, 2001. See also R. J. A. Talbert, *Roman Portable Sundials: The Empire in Your Hand*, Oxford, 2017, which is especially significant for the geographical aspects.

2 Vitruve, *De l'Architecture*, Livre IX, texte établi, traduit et commenté par J. Soubiran, Les Belles Lettres, Paris, 1969, p. 31.

3 Accademia Ercolanese, *Le Pitture Antiche d'Ercolano e conformi incisi con qualche spiegazeione III,* Naples, 1762, p. V-XVII. See also J. Drecker, *Die Theorie der Sonnenuhren*, Berlin, 1925, p. 58-60.

4 K. Körber, *Die neuen römische Funde Inschriften des Mainser Museums*, 1900, n° 202, p. 119-121. See especially J. Drecker, *Die Theorie der Sonnenuhren, op. cit.*, p. 61-64. The Ponteilla fragment will be the subject of a forthcoming article.

5 For the medallion dial in the Musei Civici di Trieste see now R. J. A. Talbert, "A lost sundial found, and the role of the hour in Roman daily life," *Indo-European Linguistics and Classical Philology* 23 (2), 2019, 971–988.

6 M. Arnaldi, K. Schaldach, "A roman cylinder dial : witness to a forgotten tradition," *Journal for the History of Astronomy*, 28, 1997, p. 107-117.

7 G. Gounaris, "Anneau astronomique solaire portative antique découvert à Philippes," *Annali dell'Istituto e Museo di Storia della Scienza di Firenze*, 5 (2), 1980, p. 3-18. This exceptional dial deserves a more extensive study.

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Fixed latitude	*Medium*	*Number of exemplars*	*Findspot, present location, language, date*
"Portici Ham"	bronze	1	Herculanum, Museo Nazionale di Napoli, Latin, 1st century
Mainz type	bone, stone	1	— Mainz, Landesmuseum Mainz, Latin, 2nd-5th century — Ponteilla, lost,
Cylindrical dial	bone, bronze	3	— Este, Museo Nazionale di Atestino, Latin, 1st century — Amiens, Musée de Picardie (France), Latin, 3rd century — Domjulien (Vosges), Musée d'Art Ancien et Contemporain d'Epinal (France), Latin, Roman imperial period
Medallion dial	bronze	6	— 1 at Rome, Museo Nazionale Romano, Latin, end of 2nd century — 2 at Aquileia (Italy), Kunsthistorisches Museum,Vienna, and Civici Musei di Trieste, Latin, 1st-4th century — 2 in Bithynia, 1 Latin, 1 Greek, AD 130 — 1 at Forbach, Hérapel, Musée de la Cour d'Or, Metz (France), Latin, 1st-4th century
Variable latitude			
Medallion dial	bronze	1	Exemplar of the Vienna Museum
Armillary	bronze	1	Philippi (Greece), Kavala Museum, Greek, 3rd-4th century
Portable meridian or latitude indicator	bronze	2	— Berteaucourt-les-Dames (France), Musée de Picardie, Latin, 2nd-3rd century — Merida (Spain), National Museum of Roman Art of Merida (Estremadura, Spain), Latin, 3rd century
"Crêt-Chatelard" type	bronze, brass, copper alloys	11	— Crêt-Chatelard (France), lost, 1st-4th century — Rome, lost, 1st-4th century — Trier (Germany), Rheinisches Landesmuseum Trier, 1st-4th century — Bratislava, Museum of the History of Science, Oxford, 1st-4th century — Unknown findspot, Science Museum, London, 2nd-6th century — Unknown findspot, ex-Time Museum, Rockford, USA, private collection, 5th century — Unknown findspot, British Museum, London, 4th-6th century — Aphrodisias, Aphrodisias Museum, 4th century — Bulgaria, private collection, replica in Römisch-Germanisches Zentralmuseum, Mainz, end of the 1st century to beginning of the 4th century — Memphis, Hermitage Museum, St. Petersburg, 4th century, lost? — Samos, Vathy Museum, Greece, 4th-6th century

Table 1. Summary of types and exemplars of ancient portable sundials known at present.

However, there exists an exemplar of a medallion dial that belongs simultaneously to the categories of fixed-latitude dials and multiple-latitude dials, as we will see below.

Certain ancient dials experienced a new glory during the Middle Ages and Renaissance, in particular the altitude cylinders, which in 19th century France were called "cadrans de berger." Conversely, other portable dials vanished, and it is a matter of some interest to understand why.

Medallion dials.

The term "mediallion dial" designates small altitude sundials 4-5 cm diameter, usable for a specific latitude, and resembling small pillboxes. The rim is raised, and on the principal face is a relief portrait, generally of an emperor. An eyehole or lateral orifice, situated in the raised rim, allows the Sun's rays to enter and fall upon a small mobile ruler in the shaded interior of the box. The apparatus is used in a vertical position oriented towards the Sun. At the base of the box is a grid in the form of an angular sector, made up of hour curves and date lines. One reads the seasonal hour at the intersection of the date line—on which is the ruler—and the hour curve. There are many variants of this altitude dial: for example in the Forbach dial each face bears the same dial and each ruler is at the center of the dial. In the Rome exemplar, the ruler is off-center, and the rear face of the medallion is decorated with a portrait of the emperor Commodus.

But it is the Vienna exemplar that is most remarkable (Fig. 1).[8] Not only is one face decorated with a portrait of the emperor Antoninus Pius (138-160) with the inscription *ANTONINVS AVG PIVS TR P COS III IMP II* (Fig. 2), but the first "box" component is like the *mater* of an astrolabe, having disks installed in it that are adapted to different latitudes. The base of this first box bears a—rather crude—tracing that unquestionably resembles the hour curves of a stereographic project (Fig. 3). These curves of course have no function in the dial; this is a "learned" ornamentation whose motivation is unknown. The other part of the box is equipped with a little stud for keeping in place the disks that are furnished with a grid applicable to several latitudes. Thus there are disks for the latitudes of Rome, Alexandria, Spain, Greece, etc. The hour grids of the disks are overall rather poor. The seasonal hour curves are represented by sometimes chaotic line segments; as for the point of convergence of the date lines, it often falls to the side of the hole for the mounting. On some of the disks (for example that for Alexandria, Fig. 4), one can still read below the date lines the names of the Roman months. This portable sundial is thus a very clever system in which the traveler places a grid-disk appropriate for the locality where he is in a kind of *mater*, rather as if one was traveling with an astrolabe equipped with multiple plates. There is a striking parallelism between these two instruments, where the astrolabe was the subject of efforts to develop a universal version that became workable in the 11th century with the *saphea* of Azarquiel.[9]

Let us study this dial from a modern viewpoint (Fig. 5). The most general case consists of a stud for fixing the disks and the ruler off-center. Let R be the radius of the box, whose center is O, and let C be the point of convergence of the date lines, and P the orifice through which the

8 This dial has been studied by E. Buchner, "Römische Medaillons als Sonnenuhren," *Chiron*, vol. 6, 1976, p. 329-348. The first mention and drawing of this dial appeared in F. Kenner, "Römische Medaillons," *Jahrbuch der Kunsthistorischen Sammlungen der Allerhöchsten Kaiserhauses*, vol. 1, Vienna, 1883, p. 84-85 ; one can clearly see in the drawing of F. Kenner that the dial bore a ruler as well as a pin, which have both vanished today. See also A. Schlieben, "Römische Reiseuhren," *Annalen des Vereins für Nassauische Alterthumskunde und Geschichtsforschung*, n ° 23, 1891, 115-128 and plate VI.

9 The principle was revived by Gemma Frisius in his *astrolabe catholique*. See R. D'Hollander, *L'Astrolabe, Histoire, Théorie, Pratique*, Institut Océanographique, Paris, 1999, p. 235-262.

Figure 1. This "box" sundial preserved at the Kunsthistorisches Museum, Vienna, came from Aquileia. Dating from the 2nd century of our era, it consists of two halves of a small bronze cylindrical box (top) containing four perforated disks (bottom). On the upper half of the box one clearly sees the small metal cylinder that enables a disk to be installed, as well as the notched hole on the side that allows the Sun's rays to pass through. The four disks bear on their recto and verso the horary drawing for the latitudes of the following cities: Alexandria Aegyptus / Africa Mauretania / Hellade Asia / Hispania Achaiia / Roma ? / Ancona Tuscia / Britannia Germania.

Figure 2. Outside face of the box, bearing a relief portrait of Antoninus Pius.

Figure 3. Inside face of the half of the box, bearing a drawing resembling a stereographic projection. The reason for its presence in this dial is unknown.

Figure 4. Disk showing the horary drawing for the latitude of Alexandria. One can clearly see that the hole is displaced relative to the drawing and that the latter is quite crude. Below the lines are the names of the months.

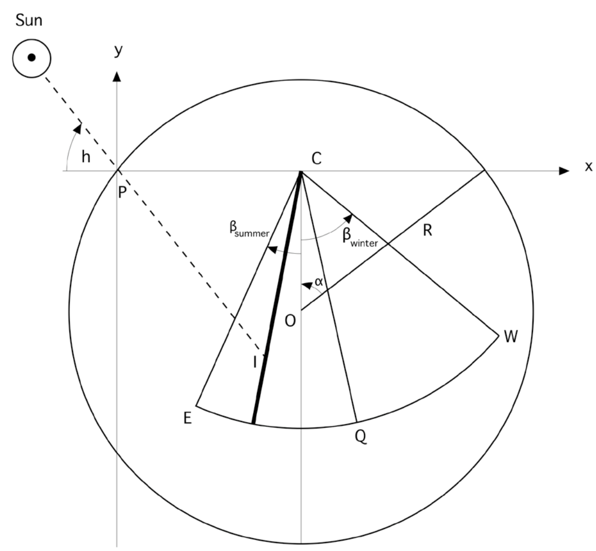

Figure 5. Diagram showing the principle of the "box" altitude dial: with it held vertically in the solar plane, the rays enter at P and strike the mobile ruler (which is very narrow); one reads off the hour at the intersection of the ruler and the drawing. The latter is contained within a triangular zone, with points E, Q, and W being the positions of the spot of light at solar noon (seasonal hour 6) respetively on the summer solstice, the equinoxes, and the winter solstice. The date lines converge on C, which is the mounting point of the ruler, while the hour arcs form a fan extending from the lowest arc (noon) to the center (sunrise/sunset).

Sun's rays enter. Let a system of axes pass through *P*, such that *x* tends rightward and *y* upward. We have:

(1) $OC = R \cos \alpha,$

where α designates the angle of aperture of the horary fan ($\alpha = 90°$ signifies that *C* coincides with *O*). One must fix the radius of the horary fan taking into account the eccentricity of C with a view to filling up the greatest possible surface in the box. Let *E*, *Q*, and *W* be the positions of the Sun at solar noon (seasonal hour 6) respectively on the summer solstice, the equinoxes, and the winter solstice. Hence we have

(2) $CE = CQ = CW = r$

This is also the radius of the mobile ruler revolving around *C*.

First of all, one aligns the ruler with the desired date. At a given moment, with the dial oriented in the plane of the Sun, a solar ray enters by the orifice *P* and strikes the ruler at *I*. The seasonal hour is read at the intersection of the date line and the hour curve. Let us seek the coordinates *x* and *y* of this intersection point.

Let *h'* be the noon altitude of the Sun:

(3) $h' = 90° - \phi + \delta$

where ϕ is the latitude of the locality and δ the declination of the Sun. The altitude *h* of the Sun for a given hour angle *H* is:

(4) $\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H$

H_0 is the semidiurnal arc, which one obtains thus:

(5) $\cos H_0 = -\tan \phi \tan \delta$

The seasonal hour k indicated by the dial (such that 0 h corresponds to sunrise, 6 h to solar noon, and 12 h to sunset) is obtained thus:

(6) $$k = \frac{6 \times (H + H_0)}{H_0}$$

from which we deduce H.

Next we have:

(7) $$\sin \gamma' = \frac{R \sin \alpha \sin h'}{r}$$

(8) $\gamma = h' - h + \gamma'$

(9) $\beta = 90° - \gamma' - h'$

(10) $$PI = \frac{R \sin \alpha \cos \beta}{\sin \gamma}$$

(11) $x = PI \cos h$

(12) $y = -PI \sin h$

β here represents the angle between a date line and the vertical through O and the point of convergence. The total opening of the angular sector equals $|\beta_{summer}| + |\beta_{winter}|$.

Numerical example: let there be a medallion dial with radius R = 7 cm, calculated for a latitude ϕ = 38°, obliquity ε = 24°. One chooses a radius for the horary fan r = 8 cm, setting α = 50° . Let us calculate the coordinates for seasonal hour k = 8 on the summer solstice (δ = +24°). We have H = 36.785° and consequently:

(13) $h' = 76°$

(14) $h = 55.787°$

(15) $\gamma' = 40.570°$

(16) $\gamma = 60.783°$

(17) $\beta = -26.570°$

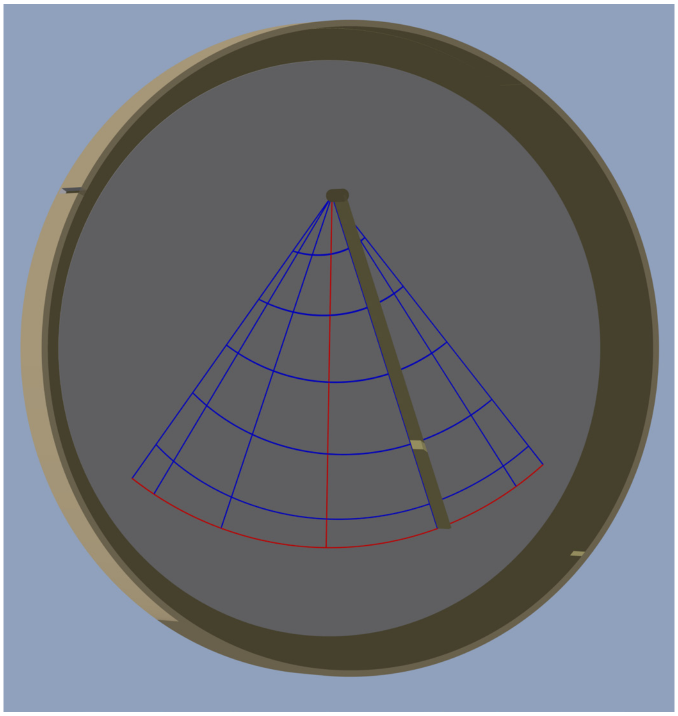

Figure 6. Modern reconstruction of the "box" altitude dial for a latitude of 38°. The rays enter by the lateral notch and strike the ruler, which is calibrated according to the month (here $\delta = -11.47°$, zodiacal signs Pisces/Scorpio). If one considers the time to be in the afternoon, it is seasonal 8 h. The red circular arc corresponds to solar noon; the red straight line in the middle corresponds to the equinoxes.

Figure 7. Bronze disk of diameter 10.4 cm, discovered in a Gallo-Roman villa at Berteaucourt-les-Dames, a locality in the north of France in Picardie, département de la Somme. The object dates from the 2nd century of our era.

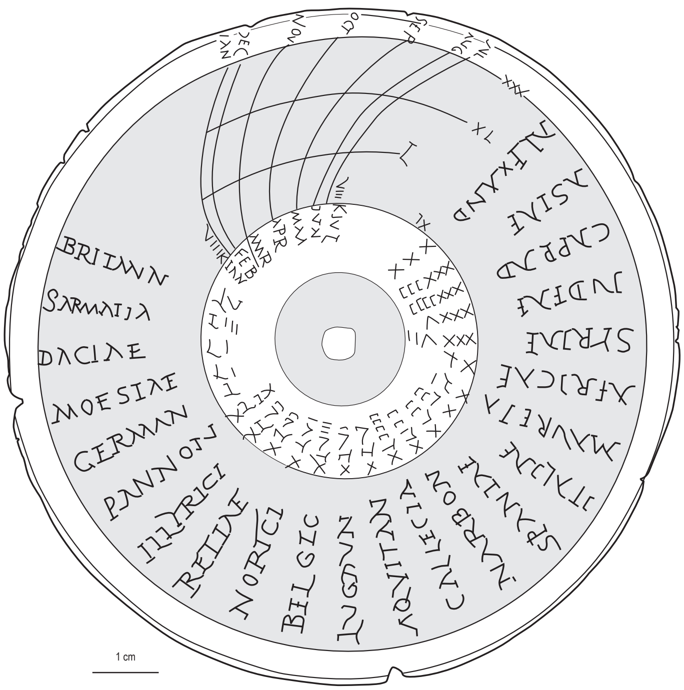

Figure 8. Drawing faithfully reproducing the inscriptions and drawing of the Berteaucourt-les-Dames disk. The upper part consists of a grid where one recognizes the names of the months; the rest of the disk is filled by a list of 23 Roman provinces, and occasionally cities, accompanied by their increasing latitudes. This progressive list begins with ALEXAND(riae) [latitude] XXX and ends with BRITANN(iae) [latitude] LX. The disk is perforated by a hole at the center.

(18) $PI = 5.495$ cm

(19) $x = 3.09$ cm

(20) $y = -4.544$ cm

(the superfluous decimals are given only for the sake of verification).

The geometrical construction is very simple. Once the radius *r* of the angular sector is defined, one obtains the intersection points of the hours on a date line while varying the solar altitude. Thus assumes that the maker has at his disposal a table giving the solar altitude as a function of the seasonal hour and the date. In fact, one can replace the orifice in the raised rim by a horizontal gnomon; the infinite shadow of the gnomon intersects the grid where one reads the hour according to the date (Fig. 6).

The grid that one obtains is quite harmonious. The hour curves are not excessively squeezed together as is the case if one draws the dial by true solar time.[10] It goes without saying that in the case of the Vienna dial, the tininess of the dials (to say nothing of their very crude execution), combined with the fact that one has to balance them vertically (which perhaps implies that there existed a suspension thread) renders them pure objects of prestige rather than real dials to indicate the time.

10 Such dials using equinoctial hours are apparently are unknown.

Portable meridians, indicators of latitude.

The two bronze disks found in the archeological excavations in 1985 at Berteaucourt-les-Dames in Picardie (France)[11] and in 1994 at Merida (Spain)[12] date respectively from the ends of the 2nd and of the 3rd centuries of our era. The archeologists at the time, contemplating their purpose, classified them as portable sundials of a new variety, without pursuing the question further. A recent study,[13] however, has made it possible to determine their functions more precisely and to bring new clarity to these little known objects.

As one can see in Fig. 7, which represents the dial of Berteaucourt-les-Dames, the object consists of a disk of about 10 cm in diameter perforated at the center by a hole (the Merida dial has diameter 13 cm). One can divide the principal face (Fig. 8) in two parts:

– the "geographical" part, with 23 names of provinces or cities sorted according to increasing latitude from *Alexandriae* (latitude 30°) to *Britanniae* (latitude 55°).[14]

– the strictly gnomonic part, which is a grid composed of four concentric curves, ascending from latitude 30° to 60° from outside to inside in steps of 10°. Seven other curves cross the latitude circles and bear labels for the twelve months of the Roman calendar, the two extreme curves being those for the solstices[15] with labels VIII K. IAN (VIII before the Kalends of January = winter solstice = December 25), et VIII K. IUL (VIII before the Kalends of July = summer solstice = June 24). The other dates doubtless correspond to the Sun's entries into the zodiacal signs.[16]

On the object's rear face is a straight line, whose significance we will see presently, which extends from the central hole to the periphery. A horizontal gnomon several centimeters long must have been lodged in this hole; it must have been long enough to cast a shadow on the disk and its diameter small enough to allow a sensitive reading.

11 J. L. Massy, *Gallia*, t. 43, fascicule 2, 1985, p. 481-482. This disk, discovered in a Gallo-Roman villa is preserved at the Musée de Picardie at Amiens. It is in much better condition than the Merida disk. See also C. Hoët-van Cauwenbergue, E. Binet, "Cadran solaire sur os à Amiens (Samarobriva)," *Cahiers Glotz*, XIX, 2008, p. 123-124.

12 J. Arce, "Viatoria pensilia. Un nuevo reloj portatil del s. III D. C. Procedente de Augusta Emerita (Mérida, España)," *Merida Tardorromana* (300-580 d. C.), 2002, p. 217-226. This disk is preserved at the National Museum of Roman Art of Merida (Estremadura). The latitude circles are effaced; six date arcs survive.

13 See D. Savoie and M. Goutaudier, "Les disques de Berteaucourt-les-Dames et de Mérida : méridiennes portatives ou indicateurs de latitude?", *Revue du Nord*, t. 94, n° 398, 2012, p. 115-119.

14 The Merida disk bears 19 names of provinces in contrast to 23 for that of Berteaucourt-les-Dames. On the latter, there are some exceptions to the increasing order of the latitudes. On the geographical labels inscribed on portable sundials, see C. Hoët-van Cauwenbergue, "Le disque de Berteaucourt-les-dames (cité des Ambiens) et les listes gravées sur les cadrans solaires portatifs pour voyageurs dans le monde romain," *Revue du Nord*, t. 94, n° 398, 2012, p. 97-114.

15 See F. K. Ginzel, *Handbuch der Mathematischen und Technischen Chronologie*, vol. 2, Leipzig, 1911, p. 179-181 et p. 282. The solstice dates given here are the classical dates in the Julian calendar that came into use in 45 BC; as for the equinoxes, they are fixed on March 25 (VIII before the Kalends of April) and September 24 (VIII before the Kalends of October). At the beginning of the 3rd century of our era, because of the shift of the Julian year relative to the tropical year, the astronomical seasons began on the following dates (see Table 5 below): vernal equinox on March 21, summer solstice on June 23, autumnal equinox on September 24, and winter solstice on December 22. Hence the user of the instrument would commit a slight error in retaining the common dates.

16 See on this point P. Brind'Amour, *Le calendrier romain*, Université d'Ottawa, 1983, p. 15-19.

Figure 9. Photo of the rear face of the Berteaucourt-les-Dames disk. A gnomon was likely inserted in the hole to cast a shadow on the grid figuring on the other face of the disk. On the rear face shown here, a groove along the radius of the disk enables one to adjust it in the direction of the zenith.

There is no horary graduation on the grid, and this allows us to conclude that the purpose of this instrument was not to show the hour throughout the day (though it could indicate noon). What the grid does clearly show is the existence of a relation between the locality's latitude and the date; hence one deduces that this instrument could have two functions, possibly at the same time:

– to indicate the solar noon for the locality

– to indicate the latitude where one is

Let us recall that in altitude sundials, the principle is to determine the solar time as a function of the Sun's altitude above the horizon, and that this requires knowledge of two constants: the latitude of the locality and the date. This results from the fact that this kind of dial is a practical and technical application of a formula of spherical trigonometry known in antiquity that relates the time H to the Sun's altitude h:

(21) $\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H$

where δ is the Sun's declination (effectively the date) and ϕ is the latitude. This formula shows very clearly that the Sun's altitude depends on three components: the place, the date, and the time. Hence an altitude sundial intended to work throughout the Roman Empire, say over a range of 30° of latitude (about 3300 km) like the disks of Berteaucourt-les-Dames and Merida, would need either a network of curves so dense as to become illegible, or a mechanism to make it easy to use, such as for example the armillary.

But in the present case, since there is no question of showing the time throughout the day but only solar noon, one can take advantage of a simplification of formula (21), observing that at

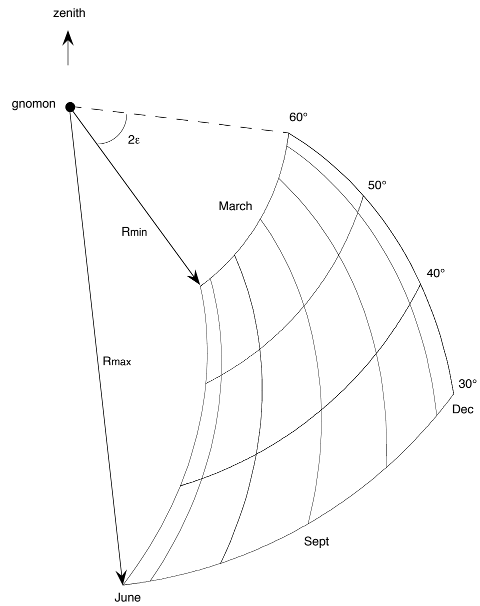

Figure 10. Modern representation of the grid. It comprises four concentric circles indicating the latitude (from 30° to 60°), crossed by seven date curves.

solar noon (H = 0°), the Sun's altitude becomes:

(22) $\sin h = \cos (\phi - \delta)$

hence

(23) $h = 90° - \phi + \delta$

The Sun's declination is obtained by the likewise classical formula:

(24) $\sin \delta = \sin \varepsilon \sin \lambda$

where λ is the Sun's ecliptic longitude, which is made to range from 0° to 360° in steps of 30°, so that one obtains the declination of the Sun at its entry into each zodiacal sign.

The grid under investigation here is a clever application of formula (23), in which one makes the Sun's declination vary as the "abscissa" and the latitude as the "ordinate." The read-off, whether of noon, or of latitude, takes place at the intersection of the shadow of a horizontal gnomon and a declination arc, while the instrument is suspended vertically and aligned with the plane of the Sun. This is corroborated by the straight line on the back of the instrument, which is perfectly aligned with respect to the grid, and which serves to orient the disk to the zenith, doubtless with the help of a suspension thread (Fig. 9). The graduated face is the one that is directed to the east.

The calculation of such an instrument is quite simple. One sets, starting from the hole, radius R_{max}, which corresponds to the minimum latitude (here 30°) and radius R_{min}, which corresponds to the maximum latitude (here 60°). Radius R for an intermediate latitude is thus obtained by a simple rule of three:

(25) $R = R_{max} - [\Delta R(\phi - 30)/30]$ where $\Delta R = R_{max} - R_{min}$

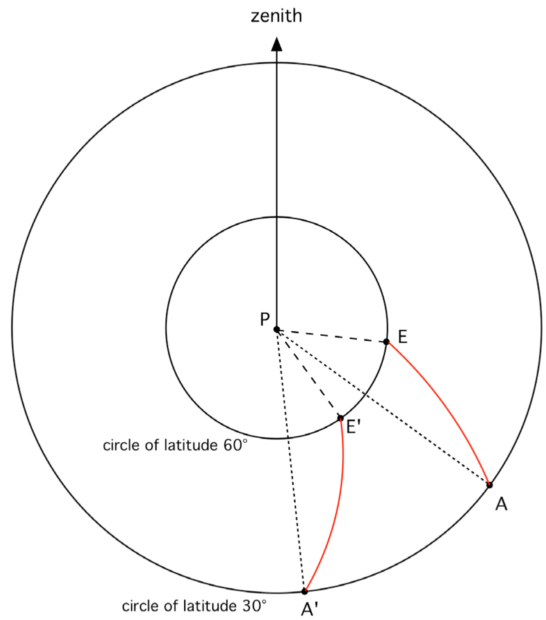

Figure 11. Geometrical drawing of the grid. Arcs EE' and AA' are respectively the arcs for latitude 60° and 30°. Arcs EA (the curve for the winter solstice) and E'A' (for the summer solstice) are in red.

Making a system of axes pass through the disk's center, with x tending northward and y towards the zenith, one draws a declination arc or a circle of latitude thus:

(26) $$x = R \cos h = R \sin (\phi - \delta)$$

(27) $$y = - R \sin h = - R \cos (\phi - \delta)$$

If one treats the latitude as variable for a given solar declination, one gets a declination arc. If one treats the declination as variable for a given latitude, one gets a latitude circle. The declination should range from $-\varepsilon$ to $+\varepsilon$ where ε is the obliquity of the ecliptic; in antiquity one generally uses $\varepsilon = 24°$. In any case one can verify on the Berteaucourt-les-Dames disk that the angle at the vertex (Fig. 10) is indeed equal to $2\,\varepsilon$, which corresponds to the difference in solar altitude between the winter and summer solstice at latitude 60°; the same angle is found again for the circle for latitude 30°.

In practice, such a grid does not require recourse to trigonometrical calculation; pure geometry allows one to draw it. One starts by establishing, with the help of the elementary formula (2), the values of the Sun's noon altitude for different latitudes. After drawing two concentric circles with a compass (Fig. 11), one uses a protractor centered on the center *P* of the circles, which is also where the gnomon will be, to trace the Sun's altitude in winter for the maximum latitude (point *E*) and then the altitude in summer for this same latitude (*E'*). One carries out the same operation for the outermost latitude circle, obtaining points *A* and *A'*. To obtain the date curves between the extremal circles, one must of course generate the altitude points for the intermediate latitudes and then join up the points by a curve.

Let us say at once that one should not look for precision instruments in these two disks: their dimensions are in the first place too slight and in the second place one should regard them more as objects of curiosity or prestige, or maybe astronomical amusement, even if they are

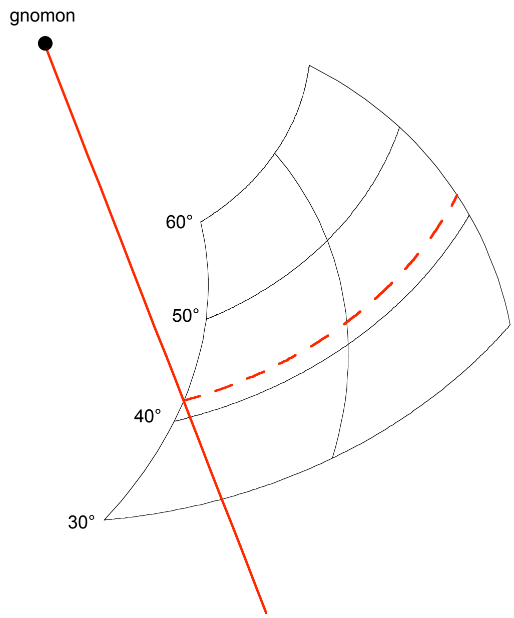

Figure 12. The gnomon's shadow (red) cuts the summer solstice curve at latitude 42° (broken circle).

scientifically quite sophisticated. In fact, what one has here are instruments that give orders of magnitude and that assume a nontrivial level of knowledge on the part of the user.

Consider first of all the case of a traveler who wants to use this indicator to determine solar noon, thus as a portable meridian. He absolutely has to know the date and the place where he is. Let us imagine that the date is the 8th day before the Kalends of July, that is, the summer solstice, and that the traveler as at Rome at a latitude of 42° as the disk indicates (Fig. 12). After a preliminary identification of the latitude circle for 42°, the traveler has to estimate *grosso modo* the moment when the gnomon's shadow cuts the latitude circle and the declination curve. One can imagine that to find such a circle a thread was provided, attached to the disk's center and furnished with a sliding bead.

It is appropriate here to note that the Sun's altitude around its culmination has little variation, especially around the solstices, so that one could easily be wrong by ±15 minutes or more, and that is without taking account of the fact that the shadow of a horizontal gnomon has a certain width. In any event it is an inconvenience that one has to underline and that affects all altitude sundials: their accuracy is mediocre at noon.

Now let us put ourselves in the hypothetical situation of a traveler who uses the disk to have an idea of the region where he is. Again he knows the date, but he also has to find a means of determining solar noon, for example by means of a gnomon, waiting for its shadow to be shortest (Fig. 13). Let us go back to the preceding example of a Roman traveler; at solar noon he has to find the intersection of the shadow with the date curve so that he can then read off the latitude. Here again, one can imagine that a thread furnished with a bead allows one better to estimate the place, or rather the region. But one should add right away that the instrument does not yield the longitude of the region or place. Hence one has to have already an idea of the place where one is, without which one might, for example, establish a latitude of 42° without knowing

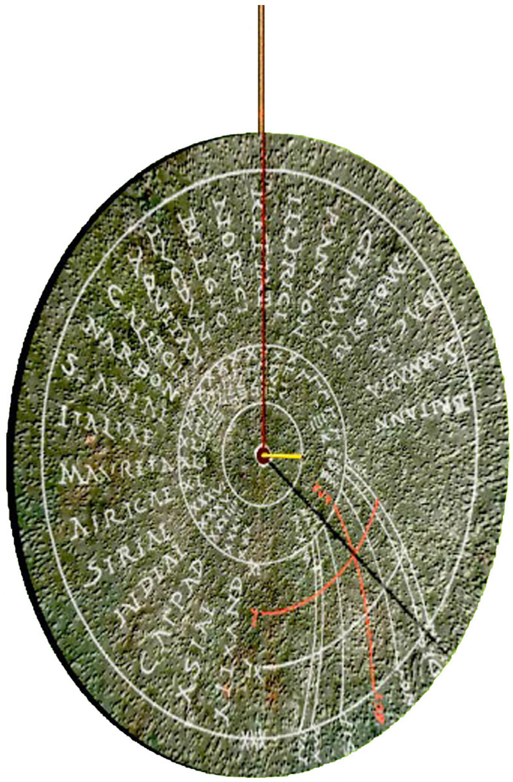

Figure 13. Method of use of the Berteaucourt-les-Dames disk. The disk is suspended in the plane of the Sun. One can here see that the horizontal gnomon casts a shadow upon the grid, cutting the curve for latitude 50° at the equinoxes (red).

whether one is in Spain or in Italy!

An attempt at a universal portable dial.

Among all the ancient portable sundials, the most numerous at present are the eleven Greek and Roman exemplars, constructed between the 1st and the end of the 6th century, of a very particular type that has tripped up many authors with regard to its functioning and hence to its theory, a bit like the case of another portable dial, in this case medieval, the *Navicula de Venetiis*.[17] These two sundials additionally have points in common, such as that of being usable at practically all latitudes and that of not being rigorously exact except on certain dates while being theoretically very accurate throughout the year. One of these dials, lost today, was discovered in France in the 19th century, at a place called Crêt-Chatelard (département de la Loire).[18]

The first to understand the working of these sundials, which had been known since the 18th century and which exhibit great variations among themselves, was the German gnomonist J.

17 See C. Eagleton, *Monks, Manuscripts and Sundials, The Navicula in Medieval England*, Brill, Boston, 2010. The calculation of the error in read-off on the Navicula was made by J. Kragten, *The Little Ship of Venise – Navicula de Venetiis*, Eindhoven, 1989.

18 This dial was described by V. Durand, followed by a restoration and—erroneous—instructions for use by general De La Noë : "Cadran solaire portatif trouvé au Crêt-Chatelard," *Bulletin et Mémoire de la Société Nationale des Antiquaires de France*, t. 7, mémoires 1896, Paris, 1897, p. 1–38. De La Noë, who does not hesitate to characterize as "crude" an earlier attempt at explication by Baldini (*cf. infra*), had absolutely no understanding of this dial, which he placed horizontally.

Figure 14. Roman exemplar found near Bratislava, dating from the 3rd century. The instrument is complete. Clearly visible are the mobile and curved read-off scale with its hour strokes, at the end of which is the gnomon. At the extremities of the date scale are the inscriptions VIII K. IAN and VIII K. IVL, which correspond to December 25 and June 24, the dates considered to be the winter and summer solstices in the Julian calendar.

Figure 15. Byzantine exemplar dating from the 5th-6th century, diameter 110 mm. On the gnomonic face can be seen the cursor, which enables the dial to be set according to the latitude, and the pendant ring for keeping the instrument vertical in a chosen direction. On the angular sectors appear abbreviations in Greek letters of the Latin names of the months for each half year, January to June on the upper part, July to December on the lower. The read-off scale is lost. On the back are the latitudes of 36 cities and provinces. [credit: Trustees of the British Museum].

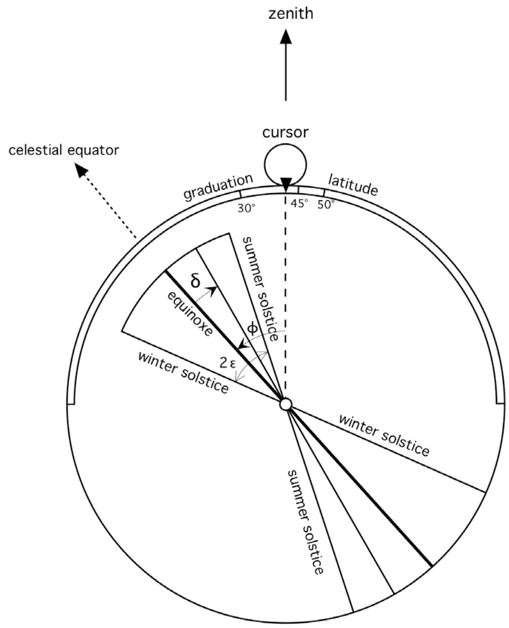

Figure 16. Astronomical indications of the gnomonic face of the portable dial, required for setting it.

Drecker at the beginning of the 20th century.[19] Several studies have been published since on the subject, the most thorough to date being that of M. T. Wright in 2000.[20]

These dials are quite simple in appearance (Fig. 14 and 15). They are disks of between 5 and 12 cm diameter. On one face names of cities or provinces are inscribed with their latitudes. On the other face are engraved labels pertaining to months (notably equinoxes and solstices). The mobile scale for reading off the time is not inscribed with numbers; it is curved, and the gnomon is at its extremity. On the disk's periphery, a sliding rail assemblage, under which the latitudes to which the instrument can be set are marked, allows a cursor or pendant ring to be slid to calibrate the dial for the place where it is to be used.

19 J. Drecker, *Die Theorie der Sonnenuhren*, *op. cit*., p. 64-66. Drecker was also the first to estimate the error in the read-off of the time as a function of the date and latitude. Two earlier authors (Baldini and Woepcke) had already studied this dial but they were completely misguided about its operation: G. Baldini, "Sopra un'antica piastra di bronzo che si suppone un orologio da sole," *Saggi di dissertazioni accademiche publicamente lette nella nobile Academia Etrusca di Cortona* , vol. III, Rome, 1741, p. 185-194 ; F. Woepcke, *Disquisitiones archaeologio-mathematicae circa solaria vetrum*, (Diss. Inaug.), Berlin, 1842, p. 14-19. In their defense, one should note that the model of dial on which they based their arguments was incomplete, the gnomon situated at the end of the read-off scale having been lost.

20 M. T. Wright, "Greek and Roman Portable Sundials – An Ancient Essay in Approximation," *Archive for History of Exact Sciences*, vol. 55, 2000, p. 177-187. Wright, who curiously does not cite Drecker, employs a Vitruvian analemma in his trigonometrical demonstrations, which is admirable. We may also cite the brief study by F. A. Stebbins, "A Roman Sundial," *Journal of the Royal Astronomical Society of Canada*, vol. 52, 1958, p. 250-254, who has the merit of having explained clearly how one used these portable dials. See also the classic and very good article by D. J. de Solla Price, "Portable Sundials in Antiquity, including an Account of a New Example from Aphrodisias," *Centaurus*, 1969, vol. 14, p. 242-266. Many authors, more specialized in archeology, have concentrated on the geographical lists that appear on the backs of the dials, which in some cases give as many as 36 places. Attention should also be drawn to an inscribed marble slab found at Aquincum, Hungary, and sometimes miscalled *tabula gromatici*, which appears to have been a set of templates for constructing this type of dial; see P. Albèri-Auber, "The Aquincum fragment," *The Compendium: Journal of the North American Sundial Society* 25 (2), 2018, 13–24.

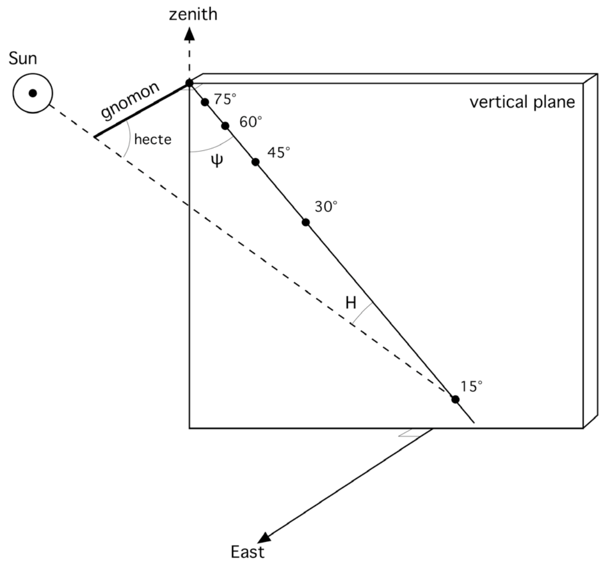

Figure 17. The portable dial is here reduced to a vertical plane oriented due east and furnished with a horizontal gnomon casting a shadow on a graduated equinoctial scale.

Let us describe with greater precision the "gnomonic" face of this dial, which, let us specify at the outset, is always used in a vertical position (Fig. 16). Its layout is in the first instance an instantiation of the celestial equator; this makes an angle with the zenith equal to the latitude of the locality. This explains why a sliding rail assemblage makes it possible to calibrate the dial in accordance with the latitude ϕ. On each side of the equator, one traces the Sun's position right to the solstices. That is, the angle between the equator and the positions specified by the calendar corresponds to the declination δ of the Sun, whose absolute value is equal to the obliquity at the two solstices.

In antiquity one uses $\varepsilon = 24°$ (*cf. infra*); one gets the Sun's declination as a function of the date, which is represented by the ecliptic longitude λ, varying by steps of 30°:

(28) $$\sin \delta = \sin \varepsilon \sin \lambda$$

The dial can be reduced initially (Fig. 17) to an equinoctial straight line drawn on a vertical plane exactly oriented towards the east, being mobile around its gnomon, and having the points for the hours drawn on it at intervals of 15° (tangent law: the distance of an hour point to the foot of a gnomon of length a is obtained as ($a/\tan H$), where H is the Sun's hour angle). This equinoctial line likewise makes an angle ψ with the vertical passing through the gnomon which is precisely equal, on the equinoxes, to the locality's latitude. As it is drawn for the moment, the dial resembles an oriental vertical sundial with gnomonic declination[21] $D = -90°$, functioning only on the equinoxes.

Let us remark at once that to draw such a dial, an ancient gnomonist in possession of Ptolemy's *Treatise on the Analemma*[22] would have noted that the horary angle H of the Sun is the

21 The gnomonic declination D is the azimuth of the line perpendicular to the plane.

22 The edition of Heiberg de 1907 is the revised edition of the Latin text and the Greek fragments that he had already published in 1895 : *Claudii Ptolemaei Opera quae exstant omnia*, Vol. II, *Opera Astronomica Minora*, ed. J. L. Heiberg, Leipzig, Teubner, 1907, p. 187-223 et *Praefatio*, p. XI-XII. This edition is the standard at present. One can find

H	H'	$90° - hecte$
−90°	−90°	−90°
−75°	−75°	−75°
−60°	−60°	−60°
−45°	−45°	−45°
−30°	−30°	−30°
−15°	−15°	−15°
0°	0°	0°

Table 2. Comparison of equinoctial and seasonal horary angles with the complement of the hectemoros for equinoxes.

complementary angle of the *hectemoros*, which last is the angle between the direction to the Sun and the cardinal east (or west) point. This *hectemoros* is independent of the latitude, since it is calculated as:

(29) $$\cos hecte = \sin H$$

It is interesting to make a tabular comparison of the equinoctial horary angle H, the Sun's seasonal hour angle H' (as in all ancient sundials, this portable dial shows the seasonal hour), and the complement of the *hectemoros*, since one observes a certain similarity, in fact a strict equivalence that—perhaps—was the origin of this portable sundial's mode of operation. On the equinoxes, the equivalence is perfect (Table 2).

Let us recall that H' is obtained thus:

(30) $$H' = \frac{H.90°}{H_0}$$

Where H_0 is the semidiurnal arc obtained as:

(31) $$\cos H_0 = -\tan \phi \tan \delta$$

The seasonal hour[23] k indicated by the dial (with 0 h corresponding to sunrise, 12 h to sunset) is:

(32) $$k = \frac{6 \times (H + H_0)}{H_0}$$

or

(33) $$k = \frac{6 \times (H' + 90)}{90}$$

a modern Italian translation facing the Latin in R. Sinisgalli, S. Vastola, *L'analemma di Tolomeo*, Edizioni Cadmo, Firenze, 1992. See O. Neugebauer, *HAMA*, Springer-Verlag, Berlin-Heidelberg-New York, 1975, t. II, p. 840-841 and D. Savoie, *Recherches sur les cadrans solaires*, Brépols, Turnhout, 2014, chap. IV.

23 If one keeps to the classical definition, the seasonal hour has no meaning beyond the polar circles (latitude 90· ± obliquity) at certain times of year because the Sun can be circumpolar.

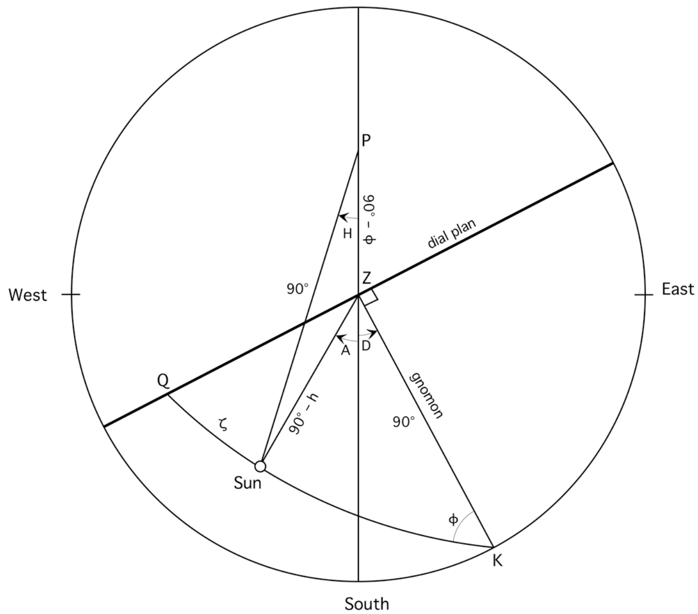

Figure 18. Representation by spherical trigonometry (viewed from the zenith) of the portable dial in use on the equinoxes. P is the north celestial pole, Z the zenith, K the tip of the gnomon.

On the equinoxes, at our latitudes, the dial at sunrise has its gnomon pointing exactly eastward; then the shadow shifts progressively during the morning along the equinoctial line until noon, when the Sun, now in the direction of geographic south, produces an infinite gnomon shadow. Throughout the morning, the plane of the sundial has stayed perfectly fixed, with the gnomon always having a gnomonic declination of 90°. But for the dial to continue to work in the afternoon, one has to pivot its vertical plane around an axis passing through the zenith. Otherwise put, the gnomon will now point towards the south-east horizon, then the south-west horizon, finally pointing exactly westward at the moment of sunset. The plane of the dial will thus have turned 180°.

One could imagine a turning about of the dial, such that the east face becomes, after solar noon, the west face. But this operation would call for a rotation equal to 2ψ of the scale and the gnomon and hence a new calibration according to the date and latitude. Now none of the dials known at present bears a double graduation in latitude and in the scale of dates that would make this kind of setting possible. Moreover the user does not know *a priori* which part of the day—morning or afternoon—he is in.

It is well to emphasize here that right from the moment that one makes the plane of the dial pivot, thus changing its gnomonic declination, the angle between the direction to the Sun and the direction in which the gnomon points is no longer the *hectemoros* but another angle that we will call ζ: this is the angle that one reads on the dial. This angle ζ can be shown to be calculated thus (Fig. 18 and 19):

(34) $$\cos\zeta = \frac{\sin h}{\cos\phi}$$

where *h* is the Sun's altitude. Since we are at the equinoxes, one has:

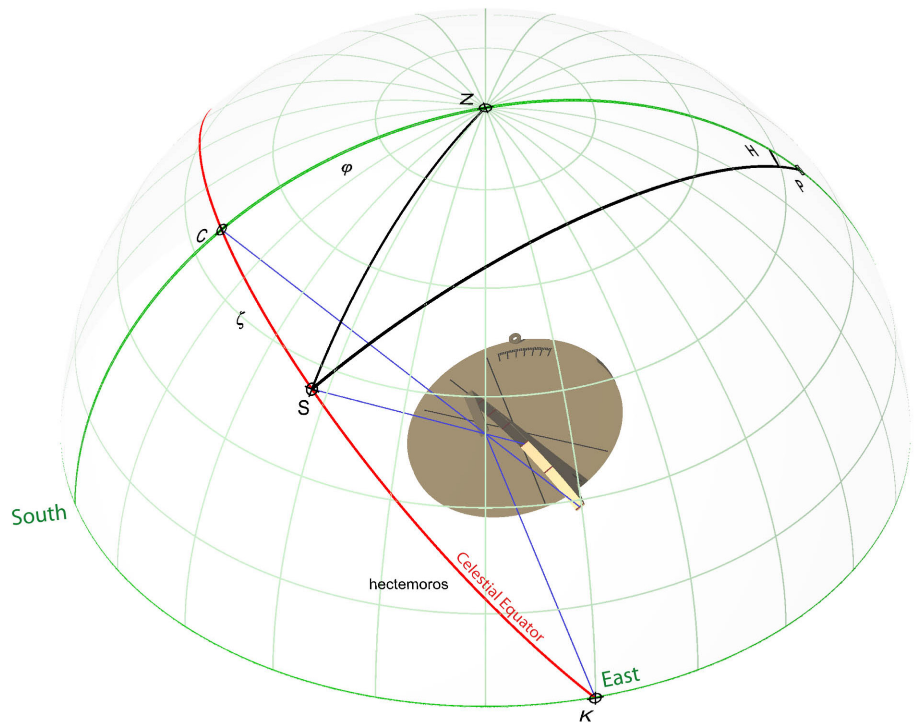

Figure 19. Representation in the celestial sphere of the dial in use in the morning on the equinoxes. The Sun S, situated on the celestial equator, illuminates the portable dial, which remains immobile. On the dial, the shadow falls upon the graduation for seasonal hour 4.

(35) $\sin h = \cos \phi \cos H,$

from which it follows that $\zeta = H$.

To sum up, on the equinoxes the operation of the dial can be divided into two phases: in the morning it is a dial with fixed horary angle is if it was a classical sundial drawn on a wall facing due east. After the Sun crosses the meridian, the user "forces" the dial by turning it in such a way that the gnomon's shadow lines up with the equinoctial line.

The relation established previously between the equinoctial hour and the *hectemoros* is only one hypothesis among others for explaining the principle of this portable dial; one could also

H	*H*'	90° – *hecte*
−105°	−84.422°	−61.935°
−90°	−72.362°	−66.0°
−75°	−60.302°	−61.935°
−60°	−48.241°	−52.293°
−45°	−36.181°	−40.239°
−30°	−24.121°	−27.179°
−15°	−12.060°	−13.677°
0°	0°	0°

Table 3. Comparison of horary angles with the complement of the hectemoros for summer solstice.

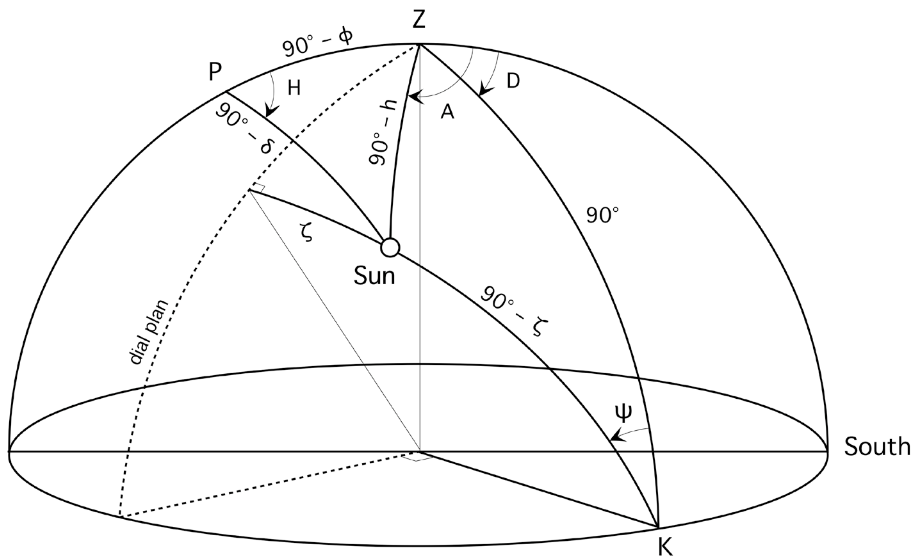

Figure 20. Representation by spherical trigonometry of the principle of the portable dial. The dial measures angle ζ whereas it is set as a function of angle ψ.

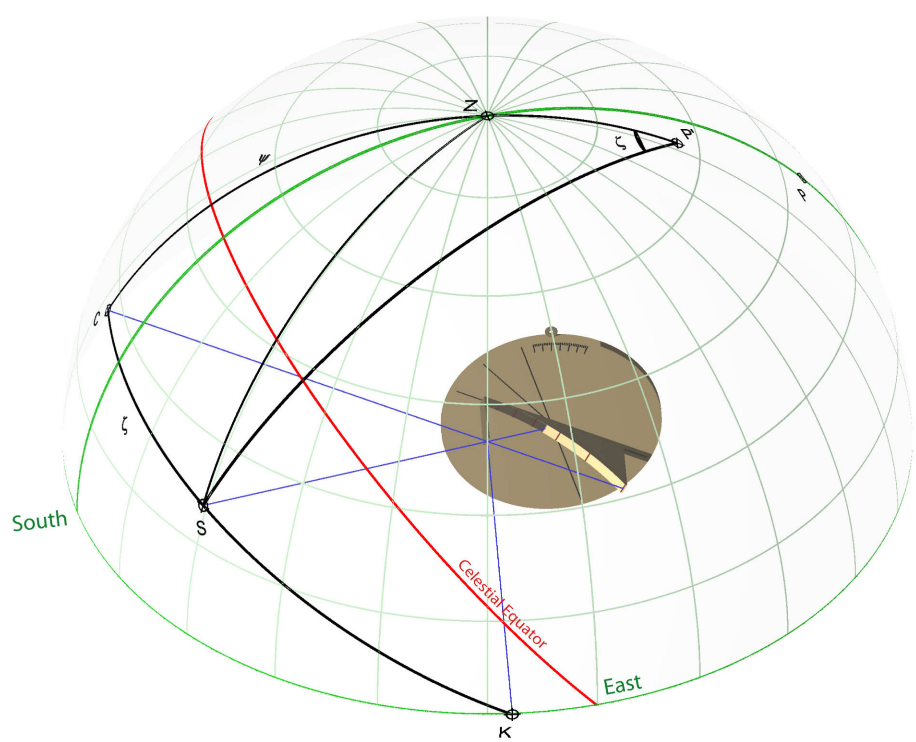

Figure 21. Representation in the celestial sphere of the portable dial in use in the morning on the winter solstice. The dial measures angle ζ which approximates a division by 6 of the semidiurnal arc described by the Sun. The Sun (at *S*) illuminates the dial, which is oriented to the southwest. The gnomon's shadow falls upon the graduation for seasonal hour 3.5.

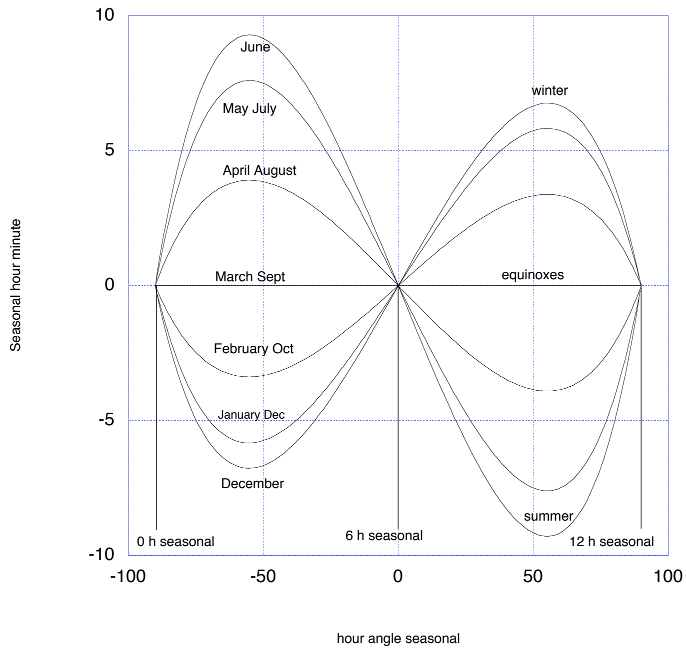

Figure 22. Error in reading the time for latitude 40°.

imagine a purely empirical approach. For that matter, we will see that a recognition in antiquity of the intrinsic error in the principle of this dial is not very probable.

If we resume the analogy between the angles but for an arbitrary date, for example for the summer solstice, for a latitude of 40° and a solar declination of +24°, we see that there is no longer an equivalence among the three aforesaid angles. One gets the following quantities for the morning, keeping in mind that cos *hecte* = sin H cos δ (Table 3).

Additionally, the gnomon's shadow no longer falls on the equinoctial line, but below it. The inventor who conceived of this dial imagined that one should modify the inclination of the equinoctial line, and that this inclination, for a given day, was equal to the noon altitude of the Sun, namely $90° - \phi + \delta$. This implies that angle ψ which the equinoctial line makes with the vertical passing through the gnomon equals $(\phi - \delta)$. The purpose of this constraint is to indicate on the dial, to the extent that this is possible, the seasonal horary angle H'. Now we have seen that the dial measures the angle ζ, and in the general case where the Sun's declination is not zero, it can be shown (Fig. 20) that:

H	H'	90° − *hecte*
−105°	−84.422°	−85.207°
−90°	−72.362°	−74.218°
−75°	−60.302°	−62.587°
−60°	−48.241°	−50.506°
−45°	−36.181°	−38.115°
−30°	−24.121°	−25.516°
−15°	−12.060°	−12.789°
0°	0°	0°

Table 4. Comparison of horary angles with the angle measured on the universal portable dial.

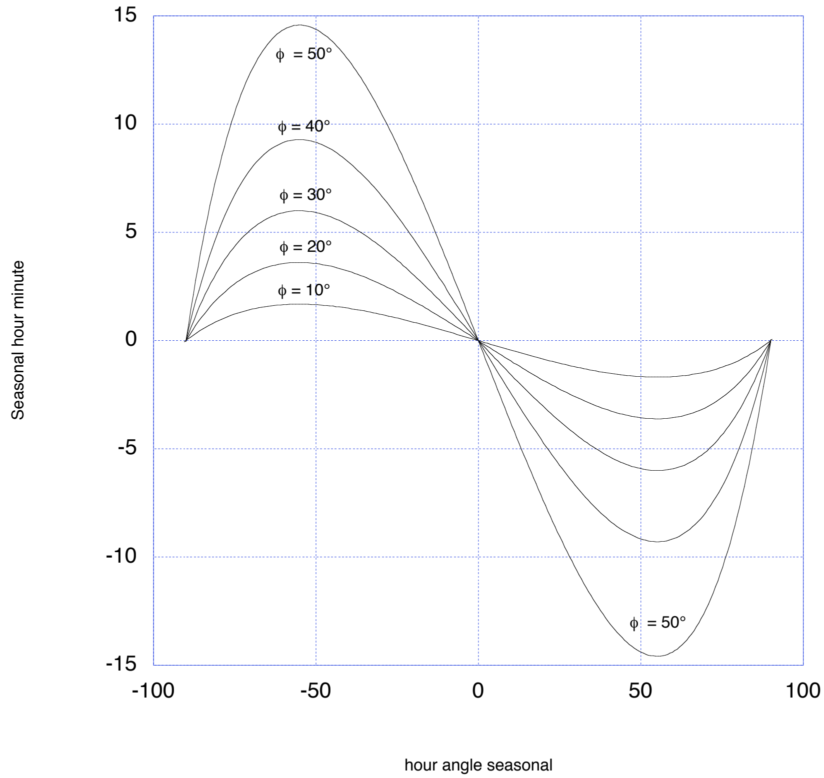

Figure 23. Deviation (H' – ζ) for different latitudes on the summer solstice.

(36) $$\cos\zeta = \frac{\sin h}{\cos\psi}$$

or, expressed in terms of the primary parameters:

(37) $$\cos\zeta = \frac{\sin\phi\sin\delta + \cos\phi\cos\delta\cos H}{\cos(\phi - \delta)}$$

such that ζ has the same sign as *H*.

The whole principle of this dial thus rests on the relationship between ζ and *H'*, using as its starting hypothesis the assumption that the deviation between these two angles (expressed on hours) would be acceptable in civil use (*cf. infra).* Let us observe that the principle of its operation is not transferable to a sundial that is supposed to show the equinoctial hour (Fig. 21). In other words, this dial is only useful in a society in which the civil practice is seasonal hours. This could explain why it fell into desuetude, unlike the other kinds of portable sundials which can indicate the seasonal or the equinoctial hour indifferently, such as the cylinder or the armillary, which continued to be used right through the Middle Ages and the Renaissance, and even beyond in the case of the cylinder. This fundamental point seems not to have been noticed: while the majority of ancient sundial types could have survived through the centuries despite the change in usage of hours, the portable dial studied here experienced an abrupt halt because it could not be adapted to equal hours.[24]

24 An example of a truly universal portable dial is that of Regiomontanus.

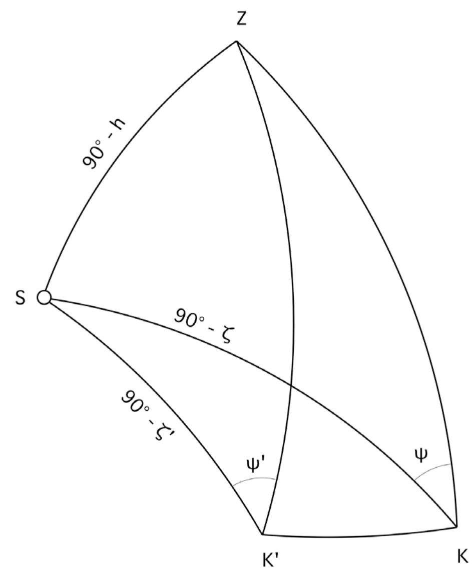

Figure 24. Principle of the calculation of error in latitude, represented in spherical trigonometry. A bad setting of the read-off arm of the dial introduces a false angle ψ', so that instead of measuring angle ζ, the dial measures ζ'.

Resuming the preceding table (for the summer solstice), we can see that the deviation between *H'* and ζ is minimized by the solution tabulated in Table 4. The greatest deviation here between *H'* and ζ (for latitude 40°) reaches a little more than 9 minutes of a seasonal hour (Fig. 22).

One should underline the great subtlety of this portable sundial, which achieves the feat of showing, with a very acceptable error, the seasonal hour, measuring it on a scale graduated in equal angles of 15°, that is, an equinoctial scale. The considerable deviation between the equinoctial horary angle *H* and angle ζ is particularly apparent in this table; for example for *H* = –75° (7h), the deviation almost reaches 50 equinoctial minutes.

When one plots the deviations (H' – ζ) as a function of latitude and as a function of declination (Fig. 23), one finds on the one hand that the greatest deviations are at the summer solstice, and on the other hand, that the deviation increases with higher latitudes.[25] But rather curiously, the deviation always seems to reach its maximum for the same seasonal horary angle in absolute value.

To highlight this feature, it is necessary to study the derivative $d(H'-\zeta')/dh$ and set it equal to zero to obtain the precise seasonal horary angle corresponding to the maximum deviation. We obtain a rather long expression that nevertheless depends only on the semidiurnal arc:

$$(38) \qquad H' = \pm 90 \frac{\arcsin\left(\dfrac{180\sqrt{\cos H_0\left[-H_0^2 + 90^2(1-\cos H_0)\right]}}{90^2 - H_0^2}\right)}{H_0}$$

In all rigor, *H'* undergoes a gentle fluctuation of a few minutes as a function of the Sun’s declination and the latitude, but always remains very close to a seasonal horary angle of ±55° (except

25 Around the polar circles, the dial becomes unusable for the most part because of the huge error caused by the latitude.

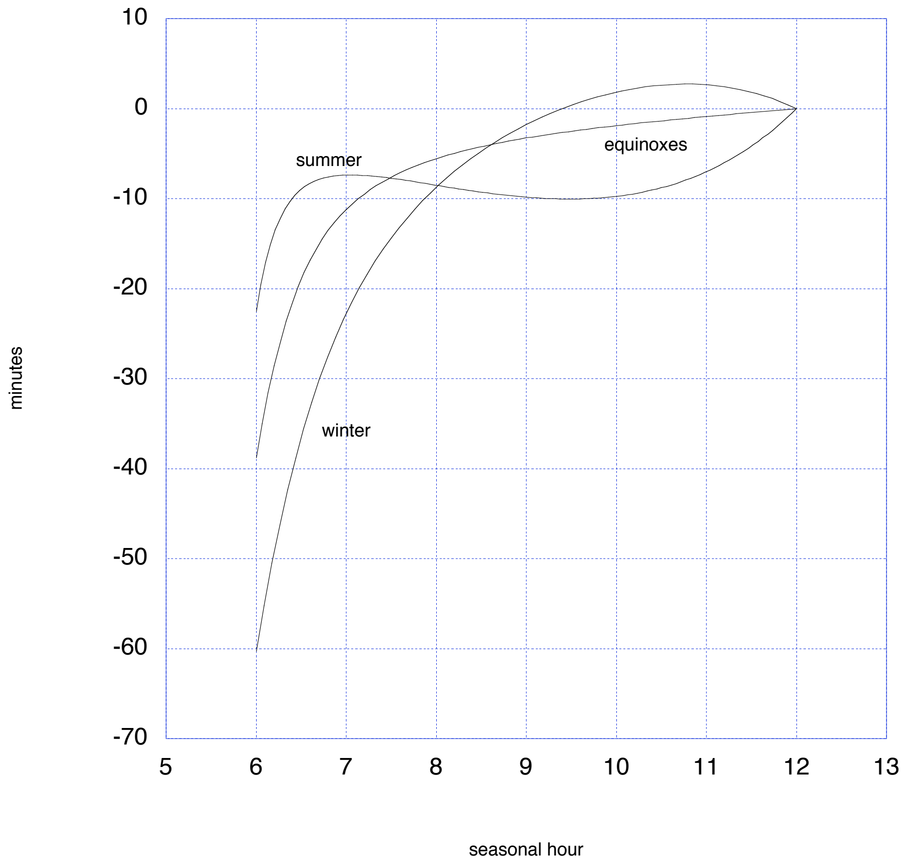

Figure 25. Error in reading the time for φ = 40°, φ' = 39°.

for high latitudes, where one observes a drop starting at ϕ = 62°). For a latitude of 40°, this corresponds to seasonal times of 2 h 19 m and 9 h 40 m in the summer, supposing one could read the time with such precision.

When the user decides to read the time on this dial, he first has to set it in latitude by positioning the cursor on the circular scale; then he must calibrate the mobile arm according to the date (this presupposes a clamp to keep the arm fixed). The user then causes the dial to turn until the shadow of the gnomon falls upon the read-off scale. If the user is uncertain about whether it is before or after solar noon, he has to make a first reading of the time and then wait before making a second reading; if the shadow shifts away from the gnomon, it is morning, while if it shifts towards the gnomon, it is afternoon.

In addition to the intrinsic error of this dial (*cf. supra*), many causes can perturb the accuracy of the indicated time and even make it impossible to read off. The first cause of error is incorrect setting of the latitude: on all the portable dials that have been found, the latitudes of cities or provinces appear on the back. If broadly speaking the latitudes correspond to reality for the cities, the same does not go for the provinces, for which the latitudinal extension can be considerable.[26]

26 There are additionally errors in the latitudes of cities, and one can find variants between the exemplars. Constantinople, for example, is places at 41° de latitude in the Aphrodisias exemplar but at 43° in the Samos, Memphis, and Rockford exemplars. See the table provided by J. V. Field and M. T. Wright, "Gears from the Byzantines : A Portable Sundial with Calendrical Gearing," *Annals of Science*, 42, 1985, p. 109-110.

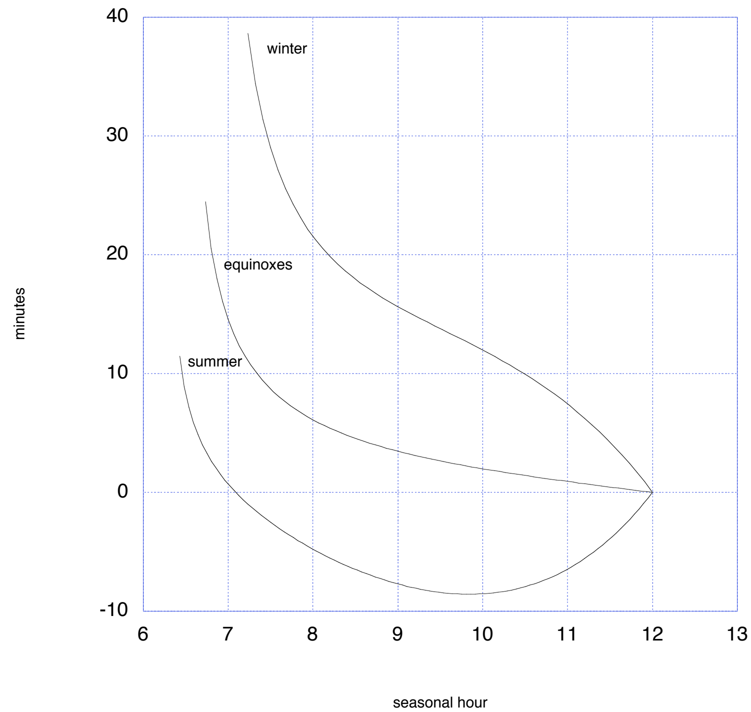

Figure 26. Error in reading the time for $\varphi = 40°$, $\varphi' = 41°$.

Getting the latitude wrong amounts to getting angle ψ wrong, hence to inclining the mobile arm at an incorrect angle ψ' such that $\psi' = (\phi' - \delta)$, where ϕ' is the supposed latitude and ϕ the correct latitude. In Fig. 24 we see that the incorrect inclination brings with it a displacement of the direction in which the gnomon points from *K* to *K'*. As a result one will read off a false angle ζ' on the read-off scale, and this angle has to be compared to the Sun's seasonal horary angle *H'* for the locality ϕ. This angle is:

(39) $$\cos\zeta' = \frac{\sin\phi\sin\delta + \cos\phi\cos\delta\cos H}{\cos\psi'}$$

In this formula, the denominator corresponds to the part set manually by the user. There are two kinds of configuration for latitude error: either the supposed latitude ϕ' is greater than the correct latitude ϕ, or it is less.

(1) $\phi > \phi'$. It is in the neighborhood of the meridian that the error is most important—in fact considerable—and the *maximum maximorum* takes place on the winter solstice (graph 3). Right at the meridian the false angle ζ' becomes:

(40) $$\cos\zeta' = \frac{\cos(\phi - \delta)}{\cos(\phi' - \delta)}$$

Fig. 25 displays the error $H' - \zeta'$ for the afternoon (the error for the morning is symmetrical but with opposite sign) when one is off by 1° in latitude (in this case the user is at 40° latitude but sets his dial for 39°). One can clearly see that between seasonal 6 h and 7 h (and hence also between 5 h and 6 h) the error is huge but decreases very quickly in such a way that one soon

Mean dates	*1st century*	*2nd century*	*3rd century*	*4th century*	*5th century*	*6th century*
Vernal equinox	March 22	March 21	March 21	March 20	March 19	March 18
Summer solstice	June 24	June 23	June 22	June 22	June 21	June 20
Autumnal equinox	Sept. 25	Sept. 24	Sept. 23	Sept. 22	Sept. 22	Sept. 21
Winter solstice	Dec. 22	Dec. 22	Dec. 21	Dec. 20	Dec. 19	Dec. 19

Table 5. Shift of dates of solstices and equinoxes in the Julian calendar.

is within the zone where the error is in the neighborhood of −10 minutes. One should note that the user can never observe seasonal 6 h (solar noon) on the dial, even by placing it in the plane of the Sun; at this moment, instead of pointing due east ($D = -90°$), the gnomon is off by several degrees (in summer the deviation can significantly exceed 20°). But since the shadow covers the read-off scale completely, the user cannot realize that he is committing an error in believing that he is before or after seasonal 6 h.

(2) $\phi < \phi'$. In the case where the latitude to which the sundial is set is greater than the correct latitude, there results a zone of hiatus in the readability of the time since it becomes impossible to use the dial in the neighborhood of seasonal 6 h (graph 4). In fact, if the mobile arm of the dial is set such that its inclination is less than the actual noon altitude, one cannot read off the time when the Sun has an altitude greater than $(90° - \psi')$, that is, when $(\cos \zeta' > 1)$.

In practice, this means that at a certain moment during the day, the user, no matter what how he turns his dial about, will never be able to bring the gnomon's shadow upon the read-off scale. This impossibility of making a reading, unlike the preceding case, allows him to grasp that there is a problem with the setting of the mobile arm.

Here again (see Fig. 26, where the error in latitude is 1°), it is at the winter solstice that the greatest deviations result between H' et ζ', and one notes that the error decreases more slowly than in the case where $\phi > \phi'$. Moreover, the dial is unusable for more than two seasonal hours during the day for a latitude around 40°.

One can take consideration of the error that one commits by using an incorrect orientation of the dial; but since the user is required to orient his dial so that the gnomon's shadow falls on the scale of hours—otherwise one cannot read off the time—the orientation is not a significant cause of error.

On the other hand, there is another source of error that has to be taken into account, namely the error in the date. The majority of examplars of the universal portable sundial bear a scale of dates, bounded by the two solstices, with the intermediate graduations corresponding to the entries of the Sun into the zodiacal signs.[27] Let us note in passing that the sole function of the symmetrical scale of dates that one finds on the exemplars is to refine the setting by means of a point situated at the tip of the read-off scale.

The solstices are generally placed on the 8th days before the Kalends of January and July, that is, respectively December 25 and June 24.[28] The indicated solstice dates are the classical dates according to the Julian calendar, which came into use in 45 BC. As for the equinoxes, they are set on

27 On this point see P. Brind'Amour, *Le calendrier romain*, Université d'Ottawa, 1983, p. 15-19.

28 F. K. Ginzel, *Handbuch der Mathematischen und Technischen Chronologie*, *op. cit*., p. 179-181 et p. 282.

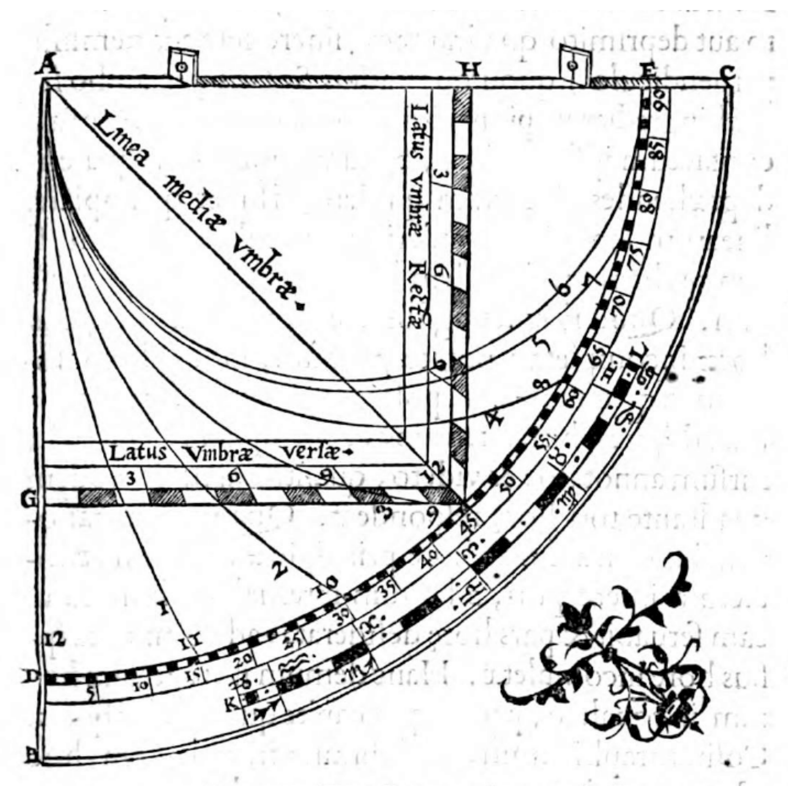

Figure 27. Illustration of the quadrans vetus from Orontius Finnaeus, *De solaribus horologiis et quadrantibus*, Paris, 1560, Book II, p. 143. On this ancient quadrant can be seen the sighting system consisting of pinnules, and the limb graduated in 90° with a dentate scale. Below the limb, the author has added a zodiacal calendar establishing the correspondence between the days of the year and the degrees of the zodiac occupied by the Sun on each day. Six circular arcs meeting at the top represent the hour curves.

March 25 (8th day before the Kalends of April) and September 24 (8th day before the Kalends of October). But because of the drift of the Julian year relative to the tropical year, the astronomical seasons underwent a shift as tabulated in Table 5 for the period from the 1st century through the end of the 6th century[29]

In addition to this shift of the seasons there is a slow diminution of the obliquity of the ecliptic, which changed from 23° 41' in the 1st century to 23° 37' in the 6th century. This has the effect of significantly modifying the inclination of the mobile arm at the solstices, and given that the dials were engraved with an obliquity of 24°, the error ends up being a little over 0° 20' in the 6th century. Making a bad estimate of the date (or of the obliquity) on this type of portable dial amounts to applying an erroneous solar declination, and it is easy to see that this reduces to the same kind of error as introducing a bad latitude: the angle ψ' here becomes $(\phi - \delta')$ where δ' is the incorrect solar declination. Since the declination varies very little around the solstices, getting the date wrong by several days is inconsequential, in particular since the graduations of the dials do not allow a setting to the precise day! For example, a 6th century user who thinks that the summer solstice falls always on June 24 is committing an error of 0° 3', which is not

29 D. Savoie, "Les dates des quatre saisons," *Observations et Travaux*, n° 19, 1989, p. 3-6. The mean dates (largest number of occurrences of the date by century) in the present table were calculated in UT according to the algorithms given by J. Meeus, *Astronomical Algorithms*, Willmann-Bell, Richmond, USA, 1998.

perceptible. On the contrary, the same user, if he thinks that the vernal equinox falls on March 25 rather than March 18, commits an error of close to 2° 30' in the declination, and this does become perceptible.

As has already been said, the dial is perfectly correct only on the equinoxes, and its error is absolutely imperceptible for at least a month before and after the equinox. Once again, our modern conception of the idea of precision for such a dial needs to be put in perspective. It would appear logical to compare the seasonal time indicated by the portable dial to that which would be indicated on an ancient dial that has been perfectly drawn and is in a functioning state, which plays the role of a kind of control-clock. In reality this appears unrealistic for many reasons. The first is tied to the physical difficulty of making such an error apparent, since ancient Greco-Roman sundials, with rare exceptions, lack any subdivision of the hour. This means that to know the time when the gnomon's shadow falls between two hour lines on the control-clock, one has to make an estimation that one cannot guarantee to be accurate to within 5 or 10 minutes. Hence unless the maximum error of the portable dial falls, by a felicitous stroke of luck, on a round hour number, it is practically certain that an error of 7 or 9 minutes is impossible to make visible. What is more, how can one be certain that the error does not arise from a bad setting in latitude or date rather than from the conceptual basis of the dial? In fact it is legitimate to ask whether the inventor or inventors of this portable dial were conscious that it was, in an ideal sense, false.

We should thus seriously consider the possibility that this dial was considered in antiquity to be perfectly exact. For lack of documents, one cannot know whether its invention resulted from a theoretical investigation or by chance or from a combination of the two. But if one recalls that it is not a precise instrument for time measurement, one has to conclude that it is doubtless the best candidate for being the famous πρὸς πᾶν κλίμα of which Vitruvius speaks,[30] and that it fulfilled in a very satisfactory way its mission of giving a good notion of the time while traveling over a vast extent of latitude. It constitutes one of the very are examples of sundials that sank into obvlivion when unequal hours progressively gave way to equinoctial hours.

The dial can be compared—without presuming *a priori* a line of descent—to another later instrument of Arabic or rather Persian origin,[31] the "ancient quadrant" (*quadrans vetus*), one of the earliest references to which goes back to Hermann le Boiteux (1013-1054),[32] and which was diffused in Europe especially thanks to Master Robert Anglès (*Robertus Anglicus*)[33] and Sacrobosco[34] (Fig. 27). This consists simply of an adaptation of a diagram of unequal hours—a grid that was in

30 Vitruvius, *De Architectura*, Book IX, chap. VIII, 1, gives a description of sundials, attributing them to inventors, and speaks of a dial "for all latitudes" (in Greek in the text). See the translation by J. Soubiran, Les Belles Lettres, Paris, 1969, p. 30.

31 See D. King, "A *Vetustissimus* Arabic treatise on the *Quadrans vetus*," *Journal for the History of Astronomy*, xxxiii, 2002, p. 237-255 et F. Charette, *Mathematical instrumentation in fourteenth-century Egypt and Syria*, Brill, Leiden-Boston, 2003, p. 211-215.

32 Study of the texts of this period, including the one allegedly by Hermann le Boiteux, enabled J.–M. Millas Vallicrosa to demonstrate that there actually existed two types of *quadrans* : the *vetustissimus*, which was older than the *vetus*. See the important study by J.–M. Millas Vallicrosa, "La introduccion del cuadrante con cursor en Europa," *Isis*, t. XVII, 1932, p. 218-258.

33 The Latin and Greek text were edited by P. Tannery, *Le Quadrant de Maître Robert Anglès*, (Montpellier, XIII[è] siècle), *Notices et extraits des manuscrits de la Bibliothèque Nationale*, Paris, t. 35, 1897, 2[è] partie, p. 561-640. See, however, the remarks and criticisms by W. R. Knorr (*cf. infra*).

34 One should consult the fundamental study by W. R. Knorr, "The Latin Sources of *Quadrans vetus* and What They Imply for Its Authorship and Date," *Texts and Contexts in Ancient and Medieval Science : studies on the occasion of John E. Murdoch's seventieth birthday*, E. Sylla and M. McVaugh, Leiden, New York, Köln, Brill, 1997, p. 23-67.

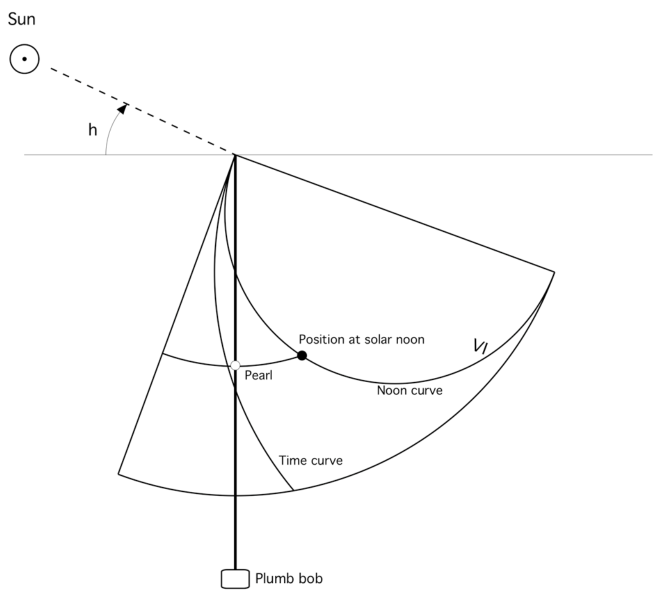

Figure 28. Principle of setting and use of the *quadrans vetus*. Knowing the latitude of the locality and the Sun's declination, one calculates the Sun's noon altitude (90° − latitude + declination), which one locates on the graduated limb. One now extends the plumb line, which cuts the circle for unequal hour VI at a point that one finds by means of a sliding bead. When this setting is complete, one sights the Sun; the position of the bead enables one to read off the unequal hour on the network of curves.

wide use notably on the backs of planispheric astrolabes—to a sighting system complemented by a weighted thread. Otherwise put, the ancient quadrant is a "universal" altitude sundial since from observation of the altitude of the Sun one deduces the unequal hour, no matter what date and latitude of the locality.

This *quadrans vetus* presents remarkable properties which call to mind those of the ancient universal portable dial: it is not rigorously exact except at the equinoxes,[35] it has to be adjusted to the noon altitude of the day, its equinoctial path is valid at (nearly) all latitudes, and it indicates the seasonal hour subject to exactly the same approximation as the ancient universal portable dial.

Let us briefly describe this sundial. It is a quadrant of a circle bearing a sighting system (pinnules on one of its sides, a plumb line fixed to the point of convergence of the hour lines, graduation of the limb in 90°), six circular arcs meeting at the instrument's top and constituting hour lines that divide the limb into six sectors of 15°. In the 12th century, a zodiacal calendar was added in the form of a mobile circular sector along the limb, allowing the solar noon altitude to be obtained as a function of the date; this is what was named the cursor. It served as a kind of table of solar declination.

On the day of observation, one calculates the altitude of the Sun's culmination at the place of observation, which one finds (for example by means of a bead sliding along the plumb line) on the line for seasonal hour VI; then one sights the Sun by holding the quadrant vertically. The entire dial is tilted, except of course for the plumb line. The sliding bead will now show the seasonal hour (Fig. 28).

35 This point did not escape J.–B. Delambre, *Histoire de l'Astronomie du Moyen Age*, Paris, 1819, p. 243-247. J. Drecker approaches the problem but in a more obscure manner in *Die Theorie der Sonnenuhren*, *op. cit.*, p. 86-89. See also R. D'Hollander, *L'Astrolabe*, *Histoire, Théorie, Pratique*, *op. cit.*, p. 213-216.

H	*T*	*T'*	*Error in minutes T – T'*
0°	0°	0°	0
15°	11.443°	12.364°	3.7
30°	22.886°	24.659°	7.1
45°	34.329°	36.810°	9.9
60°	45.773°	48.727°	11.8
75°	57.216°	60.299°	12.3
90°	68.659°	71.379°	10.9
105°	80.102°	81.771°	6.7

Table 6. Calculation of error of time determinations on the *quadrans vetus* on the summer solstice, latitude 47° 15'.

It is easy to show by plane trigonometry that the angle that one reads off on the dial is not the seasonal horary angle *T* but a different angle—let us call it *T'*—whose formula is:

(41) $$\cos T' = \frac{\sin h}{\sin h_m}$$

where h is the Sun's altitude and h_m is its noon altitude, that is, $90° - \phi + \delta$. Hence we have:

(42) $$\cos T' = \frac{\sin h}{\cos(\phi - \delta)}$$

or, again,

(43) $$\cos T' = \frac{\sin\phi \sin\delta + \cos\phi\cos\delta\cos H}{\cos(\phi - \delta)}$$

One sees at once that the expression for *T'* is the same as that for ζ in the ancient universal portable sundial. If for example one calculates the error committed on the summer solstice (δ = +23° 26') on a *quadrans vetus* calculated for the University of Puget Sound (ϕ = 47° 15' 44'') one obtains the data tabulated in Table 6.

It goes without saying that, by way of contrast, the difference (*H* – *T'*), that is, the equinoctial hour angle minus the "hour angle" of the dial yields considerable deviations: one cannot read off the equinoctial hour on such a sundial. Here one meets again one of the particularities of the ancient universal portable dial, with the difference that one could have adapted the *quadrans vetus* to equinoctial hours, but at the cost of suppressing its "universality," since such a sundial depends on the locality's latitude;[36] the hour lines remain circular arcs but they do not converge at the angle of the quadrant. Moreover, the cursor no longer has real meaning since its purpose is to render the dial usable universally.

36 The procedure for constructing a quadrant for equal hours is given, for example, by Jean Fusoris (1365-1436) : see E. Poulle, *Un constructeur d'instruments astronomiques au XV^e^ siècle : Jean Fusoris*, H. Champion, Paris, 1963, p. 71-73. A good example of a quadrant for equal hours (and unequal hours) may be found in Orontius Finnaeus, *De solaribus horologiis et quadrantibus*, Paris, 1560, Book II, p. 151.

While we wait for future archeological excavations to provide us with new portable sundials from antiquity—perhaps of a new type—we should recall the intent of these "universal" portable dials. They are witnesses first of all of remarkable originality and imagination in gnomonics, even if the principle on which they are based is relatively simple: to determine the time from the Sun's altitude. There are few scientific instruments that made it possible to deploy so many clever devices, for which there was need of a collaboratoin of applied technology, trigonometry, and geometry. The Middle Ages and the Renaissance would continue along this route opened by the gnomonists of antiquity, with dials in which esthetics assumed a major role, as in the *Navicula*, the *Capucin*, and the "Universal" of Regiomontanus. More surprisingly, one finds again even in the Age of Enlightenment this tendency to conceive of new universal portable sundials; it suffices to consult the *Supplément à l'Encyclopédie* [of Diderot and D'Alembert] and its superb plates,[37] where the author describes unpublished portable dials[38] some of which were due to the German mathematician Johann Heinrich Lambert (1728-1777).

Did these ancient universal dials serve in the "real world" to determine the time? Emmanuel Poulle, connoisseur of medieval astronomy, concluded bluntly that astronomical instruments (astrolabes, *quadrans vetus*, ...) constituted a pedagogical resource and incidentally a tool for calculation, but in no way a resource for observation. I absolutely share this view and I think that the same applies to the majority of ancient portable dials: they are objects of prestige and curiosity. As I have said above, their small dimensions make the read-off of the hour very difficult, not to say impossible; the crudeness of the drawing in the case of some of them, and in the case of others the uncertainness or inexactness of the locality where they were used and the shift in the dates of the seasons in the Julian calendar—all this makes highly improbable any precise determination of the time. Doubtless there would be much to say about this concept of "precision" in antiquity and even in the Middle Ages, and it is a safe bet that we impose modern concepts on these portable sundials that are totally anachronistic so far as concern the results that the ancients expected to obtain.[39]

37 *Supplément à l'Encyclopédie ou Dictionnaire Raisonné des Sciences, des Arts et des Métiers*, t. 2, Amsterdam, 1776, p. 97-106.

38 See Y. Massé, *De l'analemme aux cadrans de hauteur*, 2009, available from the author: 2 ruelle de la Ravine 95300 Pontoise, France.

39 Having executed an exemplar of the ancient universal sundial of 19 cm diameter in wood, I can confirm that it would yield the unequal hour with very good accuracy. However, none of the dials that have come down to us reach these dimensions; of the eleven known examples, half are 6 cm in diameter and the other half 11 cm.