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Mathematical Model of Reproduction System for Multivariate Dynamic Balance of Production and Consumption

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Abstract

In recent years, more and more elements of regulated economy used in different countries have become an object of scientific investigation. In this paper, we establish theoretical knowledge about quantitative relations and regularities of economic development. We show that the mathematical model expands opportunities for economic analysis. It also improves the quality of economic decisions. Intercorporate balance is presented as an economic-mathematical model of the reproduction process. Its expanded form represents the interconnection on production, distribution, consumption and accumulation of social product. Not only sectors of international economy, but physical and cost aspects of reproduction altogether are considered. Intercorporate balances cover only the most important types of products. Making the intercorporate balance requires the concept of a 'pure' branch. It unites all the production, regardless of departmental belonging and types of enterprises. The transition from industries to pure branches requires special transformation of actual economic data.

Keywords: Mathematical Model; Economy; Reproduction; Consumption; Balance.

1. Introduction

Economic modelling has lately become one of the most rapidly developing areas of economical science and its applications. The input-output coefficients and total input coefficients are calculated simultaneously throughout the development of long-term intercorporate models. At the same time, a large amount of work is being done to collect and process the information for the development of dynamic multi-factor models. Special scientific departments have been formed in all European countries where intercorporate models of consumption and production have to be reported periodically. These departments' aim is to adjust long-term input-output coefficients using existing intercorporate balance methods. It is possible to get national long-term balance plans and all elements of the intercorporate balance using these coefficients and a modification of the intercorporate model. (Fig. 1)

Most of the optimization calculations achieved by application of mathematical methods use report standard or report total input coefficients of state intercorporate balances. These coefficients provide information for corresponding systems of equations. This factor reduces the efficiency of economic-mathematical modelling [1, 2]. This underlines the importance of correcting total input coefficients of intercorporate balances.

A mathematical model and algorithms for realization of interconnected intercorporate balances is presented below. This model can be applied to such European regions where some elements of regulated economy are used. Consequently, the following economic division for European regions that investigate intercorporate balance is applied: Southern Europe (Italy, Spain, France and Greece), Central Europe (Germany, Austria, Switzerland, Hungary and Poland), Benelux countries (Belgium, the Netherlands and Luxembourg), and Nordic countries (Norway, Sweden, Denmark and Finland). The model can also be used for elements of economy of the US, China and some other countries. This work can be done if long-term total input coefficient matrices and long-term total product vector for the country are prepared or, at least, only the structure of this vector.

The long-term state dynamic intercorporate balances have minimum material capacity for the following reason. The balances' development has to be based on the optimal solution obtained by using the model presented below and optimization by selected criterion. Suppose that the structure of the long-term final product vector is given by the end of the planning period for the country. Then the criterion for minimization of total state gross product can be applied. Now suppose that only the structure of the long-term final product vector is given by the end of the planning period. Then the criterion for maximization of non-productive consumption as a function of production and consumption can be applied to the whole country.

Intercorporate balance represents an economic-mathematical model formed by a cross superposition of rows and columns of the table. Those are the balances of product distribution and costs of production made up according to total results. The main indicators here are the input-output coefficients and total input coefficients [3-6].

The dynamic model of the intercorporate balance characterizes the industrial relations in the national economy for a number of years and represents the reproduction process in dynamics (Figure 1). According to the intercorporate balance model, two types of calculations are performed. The first type is used when the balanced volume of production and distribution is calculated by level of final consumption. The second type includes mixed calculations and is used when the full balance of production and distribution is calculated according to the given production volume for one branch and the given final consumption for another. The matrix economic-mathematical model of the intercorporate balance has received widespread use [7]. It is a rectangular table (matrix), where the elements represent connections between economic objects. The quantitative values of these objects are calculated according to the rules of matrix theory. The matrix model represents the structure of production and distribution costs and of newly created value.

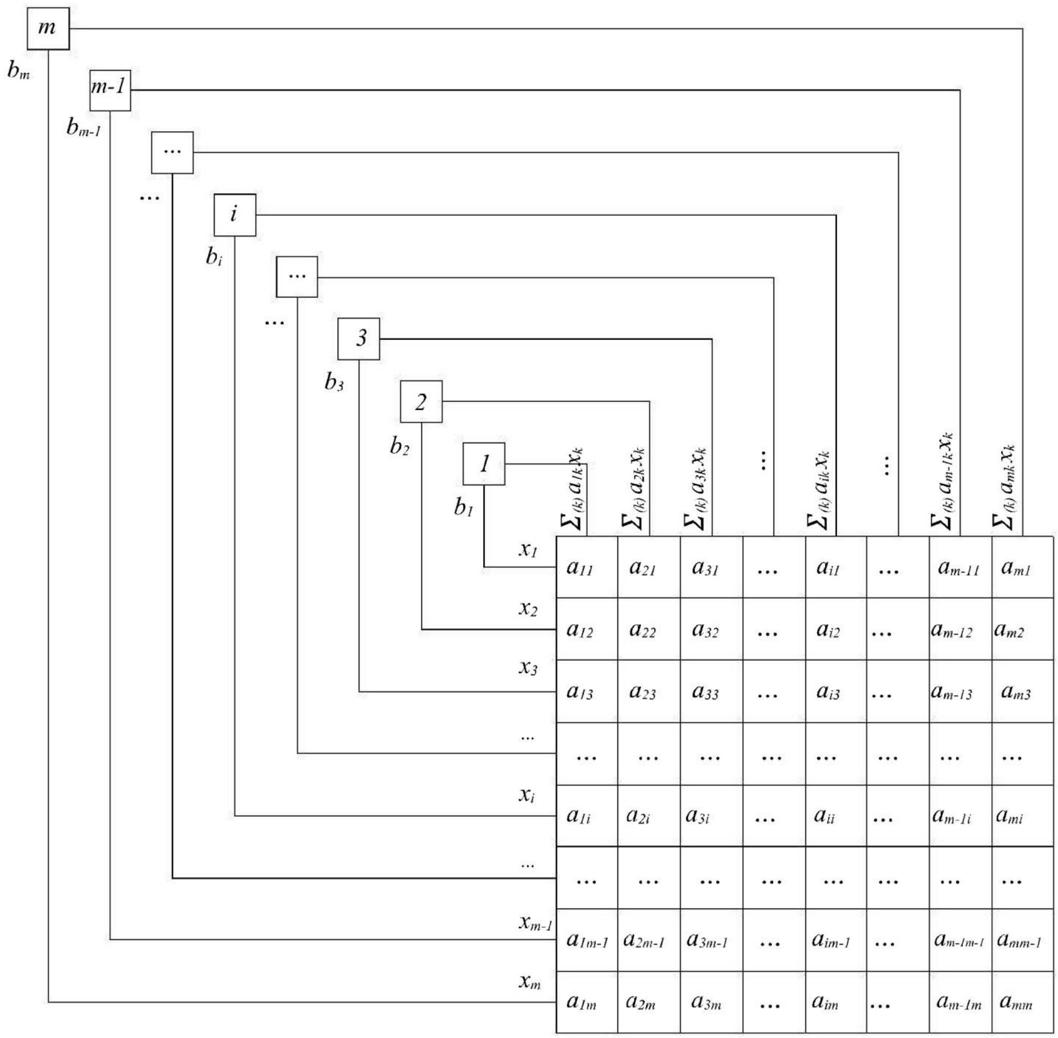

Figure 1. Extended reproduction scheme

The equations of the extended reproduction dynamic model are quite complex. It takes a lot of computational operations to find the solution. Even if initial information on structural matrices a_{ij}, b_{ij} and matrix of time delays τ_{ij} is collected, the solution of these equations can be hard to find or too approximate. This happens because of rounding and errors in calculation algorithms. The scheme of extended reproduction is shown in Figure 1. On the basis of the presented model, it is possible to conduct the statistical studies necessary for industrial branches. Furthermore, dynamic multifactor balance of production and consumption can be built.

2. Model equation

Define the coefficient matrix a_{ij} and the time delay matrix τ_{ij}. Write the system of equations for the model:

$$x_j(t) = \sum_{(i)} x_i(t - \tau_{ij}) \cdot a_{ij} + b_j(t), \tag{1}$$

where $i, j = 1, 2, \ldots m$.

If an economic system is closed and there are no foreign economic relations, external demand quantities are

$$b_j(t) = 0, where\ j = 1,2 \ldots m$$

The unknown quantities are

$$x_j(t) = 0, where\ j = 1,2 \ldots m$$

Suppose all elements τ_{ij} are commensurable values and whole multiples of a value τ. Then solution of system (1) is sufficient. The variable τ is called elementary time of delay and defined by following equality:

$$\tau_{ij} = n_{ij}\tau, \qquad i, j = 1, 2, ...m,$$

where n_{ij} are positive integers.

Suppose τ_{ij} are general variables. New results follow from the case of commensurable τ_{ij}. Let us construct theorems using the language of almost periodic functions [8, 9].

Find partial solutions of the homogeneous system.

$$x_j(t) = \sum_{(i)} x_i\left(t - n_{ij}\tau\right) \cdot a_{ij}, \tag{2}$$

Get this system from (1) where $b_j = 0$ as follows:

$$x_i(t) = X_i exp(\lambda t), \qquad i = 1, 2, ...m \tag{3}$$

Get the following system of equations for X_i and λ variables by substituting (3) and (2) and reducing common factors

$$X_j = \sum_{(i)} X_i exp\left(-\lambda\tau n_{ij}\right) \cdot a_{ij}$$

Denote

$$Z = exp(-\lambda\tau), \tag{4}$$

rewrite the system as

$$X_j = \sum_{(i)} X_i Z^{n_{ij}} a_{ij},$$

or

$$\sum_{(i)} \left(a_{ij} Z^{n_{ij}} - \delta_{ij}\right) \cdot X_i = 0, \tag{5}$$

where the Kronecker delta $\delta_{ij} = 0\ where\ i \neq j$ and 1 for $i = j$.

Select such a value of Z that system (5) determinant equals zero:

$$\det\left(a_{ij} Z^{n_{ij}} - \delta_{ij}\right) = 0 \tag{6}$$

This equation defines Z in practical research and is a high-degree algebraic equation for Z.
Suppose this equation is solved and has following roots:

$$Z_1, Z_2, Z_3, ..., Z_n$$

where N is the degree of equation (6).
Get this value set from equation (4) relating Z and λ,

$$\lambda_{1k} = -\frac{1}{\tau}\ln Z_1 + \frac{2\pi}{\tau}ki$$

$$\lambda_{2k} = -\frac{1}{\tau}\ln Z_2 + \frac{2\pi}{\tau}ki$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\lambda_{nk} = -\frac{1}{\tau}\ln Z_n + \frac{2\pi}{\tau}ki$$

where $k = 0; \pm 1; \pm 2; \pm ...,$
or shortly:

$$\lambda_{pk} = -\frac{1}{\tau}\ln Z_p + \frac{2\pi}{\tau}ki \tag{7}$$

where $p=1,\ 2,\ \dots\ N$; $k=0;\ \pm1;\ \pm2;\ \dots$; $i=\sqrt{-1}$

Values λ_{pk} are complex and there's nothing incorrect in this. Not all values used in this theory have a clear meaning in economics [10-12].

Suppose these are solutions of equations (5) where Z_p have been found

$\{X_{1i}\},\ \{X_{2i}\},\ \dots\{X_{pi}\},\ \dots.\{X_{ni}\},\quad i=1,2,\dots m.$

Partial solutions have the following form:

$$x_{pki}=X_{pi}\exp\left(\lambda_{pk}t\right)=X_{pi}Z_p^{-\frac{t}{\tau}}\exp\left(\frac{2\pi k}{\tau}it\right) \tag{8}$$

where $i=I,\ 2,\ 3,\ \dots\ m$ is the corresponding branch index;

$p=I,\ 2,\ 3,\ \dots\ N$ is the index of solutions relating to different values of Z_p;

$K=0;\ \pm I;\ \pm 2;\ \dots$ is an index of separate solutions relating to the same value of Z_p but different values of λ_{pk}.

Since system of equations (2) is linear, it follows that any linear combination of partial solutions (8) is a solution of the system.

Take random values C_{pk}, where p and k take on the same values as in formula (8). Write a general solution of the system of equations (2) as follows:

$$x_i\left(t\right)=\sum_{(p,\,k)}C_{pk}x_{pki}\left(t\right)=\sum_{(p,k)}C_{p,k}X_{pi}Z_p^{-\frac{t}{\tau}}\exp\left(\frac{2\pi k}{\tau}it\right)=\sum_{(p)}X_{pi}Z_p^{-\frac{t}{\tau}}\psi\left(t\right), \tag{9}$$

where ψ_p are periodical functions with the period τ given by Fourier series, obtained by summation over k :

$$\psi_p\left(t\right)=\sum_{(k)}C_{pk}\exp\left(\frac{2\pi k}{\tau}it\right)$$

where $p=I,\ 2,\ \dots\ N$

The following fundamental system of solutions of a homogeneous system of equations will be used further on:

$$x_{pi}\left(t\right)=X_{pi}Z_p^{-\frac{t}{\tau}}\psi_p\left(t\right) \tag{10}$$

where i is the branch index $\left(i=1,\ 2,\ \dots\ m\right)$

p is the solution index $\left(p=I,\ 2,\ \dots\ N\right)$.

Maximum value of p index (or N) surpasses the number of branches $m:N\geq m.$

3. Writing theory equations the Cauchy form. Solving problems with given initial conditions.

Define operator $\hat{\mathrm{T}}$ by the equation $\hat{\mathrm{T}}f\left(t\right)=f\left(t+\tau\right)$ for all functions of the considered class.

Let m^2 numbers ν_{ij} where $j=1,2,\dots m$ be defined by the equation $\nu_{ij}=n_{ij}-1$. Further, let us form values $\nu_j=\max\nu_{ij}$ by finding a constant $j=1,2,\dots\ m$ and a variable $i=1,2,\dots\ m$.

Find the number of intervals τ. Value of $\left(\nu_j+1\right)$ equals the number of τ which indicates the maximum delay of consumption time j delay compared to the production moment.

Equations of open dynamic system theory considered in this section are as follows:

$x_j\left(t+1\right)=\sum_{(i)}x_i\left(t-\nu_{ij}\tau\right)+b_j\left(t\right),$

where $j = 1, 2, \ldots m$. Substituting $t \to (t+1)$ and adding the right-hand sides $b_j(t)$ we get this system from system (2).

Following variables are used in forming the right-hand sides of equation of the system:

$$x_j(t - \nu_j \tau);\ x_j(t - (\nu_j - 1)\tau);\ \ldots;\ x_j(t - \tau);\ x_j(t)$$

Consider these output values j of prior periods independent. Now we shall give the following notation [12, 13]:

$$x_j(t - \nu_j \tau);\ x_{j\nu_j}(t);\ x_{j(\nu_j - 1)}(t);\ \ldots;\ x_{j1}(t);\ x_j(t)$$

Write the system in Cauchy form using the new notation. $\hat{\mathrm{T}} x_j(t) = \sum_{(i)} x_i \nu_{ij}(t) \cdot a_{ij} + b_j(t)$

$$\hat{\mathrm{T}} x_{j1}(t) = x_j(t)$$

$$\hat{\mathrm{T}} x_{j2}(t) = x_{j1}(t) \tag{11}$$

....

$$\hat{\mathrm{T}} x_{j\nu_j}(t) = x_{j\nu_j - 1}(t),$$

где $j = 1, 2, \ldots\ldots m$.

Let us introduce variables $x_{i\nu}(t)$ for a time moment t and form a column of these variables. Further, let us form a column of right-hand sides:

$$\vec{x}(t) = \left\{ \begin{array}{c} \left\{ \begin{array}{c} x_1(t) \\ x_{11}(t) \\ \ldots\ldots\ldots\ldots \\ x_{1\nu_1}(t) \end{array} \right\} \\ \left\{ \begin{array}{c} x_2(t) \\ x_{21}(t) \\ \ldots\ldots\ldots\ldots \\ x_{2\nu_2}(t) \end{array} \right\} \\ \ldots\ldots\ldots\ldots\ldots \\ \left\{ \begin{array}{c} x_j(t) \\ x_{j1}(t) \\ \ldots\ldots\ldots\ldots \\ x_{j\nu_j}(t) \end{array} \right\} \\ \ldots\ldots\ldots\ldots\ldots \\ \left\{ \begin{array}{c} x_m(t) \\ x_{m_1}(t) \\ \ldots\ldots\ldots\ldots \\ x_{m\nu_m}(t) \end{array} \right\} \end{array} \right\}; \quad B(t) = \left\{ \begin{array}{c} \left\{ \begin{array}{c} b_1(t) \\ 0 \\ \ldots\ldots\ldots \\ 0 \end{array} \right\} \\ \left\{ \begin{array}{c} b_j(t) \\ 0 \\ \ldots\ldots\ldots \\ 0 \end{array} \right\} \\ \ldots\ldots\ldots\ldots \\ \left\{ \begin{array}{c} b_j(t) \\ 0 \\ \ldots\ldots\ldots \\ 0 \end{array} \right\} \\ \ldots\ldots\ldots\ldots \\ \left\{ \begin{array}{c} b_m(t) \\ 0 \\ \ldots\ldots\ldots \\ 0 \end{array} \right\} \end{array} \right\}$$

Construct a matrix $A(t)$ from the system (II):

$$
A(t)=\begin{array}{c}
i=1\\ \\ \\ \\ \\ \\ \\ \\ \\ = \; i=2\\ \\ \\ \\ \\ \\ \\ \\ \\ i=m\\ \\ \\ \\ \\
\end{array}
\left[\begin{array}{c|cccccc|cccccc|c|cccccc}
 & & & i=1 & & & & & & i=2 & & & & & & & i=m & & \\
 & 0 & 1 & 2 & .. & v_1-1 & v_2 & 0 & 1 & 2 & & v_2-1 & v_2 & & 0 & 1 & 2 & ... & & v_m\\
0 & . & . & . & a_2v_{22} & . & . & . & . & . & a_{2v_{22}} & ... & & & . & . & . & a_{mv_{1m}} & & ...\\
1 & 2 & 0 & 0 & . & 0 & 0 & 1 & 0 & 0 & . & 0 & 0 & & 1 & 0 & 0 & . & 0 & 0\\
2 & 0 & 2 & 0 & . & 0 & 0 & 0 & 1 & 0 & . & . & . & & 0 & 1 & 0 & . & 0 & 0\\
\vdots & . & . & . & . & . & . & . & . & . & . & . & . & & . & . & . & . & . & .\\
v_1-2 & 0 & 0 & 0 & . & 0 & 0 & 0 & 0 & 0 & . & 0 & 0 & & 0 & 0 & 0 & . & 0 & 0\\
v_1 & 0 & 0 & 0 & . & 1 & 0 & 0 & 0 & 0 & . & 1 & 0 & & 0 & 0 & 0 & . & 1 & 0\\
 & & & & & & & & & & & & & ..... & & & & & & \\
0 & . & . & . & a_{2v_{21}} & . & .. & . & . & . & a_{2v_{22}} & . & .. & & . & . & . & . & a_{2v_{2m}} & \\
1 & 1 & 0 & 0 & . & 0 & 0 & 1 & 0 & 0 & : & 0 & 0 & & 1 & 0 & 0 & : & 0 & 0\\
2 & 0 & 2 & 2 & . & 0 & 0 & 0 & 1 & 0 & : & 0 & 0 & & 0 & 1 & 0 & : & 0 & 0\\
\vdots & . & . & . & . & . & . & . & . & . & . & . & . & ..... & . & . & . & . & . & .\\
v_{2-1} & 0 & 0 & 0 & . & 0 & 0 & 0 & 0 & 0 & . & 0 & 0 & & 0 & 0 & 0 & . & 0 & 0\\
v_2 & 0 & 0 & 0 & . & 1 & 0 & 0 & 0 & 0 & . & 1 & 0 & & 0 & 0 & 0 & : & 1 & 0\\
 & & . & . & . & & & & . & . & . & & & & & . & . & . & & \\
0 & . & . & . & a_{mv_{m2}} & . & .. & . & . & . & a_{mv_{m2}} & . & . & & . & . & . & . & a_m v_{mn} & \\
1 & 1 & 0 & 0 & : & 0 & 0 & 1 & 0 & 0 & . & 0 & 0 & & 1 & 0 & 0 & : & 0 & 0\\
2 & 0 & 2 & 0 & : & 0 & 0 & 0 & 1 & 0 & : & 0 & 0 & ..... & 0 & 1 & 0 & . & 0 & 0\\
. & . & . & . & : & . & . & . & . & . & . & . & . & & . & . & . & . & . & .\\
v_{m-2} & 0 & 0 & 0 & : & 0 & 0 & 0 & 0 & 0 & : & 0 & 0 & & 0 & 0 & 0 & : & 0 & 0\\
v_m & 0 & 0 & 0 & : & 1 & 0 & 0 & 0 & 0 & : & 1 & 0 & & 0 & 0 & 0 & : & 1 & 0
\end{array}\right]
$$

Generally speaking, matrix a_{ij} coefficients are time functions. That's why we have to assume $A = A(t)$. However, for short time periods A can be considered a constant. The method of finding solutions for variables $A(t)$ is described below.

For the case of constant A consider further simplification [13-15].
Write system (12) in matrix form using this notation:

$$\hat{\mathrm{T}}\vec{x}(t) = A(t)\vec{x}(t) + \vec{B}(t) \tag{12}$$

Solve the system by substitution:

$$\vec{x}(t) = \upsilon(t)\vec{V}(t)$$

where $\upsilon(t)$ is the required matrix, $\vec{V}(t)$ is the required column.

By substituting we get:

$$\upsilon(t+\tau)\vec{V}(t+\tau) - \upsilon(t+\tau)\vec{V}(t) + \upsilon(t+\tau)\vec{V}(t) = A(t)\upsilon(t)\vec{V}(t) + \vec{B}(t)$$

Let us demand that $\upsilon(t)$ and $\vec{V}(t)$ vectors satisfy the equation:

$$\left\{\begin{array}{l} \upsilon(t+\tau) = A(t)\upsilon(t) \\ \vec{V}(t+\tau) - \vec{V}(t) = \upsilon^{-1}(t)\vec{B}(t) \end{array}\right\} \tag{13}$$

In this case we can find υ and $\vec{V}$ apart, and then build the vector $\vec{x} = \upsilon\vec{V}$.

Write the following system of equations to find matrix υ where μ is any natural number:

$$\left\{\begin{array}{l} \upsilon(t+\tau) = A(t)\cdot\upsilon(t) \\ \upsilon(t+2\tau) = A(t+\tau)\cdot\upsilon(t+\tau) \\ \upsilon(t+3\tau) = A(t+2\tau)\cdot\upsilon(t+2\tau) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \upsilon(t+\mu\tau) = A(t+(\mu-1)\tau)\cdot\upsilon(t+(\mu-1)\tau) \end{array}\right\}$$

and substitute υ to succeeding equations. After the calculations we get:

$$\upsilon(t+\mu\tau) = A(t+(\mu-1)\tau)\cdot\ldots\cdot A(t+2\tau)A(t+\tau)A(t)\cdot\upsilon(t) = \prod_{(\mu>\varphi\geq 0)} A(t+\varphi\tau)\cdot\upsilon(t) \tag{14}$$

from this we get the following:

$$\upsilon^{-1}(t+\mu\tau) = \upsilon^{-1}(t)\cdot\prod_{(0\leq\varphi<\mu)} A^{-1}(t+\varphi\tau) \tag{15}$$

Matrix $A(t)$ and $A(t')$ of different time moments do not commute. The order of the factors is important:

$$\upsilon^{-1}(t+\mu\tau) = \upsilon^{-1}(t)\cdot A^{-1}(t)\cdot A^{-1}(t+\tau)\cdot A^{-1}(t+2\tau)\ldots A^{-1}(t+(\mu-1)\tau)$$

To find $\vec{V}(t)$ vector, get the following system of equations from (13):

$$\left\{\begin{array}{l} \vec{V}(t+\tau) - \vec{V}(t) = \upsilon^{-1}(t)\cdot\vec{B}(t) \\ \vec{V}(t+2\tau) - \vec{V}(t+\tau) = \upsilon^{-1}(t+\tau)\cdot\vec{B}(t+\tau) \\ \vec{V}(t+3\tau) - \vec{V}(t+2\tau) = \upsilon^{-1}(t+2\tau)\cdot\vec{B}(t+2\tau) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \vec{V}(t+\mu\tau) - \vec{V}(t+(\mu-1)\tau) = \upsilon^{-1}(t+(\mu-1)\tau)\cdot\vec{B}(t+(\mu-1)\tau) \end{array}\right\}$$

and add term by term. After cancelling common factors we get

$$\vec{V}(t+\mu\tau) = \vec{V}(t) + \upsilon^{-1}(t)\cdot\vec{B}(t) + \upsilon^{-1}(t+\tau)\cdot\vec{B}(t+\tau) + \ldots + \upsilon^{-2}(t+(\mu-1)\tau)\cdot\vec{B}(t+(\mu-1)\tau)$$

Or shortly:

$$\vec{V}(t+\mu\tau)=\vec{V}(t)+\sum_{(0\le\lambda<\mu)}\upsilon^{-1}(t+\lambda\tau)\cdot\vec{B}(t+\lambda\tau) \quad (16)$$

Values of υ and $\vec{V}$ are found; construct the solution:

$$\vec{x}(t+\mu\tau)=\upsilon(t+\mu\tau)\cdot\vec{V}(t+\mu\tau)=\prod_{(\mu>\varphi\ge 0)}A(t+\varphi\tau)\cdot\upsilon(t)\cdot\vec{V}(t)+$$

$$+\prod_{(\mu>\varphi\ge 0)}A(t+\varphi\tau)\cdot\upsilon(t)\cdot\sum_{(0\le\lambda<\mu)}\left[\upsilon^{-1}(t)\prod_{(0\le\xi<\lambda)}A^{-1}(t+\xi\tau)\right]\cdot\vec{B}(t+\lambda\tau)$$

Simplify this equation. Substitute $\upsilon(t)\vec{V}(t)$ for $x(t)$ in the first term.

Multiply c by $\upsilon^{-1}(t)$ in the second term and get the identity matrix. Let us remark that for $\mu>\lambda$:

$$\prod_{(\mu>\varphi\ge 0)}A(t+\mu\tau)\cdot\prod_{(0\le\xi<\lambda)}A^{-1}(t+\xi\tau)=\prod_{(\mu>\varphi\ge\lambda)}A(t+\varphi\tau)$$

Taking this into account, find the final solution:

$$\vec{x}(t+\mu\tau)=\prod_{(\mu>\varphi\ge 0)}A(t+\varphi\tau)\cdot\vec{x}(t)+\sum_{(0\le\lambda<\mu)}\left[\prod_{(\mu>\varphi\ge\lambda)}A(t+\varphi\tau)\right]\cdot\vec{B}(t+\lambda\tau) \quad (17)$$

If A is time independent, then this expression can be simplified:

$$\vec{x}(t+\mu\tau)=A^{\mu}x(t)+\sum_{(0\le\lambda<\mu)}A^{\mu-\lambda-1}\cdot\vec{B}(t+\lambda\tau) \quad (18)$$

Equations (17) and (18) lead to the solution of Cauchy problem. This problem is to find the state of a dynamical system given the initial state of the system. In planning, solutions of Cauchy problems are used for finding plans for future years with given economic index of previous years [15, 16].

Inverse of matrices A and finding their eigenvectors and eigenvalues is not necessary for the use of formulas (17) and (18). Only multiplication of matrices is suggested. Even though this simplifies calculations, there are still certain drawbacks. Accuracy of calculations will decrease step by step if A are large matrices.

For the case of time independent matrix A, these difficulties can be avoided by using general solutions found in section 2. However, this leads to difficulties with inversion of large matrices. The solution plan will be implemented below.

4. Solving Cauchy problem and finding partial solutions for certain initial conditions supposed A is a time independent matrix.

If A is constant, then solution of equation (13) for this matrix A exists. It can be found using the fundamental system of solutions (10).

In fact, we can build the fundamental system of solutions according to (10) and (11) :

$T\vec{x}(t)=A\vec{x}(t)$

in the following form:

$$\left\{\begin{array}{l} x_{pi}(t)= X_{pi}Z_p^{-\frac{t}{\tau}}\psi_p^{(t)} \\ x_{pi1}(t)= Z_p^{-1}X_{pi}Z_p^{-\frac{t}{\tau}}\psi_p^{(t)} \\ x_{pi2}(t)= Z_p^{-2}X_{pi}Z_p^{-\frac{t}{\tau}}\psi_p^{(t)} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ x_{piv_i}(t)= Z_p^{-v_i}X_{pi}Z_p^{-\frac{t}{\tau}}\psi_p^{(t)} \end{array}\right\}, \quad (19)$$

where $p=1,2,\ldots N$ is the eigenvalue index of matrix A;

$i=1,2,\ldots m$ is the branch index;

$v=1,2,\ldots v_i$ is the solution component index for fixed i i.

System (19) contains N solutions. Every solution leads to the following eigenvector:

$$\vec{x}_p\left(t\right)=\vec{Y}_p Z_p^{-\frac{t}{\tau}}\cdot\psi_p^{(t)} \quad , \tag{21}$$

where $\vec{Y}_p$ is the following column:

$$\vec{Y}_p = \begin{array}{lcl}
i=1 & \begin{array}{c}0\\1\\2\\\vdots\\\nu_1\end{array} & \begin{pmatrix} \cdot\quad X_{p1} \\ Z_p^{-1}X_{p1} \\ Z_p^{-2}X_{p1} \\ \ldots\ldots\ldots\ldots \\ Z_p^{-\nu_1}X_{p1} \end{pmatrix} \\
 & & \ldots\ldots\ldots\ldots\ldots\ldots \\
i & \begin{array}{c}0\\1\\2\\\vdots\\\nu_i\end{array} & \begin{pmatrix} X_{pi} \\ Z_p^{-1}X_{pi} \\ Z_p^{-2}X_{pi} \\ \ldots\ldots\ldots\ldots \\ Z_p^{-\nu i}X_{pi} \end{pmatrix} \\
 & & \ldots\ldots\ldots\ldots\ldots\ldots \\
i=m & \begin{array}{c}0\\1\\2\\\vdots\\\nu_m\end{array} & \begin{pmatrix} X_{pm} \\ Z_p^{-1}X_{pm} \\ Z_p^{-2}X_{pm} \\ \ldots\ldots\ldots\ldots \\ Z_p^{-\nu_m}X_{pm} \end{pmatrix}
\end{array} \tag{22}$$

Matrix $V\left(t\right)$ is the solution of the following equation:

$$TV\left(t\right)=AV\left(t\right)$$

It is possible to rewrite this matrix as follows:

$$V\left(t\right)=\left[\vec{x}_1\left(t\right),\ \vec{x}_2\left(t\right),\ \ldots,\ \vec{x}_p\left(t\right),\ \ldots,\ \vec{x}_n\left(t\right)\right]=\left[\vec{Y}_1 Z_1^{-\frac{t}{\tau}}\psi_1\left(t\right),\ \vec{Y}_2 Z_2^{-\frac{t}{\tau}}\psi_2,\ \ldots\ ,\vec{Y}_p Z_p^{-\frac{t}{\tau}}\psi_p,\ \ldots\ ,\vec{Y}_n Z_n^{-\frac{t}{\tau}}\psi_n\right]$$

Inverse matrix $V^{-1}\left(t\right)$ will be used further. Find the determinant of $V\left(t\right)$:

$$\det V\left(t\right)=\det\left[\vec{Y}_1 Z_1^{-\frac{t}{\tau}};\ \ldots;\ \vec{Y}_p Z_p^{-\frac{t}{\tau}}\psi_p;\ \ldots;\ \vec{Y}_n Z_n^{-\frac{t}{\tau}}\psi_n\right]=$$

$$=Z_1^{-\frac{t}{\tau}}\psi_1\cdot\ldots\cdot Z_p^{-\frac{t}{\tau}}\psi_p\cdot\ldots\cdot Z_n^{-\frac{t}{\tau}}\psi_n\cdot\det\left[\vec{Y}_1;\ \ldots;\ \vec{Y}_n\right]=Z_1^{-\frac{t}{\tau}}Z_2^{-\frac{t}{\tau}}\cdot\ldots\cdot Z_n^{-\frac{t}{\tau}}\cdot\psi_1\psi_2\ldots\psi_n\cdot\det Y \tag{24}$$

where the following matrix is denoted by Y :

$$Y=\left[\vec{Y}_1,\ \vec{Y}_2,\ \ldots,\ \vec{Y}_p,\ \ldots\vec{Y}_n\right] \tag{25}$$

Delete column p and row i, v from the matrix Y and denote the new matrix by $Y_p,\ iv$.

In this case co-factor $D_{p,\,iv}\left(t\right)$ of the element $V_{p,\,iv}$ of matrix V occupying column p and row i, v equals:

$$D_{pjiv}(t) = Z_1^{-\frac{t}{\tau}} Z_2^{-\frac{t}{\tau}} \times \ldots \times Z_n^{-\frac{t}{\tau}} \cdot \psi_1 \ldots \psi_n \cdot \det Y_{p,\,iv} \tag{26}$$

Here $Z_p^{-\frac{t}{\tau}} \psi_p$ is absent.

Using the rules of matrix inversion, get the following rule for forming elements of a matrix $V^{-1}(t)$:

Element V_{iv}^{-1}, p of the matrix $V^{-1}(t)$ occupying column i , v and row p equals:

$$V_{iv,\,p}^{-1}(t) = (-1)^{i+v+p} \cdot \frac{D_{p_1 iv}(t)}{\det V(t)}$$

Substituting value of $D(t)$ and $\det V$ from (24) and (26) to this expression) and following above-noted actions, find:

$$V_{iv,\,p}^{-1}(t) = (-1)^{i+v+p} Z_p^{t/\tau} \psi_p(t) \left(\frac{\det Y_{p,\,iv}}{\det Y} \right) \tag{27}$$

This expression denotes elements of matrix $V^{-1}(t)$. Matrix $V^{-1}(t)$ can be written as a column where every element is a row:

$$V^{-1}(t) \begin{bmatrix} \left[V_{iv,\,p=1}^{-1}(t) \right] \\ \left[V_{iv,\,p=2}^{-1}(t) \right] \\ \cdots\cdots\cdots\cdots \\ \left[V_{iv,\,p}^{-1}(t) \right] \\ \cdots\cdots\cdots\cdots \\ \left[V_{iv,\,p=N}^{-1}(t) \right] \end{bmatrix}$$

Use special notation for rows of matrix $V^{-1}(t)$:

$$\vec{Y}_p(t) = \tilde{Y}_p \cdot Z_p^{\frac{t}{\tau}} \psi_p^{-1}(t), \tag{28}$$

where rows $\tilde{Y}_p$ are denoted by equations:

$$\tilde{Y}_p = \left[(-1)^{i+v+p} \cdot \frac{\det Y_{p,\,iv}}{\det Y} \right] \tag{29}$$

Now matrix $V^{-1}(t)$ has the following form:

$$V^{-1}(t) = \begin{bmatrix} \vec{Y}_1(t) \\ \vec{Y}_2(t) \\ \cdots\cdots \\ \vec{Y}_p(t) \\ \cdots\cdots \\ \vec{Y}_n(t) \end{bmatrix} = \begin{bmatrix} \tilde{Y}_1 Z_1^{\frac{t}{\tau}} \psi_1^{-1}(t) \\ \tilde{Y}_2 Z_2^{\frac{t}{\tau}} \psi_2^{-1}(t) \\ \cdots\cdots\cdots\cdots \\ \tilde{Y}_p Z_p^{\frac{t}{\tau}} \psi_p^{-1}(t) \\ \cdots\cdots\cdots\cdots \\ \tilde{Y}_n Z_n^{\frac{t}{\tau}} \psi_n^{-1}(t) \end{bmatrix} \tag{30}$$

Matrices (23) and (30) are mutually invertible.

Consequently, $V \cdot V^{-1} = E$:

$$V(t)\times V^{-1}(t)=\left[\vec{Y}_1 Z_1^{-\frac{t}{\tau}}\psi_1,\ \ldots\ \vec{Y}_n Z^{-\frac{t}{\tau}}\psi_n\right]\times\begin{bmatrix}\tilde{Y}_1 Z_1^{\frac{t}{\tau}}\psi_1^{-1}\\ \vdots\\ \tilde{Y}_n Z_n^{\frac{t}{\tau}}\psi_n^{-1}\end{bmatrix}=\vec{Y}_1\times\tilde{Y}_1+\ldots+\vec{Y}_n\times\tilde{Y}_n=E$$

We have proved that:

$$\vec{Y}_1\times\tilde{Y}_1+\ldots+\vec{Y}_n\times\tilde{Y}_n=E \tag{31}$$

It means that matrices

$$Y=\left[\vec{Y}_1,\ \ldots\ \vec{Y}_n\right]\ \text{ and }\ \tilde{Y}=\begin{bmatrix}\tilde{Y}_1\\ \vdots\\ \tilde{Y}_n\end{bmatrix} \tag{32}$$

are orthogonal.

Formulate algorithm of finding $V(t)$ and $V^{-1}(t)$:

1. Build two mutually orthogonal matrices Y and $\tilde{Y}$. Form Y by using formula (32) from columns $\vec{Y}_p$ (22). Get $\tilde{Y}$ by inversion of Y and decompose into rows $\tilde{Y}_p$ also by using formula (32).
2. Formula (23) defines matrix $V(t)$. Formula (30) defines matrix $V^{-1}(t)$.

Use formulas (16), (23) and (30) to find solution $\vec{x}(t)$ of Cauchy problem.
We have:

$$\begin{aligned}x(t+\mu\tau)&=V(t+\mu\tau)\cdot\vec{v}(t+\mu\tau)=V(t+\mu\tau)\left[\vec{v}(t)+\sum_{(0\le\lambda<\mu)}V^{-1}(t+\lambda\tau)\vec{B}(t+\lambda\tau)\right]=\\&=V(t+\mu\tau)\left[V^{-1}(t)\cdot\vec{x}(t)+\sum_{(0\le\lambda<\mu)}V^{-1}(t+\lambda\tau)\vec{B}(t+\lambda\tau)\right]=\\&=V(t+\mu\tau)\cdot V^{-1}(t)\cdot\vec{x}(t)+\sum_{(0\le\lambda<\mu)}\left\{V(t+\mu\tau)V^{-1}(t+\lambda\tau)\right\}\vec{B}(t+\lambda\tau)\end{aligned} \tag{33}$$

The problem is reduced to calculating paired product of matrices V and V^{-1} that relate to different time moments.

Calculate the product:

$$\begin{aligned}V(t+\mu\tau):V^{-1}(t+\lambda\tau)&=\left[\vec{Y}_1\ Z_1^{-\frac{t}{\tau}-\mu}\psi_1(t),\ \ldots,\ \vec{Y}_n Z_n^{-\frac{t}{\tau}-\mu}\psi_n(t)\right]\times\begin{bmatrix}\tilde{Y}_1 Z_1^{+\frac{t}{\tau}+\lambda}\psi_1^{-1}(t)\\ \vdots\\ \tilde{Y}_n Z_n^{+\frac{t}{\tau}+\lambda}\psi_n^{-1}(t)\end{bmatrix}=\\&=\tilde{Y}_1\times\tilde{Y}_1\ Z_1^{(\lambda-\mu)}+\ldots+\tilde{Y}_n\times\tilde{Y}_n Z_n^{(\lambda-\mu)}\end{aligned} \tag{34}$$

The product of matrices V and V^{-1} that relate to different time moments does not depend on certain periodic functions
$\psi_1(t),\ \ldots,\ \psi_n(t)$.

Cauchy problem formula follows from equations (33) and (34):

$$x(t+\mu\tau) = \left[\vec{Y}_1 \times \tilde{Y}_1 Z_1^{-\mu} + \dots \vec{Y}_n \times \tilde{Y}_n Z_n^{-\mu}\right] \cdot \vec{x}(t) +$$
$$+ \sum_{(0 \le \lambda < \mu)} \left\{ \vec{Y}_1 \times \tilde{Y}_1 Z_1^{(\lambda-\mu)} + \dots + \vec{Y}_n \times \tilde{Y}_n Z_n^{(\lambda-\mu)} \right\} \vec{B}(t+\lambda\tau) \quad (35)$$

Let us introduce rhythmic weekly plans (suppose τ = 7 days) to industry's working practice. Let us organize continuous planning where production plan for every succeeding time period τ is derived from results of a preceding time period τ .

Continuous planning requires a balanced consumption system and superior precision when calculating industrial capacity. In the regarded economic model this takes place in the following case. Not only values $x_j(t)$ of production and consumption, but also values of industrial capacity and product stock for preceding time intervals τ : $x_{j1}(t)$, $x_{j2}(t)$ and so on should be included into the population of values $\vec{x}$ declared by dynamical model vector. In this scheme time shifts equal to an elementary τ . This meets the condition of continuous planning and represents it as a system of marginal plans.

Smallest stock system with only the most necessary capacity and stock is required to get maximum time gain. In the regarded economic model capacity of incomplete products and complete product stock necessary for extended production are reduced to minimum estimated quality. Values v_j are measured for time intervals when capacity and stock are used and when previous production level *j* influences production plan for every product.

5. Conclusion

The suggested model allows achieving optimal distribution of investments by production branches and countries. Furthermore, optimal proportions of industrial corporation employment by the end of planning period can be found. Not only the corrected matrices and total input coefficients are required for this, but also the following information:

1) volume of fixed assets by branches and sub-branches by the beginning of the planning period;
2) average annual fixed assets retirement coefficients for a considered time period;
3) labor coefficient of gross output;
4) volume of capital investments and labor force for the whole country by the end of the planning period.

The suggested model allows optimizing international distribution of import and export, production and consumption, industrial capacity and product stock. The latter is possible if the products of the same time period are similar to those in reported regional intercorporate balances.

If internationally connected long-term balances have minimum material capacity, then they can be developed without accuracy loss. Accuracy of initial information for the model makes this possible.

The model determines international import and export of goods and distribution to countries. Solving problems of linear programming with thousands of variables and thousands of equations is necessary to develop connected perspective state interconnected balances. That is why it is necessary to create a special system of programs. This system would automate variant calculations and quick recalculation of all international long-term intercorporate balances. This can be used when some values are changed or for checking the balances' insensitivity to a defined coefficient given as information of big inaccuracy.

It takes several years to report an intercorporate balance. However, the use of computers reduces this time to several days for all countries at once. That's why correction of long-term balance direct costs coefficients for all countries must be considered an economical problem. Countries of a particular region must report long-term intercorporate coefficients partly in hard copy and partly in electronic copy.

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