Matlab Codes for Two-Dimensional Scattering Surface Reconstruction Using Broadband Acoustic Data

Giulio Dolcetti[a], Anton Krynkin[b]

[a]Department of Civil and Structural Engineering, The University of Sheffield, Sheffield, United Kingdom, g.dolcetti@sheffield.ac.uk

[b]Department of Mechanical Engineering, The University of Sheffield, Sheffield, United Kingdom, a.krynkin@sheffield.ac.uk

July 24, 2020

Abstract

This documentation supplements a set of Matlab scripts to reconstruct the shape of a two-dimensional rough surface based on scattered acoustic field data. The method is based on the approach introduced by Krynkin et al. (2016) (Krynkin et al., 2016, An airborne acoustic method to reconstruct a dynamically rough surface, J. Ac. Soc. Am. 140(3) `https://doi.org/10.1121/1.4962559`). The present implementation is based on the work by Dolcetti et al. (2020) (Dolcetti et al., 2020, Robust Reconstruction of Scattering Surfaces Using a Linear Microphone Array, submitted to Journal of Sound and Vibration). This includes some improvements to the previous algorithms, including a multi-frequency extension aimed at improving the robustness of the reconstruction using broadband data. The algorithms apply to smooth sound-hard rough surfaces that satisfy the applicability of the Kirchhoff approximation. Input data can be either experimental or numerical. Algorithms to create random realisations of a rough surface with a power-function spatial spectrum, and to estimate the corresponding synthetic scattered sound field based on the Kirchhoff approximation, are included, together with a working example.

1 Generalities:

The user is referred to Dolcetti et al. [2020] for detailed information about the theory and sensitivity to the choice of the algorithm and parameters. Here only the main equations and meaning of the mathematical symbols are reported.

1.1 Problem Statement:

Sound is produced by a source with co-ordinates vector $\mathbf{S} = (x_s, 0, z_0)$ in the half plane $z_0 > 0$, and scattered by a sound-hard surface with profile $z = \zeta(x)$. The surface elevation is constant along the direction y. The scattered sound field $P(\mathbf{M}_m)$ is recorded at a set of N_m microphones with co-ordinates $\mathbf{M}_m = (x_m, 0, z_m)$. Assuming the validity of the Kirchhoff approximation [Thorsos, 1988], and the smallness of $\zeta/z_0 \ll 1$, $\zeta/z_m \ll 1$, an estimate of the surface profile $\zeta(x_r)$ at a set of points x_r is obtained from a measurement of $P(\mathbf{M}_m)$ [Krynkin et al., 2016]. The solution is found by means of a singular value decomposition of the linearised Kirchhoff integral equation, after a stationary phase expansion along the y-direction (parallel to the surface crest). The number of microphones N_m does not need to be as large as the number of reconstruction locations N_r.

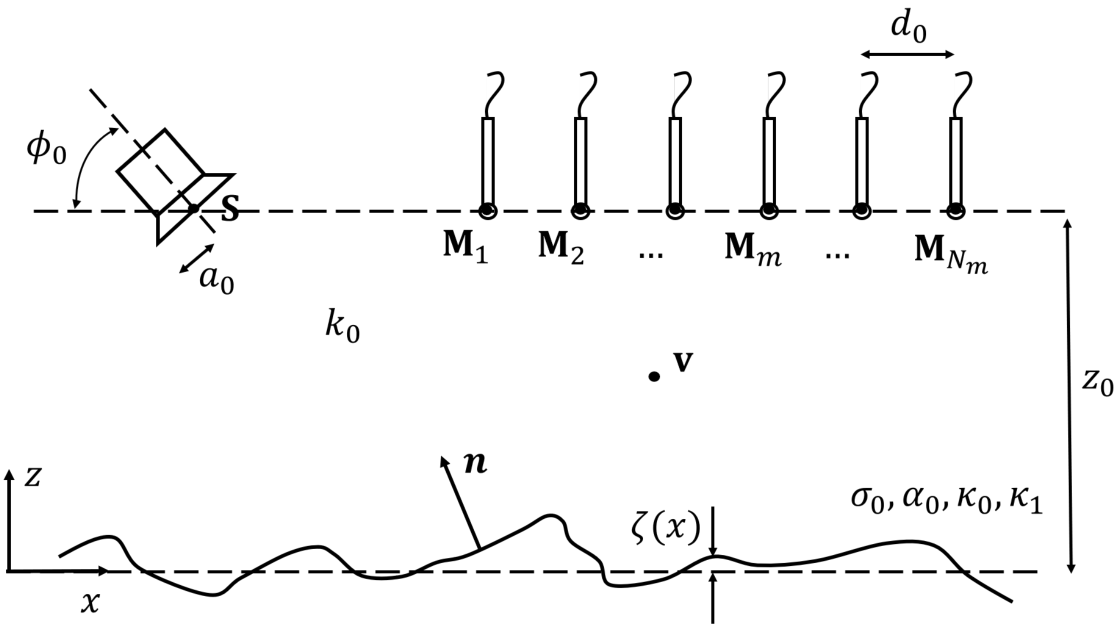

Figure 1: Schematic representation of the surface scattering problem geometry, and characteristic parameters: acoustic wavenumber k_0, source inclination angle ϕ_0, equivalent piston radius a_0, microphone array separation d_0 and height z_0, surface standard deviation σ_0, spectrum slope α_0, saturation wavenumber κ_0, and short-scale cut-off wavenumber, κ_1. The rough surface is described by the function $z = \zeta(x)$, with normal vector $\mathbf{n}$. (taken from Dolcetti et al. [2020])

1.2 Kirchhoff Approximation:

Theoretically, the complex potential can be calculated according to the Kirchhoff approximation [Dolcetti et al., 2020, Eq.5-7]:

$$P(\mathbf{M}_m, k) \approx \int \mathcal{A}(\mathbf{S}, \mathbf{M}_m, x, k) \exp\left[-\mathrm{i} q_z(\mathbf{M}_m, \mathbf{S}, x, k)\zeta(x)\right] \mathrm{d}x, \tag{1}$$

where

$$q_z(\mathbf{M}_m, \mathbf{S}, x, k) = k\left[\frac{z_m}{R(\mathbf{M}_m, \boldsymbol{\rho}_0)} + \frac{z_0}{R(\mathbf{S}, \boldsymbol{\rho}_0)}\right], \tag{2}$$

$\boldsymbol{\rho}_0$ is the projection of $\boldsymbol{\rho}$ onto the x-axis, $\boldsymbol{\rho}_0 = (x, 0)$, and

$$\mathcal{A}(\mathbf{S}, \mathbf{M}_m, x, k) = \frac{\mathrm{e}^{-\mathrm{i}\pi/4}}{k\sqrt{k8\pi}} \frac{D(\theta(\boldsymbol{\rho}_0 - \mathbf{S})) \exp\left\{\mathrm{i}k\left[R(\mathbf{M}_m, \boldsymbol{\rho}_0) + R(\mathbf{S}, \boldsymbol{\rho}_0)\right]\right\}}{\sqrt{R(\mathbf{S}, \boldsymbol{\rho}_0) R(\mathbf{M}_m, \boldsymbol{\rho}_0)}\sqrt{\left[R(\mathbf{S}, \boldsymbol{\rho}_0) + R(\mathbf{M}_m, \boldsymbol{\rho}_0)\right]}} q_z(\mathbf{M}_m, \mathbf{S}, x, k). \tag{3}$$

$k = 2\pi/\lambda$ is the acoustic wavenumber, λ is the wavelength, $R(\mathbf{M}_m, \boldsymbol{\rho}_0)$ is the distance from the m-th receiver to a point $\boldsymbol{\rho}_0$ on the surface, and $R(\mathbf{S}, \boldsymbol{\rho}_0)$ is the distance from the source to a point $\boldsymbol{\rho}_0$ on the surface. $D(\theta(\boldsymbol{\rho}_0 - \mathbf{S})$ is the source directivity pattern, which is approximated by the directivity of an ideal piston with radius a_0 and with an infinite baffle [Morse and Ingard, 1986, p. 381],

$$D(\theta) = 2\frac{J_1(ka_0 \sin(\theta))}{ka_0 \sin(\theta)}, \tag{4}$$

where J_n is the Bessel function of the first kind, and θ is the angle from the axis of the piston. The result of Eq. (1) is estimated by the Matlab script *KirchhoffScattering2D.m*, where the integral is approximated with a quadrature method applied to a discretised surface.

1.3 Input Data:

The input data must be given in the form of a non-dimensionalised complex signal amplitude. This can be calculated from a measured acoustic pressure time series by means of a Fourier transform. In this case, it is suggested to divide the measured complex Fourier amplitudes at each frequency by that of a reference signal, for example the direct output from the speaker or a reference microphone, in order to synchronise the phase and remove the speaker sensitivity. Correct scaling of the measurements should be ensured prior to any reconstruction attempt by comparing experimental and synthetic data obtained for a simple case, for instance a flat reflecting surface.

1.4 Scaling and Calibration:

For a correct implementation of the inversion, the measured data should be scaled consistently with Eq. (1)-(3). A calibration procedure has been suggested by Dolcetti et al. [2020], as follows: (i) measure the reflection from a flat surface at wavenumber k and at all receivers $\mathbf{M}_m$, $\tilde{P}_0(\mathbf{M}_m, k)$; (ii) calculate the acoustic potential reflected by a flat surface, $P_0(\mathbf{M}_m, k)$, theoretically (one can use the Matlab script *KirchhoffScattering2D.m*, after setting the z-component of the surface co-ordinates matrix *CoordSurface* to zero); (iii) calculate a complex scaling factor $\mathcal{C}(\mathbf{M}_m, k) = P_0(\mathbf{M}_m, k)/\tilde{P}_0(\mathbf{M}_m, k)$. Each subsequent measurement obtained with a rough surface can be correctly rescaled multiplying it by the factor $\mathcal{C}(\mathbf{M}_m, k)$. In principle, the rescaling corrects for the amplitude and phase mismatch of each microphone, for the microphone sensitivity at each frequency, and for the different variables used within the algorithms (acoustic potential vs acoustic pressure). Once it has been calculated, it remains valid at least in principle even after a change of the array geometry, as long as the correct factor is applied to each microphone and the source excitation signal is the same. However, it is strongly recommended to measure the scaling factor using a setup as similar as possible to the one used for the measurements. In this way, the calibration procedure can also limit the uncertainties due to the array geometry.

1.5 Algorithms:

The scripts allow to choose among three different reconstruction algorithms. These are explained with more detail by Dolcetti et al. [2020]. The choice of the algorithm is made in the argument of the script *Reconstruction_MultiFrequency_2D.mat*. The three options are:

- 'SA': standard (short array) algorithm, as described by Krynkin et al. [2016];
- 'SA0': debiased version of 'SA';
- 'SP': small-perturbation version of 'SA'.

A thorough comparison of the three algorithms can be found in Dolcetti et al. [2020]. The 'SA' algorithm is affected by a significant bias for surfaces with small relative rms roughness amplitude σ_0/λ. The 'SP' algorithm, instead, works best when $\sigma_0/\lambda < 0.1$. The 'SA0' algorithm was found to be the most accurate over a wide range of σ_0/λ. The 'SP' and 'SA0' algorithms require the flat-surface potential P_0 as input. This can be calculated with the script *KirchhoffScattering2D.m*, after setting the z-component of the surface co-ordinates matrix *CoordSurface* to zero.

1.6 Multiple-Frequency Extension:

All three algorithms can be applied either in single-frequency or multiple-frequency mode. Multiple-frequency mode combines information at multiple frequencies to constrain the inversion, to produce more robust results without significantly affecting the measurement resolution. The method is described in more detail in Dolcetti et al. [2020], where it was first introduced. To be able to apply the multiple-frequency extension, data at multiple frequencies must be available. These can be obtained sequentially, one frequency at a time, or simultaneously, for example by means of a broadband excitation. In the latter case, the complex amplitude at each frequency can be calculated by means of a Fourier transform. The data frequency bands do not need to be contiguous.

The multiple-frequency extension is slower, but can be useful especially in the presence of noise and for relatively high frequencies, when standard methods can become unreliable and occasionally fail. The extension is selected automatically based on the form of the input data matrix within the script *Reconstruction_MultiFrequency_2D.mat*. If the input frequency array *f0* has dimensions 1×1, then the 'single-frequency' algorithms are used. If the input frequency array *f0* has length N_k, then the 'multiple-frequency' extension is used. In this case, the input acoustic data matrices *U2D_Kir* and *U02D_Kir* must have dimensions $N_m \times N_k$, i.e., each column must correspond to the data at a separate frequency.

Whenever $N_m N_k > N_r$, where N_r is the number of grid points that discretise the surface, the problem changes from being under-determined to over-determined. In this case, the method inversion changes from a pseudo inversion obtained via a singular value decomposition (*svd*), to an iterative least-squares inversion based on the standard *lsqr* Matlab script. In the latter case, the accuracy can be substantially higher, and the calculations are generally faster.

2 Scripts:

Four separate scripts are included:

- *Example.m*: contains a working example of reconstruction based on synthetic data obtained for a random surface with power-function spatial spectrum;
- *SurfaceInversion_MultiFrequency_2D.m*: estimates the shape of the scattering surface based on acoustic scattering data and setup geometry information;
- *KirchhoffScattering2D.m*: estimates the acoustic potential scattered by a rough surface. Setup geometry and surface shape must be provided as input;
- *RandomSurfaceGenerator.m*: creates random surface profiles with a power-function spatial spectrum, based on a choice of surface parameters.

The scripts *SurfaceInversion_MultiFrequency_2D.m*, *KirchhoffScattering2D.m*, and *RandomSurfaceGenerator.m* can be run independently.

2.1 *Example.m*

This is an example script, that allows to calculate the inverse surface profile based on all three methods, using synthetic data calculated with the Kirchhoff approximation for a surface with a power-function spatial spectrum. Each time the script is run, a different random surface realisation is generated based on script *RandomSurfaceGenerator.m*. Synthetic acoustic potential data scattered by the rough surface realisation, *P*, and reflected by a flat surface, *P0*, are generated with script *KirchhoffScattering2D.m*, based on the defined array geometry. The synthetic data are then inverted to estimate the surface shape using script *SurfaceInversion_MultiFrequency_2D.m*, and this is finally compared to the known original surface shape visually. Amplitude and phase noise can be added to the synthetic data according to

$$P_\nu(\mathbf{M}_m, k) = P(\mathbf{M}_m, k)\,(1+\xi)\exp\left(\mathrm{i}\chi 2\pi\right), \tag{5}$$

where P_ν is the signal with noise, and ξ and χ are Gaussian distributed random variables. The parameters *AmpNoise* (σ_ξ) and *PhaNoise* (σ_χ) correspond to the standard deviations of the distributions of ξ and χ, and can be modified within the script.

All simulation parameters summarised below can be modified within the script. The effect of each parameter is described by Dolcetti et al. [2020]. The corresponding symbols are listed below in the tables.

2.1.1 Surface parameters:

name	symbol	dimensions	description
sigma0	σ_0	m	Average standard deviation of the surface profile.
kappa0	κ_0	rad/m	Wavenumber of the surface saturation scale.
kappa1	κ_1	rad/m	Wavenumber of the surface cut-off scale.
alpha0	α_0	–	Exponent of the surface spatial power spectrum.
Dx	Δx	m	Surface discretisation grid scale.
N	N	–	Number of surface grid points.

2.1.2 Signal parameters:

name	symbol	dimensions	description
f0	f_0	Hz	Excitation frequency. For multiple-frequency, f_0 is the centre-band frequency.
Wf	W_f	Hz	Frequency half-bandwidth. If $W_f = 0$, then the single-frequency approach is used.
Nf	N_f	–	Number of frequency bands between $f_0 - W_f$ and $f_0 + W_f$.

2.1.3 Array parameters:

name	symbol	dimensions	description
a0	a_0	m	Equivalent source radius.
psi0	ψ_0	rad	Source inclination from the horizontal.
z0	z_0	m	Source height from the surface.
zM	z_m	m	Receivers height from the surface.
d0	d_0	m	Spacing between receivers.
Nm	N_m	–	Number of receivers.

2.1.4 Noise parameters:

name	symbol	dimensions	description
AmpNoise	σ_ξ	–	Relative amplitude noise standard deviation.
PhaNoise	σ_χ	–	Relative phase noise standard deviation.

2.1.5 Reconstruction parameters:

name	symbol	dimensions	description
Dxr	Δx_r	m	Reconstruction grid size.

The size of the reconstruction grid is set to 6Λ, by default, where Λ is the effective reconstruction domain [Dolcetti et al., 2020].

2.2 *SurfaceInversion_MultiFrequency_2D.m*

This code estimates the shape of a 2D scattering surface from scattered acoustic data, information about the array geometry, and excitation signal. For the under-determined problem ($N_m \times N_f < N_r$) the solution is calculated by a singular value decomposition, with Tikhonov regularisation, and the regularisation parameter is identified by means of a generalised cross-validation technique. For the over-determined problem ($N_m \times N_f > N_r$) the solution is calculated by an iterative least squares minimisation algorithm based on the standard Matlab *lsqr* script, with a tolerance value of 0.01.

2.2.1 Input:

name	symbol	dim.	size	description
f0	f	Hz	$1 \times N_k$	Excitation frequency array. If $N_k > 1$, the multiple-frequency extension is applied.
CoordMic_dimensional	$\mathbf{M}$	m	$N_m \times 3$	Matrix of microphones co-ordinates. $(:,1)$: x-co-ordinate; $(:,2)$: y-co-ordinate; $(:,3)$: z-co-ordinate.
CoordSource_dimensional	$\mathbf{S}$	m	1×5	Array of source co-ordinates. $(:,1)$: x-co-ordinate; $(:,2)$: y-co-ordinate; $(:,3)$: z-co-ordinate; $(:,4)$: source inclination angle ψ_0 (rad); $(:,5)$: equivalent source radius a_0 (m).
xRec_dimensional	x_r	m	$N_r \times 1$	Array of reconstruction locations.
TypeExp	-	-	-	'SA': short-array method; 'SA0': de-biased short-array method; 'SP': small perturbation method.
U2D_Kir	P	(-)	$N_m \times N_f$	Matrix of scattered potentials. *U2D_Kir*(j,i) is the potential at the j-th microphone, at the i-th frequency.
U02D_Kir	P_0	(-)	$N_m \times N_f$	Matrix of scattered potentials for a flat surface. *U02D_Kir*(j,i) is the potential at the j-th microphone, at the i-th frequency.

2.2.2 Output:

name	symbol	dim.	size	description
zRec	ζ	m	$N_r \times 1$	Array of reconstructed surface elevation.

2.3 *KirchhoffScattering2D.m*

This script estimates the scattered acoustic potential based on the Kirchhoff approximation, according to Eq. (1). The equation is derived for a surface with constant elevation in the y-direction. The surface shape, array geometry, and excitation signal must be provided as input.

2.3.1 Input:

name	symbol	dim.	size	description
CoordMic	$\mathbf{M}$	m	$N_m \times 3$	Matrix of microphones co-ordinates. $(:,1)$: x-co-ordinate; $(:,2)$: y-co-ordinate; $(:,3)$: z-co-ordinate. *CoordMic*$(:,2)$ must be equal to 0 for the 2D case.
CoordSource	$\mathbf{S}$	m	1×5	Array of source co-ordinates. $(:,1)$: x-co-ordinate; $(:,2)$: y-co-ordinate; $(:,3)$: z-co-ordinate; $(:,4)$: source inclination angle ψ_0 (rad); $(:,5)$: equivalent source radius a_0 (m). *CoordSource*$(:,2)$ must be equal to 0 for the 2D case.
CoordSurface	$\boldsymbol{\rho}_0$	m	$N \times 3$	Array of surface co-ordinates. $(:,1)$: x-co-ordinate; $(:,2)$: y-co-ordinate; $(:,3)$: z-co-ordinate. *CoordSurface*$(:,2)$ must be equal to 0 for the 2D case.
lambda	λ	m	1×1	Acoustic wavelength.

2.3.2 Output:

name	symbol	dim.	size	description
U	P	(-)	$N_m \times 1$	Array of scattered acoustic potential.

2.4 *RandomSurfaceGenerator.m*

This script generates a random realisation of a discretised surface with power-function spatial spectrum each time it is run. The average surface power-spectrum is of the form

$$\Psi(\kappa) = \begin{cases} \sigma_0^2 C, & \text{where } \kappa < \kappa_0, \\ \sigma_0^2 C \left(\frac{\kappa}{\kappa_0}\right)^{-\alpha_0}, & \text{where } \kappa_0 \le \kappa \le \kappa_1 \\ 0, & \text{where } \kappa > \kappa_1, \end{cases} \tag{6}$$

where σ_0 is the surface standard deviation, C is a constant scaling factor, α_0 is the spectrum exponent, κ_0 is the saturation wavenumber, and κ_1 is the short-scale cut-off wavenumber. The surface extends for a size of $N\Delta x$, where Δx is the grid size.

2.4.1 Input:

name	symbol	dim.	size	description
sigma_0	σ_0	m	1×1	Surface standard deviation.
kappa_0	κ_0	rad/m	1×1	Spectrum saturation wavenumber.
kappa_1	κ_1	rad/m	1×1	Short-scale surface cut-off wavenumber.
alpha_0	α_0	(–)	1×1	Power spectrum slope.
Dx	Δx	m	1×1	Surface discretisation grid size.
N	N	(–)	1×1	Number of surface grid points.

2.4.2 Output:

name	symbol	dim.	size	description
x	x	m	$N \times 1$	Surface x-co-ordinate.
z	ζ	m	$N \times 1$	Surface z-co-ordinate.

Acknowledgements

This work was supported by a Knowledge Exchange Support Fund provided by the UK Engineering and Physical Sciences Research Council (EPSRC). Giulio Dolcetti is funded by the UK EPSRC Grant EP/R022275/1.

References

G. Dolcetti, M. Alkmim, J. Cuenca, L. De Ryck, and A. Krynkin. Robust reconstruction of scattering surfaces using a linear microphone array. 2020. Submitted to Journal of Sound and Vibration.

A. Krynkin, K. V. Horoshenkov, and T. Van Renterghem. An airborne acoustic method to reconstruct a dynamically rough flow surface. *The Journal of the Acoustical Society of America*, 140(3):2064–2073, 2016.

P. M. Morse and K. U. Ingard. *Theoretical Acoustics*. Princeton university press, 1986.

E. I. Thorsos. The validity of the Kirchhoff approximation for rough surface scattering using a Gaussian roughness spectrum. *The Journal of the Acoustical Society of America*, 83(1):78–92, 1988.