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## APPROXIMATE MAIN VALUE PERFORMANCE ANALYSIS OF COMPUTING PROCESS USING SHPN WITH FUZZY PARAMETERS

Emilian Guțuleac<sup>\*</sup>, ORCID ID: 0000-0001-6839-514X,  
Sergiu Zaporojan, ORCID ID: 0000-0001-5928-4229,  
Victor Moraru, ORCID ID: 0000-0002-5454-8341,  
Alexei Sclifos, ORCID ID: 0000-0003-4531-7944,  
Andrei Furtună, ORCID ID: 0000-0003-0057-2407

*Technical University of Moldova, 168 Stefan cel Mare Blvd., MD-2004 Chisinau, Republic of Moldova*

*\*Corresponding author: Emilian Guțuleac, [emilian.gutuleac@calc.utm.md](mailto:emilian.gutuleac@calc.utm.md)*

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**Abstract.** Stochastic fluid models are a class of analytic models that have recently drawn the attention of many researchers for the modeling and performance evaluation of complex computer systems and networks (CSN). In this paper we present an approximated main-value analysis method of stochastic hybrid Petri nets (SHPN) models with fuzzy parameters (FSHPN) for performances evaluation of data continuous transmission CSN virtual channels. The method is based on the main-value analytical solution of one buffer finite FSHPN sub-models, also referred dipoles as building blocks. We develop a fixed point iterative algorithm to accurately estimate performance measures of buffer pipe-line FSHPN models such as throughput and mean buffer contents. The accuracy of the proposed method has been validated by simulation experiments.

**Keywords:** *aggregation, approximation, computer system and networks, decomposition, evaluation, fuzzy numbers, hybrid, Petri nets, stochastic, throughput.*

### 1. Introduction

Formal modeling and techniques for performance evaluation and dependability measures are widely used while designing and analyzing many types of computer systems and computer networks (CSN) with discrete-continuous process applications, based on which design gaps can be identified at the early stage of their life cycle [1, 2]. As a result, these issues can be eliminated earlier and the costs of troubleshooting and maintenance can be significantly reduced.

The development of appropriate models is crucial for the design, performance and dependability analysis and control of dynamic CSN. The usefulness of a model depends on both its capacity to capture the relevant features of the system, and its capacity for mathematical analysis. These capacities of the model largely rely on the adopted modeling formalism, i.e. on the set of modeling principles and rules that are used to build the model. Also, randomness and uncertainty are two major problems one faces while modeling

nonlinear dynamics of CSN. Stochastic and fuzzy methods are used to cope with these problems [3].

Among the most popular modeling formalisms, due to the advantage of quick construction and numerical analysis, extended Petri net (PN) [4], such as generalized stochastic Petri nets (GSPN) [5] and stochastic reward nets (SRN) [6] formalisms, are recognized paradigms for performance and dependability analysis of various systems such as distributed computing network, network communication protocols, industrial systems etc. In addition, analytical modeling using these formalisms is a less costly and more efficient method compared to simulation techniques [2]. It generally provides the best insight into the effects of various parameters and their interactions. With respect to other more popular techniques of graphical system representation, these formalisms are particularly suited to represent in natural way logical interactions among parts or activities in a system. Typical situations that can be modeled by PN are synchronization, sequentially, concurrency conflict, etc. Also, some advantages of GSPNs and SRNs are that they allow graphical editing of the model, they are easy to understand, they are powerful and flexible, and they can be simulated automatically by a computer.

Despite the existence of an impressive amount of results in the literature for the analysis of GSPNs and SRNs models, many analysis techniques cannot be applied to large systems due to the so called *state explosion problem* (i.e., the set of reachable markings increases exponentially) [5, 6]. *For this reason*, the *fluidification* (i.e., getting continuous-state approximations) has been proposed as a relaxation technique that overcomes the state explosion problem by introducing *fluid stochastic Petri nets* (FSPN) [7] and *stochastic hybrid Petri nets* (SHPN) [8, 9]. The idea consists in the analysis of the GSPN or SRN models via a relaxed continuous version, reducing thus the computational efforts and, at the same time, hoping that the information obtained from the continuous model be valid for the original discrete one.

Moreover, uncertainty handling is a central topic in performance and dependability analysis of CSN. The task of CSN modeling and analysis is often hampered by the lack of detailed system parameter information, a limited knowledge of the system behavior to be modeled and the difficulty of obtaining accurate data. This often results in a number of uncertain system parameters that need to be incorporated into a mathematical model. Although, the most common approach used to represent uncertainty in modeling systems are based on stochastic process theory, such models are not always suitable to describe all dimensions of uncertainty. Especially the inaccuracy of data which, for example, is the result of limited measurement accuracy which is not statistical in nature and cannot be described using probability measures [3]. Meanwhile, uncertainty in many CSN performance and dependability modeling is unavoidable. Hence, to deal with uncertain information in stochastic models, the fuzzy set theory [10, 11] and the concept of fuzzy numbers is well suited to use for handling vague conditions that include behavior uncertainties and parameter inaccuracies.

GSPN, SRN, FSPN and SHPN formalisms assume the availability of cert values of quantitative parameters. To deal with uncertainty the GSPN with fuzzy parameters approaches have been proposed in papers [10]. In these approaches, the transition firing rates as well as states probabilities can be considered as fuzzy numbers.

In this context, it is necessary to enhance the FSPN and SHPN formalisms with fuzzy parameters that will allow adapting all available data, even if they are uncertain, to use and

build useful models. Although, FSPNs and SHPNs are prominent formalisms applied for modeling and performance evaluation of hybrid dynamic systems with discrete and continuous components, that interact with each other, an analytical evaluation of the performance measures of these models requires the solution of a complex first-order hyperbolic partial differential equations, combined with boundary algebraic equations, whose numerical analysis becomes a very difficult task [3].

An FSPNs or SHPN model of a hybrid system with  $N \geq 4$  continuous places (buffers) cannot be accurately analyzed analytically [7, 9, 12], thus justifying the development of approximation methods for analysis of these types models.

To tackle this issue, in this paper, we present a decomposition mean-value analytical method for approximate performance analysis of continuous data transmission network virtual channel, modeled by SHPN with fuzzy parameters and finite capacity buffers. The basic idea is the following: the overall SHPN model is decoupled into simpler and more tractable SHPN sub-models. We use the mean-value analytical solution of two-stage SHPN sub-model with one buffer as a building block, called dipole, and the performance measures obtained from the solution of the sub-models are then used to compute those concerning the overall model. In order to take into account the propagating effect of partial and complete blocking and starvation phenomena of servers throughout the system, analytical decomposition expressions for iterative modification parameters of each dipole are provided similarly as proposed in [13, 14]. Based on this method, a *fixed point iterative procedure* for efficient estimation, with a given accuracy, of the performance measures of a SHPN with fuzzy parameters and pipe – line finite capacity buffers is developed. Numerical results and simulation prove the good accuracy of the developed method.

## 2. Stochastic hybrid petri nets with fuzzy parameters

### 2.1. Elements of fuzzy sets theory and fuzzy numbers

The fuzzy sets theory (FST) and concepts with fuzzy numbers [15] appears from the need to deal with imprecise, vague or partially true information to exact values but the range of possible values are known. In classical set theory, membership of an element in a set is considered to be binary. However, FST allows an element of a set to have a membership value from the interval  $[0, 1]$ . To facilitate the exposition of the proposed approach, in this subsection we present some basic elements of fuzzy sets, triangular fuzzy numbers (TFN) and arithmetic algebraic operations according to [15], required for defining SHPNs with fuzzy parameters.

A fuzzy subset  $\tilde{A}$  of the set of real numbers  $\mathbb{R}$  is defined by its membership function  $\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0, 1]$ , which assigns a real number  $\mu_{\tilde{A}}$  in the interval  $[0, 1]$  to each element  $\tilde{x} \in \tilde{\mathbb{R}}$ , where the value of  $\mu_{\tilde{A}}$  at  $x$  shows the grade of membership of  $x$  in  $\tilde{A}$ .

A fuzzy number is a fuzzy set  $\tilde{a} : \mathbb{R} \rightarrow [0, 1]$  which satisfies: 1)  $\tilde{a}$  is upper semicontinuous; 2)  $\tilde{a}(x)$  outside some interval  $[0, 1]$ ; 3) there are real numbers  $a, b$  in  $[c, d]$  for which: (i)  $\tilde{a}(x)$  is monotonic increasing on  $[c, a]$ ; (ii)  $\tilde{a}(x)$  is monotonic decreasing on  $[b, d]$ ; (iii)  $\tilde{a}(x) = 1$ , for  $a \leq x \leq b$ . Fuzzy numbers can be considered from two different points of view: with their membership function or with their  $\alpha$ -cuts. The two ways of considering fuzzy numbers are equivalent, and, depending on the details we want to study, one can be better than the other.

In this paper, we consider a fuzzy number in the form of  $\tilde{a} = (a, \delta, \beta)$ , where  $a$  is the center,  $\delta$  is the left width, and  $\beta$  is the right width. For arbitrary fuzzy number  $\tilde{a} = (a, \delta, \beta)$ , the membership function is of the following form:

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 - (a - x) / \delta, & a - \delta \leq x \leq a, \\ 1 - (x - a) / \beta, & a \leq x \leq a + \beta, \\ 0, & \text{otherwise.} \end{cases}$$

On the other hand, a parametric  $\alpha$ -cut fuzzy number  $\tilde{a} = [a^-(\alpha), a^+(\alpha)]$  can be represented as:  $a^-(\alpha) = a - (1 - \alpha)\delta$  and  $a^+(\alpha) = a + (1 - \alpha)\beta$ .

Two types of fuzzy numbers are most commonly found in real applications: trapezoidal fuzzy numbers and triangular fuzzy numbers (TFN). The use of these types of fuzzy numbers is more appropriate, one reason being the computing complexity. Between these, TFN are often used because the computation is simplified and the results are easier interpreted and implemented.

The arithmetic algebraic operations for TFN  $\tilde{a} = (a, \delta, \beta)$  and  $\tilde{b} = (b, \gamma, \varepsilon)$  numbers are as follows [15]: 1) Addition  $\tilde{c} = \tilde{a} \oplus \tilde{b} = (a + b, \delta + \gamma, \beta + \varepsilon)$ ; 2) Subtraction  $\tilde{c} = \tilde{a} \ominus \tilde{b} = (a - b, \delta + \varepsilon, \beta + \gamma)$ ; 3) Multiplication  $\tilde{a}$  positive with  $\tilde{b}$  positive  $\tilde{c} = \tilde{a} \otimes \tilde{b} = (a \cdot b, a \cdot \gamma + b \cdot \delta, a \cdot \varepsilon + b \cdot \beta)$ ; 4) Scalar product:  $\tilde{c} = \kappa \otimes \tilde{a} = (\kappa \cdot a, \kappa \cdot \delta, \kappa \cdot \beta)$ ,  $\kappa \geq 0$ .

The inverse for triangular fuzzy number is:  $\tilde{c} = 1/\tilde{a} = (1/a, \beta/a^2, \delta/a^2)$ .

## 2.2. Basic concepts and elements of SHPN with fuzzy parameters

We assume that the readers are familiar with the basic concepts of GSPN, FSPN or SHPN. A comprehensive theoretical treatment of these topics is outside the scope of this paper. For more comprehensive details to SHPN we let the readers refer to [8, 9]. In this section we provide a brief presentation of the SHPN with fuzzy parameters, called FSHPN.

Let  $IN_+$  be the set of nonnegative integers, and  $IR_+$  is the set of positive real numbers.

The particularities of the SHPN, underlying of FSHPN, consist in the fact that both the set  $P$  of places constitutes a partition  $P = P_d \cup P_c$ ,  $P_d \cap P_c = \emptyset$ , where  $P_d$  is the set of discrete places and  $P_c$  is the set of continuous places (buffers), which can have a finite capacity. The set  $T$  of timed transitions constitutes a partition  $T = T_d \cup T_c$ ,  $T_d \cap T_c = \emptyset$ , where  $T_d$  is the set of discrete transitions  $T_c$  and is the set of continuous transitions. The discrete places  $p_k$  can contain an integer number of tokens, and the continuous places can contain a real quantity of fluid. The discrete places graphically are represented by simple circles, and the continuous ones by two concentric circles. The set of discrete transitions  $T_d$  in turn is partitioned into a subset of timed transitions  $T_t$  (resp. immediate  $T_0$ ),  $T_d = T_t \cup T_0$ ,  $T_t \cap T_0 = \emptyset$ , which are represented by thick black bars (resp. thin black bars), and the continuous ones are represented by small rectangles. A positive integer marking-dependent priority function  $Pri(t_j) \in IN_+$  is associated with each one transition  $t_j \in T_d$  for which  $Pri(T_0) > Pri(T_t)$ . Also, with each transition  $t_j \in T$ , a guard function  $g(t_j) \in T$  is associated, which is a marking-

dependent Boolean function, that condition the enabling and selection of the triggering of these transitions.

The current marking  $(M, \vec{x})$  of SHPN is a column vector that shows respectively the common distribution of tokens in discrete places and fluid levels in buffers. The discrete part of this marking  $M$  is represented by the column vector containing a positive integer number of tokens  $m_n = M(p_n)$  in the discrete places  $p_n$ , and  $\vec{x}$  is the continuous marking column vector of the buffers  $b_i$  in which it represents the fluid level in the buffers, which are a real positive quantities. Each  $b_i$  has a maximum storage capacity  $h_i \in \mathbb{R}_+$  of the respective fluid. The discrete tokens are consumed and produced in accordance with the rules for triggering enabled discrete transitions through normal discrete arcs. The continuous fluid flows are consumed and produced in accordance with the firing rules of enabled continuous transitions through normal fluid arcs or fluid setting arcs [8]. So, when the respective continuous transition  $u_k \in T_c$  is enabled, it continuously change the fluid level  $x_i$  of the incident buffers with speed  $v_k(M, \vec{x}) \in \mathbb{R}_+$ .

The graphical representation of all SHPN primitives is given in Figure 1

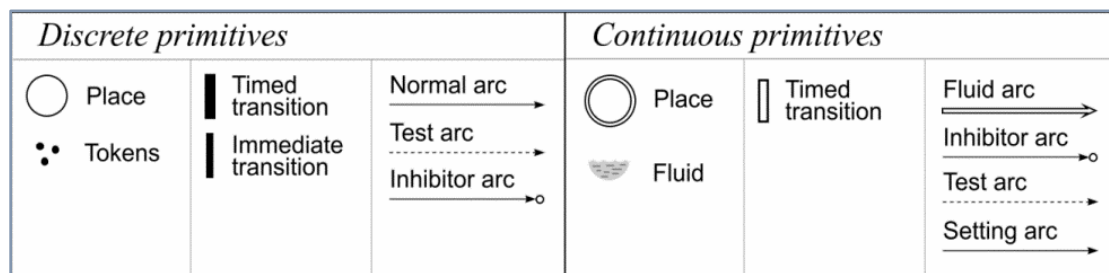


Figure 1. All primitives of a SHPN models.

Inhibitory arcs (resp. test arcs) disable (resp. activate) a transition when a certain number of tokens or a certain amount of fluid is present in a discrete or continuous place, respectively.

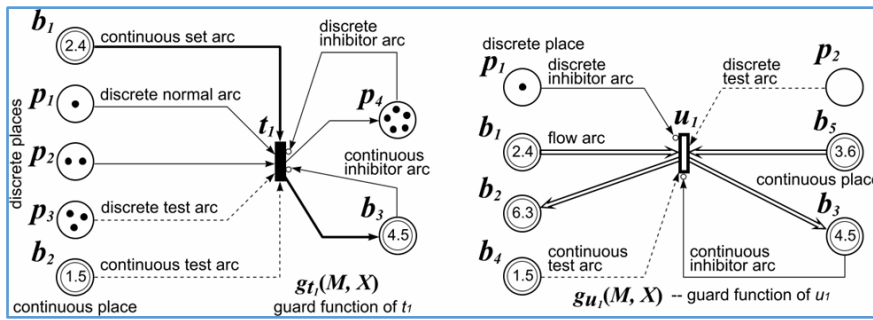
The firing delay of timed discrete transitions  $t_i \in T_t$  are defined by firing fuzzy rates  $\lambda_i(M, \vec{x}) \in \mathbb{R}_+$ , which may depend on the current marking of SHPN. With the immediate discrete transitions  $t_j \in T_0$  are associated fuzzy weights  $w_j(M, \vec{x}) \in \mathbb{R}_+$ , based on which their firing fuzzy probabilities are calculated. The fluid content in buffers is changed either by continuous transitions  $u_k$  depending on their firing fuzzy speed, which may depend on the current discrete marking of the SHPN, that it can be directly established by a setting cardinality of arcs at a certain value, when a discrete transition will be fired.

Figure 2 shows the possible connections between discrete and continuous places with a discrete (resp. continuous) transition through different types of arcs.

The detailed definition of the SHPN, its quantitative attributes, the enabling and firing rules of the respective transitions by the current marking are presented in [8, 9].

A FSHPN model is described by a underlying SHPN in which its parameters  $\tilde{\lambda}_i(M, \vec{x})$ ,  $w_j(M, \vec{x})$  and  $v_k(M, \vec{x})$  are the respective fuzzy numbers [10, 15]. These fuzzy parameters of FSHPN are noted in the following:  $\tilde{\lambda}_i(M, \vec{x})$ ,  $w_j(M, \vec{x})$  and  $\tilde{v}_k(M, \vec{x})$ .

Modelling complex CSN using FSHPN network models allows us to better understand their behaviour, to estimate and ultimately to improve their performance.



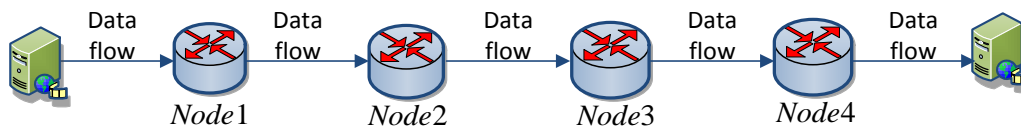
**Figure 2.** The possible connections between discrete and continuous places with discrete or transitions through different types of arcs.

Due to the restriction space for the presentation of proposed method in this paper, we will limit to the fact that only one buffer is incident forward (resp. backward) to the continuous transitions of an *FSHPN* model, i. e.  $|\bullet u_i| = 1$  and/or  $|u_i \bullet| = 1$  [8]. Also, we consider the *FSHPN* models whose continuous markings have no influence to the discrete ones, i.e. every discrete marking determines completely both the set of enabled transitions, their firing rates and the fluid flow speeds of the incoming and outgoing arcs for each buffers.

### 3. Approximate mean-value performance analysis method of continuous flow pipeline discrete - computing process with *FSHPN*

#### 3.1. Modeling of continuous data transmission network virtual channel with *FSHPN*

To present the method of approximate mean-value performance analysis of pipe-line discrete-continuous computing process with *FSHPN* models we will consider a virtual channel of computer network (VCN) [16 -19] for continuous pipe-line transmission data flow consisting of  $N$  node, called servers  $S_{V_1}, S_{V_2}, \dots, S_{V_N}$ , and  $N-1$  buffers  $b_1, b_2, \dots, b_{N-1}$ . Each  $S_{V_i}$  interacts and cooperates with each other through a continuous data storage buffer  $b_i$  with a storage capacity  $h_i$  [8]. Figure 1 shows such a virtual channel of a computer network.

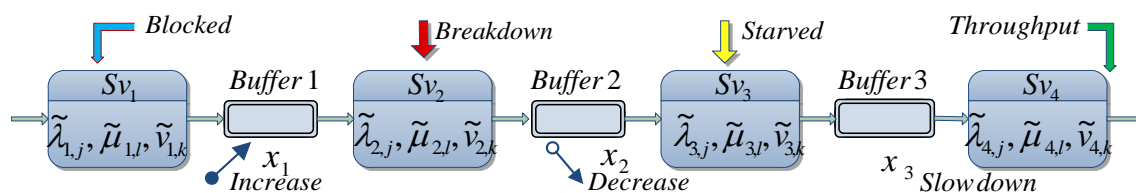


**Figure 3.** A virtual channel of computer network.

*Modeling assumptions of an VCN with FSHPN.* The construction of the model of VCN in the form of *FSHPN* is based on the adoption of the following hypotheses: 1) It is considered that the data flow that processed and transmitted by VCN is continuous, interpreted as fluid; 2) The data flow enters to the server  $S_{V_i}$  is being processed in *active state* and stored in the buffer  $b_i$  with a finite capacity  $h_i$ , then from this buffer it passes to the server  $S_{V_{i+1}}$  and finally from the server  $S_{V_N}$  it leaves the system; 3) When a server  $S_{V_i}$  goes into the *passive state* (for example, it is faulty) in a certain way, it does not process and transmit the data flow; 4) The first server  $S_{V_1}$  is never starved and the last one  $S_{V_N}$  is *never blocked*; 4) Servers  $S_{V_i}$  have different data flow processing speeds  $\tilde{v}_{i,l}$ ; 5) In the initial state, all the servers are operational (they are in *active states*); 6) At the same time a server cannot be in

more than one passive state (failure modes). Even if a server that is in a given failure mode cannot fail in another way before it is repaired; 7) The respective rates of change of the discrete local states follow exponential distributions. 8) The discrete part of *FSHPN* is bounded, live and reversible, i.e. it underlying *FGSPN* has a stationary regime [8, 9].

In these modeling assumptions the behavior of such a VCN is as follows. Suppose the server  $Sv_2$  is in a *passive state*, and the other servers are still functional i.e. in *active states* (see Figure 4). This causes the amount  $x_2$  of fluid to decrease in the buffer  $b_2$  while the amount  $x_1$  of fluid in the buffer  $b_1$  increases. After the buffer  $b_2$  is empty, the server  $Sv_3$  will be starved. On the other hand, after the buffer  $b_1$  is full, the server  $Sv_1$  will be blocked. Moreover, it is also possible for servers to change the active state to a passive state. As an example, suppose the servers  $Sv_1$  and  $Sv_2$  are also functional. Suppose the server  $Sv_2$  is running faster than the server  $Sv_1$ . When the buffer between these servers is empty, the server  $Sv_2$  will slow down processing and run at the same speed as the server  $Sv_1$ . The case when the server  $Sv_1$  is running faster than the server  $Sv_2$  is similar. When all the servers can be either in passive or active state and they operate at different speeds, the throughput of this pipe-line system depends on the rate of change of states, the duration of presence in the passive or active states and the processing speed of the fluid by the servers. An illustration of the dynamic behavior of this type of VCN is given in Figure 4.



**Figure 4.** An illustration of the dynamic of data flow transmission through VCN.

Next, for the convenience of presenting and analyzing the *FSHPN* models that describe the behavior of  $Sv_i$ , we will use the notations of the attributes of these models rendered by respective symbols that have two indices: the first index  $i$  indicates the number of order of belonging to the submodel *FSHPN* <sub>$i$</sub>  that describes the behavior of  $Sv_i$ , and the second index indicates the order number of an attribute that belongs to this submodel, for example, discrete place  $p_{i,k}$ , discrete transition  $t_{i,j}$  or continuous transition  $u_{i,l}$ , etc.

Figure 5 shows an example of a *FSHPN1* model that describes the behavior of a VCN with  $N = 3$  servers and 2 buffers. The discrete part of *FSHPN1* describes the algorithm for controlling the behavior of the continuous part of data flows processing.

In the model *FSHPN1* shows in Figure 5, the notations used to list the discrete places, the discrete and continuous transitions have the following meanings:

$$pi\_k = p_{i,k}, k = 1, \dots, 8; ti\_j = t_{i,j}, j = 1, \dots, 8 \text{ and } ui\_l = t_{i,l}, l = 1, 2.$$

The nominal fuzzy speeds  $\tilde{v}_{i,l}$  of continuous data processing are associated with the respective continuous transitions  $u_{i,l}$ , and the fuzzy rates  $\tilde{\lambda}_{i,j}(M)$  change of the discrete part states (for example, *failure* or *repair*; *active* or *passive*; *available* or *unavailable*) are respectively associated with the discrete timed transitions  $t_{i,j}$ . The respective local active

or passive state of  $Sv_i$  is shown by marking  $m_{i,k} = M(p_{i,k}) = 1, k = 1, \dots, k_d$  of discrete places  $P_{i,k}$ .

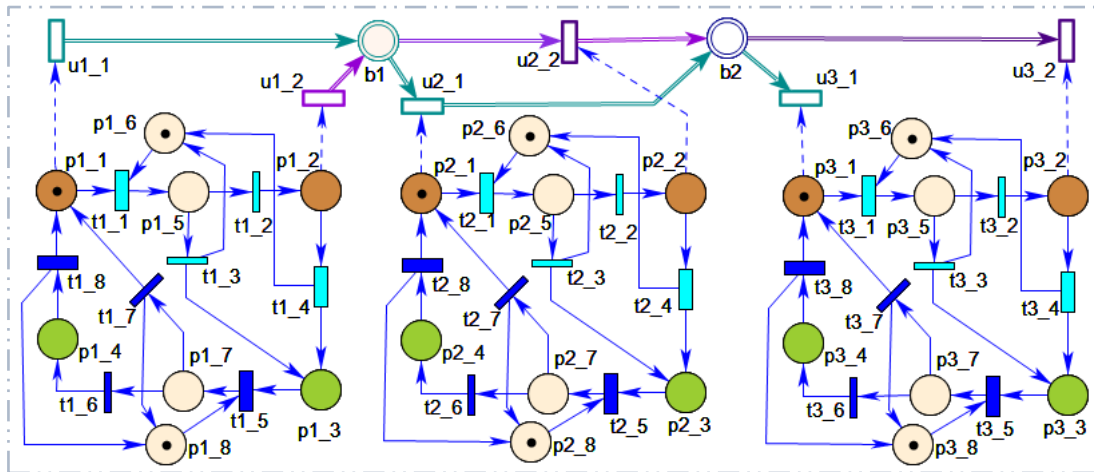


Figure 5. Example of *FSHPN1* model of a VCN with 3 servers and 2 buffers.

As already mentioned, a model *FSPN*, *SHPN* and also *FSHPN* that has  $N \geq 4$  buffers cannot be accurately analyzed analytically [9, 12], we will further develop a method of mean-value aggregate decomposition and approximation of the average performance measures of VCN and the distribution of the average content of buffers.

The proposed method was inspired by the studies of the production lines presented in the works [13 - 15]. For this we decompose VCN into  $N - 1$  producer-consumer subsystems  $DP_i$ , called dipoles. Each dipole  $DP_i$  consists of a  $SA_i$  fluid-producing server in the buffer  $b_i$  and a  $SD_i$  fluid-consuming server from the buffer  $b_i$  between these servers.

Figure 6 shows such an dipole  $DP_i$  of VCN whose behavior is presented in Figure 4.

In order to evaluate the performance of each dipole  $DP_i$  and then of the entire VCN system, we will build the aggregate model  $FSHPN_{Sv_i}$  of each server  $Sv_i$  and also the model  $FSHPN_i$  of each dipole  $DP_i$ .

Figure 7 shows the model  $FSHPN_i$  of such dipole  $DP_i$  of the virtual channel described by the model *FSHPN1* presented in Figure 5.

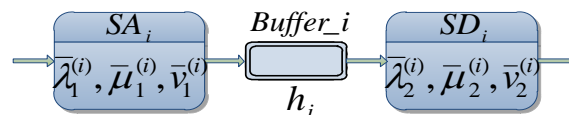


Figure 6. Dipole  $DP_i$  of a VCN whose behavior is shown in Figure 4.

Next, for the convenience of presentation, exposing and interpreting the behaviour of the  $FSHPN_i$  model of a dipole  $DP_i$  in Figure 7, for upstream (producer)  $SA_i = Sv_i$  at the buffer  $b_i$  we will use the following notations of discrete places, discrete and continuous transitions:  $pAi\_k = p_{i,k}, k = 1, \dots, 8$ ;  $tAi\_j = t_{i,j}, j = 1, \dots, 8$ ; and  $uAi\_l = t_{i,l}, l = 1, 2$ , and for downstream (consumer)  $SD_i = Sv_{i+1}$  at the buffer  $b_i$  we will use the following notations:  $pAi\_k = p_{i,k}, k = 1, \dots, 8$ ;  $tAi\_j = t_{i,j}, j = 1, \dots, 8$ ; and,  $uAi\_l = t_{i,l}, l = 1, 2$ .



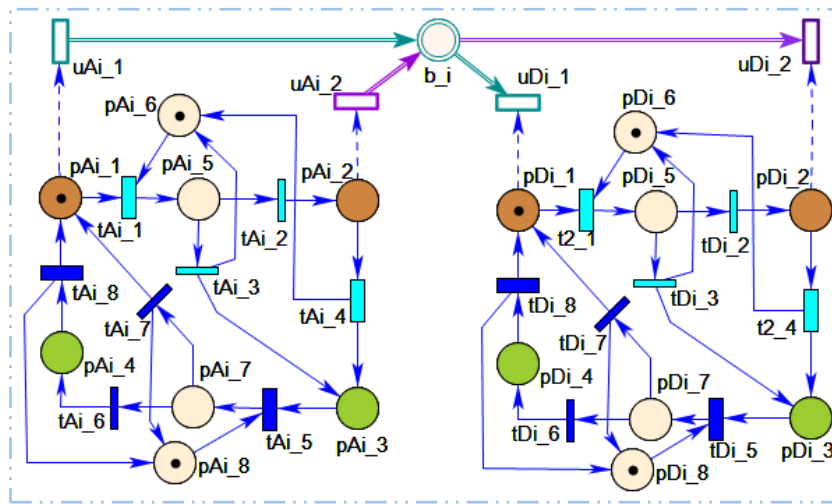


Figure 7. The model  $FSHPN1_i$  of a dipole  $DP_i$  of  $FSHPN1$  shown in figure 5.

### 3.2. Aggregate $FSHPN$ model of a dipole

As an example, the aggregate  $FSHPN2_i$  model of a dipole  $DP_i$ , described by the model  $FSHPN1_i$  in Figure 7, is shown in Figure 8. In this model, the aggregate quantitative fuzzy parameters are determined based on the fuzzy steady state probabilities  $\tilde{\pi}_{i,k}$  of presence in the respective accessible markings  $M_{i,k}$  of  $FGSPN_{Sv_i}$  that describe the behavior of the discrete part  $FSGPN_{Sv_i}$  of analyzed model  $FSHPN_{Sv_i}$ .

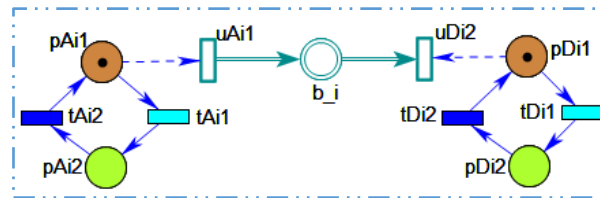


Figure 8. The aggregate model  $FSHPN2_i$  of a dipole  $DP_i$ .

To specify and determine the values of the quantitative fuzzy parameters of the aggregate model  $SHPN2_i$ , for the upstream (producer)  $Sv_i$  at the buffer  $b_i = b_i$  we use the notations:  $pAi1 = p_1^i$ ,  $pAi2 = p_2^i$ ,  $tAi1 = t_1^i$ ,  $tAi2 = t_2^i$  and  $uAi1 = u_1^i$ , again for the downstream (consumer)  $Sv_{i+1}$  at the buffer  $b_i = b_i$  we use the notations:  $pDi1 = p_1^{i+1}$ ,  $pDi2 = p_2^{i+1}$ ,  $tDi1 = t_1^{i+1}$ ,  $tDi2 = t_2^{i+1}$  and  $uDi1 = u_2^i$ . The meanings of discrete places:  $p_1^i$  (resp.  $p_1^{i+1}$ ) - is respectively the active local state in which  $Sv_i$  (resp.  $Sv_{i+1}$ ) produces (resp. consumes) fluid with the aggregate speed  $\bar{v}_1^{(i)}$  (resp.  $\bar{v}_2^{(i)}$ ) in (resp. from) the buffer  $b_i$ , and  $p_2^i$  (resp.  $p_2^{i+1}$ ) - is respectively the passive local state in which (resp.) does not produce (resp. does not consume) fluid in (resp. from) the buffer  $b_i$ . The meanings of the discrete transitions:  $t_1^i$  (resp.  $t_1^{i+1}$ ) - represents respectively the activity in which  $Sv_i$  (resp.  $Sv_{i+1}$ ) changes its active state into the passive one with the aggregate rate  $\bar{\lambda}_1^{(i)}$  (resp.  $\bar{\lambda}_2^{(i)}$ ), and  $t_2^i$  (resp.  $t_2^{i+1}$ ) - represents respectively the activity in which  $Sv_i$  (resp.  $Sv_{i+1}$ ) changes the passive state in the active one with the aggregate rate  $\mu_1^{(i)}$  (resp.  $\mu_2^{(i)}$ ).

The fully linear system of Chapman-Kolmogorov equations (FFLSE), that describes the continuous time Markov chain (CTMC) behaviour of discrete part  $FGSPN_{Sv_i}$  model of server  $Sv_i$ , is generated using the VPNP [19] or PIPE 4.3 [20] ToolKits. After obtaining parametric steady state probabilities  $\pi_{i,k}$ , they have to be converted to their corresponding fuzzy form  $\tilde{\pi}_{i,k}$ . To determine the steady state distribution fuzzy probability  $\tilde{\pi}_{i,k}$  of this CTMC must solve this FFLSE by applying fuzzified transition firing rates. The matrix form of this FFLSE is  $\tilde{\mathbf{A}} \otimes \tilde{\mathbf{x}} = \tilde{\mathbf{b}}$ , where  $\tilde{\mathbf{A}} = (\tilde{\mathbf{a}}_{i,j}) = (\tilde{\alpha}_{i,j}, \tilde{\delta}_{i,j}, \tilde{\beta}_{i,j})$  is a  $n \times n$  fuzzy matrix in which each element of  $\tilde{\mathbf{A}}$  is a fuzzy number, and  $\tilde{\mathbf{x}} = (\tilde{x}_1, \dots, \tilde{x}_n)$ ,  $\tilde{\mathbf{b}} = (\tilde{b}_1, \dots, \tilde{b}_n)$  are fuzzy vectors  $n \times 1$ . We may represent fuzzy matrix  $\tilde{\mathbf{A}} = (\tilde{\mathbf{a}}_{i,j})_{n \times n}$  with new notation  $\tilde{\mathbf{A}} = (\mathbf{A}, \mathbf{M}, \mathbf{N})$ , where  $\mathbf{A} = (a_{i,j})$ ,  $\mathbf{M} = (\delta_{i,j})$ , and  $\mathbf{N} = (\beta_{i,j})$  are three  $n \times n$  crisp matrices [21, 22]. Using the algebra multiplication of two fuzzy numbers on the  $\tilde{\mathbf{A}} \otimes \tilde{\mathbf{x}} = \tilde{\mathbf{b}}$ , where  $\tilde{\mathbf{A}} = (\mathbf{A}, \mathbf{M}, \mathbf{N})$ ,  $\tilde{\mathbf{x}} = (\mathbf{x}, \mathbf{y}, \mathbf{z})$  and  $\tilde{\mathbf{b}} = (\mathbf{b}, \mathbf{g}, \mathbf{d})$ , we obtain the formula  $(\mathbf{A} \cdot \mathbf{x}, \mathbf{A} \cdot \mathbf{y} + \mathbf{M} \cdot \mathbf{x}, \mathbf{A} \cdot \mathbf{z} + \mathbf{N} \cdot \mathbf{x}) = (\mathbf{b}, \mathbf{g}, \mathbf{d})$ , whereby the solution  $\tilde{\mathbf{x}} = (\mathbf{x}, \mathbf{y}, \mathbf{z})$  is determined by solving the following three systems of equations in crisp numbers [21, 22]:

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}; \quad \mathbf{A} \cdot \mathbf{y} = \mathbf{g} - \mathbf{M} \cdot \mathbf{x}; \quad \mathbf{A} \cdot \mathbf{z} = \mathbf{d} - \mathbf{N} \cdot \mathbf{x}. \quad (1)$$

Let  $M_{i,k}[u_{i,l} >]$  is the continuous transition  $u_{i,l}$  enabled by the current marking  $M_{i,k}$  of the discrete part of the analyzed model  $FSPN$  [8, 9]. In this case the average aggregate data processing fuzzy speed  $\bar{v}^{(i)}$  of  $Sv_i$  is determined by following expressions:

$$\bar{v}^{(i)} = \left( \sum_{\forall M_{i,k}[u_{i,j} >]} (\tilde{v}_{i,l}(M_{i,k}) \cdot \tilde{\pi}_{i,k}) \right) / \sum_{\forall M_{i,k}[u_{i,j} >]} \tilde{\pi}_{i,k}, \quad i = \overline{1, N}. \quad (2)$$

Also, let  $M_{i,k}[t_{i,j} >]$  it is the timed discrete transition  $t_{i,j}$  enabled by the current marking  $M_{i,k}$ .

The average aggregate fuzzy rate  $\bar{\lambda}^{(i)}$  of transition  $Sv_i$  from active to passive state is:

$$\bar{\lambda}^{(i)} = \left( \sum_{\substack{\forall (M_{i,k}[t_{i,j} >]) \wedge \\ (M_{i,k}[u_{i,l} >])}} \tilde{\lambda}_{i,j}(M_{i,k}) \cdot \tilde{\pi}_{i,k} \right) / \left( \sum_{\substack{\forall (M_{i,k}[t_{i,j} >]) \wedge \\ (M_{i,k}[u_{i,l} >])}} \tilde{\pi}_{i,k} \right), \quad i = \overline{1, N}. \quad (3)$$

Also, the aggregate average fuzzy rate  $\bar{\mu}^{(i)}$  of transition  $Sv_i$  from passive to active state is:

$$\bar{\mu}^{(i)} = \left( \sum_{\forall (M_{i,n}[t_{i,m} >])} \tilde{\mu}_{i,m}(M_{i,n}) \cdot \tilde{\pi}_{i,n} \right) / \left( \sum_{M_{i,k}[t_{i,j} >]} \tilde{\pi}_{i,n} \right), \quad i = \overline{1, N}. \quad (4)$$

Thus, determining the fuzzy parameters calculated according to the expressions (1) ... (3), we identify the respective fuzzy parameters of the aggregate model  $SHPN_2$  that describes the dipole  $DP_i$  behavior shown in Figure 8:

$$\bar{v}_1^{(i)} = \bar{v}^{(i)}, \quad \bar{v}_2^{(i)} = \bar{v}^{(i+1)}, \quad \bar{\lambda}_1^{(i)} = \bar{\lambda}^{(i)}, \quad \bar{\lambda}_2^{(i)} = \bar{\lambda}^{(i+1)}, \quad \bar{\mu}_1^{(i)} = \bar{\mu}^{(i)} \quad \text{și} \quad \bar{\mu}_2^{(i)} = \bar{\mu}^{(i+1)}.$$

Based on these parameters, we can build the aggregate hybrid FCMTc to realize the performance evaluation of a dipole  $DP_i$ , and then they are used to evaluate the performance measures of the entire analyzed FSHPN model. For this, in section 5 we present an algorithm in a fixed point iteration scheme for the analysis of a this type FSHPN models.

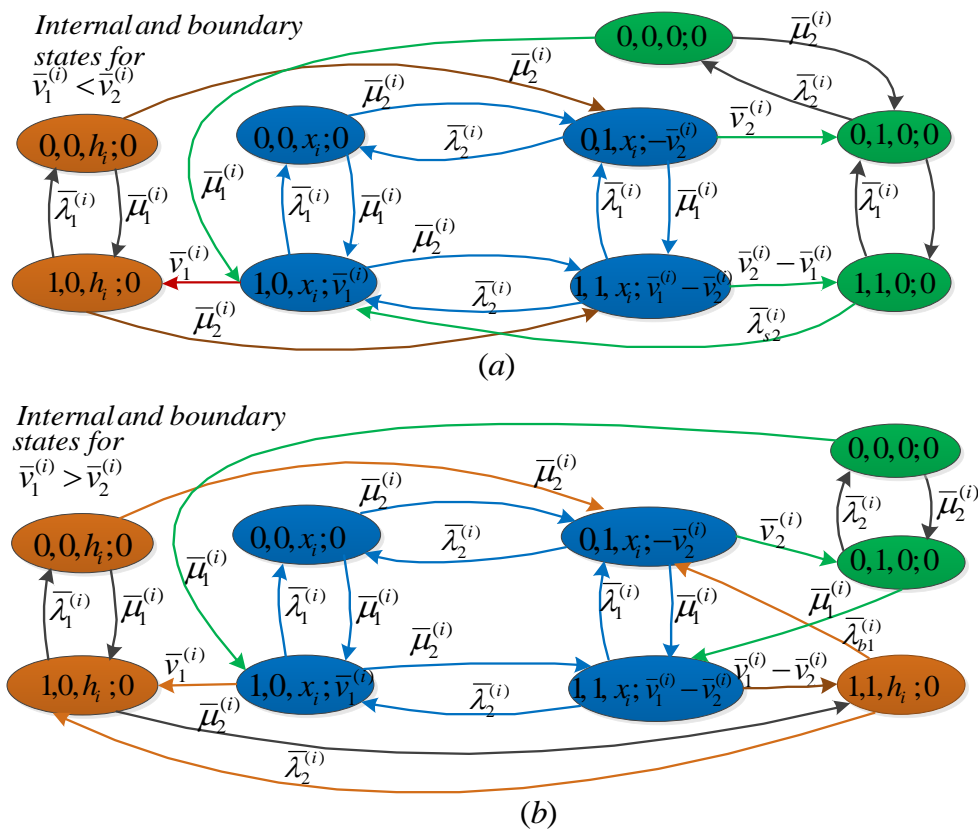
**4. Aggregate HCTMC and performance analysis of a dipole**

**4.1. Aggregate HCTMC of a dipole**

The aim of this section is to derive the formulas of the most important steady state performance measures of a dipole  $DP_i$ .

The behavior of  $SA_i$  and  $SD_i$  with two respectively aggregate states (active  $s_1$  or passive  $s_2$  state) of a dipole  $DP_i$ ,  $i = 1, 2, \dots, N - 1$  of the model FSHPN  $2_i$  is described by the aggregate fuzzy hybrid CTMC (HCTMC  $2_i$ ). The states of HCTMC  $2_i$  are described by the 4-tuple  $(s_1, s_2, x_i; \bar{V}_k^{(i)})$ , where  $s_1, s_2 \in \{0, 1\}, k = \bar{0}, \bar{3}$ , and  $\bar{V}_k^{(i)} = s_1 \cdot \bar{v}_1^{(i)} - s_2 \cdot \bar{v}_2^{(i)}$  is the dynamic balance [??] that determine the speed of changing the fluid stock level  $x_i$ ,  $0 \leq x_i \leq h_i$ , in the buffer  $b_i$ .

The state transition diagram of this HCTMC  $2_i$  depends on the concrete relationships between the values of  $\bar{v}_1^{(i)}$  and  $\bar{v}_2^{(i)}$ , which can be:  $\bar{v}_1^{(i)} = \bar{v}_2^{(i)}$ ,  $\bar{v}_1^{(i)} > \bar{v}_2^{(i)}$  or  $\bar{v}_1^{(i)} < \bar{v}_2^{(i)}$ . The method of constructing these types of transition diagrams and writing on their basis the Chapman-Kolmogorov equilibrium equations is described in [9]. Figure 9a shows the transition diagram of HCTMC  $2_i$  for the internal states ( $0 < x_i < h_i$ ) with low ( $x_i = 0$ ) and high ( $x_i = h_i$ ) boundary states for  $\bar{v}_1^{(i)} < \bar{v}_2^{(i)}$ , and in Figure 9b the one is presented for  $\bar{v}_1^{(i)} > \bar{v}_2^{(i)}$ .



**Figure 9.** State transition diagrams of HCTMC  $2_i$ , which describes the behaviour of FSHPN  $2_i$ : (a) internal and boundary states with  $\bar{v}_1^{(i)} < \bar{v}_2^{(i)}$ ; (b) internal and boundary states with  $\bar{v}_1^{(i)} > \bar{v}_2^{(i)}$ .

#### 4.2. Aggregate HCTMC equations describing the evolution of a dipole

As, all the proposed models are bounded and the initial marking is a home state, the underlying HCTMC  $2_i$  of a dipole  $DP_i$  are ergodic for the different behavior fuzzy parameters.

The steady state density probability equations in the internal states of HCTMC  $2_i$ . When the buffer  $b_i$  is neither empty nor full, the stock level  $x_i$ ,  $0 < x_i < h_i$ , of this buffer varies with density probability  $f_{s1,s2}^{(i)}(x_i)$  depending on the current internal state of the buffer  $b_i$  and adjacent servers. Because the level of the stock in  $b_i$  can vary very little over a short period of time, the behavior of the dipole  $DP_i$  in steady state regime for internal states is described by the following system of ordinary differential equations [7]:

$$\begin{aligned} 0 &= -(\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)})f_{0,0}^{(i)}(x_i) + \bar{\lambda}_2^{(i)}f_{0,1}^{(i)}(x_i) + \bar{\lambda}_1^{(i)}f_{1,0}^{(i)}(x_i), \\ -\bar{v}_2^{(i)}\frac{\partial f_{0,1}^{(i)}(x_i)}{\partial x_i} &= -(\bar{\mu}_1^{(i)} + \bar{\lambda}_2^{(i)})f_{0,1}^{(i)}(x_i) + \bar{\lambda}_1^{(i)}f_{1,1}^{(i)}(x_i) + \bar{\mu}_2^{(i)}f_{0,0}^{(i)}(x_i), \end{aligned} \quad (5)$$

$$\bar{v}_1^{(i)}\frac{\partial f_{1,0}^{(i)}(x_i)}{\partial x_i} = -(\bar{\lambda}_1^{(i)} + \bar{\mu}_2^{(i)})f_{1,0}^{(i)}(x_i) + \bar{\lambda}_2^{(i)}f_{1,1}^{(i)}(x_i) + \bar{\mu}_1^{(i)}f_{0,0}^{(i)}(x_i),$$

$$(\bar{v}_1^{(i)} - \bar{v}_2^{(i)})\frac{\partial f_{1,1}^{(i)}(x_i)}{\partial x_i} = -(\bar{\lambda}_1^{(i)} + \bar{\lambda}_2^{(i)})f_{1,1}^{(i)}(x_i) + \bar{\mu}_1^{(i)}f_{0,1}^{(i)}(x_i) + \bar{\mu}_2^{(i)}f_{1,0}^{(i)}(x_i). \quad (6)$$

The steady-state fuzzy probability equations in the boundary states of HCTMC  $2_i$ . While the internal behavior of the dipole  $DP_i$  can be described by probability densities, it can reach a certain boundary state with a fuzzy probability  $\rho_{s1,s2}^{(i)}(0)$  or  $\rho_{s1,s2}^{(i)}(h_i)$  different from zero. For example, if  $\bar{v}_1^{(i)} > \bar{v}_2^{(i)}$  (resp.  $\bar{v}_1^{(i)} < \bar{v}_2^{(i)}$ ) and both servers are operational, the stock  $x_i$  of buffer  $b_i$  tends to increase (resp. to decrease). If this situation lasts long enough, the buffer  $b_i$  will be full (resp. empty), we have  $x_i = h_i$  (resp.  $x_i = 0$ ) and the dynamic balance of stock in buffer  $b_i$  is  $(\bar{v}_1^{(i)} - \bar{v}_2^{(i)}) = 0$ . In this case it will reach the state  $x_i = h_i$  and it will remain there until one of the two servers  $Sv_i$  or  $Sv_{i+1}$  it will not go into passive state.

From the transition diagrams presented in Figures 6 and Figures 7 respectively, for the lower boundary,  $x_i = 0$ , according to the method of writing the equilibrium equations of the steady-state probabilities described in [9], we obtain:

- for  $\bar{v}_1^{(i)} > \bar{v}_2^{(i)}$  we have:

$$\text{if } x_i \rightarrow 0: (\bar{\mu}_1^{(i)} + \bar{\lambda}_2^{(i)}) \cdot \rho_{0,1}^{(i)}(0) = \bar{\mu}_2^{(i)} \cdot \rho_{0,1}^{(i)}(0) + \bar{v}_2^{(i)} \cdot f_{0,1}^{(i)}(0), \quad (7)$$

$$(\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot \rho_{0,0}^{(i)}(0) = \bar{\lambda}_2^{(i)} \cdot \rho_{0,1}^{(i)}(0),$$

$$\text{- if } 0 \rightarrow x_i: \bar{v}_2^{(i)} \cdot f_{1,0}^{(i)}(0) = \bar{\mu}_1^{(i)} \cdot \rho_{0,0}^{(i)}(0), \quad (\bar{v}_1^{(i)} - \bar{v}_2^{(i)})f_{1,1}^{(i)}(0) = \bar{\mu}_1^{(i)} \rho_{0,1}^{(i)}(0);$$

- for  $\bar{v}_1^{(i)} < \bar{v}_2^{(i)}$  we have:

$$\text{- if } x_i \rightarrow 0: (\bar{\mu}_1^{(i)} + \bar{\lambda}_2^{(i)}) \cdot \rho_{0,1}^{(i)}(0) = \bar{\mu}_2^{(i)} \cdot \rho_{0,0}^{(i)}(0) + \bar{\lambda}_1^{(i)} \cdot \rho_{1,1}^{(i)}(0) + \bar{v}_2^{(i)} \cdot f_{0,1}^{(i)}(0),$$

$$(\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot \rho_{0,0}^{(i)}(0) = \bar{\lambda}_2^{(i)} \cdot \rho_{0,1}^{(i)}(0), \tag{8}$$

$$(\bar{\lambda}_1^{(i)} + \bar{\lambda}_{s_2}^{(i)}) \cdot \rho_{1,1}^{(i)}(0) = \bar{\mu}_1^{(i)} \cdot \rho_{0,1}^{(i)}(0) + (\bar{v}_2^{(i)} - \bar{v}_1^{(i)}) \cdot f_{1,1}^{(i)}(0),$$

if  $0 \rightarrow x_i$ :  $\bar{v}_1^{(i)} \cdot f_{1,0}^{(i)}(0) = \bar{\mu}_1^{(i)} \cdot \rho_{0,0}^{(i)}(0) + \bar{\lambda}_{s_2}^{(i)} \cdot \rho_{1,1}^{(i)}(0)$ , where:  $\bar{\lambda}_{s_2}^{(i)} = \bar{\lambda}_2^{(i)} \cdot \bar{v}_1^{(i)} / \bar{v}_2^{(i)}$ .

Similarly, for the *upper boundary*,  $x_i = h_i$ , we obtain the following steady-state equations:

- for  $\bar{v}_1^{(i)} > \bar{v}_2^{(i)}$  we have:

- if  $x_i \rightarrow h_i$ :  $(\bar{\lambda}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot \rho_{1,0}^{(i)}(h_i) = \bar{\lambda}_2^{(i)} \cdot \rho_{1,1}^{(i)}(h_i) + \bar{\mu}_1^{(i)} \cdot \rho_{0,0}^{(i)}(h_i) + \bar{v}_1^{(i)} \cdot f_{0,1}^{(i)}(h_i)$ ,

$$(\bar{\lambda}_{b_1}^{(i)} + \bar{\lambda}_2^{(i)}) \cdot \rho_{1,1}^{(i)}(h_i) = \bar{\mu}_2^{(i)} \cdot \rho_{1,0}^{(i)}(h_i) + (\bar{v}_1^{(i)} - \bar{v}_2^{(i)}) \cdot f_{1,1}^{(i)}(h_i), \tag{9}$$

$$(\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot \rho_{0,0}^{(i)}(h_i) = \bar{\lambda}_1^{(i)} \cdot \rho_{1,0}^{(i)}(h_i),$$

- if  $h_i \rightarrow x_i$ :  $\bar{v}_2^{(i)} \cdot f_{0,1}^{(i)}(h_i) = \bar{\lambda}_{b_1}^{(i)} \cdot \rho_{1,1}^{(i)}(h_i) + \bar{\mu}_2^{(i)} \cdot \rho_{0,0}^{(i)}(h_i)$ , where:  $\bar{\lambda}_{b_1}^{(i)} = \bar{\lambda}_2^{(i)} \cdot \bar{v}_1^{(i)} / \bar{v}_2^{(i)}$ ;

- for  $\bar{v}_1^{(i)} < \bar{v}_2^{(i)}$  we have:

- if  $x_i \rightarrow h_i$ :  $(\bar{\lambda}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot \rho_{1,0}^{(i)}(h_i) = \bar{\mu}_1^{(i)} \cdot \rho_{0,0}^{(i)}(h_i) + \bar{v}_1^{(i)} \cdot f_{1,0}^{(i)}(h_i)$ ,

$$(\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot \rho_{0,0}^{(i)}(h_i) = \bar{\lambda}_1^{(i)} \cdot \rho_{0,1}^{(i)}(h_i), \tag{10}$$

- if  $h_i \rightarrow x_i$ :  $(\bar{v}_2^{(i)} - \bar{v}_1^{(i)}) \cdot f_{1,1}^{(i)}(h_i) = \bar{\mu}_2^{(i)} \cdot \rho_{1,0}^{(i)}(h_i)$ ,  $\bar{v}_2^{(i)} \cdot f_{0,1}^{(i)}(h_i) = \bar{\mu}_2^{(i)} \cdot \rho_{0,0}^{(i)}(h_i)$ .

*The normalization equation.* Since the sum of all probabilities must be equal to 1, we have:

$$\sum_{s_1=0,1} \sum_{s_2=0,1} [\int_0^{h_i} f_{s_1,s_2}^{(i)}(x_i) dx_i + \rho_{s_1,s_2}^{(i)}(0) + \rho_{s_1,s_2}^{(i)}(h_i)] = 1. \tag{11}$$

*Performance measures of the system:*

- *The average level  $\bar{x}_i$  of stock in buffer  $b_i$ .* This QoS indicator is determined can be determined from the following expression:

$$\bar{x}_i = \sum_{s_1=0,1} \sum_{s_2=0,1} [\int_0^{h_i} x_i \cdot f_{s_1,s_2}^{(i)}(x_i) dx_i + h_i \cdot \rho_{s_1,s_2}^{(i)}(h_i)]. \tag{12}$$

- *The average data processing productivity (throughput)  $\bar{\eta}_{v_2}^{(i)}$*  by  $DP_i$  represents the *average speed* at which the fluid leaves the downstream server  $SD_i = S_{v_{i+1}}$  of the buffer  $b_i$ . When the stock is not empty and  $SD_i$  it is operational, its speed is  $\bar{v}_2^{(i)}$ , and when the stock is empty and both servers  $SA_i$  and  $SD_i$  are operational, then its processing speed is equal to  $\bar{v}_1^{(i)}$ , so the  $\bar{\eta}_{v_2}^{(i)}$  is:

$$\bar{\eta}_{v_2}^{(i)} = \bar{v}_2^{(i)} [\int_0^{h_i} (f_{0,1}^{(i)}(x_i) + f_{1,1}^{(i)}(x_i)) dx_i + \rho_{1,1}^{(i)}(h_i)] + \min(\bar{v}_1^{(i)}, \bar{v}_2^{(i)}) \cdot \rho_{1,1}^{(i)}(0). \tag{13}$$

Similarly, we calculate the throughput  $\bar{\eta}_1^{(i)}$  of  $DP_i$ , which represents the average speed at which the fluid enters the buffer  $b_i$ .

We mention that the conservation of the flux in the dipole  $DP_i$  considered for  $\bar{\eta}_{v1}^{(i)} = \bar{\eta}_{v2}^{(i)}$ ,  $0 \leq x_i \leq h_i$ , is given by the following equation:

$$(\bar{v}_2^{(i)} - \bar{v}_1^{(i)})f_{1,1}(x_i) + \bar{v}_2^{(i)}f_{0,1}(x_i) - \bar{v}_1^{(i)}f_{1,0}(x_i) = 0. \quad (14)$$

### 4. 3. Performances evaluation of a dipole

In order to evaluate the performances measures of a dipole  $DP_i$ , it is necessary to consider and analyze separately the following three cases:  $\bar{v}_1^{(i)} = \bar{v}_2^{(i)} = \bar{v}^{(i)}$ ,  $\bar{v}_1^{(i)} > \bar{v}_2^{(i)}$  and  $\bar{v}_1^{(i)} < \bar{v}_2^{(i)}$ .

Due to the space restriction we will further analyze only the  $\bar{v}_1^{(i)} \neq \bar{v}_2^{(i)}$ . For  $\bar{v}_1^{(i)} \neq \bar{v}_2^{(i)}$  from equations (12) and (13) the analytical solutions of system equations (12) and (12), which describe the evolution of  $HCTMC 2_i$ , taking into account the boundary equations (12) for two separate cases:  $\bar{v}_1^{(i)} > \bar{v}_2^{(i)}$  and  $\bar{v}_1^{(i)} < \bar{v}_2^{(i)}$ , are determined by the following expressions:

- internal states,  $0 < x_i < h_i$ :

$$f_{1,0}^{(i)}(x_i) = C_0^{(i)} \cdot A_1^{(i)} \cdot e^{r^{(i)} \cdot x_i}, \quad f_{0,1}^{(i)}(x_i) = C_0^{(i)} \cdot e^{r^{(i)} \cdot x_i}, \quad (15)$$

$$f_{0,0}^{(i)}(x_i) = C_0^{(i)} \cdot A_2^{(i)} \cdot e^{r^{(i)} \cdot x_i}, \quad f_{1,1}^{(i)}(x_i) = C_0^{(i)} \cdot A_3^{(i)} \cdot e^{r^{(i)} \cdot x_i};$$

- boundary states for  $x_i = 0$ :  $\rho_{1,0}^{(i)}(0) = 0$ ,

$$\rho_{0,0}^{(i)}(0) = \begin{cases} C_0^{(i)} \cdot \bar{v}_1^{(i)} \cdot A_1^{(i)} / \bar{\mu}_1^{(i)} & \text{if } \bar{v}_1^{(i)} > \bar{v}_2^{(i)} \\ C_0^{(i)} \cdot \bar{\lambda}_2^{(i)} \cdot C_2^{(i)} / ((\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot A_4^{(i)}) & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases}, \quad (16)$$

$$\rho_{0,1}^{(i)}(0) = \begin{cases} C_0^{(i)} \cdot \bar{v}_1^{(i)} \cdot A_1^{(i)} \cdot (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) / (\bar{\lambda}_2^{(i)} \cdot \bar{\mu}_1^{(i)}) & \text{if } \bar{v}_1^{(i)} > \bar{v}_2^{(i)} \\ C_0^{(i)} \cdot C_2^{(i)} / A_4^{(i)} & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases},$$

$$\rho_{1,1}^{(i)}(0) = \begin{cases} 0 & \text{if } \bar{v}_1^{(i)} > \bar{v}_2^{(i)} \\ C_0^{(i)} \cdot C_1^{(i)} / A_4^{(i)} & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases};$$

- boundary states for  $x_i = h_i$ :  $\rho_{0,1}^{(i)}(h_i) = 0$ ,

$$\rho_{0,0}^{(i)}(h_i) = \begin{cases} C_0^{(i)} \cdot C_2^{(i)} \cdot \bar{\lambda}_1^{(i)} / ((\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot A_4^{(i)}) & \text{if } \bar{v}_1^{(i)} > \bar{v}_2^{(i)} \\ C_0^{(i)} \cdot \bar{v}_2^{(i)} \cdot e^{r^{(i)} \cdot x_i} / \bar{\mu}_2^{(i)} & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases},$$

$$\rho_{1,0}^{(i)}(h_i) = \begin{cases} C_0^{(i)} \cdot C_2^{(i)} / A_4^{(i)} & \text{if } \bar{v}_1^{(i)} > \bar{v}_2^{(i)} \\ C_0^{(i)} \cdot (\bar{v}_2^{(i)} - \bar{v}_1^{(i)}) \cdot A_3^{(i)} \cdot e^{r^{(i)} \cdot x_i} / \bar{\mu}_2^{(i)} & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases}, \quad (17)$$

$$\rho_{1,1}^{(i)}(h_i) = \begin{cases} C_0^{(i)} \cdot C_1^{(i)} / A_4^{(i)} & \text{if } \bar{v}_1^{(i)} > \bar{v}_2^{(i)} \\ 0 & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases},$$

where:

$$r^{(i)} = \begin{cases} (E_1^{(i)} \cdot E_4^{(i)} - E_2^{(i)} \cdot E_3^{(i)}) / (E_1^{(i)} - E_3^{(i)}) & \text{if } \bar{v}_1^{(i)} > v_2^{(i)} \\ (E_1^{(i)} \cdot E_4^{(i)} - E_2^{(i)} \cdot E_3^{(i)}) / (E_3^{(i)} - E_1^{(i)}) & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases}$$

$$A_2^{(i)} = [\bar{\lambda}_2^{(i)} \cdot (E_1^{(i)} - E_3^{(i)}) + \bar{\lambda}_1^{(i)} \cdot (E_4^{(i)} - E_2^{(i)})] / [(E_1^{(i)} - E_3^{(i)}) \cdot (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)})],$$

$$A_3^{(i)} = [\bar{v}_2^{(i)} \cdot (E_1^{(i)} - E_3^{(i)}) + \bar{v}_1^{(i)} \cdot (E_4^{(i)} - E_2^{(i)})] / [(E_1^{(i)} - E_3^{(i)}) \cdot (\bar{v}_1^{(i)} - \bar{v}_2^{(i)})],$$

$$E_1^{(i)} = (\bar{\lambda}_2^{(i)} \cdot \omega_1^{(i)} - \bar{\mu}_2^{(i)} \cdot \omega_2^{(i)}) / \bar{v}_1^{(i)}, \quad E_2^{(i)} = -\bar{\lambda}_2^{(i)} \cdot \omega_1^{(i)} / \bar{v}_1^{(i)}, \quad E_3^{(i)} = -\bar{\lambda}_1^{(i)} \cdot \omega_1^{(i)} / \bar{v}_2^{(i)}$$

$$E_4^{(i)} = (\bar{\lambda}_1^{(i)} \cdot \omega_1^{(i)} + \bar{\mu}_1^{(i)} \cdot \omega_2^{(i)}) / \bar{v}_2^{(i)}, \quad A_1^{(i)} = (E_2^{(i)} - E_4^{(i)}) / (E_3^{(i)} - E_1^{(i)}),$$

$$\omega_1^{(i)} = (\bar{v}_1^{(i)} \cdot \bar{\mu}_1^{(i)} + \bar{v}_2^{(i)} \cdot \bar{\mu}_2^{(i)}) / ((\bar{v}_2^{(i)} - \bar{v}_1^{(i)}) (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)})), \quad \omega_2^{(i)} = 1 + (\bar{\lambda}_1^{(i)} + \bar{\lambda}_2^{(i)}) / (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}),$$

$$A_4^{(i)} = \begin{cases} \bar{\lambda}_1^{(i)} \cdot (\bar{\lambda}_2^{(i)} + (\bar{\lambda}_1^{(i)} + \bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot \bar{v}_2^{(i)} / \bar{v}_1^{(i)}) & \text{if } \bar{v}_1^{(i)} > v_2^{(i)} \\ \bar{\lambda}_2^{(i)} \cdot (\bar{\lambda}_1^{(i)} + (\bar{\lambda}_2^{(i)} + \bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot \bar{v}_1^{(i)} / \bar{v}_2^{(i)}) & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases}$$

$$C_1^{(i)} = \begin{cases} [\bar{v}_1^{(i)} \cdot (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) + (\bar{\lambda}_1^{(i)} + \bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot (\bar{v}_1^{(i)} - \bar{v}_2^{(i)}) \cdot A_3^{(i)}] \cdot e^{r^{(i)} \cdot h_i} & \text{if } \bar{v}_1^{(i)} > v_2^{(i)} \\ \bar{v}_2^{(i)} \cdot (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) + (\bar{\lambda}_2^{(i)} + \bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot (\bar{v}_2^{(i)} - \bar{v}_1^{(i)}) \cdot A_3^{(i)} & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases}$$

$$C_2^{(i)} = \begin{cases} (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot [\bar{\lambda}_1^{(i)} \cdot \bar{v}_2^{(i)} + \bar{\lambda}_2^{(i)} \cdot \bar{v}_1^{(i)} + \bar{\lambda}_2^{(i)} \cdot (\bar{v}_1^{(i)} - \bar{v}_2^{(i)}) \cdot A_3^{(i)}] \cdot e^{r^{(i)} \cdot h_i} / \bar{\mu}_2^{(i)} & \text{if } \bar{v}_1^{(i)} > v_2^{(i)} \\ (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot [\bar{\lambda}_1^{(i)} \cdot \bar{v}_2^{(i)} + \bar{\lambda}_2^{(i)} \cdot \bar{v}_1^{(i)} + \bar{\lambda}_1^{(i)} \cdot (\bar{v}_2^{(i)} - \bar{v}_1^{(i)}) \cdot A_3^{(i)}] / \bar{\mu}_1^{(i)} & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases}$$

$$D_1^{(i)} = (1 + A_1^{(i)} + A_2^{(i)} + A_3^{(i)}) \cdot (e^{r^{(i)} \cdot h_i} - 1) / r^{(i)}, \quad D_2^{(i)} = \bar{v}_1^{(i)} \cdot A_1^{(i)} \cdot (1 + (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) / \bar{\lambda}_2^{(i)}),$$

$$D_3^{(i)} = C_1^{(i)} + C_2^{(i)} \cdot (1 + \bar{\lambda}_1^{(i)} / (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)})),$$

$$A_4^{(i)} = \begin{cases} \bar{\lambda}_1^{(i)} \cdot (\bar{\lambda}_2^{(i)} + (\bar{\lambda}_1^{(i)} + \bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot \bar{v}_2^{(i)} / \bar{v}_1^{(i)}) & \text{if } \bar{v}_1^{(i)} > v_2^{(i)} \\ \bar{\lambda}_2^{(i)} \cdot (\bar{\lambda}_1^{(i)} + (\bar{\lambda}_2^{(i)} + \bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}) \cdot \bar{v}_1^{(i)} / \bar{v}_2^{(i)}) & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases}$$

$$D_4^{(i)} = (1 + A_1^{(i)} + A_2^{(i)} + A_3^{(i)}) \cdot [(r^{(i)} \cdot h_i - 1) \cdot e^{r^{(i)} \cdot h_i} + 1] / (r^{(i)})^2,$$

$$G_3^{(i)} = [C_1^{(i)} + C_2^{(i)} \cdot (1 + \bar{\lambda}_2^{(i)} / (\bar{\mu}_1^{(i)} + \bar{\mu}_2^{(i)}))] / A_4^{(i)}, \quad G_4^{(i)} = [\bar{v}_2^{(i)} \cdot A_2^{(i)} + (\bar{v}_2^{(i)} - \bar{v}_1^{(i)}) \cdot A_3^{(i)}] \cdot e^{r^{(i)} \cdot h_i} / \bar{\mu}_2^{(i)},$$

$$C_0^{(i)} = \begin{cases} 1 / (D_1^{(i)} + D_2^{(i)} + D_3^{(i)}) & \text{if } \bar{v}_1^{(i)} > \bar{v}_2^{(i)} \\ 1 / (D_1^{(i)} + G_3^{(i)} + G_4^{(i)}) & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases}.$$

For  $r^{(i)} \neq 0$  and  $\bar{v}_1^{(i)} \neq \bar{v}_2^{(i)}$  based on the above expressions we determine the following performance measures of isolated  $DP_i$ :

- The average throughput  $\bar{\eta}_{v_2}^{(i)}$  of data processing is:

$$\bar{\eta}_{v_2}^{(i)} = \begin{cases} C_0^{(i)} \cdot \bar{v}_2^{(i)} \cdot (1 + A_3^{(i)}) \cdot (e^{r^{(i)} \cdot h_i} - 1) / r^{(i)} & \text{if } \bar{v}_1^{(i)} > v_2^{(i)} \\ C_0^{(i)} \cdot [(\bar{v}_2^{(i)} \cdot (1 + A_3^{(i)}) \cdot (e^{r^{(i)} \cdot h_i} - 1) / r^{(i)}) + \bar{v}_1^{(i)} \cdot C_1^{(i)} / A_4^{(i)}] & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases}; \quad (18)$$

- The average level  $\bar{x}_i = [x_i^-, x_i^+]$  of stock in buffer  $b_i$  is given by the expression:

$$\bar{x}_i = \begin{cases} C_0^{(i)} \cdot (h_i \cdot D_3^{(i)} + D_4^{(i)}) & \text{if } \bar{v}_1^{(i)} > \bar{v}_2^{(i)} \\ C_0^{(i)} \cdot (h_i \cdot G_4^{(i)} + D_4^{(i)}) & \text{if } \bar{v}_1^{(i)} < \bar{v}_2^{(i)} \end{cases}; \quad (19)$$

- The  $Sv_{i+1}$  starvation probability  $\rho_{ST_2}^{(i)} = [\rho_{ST_2}^{-(i)}, \rho_{ST_2}^{+(i)}]$  of dipole  $DP_i$  is:

$$\rho_{ST_2}^{(i)} = \sum_{s_1=0,1} \sum_{s_2=0,1} \rho_{s_1,s_2}^{(i)}(0);$$

- The  $Sv_i$  blocking probability  $\rho_{BL_1}^{(i)} = [\rho_{BL_1}^{-(i)}, \rho_{BL_1}^{+(i)}]$  of dipole  $DP_i$  is:

$$\rho_{BL_1}^{(i)} = \sum_{s_1=0,1} \sum_{s_2=0,1} \rho_{s_1,s_2}^{(i)}(h_i);$$

- The  $Sv_{i+1}$  average temporal redundancy  $\bar{\tau}_i = [\bar{\tau}_i^-, \bar{\tau}_i^+]$  of dipole  $DP_i$  is:  $\bar{\tau}_i = \bar{x}_i / \bar{\eta}_{v_2}^{(i)}$ .

#### 4.4. Numerical case study for performances analysis of a dipole

Next we will present a case study to show the use of the approach presented in this paper. The numerical analysis of some performance measures is based on the use of the  $HCTMC 2_i$  model, using the knowledge of the experts in the field [10]. As example for the fuzzy rates  $\tilde{\lambda}_j$  of discrete transitions and speeds  $\tilde{v}_i$  of continuous transitions, we establish the following TNFs values:  $\tilde{a} = (a, \delta, \beta)$ , on the other hand, a parametric  $\alpha$ -cut fuzzy number  $\tilde{a} = [a^-(\alpha), a^+(\alpha)]$ ,  $\tilde{a} \in \{\tilde{\lambda}_j, \tilde{v}_i\}$  can be represented as:  $a^-(\alpha) = a - (1 - \alpha)\delta$  and  $a^+(\alpha) = a + (1 - \alpha)\beta$ .

The transition diagrams of the accessibility graph ( $AcG1_i$ ) with symbolic marking [4, 5] and of the fuzzy Markov continuous time chain ( $FCMTC1_i$ ), which describes the behavior of the discrete part of  $FGSPN_{Sv_i}$  model in Figure 7, are shown respectively in Figure 10a and Figure 10b.

The steady state fully fuzzy linear system of Chapman-Kolmogorov equations, that describes the Markov process behavior  $FCMTC1_i$  of discrete part  $FGSPN_{Sv_i}$ , is:

$$\tilde{\lambda}_{i,1} \cdot \tilde{\pi}_{i,0} - \tilde{\lambda}_{i,5} \cdot \tilde{\sigma}_{i,7} \cdot \tilde{\pi}_{i,2} - \tilde{\lambda}_{i,8} \cdot \tilde{\pi}_{i,3} = 0; \quad \tilde{\lambda}_{i,4} \cdot \tilde{\pi}_{i,1} - \tilde{\lambda}_{i,1} \cdot \tilde{\sigma}_{i,2} \cdot \tilde{\pi}_{i,0} = 0; \quad (20)$$

$$\tilde{\lambda}_{i,5} \cdot \tilde{\pi}_{i,2} - \tilde{\lambda}_{i,1} \cdot \tilde{\sigma}_{i,3} \cdot \tilde{\pi}_{i,0} - \tilde{\lambda}_{i,4} \cdot \tilde{\pi}_{i,1} = 0; \quad \tilde{\pi}_{i,0} + \tilde{\pi}_{i,1} + \tilde{\pi}_{i,2} + \tilde{\pi}_{i,3} = 1; \quad \tilde{\sigma}_{i,2} + \tilde{\sigma}_{i,3} = 1; \quad \tilde{\sigma}_{i,6} + \tilde{\sigma}_{i,7} = 1.$$

where:  $0 < \tilde{\sigma}_{i,2} = \tilde{w}_{i,2} / (\tilde{w}_{i,2} + \tilde{w}_{i,3}) < 1, \quad 0 < \tilde{\sigma}_{i,7} = \tilde{w}_{i,7} / (\tilde{w}_{i,6} + \tilde{w}_{i,7}) < 1.$

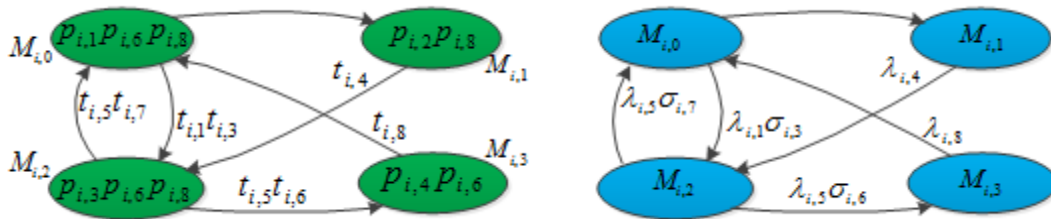


Figure 10. The states transition diagrams of the discrete part  $FSHPN1_i$ : a)  $AcG1_i$ ; b)  $FCMTC1_i$ .

To illustrate this approach, we analyzed the  $FSHPN1_i$  model of dipole  $DP_i$  with the following value of the fuzzy parameters for  $Sv_i$  and  $Sv_{i+1}$ :



$$\begin{aligned} \tilde{\lambda}_{i,1} &= (0.3, 0.1, 0.1), \quad \tilde{\lambda}_{i,4} = (0.8, 0.3, 0.2), \quad \tilde{\lambda}_{i,5} = (0.5, 0.3, 0.1), \quad \tilde{\lambda}_{i,8} = (0.9, 0.2, 0.2), \quad \tilde{\sigma}_{i,2} = (0.8, 0.0, 0.0), \\ \tilde{\sigma}_{i,3} &= (0.2, 0.0, 0.0), \quad \tilde{\sigma}_{i,7} = (0.2, 0.0, 0.0), \quad \tilde{v}_{i,1} = (5.0, 2.0, 3.0), \quad \tilde{v}_{i,2} = (3.0, 1.0, 2.0); \\ \tilde{\lambda}_{i+1,1} &= (0.2, 0.1, 0.3), \quad \tilde{\lambda}_{i+1,4} = (0.6, 0.2, 0.2), \quad \tilde{\lambda}_{i+1,5} = (0.6, 0.2, 0.3), \quad \tilde{\lambda}_{i+1,8} = (0.8, 0.3, 0.2), \quad \tilde{\sigma}_{i+1,2} = (0.7, 0.0, 0.0), \\ \tilde{\sigma}_{i+1,3} &= (0.3, 0.0, 0.0), \quad \tilde{\sigma}_{i+1,7} = (0.3, 0.0, 0.0), \quad \tilde{v}_{i+1,1} = (4.0, 2.0, 1.0), \quad \tilde{v}_{i+1,2} = (2.0, 1.0, 2.0). \end{aligned}$$

For these fuzzy parameters, the solutions  $\tilde{\pi}_{i,j} = [\tilde{\pi}_{i,j}^-(\alpha), \tilde{\pi}_{i,j}^+(\alpha)]$ ,  $i = 1, N - 1$  and  $j = i + 1$  of Chapman-Kolmogorov equation system (1) in  $\alpha$ -cut parametric fuzzy numbers are obtained based on the methods presented in [10] and using respective MatLab Tool programs. These solutions, applicable to  $0.34 \leq \alpha \leq 1$ , are the following:

$$\begin{aligned} \tilde{\pi}_{i,0} &= [(0.46153846 - (1 - \alpha) \cdot 0.541001972), (0.46153846 + (1 - \alpha) \cdot 0.489067061)], \\ \tilde{\pi}_{i,1} &= [(0.138461538 - (1 - \alpha) \cdot 0.111531361), (0.138461538 + (1 - \alpha) \cdot 0.112681657)], \\ \tilde{\pi}_{i,2} &= [(0.276923076 - (1 - \alpha) \cdot 0.162139645), (0.276923076 + (1 - \alpha) \cdot 0.238055621)], \\ \tilde{\pi}_{i,3} &= [(0.123076923 - (1 - \alpha) \cdot 0.185327022), (0.123076923 + (1 - \alpha) \cdot 0.160195661)]; \\ \tilde{\pi}_{i+1,0} &= [(0.574162671 - (1 - \alpha) \cdot 0.534709159), (0.574162671 + (1 - \alpha) \cdot 0.534709159)], (21) \\ \tilde{\pi}_{i+1,1} &= [(0.133971292 - (1 - \alpha) \cdot 0.096212717), (0.133971292 + (1 - \alpha) \cdot 0.078611624)], \\ \tilde{\pi}_{i+1,2} &= [(0.191387568 - (1 - \alpha) \cdot 0.138088202), (0.191387568 + (1 - \alpha) \cdot 0.081045858)], \\ \tilde{\pi}_{i+1,3} &= [(0.100478409 - (1 - \alpha) \cdot 0.145546670), (0.100478409 + (1 - \alpha) \cdot 0.305633339)]; \\ \tilde{v}_{i,1} &= [(5.0 - (1 - \alpha) \cdot 2.0), (5.0 + (1 - \alpha) \cdot 3.0)]; \quad \tilde{v}_{i,2} = [(3.0 - (1 - \alpha) \cdot 1.0), (3.0 + (1 - \alpha) \cdot 2.0)], \\ \tilde{v}_{i+1,1} &= [(4.0 - (1 - \alpha) \cdot 2.0), (4.0 + (1 - \alpha) \cdot 1.0)]; \quad \tilde{v}_{i+1,2} = [(2.0 - (1 - \alpha) \cdot 1.0), (2.0 + (1 - \alpha) \cdot 2.0)]. \end{aligned}$$

Thus, using the relations (1), (2) and (3) the respective fuzzy parameters of the aggregate model  $FSHPN2_i$  are calculated giving the following expressions:

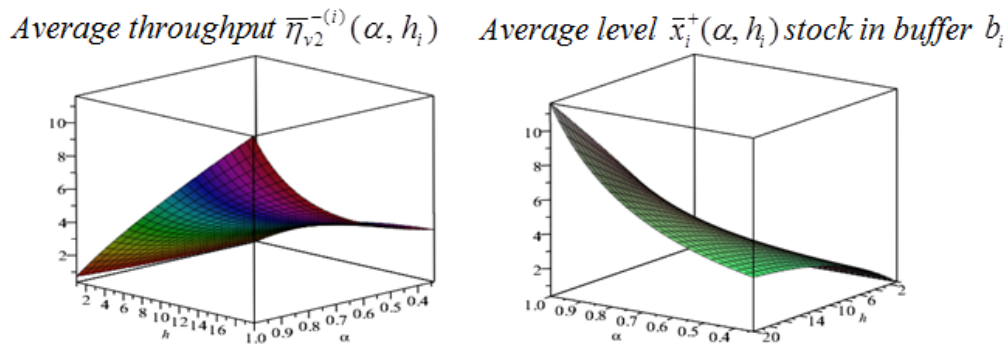
$$\begin{aligned} \bar{v}_1^{(i)} &= (\tilde{v}_{i,1} \tilde{\pi}_{i,0} + \tilde{v}_{i,2} \tilde{\pi}_{i,1}) / (\tilde{\pi}_{i,0} + \tilde{\pi}_{i,1}), \quad \bar{v}_2^{(i)} = (\tilde{v}_{i+1,1} \tilde{\pi}_{i+1,0} + \tilde{v}_{i+1,2} \tilde{\pi}_{i+1,1}) / (\tilde{\pi}_{i+1,0} + \tilde{\pi}_{i+1,1}), \\ \bar{\lambda}_1^{(i)} &= (\tilde{\lambda}_{i,1} \tilde{\sigma}_{i,3} \tilde{\pi}_{i,0} + \tilde{\lambda}_{i,4} \tilde{\pi}_{i,1}) / (\tilde{\pi}_{i,0} + \tilde{\pi}_{i,1}), \quad \bar{\lambda}_2^{(i)} = (\tilde{\lambda}_{i+1,1} \tilde{\sigma}_{i+1,3} \tilde{\pi}_{i+1,0} + \tilde{\lambda}_{i+1,4} \tilde{\pi}_{i+1,1}) / (\tilde{\pi}_{i+1,0} + \tilde{\pi}_{i+1,1}), (22) \\ \bar{\mu}_1^{(i)} &= (\tilde{\lambda}_{i,5} \tilde{\sigma}_{i,7} \tilde{\pi}_{i,2} + \tilde{\lambda}_{i,8} \tilde{\pi}_{i,3}) / (\tilde{\pi}_{i,2} + \tilde{\pi}_{i,3}), \quad \bar{\mu}_2^{(i)} = (\tilde{\lambda}_{i+1,5} \tilde{\sigma}_{i+1,7} \tilde{\pi}_{i+1,2} + \tilde{\lambda}_{i+1,8} \tilde{\pi}_{i+1,3}) / (\tilde{\pi}_{i+1,2} + \tilde{\pi}_{i+1,3}). \end{aligned}$$

Based on the expressions (22) the fuzzy parameters of the  $FSHPN2_i$  model are:

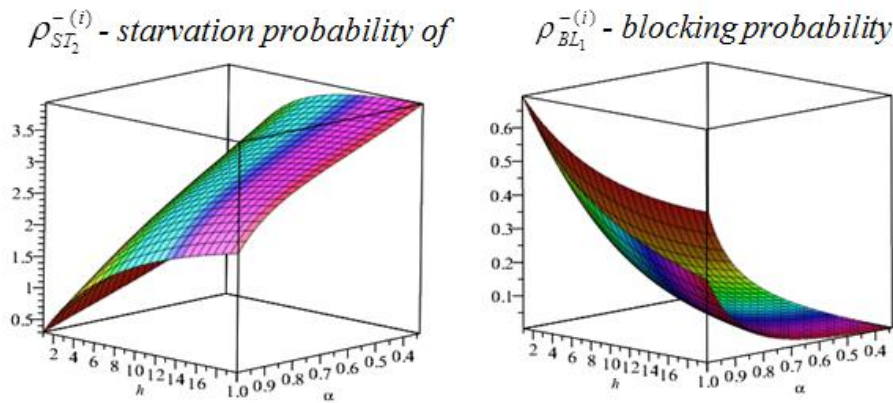
$$\begin{aligned} \bar{v}_1^{(i)} &= [(4.538487178 - (1 - \alpha) \cdot 3.512342402), (4.538487178 + (1 - \alpha) \cdot 2.626924617)]; \\ \bar{v}_2^{(i)} &= [(3.621621621 - (1 - \alpha) \cdot 0.237395468), (3.621621621 + (1 - \alpha) \cdot 3.160870372)]; \\ \bar{\lambda}_1^{(i)} &= [(0.259415519 - (1 - \alpha) \cdot 0.230759743), (0.230759743 + (1 - \alpha) \cdot 1.510542438)]; (23) \\ \bar{\lambda}_2^{(i)} &= [(0.145945946 - (1 - \alpha) \cdot 0.133378811), (0.145945946 + (1 - \alpha) \cdot 0.123051021)]; \\ \bar{\mu}_1^{(i)} &= [(0.346153847 - (1 - \alpha) \cdot 0.063555446), (0.346153847 + (1 - \alpha) \cdot 0.063555446)]. \end{aligned}$$

Using these TFN values we can respectively determine the specified performance measures of the dipole  $DP_i$ . For instance, Figure 11 and Figure 12 show the effects of the  $\alpha$ -cut and buffer capacity  $h_i$  on the average throughput  $\bar{\eta}_{v2}^{+(i)}(\alpha, h_i)$ , average level stock

$\bar{x}_i^-(\alpha, h_i)$ , starvation probability  $\rho_{ST_2}^{+(i)}(\alpha, h_i)$  of  $S_{v_{i+1}}$  and the blocking probability  $\rho_{BL_1}^{+(i)}(\alpha, h_i)$  of  $S_{v_i}$ .



**Figure 11.** The average throughput  $\bar{\eta}_{v2}^{+}(i)(\alpha, h_i)$  and the average level stock  $\bar{x}_i^{-}(\alpha, h_i)$  in buffer  $b_i$  of dipole  $DP_i$ .



**Figure 12.** The starvation probability  $\rho_{ST_2}^{+}(i)(\alpha, h_i)$  of  $Sv_{i+1}$  and the blocking probability  $\rho_{BL_1}^{+}(i)(\alpha, h_i)$  of  $Sv_i$ .

## 5. Performance evaluation of buffer pipe-line FSHPN models

### 5.1. Update parameters of dipoles

Value evaluation of the  $DP_i$  aggregate parameters interaction depends with the adjacent dipoles. In order to take into account the behaviour interdependence of the adjacent dipoles with  $DP_i$  it is necessary to include the *starvation states* of server  $SA_i$  and the *blocking states* of the server  $SD_i$  in their passive states [13, 14]. That is why it is necessary that the rates of change of states and the processing speed of the respective servers have to be adjusted. The rest of this subsection refers to the determination of the quantitative parameters of these aggregate passive states. Recall that  $SA_i$  of the considered dipole  $DP_i$  corresponds to the upstream server  $SD_{i-1}$  of the dipole  $DP_i$ , and  $SD_i$  corresponds to the downstream server  $SA_{i+1}$  of the dipole  $DP_i$ . First, we determine the aggregate average parameters for the server  $SA_i$ , after which the server  $SD_i$  parameters will be identified in a similar way. For this we use *upstream* and *downstream* dipole information, assuming we have all the necessary information. Then we get the aggregate rates of change of the respective states for  $SA_i$  by linking them together. So, for  $SA_i$ , the behavior aggregate state space contain three aggregate states: state where  $SA_i$  is *active* and producing fluid flow; *passiv* state where  $SA_i$  is not producing fluid flow because of a breakdown of the underlying server  $Sv_i$  (*down*), and state where  $SA_i$  is not producing fluid flow because it has no input (*starved*). These three aggregate states are formally defined as follows: 1) the  $SA_i$  is in *active state*

when  $Sv_i$  is active, and either buffer  $b_{i-1}$  is not empty and  $SA_{i-1}$  is active; 2) the  $SA_i$  is in passive state when  $Sv_i$  is passive;  $SA_i$  is starved when  $Sv_i$  is active,  $b_{i-1}$  is empty, and  $SA_{i-1}$  is in passive state or starved. For  $SA_i$  (resp.  $SD_i$ ) of the dipole  $DP_i$ , we determine the aggregate rate  $\bar{\lambda}_A^{(i)}$  (resp.  $\bar{\lambda}_D^{(i)}$ ) transition from the active state to the passive state and the aggregate rate  $\bar{\mu}_A^{(i)}$  (resp.  $\bar{\mu}_D^{(i)}$ ) transition from the passive state to the active state. Also, we determine the average aggregate speed  $\bar{v}_A^{(i)}$  (resp.  $\bar{v}_D^{(i)}$ ) of data processing by the respective server in the active state, which is not necessarily equal to the specified maximum speed, because it may be the case that the server sometimes has to adjust its speed because of a slower upstream server and an empty buffer. In this case, we use the following information from the upstream dipole (or downstream dipole):

- $\bar{\pi}_{j,j'}^{(i-1)}$  (resp.  $\bar{\pi}_{j,j'}^{(i+1)}$ ) is the probability that  $SA_{i-1}$  (resp.  $SA_{i+1}$ ) is in the  $j$  state and  $SD_{i-1}$  (resp.  $SD_{i+1}$ ) is in the  $j'$  state, ( $j, j' \in \{1, 0\}$ , where, 1 = "active" and 0 = "passive");
- $\rho_{j,j'}^{(i-1)}(0)$  (resp.  $\rho_{j,j'}^{(i+1)}(h_{i+1})$ ) is the probability that  $SA_{i-1}$  (resp.  $SA_{i+1}$ ) is in the  $j$  state, and  $SD_{i-1}$  (resp.  $SD_{i+1}$ ) is in the  $j'$  state and  $b_{i-1}$  is empty (resp.  $b_{i+1}$  is full);
- $f_{j,j'}^{(i-1)}(0)$  (resp.  $f_{j,j'}^{(i+1)}(h_{i+1})$ ) is the probability density of the content of the downstream dipole buffer when  $SA_{i-1}$  (resp.  $SA_{i+1}$ ) is in the  $j$  state and  $SD_{i-1}$  (resp.  $SD_{i+1}$ ) is in the  $j'$  state and  $b_{i-1}$  is empty (resp.  $b_{i+1}$  is full);
- $\bar{\lambda}_A^{(i-1)}(j, j')$  (resp.  $\bar{\lambda}_D^{(i+1)}(j, j')$ ) is the rate transition from  $j$  to  $j'$  states of  $SA_{i-1}$  (resp.  $SD_{i+1}$ ).

We can distinguish three different ways in which the server  $SA_i$  can move from the active aggregate state to the passive one. The first mode is caused by an active change current state of  $SA_i$ , and the second and third mode is caused by upstream server  $SA_{i-1}$  of the dipole  $DP_{i-1}$  and an empty buffer  $b_{i-1}$ . In this case the average aggregate rate  $\hat{\lambda}_A^{(i)}$  of transition from the active to the passive state is made up of three parts [13, 14] given by the expression (24):

$$\hat{\lambda}_A^{(i)} = \hat{\lambda}_{A,1}^{(i)} + \hat{\lambda}_{A,2}^{(i)} + \hat{\lambda}_{A,3}^{(i)}. \tag{24}$$

The first part corresponds to a change in active current state of  $SA_i$ , which occurs with the rate  $\hat{\lambda}_{A,1}^{(i)} = \bar{\lambda}_1^{(i)}$ . Secondly,  $SA_i$  cannot be active when upstream  $SA_{i-1}$  is passive and the buffer  $b_{i-1}$  is empty. The number of jumps of this type of events per unit of time is obtained from the upstream dipole  $DP_{i-1}$  as determined by the value  $f_{0,1}^{(i-1)}(0)$ . This value is multiplied by the speed  $\bar{v}_2^{(i-1)}$  at which the  $SD_{i-1}$  consumes fluid from the buffer  $b_{i-1}$ . This fact is conditioned by the aggregate probability  $\hat{\pi}_A^{(i-1)}$  that the upstream server  $SD_{i-1}$  is capable of producing, that is  $\hat{\pi}_A^{(i-1)} = \bar{\pi}_{1,1}^{(i-1)} + \bar{\pi}_{0,1}^{(i-1)} - \rho_{0,1}^{(i-1)}(0)$ , based on which we obtain:

$$\hat{\lambda}_{A,2}^{(i)} = f_{0,1}^{(i-1)}(0) \cdot \bar{v}_2^{(i-1)} / \hat{\pi}_A^{(i-1)}.$$

The last type of jump from the active (operational) state to the passive state occurs when the upstream buffer  $b_{i-1}$  is empty and the server  $SA_{i-1}$  is active. This case occur with

probability  $\rho_{1,1}^{(i-1)}(0)$ . If in this case the upstream server  $SA_{i-1}$  goes into the *passive* (non-operational) state, which happens with the average rate  $\bar{\lambda}_2^{(i-1)}$ , the considered server  $SA_i$  also goes into the *passive* state. This fact is conditioned by the probability that the upstream server  $SD_{i-1}$  can produce, that's how we get:

$$\hat{\lambda}_{A,3}^{(i)} = \rho_{0,1}^{(i-1)}(0) \cdot \bar{\lambda}_2^{(i-1)} / \hat{\pi}_A^{(i-1)}.$$

Similarly, to determine the *average delay*  $1/\hat{\mu}_A^{(i)}$  of stay in the passive state of  $SD_{i-1}$ , we use the fact that a jump from the active to the passive state is determined by the probability  $\bar{\lambda}_{A,1}^{(i)} / \hat{\lambda}_A^{(i)}$  of a change from the active state to the passive state and the probability  $(\hat{\lambda}_{A,2}^{(i)} + \hat{\lambda}_{A,3}^{(i)}) / \hat{\lambda}_A^{(i)}$  of starvation. In the first case, we have an average wait time  $1/\hat{\mu}_A^{(i)}$  in passive state and in the second case we get the average wait time  $1/\bar{\lambda}_1^{(i-1)}$  in passive state of the upstream server  $SD_{i-1}$ . Thus, the estimated *average delay*  $1/\hat{\mu}_A^{(i)}$  presence in the passive state of the upstream server  $SD_{i-1}$  is a weighted average for these two probabilities and it is given by the expression (25):

$$\frac{1}{\hat{\mu}_A^{(i)}} = \frac{\hat{\lambda}_{A,1}^{(i)}}{\hat{\lambda}_A^{(i)}} \cdot \frac{1}{\bar{\mu}_1^{(i)}} + \frac{\hat{\lambda}_{A,2}^{(i)} + \hat{\lambda}_{A,3}^{(i)}}{\hat{\lambda}_A^{(i)}} \cdot \frac{1}{\bar{\lambda}_1^{(i-1)}}. \quad (25)$$

The average aggregate speed  $\hat{v}_A^{(i)}$  of data processing by  $SA_i$  in the active state is a weighted average of two possible speeds, being conditioned by the fact that  $SD_{i-1}$  produces (and thus  $SA_i$  is active) and the fact that the upstream buffer  $b_{i-1}$  is empty with probability  $\rho_{1,1}^{(i-1)}(0) / \hat{\pi}_{A,2}^{(i-1)}$ . In this case, the server  $SA_i$  must adjust speed to speed of  $SA_{i-1}$ , that produces speed  $\hat{v}_A^{(i-1)}$ . Also, with the complementary probability, the upstream server is capable of producing at maximum speed  $\bar{v}_i$ . Thus, we obtain  $\hat{v}_A^{(i)}$ :

$$\hat{v}_A^{(i)} = \bar{v}_i - (\bar{v}_i - \hat{v}_A^{(i-1)}) \cdot \rho_{1,1}^{(i-1)}(0) / \hat{\pi}_A^{(i-1)}. \quad (26)$$

Similarly, as for the server  $SA_i$ , we can divide in three parts the aggregate average rate transition from the active state to the passive state of the server  $SD_i$  we obtain  $\hat{\lambda}_D^{(i)}$ :

$$\hat{\lambda}_D^{(i)} = \hat{\lambda}_{D,1}^{(i)} + \hat{\lambda}_{D,2}^{(i)} + \hat{\lambda}_{D,3}^{(i)}. \quad (27)$$

The first part corresponds to a current change of the server  $SD_i$  from the active state to the passive one, which occurs with the rate  $\hat{\lambda}_{D,1}^{(i)} = \bar{\lambda}_2^{(i)}$ , and the second and third part are jumps caused by blocking this server when its downstream buffer  $b_{i+1}$  is *full*.

As the analysis follows along the same SCP, in the same way as for the server  $SA_i$ , there we will present only the expressions of the *jump aggregate rates* between states and of the respective aggregate average speed in this case for  $SD_i$ :

$$\hat{\lambda}_{D,1}^{(i)} = \bar{\lambda}_2^{(i)}, \quad \hat{\lambda}_{D,2}^{(i)} = f_{1,0}^{(i+1)}(h_{i+1}) \cdot \hat{v}_D^{(i+1)} / \hat{\pi}_D^{(i+1)},$$

$$\hat{\lambda}_{D,3}^{(i)} = \rho_{1,1}^{(i+1)}(h_{i+1}) \cdot \bar{\lambda}_2^{(i+1)}(j, j') / \hat{\pi}_D^{(i+1)},$$

$$\frac{1}{\hat{\mu}_D^{(i)}} = \frac{\hat{\lambda}_{D,1}^{(i)}}{\hat{\lambda}_D^{(i)}} \cdot \frac{1}{\bar{\mu}_2^{i+1}} + \frac{\hat{\lambda}_{D,2}^{(i)} + \hat{\lambda}_{D,3}^{(i)}}{\hat{\lambda}_D^{(i)}} \cdot \frac{1}{\bar{\lambda}_2^{(i+1)}}, \tag{28}$$

$$\hat{v}_D^{(i)} = \bar{v}_{i+1} - (\bar{v}_{i+1} - \hat{v}_D^{(i+1)}) \cdot \rho_{1,1}^{(i+1)}(h_{i+1}) / \hat{\pi}_D^{(i+1)}, \text{ where:}$$

$$\hat{\pi}_D^{(i+1)} = \pi_{1,1}^{(i+1)} + \pi_{1,0}^{(i+1)} - \rho_{1,0}^{(i+1)}(h_{i+1}).$$

The aggregate quantitative parameters of each particular dipole thus determined are then used to evaluate the performance measures of the entire analyzed *FSHPN* model. For this, we present an algorithm in a fixed point iteration scheme for the analysis of a *FSHPN*, which is similar to the one in the papers [13, 14].

**5.2. The fixed point iterative algorithm for the performance evaluation of *FSHPNs***

Based on the use of the method described in sections 3 and 4 to properly obtain the average throughput of and the average buffer level  $\bar{x}_i$  estimation, modeled by SHPN, we apply the following **fixed point iterative algorithm**:

**Start. Step 0. Initialization:** We initially assume that each dipole  $DP_i, i=1, \dots, N-1$ , is not affected by starvation or blocking due to upstream or downstream dipoles. The  $\tilde{\lambda}_{i,j}(M)$  and  $\tilde{v}_{i,l}$  of  $Sv_i, i=1, \dots, N$  are set accordingly. Also, we set the initial value of the iteration counter  $k=0$ .

**Step 1.** Using the PIPE 4.3 ToolKit [20], we calculate the stationary probabilities  $\pi_j$  of stay in the accessible marking  $M_j$  of the initial GSPN discrete part of the analyzed *FSHPN* model. For each dipole  $DP_i, i=1, 2, \dots, N-1$  in the base of  $\pi_j$  we calculate the average rates  $\bar{\lambda}_1^{(i)}, \bar{\lambda}_2^{(i)}, \bar{\mu}_1^{(i)}, \bar{\mu}_2^{(i)}$  of change of the current states and the average data processing speeds  $\bar{v}_1^{(i)}, \bar{v}_2^{(i)}$ . Depending on the relations between the values  $\bar{v}_1^{(i)}$  and  $\bar{v}_2^{(i)}$ , based on the expressions (9) and (16) - (20) we calculate the values of the respective aggregate parameters of  $DP_i$ :

$$\gamma_k^i = \bar{\eta}_{v2}^{(i)} \text{ and } \hat{\lambda}_A^{(i)}, \hat{\lambda}_D^{(i)}, \hat{\mu}_A^{(i)}, \hat{\mu}_D^{(i)}, \hat{v}_A^{(i)}, \hat{v}_D^{(i)}.$$

**Step 2.** For each dipole  $DP_i, i=1, 2, \dots, N-1$  we perform the following operations:

2.1) We increase the iteration counter by one, i.e.  $k = k + 1$ ;

2.2) We set  $\bar{\lambda}_1^{(i)} = \hat{\lambda}_A^{(i)}, \bar{\lambda}_2^{(i)} = \hat{\lambda}_D^{(i)}, \bar{\mu}_1^{(i)} = \hat{\mu}_A^{(i)}, \bar{\mu}_2^{(i+1)} = \hat{\mu}_D^{(i)}, \bar{v}_1^{(i)} = \hat{v}_A^{(i)}$  and  $\bar{v}_2^{(i)} = \hat{v}_D^{(i)}$ ;

2.3) Depending on the relations between the values  $\bar{v}_1^{(i)}$  and  $\bar{v}_2^{(i)}$  for these new parameters values of dipole  $DP_i$ , we calculate the stationary probabilities  $\pi_{j,j'}^{(i)}$ , the boundary probabilities  $\rho_{j,j'}^{(i)}(0)$  and  $\rho_{j,j'}^{(i)}(h_i)$ , the probability density function  $f_{j,j'}^{(i)}(x_i)$  of the buffer  $b_i$  content and the average throughput  $\gamma_k^i = \bar{\eta}_{v2}^{(i)}$  estimated at the  $k$  iteration, based on which we calculate:  $\hat{\lambda}_A^{(i)}, \hat{\lambda}_D^{(i)}, \hat{\mu}_A^{(i)}, \hat{\mu}_D^{(i)}, \hat{v}_A^{(i)}, \hat{v}_D^{(i)}$ ;

2.4) If,  $i < N - 1$  we adjust the aggregate rates  $\hat{\lambda}_A^{(i+1)}$  and aggregate speeds  $\hat{v}_A^{(i+1)}$  of the down-stream dipole  $DP_{i+1}$ , using the respective expressions in subsection IV;

2.5) If,  $i > 1$  we adjust the parameters  $\hat{\lambda}_D^{(i-1)}$  and speed  $\bar{v}_D^{(i-1)}$  of the upstream dipole  $DP_{i+1}$ , using the respective expressions in subsection IV.

**Step 3. Convergence check.** Repeat Step 2 until we get convergence for all estimats productivities  $\gamma_k^{(i)}$  of *FSHPN* model. That if  $\min_{\forall i} (|\gamma_k^{(i)} - \gamma_{k-1}^{(i)}|) / \gamma_k^{(i)} > \varepsilon$  for a very small speci-fied precision value  $\varepsilon$ , we get to do another iteration; otherwise we go to *step 4*.

**Step 4.** For each dipole  $DP_i$ ,  $i = 1, 2, \dots, N - 1$ , based on the quantitative parameters being obtained at the last iteration, we calculate, according to the expression (8), the average fluid level  $\bar{x}_i$  in the buffer  $b_i$ , then we **stop** the calculations and show the specified results. **Stop.**

To validate the developed method in this paper, a large set of tests was used in which the *average throughput* of the *FSHPN1* model and the average content of the respective buffers were approximated with those obtained by simulation methods with discrete events using NS-2 simulator [23]. It was found that the values  $\gamma_k^{(i)}$  of all dipoles converge to the same values for which the maximum error of the developed method while estimating the average *throughput* of the system is less than 2.60%, corresponding to the system with six pipe-line dipoles  $DP_i$ , and  $1 \leq h_i \leq 10$  of buffers. Concerning the average buffer level  $\bar{x}_i$  estimation, the maximum error is below 3%. The accuracy of the insensitive method appears to the number of buffers and the number of timed transitions per servers  $Sv_i$ .

## Conclusions

In this paper we present an approximated main-value analysis method of stochastic hybrid Petri nets (SHPN) models with fuzzy parameters, called FSHPN, for performance evaluation of continuous data transmission CSN virtual channels. The method is based on the main-value analytical solution of one buffer finite FSHPN sub-models, also referred dipoles as building blocks. We develop an iterative fixed point algorithm to accurately estimate performance measures of buffer pipe-line FSHPN models such as throughput and mean buffer contents. The approach is general and it can be applied to several pipe-line dipoles without mining the accuracy in the estimation. The accuracy of the proposed method has been validated by numerical simulation experiments. The use of a large set of numerical tests shows that performance evaluation result of FSHPN models by using the proposed method is quite close to those obtained by simulation, and proposed method is less time-consuming. Further research can also include the adaption of this method to *SHPN* models with intuitionistic fuzzy parameters and/or more buffers are incident forward/backward to the continuous transitions, i. e.  $|u_i^*| > 1$  and/or  $|u_i^*| > 1$ . Also, this approach can be further generalized to systems performance analysis from domains with dynamically reconfigurable behaviour characteristics that are conditioned by events specified at the design stage.

**Conflicts of Interest:** The authors declare no conflict of interest.

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