Improved Power Flow Methods for DC Grids

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***Abstract*—This paper presents a steady-state model and associated power flow equations that can be applied to any dc grid. State-of-art power flow methods and two newly proposed methods are discussed and applied to the proposed steady-state model. A standardized IEEE test feeder is used to benchmark the power flow methods with respect to accuracy, convergence and computational efficiency. It is shown that the two new methods have a superior performance compared to the existing techniques for the steady-state analysis of most common dc grids, providing up to a 93 % increase in computational efficiency for the system that was analyzed in this paper. Therefore, it is demonstrated in this paper that these power flow techniques can be used for the operation, planning, optimization, market simulation, and security assessment of practical dc grids.**

***Index Terms*—DC Grids, Modelling, Operation, Power Flow, Steady-State**

I. Introduction

RESEARCH into dc systems has increased rapidly in industry and academia over the last decade [1]. Renewable energy resources, such as photovoltaic (PV) panels, wind turbines, batteries and electric vehicles (EVs) are playing a vital role in the energy transition from traditional energy sources [2]–[4]. Since all of these technologies have dc voltages in their conversion steps, the implementation of dc grids at low and medium voltage is technically and economically viable [5]–[7]. Moreover, many loads are becoming dc-based such as LED lighting, USB type-C charging, and dc data centers [8]–[10].

Power flow methods determine the steady-state operating conditions of a power system. In general, the main goal of power flow techniques is to determine all the bus voltages, line currents, and power flows of a system, given the injected or consumed power at each node [11], [12]. Power flow analysis is widely used for the operation and planning of electrical power systems, but can also be used for more complex applications such as stability analysis, economic system optimization, flow-based market simulations and N-1 security assessments [12], [13].

Several dc power flow methods are presented in the technical literature. Most commonly, ac and dc power flow solutions are determined iteratively by utilizing analytical methods such as Gauss-Seidel (GS), Newton-Raphson (NR), Backward-Forward (BF) sweep methods, or by incorporating the system's equation into an optimization problem (OP) [12]–[15]. However, a Quadratic Solver (QS) can also be used to find the power flow solution by directly solving the quadratic equations [16].

Because all of the existing methods are derived from the power flow analysis of conventional ac systems, power flow methods developed for dc grids have the potential to provide significant improvements in terms of computational efficiency. Furthermore, no comparison exists of the application of existing methods to dc grids. Thus, this paper applies all these methods to dc grids, proposes two novel techniques, and compares them with respect to accuracy, convergence and computational effort.

Based on the previous discussion, the main contributions of this paper are three-fold. First, a steady-state model that can be used to represent any dc grid is presented, showing how the power flow equations can be derived from this model. Second, the most common existing power flow methods for dc grids are discussed in detail, showing how they can be applied to dc grids. Finally, two novel power flow methods are proposed, demonstrating that they are numerically superior to existing methods in terms of accuracy, convergence, and computational burden.

The remainder of this paper is organized as follows: In Section II, the steady-state model of dc grids is presented. In Section III, the different existing power flow methods are discussed in detail, and two novel methods are presented. In Section IV, the different power flow methods are compared with respect to accuracy, convergence and computational cost, using a realistic dc test system. Lastly, in Section V, the main conclusions are drawn.

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II. DC Grid Steady-State Model

An example of a generalized dc grid is shown in Fig. 1. Any dc grid can be fully described by its n nodes and l lines with m conductors.

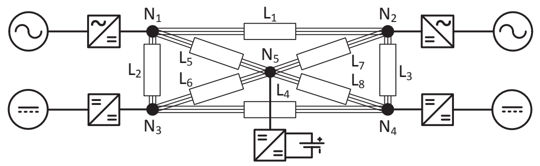

Fig. 1. Example of a dc grid with 5 nodes, 8 lines and 5 converters.

A. Distribution and Transmission Line Model

The lumped element models shown in Fig. 2 are commonly used to model distribution and transmission lines. This lumped approach provides reasonable accuracy when the wavelength of the signals are much longer than the length of the lines [17], as is the case in steady-state conditions.

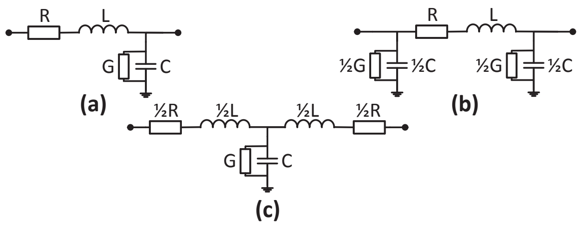

Fig. 2. (a) Gamma, (b) pi, and (c) T lumped element line models.

Conveniently, in a dc system, all the lumped element models can be reduced to the same steady-state model, which consists of a simple resistor, i.e., the inductive and capacitive components can be neglected. Furthermore, the conductance G can also be neglected, since most systems have very low conductance, which is especially true for the typical low and medium voltage dc grids.

B. DC Grid Model

For monopolar systems, the incidence matrix γ describes the interconnection of the nodes in the grid and is given by:

$$\gamma(j,i) = \begin{cases} 1 & \text{if } I_j \text{ is flowing from node } i \\ -1 & \text{if } I_j \text{ is flowing to node } i \end{cases}, \quad (1)$$

where the index i is used to indicate a node, and the index j is used to indicate a line. Consequently, the current I_j is the current flowing in line j.

Unipolar and bipolar dc systems have multiple conductors in each line, which have different potentials and carry different currents. Therefore, the multi-conductor incidence matrix $\mathbf{\Gamma}$ differentiates between conductors and is derived as follows:

$$\mathbf{\Gamma}\left((j-1)\,m+k,(i-1)\,m+k\right) = \boldsymbol{\gamma}\left(j,i\right), \quad (2)$$

where the index k is used to indicate a specific conductor, and one must cycle through all line, node and conductor indices to find the elements of this matrix [18].

If all the resistances of the conductors in the dc systems' lines are put in a diagonal matrix $\boldsymbol{R}$, the currents in the system's lines can be defined as:

$$\boldsymbol{I_L} = \boldsymbol{R}^{-1}\mathbf{\Gamma}\boldsymbol{U_N}, \quad (3)$$

where $\boldsymbol{U_N}$ is the vector containing the voltages at each node, and $\boldsymbol{I_L}$ is the vector containing the currents in each line.

According to Kirchhoff's law, the sum of the currents flowing into each node must equal 0. Therefore, the current flowing from the power electronic converters into each node, defined as $\boldsymbol{I_N}$, must be equal to the current flowing out of that node via the connected lines. Thus:

$$\boldsymbol{I_N} = \mathbf{\Gamma}^{\boldsymbol{T}}\boldsymbol{I_L} = \mathbf{\Gamma}^{\boldsymbol{T}}\boldsymbol{R}^{-1}\mathbf{\Gamma}\boldsymbol{U_N} = \boldsymbol{Y}\boldsymbol{U_N}, \quad (4)$$

where $\boldsymbol{Y}$ is the admittance matrix of the dc system.

C. Power Flow Formulation

When the power in each node is used instead of the injected current, the system's equations become:

$$\boldsymbol{P_N} = \langle \boldsymbol{U_N}, \boldsymbol{Y}\boldsymbol{U_N} \rangle, \quad (5)$$

where $\langle \cdot,\cdot \rangle$ represents the scalar product of two vectors. From this equation, it is clear that the power flow equations are quadratic, and can hence not be explicitly solved for larger networks, requiring numerical techniques. More importantly, the admittance matrix is singular and can therefore not be inverted or factorized, because if only the currents are defined in the system, an infinite number of solutions exist for the node voltages. Therefore, a slack node, i.e., a node with a constant voltage, is required. However, at least one node that establishes a given relationship with a specified voltage will also yield a single solution, thus making the system solvable.

III. DC Grid Power Flow Methods

In this section, the most commonly used methods for solving dc power flow problems are discussed in detail. Two novel power flow methods that arise from the dc system's equations are also presented. All methods presented in this section provide an adequate numerical approximation of the power flow solution, since determining the explicit solution becomes intractable for large systems.

A. Quadratic Solver (QS)

Equation (5) can be expanded as:

$$P_i = U_i \sum_{j=1}^{n} Y_{ij} U_j, \quad (6)$$

where Y_{ij} refers to the element in row i and column j of the admittance matrix $\boldsymbol{Y}$. In matrix form, this equation becomes:

$$\boldsymbol{P_N} = \begin{bmatrix} U_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & U_n \end{bmatrix} \boldsymbol{Y}\boldsymbol{U_N}. \quad (7)$$

To solve these equations directly, often Newton or Quasi-Newton methods are used to find the solution [16]. In this paper the Newton search algorithm is utilized.

B. Optimisation Problem (OP)

The power flow problem can also be adapted into a Quadratically Constrained Quadratic Problem (QCQP) as follows:

$$\min \sum_{i=1}^{n} \epsilon_i^2, \tag{8}$$

$$\text{s.t.} \quad \epsilon_i = P_i - U_i \sum_{j=1}^{n} Y_{ij} U_j. \tag{9}$$

Methods to solve these types of problems include the interior point, augmented Lagrangian, and the Simplex algorithms [19]–[21]. In this paper, an interior point solver is used, to which the Hessian and the Gradient matrices are provided to improve convergence.

Alternatively, semidefinite or second order cone programming can also be used to incorporate the equations into an optimization problem.

C. Gauss-Seidel (GS)

The GS method utilizes a simple fixed-point iteration [12], [22]. It is based on the equations for each individual node voltage, iterating on a node by node basis until the convergence criteria are met. The equations for the voltage at each node, for iteration k, are given by:

$$U_i^{k+1} = \frac{1}{Y_{ii}} \left(\frac{P_i}{U_i^k} - \sum_{j=1}^{i-1} Y_{ij} U_j^{k+1} - \sum_{j=i+1}^{n} Y_{ij} U_j^k \right). \tag{10}$$

In matrix form this equation becomes:

$$U_i^{k+1} = \frac{1}{Y_{ii}} \left(\frac{P_i}{U_i^k} - \boldsymbol{Y_i^*} \boldsymbol{U_N} \right), \tag{11}$$

where $\boldsymbol{Y^*}$ is the admittance matrix where the diagonal entries are removed, and $\boldsymbol{Y_i^*}$ represents the i-th row of this matrix.

In general, the GS method is easy to implement, but the convergence is slow compared to other methods. Therefore, an accelerating factor α is often used to improve convergence [12]. The algorithm is then appended with:

$$U_i^{k+1} = U_i^k + \alpha \left(U_i^{k+1} - U_i^k \right), \tag{12}$$

where usually an α between 1.4 and 1.6 is used.

D. Newton-Raphson (NR)

The NR method and its many variations is the most widely used computational method in industry [12], [23]–[25]. For this method, the mismatch between the specified power and the calculated power can be defined as:

$$\Delta P_{N,i} = P_i - U_i \sum_{j=1}^{n} Y_{ij} U_j. \tag{13}$$

Based on this mismatch equation, the Jacobian matrix $\boldsymbol{J}$ is used to determine the next iteration of the node voltages according to:

$$\boldsymbol{U_N}^{k+1} = \boldsymbol{U_N}^k + \boldsymbol{J}^{-1} \Delta \boldsymbol{P_N}, \tag{14}$$

where:

$$\begin{aligned} \boldsymbol{J} &= \begin{bmatrix} \frac{\partial P_1}{\partial U_1} & \cdots & \frac{\partial P_1}{\partial U_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial U_1} & \cdots & \frac{\partial P_n}{\partial U_n} \end{bmatrix} \\ &= \boldsymbol{Y} \begin{bmatrix} U_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & U_n \end{bmatrix} + \text{diag}(\boldsymbol{Y U_N}), \end{aligned} \tag{15}$$

and diag() defines a diagonal matrix from the elements of a vector.

Since the partial derivatives are taken into account and the power flow equations are quadratic, the NR converges relatively fast. However, every iteration requires a refactorization of the Jacobian leading to increased computational effort per iteration, although strategies could be used to reduce this computational burden as is done for ac power flow techniques [12].

E. Backward-Forward (BF)

Another method that has been successfully implemented for radial or weakly meshed dc grids is the BF sweep method [26]–[28], where at every iteration, backward and forward sweeps are carried out. For the backward sweep, the nodes voltages are considered constant, and therefore the current from each converter can be represented as:

$$I_i^k = \frac{P_i}{U_i^k}. \tag{16}$$

Next, the algorithm iterates through all the lines from downstream to upstream, where for every line j connecting node a (downstream) to node b (upstream), the current in line j and the current flowing into node b can be found as follows:

$$I_j^k = I_a^k, \tag{17}$$

$$I_b^k = I_b^k + I_a^k. \tag{18}$$

Consequently, the node current I_b is the sum of the currents in downstream lines, and the current in every line is the cumulative current in its downstream node. For the forward sweep, the line currents are considered constant and the node voltages are calculated. The algorithm iterates again through all the lines, but now from upstream to downstream, with the node voltages given by:

$$U_a^{k+1} = U_b^{k+1} - I_j^k R_j. \tag{19}$$

The main advantages of the BF method are its simplicity and convergence. However, a clear downstream-upstream hierarchy of the lines in the system is required. Moreover, the method only converges satisfactorily for radial or weakly meshed dc grids.

F. Direct Matrix-Current Approximation (DM-CA)

Here, a novel power flow method is presented that combines the strengths of the NR, BF, and interior point methods to solve the quadratic power flow problem. For every iteration, the constant power loads are linearized as a constant current load, utilizing the node voltages from the previous iteration. The resulting system is linear and the resulting node voltages can be solved explicitly.

It was mentioned before that for the admittance matrix to be invertible, one or more of the voltages in the system must be referenced to a pre-determined voltage. If one or more of the nodes in the system are a slack node (have a constant voltage), the currents in the lines are given by:

$$\boldsymbol{I_L} = \boldsymbol{R^{-1}}\check{\boldsymbol{\Gamma}}\check{\boldsymbol{U}}_{\boldsymbol{N}} + \boldsymbol{R^{-1}}\hat{\boldsymbol{\Gamma}}\hat{\boldsymbol{U}}_{\boldsymbol{N}}, \tag{20}$$

where $\check{\boldsymbol{U}}_{\boldsymbol{N}}$ contains the unknown node voltages; $\hat{\boldsymbol{U}}_{\boldsymbol{N}}$ contains the known node voltages; $\check{\boldsymbol{\Gamma}}$ contains the columns of the incidence matrix referring to the unknown node voltages; and $\hat{\boldsymbol{\Gamma}}$ contains the columns of the incidence matrix referring to the known node voltages. Therefore, the currents flowing from the converters into nodes where voltage is not defined, must be equal to:

$$\check{\boldsymbol{I}}_{\boldsymbol{N}} = \check{\boldsymbol{\Gamma}}^{\boldsymbol{T}}\boldsymbol{R^{-1}}\check{\boldsymbol{\Gamma}}\check{\boldsymbol{U}}_{\boldsymbol{N}} + \check{\boldsymbol{\Gamma}}^{\boldsymbol{T}}\boldsymbol{R^{-1}}\hat{\boldsymbol{\Gamma}}\hat{\boldsymbol{U}}_{\boldsymbol{N}} = \check{\boldsymbol{Y}}\check{\boldsymbol{U}}_{\boldsymbol{N}} + \boldsymbol{I_0}. \tag{21}$$

Based on the the BF method and (21), the unknown voltages for each iteration can then be calculated as follows:

$$\check{\boldsymbol{U}}_{\boldsymbol{N}}^{\boldsymbol{k+1}} = \check{\boldsymbol{Y}}^{\boldsymbol{-1}}\left(\begin{bmatrix}\frac{P_1}{U_1^k}\\ \vdots \\ \frac{P_n}{U_n^k}\end{bmatrix} - \boldsymbol{I_0}\right). \tag{22}$$

This method directly uses the system's matrices instead of the Jacobian, and approximates the constant power nodes as a current source. Therefore, this method is referred here as the DM-CA method.

The main advantage of this method is that the matrix $\check{\boldsymbol{Y}}$ remains constant throughout the iterations, and therefore only has to be factorized once. Only the injected current for each node, P_i/U_i, and the product with the factorized admittance matrix has to be determined every iteration. Therefore, the complexity of this method mostly depends on one factorization of the admittance matrix and multiple matrix multiplications of this matrix.

G. Direct Matrix-Impedance Approximation (DM-IA)

Another novel power flow technique is proposed here, where the constant current model (21) is modified by adding a parallel impedance. Therefore, the current flowing from each constant power converter is approximated by:

$$I_i^{k+1} \approx \frac{2P_i}{U_i^k} - \frac{P_i}{\left(U_i^k\right)^2}U_i^{k+1} = \frac{2P_i}{U_i^k} - Z_i^k U_i^{k+1}. \tag{23}$$

Consequently, the current flowing from the converters into each node can be given as follows:

$$\check{\boldsymbol{I}}_{\boldsymbol{N}} = \check{\boldsymbol{Z}}^{-1}\check{\boldsymbol{U}}_{\boldsymbol{N}} + \check{\boldsymbol{Y}}\check{\boldsymbol{U}}_{\boldsymbol{N}} + \boldsymbol{I_0}, \tag{24}$$

where $\check{\boldsymbol{Z}}$ is a diagonal impedance matrix with elements determined from (23). The voltages at each iteration can then be determined by utilizing:

$$\check{\boldsymbol{U}}_{\boldsymbol{N}}^{\boldsymbol{k+1}} = \left(\check{\boldsymbol{Z}}^{\boldsymbol{k}^{-1}} + \check{\boldsymbol{Y}}\right)^{-1}\left(\begin{bmatrix}\frac{2P_1}{U_1^k}\\ \vdots \\ \frac{2P_n}{U_n^k}\end{bmatrix} - \boldsymbol{I_0}\right). \tag{25}$$

Since this method adds an impedance to the approximation of the constant power nodes, this method is referred here as the DM-IA method. The main advantage of this method over the DM-CA is that its iterations converge faster, since it also takes into account the gradient from the constant power converters' behavior. However, this comes at the cost of having to factorize $\check{\boldsymbol{Z}}^{\boldsymbol{k}^{-1}} + \check{\boldsymbol{Y}}$ at every iteration, thus increasing the complexity of every iteration. Both the DM-CA and DM-IA methods give a numerical approximation of the power flow solution with an error dependent on the convergence criteria.

An advantage of both DM methods is that they can deal with a broader set of grids than those with only slack and constant power nodes. In this case, any linear node behavior can be modelled by a linear combination of a constant voltage, impedance, or current node. Furthermore, non-linear behavior can be approximated by a constant current and a constant impedance that are updated every iteration, as was done for the constant power nodes. However, for the sake of convergence, every grid has to have at least one slack node, or a node with an impedance.

IV. Power Flow Methods Benchmark

In this section, the power flow methods presented in the previous section are compared with respect to accuracy, convergence, and computational effort. Accuracy is defined here as a Root Mean Square Error (RMSE) with respect to the actual solution of the power flow problem. For the iterative methods, the convergence is given by the number of iterations that are required to achieve a convergence criteria, with computational effort being measured as the required computational time to converge.

For the iterative power flow methods, the iterative process stops when the solution converges with a desired tolerance according to

$$\left|\frac{U_i^{k+1} - U_i^k}{U_i^k}\right| < \epsilon \qquad \forall i, \tag{26}$$

where ϵ is the desired tolerance. Note that a set tolerance does not always guarantee a similar accuracy for all methods, as discussed next.

The results in this section are obtained by implementing the power flow methods in Matlab 2017b, and run on a computer with Windows 7, an Intel Xeon E5-1620 processor, and 8 GB of RAM.

A. IEEE Test Feeder

To compare the power flow methods the IEEE European Low Voltage Test Feeder [29] is used, as is illustrated in Fig. 3, and consists of 111 nodes and 112 lines. The ac feeder is a representative neighborhood grid that includes household load profiles and line parameters, and it is assumed here to be a dc feeder with the same line parameters. In this case, 10.000 simulations of one day are carried out, where a day consists of 96 time steps of 15 minutes. In addition to the 55 households included in the test feeder, 15 photovoltaic (PV) systems and 15 electric vehicles (EVs) are randomly distributed among the households for every simulation. A convergence tolerance of 10^{-6} is used.

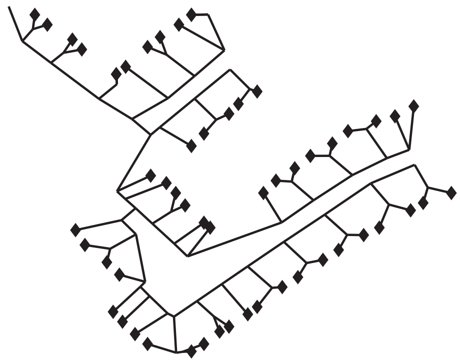

Fig. 3. IEEE European Low Voltage Test Feeder [29].

The power consumption from each household is randomly determined, assuming a uniform Probability Density Function (PDF) from the provided load profiles in the test feeder at every time step. Furthermore, the PV production is simulated using a Gaussian PDF, with a variance of 1/6 of the expected value. Additionally, the arrival time of the EVs is simulated by a Gaussian PDF with a mean at 18:00 and a standard deviation of 1.2 hours, while the charging time is defined as a Weibull distribution with $k = 2.022$ and $\lambda = 2.837$ [30], [31], resulting in a Gaussian-like PDF for the probability that a vehicle is charging with a constant power of 3 kW. The expected power for all these grid elements are shown in Fig. 4.

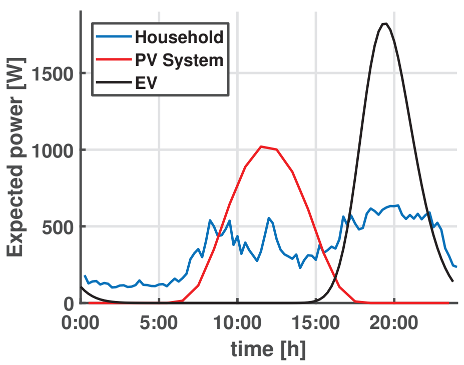

Fig. 4. Expected power for the IEEE test feeder load profiles, PV systems, and EVs.

B. Numerical Results

For the first step in the power flow calculations, an initial guess of 350 V is used at all nodes. Furthermore, the solution of each time step t is used as initial guess for the next time step $(t + 1)$. Note that, because this system is relatively large and the matrices are sparse, LU factorization significantly reduces the average computation times.

The RMSE, average number of iterations, and average computation time (per simulation of a day) for the various power flow methods applied to the IEEE test feeder are shown in Table I. Observe that the OP and DM-IA methods converge faster (have less iterations on average), since both these methods incorporate the non-linear behavior of the constant power loads. Besides the GS method (which is notorious for slow convergence) and QS method (which is not an iterative method), the other methods exhibit similar convergence.

TABLE I
COMPUTATIONAL METRICS WITH $\epsilon = 10^{-6}$

Method	RMSE [p.u.]	Average Iterations	Average Time [s]
GS	0.000189	367	2.87
NR	$1.1 * 10^{-9}$	2.74	0.0445
BF	$3.0 * 10^{-9}$	2.88	0.109
DM-CA	$2.9 * 10^{-9}$	2.87	0.0031
DM-IA	$2.7 * 10^{-14}$	2.00	0.0175
QS	$5.3 * 10^{-15}$	N/A	240
OP	$4.1 * 10^{-10}$	2.00	9.54

Notice that the DM methods require the least computational effort of all the power flow methods. Moreover, even though the DM-IA converges faster than the DM-CA method, the DM-CA method requires the least computational effort of all methods. This is because, for the DM-CA method, the factorized admittance matrix is re-used for every iteration and every time step. Also note that, due to the slow convergence and many iterations of the GS method, the GS does not achieve the level of accuracy that one would expect with these convergence criteria. Consequently, these criteria should be adjusted for the GS method if higher levels of accuracy are required.

For the simulation, a convergence tolerance $\epsilon = 10^{-6}$ was used. However, to ensure that a comprehensive comparison of the different power flow methods is given, the RMSE, average number of iterations, and average computation time for the same simulation with $\epsilon = 10^{-3}$ are given in Table II. Note that, as expected, for all methods, the average number of required iterations decreases when the convergence tolerance is substantially increased. Nevertheless, the results are consistent with the previous simulations.

TABLE II
COMPUTATIONAL METRICS WITH $\epsilon = 10^{-3}$

Method	RMSE [p.u.]	Average Iterations	Average Time [s]
GS	0.00841	1.028	0.0141
NR	$5.9 * 10^{-7}$	1.684	0.0277
BF	$1.1 * 10^{-6}$	1.683	0.0705
DM-CA	$1.1 * 10^{-6}$	1.683	0.0022
DM-IA	$6.0 * 10^{-10}$	1.684	0.0141
QS	$2.4 * 10^{-12}$	N/A	240
OP	$3.4 * 10^{-10}$	2.003	9.59

V. CONCLUSIONS

The existing literature discusses several iterative power flow methods for dc grids such as the GS, NR, and BF methods, but it is shown that the problem can also be formulated and solved as an OP or by using a QS. In this paper, a steady-state power flow model for dc grids was presented. Furthermore, state-of-art power flow methods were discussed and applied to the dc grid model, and two novel power flow methods were proposed. Finally, the existing and novel power flow methods were benchmarked with respect to accuracy, convergence, and computational effort.

The results show that the proposed DM-CA method requires the least computational effort overall (up to 93% less than the NR method). However, this comes at a cost of diminished convergence. Furthermore, the proposed DM-IA method shows improved convergence and requires up to 60% less computational effort compared to the NR method. Therefore, it is shown that both DM methods have superior performance compared to existing techniques for the steady-state analysis of most dc grids, and hence can be used for planning, optimization, and analysis purposes.

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