The effect of surface gravity waves on the measurement of river surface velocity

G. Dolcetti[1#], B. Hortobágyi[2], M. Perks[2], S. Tait[1]

[1] The University of Sheffield, Department of Civil and Structural Engineering, Sheffield, United Kingdom

[2] Newcastle University, School of Geography, Politics and Sociology, Newcastle, United Kingdom

[#] author for correspondence: g.dolcetti@sheffield.ac.uk

Key points:

- A video of a river surface shows evidence of water waves;
- The characteristics of these waves can be quantified and predicted;
- Water waves have a clear, predictable effect on the space-time image correlation;
- Correlation-based surface velocimetry (e.g., LSPIV) can be very inaccurate if applied without tracers and in the presence of waves;
- A more robust estimate of the velocity can be obtained from a knowledge of the characteristic surface length scales.

Methodology:

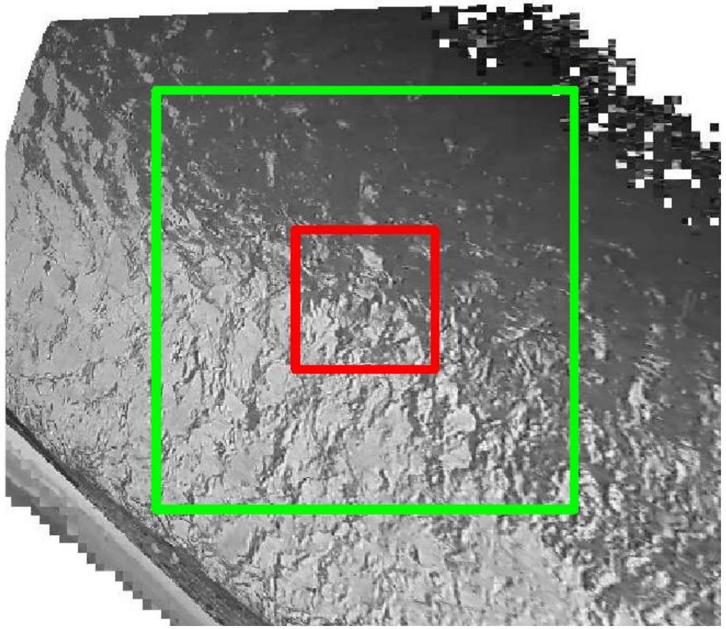
- Measurement of a 5-minutes long video of a small river (River Sheaf, Sheffield, UK) with a fixed low-cost camera;
- Fourier analysis to identify gravity waves and their characteristic spatial and temporal scales;
- Analytical model of the spatio-temporal correlation function based on a simplified Fourier spectrum;
- Comparison with measured average cross-correlation between video frames;
- Application of simplified PIV algorithm to measured and modelled correlations, to estimate surface velocity.

Main results:

- The surface deformations that can be seen in the video are mostly gravity-waves with a characteristic wavelength $\lambda_0 \approx 2\pi F^2 d$;
- These waves are stationary (standing) waves when they propagate against the flow, but can travel in all directions;
- Because of these waves, the spatio-temporal image correlation fluctuates in time (frequency $f_0 \approx U/\lambda_0$) and in space (period λ_0);
- If the velocity is estimated from a peak of the space-time correlation, the error is large and depends strongly on the time-separation between images (frame rate).

Measurement of a 5-minutes long video of a small river (River Sheaf, Sheffield, UK) with a fixed low-cost camera

Orthorectified



- River is approximately straight with a trapezoidal cross-section, 9.2 m width.
- Sharp-crested measurement weir 25 m downstream of section.
- Resolution 1920 x 1080 pixels, frame rate 20 fps. Pixel size of the rectified image 20 cm.

Fourier analysis to identify gravity waves and their characteristic spatial and temporal scales

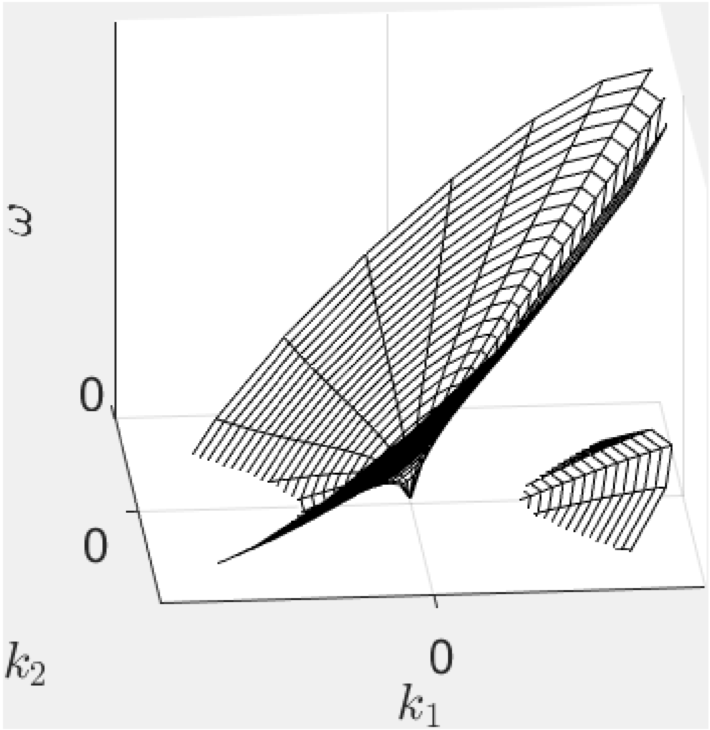
- The Fourier analysis identifies different types of waves based on their…

Frequency: $\omega = \frac{2\pi}{T}$ $[T^{-1}]$

Wavenumber: $k = \frac{2\pi}{\lambda}$ $[L^{-1}]$

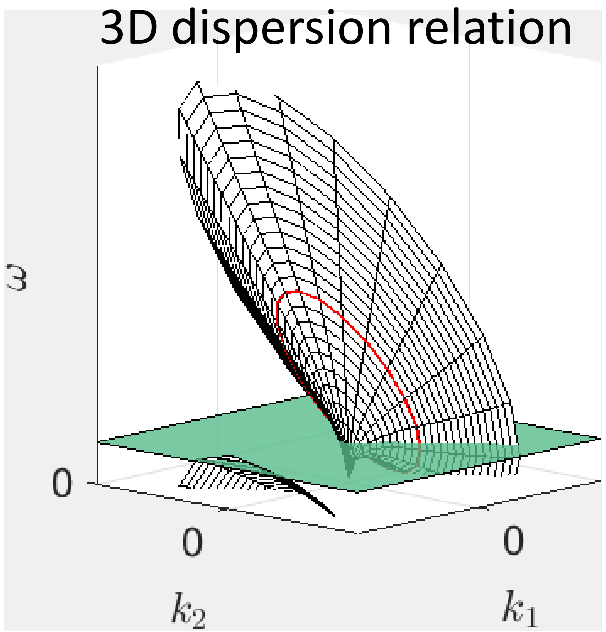
- For gravity waves in a flow with depth d and speed U:

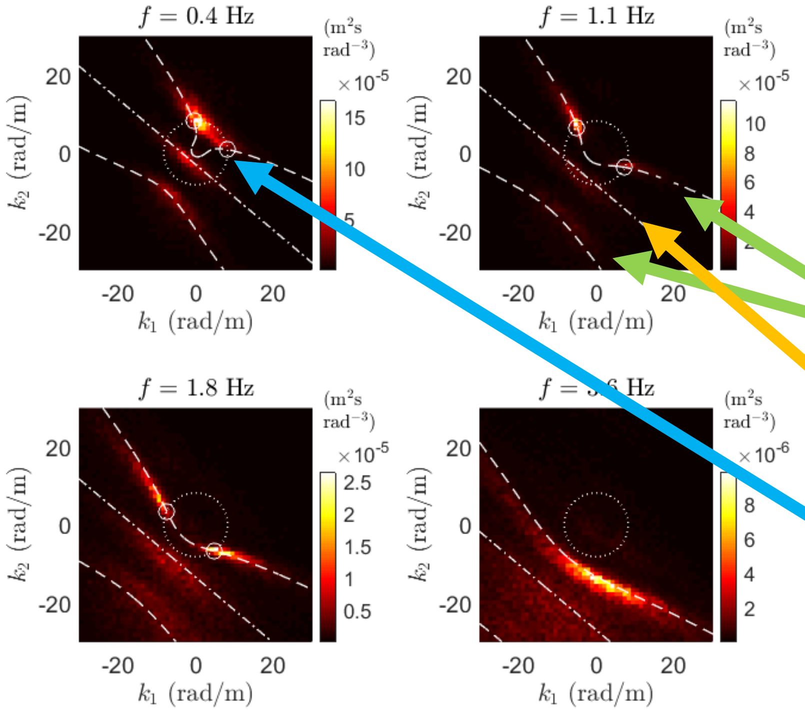
$$\omega(\boldsymbol{k}) = \boldsymbol{k} \cdot \boldsymbol{U} \pm \sqrt{\left(gk + \frac{\gamma k^3}{\rho}\right)\tanh(kd)}$$



A stationary (standing) wave has wavelength $\lambda_0 = 2\pi/k_0 \approx 2\pi F^2 d$, and $\omega(\boldsymbol{k}_0) = 0$

Fourier analysis to identify gravity waves and their characteristic spatial and temporal scales





Constant-frequency sections through the <u>3D frequency-wavenumber spectrum of the video.</u>

- Dashed: gravity waves theory
- Dashed-dotted: rigidly advected surface patterns
- Circle: waves with characteristic wavelength λ_0

- The <u>surface is dominated by gravity waves with wavelength</u> $\lambda_0 \approx 2\pi F^2 d$;
- If these waves are directed against the current, they are stationary (standing), however the waves seen here can also move in all directions relative to the mean flow;
- Some <u>additional patterns</u> (turbulence-induced surface deformations, floating debris, etc.) actually move at the <u>speed of the flow</u>. These are <u>much weaker</u> than the gravity waves.

Analytical model of the spatio-temporal correlation function based on a simplified Fourier spectrum

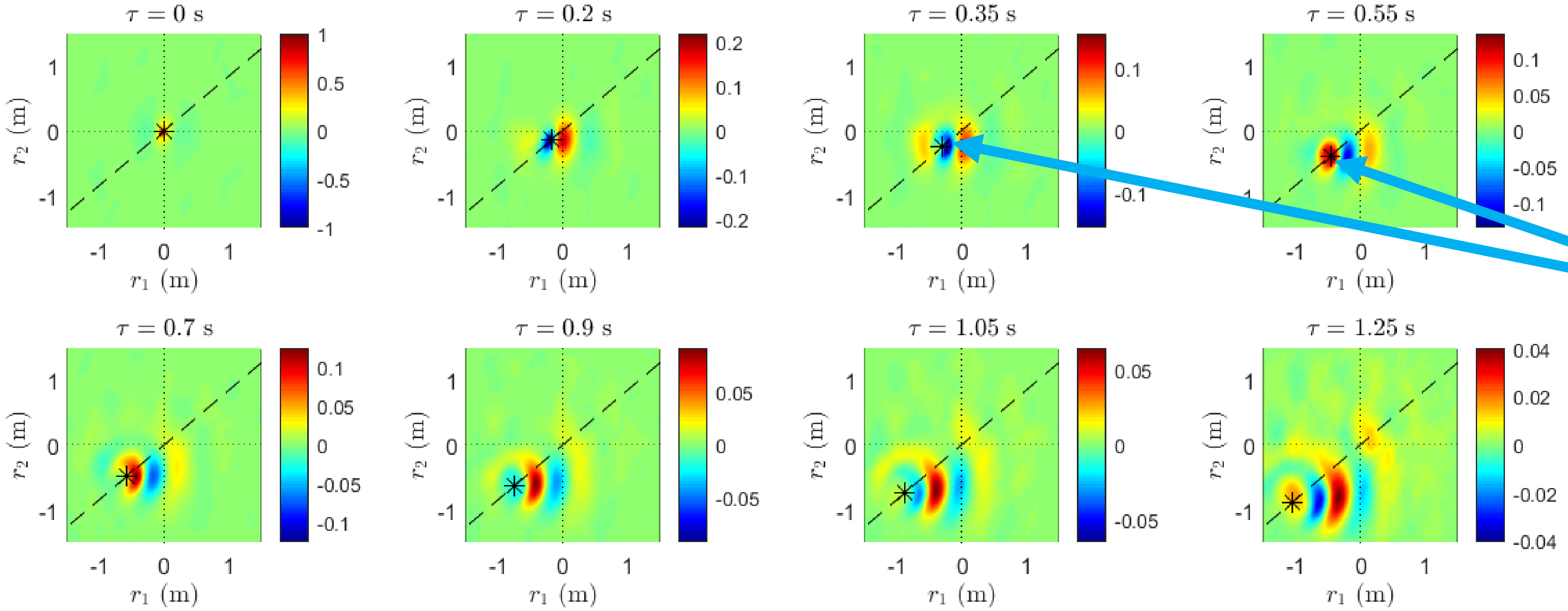
- We can build a model of the space-time correlation of the water surface based on a cosine transform of the surface spectrum (Wiener Khinchin's theorem).

- Considering a distribution of waves with wavelength λ_0 that propagate in all directions,

$$W(r,\tau) = \pi a_0 \, \mathrm{J}_0(k_0|r - U\tau|) \cos[\omega_i(k_0)\tau] + \pi \sum_{n=1}^{\infty} a_n (-1)^{n/2} \cos(n\beta) \left\{ \mathrm{J}_n(k_0|r - U\tau|) e^{-\mathrm{i}\omega_i(k_0)\tau} + \mathrm{J}_n(-k_0|r - U\tau|) e^{\mathrm{i}\omega_i(k_0)\tau} \right\}$$

- a_n: coefficients of a cosine series that describes the variation of wave amplitude with direction of propagation
- $\mathrm{J}_0(k_0|r - U\tau|)$: spatially fluctuating function, wavelength λ_0
- $\cos[\omega_i(k_0)\tau]$: temporally fluctuating function, frequency $\omega_i(k_0)/2\pi \approx U/\lambda_0$

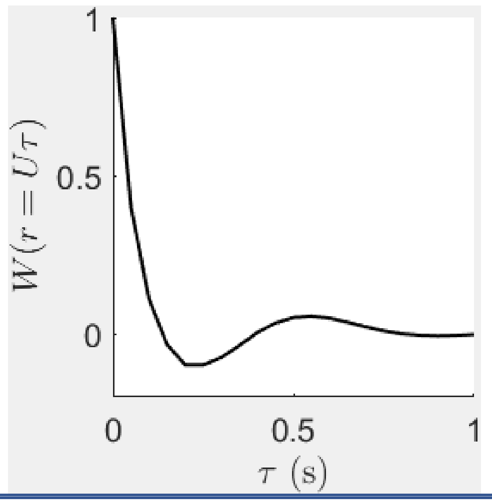
Comparison with measured average cross-correlation between video frames - Measured



<u>Measured Average spatial correlation between 2 images with time-separation τ</u>

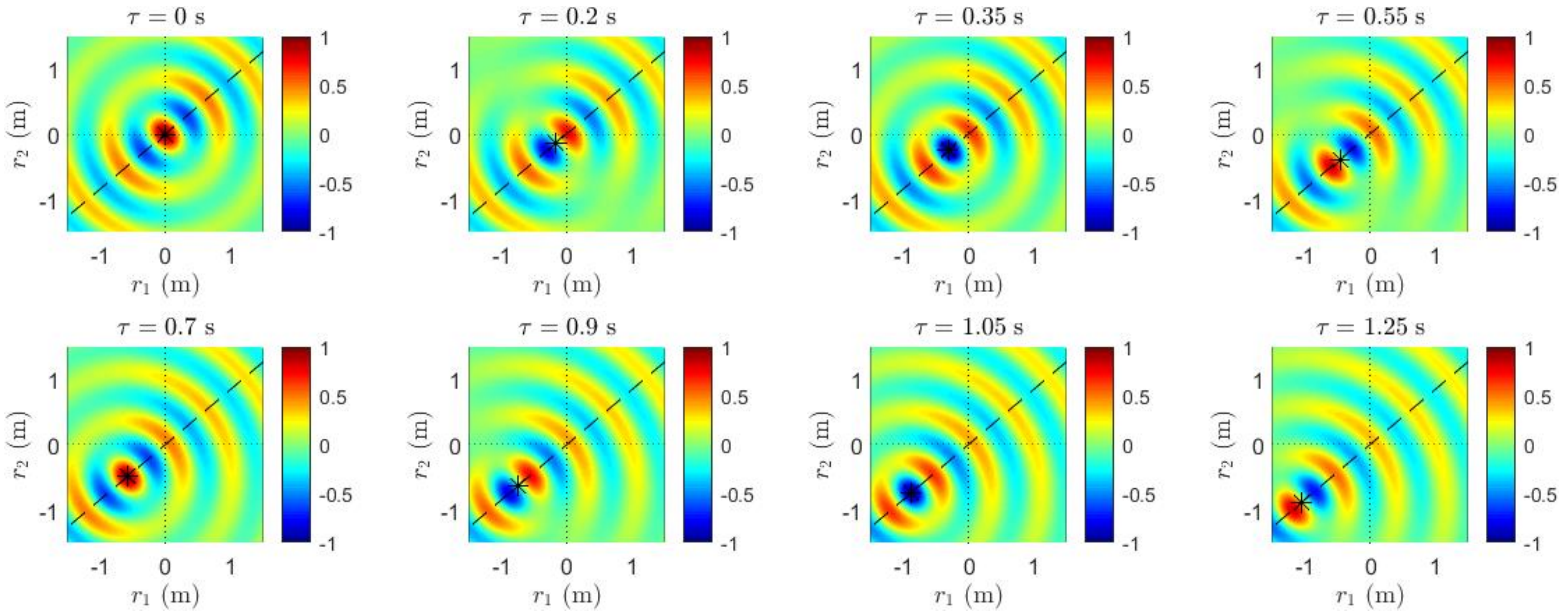
- Asterisk: expected location of the peak of correlation, $\boldsymbol{r} = \boldsymbol{U}\tau$

Correlation at $\boldsymbol{r} = \boldsymbol{U}\tau$



- <u>If the surface moved at the flow velocity, there should be a correlation maximum</u> at $\boldsymbol{r} = \boldsymbol{U}\tau$
- Instead, the correlation at $\boldsymbol{r} = \boldsymbol{U}\tau$ oscillates in time
- The correlation also fluctuates in space, like a `ring wave' with centre at $\boldsymbol{r} = \boldsymbol{U}\tau$

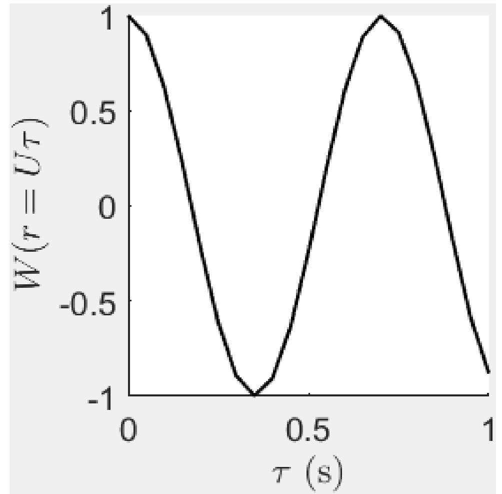
Comparison with measured average cross-correlation between video frames - Model



Modelled Average spatial correlation between 2 images with time-separation τ

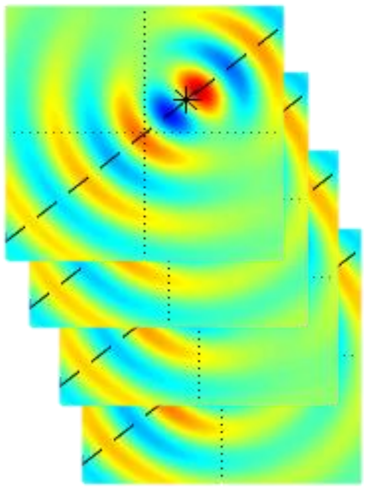
- Asterisk: expected location of the peak of correlation, $\boldsymbol{r} = \boldsymbol{U}\tau$

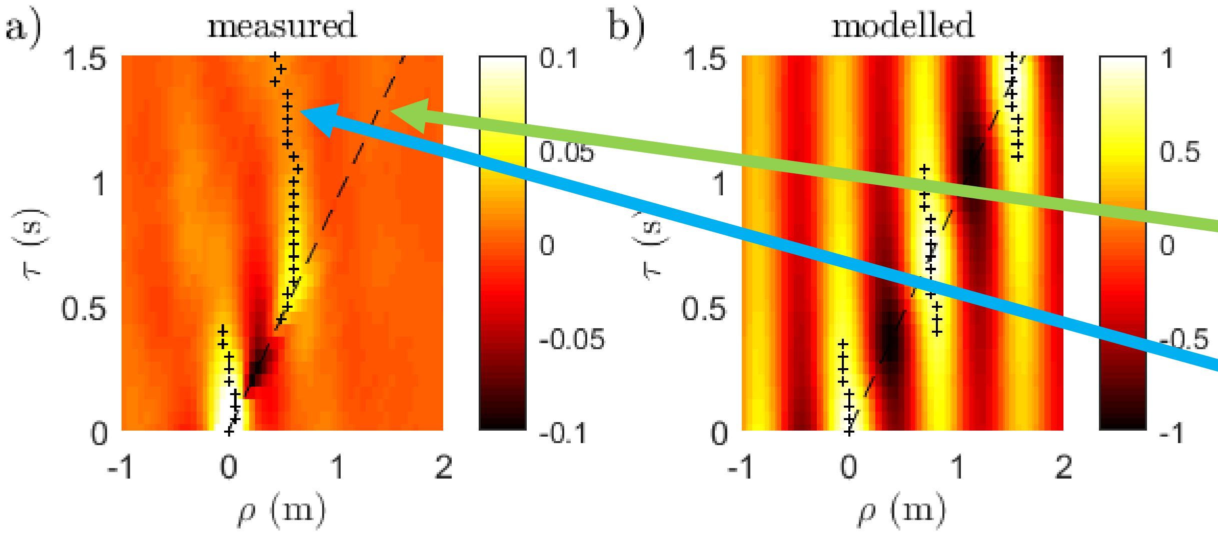
Correlation at $\boldsymbol{r} = \boldsymbol{U}\tau$



- The model indicates the signature of gravity waves with characteristic wavelength λ_0;
- The model correlation oscillates in time and in space, like the measurements, but does not decay (no noise and/or dissipation);
- The oscillations can be predicted, and are to be attributed to gravity waves.

Application of simplified PIV algorithm to measured and modelled correlations to estimate surface velocity



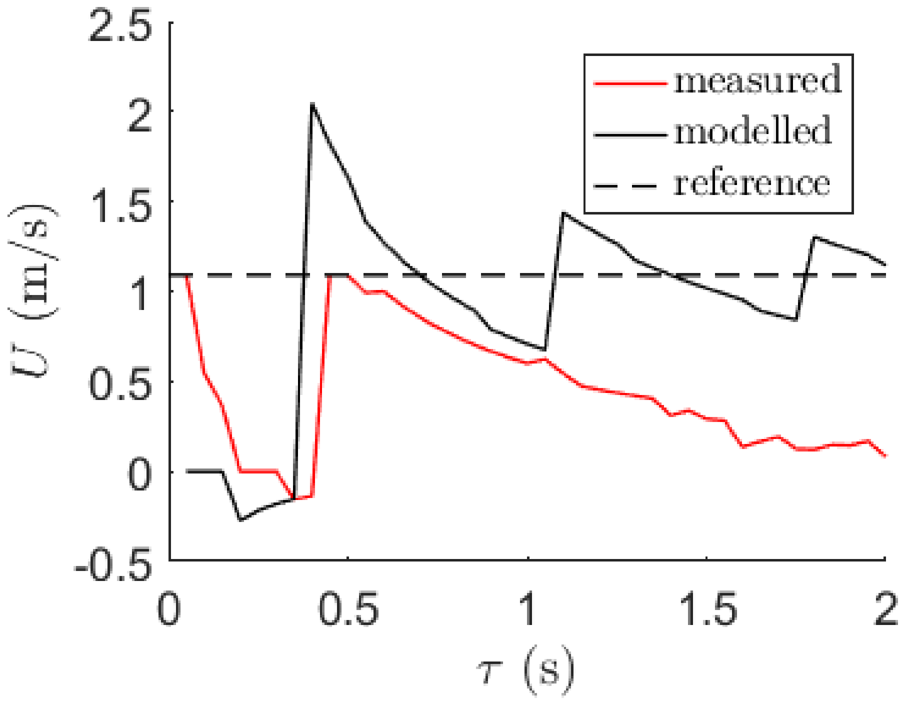


Spatio-temporal correlation on the plane parallel to the flow velocity

- (dashed): expected location of the correlation peak, $\rho = U\tau$
- (crosses): actual location of the correlation peak

- The maximum of the correlation is calculated for different time separations between frames;
- If the surface was moving at the flow velocity, the peak would be at $\rho = U\tau$;
- The measured correlation oscillates with period $\lambda_0 = 0.77$ m.
- Because of wave-related oscillations of the correlation function, the peak of the correlation does not coincide with its expected location, except for very short τ.

Application of simplified PIV algorithm to measured and modelled correlations to estimate surface velocity

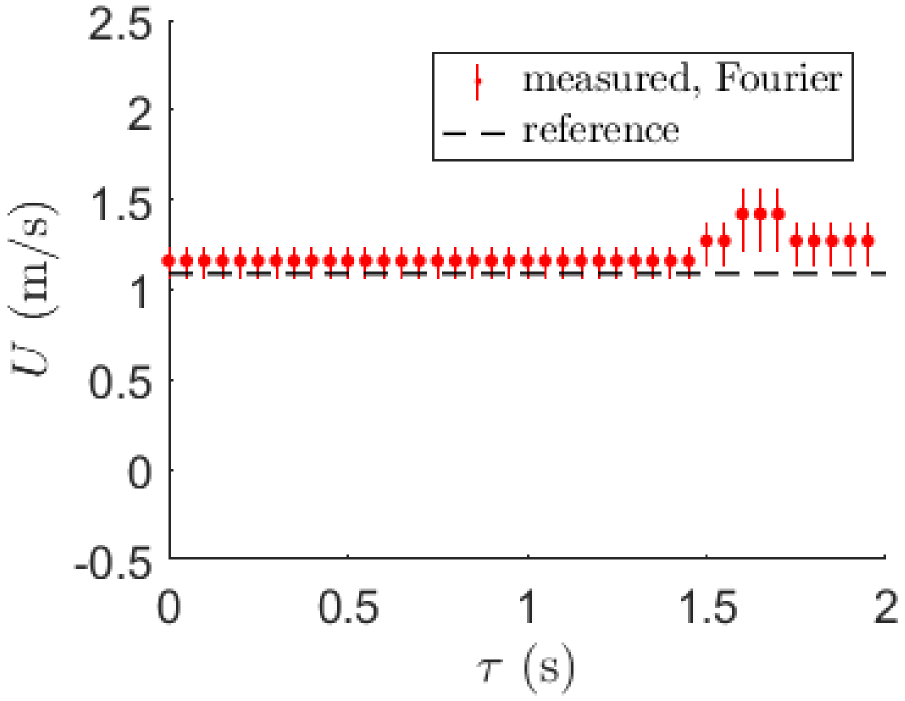


Surface velocity, calculated from the peak of the correlation, i.e., $U_W = \rho_W/\tau$ as a function of time separation τ

- Measured: calculated from videos
- Modelled: calculated from model of the surface correlation

- If the flow velocity is calculated from a peak of the correlation between frames, errors are large and the estimates vary greatly with the time-separation;
- These variations are due to the presence of water waves.

Application of simplified PIV algorithm to measured and modelled correlations to estimate surface velocity



- Surface velocity calculated from proposed Fourier-based analysis, $U \approx \sqrt{g\lambda_0/2\pi}$, from the fluctuation wavelength λ_0

- An alternative is to find the period of the correlation, λ_0, using Fourier analysis
- Flow velocity can be estimated as $U \approx \sqrt{g\lambda_0/2\pi}$ (deep water gravity waves assumption)
- Alternative method is more robust, but requires well-defined waves

Comments and Final Remarks

- The observed behaviour of surface waves agrees with laboratory experiments (Dolcetti et al. 2016, Dolcetti & García Nava 2019) carried out for relatively shallow and fast flows (Froude numbers 0.3 - 0.7);
- Waves may be less visible, or secondary compared to other surface deformations in different types of flows (very low or high Froude numbers, smooth beds);
- In all cases, application of standard correlation-based surface velocimetry techniques is not advised without easily identifiable tracers and/or in the presence of waves, because the velocity of waves differs from that of the flow;
- The Fourier analysis described here can act as a useful tool to identify conditions where standard correlation-based surface velocimetry techniques are not viable;
- The surface model described here could help quantify the uncertainties of video-based velocimetry for different surface appearances.

Further Reading

- Behaviour of gravity waves in open channel flows:
  - Dolcetti, G. & García Nava, H. (2019). Wavelet spectral analysis of the free surface of turbulent flows. Journal of Hydraulic Research 57(2): 211-226. (https://doi.org/10.1080/00221686.2018.1478896)
  - Dolcetti, G., Horoshenkov, K. V., Krynkin, A. & Tait, S. J. (2016). Frequency-wavenumber spectrum of the free surface of shallow turbulent flows over a rough boundary. Physics of Fluids 28(10): 105105. (https://doi.org/10.1063/1.4964926)

- Gravity waves effect on video-based surface velocimetry:
  - Benetazzo, A., Gamba, M. & Barbariol, F. (2017). Unseeded large scale PIV measurements corrected for the capillary-gravity wave dynamics. Rendiconti Lincei 28(2): 393-404. (DOI: 10.1007/s12210-017-0606-2)
  - Tauro, F., Porfiri, M. & Grimaldi, S. (2014). Orienting the camera and firing lasers to enhance large scale particle image velocimetry for streamflow monitoring. Water Resources Research 50(9): 7470-7483. (https://doi.org/10.1002/2014WR015952)

Contacts

- E-mail: g.dolcetti@sheffield.ac.uk, s.tait@sheffield.ac.uk
- Twitter: @dolcetti_g
- ResearchGate: https://www.researchgate.net/profile/Giulio_Dolcetti2