Line Parameters Estimation in Presence of Uncalibrated Instrument Transformers

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***Abstract*—This paper presents a method to estimate parameters of a 3-phase line segment using PMU data. The novelty of this method is that it is capable of giving accurate estimates even in the presence of non-calibrated instrument transformers at both ends of the line whose ratio and phase correction coefficients are unknown. To do so, this method adds extra parameters in the regression model. These added parameters account for the errors present in the non-calibrated instrument transformers. In case the instrument transformers are calibrated at one end of the line, then the correction coefficients at the other end could also be estimated. The presented method does not require reversal of current flow direction in the line as a necessary condition. Results from simulated and laboratory experiments are presented to show the efficacy of the method. A discussion about analyzing the obtained results is also presented.**

***Index Terms*—Distribution Grid Monitoring, Line Parameter Estimation, PMU, Power System Instrumentation**

I. Introduction and Background

Accurate estimates of line parameters could be useful in improving the performance of power system monitoring, protection and control applications including dynamic line rating [1], accurate relay settings and fault location [2]. Traditionally, the line parameters (resistance (R) reactance (X) and susceptance (B) have been calculated using the standard formulas utilizing the available information and assumptions on the physical attributes of the line segment like line lengths, conductor dimension and tower geometry [3]. These methods are static in nature as the used specifications are considered constant while the line parameters could change based on ambient temperature and loading levels. In the recent past, methods have been presented that utilize PMU and SCADA data to estimate the line parameters [2], [3]. These methods estimate line parameters utilizing voltage and current phasors at both ends of the line segment and could give frequent parameter estimates [3], [4].

As mentioned in [4], factors like unbalanced loading levels and some independence in the loading of the 3 phases are the necessary conditions for convergence of the algorithm. However after successful convergence, the accuracy of the line parameter estimates is of great importance.

The accuracy is a qualitative characteristics which is made up of components *trueness* and *precision*. Quantitive estimates of trueness and precision are given by *bias* and *standard deviation* respectively [5]. Combining the bias and standard deviation gives us the *uncertainty* associated with the estimated parameters. This uncertainty in the line parameter estimates is dependent on the quality of PMU (synchrophasor) data. However, synchrophasor data in-turn depends on the accuracy of the overall instrumentation channel feeding voltage and current signals to the PMUs [6]. For example, small random measurement errors at CTs (Current Transformer) and VTs (Voltage Transformer) and PMU's voltage and current phasors would contribute towards high standard deviation in the parameter estimates. While, the ratio and phase errors in the non-calibrated CTs and VTs would contribute towards a bias in the parameter estimates.

This paper shows the drawbacks in old method of overlooking these bias errors and how it effects the overall accuracy of the line parameters estimates. Then, a new method is proposed to eliminate these drawbacks and to estimate accurate line parameters without any prior information about the class or calibration of CTs and VTs at both ends of the line. The proposed method adds some extra parameters in the regression model which account for the bias errors present in the non-calibrated instrument transformers. The novelty of this method is that it gives highly-accurate estimates of 3-phase line parameters even in presence of non-calibrated

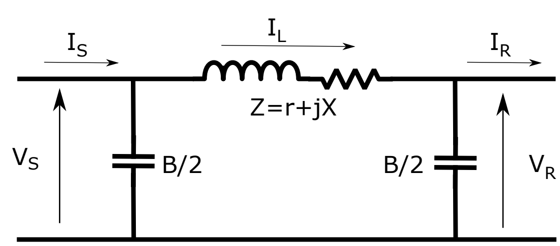

Fig. 1. Nominal Pi model for a medium length medium voltage line

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instrument transformers at both ends. In the past, methods have been proposed to estimate the line parameters and correction coefficients for CT VT errors [7]. However, these methods were confined to single phase. The mutual components of impedance and susceptance were ignored and hence the total number of unknowns are reduced significantly. However, in case of unbalance and independence in 3-phase loading levels, these methods would not be applicable for three phase systems.

The remainder of this paper is arranged as follows. Section II discusses the old method and the drawbacks with it to estimate line parameters. Section III presents the proposed method to overcome these drawbacks in a 3-phase system. Section IV presents the results and compares the results obtained from simulation and laboratory based tests. Conclusions are presented in Section V.

II. Overview of Existing Method

In previous approaches for 3-phase line segments, unknown line parameters are estimated using multiple linear regression based estimation. Based on the assumed line model shown in Fig. 1, and basic equations (1) and (2), multiple real and imaginary equations are formulated using the measured voltage and current signals and unknown line parameters Z_{abc} and B_{abc}.

$$I^S_{abc} = I^R_{abc} + \frac{B_{abc}}{2}\left(V^S_{abc} + V^R_{abc}\right) \quad (1)$$

$$V^S_{abc} = V^R_{abc} + Z_{abc}\left(I^R_{abc} + \frac{B_{abc}}{2}V^R_{abc}\right) \quad (2)$$

Where, superscript S/R denotes the sending and receiving end of the line and subscript ABC denote the 3-phase system. Z_{abc} and B_{abc} are the three phase impedance and shunt susceptance of the line. The real and imaginary terms of (1) and (2) are separated into a set of 12 equations. The separated real and imaginary equations represent a linear system with measured output and input data. These equations are then solved in linear least square sense to estimate the unknown Z_{abc} and B_{abc} parameters. The set of linear equations with n parameters can be written as:

$$z_i = h_{1i}\theta_1 + h_{2i}\theta_2 + ... + h_{ni}\theta_n + \epsilon_i \quad (3)$$

where, z_i is the measured quantity, h_{ji} is the value of known variables and θ_j are the line parameters to be estimated. ϵ_i is the error term calculated as the difference between the measured quantity and the linear equation formed by parameters and independent variables. In matrix form, the whole system of linear equations can be written as:

$$Z = H\Theta + \mathcal{E} \quad (4)$$

where, Z is the measurement vector and Θ is the parameters vector. H is the relation matrix formed of independent variables (which are also measurements in this case).

The estimates for parameter θ is given by the equation:

$$\Theta = \frac{covariance(Z,H)}{variance(H)} \quad (5)$$

In matrix form, the same can be represented as:

$$\Theta = (H^T H)^{-1} H^T Z \quad (6)$$

To get an optimal estimate from linear regression, it is assumed that:

a Zero mean random error. $n \times 1$ error vector (residuals) has a distribution with zero mean such that $E[\epsilon_i] = 0$.
b No heteroscedasticity. All the diagonal elements of the covariance matrix of the errors $E[\epsilon\epsilon']$ are equal to σ^2 that is $E[\epsilon_i^2] = \sigma^2$.
c No correlation. The off-diagonal elements of the covariance matrix of the errors $E[\epsilon\epsilon']$ are all equal to 0.
d Normality. The error vector is normally distributed.
e Constant parameter. The elements of the parameter vector Θ are fixed.

Assumptions a, b, c and d can be summarized in matrix notation as:

$$\epsilon = N(0, \sigma^2 I) \quad (7)$$

The precision part of uncertainty in parameter estimates is due to the random measurement errors and causes higher standard deviation of the estimates. It can be quantified as the confidence interval (CI). Estimating the trueness of the parameter estimates is tricky, because generally there is no known true value of the parameters. Bias in the estimates which is the measure of trueness depends on the validity of the assumptions made while solving the linear regression problem. If all the assumptions hold true then it is an indication that the linear model relating the vector Z and matrix H using the parameter vector Θ is correct. If the residuals adheres to the assumptions above, then this would suggest that the bias part of the estimation uncertainty is low. At the same time, if the standard deviation of the expected parameters is low as well, then the total uncertainty would be low. However, if the residuals do not adhere to the assumptions, then it suggests a fundamental error in the assumed process model or data acquisition and the estimates can be rejected even when the estimated CI is very narrow. The same approach is utilized in this paper to assess the model and estimation results.

In real field measurements each of these assumptions might not hold true. Apart from the random measurement errors, field CTs and VTs have a steady bias error in their measurement due to the inherent ratio and phase errors of different magnitudes. The elements forming the independent and dependent elements for the system of equations (as shown in (3)) are real and imaginary parts of 3-phase current and voltage phasors. The presence of any ratio and phase errors in the measurement chain would make the difference between actual and measured quantities depend on the magnitude of ratio and phase errors in a non-linear sense. Thus if we use the same process model as in the old methods to link the output and the input measurements then the error vector would not be normally distributed and the expected value of errors might also be non zero. This would mean that the model of the linear equations does not explain the relationship between the measured dependent and independent variables. This would result in bias in parameter estimates. The following section presents the proposed method to overcome this problem.

III. Improved Method

The new algorithm presented in this paper eliminates the bottleneck of unknown correction coefficients for CTs and VTs while estimating line parameters. The idea is to add more parameters which would explain the measurement errors in the CTs and VTs. Using the correction coefficients at both ends, (1) and (2) for a 3-phase system are written in the following form:

$$Ci^S * I^S - Ci^R * I^R = \frac{B}{2}\left(Cv^S * V^S + Cv^R * V^R\right) \quad (8)$$

$$Cv^S * V^S - Cv^R * V^R = Z\left(Ci^R * I^R + \frac{B}{2} Cv^R * V^R\right) \quad (9)$$

where, Ci^S, Ci^R, Cv^S, Cv^R are the three phase correction coefficients for the ratio and phase errors of CTs and VTs at both ends of the line. These correction coefficients could be represented in the form $|M|e^{j\phi}$, where M is coefficient for ratio error and ϕ is the phase error.

Equations (8) and (9) are the difference equations of measured voltage and current phasors represented separately in real and imaginary parts. These equations can be are rewritten using a new set of *Adjusted Correction Coefficients*.

$$I^S - Ci * I^R = \frac{B}{2}\left(V^S + Cv * V^R\right) \quad (10)$$

$$V^S - Cv * V^R = Z\left(Ci * I^R + \frac{B}{2} Cv * V^R\right) \quad (11)$$

where, Ci and Cv are the adjusted coefficients. These adjusted coefficients are assumed to be the net effect of individual correction coefficients at each end when applied in the left hand side of the (8) and (9). For example,

$$Cv^S * V^S - Cv^R * V^R = V^S - Cv * V^R \quad (12)$$

The new equations (10) and (11) are representation of (8) and (9) assuming that the measurements at one end of the line (the sending end in this case) are error free. The measurements of the receiving ends are corrected using the adjusted correction coefficients. The mutual susceptance was ignored and only self susceptance was estimated. Overall 21 parameters needed to be estimated:

- Adjusted CTs correction coefficients: $[Ci_a,\ Ci_b,\ Ci_c]$.
- Adjusted VTs correction coefficients : $[Cv_a,\ Cv_b,\ Cv_c]$.
- self susceptance: $[b_{aa},\ b_{bb},\ b_{cc}]$.
- Self and mutual resistance: $[r_{aa},\ r_{ab},\ r_{ac},\ r_{bb},\ r_{bc},\ r_{cc}]$.
- Self and mutual reactance: $[x_{aa},\ x_{ab},\ x_{ac},\ x_{bb},\ x_{bc},\ x_{cc}]$.

This method could be used to calibrate the CTs and VTs using one calibrated end as reference. Hence if the CTs and VTs on the one side of the line are calibrated and their correction coefficients are known, then the estimated values of the adjusted correction coefficients would be the correction coefficients for the CTs and VTs of the other end of the line.

From here, the parameter estimation process was divided in two parts. Equation (10) was written as two separate equations for real and imaginary parts. This was done for all the three phases resulting in six equations. All the measured data was arranged according to these six equations forming a set of over-determined system of linear equations. Parameters Cv, Ci and B for all phases were estimated using the analytical solution given by (6). Now, these estimated parameters were substituted in (11) and the remaining parameters of Z matrix (R_{abc} and X_{abc}) were estimated. The next section presents the results and comparison with the previous method.

IV. Results and Comparison

First, both the old and the proposed algorithms were tested in simulation mode. A 20 kV, 10 km cable was simulated. A power flow profile based on random walk was subjected on to the lines. As per [4], the power flow variation in each phase was kept independent of each other. Even though here we knew the exact values of the parameters of simulated 3-phase cables, in the real field measurements, we only have a rough estimate about their range. In the simulation based tests, initially the obtained results were analyzed based on the analysis of the residuals. If the residual vector seems to satisfactorily pass the tests to check the assumptions mentioned earlier, then we look further to the CI for expected parameters. In the end we give an error percentage which is the percentage difference between the actual parameters and its expected values.

In the simulation based tests, all the other operating and measurement conditions were kept same for the purpose of fair comparison. The number of samples collected, the variance in the power flow process and the noise levels in the measurement process were kept same across all the tests. The random measurement noise errors by CTs and VTs are assumed to be averaged and filtered out by the sampling and phasor estimation process of the PMUs [8]. Random errors in PMU phasor estimates were assumed to be 0.1% in magnitude and 0.05° in phase. Steady state linear Kalman filter was used to filter out the PMU measurements before feeding them to the algorithm. The Kalman filter assumes a pseudo-steady state linear system model with the state transition matrix being $n \times n$ identity matrix where n is the number of states (3-ph voltage and current magnitude and phase values in this case). The process noise covariance is unknown and is initialized to a very high value. It is updated after a set window length based upon the covariance in the system states in recent history. Measurement noise covariance matrix is a diagonal matrix with known measurement error variance as its elements.

After filtering the phasor estimates, the data is used in estimation algorithm. To examine the results, the residuals were checked for the assumptions of normality and homoscedasticity. To test normality for large data set, the best way is to apply a more visual approach and do a QQ-plot to see if the residual looks normal enough. A QQ-plot displays the quantiles of the data under test versus the expected quantile values of a normal distribution [9]. In this case, if the distribution of residual is normal, then the plotted residuals in the QQ-plot appears linear. Visual tests can also be done to check for homoscedasticity and see that the variance of the residuals do not vary at different measurement values. The same approach was adopted for the following tests.

Another method to check normality is using the Shapiro-Wilk test and homoscedasticity is using the Engle's ARCH test. These test are only reliable for smaller sample size. Predefined MATLAB function were used to perform these tests [10], [11]. For this the data can be randomly re-sampled with replacement into a number of equal size data sets. Each of these data sets are then used as input to the parameter estimation. The parameters from the data set whose residual clears these tests with highest probability (given by p-value) are accepted. After reducing the number of samples by re-sampling data from a big sample size, the new smaller samples are more likely to appear normally distributed than the original set. One more point to keep in mind is that due to lower number of samples, the standard deviation and hence the uncertainty associated with the parameter estimates increases and the CI to becomes wider.

In real-field scenario a narrow CI caused by large sample size makes the estimates highly precise. The accuracy of the estimates is characterized by both trueness and precision and high precision does not necessary mean trueness of the parameter estimates. That is, high precision does not necessarily mean that there is little difference between the estimates and their *unknown* true values. Hence, with a large sample size, the narrow CIs of the estimated parameters could be highly misleading about the accuracy of the estimates. It can be the case that the CI of an estimate is very narrow indicating a precise estimate but the estimate actually is totally wrong due to the wrong model assumption or error in measurements.

If residuals from none of the data sets satisfy the criteria for normality and homoscedasticity then it is an indication that the model and the measurements do not explain the system correctly. In that case, we must either do re-measurements or change the equation model. Simulation and lab tests done to show the results of the proposed method and compare it with the old method are presented below.

A. Test 1: Simulation with known correction coefficients

Properties of class 0.2 CTs and VTs were used in this test. It was assumed that the CTs and VTs are calibrated and correction coefficients of all them were known. The older and the new proposed method were applied to the filtered PMU data. The number of distinct samples for each measurement was same. The analysis for results obtained is presented below.

First the residuals are analyzed. Visual analysis tests using the QQ-plot suggested that the residuals of both methods were normal and no heteroscedasticity was found. The QQ-plot for both the tests are shown in Fig. 2. The plotted residuals for both the methods in this case are linear and on the expected line for a normal distribution. This suggests that the result from both methods would not have any significant bias and the CI would be a reliable measure of overall accuracy.

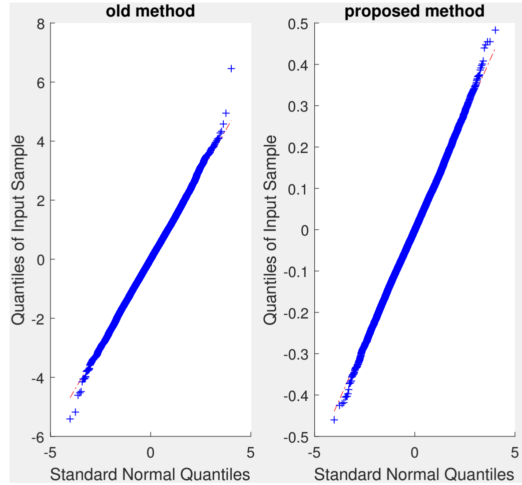

Fig. 2. Test 1: QQ plot of residuals for both the methods

Next, estimates were assessed based on the CI. The lower and upper limits of the CI around the estimated parameters have been calculated considering the parameters have a normal distribution. The significance level (α) was kept 0.05. In the old method, in case when the estimates of susceptance (B) is required then the linear equations are arranged in such a manner that the parameters estimated are the elements of admittance (*siemens*) instead of impedance (*ohms*). The impedance parameters were achieved after inverting the admittance matrix. Thus, the confidence interval of the admittance parameters cannot be used as the CI for impedance parameters. Also the parameters differ from each other in order of magnitudes. So to simplify the result analysis and comparison process, the confidence intervals range are mentioned as the percentage deviation from the expected value of the parameters. The results from Test 1 are presented in Tables I-IV.

It was established that as there were no ratio and phase errors in the CTs and VTs and the parameters estimates would be free from bias errors. Hence, the precision of the estimates given by the CI would suggest the overall accuracy. From the Tables I-III, it can be inferred that the estimates of resistance and reactance seems more accurate as they have a narrow CIs meaning low uncertainty. The estimates for susceptance have wide CIs in both the methods. This can be explained for the fact that the lines being only 10 km long have very small charging current and there is not enough difference between the currents at the both end. So in presence of measurement

TABLE I
TEST 1: COMPARISON OF CIS FOR REAL PART OF SELF-ADMITTANCE PARAMETERS FOR OLD METHOD AND SELF-RESISTANCE PARAMETERS FOR PROPOSED METHOD

	Confidence Interval Width		
	Phase A	Phase B	Phase C
Old Method	±0.58%	±0.53%	±0.3%
Proposed Method	±1.01%	±1.37%	±1.11%

TABLE II

TEST 1: COMPARISON OF CIS FOR IMAGINARY PART OF SELF-ADMITTANCE PARAMETERS FOR OLD METHOD AND SELF-REACTANCE PARAMETERS FOR PROPOSED METHOD

	Confidence Interval Width		
	Phase A	Phase B	Phase C
Old Method	$\pm 0.43\%$	$\pm 0.41\%$	$\pm 0.27\%$
Proposed Method	$\pm 1.53\%$	$\pm 2.03\%$	$\pm 1.63\%$

TABLE III

TEST 1: COMPARISON OF CIS FOR SELF-SUSCEPTANCE PARAMETERS

	Confidence Interval Width		
	Phase A	Phase B	Phase C
Old Method	$\pm 38.61\%$	$\pm 39.01\%$	$\pm 38.49\%$
Proposed Method	$\pm 55.36\%$	$\pm 32.99\%$	$\pm 45.17\%$

noise, the power and variance of the current difference signals is insufficient for precise identification of B parameters. It can be shown that for longer lines and similar noise conditions and power flow, the accuracy of estimates for B and R and X would be better. Similarly application on data from short lines in similar conditions would result in less precise estimates. More information about the effects of varying different factors can be found out in [1].

The calculated percentage errors for various parameters are shown in Table IV. The R and X estimates were expected to be more accurate than the B estimates. The results are showing that the new method works similar to the older one especially for R and X estimates. The CI even though small is bigger than the error percentage. For B estimate, even though the CI for both methods were of same magnitude, the accuracy of older method was better when compared to the proposed method. But this can only be confirmed when we have an accurate value as a reference. Now the next test shows the effect of unknown calibration coefficients (not-calibrated CTs/VTs) on the parameter estimates.

B. Test 2: Simulation with unknown correction coefficients

Properties of class 1.0 CTs and VTs are used and it was assumed that that they are not calibrated. That means the correction coefficients are not known and the actual ratio and phase errors could be anywhere in the range given by the class.

TABLE IV

TEST1: ERROR WHEN COMPARED TO THE ACTUAL PARAMETERS

	Parameters	Phase A	Phase B	Phase C
Old Method	R	0.58%	0.62%	0.25%
	X	0.07%	1.36%	0.48%
	B	5.14%	5.21%	5.17%
Proposed Method	R	1.37%	0.84%	0.81%
	X	1.15%	1.32%	0.27%
	B	5.20%	1.96%	26.48%

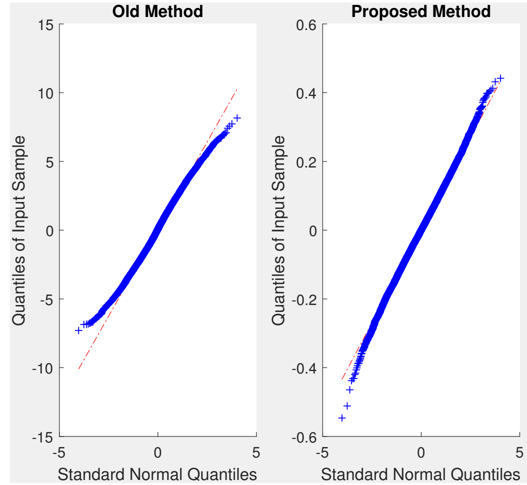

Fig. 3. Test 2: QQ plot of residuals for both the methods

The same power flow profile was used and current and voltage signals were recorded and filtered. The data were fed to both the algorithms and line parameters were estimated. The QQ-plots for residuals for both the methods are plotted in Fig. 3.

The left plot shows the residuals from the old method and the points are not linear along the line of normal distribution. This suggests that the residuals of the old method are not normal in this case when the correction coefficients of the CTs and VTs are unknown. On the other hand, the residuals from the proposed method seem fairly linear along the line of normal distribution suggesting normality. This means the result from the old method would be biased and the CI would not be reliable as a measure of accuracy. To verify our inference about the results accuracy and reliability of the CIs, the results are presented in Tables V-VIII.

From the Tables V-VIII, we see that the effect of reduced

TABLE V

TEST 2: COMPARISON OF CIS FOR REAL PART OF SELF-ADMITTANCE PARAMETERS FOR OLD METHOD AND SELF-RESISTANCE PARAMETERS FOR PROPOSED METHOD

	Confidence Interval Width		
	Phase A	Phase B	Phase C
Old Method	$\pm 0.74\%$	$\pm 1.49\%$	$\pm 0.88\%$
Proposed Method	$\pm 1.28\%$	$\pm 1.02\%$	$\pm 1.25\%$

TABLE VI

TEST 2: COMPARISON OF CIS FOR IMAGINARY PART OF SELF-ADMITTANCE PARAMETERS FOR OLD METHOD AND SELF-REACTANCE PARAMETERS FOR PROPOSED METHOD

	Confidence Interval Width		
	Phase A	Phase B	Phase C
Old Method	$\pm 0.69\%$	$\pm 0.87\%$	$\pm 0.29\%$
Proposed Method	$\pm 2.19\%$	$\pm 1.62\%$	$\pm 1.93\%$

normality of residuals. In the old method, the residuals are visibly not normal and as predicted, the accuracy of the estimated parameters is not in accordance with the suggested CI given by the algorithm. The CI for each parameter still remains similar to the values obtained in Test 1. On the other hand, when compared to the reference value of the parameters, the percentage errors have increased multiple folds. This means that the accurate parameters lie outside the confidence intervals that too by a big margin.

In case of the proposed method, the QQ-plot on a visual inspection still looks quite similar to the QQ-plot obtained for Test 1. The CI and the actual percentage error have comparable magnitude. This suggests that the proposed method reduces the bias in parameters caused by ratio and phase errors in the CTs and VTs. The high number of samples used in the algorithm causes the CI to be narrow.

The other method of re-sampling the samples into 5 smaller lengths to test normality and estimate parameters was also tested. The sample set of independent and dependent measurements were divided randomly and with replacement into 500 samples each. The residuals of each sample were analyzed using Shapiro-Wilk test and the sample-set whose residuals had the highest probability of being normally distributed was chosen. The estimated results from the sample-set which produced the residual vector that is most probable of being normally distributed is presented in Tables IX and X. All the sample sets produced residuals whose probability of being normally distributed was great than 10%. The highest probability was of sample set 5 with a probability of 55.8%.

With a sample size reduced by a factor of 700 did not have a drastic effect on *actual accuracy* for the estimates of R and X. On the other hand, as expected the CI for all the parameters became wider as expected. The next subsection presents a test done in the university laboratory.

TABLE VII
TEST 2: COMPARISON OF CIS FOR SELF-SUSCEPTANCE PARAMETERS

	Confidence Interval Width		
	Phase A	Phase B	Phase C
Old Method	±57.74%	±64.59%	±72.86%
Proposed Method	±46.73%	±41.73%	±58.73%

TABLE VIII
TEST 2: ESTIMATION ERROR USING ALL THE SAMPLES WHEN COMPARED TO THE ACTUAL PARAMETERS

	Parameters	Phase A	Phase B	Phase C
Old Method	R	13.86%	3.93%	10.78%
	X	18.03%	2.23%	29.80%
	B	223.60%	210.18%	195.93%
Proposed Method	R	3.03%	0.71%	2.54%
	X	8.63%	6.56%	0.1.75%
	B	4.60%	22.96%	3.68%

TABLE IX
TEST 2: CIS FOR SELF-RESISTANCE, REACTANCE AND SUSCEPTANCE PARAMETERS FOR PROPOSED METHOD USING 500 SAMPLES

Parameter	Confidence Interval Width		
	Phase A	Phase B	Phase C
R	±5.71%	±7.46%	±5.53%
X	±8.94%	±12.40%	±9.29%
B	±270.03%	±752.10%	±4346%

TABLE X
TEST 2: ESTIMATION ERROR USING 500 SAMPLES WHEN COMPARED TO THE ACTUAL PARAMETERS

	Parameters	Phase A	Phase B	Phase C
Proposed Method	R	2.30%	1.37%	3.59%
	X	5.48%	7.40%	3.42%
	B	15.54%	124.52%	107.13%

C. Test 3: Laboratory tests with unknown correction coefficients

The power quality laboratory at the university has a 4-core (3ph+1N) 70 mm^2 Al cable feeding a flexible power source to a number of modeled household connections via short 16 mm^2 (3ph+1N) Cu cables. The new proposed method was tested for its accuracy while estimating the impedance parameters of combination of the big Al cable and the Cu cable till the last household. The exact length of the main Al and smaller Cu cables are unknown. To set a reference, the DC resistance of the combined cable was measured at DC current levels from 1 A to 10 A. The voltage drop was measured by two multimeters at the start and the end of the cables. The reference DC resistance between the two ends of the cable system was 0.0935 ohms.

Voltage waveforms are measured at two ends of the line using two National Instruments voltage acquisition devices based on cRIO-9038 chassis. The line current was measured by a rogowsky coil. All the input channels of the cRIO chassis were times synchronized with an accuracy of ± 200 ns. Since the equivalent length of the cables is very small, the effect of charging capacitance would not come into picture. It was assumed that there would not be any measurable difference between the current at two ends and hence current measurement was done only at one end and the current difference equation (1) was ignored. Voltage and current phasors were calculated using the synchronized waveforms. Only the voltage difference equation (2) and hence (11) was used to make the system of linear equations. The 3-phase voltage difference equation can be written in matrix form as:

$$\begin{bmatrix}\delta V_a\\ \delta V_b\\ \delta V_c\end{bmatrix} = \begin{bmatrix} r_{aa}+jx_{aa} & r_{ab}+jx_{ab} & r_{ac}+jx_{ac}\\ r_{ab}+jx_{ab} & r_{bb}+jx_{bb} & r_{bc}+jx_{bc}\\ r_{ac}+jx_{ac} & r_{bc}+jx_{bc} & r_{cc}+jx_{cc}\end{bmatrix} * \begin{bmatrix}I_a\\ I_b\\ I_c\end{bmatrix}$$

that is :

$$\delta V_{abc} = Z_{abc} * I_{abc} \tag{13}$$

Figure 4 gives a basic overview of the lab cable and measurement set-up. At the source end, there is a flexible and controllable voltage source and at the load end, there is a controllable load bank. This set-up enables the testing of the parameter estimation algorithm under various conditions which affect the estimation accuracy such load unbalance, power flow variance and at different signal to noise ratio. This paper only presents the performance of the proposed method in the presence of non-calibrated measurement sensors.

As it was mentioned before, the underlying model of the system and the measurement describing the system are most important to estimate the parameters. For this lab cable, it was assumed that the

- Self-impedance of all the three phases and the return path (neutral) is same.
- Self resistance of single core of the cable (all phases and neutral) : r_s.
- Self reactance of single core of the cable (all phases and neutral) : x_s.
- Mutual coupling between all the phases is the same : x_m.
- Assuming low neutral current, the mutual coupling effect of neutral current on other phases are ignored.

Also from point of view of the measurement model is that:

- Sending (source) side voltage, is measured with respect to ground.
- Receiving (load) side voltage, is measured with respect to neutral.

The voltage difference equation then can be written with the help of above assumed r_s, x_s and x_m parameters. The current in neutral (I_n) is the summation of I_a, I_b and I_c. These conditions make the voltage difference equations as:

$$\delta V_a = I_a*(2r_s+2jx_s)+I_b*(r_s+jx_s+jx_m)+I_c*(r_s+jx_s+jx_m) \quad (14)$$

$$\delta V_b = I_a*(r_s+jx_s+jx_m)+I_b*(2r_s+2jx_s)+I_c*(r_s+jx_s+jx_m) \quad (15)$$

$$\delta V_c = I_a*(r_s+jx_s+jx_m)+I_b*(r_s+jx_s+jx_m)+I_c*(2r_s+2jx_s) \quad (16)$$

Using the equations (14)-(16), we can write the Z_{abc} in the form:

$$\tilde{Z}_{abc} = \begin{bmatrix} 2r_s + j2x_s & r_s + j\tilde{x}_m & r_s + j\tilde{x}_m \\ r_s + j\tilde{x}_m & 2r_s + j2x_s & r_s + j\tilde{x}_m \\ r_s + j\tilde{x}_m & r_s + j\tilde{x}_m & 2r_s + j2x_s \end{bmatrix} \quad (17)$$

Where the effective mutual reactance in phasor form $\tilde{x}_m = x_s + x_m$. Comparing $\tilde{Z}_{abc}$ with Z_{abc} matrix it can be seen that:

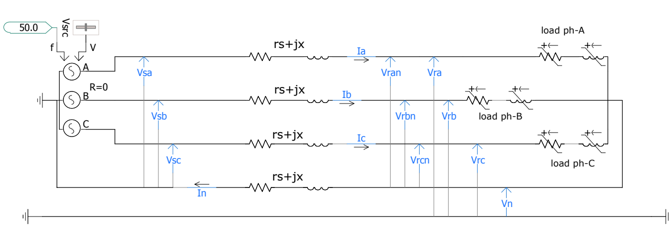

Fig. 4. Lab cable and measurement set-up.

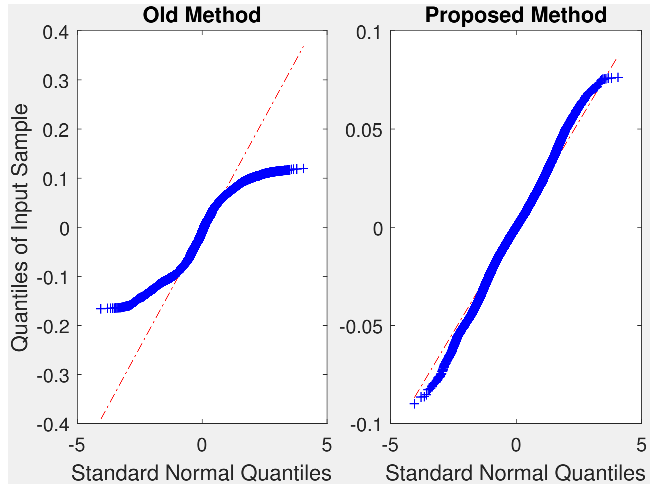

Fig. 5. Test 3: QQ-plot of the residuals for lab test data

$r_{aa} = r_{bb} = r_{cc} = 2r_s$
$r_{ab} = r_{bc} = r_{ac} = r_s$
$x_{aa} = x_{bb} = x_{cc} = 2x_s$ and,
$x_{ab} = x_{bc} = x_{ac} = \tilde{x}_m$

So to for accurate construction of the impedance matrix in phasor form (Z_{abc}) the parameters required to be estimated are: $r_s\ x_s\ and\ \tilde{x}_m$.

Without any information of present bias in the measurement sensors, the two methods were utilized for estimation of parameters of the cable system. No data re-sampling was done and the whole sample size was utilized. The QQ-plots of the residuals of the two tests are presented in Figure 5. The left side plot from the old method indicates that there is an unaccounted bias error present in the measurement system. It suggests that the results obtained from the old method would be less accurate than the results from the proposed method. The results in terms of expected values of the parameters and the CI (% of the mean value) are presented in the Table XI.

The $\tilde{Z}_{abc}$ matrix can composed with the estimated parameters using (17).

$$\tilde{Z}_{abc} = \begin{bmatrix} 0.190 + j0.0216 & 0.095 + j.0124 & 0.095 + j.0124 \\ 0.095 + j0.0124 & 0.190 + j.0216 & 0.095 + j.0124 \\ 0.095 + j0.0124 & 0.095 + j.0124 & 0.190 + j.0216 \end{bmatrix}$$

In sequence components it cab be written as,

$$\tilde{Z}_{012} = \begin{bmatrix} 0.380 + j0.046 \\ 0.095 + j0.009 \\ 0.095 + j0.009 \end{bmatrix}$$

TABLE XI
TEST 3: EXPECTED VALUE OF PARAMETERS FOR TOTAL NUMBER OF SAMPLES USING BOTH METHODS

Method	$r_s\ (ohms)$	$x_s\ (ohms)$	$\tilde{x}_m\ (ohms)$
Old	0.0966	0.0117	0.0120
Proposed	0.0951	0.0108	0.0124

The positive sequence resistance is quite close to the reference value measured by the DC measurement system. Also the fact that the zero sequence impedance is about 4 times the positive sequence impedance (especially for the resistance estimate) suggests also that the estimates are supporting the cable model assumed. The 3-phase 4-wire system with neutral as a return path it is known that,

$$Z_{0seq} = Z_{phase} + 3 * Z_{neutral} \tag{18}$$

And in this case, $Z_{phase} = Z_{neutral}$

The robustness of the proposed algorithm can also be supported by some extra tests done on manipulated lab measurements. Lab tests data was manipulated by adding bias in measured voltage signals at both ends. Three tests were done by adding a random bias to each phase of the voltage signals at both the ends. The maximum bias in terms of percentage of the original signals were 0.1%, 0.5% and 1.0%. The proposed method was applied to a randomly with-repetition sampled 500 samples. Residuals from all the sample-sets passed the normality tests and the best results were chosen based on the probability of normality of the residuals. The expected estimates and their respective CIs are mentioned in the Table XII.

TABLE XII
TEST 3: ESTIMATES AND CIS FOR SELF AND MUTUAL COMPONENTS OF $\tilde{Z}_{abc}$ USING PROPOSED METHOD WITH 500 SAMPLES

	$r_{aa}/r_{bb}/r_{cc}$		$x_{aa}/x_{bb}/x_{cc}$		$x_{ab}/x_{ac}/x_{bc}$	
max bias	mean	CI %	mean	CI %	mean	CI %
0.1 %	0.1875	±1.82	0.0193	±17.88	0.0126	±10.57
0.5 %	0.1890	±1.80	0.0214	±16.95	0.0124	±11.39
1.0 %	0.1867	±1.64	0.0216	±16.42	0.0129	±11.39

From the above results it was shown that the proposed method is robust in presence of bias errors. The estimates and the CIs did not vary a lot even in presence of increasing bias errors. In presence of the same bias, the old method could not clear the test for normality. With only 500 samples, the estimates from the old method had a very wide CI suggesting that the estimates were unreliable.

V. CONCLUSION

The paper presents a new method for estimating the line parameters R, X and B. In conditions where the correction coefficients of CTs and VTs are unknown, the proposed method can estimate self and mutual components of resistance and inductive reactance along with self susceptance with a better accuracy when compared to the generally applied old method. One drawback is that it can't estimate the mutual components of the line susceptance B. This method is also suitable for making real time estimates of the line parameters. Validation of the parameter estimates by analyzing the residuals was discussed and shown in detail. The results were also analyzed with the point of view of bias and standard deviation. In the end results from a laboratory test to estimate the parameter of a cable system was presented. It was again shown that the proposed method was able to achieve more reliable and precise estimates in the presence of bias errors. This means that the proposed method is better suited for real-field 3-phase line parameter estimation where the correction coefficient of the instrument transformers is not known.

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