

# Energy Storage Placement in the Transmission Network: A Robust Optimization Approach

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**Abstract**— The market liberalization and renewable energy integration, increasing the uncertainty in operational planning, which implies more significant and more frequent deviations from schedules. A lack of proper decision-making tool, such as OPF, can impede the integration of renewable energy and system flexibility. A promising approach to model the OPF dealing with uncertainty is robust optimization (RO). Typically, a robust optimization model has a dual goal, generating performance as well as system reliability under uncertainty. This paper discusses an RO-based AC-OPF model for finding the optimal location of the energy storage system in the transmission network. An AC multi-temporal OPF algorithm that adopts a convex relaxation of the power flow equations to guarantee precise and optimal solutions with high algorithmic performance has been used. The developed model has been validated for storage planning on the IEEE-14 bus network.

**Keywords** — energy storage, robust optimization, uncertainty, transmission network planning.

## I. INTRODUCTION

With high intermittent energy penetration, more operating reserves are required, and the ancillary service cost increases. Also, due to the ramp rate limitation of conventional generators, fast wind power fluctuations can hardly be compensated within a short period.

Grid-scale storage contains a number of technologies that cover a wide range of time-scales, costs, spatial footprints, and capacity. Identifying the appropriate size, siting, and technology is imperative for getting the full potential of storage in renewable energy integrated power grids [1]. Many of these problems are application-specific, and therefore, the study linked to storage capacity requirements has often concentrated on specific problem sets. For instance, isolated systems with high renewable energy penetration have been explored in several situations [2]–[4]. Other studies have dedicated to characterizing the amount of storage required to compensate for short [5] and long-term [6] energy shortages and to substitute fast response generators in single bus systems.

Regarding optimal siting of the energy storage problem, many references proposed that the energy storage systems (ESS) should be installed on the site coupling with Wind Farms (WFs) [1]. It has been shown that the potential siting of ESS can bring benefits such as deferring or avoiding capacity and transmission upgrade, reduced transmission and distribution losses and more robust system reliability [7]–[10].

While the majority of the works on storage planning used a DC network model [10], [11]–[14], there have been some noteworthy contributions that adopt an AC formulation [15]–[18]. In [15] developed a dynamic AC-OPF planning solution where electricity from wind energy is stored at night and released during the day. The model based on AC OPF leads to increased accuracy in the planning decision. However, the resulting nonlinear programming model may unveil

convergence problems; moreover, there is no guarantee that the solution is global. In [16]–[18] proposed relaxed models based on semidefinite programming (SDP) relaxation of the dynamic storage planning problem through an AC model. The solution to the SDP relaxation is globally optimal and feasible to the planning problem providing that the duality gap is zero. Besides, the SDP solution does not scale well with the problem size. The exploitation of the AC model, therefore, necessitates further research and development for general use in a planning tool.

In regard of power system planning, the scenarios are commonly obtained from load forecasts, which are formed based on load demand and econometric models that use historical data, together with economic forecasts and inputs from government sources. Utilities perform annual long-term load forecasting yearly for planning and investment. That being said, planning tools need to consider uncertainty in the model. One of the promising approaches is robust optimization. The necessity for robust optimization solutions has been acknowledged in several applications [19]–[22]. It is also favored in storage investment planning over stochastic optimization, where practical applications require accurate statistical models that are usually unavailable [23].

In this paper, an energy storage planning tool based on robust optimization has been proposed. In order to have an efficient algorithm and to maintain convex formulation, a second-order cone programming (SOCP) relaxation adopted. The methodology has been tested on the IEEE-14 bus network.

## II. MATHEMATICAL FORMULATION

The proposed methodology for storage investment in the transmission network based on a multi-period AC OPF algorithm given by constraints (1)–(22). The objective function of storage planning consists of the daily cost of operating conventional generations ( $P_n^g$ ), the storage investment cost ( $SC_n$ ) per interest period and penalty terms of wind curtailment ( $P_n^{wc}$ ). Therefore, the aim is to reduce the conventional generation cost, storage investment, and wind curtailment.  $C_n^g$  and  $C_{wc}$  are the specific costs of conventional generator production and the wind curtailment penalty, respectively. The storage investment cost ( $SC_n$ ) is proportional to the number of storage installed ( $\Gamma_{n,ESS}$ ) at each node  $n$  and their size in terms of rated power and rated energy. The total storage investment cost appears in the objective function multiplied by a factor  $K_s$  that defines the capital recovery factor of storage investment.

Furthermore, in this study, the operational cost of batteries has been disregarded, as well as the impact of the charging/discharging cycles on the batteries life. Actually, a term that takes into account the depreciation of the storages due to their use could be included, but in this paper, this cost is assumed negligible. The model encapsulates the voltage drop of each node by constraints (2), while it does not

explicitly consider the network energy losses, because assumed within the permissible range and thus not valuable as a penalty for the system operator.

$$\min C_{tot} = \min \left\{ \sum_{n=1}^N \sum_{t=1}^{24} C_n^g * P_n^g(t) + \sum_{n=1}^N \sum_{t=1}^{24} C_{wc} * P_n^{wc}(t) + \sum_{n=1}^N \sum_{t=1}^{24} VOLL * L_n^{sh}(t) + \frac{K_s}{365} \sum_{n=1}^N SC_n \right\} \quad (1)$$

Here, the VOLL represents the value of lost load and  $L_n^{sh}$  is the amount of load shedding.

The voltage drop and corresponding current flow in the branch  $mn$  can be calculated by (2) and (3) respectively

$$V_m - V_n = I_{mn}(R_{mn} + jX_{mn}) \quad (2)$$

$$I_{mn} = \left( \frac{P_{mn} + jQ_{mn}}{V_n} \right)^* \quad (3)$$

The current flow on the branch  $mn$  can be placed in (2) to obtain the following equation:

$$(V_m - V_n)V_n^* = (P_{mn} - jQ_{mn})(R_{mn} + jX_{mn}) \quad (4)$$

Considering the associated voltage angle of each bus (4) can be written as

$$V_m V_n (\cos \theta_{mn} + j \sin \theta_{mn}) - V_n^2 = (P_{mn} - jQ_{mn})(R_{mn} + jX_{mn}) \quad (5)$$

From (5), identifying the real and imaginary parts and squaring them, the following equation can be derived, which can be used to obtain the voltage across the branch  $mn$

$$V_m^2 = V_n^2 - 2(R_{mn}P_{mn} + X_{mn}Q_{mn}) + (R_{mn}^2 + X_{mn}^2)I_{mn}^2 \quad (6)$$

The magnitude of the current flow  $I_{mn}^2$  can be obtained as

$$I_{mn}^2 = \frac{P_{mn}^2 + Q_{mn}^2}{V_n^2} \quad (7)$$

Here (8)-(9) represent the active and reactive power flow of this optimal power flow problem

$$P_n^g(t) + P_n^d(t) - P_n^c(t) - P_n^{wc}(t) - PD_n(t) + P_w(t) - R_{mn}I_{mn}^2 = \sum_{m \in \theta_{12}} P_{mn}(t) \quad (8)$$

$$Q_n^g(t) - QD_n(t) - x_{mn}I_{mn}^2 = \sum_{m \in \theta_{12}} Q_{mn}(t) \quad (9)$$

$$P_{mn}^2(t) + Q_{mn}^2(t) = S_l^2(t) \quad (10)$$

$$i_{mn}(t)v_{mn}(t) = S_l^2(t) \quad (11)$$

In this multi-temporal AC OPF model, the Second Order Cone Programming (SOCP) convex relaxation has been used. This relaxation necessitates the relaxation of certain equality constraints and replacing certain quadratic terms for linear

terms. The equality constraints in this model, constraint (7), (10) and (11) are relaxed ultimately by relaxing the magnitude of currents within each branch and using a conic formation on the limitation of exchanged active power. For linearization purposes, voltage and current magnitude has been considered as  $I_{mn}^2 = i_{mn}$  and  $V_{mn}^2 = v_{mn}$  with including new variables ( $i_{mn}, v_{mn}$ ) in order to replace the quadratic terms and successfully formulate the SOCP problem.

$$V_{min}^2 \leq v_n(t) \leq V_{max}^2 \quad (12)$$

$$P_n^{min} \leq P_n^g(t) \leq P_n^{max} \quad (13)$$

$$Q_n^{min} \leq Q_n^g(t) \leq Q_n^{max} \quad (14)$$

$$-RR_n \leq P_n^g(t) - P_n^g(t-1) \leq RR_n \quad (15)$$

The constraint (12) provides the voltage limits of each bus. The constraints (13)-(15) are associated with conventional generators, which includes the limits of active and reactive power capacity and ramping capability of each unit.

**The Storage dynamics**

$$SOC_n(t) = SOC_n(t-1) + \left( P_n^c(t) * \eta_c - \frac{P_n^d(t)}{\eta_c} \right) \quad (16)$$

$$0 \leq P_n^c(t) \leq \alpha_n^c * P_n^{c,max}(t) \quad (17)$$

$$0 \leq P_n^d(t) \leq \alpha_n^d * P_n^{d,max}(t) \quad (18)$$

$$SOC_{n,min} \leq SOC_n(t) \leq SOC_{n,max} \quad (19)$$

$$\alpha_n^c(t) + \alpha_n^d \leq 1 \quad (20)$$

$$\alpha_n^c(t) \in [0 \text{ or } 1] \text{ and } \alpha_n^d \in [0 \text{ or } 1] \quad (21)$$

$$\sum_n N_{ESS,n} \leq N_{ESS,n}$$

The state of charge (SOC) of energy storage devices is illustrated by (16) that accounts the initial state of charge and charging and discharging efficiencies, constraints (17)-(19) are the charging and discharging limits of the storage unit and SOC of each node at time  $t$ . The variables  $\alpha_n^c$  and  $\alpha_n^d$  are the binary variables, during charging  $\alpha_n^c = 1$  and 0 otherwise. The similar analogy goes for discharging period. To restrict the simultaneous charging and discharging of the storage system, constraint (20) has been imposed. It is worth mentioning that during the estimation of charging and discharging power of storage unit, a quadratic term has arisen due to the multiplication of binary and integer variables.

The constraint (21) represents the storage investment budget in terms of the allowed storage unit. An additional constraint is added to avoid daily accumulation effects by forcing the state of charge of the first and last time step of a day to be equal as represented by (22), where the T represents the last time step of the day.

$$SOC_{n,0} = SOC_{n,T} \quad (22)$$

### III. UNCERTAINTY MODELING

Since it is utmost interest to involve the uncertainties in the model while studying the investment of new technologies such as energy storage in the network, the robust optimization (RO), a promising method to deal with uncertainty, has been used. Unlike stochastic optimization, which assumes to have an exact probability distribution of the uncertain data to be known, RO only assumes that the uncertain data resides in a defined uncertainty set. As it can be understood that the choice of uncertainty set is significant in order to have an acceptable solution within a given conservatism.

Therefore, the first step to applying the robust approach is to specify the uncertainty set. The uncertainty set is the set of

values for the uncertain parameters that are taken into consideration in the robust model. The selection of uncertainty solely depends on the available information of uncertain parameters and the level of robustness the acceptable by the decision-maker. It is often reasonable to make a compromise between the robustness against each physical realization of the uncertain parameter and the size of the uncertainty set. The most robust uncertainty set which guarantees that the constraints are never violated is the box uncertainty set. However, this kind of sets only considers the worst scenarios that often make the model very conservative and leads to an unacceptable solution. The box uncertainty set can be represented as:

$$U = \{\xi \in R^L : \|\xi_\infty\| \leq 1\}$$

Here  $\xi$  is the only knowledge available known as perturbation vector that varies inside a given interval.  $R^L$  represents the real number with the dimension of  $L$  and  $\|\xi_\infty\|$  defined the continuous uniform norm of  $\xi$ . Since box uncertainty set is often too pessimistic, two other uncertainty sets are used in practice. One of them is the ellipsoidal uncertainty set and polyhedral uncertainty set. The ellipsoidal uncertainty set can be defined as,

$$U = \{\xi \in R^L : \|\xi_\infty\| \leq \Omega\}$$

The ellipsoidal uncertainty set can be assumed as the sphere of radius  $\Omega$  centered at the origin. This kind of uncertainty sets leads to a better objective value for a fixed probability guarantee. However, it could lead to a quadratic constraint from a linear problem. Another uncertainty set which considers being tractable from a computational point of view is polyhedral uncertainty set, and it can be expressed as,

$$U = \{\xi \in R^L : \|\xi_\infty\| \leq \Gamma\}$$

This type of uncertainty set, also called as a budgeted uncertainty set since the level of robustness can be adjusted with  $\Gamma$ . It is essential to accurately select the budget of uncertainty,  $\Gamma$  in order to have a reasonable solution maintaining sufficient robustness of the model. In this work, the polyhedral uncertainty set has been adopted since it produces sufficiently robust solution if the budget of uncertainty is chosen based on the uncertainty level one wants to accept.

#### IV. SOLUTION APPROACH

Robust optimization problem usually contains an infinite number of constraints due to imposing worst-case formulation and hard constraints. Therefore it is often computationally intractable of its present form. Usually, there are two ways to deal with this kind of situation. One of the ways is to apply robust reformulation techniques to make the formulation immune of all the uncertain parameters.

Moreover, another way is to apply the adversarial approach. This approach starts with a finite set of scenarios, which only contains the nominal scenarios. Then the robust optimization problem will be solved for the finite set of scenarios; if the resulting solution is not robust, it is necessary to search for a scenario that maximizes the infeasibility. When the scenario of maximum infeasibility is found, this scenario will be added to the uncertainty set and solve the robust optimization problem.

In this work, the analytical robust reformulation approach has been adopted. Since the uncertainty is constraint-wise, the

reformulation will only deal with constraints that contain the uncertain parameters.

The structure of the model is the same, and the objective function is identical as in the deterministic model. The changes will arise in the constraints containing the uncertainty. In this work, two uncertain parameters are considered: the forecasted real power of wind farms  $P_w$  and demand  $PD_n$ . The uncertain parameter will be expressed with tilde ( $\sim$ ),  $\widetilde{P}_{w,t}$  and  $\widetilde{P}_{D,t}$  represent the uncertain wind power output and load demand at each time step  $t$ , respectively. The deviation from the predicted values and its bound for wind generation and load demand can be expressed respectively as  $\Delta P_w$  and  $\Delta P_D$ . The polyhedral uncertainty set for wind power output is defined as:

$$U^w = \{\widetilde{P}_{w,t} : \xi_{w,t} \in R^L \text{ s.t. } \|\xi_\infty\| \leq \Gamma_w\}$$

$$\widetilde{P}_{w,t} \in [P_{w,t} + \Delta P_w \xi_{w,t}]$$

Where  $\xi_w$  is the degree of uncertainty of the wind power output. In other words, it can be considered as the quantification of the actual deviation from the forecasted value  $P_w$  and it belongs to the interval  $[-1; 1]$ ;  $\Gamma_w$  is the budget of uncertainty of the total wind generation of the system. The value of  $\Gamma_w$  lies between 0 to 1, where 0 being the deterministic case and 1 defines the most robust case.

The similar formulation will be applied to load demand uncertainty. The uncertainty set of load demand expressed as:

$$U^D = \{\widetilde{P}_{D,t} : \xi_{D,t} \in R^L \text{ s.t. } \|\xi_\infty\| \leq \Gamma_D\}$$

$$\widetilde{P}_{D,t} \in [P_{D,t} + \Delta P_{D,t} \xi_{D,t}]$$

The representation and the associated value of the parameters are similar to wind generation.  $\xi_D$  ( $-1 \leq \xi_D \leq 1$ ) and  $\Gamma_D$  ( $0 \leq \Gamma_D \leq 1$ ) are the degree and budget of the uncertainty of load demand, respectively. The larger  $\Gamma$ , the larger the uncertainty set, and the larger the worst case value of the uncertain component of the constraint.

The reformulation process is composed of three main steps, (i) Worst case reformulation, (ii) Duality and (iii) Robust counterpart.

##### Worst case reformulation

First, it is vital to identify the maximum deviation from the nominal value and rewrite the constraints that are affected by the uncertain parameters such as a way that it considers the possible worst case. In the worst case of maximum load and minimum wind, the value of  $\xi_D$  and  $\xi_w$  will be 1 and -1 respectively. Moreover, each uncertain parameter will be solved separately.

##### Demand:

$$\max(\Delta P_{D,t} * \xi_{D,t}) \quad (23)$$

$$\text{s.t. } |\xi_{D,t}| \leq \Gamma_D \quad (24)$$

$$0 \leq |\xi_{D,t}| \leq 1 \quad (25)$$

In order to eliminate the absolute term in the above formulation, an additional variable  $M_{D,t}$  ( $M_{D,t} \geq 0$ ) can be introduced and the new formulation becomes,

$$\max(\Delta P_{D,t} * \xi_{D,t}) \quad (26)$$

$$\text{s.t. } M_{D,t} \leq \Gamma_D \quad (27)$$

$$M_{D,t} \geq \xi_{D,t} \quad (28)$$

$$-1 \leq \xi_{D,t} \leq 1 \quad (29)$$

##### Wind:

$$\min(\Delta P_{w,t} * \xi_{w,t}) \quad (30)$$

$$\text{s.t. } |\xi_{w,t}| \leq \Gamma_w \quad (31)$$

$$|\xi_{w,t}| \leq 1 \quad \forall w \quad (32)$$

As mentioned above for demand, a new variable  $M_w (M_w \leq 0)$  will be added to relax the absolute term.

$$\min(\Delta P_{w,t} * \xi_{w,t}) \quad (33)$$

$$s. t. M_{w,t} \geq -\Gamma_w \quad (34)$$

$$M_{w,t} \leq \xi_{w,t} \quad \forall w \quad (35)$$

$$-1 \leq \xi_{w,t} \leq 1 \quad \forall w \quad (36)$$

Once the worst case has been considered in the formulation that means the above formulation maximizes the deviation of the demand and minimizes the deviation of wind and optimized utilizing inner minimization/maximization problems. In order to make the problem tractable, the next step is to find the dual of the inner minimization problems.

*Forming Dual:*

Since the inner minimization/maximization problems and its dual yield the same optimal objective value by strong duality theorem, therefore, the problem described above can be reformulated as follows:

Demand:

$$\min\{Q_{D,t} * \Gamma_D + G_{D,t} + I_{D,t}\} \quad (37)$$

$$s. t. R_{D,t} + G_{D,t} - I_{D,t} = \Delta P_{D,t} \quad (38)$$

$$Q_{D,t} - R_{D,t} \geq 0 \quad (39)$$

Where  $Q_D, G_D, I_D$  and  $R_D$  are the dual positive variables respectively associated with the primal optimization problem. It is worth mentioning that the dual variables do not have any physical meaning.

Wind:

$$\max\{-Q_{w,t} * \Gamma_w - G_{w,t} - I_{w,t}\} \quad (40)$$

$$s. t. R_{w,t} - G_{w,t} + I_{w,t} = \Delta P_{w,t} \quad \forall w \quad (41)$$

$$Q_{w,t} - R_{w,t} \geq 0 \quad \forall w \quad (42)$$

If the different worst-case has been considered, the only changes in the formulation would be the sign of the variables, and the structure of the formulation would be the same.

*Forming the robust counterpart:*

It is worthy to point out that it is possible to omit the minimization/maximization terms since it is sufficient that the constraints hold for the defined uncertainty limit. The inner optimization problems for demand and wind are set to be integrated into the main deterministic model. To obtaining the robust counterpart of the deterministic model, the objective functions of the dual form have to be added in the respective constraint, and the constraints of the dual form will need to be included in the algorithm. All the constraint containing uncertain parameter can be replaced with linear constraints without uncertainty and converted into a mixed-integer form. Since in the main deterministic model, only one constraint is affected by uncertainty, namely the power balance equation (8), the reformulated power balance equation will become,

$$\begin{aligned} & P_n^g(t) + P_n^d(t) - P_n^c(t) - P_n^{wc}(t) - PD_n(t) - Q_{D,t} \\ & \quad * \Gamma_D + G_{D,t} + I_{D,t} + P_w(t) - Q_{w,t} \\ & \quad * \Gamma_w - G_{w,t} - I_{w,t} - R_{mn} I_{mn}^2 \\ & \geq \sum_{m \in \theta_{12}} P_{mn}(t) \end{aligned} \quad (43)$$

In addition, the additional constraints for wind and load demand that will be added in the algorithm are:

$$R_{D,t} + G_{D,t} - I_{D,t} = \Delta P_{D,t} \quad (44)$$

$$Q_{D,t} - R_{D,t} \geq 0 \quad (45)$$

$$R_{w,t} - G_{w,t} + I_{w,t} = \Delta P_{w,t} \quad \forall w \quad (46)$$

$$Q_{w,t} - R_{w,t} \geq 0 \quad \forall w \quad (47)$$

The new model (1-22, 43-47) does not contain any uncertainty and is a mixed-integer second-order conic programming (MISOCP) problem that can be solved efficiently using CPLEX that uses a branch and cut algorithm to find the integer feasible solution.

## V. CASE STUDY

The numerical studies in this section are based on the IEEE14-bus benchmark system (Figure 1). The time horizon of 24h has been considered with a time step of 1h. The IEEE 14-bus network has five wind farms connected at nodes 1, 2, 3, 6 and 8, with each having an installed capacity of 30MW. The wind generation profiles were simulated by assuming that the wind speed is Weibull distributed with a scale factor of 11.01 m/s and a shape factor of 2 m/s. The load profile, expressed as a percentage of the annual peak load in the original data set, was computed using specific multiplying coefficients for the week of the year, for the day of the week, and the hour of the day.

The mathematical formulation of the robust algorithm for an AC OPF based energy storage planning tool has been programmed in GAMS and solved using CPLEX on a 2.30 GHz personal computer with 4GB RAM. The simulations have been performed with an optimality gap of 0.001. In this experimental study, the worst case has been considered when the load is high ( $\xi_{D,t} = 1$ ), and wind generation is low ( $\xi_{w,t} = -1$ ).

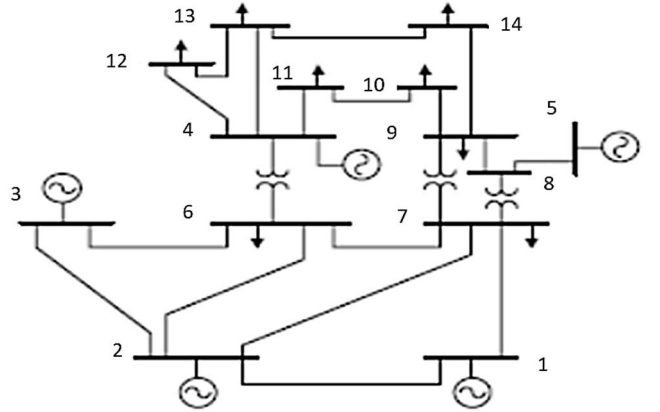


Figure 1 IEEE-14 bus network

### Daily Operational Cost

Table 1 summarizes the daily operational cost of the dispatch from conventional generators for three different scenarios. A significant reduction of daily operational cost has been observed with the inclusion of storage. The storage devices in this system help to avoid wind curtailment and reduce conventional energy usage. As described in the next section, Storage requirements for a robust problem would always be higher than a deterministic case. Hence, the operational cost would always be higher. In the deterministic model, consideration of uncertainty is avoided by assuming perfect information for all parameters. However, even considering the robust scenario that includes the worst case, the operational cost is much lower compared to the no storage case.

TABLE 1 DAILY OPERATIONAL COST OF 14-BUS NETWORK

Cases	Daily Operational Cost(€)
No storage	120104.39
With Storage(Deterministic)	16153.74
With Storage(Robust)	26741.42

*Storage Allocation*

All the buses of this 14-bus network were assumed candidates for storage placement. The available storage devices were considered of 20MW/20MWh storage capacity. The efficiencies for charging and discharging were considered 90% each, which gives an overall roundtrip efficiency of 81%. The cost Lithium-ion technology in this study has been considered as €200 for each kW of rated power, €400 for each kWh of rated energy and capital recovery factor,  $K_s = 0.01$  assuming a planning horizon of 10 years.

TABLE 2 DAILY OPERATIONAL COST AND ENERGY STORAGE ALLOCATION (WITH VARYING ROBUSTNESS)

Budget of Uncertainty	Number of Storage	Location	Average Daily Dispatch cost(€)
$\Gamma = 0$	4	3,4,10,14	16153
$\Gamma = 0.25$	5	3,4,9,10,14	18428
$\Gamma = 0.50$	7	3,4,6,7,9,10,14	21057
$\Gamma = 0.75$	8	3,4,6,7,9,10,13,14	23959
$\Gamma = 1$	10	3,4,6,7,9,10,12,13,14	26741

In table 2, it shows how the investment of storage and daily operational cost change with changing the robustness. The deterministic case ( $\Gamma = 0$ ) suggests 4 storage devices be installed in the network. As the budget of uncertainty increases, the number of storage increases and more charging/discharging cycles that leads to increased energy losses, therefore the daily operational cost also increases.

Since different scenario can be generated with the different budget of uncertainty, and the algorithm provides the storage and operational cost information, it would be useful for the decision-maker to take a compromised decision. The most robust case ( $\Gamma = 1$ ) may be avoided since it considers the worst case, and it may be very unlikely to happen.

*Contribution of Storage during the peak load*

In order to comprehensively understand the contribution of storage in the power flow, node 4 of Fig. 1 has been selected since it is the most representative node in terms of line loading during the peak period. As it has shown in figure 2 and 3, the apparent power flows reach their maximum capacity during the peak period. The storage helps to reduce the peak line flows. Besides, it helps to distribute the power flow in such a way that no branch reaches its maximum capability. Though the deterministic case mostly flattens the load curve, the robust case is more fluctuating to maintain the power flow in the limit.

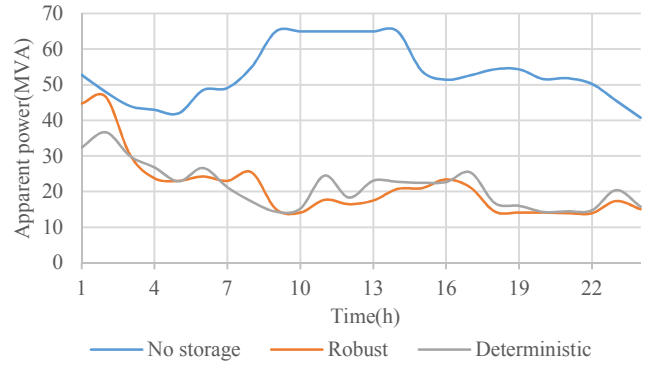


Figure 2 Power flow at branch 4-2 (with capacity 65MVA)

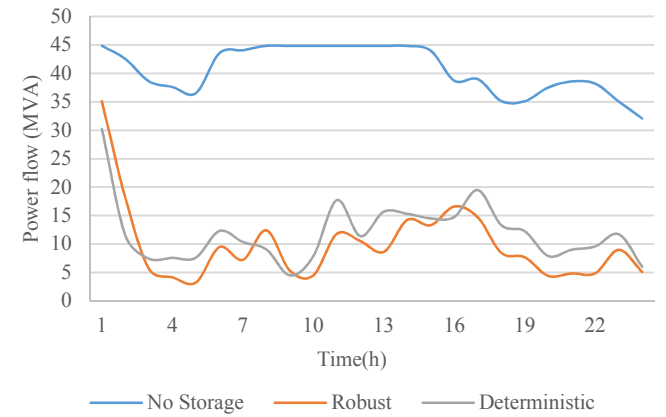


Figure 3 Power flow at branch 4-5 (with capacity 45MVA)

*Storage Activity*

In both deterministic and robust cases, the storage at bus 9 behaves similarly except during the peak period where storage activity changes abruptly for the robust case. It is worthwhile to mention that in the case of planning the robust solution is quite different from the deterministic one. Since, the number of storage system increases in the robust scenario, the operation of individual storage in the network remain analogous for both robust and deterministic cases. Therefore, the pattern of charging and discharging is quite similar. Although the real change of operation due to the robust approach can be understood from the daily, operational cost that changes with robustness.

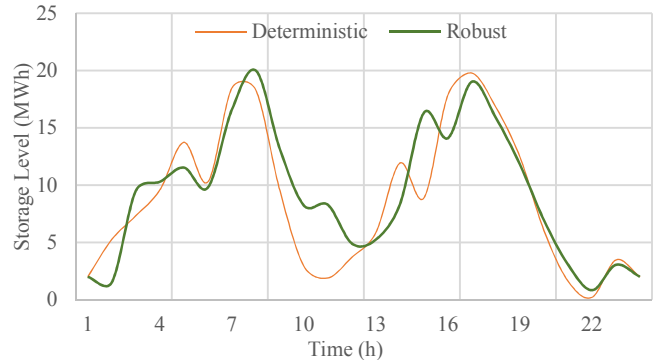


Figure 4 Storage Activity at Bus 9

## VI. CONCLUSION

In this study, a systematic approach of applying robust optimization on an AC OPF based siting of energy storage devices in the transmission network has been demonstrated. Since DC OPF neglects the transmission losses and may lead to an infeasible planning solution, the use of AC OPF in this study aimed at increasing accuracy in the planning.

Preliminary results prove that inclusion of load and wind uncertainties in the problem significantly increases the operational and future planning costs of the system, which indicates the need to include uncertainties in planning.

Since the polyhedral uncertainty set has been exploited to represent the uncertainty sets, it allows having the flexibility to do a trade-off between economic efficiency and conservatism. The integration of this kind of flexibility also helped to observe the scenario other than the worst-case scenario that most of the robust optimization problem does not consider.

The robust optimization approach aims at efficiently incorporating the uncertainty in the model. Unlike many robust optimization problems, which mainly considers the worst-case scenario thus do not provide the optimal solution rather a very conservative solution which could be unrealistic based on the problem. However, the analytical reformulation technique helped to find the robust counterpart of the original problem that was solved with less computational burden using CPLEX.

Since planning involves limited economic budget and resources, this study will provide a comprehensive view which is a combination of different scenario (budget of uncertainty).

It should be mentioned that this proposed methodology can be used with different energy storage technologies by considering related costs and performance parameters such as charging/discharging efficiencies.

## ACKNOWLEDGMENT

Nayeem Chowdhury has been funded from the European Union's Horizon 2020 research and innovation programme under Grant Agreement No 676042.

The contribution of G. Pisano to this paper has been conducted within the R&D project "Cagliari2020" partially funded by the Italian University and Research Ministry (grant# MIUR\_PON04a2\_00381).

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