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COST EVALUATION OF A TWO-ECHELON INVENTORY SYSTEM WITH LOST SALES AND NON-IDENTICAL RETAILERS

Abstract. *The inventory system under consideration consists of one central warehouse and an arbitrary number of non-identical retailers controlled by continuous review policy* (R, Q)*. It is assumed Independent Poisson demands with constant transportation times for the retailers and constant lead time for replenishing orders from an external supplier for the warehouse. Unsatisfied demands are assumed lost at the retailers and unsatisfied retailer orders are backordered at the warehouse. An approximate cost function is developed to find optimal reorder points for given batch sizes in all installations and the related accuracy is assessed through simulation. The proposed method is an extension to the approximate assumption of Poisson demand on the warehouse previously and adds more approximations to tackle retailer's lead time complexity.*

Keywords: *Inventory; Multi-echelon; Lost sales; Non-identical retailers; Poisson demand.*

JEL Classification: M11, C61, C63

1. Introduction

A two-echelon inventory system consists of one central warehouse and an arbitrary number of non-identical retailers, is considered. Retailers demand for a consumable item at the warehouse. The inventory control policy is assumed continuous review policy *(R,Q)* in all installations which means that once the inventory position reaches a predetermined value of R an order of size Q is placed. The demand processes is assumed independent Poisson at the retailers and unsatisfied demands are lost. The transportation time of each order placed by the retailers is assumed constant. A constant lead time for replenishing the warehouse orders from an external supplier is assumed and unsatisfied retailers orders is backordered at the warehouse and all backordered orders are filled according to a FIFO-policy. The reorder point and batch size of the warehouse are assumed integer multiples of the retailers identical batch size.

Reviewing researches closer to the area of divergent multi echelon inventory systems whose structure typically consisting of one central warehouse (depot) and an arbitrary number of retailers (bases), implies that multi-echelon inventory systems literature is rich enough. Sherbrooke (1986) is one of the initial researches in this area. Assuming $(S-1,S)$ policies in a Depot-Base system for a repairable item, the average unit years of inventory and stockout at the bases is estimated. Most of the researches in the 1980s concentrated on the repairable items in a Depot-Base system. Graves (1985) determined the stocking levels in such a system. Moinzadeh and Lee (1986) considered the issue of determining the optimal order batch size and stocking levels at the stocking locations using a power approximation, Lee and Moinzadeh (1987) generalized previous models on multi-echelon repairable inventory systems to cover the cases of batch ordering and batch shipment. On consumable items, Deuermeyer and Schwarz (1981) proposed a simple approximation for a complex multi-echelon system (one warehouse and multiple retailers) assuming backordering of unsatisfied demands in all installations with a batch ordering policy .Svoronos and Zipkin (1988) proposed several refinements on the latter paper considering second-moment information (standard deviation as well as mean) in their approximations.

The literature on multi-echelon inventory systems with consumable items continued in the 1990s. Axsäter (1990) provided a simple recursive procedure for determining the holding and stockout costs of a system consisting of one central warehouse and multiple retailers with (S-1, S) policy, independent Poisson demands at the retailers ,backordered demand during stockouts in all installations and constant lead times. Axsäter (1993) proposed exact and approximate methods for evaluating the previous system for the case of a general batch size in all installations but with identical retailers. For the case of non-identical retailers and a general batch size, Axsäter (1998) proposed methods for the exact evaluation of two retailers 'case and the approximate evaluation for the case of more than two retailers. Forsberg (1996) presented a method for exactly evaluating the costs of the system with one central warehouse and a number of different retailers using a different approach. Axsäter & Marklund (2008) considered the two-echelon inventory system and derived a new policy for the warehouse ordering, which was optimal in the broad class of position-based policies relying on complete information about the retailer inventory positions, transportation times, cost structures and demand distributions at all facilities. The exact analysis of the new policy included a method for determining the expected total inventory holding and backorder costs for the entire system.

The common assumption of the above papers is that demand during stockout at the retailers, are backordered. However, on some conditions for example in non-captive markets demands may be lost. Andersson and Mechiors (2002) proposed an approximate cost function for the structure of one central warehouse and arbitrary number of identical retailers assuming lost sales during the stock out at the retailers and $(S-1,S)$ control policy in all installations. Unsatisfied retailers, orders are backordered at the warehouse as the former researches. Seifbarghy and Akbari

Jokar (2006) considered the inventory system with one central warehouse and many identical retailers controlled by continuous review policy. They assumed independent Poisson demands with constant transportation times for the retailers and a constant lead time for replenishing orders from an external supplier for the warehouse. An approximation cost function to find optimal reorder points for given batch sizes in all installations was proposed. Non-identical retailers case with lost sales during stockout at the retailers is addressed as an extension by Seifbarghy and Akbari Jokar (2006). This is exactly what is considered in this paper.

Field of inventory management and systems has been exposed a lot of interest by authors such as: Nita H. Shah and Chirag J. Trivedi (2007) develop an order level lot size inventory model for exponentially deteriorating inventory with random lead time and supported it with a numerical example. Cristina Fulga and Florentin Setban (2008) present a method to solve a deteriorating multi-item inventory model with limited storage space and an assurance stock. The demand rate for the items is finite, the items deteriorate at constant rates and are replenished instantaneously. The model is solved by a non-linear programming method. Nitah H. Shah (2006) use an EOQ model and a comparison between existing deteriorating items models and effect of various parameters on the total cost of an inventory system has been studied by him. Nita H. Shah (2008) presents inventory policies for deteriorating items under incentives of price discount for one time only. It is quite a common practice to offer special discount to motivate the buyer to order in larger than regular order quantities. Such special sales are available for a limited time only.

Our paper will be considered the inventory management from different aspects.

We now outline the contents of this paper. In Section 2 the problem formulation is given. Section 3 contains the review of two special single echelon problems. Section 4 explains problem complexities. In Section 5 the approximate total cost of the inventory system is presented. In Section 6 a genetic algorithm is proposed to optimize the total cost and finding optimal reorder points. In section 7 some numerical problems is given to measure the accuracy of the approximation and in Section 8 some conclusions and further research is given.

2. Problem Formulation

The common batch size of the retailers and the batch size of the central warehouse are assumed predetermined as many similar previous works such as Axsäter (1993 and 1998), Deuermeyer and Schwarz (1981) have done before to simplify the problem. The objective is to find the optimal reorder points through minimizing the total holding cost of the warehouse and retailers and stockout costs of retailers. The notation is as follows:

N : Number of retailers

λ_i : Demand rate at retailer i

λ_O : Demand rate at the warehouse

L_i : Transportation time for deliveries from the warehouse to retailer i

L_O : Lead time of the warehouse orders

Q_r : Common batch size of a retailer
Q_O : Batch size of the warehouse
R_i : Reorder point of retailer i
R_O : Reorder point of the warehouse (an integer multiple of Q_r)
h_r : Holding cost per unit per unit time at a retailer
h_O : Holding cost per unit per unit time at the warehouse
π_r : Penalty cost per unit of lost sale at a retailer
C_i : Cost per unit time of retailer i in steady state
C_O : Cost per unit time of the warehouse in steady state
TC : Total cost of the inventory system per unit time in steady state

3. Review of two special cases

3.1. Review of exact solution for backordering problem with Poisson demand

Considering a single echelon inventory system with continuous review control policy, reorder point of R and batch size of Q, constant lead time for replenishing orders, demand generated by a Poisson process and backordered demand during a stockout, Hadley and Whitin (1963) developed formulae for the average stock level ($D(Q,R)$), for the average stockout level ($B(Q,R)$) and for the average number of backorders per unit time ($E(Q,R)$). Assuming linear unit costs of holding and stockout, they obtained the related cost per unit time. The formulae are as follows:

$$B(Q,R) = \frac{1}{Q}[\beta(R) - \beta(R+Q)], \tag{1}$$

Where

$$\beta(\nu) = \frac{(\lambda L)^2}{2} P(\nu-1;\lambda L) - (\lambda L)\nu P(\nu;\lambda L) + \frac{\nu(\nu+1)}{2} P(\nu+1;\lambda L) \tag{2}$$

$$D(Q,R) = \frac{(Q+1)}{2} + R - \lambda L + B(Q,R). \tag{3}$$

$$E(Q,R) = \lambda.P_{out} = \frac{\lambda}{Q}\left[\alpha(R) - \alpha(R+Q)\right], \tag{4}$$

where

$$\alpha(\nu) = \lambda L\, P(\nu;\lambda L) - \nu\, P(\nu+1;\lambda L). \tag{5}$$

The parameters in the above formulae are as follows:

Q: Ordering batch size of continuous review policy

R: Reorder point of continuous review policy

λ: Demand rate (mean of Poisson demand distribution)

L: Constant lead time

P_{out} : The probability that there is no stock on hand

and

$$P(x,\lambda L)=\sum_{i=x}^{\infty}e^{-\lambda L}\left(\frac{(\lambda L)^{i}}{i!}\right) \qquad x=0,1,2,3,\ ... \tag{6}$$

3.2. Review of exact solution for lost sale problem with Poisson demand

Considering the same system with $R<Q$ (we us this assumption to assure of not having more than one order outstanding at a time), Hadley and Within (1963) developed formulae for the average stock level (D) and for the average number of lost sale incurred per unit time (E). Assuming linear unit costs of holding and stockout, they obtained the related annual cost. The formulae and parameters are as follows:

$$E=\frac{\lambda}{Q+\lambda\hat{T}}\lambda\hat{T}, \tag{7}$$

$$D=\frac{\lambda}{Q+\lambda\hat{T}}\left[\frac{Q(Q+1)}{2\lambda}+\frac{QR}{\lambda}-QL\right]+\frac{Q}{\lambda}E, \tag{8}$$

Where

$$\hat{T}=LP(R;\lambda L)-\frac{R}{\lambda}P(R+1;\lambda L) \tag{9}$$

and

$$T=\frac{Q+\lambda\hat{T}}{\lambda}. \tag{10}$$

The parameters in the above formulae are as follows:

Q: Ordering batch size of continuous review policy

R: Reorder point of continuous review policy

λ: Demand rate (mean of Poisson demand distribution)

L: Constant lead time

$\hat{T}$: The expected length of time per cycle that the system is out of stock

T: Time per cycle

and (6).

4. Complexities of the problem

4.1. Demand analysis in the warehouse

As Seifbarghy and Akbari Jokar (2006) mention what makes the multi-echelon inventory problems difficult to solve is how to exactly or approximately determine the type of demand at higher echelons and real replenishment time of orders, from downstream echelons to higher ones, because of possible stockouts in the higher ones. In the under study model, higher echelon is the central warehouse and downstream echelon is the retailers. On the type of demand at the central warehouse, we extend the given approximation by Seifbarghy and Akbari Jokar (2206) for the case of non-identical retailers. They mention that demand at the warehouse could be well approximated by a Poisson process with mean rate λ'_O which is computed as (11).

$$\lambda'_O = \frac{N\lambda_r}{Q_r + \lambda_r.\hat{T}_r}, \tag{11}$$

in terms of the identical batch size of Q_r. *N* is the number of retailers, λ_r is the demand rate at a retailer (retailers are identical) and $\hat{T}_r$ is the expected length of time per cycle that a retailer is out of stock. Such an approximation with a little difference had been suggested by Moinzadeh and Lee (1986), Muckstadt (1977), Deuermeyer and Schwarz (1981), Albin (1982), and Zipkin (1986) for the case of backordering of unsatisfied demand at the retailers.

Considering the notation defined in Section 2, demand at the warehouse can be approximated by a Poisson process with mean rate λ_O which is computed as (12).

$$\lambda_O = \sum_{i=1}^{n} \frac{\lambda_i}{Q_r + \lambda_i.\hat{T}_i}, \tag{12}$$

in terms of the identical batch size of Q_r. Noting the Section 3.2, $\hat{T}_i$ which is the expected length of time per cycle that retailer *i* is out of stock, is computed as (13).

$$\hat{T}_i = L_i P(R_i, \lambda_i L_i) - \frac{R_i}{\lambda_i} P(R_i + 1, \lambda_i L_i) \tag{13}$$

4.2. Approximating the retailers lead time

Retailers at the first echelon of the model experience independent Poisson demand processes. Demand during a stockout is assumed lost. Each order that is placed at the warehouse by each retailer has a minimum lead time equal to the transportation time.

Seifbarghy and Akbari Jokar (2006) express that the effective lead time of each retailer's order consists of two components: first the transportation time of the orders from the warehouse into the retailer; second an additional waiting time which results from a stockout in the warehouse. Based on the approximation of demand at the warehouse, the warehouse behaves just like an inventory system of type described in Section 3.1. From Little's formula in queuing theory (as Andersson and Malchiors [2] use it), they use the expression for the average stockout level given by Formula (1) and (2) to obtain the average waiting time of each retailer order as given by Formula (14).

$$\bar{W}' = \frac{B\left(Q_O/Q_r\,,R_O/Q_r\right)}{\lambda'_O}, \tag{14}$$

Where $\bar{W}'$ is the average waiting time of each retailer order and $B\left(Q_O/Q_r\,,R_O/Q_r\right)$ is the average stockout level at the warehouse (Formula (1)). The batch size and reorder point of the warehouse (Q_O and R_O) are assumed integer multiples of the identical batch size of the retailers $\left(Q_r\right)$.

$\bar{W}$ is added to the transportation time of each retailer to make the approximate constant lead time for the orders and it can be used for evaluating the retailer costs (Seifbarghy and Akbari Jokar (2206)).

In the approximation proposed in this paper for the case of non-identical retailers, the retailers' costs are not computed based on an effective lead time. In the other hand, C_i is composed of two parts. The first part is for the orders that do not incur stockout at the warehouse for which the warehouse freights once receiving the order and the real lead time is the transportation time. The second part is for the orders incurring stockout at the warehouse. Since λ_O is the demand rate at the warehouse and P_{out} is the probability that the warehouse to be in stockout, the number of such orders per unit time is $\lambda_O.P_{out}$. The average waiting of each retailer order which incurs stockout ($\bar{W}$), is given by (15).

$$\bar{W} = \frac{B\left(Q_O/Q_r\,,R_O/Q_r\right)}{\lambda_O.P_{out}} \tag{15}$$

Noting Formula (4) and considering that the batch size and reorder point of the warehouse (Q_O and R_O) are assumed integer multiples of the identical batch size of the retailers $\left(Q_r\right)$, $\bar{W}$ can be computed as

$$\bar{W} = \frac{B\left(Q_O/Q_r\,,R_O/Q_r\right)}{E\left(Q_O/Q_r\,,R_O/Q_r\right)}, \tag{16}$$

where

$$B(Q_O/Q_r,R_O/Q_r) = \frac{1}{Q_O/Q_r}[\beta(R_O/Q_r) - \beta(R_O/Q_r + Q_O/Q_r)], \tag{17}$$

$$E(Q_O/Q_r,R_O/Q_r) = \frac{\lambda_O}{Q_O/Q_r}\left[\alpha(R_O/Q_r) - \alpha(R_O/Q_r + Q_O/Q_r)\right]. \tag{18}$$

The functions α and β are as in (2) and (5).

The lead time for the orders which incur stockout at the warehouse (y_i) can be well approximated with a uniform distribution with the lower and upper bounds of

L_i and $L_i + 2\overline{W}$. It is clear that the probability distribution function of the lead time is $\frac{1}{2\overline{W}}$ and the expected value of the lead time is $L_i + \overline{W}$.

The cost per unit time of retailer i in steady state (C_i) is computed as

$$C_i = (1 - P_{out}).(\pi_r.E_i + h_r.D_i) + P_{out}.\int_{L_i}^{L_i+2\overline{W}} (\pi_r.E_i' + h_r.D_i').\frac{1}{2\overline{W}}dy_i, \quad (19)$$

where

$$P_{out} = \frac{E(Q_O / Q_r, R_O / Q_r)}{\lambda_O}, \quad (20)$$

$$E_i = \frac{\lambda_i}{Q_r + \lambda_i \hat{T}_i} \lambda_i \hat{T}_i, \quad (21)$$

$$D_i = \frac{\lambda_i}{Q_r + \lambda_i \hat{T}_i}\left[\frac{Q_r(Q_r+1)}{2\lambda_i} + \frac{Q_r R_i}{\lambda_i} - Q_r L_i\right] + \frac{Q_r}{\lambda_i}E_i \quad (22)$$

and $\hat{T}_i$ is obtained from (13). E_i' and D_i' are obtainable as E_i and D_i in (21) and (22) replacing y_i with L_i.

5. Approximate total cost

The total cost of the inventory system (TC) consists of the retailers' inventory holding and stockout costs and the warehouse' inventory holding cost and is computed as

$$TC = C_O + \sum_{i=1}^{n} C_i \quad (23)$$

The warehouse' inventory holding cost is as in (24):

$$C_O = h_O.D(Q_O/Q_r, R_O/Q_r).Q_r \quad (24)$$

In the above formula $D(Q_O/Q_r, R_O/Q_r)$ is the average stock level in the warehouse and is as in (25), using Formula (3) in Section 3.1 and noting that Q_O should be an integer multiple of Q_r.

$$D(Q_O / Q_r, R_O / Q_r) = \frac{(Q_O / Q_r + 1)}{2} + R_O / Q_r - \lambda_O L_O + B(Q_O / Q_r, R_O / Q_r). \quad (25)$$

In the above formula, λ_O is obtained from (12) considering (13) and $B(Q_O / Q_r,\ R_O / Q_r)$ is obtained from (17).

C_i is as in (19) considering Formulae (20), (21), (22) and (13).

It is clear that the optimal reorder points in all installations should be found through optimizing the total inventory system cost (TC). Since TC belongs to Nonlinear Integer Programming (NIP) problems, a GA based heuristic is proposed to evolve optimal or near to optimal values of the reorder points R_O and R_i $for\ i = 1,...,N$.

6. Computational Procedure

Genetic Algorithm (GA) is a class of evolutionary algorithms and is based on a population of solutions. GA is a generic optimization method which can be applied to almost every problem. The feasible solutions of the problem are usually represented as strings of binary or real numbers called chromosomes. Each chromosome has a fitness value that corresponds to the objective function value of the associated solution. Initially there is a population of chromosomes randomly generated. Then, a number of chromosomes are selected as parents for mating in order to produce new chromosomes (solutions) called offspring. The mating of parents is done applying the GA operators, such as crossover and mutation. The selection of parents and producing offspring are repeated until the stopping rule (for example a certain number of iterations) is satisfied.

Before giving a general outline of the proposed genetic algorithm, some additional notation is defined as follows:

Population_size: Size of the population of solutions that remains constant during the algorithm performance.

Max_iteration: Number of generations which should be produced until the algorithm stops.

p_c : Crossover rate (which is the probability of selecting a chromosome in each generation for performing crossover)

p_m : Mutation rate (which is the probability of selecting a gene or bit inside a chromosome for mutating)

Fitness_function: Fitness value or the objective function value

The general outline of the proposed GA is as follows:

Step 0: Initialize *Population_size*, *Max_iteration,* p_c and p_m .

Step 1: Randomly generate the initial population.

Step 2: Repeat until the *Max_iteration*:

Step 2.1: Perform the reproduction operator according to the roulette wheel rule and make a newer population.

Step 2.2: Select the parent chromosomes from the population with probability p_c .

Step 2.3: crossover:

a. Determine the pairs of parents among the parent chromosomes.
b. Apply the crossover operator to produce two offspring for each pair.
c. Replace each offspring in the population instead of the parents.

Step 2.4: Apply the mutation operator on the population with probability p_m.

Step 2.5: Calculate the *Fitness_function* for each chromosome and save the best value in *bv* (best value).

Step 3: Print *bv*.

In the proposed GA, each chromosome is represented by an *N+1* dimensional vector as $\left[R_O\, R_1\, R_2 \ldots R_N\right]$ which *N* is the number of retailers and the values are reorder points of the warehouse and *N* retailers respectively. The *Population_size*, p_c and p_m are assumed 100, 0.8 and 0.01 respectively. *Max_iteration* is assumed equal to 2000.

It is necessary to state that the reorder point of a retailer is bounded by 0 and Q_r which means $0 \le R_i \prec Q_r$, since here should not be more than one order outstanding in each retailer at any time and this constraint satisfies this condition for a continuous review inventory system with lost sales (Hadley and Whitin (1963)). Since there are *N* retailers in the model and none of them can have more than one order outstanding, R_O has a lower bound equal to ($-N.Q_r$).

7. Numerical Results

In order to determine the power of our approximation we have designed a set of 24 numerical problems. We also simulated each numerical problem 10 times (having 10 runs), for the optimal reorder points obtained from the approximate model, using GPSS/H simulation software. The simulation time length of each run is 110,000 unit times with 10,000 unit times as a ‘‘run in’’ period.

Different starting random number seeds were employed for each problem. All of the results show that this length of time is sufficient for the system to reach a steady state. This is also clear from the standard deviation of the total system cost. The numerical problems are as in Table 1 and Table 2.

In Table 1, the retailers are considered with different demand rates which are randomly generated among 0.5, 1, 1.5 and 2 but with equal transportation times which is assumed one time unit, $L_i = 1, for\, i = 1,2,3,4$. In Table 2 the retailers are considered with different transportation times which are randomly generated among 0.5, 1, 1.5 and 2 but with equal demand rates which is assumed one per time unit, $\lambda_i = 1, for\, i = 1,2,3,4$.

There are different stockout costs to assess the accuracy of the approximations for the various service levels. The number of retailers is four. The holding costs of the warehouse and retailers per unit per unit time are assumed to be 1, $h_O = h_r = 1$, and the lead time of the warehouse is assumed to be 1, $L_O = 1$.

Table 1

Numerical examples with different demand rates but equal transportation times.

No	π_r	Q_o	Q_r	λ_1	λ_2	λ_3	λ_4
1	25	128	8	1	2	0.5	1.5
2	25	128	16	1.5	1.5	2	1
3	25	256	8	2	1.5	1	1
4	25	256	16	0.5	0.5	2	2
5	50	128	8	1	2	0.5	1.5
6	50	128	16	1.5	1.5	2	1
7	50	256	8	2	1.5	1	1
8	50	256	16	0.5	0.5	2	2
9	100	128	8	1	2	0.5	1.5
10	100	128	16	1.5	1.5	2	1
11	100	256	8	2	1.5	1	1
12	100	256	16	0.5	0.5	2	2

Table 2

Numerical examples with different transportation times but equal demand rates

No	π_r	Q_o	Q_r	L_1	L_2	L_3	L_4
13	25	128	8	1	1	2	0.5
14	25	128	16	1	1.5	1.5	2
15	25	256	8	1	2	1.5	1
16	25	256	16	1	0.5	0.5	2
17	50	128	8	1	1	2	0.5
18	50	128	16	1	1.5	1.5	2
19	50	256	8	1	2	1.5	1
20	50	256	16	1	0.5	0.5	2
21	100	128	8	1	1	2	0.5
22	100	128	16	1	1.5	1.5	2
23	100	256	8	1	2	1.5	1
24	100	256	16	1	0.5	0.5	2

The total cost results are as in Tables 3 and Table 4. As can be seen from the tables the errors in the approximate total cost are small in comparison with the simulated values. The mean error is 2.73.

Table 3.
Approximated and simulated total cost results for the numerical problems in Table 1.

No	R_1	R_2	R_3	R_4	R_O	*Approximate TC*	*Simulated TC*	*Error %*	**Mean Error**	**St dev**
1	2	6	1	3	-16	79.1500	76.5337	3.41	2.74	2.1213
2	5	6	8	3	-32	92.2441	88.2688	4.50		
3	6	4	2	2	-24	139.5524	134.8256	3.50		
4	1	1	8	5	-32	150.3781	147.5375	1.92		
5	2	5	1	3	-8	83.1585	82.4498	0.85		
6	3	3	4	1	-16	95.3382	93.7210	1.72		
7	6	4	2	3	-16	144.0123	142.0288	1.39		
8	2	2	11	7	-32	157.0572	154.0357	1.96		
9	2	6	2	4	-8	86.3791	85.4253	1.11		
10	2	3	6	4	-16	101.7156	99.1507	2.58		
11	7	5	4	3	-16	148.1674	145.7633	1.64		
12	1	0	6	3	-16	161.7296	160.6039	0.70		

Table 4
Approximated and simulated total cost results for the numerical problems in Table 2.

No	R_1	R_2	R_3	R_4	R_O	*Approximate TC*	*Simulated TC*	*Error %*	**Mean Error**	**St dev**
13	2	3	2	4	-16	78.4930	75.9617	3.33	2.72	2.3520
14	3	5	4	4	-32	90.9657	84.0427	8.23		
15	4	3	3	3	-24	138.5116	134.2799	3.15		
16	2	3	3	3	-32	148.3618	144.4759	2.68		
17	3	4	3	4	-16	82.5648	78.9210	4.61		
18	2	4	3	2	-16	94.8567	93.9444	0.97		
19	4	5	3	3	-16	143.6549	142.2749	0.96		
20	2	2	5	4	-32	154.5707	146.9175	5.20		
21	2	4	2	3	-8	85.9030	84.9153	1.16		
22	3	3	4	4	-16	97.5928	96.5831	1.04		
23	6	4	4	6	-16	148.3127	146.8043	1.02		
24	2	2	4	4	-16	159.1857	158.7168	0.29		

8. Conclusions and further research

An approximate cost function for a two-echelon inventory system consisting of one warehouse and an arbitrary number of non-identical retailers where unsatisfied demand at the retailers is lost and the control policy is continuous review has been developed. The main assumptions of this research are having non-identical retailers in the model and lost sales during a stockout at the retailers. Demand distribution has been approximated as Poisson at the warehouse and the retailers' costs are not computed based on an effective lead time. In the other hand, it is composed of two parts. The first part is for the orders that do not incur stockout at the warehouse for which the warehouse freights once receiving the order and the real lead time is the transportation time. The second part is for the orders incurring stockout at the warehouse. The numerical results show that the approximation is good enough and the mean error is around 2.73 %.

Future research is to use a service level objective for determining the optimal control policy. The inventory control policies could be changed and some parameters such as transportation time the orders from the central warehouse to the retailers and the warehouse lead time could be assumed stochastic.

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