README: Simulating Temperature Transients in First Year Sea Ice

Nitay Ben-Shachar

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This package solves a 1D transient thermodynamic model for the temperature profile of sea ice/snow, based primarily on the air temperatures, which can be solved to arbitrary resolution both spatially and temporally, and has been coded in MATLAB. The growth of sea ice is determined by balancing heat fluxes at the ice-ocean interface. An oceanic heat flux and snow layer can be incorporated as specified by input files but snow is assumed to have constant density. A sharp interface between the ice and ocean is assumed. Salt transport is accounted for only in its variation of the ice's physical parameters.

The model has been validated using temperature array data taken in McMurdo Sounds, Antarctica in 1997 and has achieved an agreement to within 0.4 ^{o}C with the entire thermistor array when some oceanic heat flux was accounted for. More details are provided in the manuscript submitted to JGR: Oceans.

This program was coded and tested with MATLAB R2019b and possibly uses functionality not available from earlier releases.

1 Model Details

The model used here is a 1D transient thermodynamical model. The goal of the model is to obtain a solution to the temperature profile in the sea ice and snow, and hence being able to estimate the ice growth rate and thickness over time, primarily based on the air temperatures. As we are modelling temperature transients, the core of the model is the heat equation which is solved throughout the sea ice/snow to determine the evolution of the temperature profiles. The heat equation is solved by making use of the Method of Lines and the mathematical technique of freezing the boundary, a very appropriate name in this situation. Both techniques are briefly detailed below, but are described in more details in the manuscript.

Freezing the Boundary is a mathematical method especially useful in analysing systems with moving boundaries. It has been incorporated in this model for mathematical ease and elegance. Rather than adding simulation calculation points as the ice grows, we solve an equivalent mathematical problem over a fixed depth interval by defining the depth-like variables $\chi, \xi = \frac{z}{H(t)}$ where $H(t)$ is the ice/snow depth. By definition, the slab of sea ice always lies in corresponding values of χ, ξ between 0 and 1, and so solving the (modified) heat equation in terms of χ, ξ takes into account any sea ice growth or snow dump/melt.

In using the Method of Lines we first discritizing over space, meaning that some level of resolution must be chosen for the model. The partial differential equation is then converted to a set of coupled ordinary differential equations, each describing the temperature evolution at a fixed value of χ, ξ. The temperature profile is linearly interpolated over time and space from the solutions of each ordinary differential equation. The spatial resolution of the model can be chosen to be arbitrarily high/low, with discretizing at 5cm intervals usually being sufficiently accurate.

2 Using the Package

In order to run the program, at least two files must be provided in the input folder. These files are:

airTemps.csv: This file contains the air temperatures over time. Provide the time in days in the first column and the corresponding air temperature in degrees C in the second column.

salinity.csv: This file contains the salinity profile of the sea ice. The first column should be the depth (in meters) corresponding to the salinity (in grams per gram) in the second column. The depths should all be negative numbers, corresponding to depths below the sea level. If the salinity profile is not known leave this file blank and the salinity will default to 5ppt.

(optional) snow.csv: This file contains the snow depth over time. The first column should contain the time in days corresponding to the snow depth (m) in the right column. The snow depths are then interpolated linearly between the provided values.

(optional) oceanFlux.csv: This file contains the ocean flux over time. The first column should contain the time in days corresponding to the ocean flux (W/(m K)) in the right column. The ocean flux is interpolated linearly between the provided values, and defaults to zero outside the provided time range or if the file is not supplied.

As well as the above files, some parameters and initial conditions should be adjusted in the file seaIceModel.m. These values are clearly labeled near the top of the file.

num_of_points_snow/ice: These constants determine the number of calculation points that are used in discritizing over the depth of the snow/ice for the simulation. A constant number of points is used meaning that the resolution of the model changes over time, for example, setting num_of_points_ice = 10 for a simulation were the ice slab grows from -0.2m to -0.5m means that the model's resolution changes from 50points/meter to 20 points/meter. If the output temperature profiles seem unrealistic, for example with sharp changes, a likely solution would be to increase the spatial resolution of the simulation by increasing these constants.

odeset: This constant determines the accuracy that the ODE solver takes, so determines the temporal accuracy of the simulation. MATLAB's stiff ODE solver `ode15s` is used to determine the numerical solutions to the temperature transients of the sea ice/snow and choose a time resolution that corresponds to an arbitrarily accurate solution specified by odeset. 'RelTol' determines the percentage error tolerated for each variable. 'AbsTol' determines the absolute numerical error allowed for each variable. See the MATLAB documentation for further settings that could be selected.

final_t: This is the number of seconds that the solver should run for. Select the duration for which the solution is desired. Time being zero is set to the first line in each input file (airTemps.csv, snow.csv).

time_offset: The solver will begin the solution from the point time_offset, with t=0 being defined at the beginning of the input files.

initial_depth: This is the initial depth of the ice, at the point in time specified by time_offset. This value should be negative and measured in meters.

present_dt: This is the temporal resolution (in seconds) in which the solutions will be reported.

z_res_ice: This is the spatial resolution (in meters) in which the sea ice temperature profile solutions will be reported.

z_res_snow: This is the spatial resolution (in meters) in which the snow temperature profile solutions will be reported.

Note: Increasing present_dt, z_res_ice and z_res_snow does not increase the accuracy of the solutions, only the resolution in which the solution is reported. To increase the accuracy increase the number of calculation points in the model and decrease the error tolerance (odeset).

T_freezing: This is the freezing temperature of the sea water, taken to be constant. This can be adjusted if the salinity of the water is well known. Further, it can be extended into a subroutine that adjusts the freezing point depending on the salinity, depth or any other variables of interest.

V_a: This is the proportion of incorporated air in the freezing sea ice, taken to be constant. Again, this can be made to depend on any number of parameters via a subroutine.

Once all the necessary variables have been provided, with the MATLAB working directory set to the folder containing the file seaIceModel.m, run the simulation by typing model_output = seaIceModel() into the MATLAB command line. The solutions will be stored in the variable model_output as detailed here:

model_output.time: A 1D array containing the values of time (in seconds), starting from 0 corresponding to the time of time_offset, at which the remainder of the solutions are given. This temporal resolution can be adjusted with the constant present_dt.

model_output.z_ice: A 1D array containing the depths associated with the values presented in the temperature profile of ice.

model_output.z_snow: A 1D array containing the depths associated with the values presented in the temperature profile of snow.

model_output.temp_profile_snow: A 2D array containing the solution to the temperature profile in the snow as a function of time. The temperature profile at a specific time is given by temp_profile_snow(time_index, :) where time_index is an integer corresponding to the time of interest. The temperature profile is calculated at time increments of present_dt and so time_index=$\frac{\texttt{time}}{\texttt{present\_dt}}$. Points in the air are filled with a dummy value of 0.

model_output.snow_depth: A 1D array containing the snow depth over the desired times. This will merely be an interpolation of the provided snow thickness at the points of interest.

model_output.temp_profile_ice: A 2D array with the corresponding solution of the ice temperature profile at the times of interest, similar to temp_profile_snow. Points in the water are filled with a dummy value of the freezing temperature.

model_output.ice_depth: A 1D array containing the solution to the ice depth over time.

3 Files

seaIceModel.m This is the central function that administrates the rest of the simulation. It has three primary purposes being initialising all the necessary variables, obtaining a solution with the user provided data and finally re-organizing the solution in the necessary format.

3.1 input

This is the folder under which the user should place the necessary data needed for the simulation. Four files should be placed here: salinity.csv, snow.csv, oceanFlux.csv and airTemps.csv. Each should be formatted as detailed above in section 2.

3.2 user

In this folder are the subroutines associated with handling the user data.

calcAirTemp.m This function returns the air temperature at a specific time by interpolating between the user specified air temperatures. The request time should be measured in seconds and begins at zero at the first time specified in the airTemps.csv. If time_offset was initialised to anything but zero this is accounted for.

calcOceanFlux.m This function returns the ocean flux at a given time (seconds), handling the input from oceanFlux.csv, by interpolating on the ocean fluxes given in oceanFlux.csv.

calcSalinity.m This function returns the salinity at a given depth, handling the input from salinity.csv, by interpolating on the salinities given in salinity.csv.

snowThickness.m This function returns the snow thickness as a given time by interpolating between the user specified snow thicknesses in snow.csv.

snowGrowthRate.m This function returns the growth rate of the snow at a particular time by linearly interpolating between the snow thicknesses provided and returning the slope of the linear interpolation it lies on.

3.3 params

This folder contains the functions that return the physical parameters of the sea ice/snow based on the local conditions (such as temperature, salinity etc.). The formulas are all provided in appendix A along with references.

3.4 utils

importUserData.m This function imports the data entered in the 'input' folder and saves it in matrices ready to be used. It also preforms some preliminary checks to ensure the data was entered in the correct format.

surfaceTemp.m This function returns the temperature between the ice and the snow in order to balance the heat flows and ensure the interface is in thermal equilibrium. This is to ensure equation 10 of the manuscript is satisfied.

iterateDEs.m This is the function that returns the rate of change of the temperature profiles and ice depth for a given temperature profile of the snow and ice. This is the function passed to the MATLAB ODE solver. This function finds the rate of change of each point in the snow/ice profile via the heat equation (equations 7 and 8 in the manuscrpit). The ice growth rate is determined with the boundary condition given in equation 9 in the manuscript.

4 Troubleshooting

4.1 Unable to reduce time step below minimum amount

If a MATLAB warning that the minimum time step was hit while trying to solve the ODEs, the resolution of the simulation must be increased by increasing the number of calculation points num_of_points_ice/snow.

4.2 Sharp jump in the snow temperature profile

This is due to the solver completely jumping over a critical time where the snow depth changed rapidly. This can be fixed by reducing the error tolerance or adding a minimum time step being a fraction of the length of the rapid snow melt/dump, however, these methods can make the solver run for a long time. If only one major jump is present, the solver can be separated into three solutions, where the first runs from the beginning to just before the major jump and uses a large minimum time step. The second covers the small region associated with the large jump using a small maximum time step, and the last covers the remainder of the simulation with a large maximum time step. Each solution would use the final output of the previous as its initial variables, and all three solutions are stitched together. See the MATLAB documentation for odeset for possible ways of increasing the solver accuracy and ensuring that it doesn't skip the points of interest.

A Physical Parameters and Constants

All quantities are given in SI units. Temperature is measured in degrees Celsius. Salinity is given in units of grams per gram, note that this is a factor of 1000 smaller than other standard units such as psu, ppt (parts per thousand).

A.1 Ice Properties

- Thermal Conductivity [3] [W/(m K)]: $k_{\texttt{ice}} = \frac{\rho}{\rho_i}\left[2.11 - 0.011T + 0.09\frac{S}{T} - \frac{\rho-\rho_i}{1000}\right]$
- Heat Capacity [4, 2] [J/(kg K)]:
  $c_{\texttt{ice}} = 1000\left(2.113 + 0.0075T - 0.0034S \times 1000 + 0.00008ST \times 1000 + 18.04\frac{S\times 1000}{T^2}\right)$
- Density [4] [kg/(m^3)] $\rho_{\texttt{ice}} = (1 - V_a)\left(1 - \frac{4.51S}{T}\right)917$
- Latent Heat of Fusion [4] [J/kg]: $L_{\texttt{ice}} = 4184\left(79.68 - 0.505T - 27.3S + 4311.5\frac{S}{T}\right)$

A.2 Snow Properties

- Density [1] [kg/m^3]: $\rho_{\texttt{snow}} = 330$
- Thermal Conductivity [4] [W/(m K)]: $k_{\texttt{snow}} = 0.0688\exp\left(0.0088T + 4.6682\frac{\rho_{\texttt{snow}}}{1000}\right)$
- Heat Capacity [4] [J/(kg K)]: $c_{\texttt{snow}} = (2.7442 + 0.1282(T + 273.15)) \times \frac{18.02}{1000}$

References

[1] Gary A. Maykut and Norbert Untersteiner. "Some results from a time-dependent thermodynamic model of sea ice". In: *Journal of Geophysical Research (1896-1977)* 76.6 (1971), pp. 1550–1575. DOI: 10.1029/JC076i006p01550. URL: https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/JC076i006p01550.

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