<u>Title:</u> NMR experimental data on magnon-BEC time crystals in superfluid <sup>3</sup>He in the presence of a free surface

**Description:** This dataset supplements described experimental results in the article "AC Josephson effect between two superfluid time crystals". The dataset contains experimental measurements of magnon-BEC time crystals in the B phase of superfluid <sup>3</sup>He in a cylindrical sample container in the presence of a free surface. Two spatially separate time crystals are observed: one in the bulk of the liquid, and one touching the free surface located above the bulk crystal.

A nuclear demagnetization cryostat and NMR spectrometer were used in the experiments. Experimental measurements were carried out using the Low Temperature Laboratory, which is part of the OtaNano research infrastructure of Aalto University and an access site in the European Microkelvin Platform.

The <sup>3</sup>He sample was confined within a 150-mm-long cylindrical container with  $\emptyset$ 6mm inner diameter, made from quartz glass. The experimental volume is connected to another volume of bulk B phase, used for thermometry and coupling to nuclear demagnetization stage. This volume contains a commercial quartz tuning fork with 32 kHz resonance frequency. Static magnetic field of 17–25 mT corresponding to NMR frequencies of 560–833kHz is created using a coil oriented parallel to the axis of the sample container. The NMR pick-up coil, oriented perpendicular to both main magnets, is a part of a tuned tank circuit with quality factor roughly  $Q \sim 100$ . We use a cold preamplifier, thermalized to a bath of liquid helium. The signal is then fed to a room temperature preamplifier, and from that to a lock-in amplifier which mixes down the signal w.r.t. the refernce frequency. The lock-in output signal is then digitised ad 48000Hz sampling rate. The sample can be cooled down to ~130 mK using ROTA nuclear demagnetization refrigerator. The earth's magnetic field is compensated using two saddle-shaped coils installed around the refrigerator. In rotation, the total heat leak to the sample remains below 20 pW.

The data presented here was measured in 25mT field corresponding to ~833kHz Larmor frequency. The current in the pinch coil used to control the magnetic trap shape (4 turns, diameter ~1cm) was 180mA. The excitation pulse used was at 831750Hz, pulse length was 2000 cycles, and the pulse amplitude was adjusted to populate both time crystals. A cold premplifier and a room temperature preamplifier was used. The maximum of the recorded signal corresponds to about 20 degree tilt angle of the magnetisation at the maximum of the time crystal wave function. The lock-in amplifier reference frequency was 834000Hz. The thermometer fork width was 106mHz, while the intrinsic width of the fork was 75.6mHz.

## Content of the dataset parts:

1. [raw\_data.flac] Raw data file for Figure2 in the article. The file is in flac format, as recorded from the pick-up coil after preamplification at 48kHz sampling rate. Right channel is the measured signal, and the unit is 0.1 x Volts. Left channel was not used in this experiment. Note that the frequency axis is inverted as the lock-in reference frequency is higher than the signal frequency. The Larmor frequency is 1500Hz in the units of the raw data.

2. [Fig2.csv] Windowed Fourier analysis of the raw data in csv format. First column is frequency separation from the Larmor frequency in Hz as indicated by the header. First row is time in seconds, also indicated by a header. The corresponding absolute values of the FFT amplitudes [abs(FFT)] for each (frequency,time) pair are given by the matrix in mV enveloped by the first column and the first row. The Blackman time window width used is +-0.3125s, and the time step between the overlapping

windows is 0.0625s. The raw data frequency spectrum was corrected according to the measured lock-in amplification frequency profile (Stanford SR844, 6db/octave, time constant 100µs).

3. [Fig3\_panel\_A.csv] Frequency difference of the two time crystals as extracted from part 2. First column is time in seconds, second column is the difference of the precession frequencies of the two time crystals in Hz, third column is the frequency difference between the bulk time crystal and the side band of the bulk time crystal in Hz.

4. [Fig3\_panel\_B.csv] Azimuthal angle of the time crystal precession in the frame co-rotating with the precession. The azimuthal angle is extracted by feeding the raw signal (dataset part 1) to a numerical lock-in amplifier, where the reference frequency is extracted from the Fourier analysis by tracing the maximum of the two peaks that correspond to the precessing magnetisation in the time crystals (dataset part 2). First column is time in seconds, second column is surface crystal azimuthal angle in degrees, third column is bulk crystal azimuthal angle in degrees. The time window used here is +- 0.2083s, and the time step between the overlapping windows is 0.0417s.

5. [Fig3\_panel\_C.csv] Amplitudes of the surface and bulk time crystal signals, extracted by fitting the raw signal (dataset part 1) directly with a sine curve in short time windows. The raw data was filtered to a narrow frequency band around each time crystal trace with a FFT filter. First column is time in seconds, second column is bulk time crystal amplitude in mV, third column is surface crystal amplitude in mV. The time window used in the direct fit is +-5ms, and the time step between the overlapping windows is 5ms.

6. [Fig3\_panel\_D.csv] Numerical quasi-equilibrium simulation of the time crystal population oscillations. The simulation codes and advice for using them is available from the corresponding author of the above-mentioned article upon reasonable request. First column is population oscillation periods, second column is the signal amplitude from the bulk crystal in mV, third column is the signal amplitude from the surface crystal in mV.