$$\boldsymbol{X} = x\boldsymbol{e_x} + y\boldsymbol{e_y} + z\boldsymbol{e_z}$$

$$\boldsymbol{v} = v^{x}\boldsymbol{e_x} + v^{y}\boldsymbol{e_y} + v^{z}\boldsymbol{e_z}$$

$$\boldsymbol{A} = A^{x}\boldsymbol{e_x} + A^{y}\boldsymbol{e_y} + A^{z}\boldsymbol{e_z}$$

$$\boldsymbol{v} \cdot \nabla = v^{x}\frac{\partial}{\partial x} + v^{y}\frac{\partial}{\partial y} + v^{z}\frac{\partial}{\partial z}$$

$$\nabla^{2} = \frac{\partial^{2}}{\partial x^{2}} + \frac{\partial^{2}}{\partial y^{2}} + \frac{\partial^{2}}{\partial z^{2}}$$

$$\boldsymbol{v} \cdot \nabla f = v^{x}\partial_{x} f + v^{y}\partial_{y} f + v^{z}\partial_{z} f$$

$$\nabla^{2} f = \partial^{2}_{x} f + \partial^{2}_{y} f + \partial^{2}_{z} f$$

$$\nabla^{2}\boldsymbol{A} = \left(\partial^{2}_{x} A^{x} + \partial^{2}_{y} A^{x} + \partial^{2}_{z} A^{x}\right)\boldsymbol{e_x} + \left(\partial^{2}_{x} A^{y} + \partial^{2}_{y} A^{y} + \partial^{2}_{z} A^{y}\right)\boldsymbol{e_y} + \left(\partial^{2}_{x} A^{z} + \partial^{2}_{y} A^{z} + \partial^{2}_{z} A^{z}\right)\boldsymbol{e_z}$$

$$\bar{\nabla} \cdot v = v^{x}\frac{\partial}{\partial x} + v^{y}\frac{\partial}{\partial y} + v^{z}\frac{\partial}{\partial z}$$

$$\boldsymbol{X} \cdot \nabla = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}$$

$$\bar{\nabla} \cdot \boldsymbol{X} = 3 + x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}$$

$$\boldsymbol{X} \cdot \nabla - \bar{\nabla} \cdot \boldsymbol{X} = -3$$

$$\nabla^{2} = \nabla \cdot \nabla = \frac{2}{r}\frac{\partial}{\partial r} + \frac{\partial^{2}}{\partial r^{2}} + \frac{1}{r^{2}\tan\left(\theta\right)}\frac{\partial}{\partial \theta} + r^{-2}\frac{\partial^{2}}{\partial \theta^{2}} + \frac{1}{r^{2}\sin^{2}\left(\theta\right)}\frac{\partial^{2}}{\partial \phi^{2}}$$

$$\left(\nabla^{2}\right) f = \frac{1}{r^{2}}\left(r^{2}\partial^{2}_{r} f + 2r\partial_{r} f + \partial^{2}_{\theta} f + \frac{\partial_{\theta} f}{\tan\left(\theta\right)} + \frac{\partial^{2}_{\phi} f}{\sin^{2}\left(\theta\right)}\right)$$

$$\nabla \cdot \left(\nabla f\right) = \frac{1}{r^{2}}\left(r^{2}\partial^{2}_{r} f + 2r\partial_{r} f + \partial^{2}_{\theta} f + \frac{\partial_{\theta} f}{\tan\left(\theta\right)} + \frac{\partial^{2}_{\phi} f}{\sin^{2}\left(\theta\right)}\right)$$