$$f = f$$

$$A = A^{r}\boldsymbol{e_{r}} + A^{\theta}\boldsymbol{e_{\theta}} + A^{\phi}\boldsymbol{e_{\phi}}$$

$$B = B^{r\theta}\boldsymbol{e_{r}}\wedge\boldsymbol{e_{\theta}} + B^{r\phi}\boldsymbol{e_{r}}\wedge\boldsymbol{e_{\phi}} + B^{\phi\phi}\boldsymbol{e_{\theta}}\wedge\boldsymbol{e_{\phi}}$$

$$\boldsymbol{\nabla} f = \partial_{r} f\boldsymbol{e_{r}} + \frac{1}{r^{2}}\partial_{\theta} f\boldsymbol{e_{\theta}} + \frac{\partial_{\phi} f}{r^{2}\sin^{2}{\left(\theta \right)}}\boldsymbol{e_{\phi}}$$

$$\boldsymbol{\nabla} \cdot A = \frac{A^{\theta}}{\tan{\left(\theta \right)}} + \partial_{\phi} A^{\phi} + \partial_{r} A^{r} + \partial_{\theta} A^{\theta} + \frac{2A^{r}}{r}$$

$$\boldsymbol{\nabla} \times A = -I(\boldsymbol{\nabla} \wedge A) = r^{2}\left(\left(- \frac{1}{2}\cos{\left(2\theta \right)} + \frac{1}{2}\right)\partial_{\theta} A^{\phi} + A^{\phi}\sin{\left(2\theta \right)} - \partial_{\phi} A^{\theta}\right)\boldsymbol{e_{r}} + \left(\left(- r^{2}\partial_{r} A^{\phi} - 2rA^{\phi}\right)\sin^{2}{\left(\theta \right)} + \partial_{\phi} A^{r}\right)\boldsymbol{e_{\theta}} + \left(r^{2}\partial_{r} A^{\theta} + 2rA^{\theta} - \partial_{\theta} A^{r}\right)\boldsymbol{e_{\phi}}$$

$$\nabla^{2} f = \frac{1}{r^{2}}\left(r^{2}\partial^{2}_{r} f + 2r\partial_{r} f + \partial^{2}_{\theta} f + \frac{\partial_{\theta} f}{\tan{\left(\theta \right)}} + \frac{\partial^{2}_{\phi} f}{\sin^{2}{\left(\theta \right)}}\right)$$

$$\boldsymbol{\nabla} \wedge B = \frac{1}{r^{2}}\left(r^{2}\partial_{r} B^{\phi\phi} + 4rB^{\phi\phi} - \frac{2B^{r\phi}}{\tan{\left(\theta \right)}} - \partial_{\theta} B^{r\phi} + \frac{\partial_{\phi} B^{r\theta}}{\sin^{2}{\left(\theta \right)}}\right)\boldsymbol{e_{r}}\wedge\boldsymbol{e_{\theta}}\wedge\boldsymbol{e_{\phi}}$$

Derivatives in Paraboloidal Coordinates

$$f = f$$

$$A = A^{u}\boldsymbol{e_{u}} + A^{v}\boldsymbol{e_{v}} + A^{\phi}\boldsymbol{e_{\phi}}$$

$$B = B^{uv}\boldsymbol{e_{u}}\wedge\boldsymbol{e_{v}} + B^{u\phi}\boldsymbol{e_{u}}\wedge\boldsymbol{e_{\phi}} + B^{v\phi}\boldsymbol{e_{v}}\wedge\boldsymbol{e_{\phi}}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{u} f}{\sqrt{u^{2} + v^{2}}}\boldsymbol{e_{u}} + \frac{\partial_{v} f}{\sqrt{u^{2} + v^{2}}}\boldsymbol{e_{v}} + \frac{\partial_{\phi} f}{uv}\boldsymbol{e_{\phi}}$$

$$\boldsymbol{\nabla} \cdot A = \left(\frac{u}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{u\sqrt{u^{2} + v^{2}}}\right)A^{u} + \left(\frac{v}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{v\sqrt{u^{2} + v^{2}}}\right)A^{v} + \frac{\partial_{u} A^{u}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{v} A^{v}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{\phi} A^{\phi}}{uv}$$

$$\begin{aligned} \boldsymbol{\nabla} \times A = -I(\boldsymbol{\nabla} \wedge A) = & \frac{1}{uv\left(u^{2} + v^{2}\right)}\left(uv\sqrt{u^{2} + v^{2}}\partial_{v} A^{\phi} + u\sqrt{u^{2} + v^{2}}A^{\phi} + \left(- u^{2} - v^{2}\right)\partial_{\phi} A^{v}\right)\boldsymbol{e_{u}} \\ & + \frac{1}{uv\left(u^{2} + v^{2}\right)}\left(- uv\sqrt{u^{2} + v^{2}}\partial_{u} A^{\phi} - v\sqrt{u^{2} + v^{2}}A^{\phi} + \left(u^{2} + v^{2}\right)\partial_{\phi} A^{u}\right)\boldsymbol{e_{v}} \\ & + \frac{1}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}}\left(uA^{v} - vA^{u} + \left(u^{2} + v^{2}\right)\left(- \partial_{v} A^{u} + \partial_{u} A^{v}\right)\right)\boldsymbol{e_{\phi}} \end{aligned}$$

$$\boldsymbol{\nabla} \wedge B = \left(\left(\frac{u}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{1}{u\sqrt{u^{2} + v^{2}}}\right)B^{v\phi} + \left(- \frac{v}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} - \frac{1}{v\sqrt{u^{2} + v^{2}}}\right)B^{u\phi} - \frac{\partial_{v} B^{u\phi}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{u} B^{v\phi}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{\phi} B^{uv}}{uv}\right)\boldsymbol{e_{u}}\wedge\boldsymbol{e_{v}}\wedge\boldsymbol{e_{\phi}}$$

$$f = f$$

$$A = A^{u}\boldsymbol{e_{u}} + A^{v}\boldsymbol{e_{v}} + A^{z}\boldsymbol{e_{z}}$$

$$B = B^{uv}\boldsymbol{e_{u}}\wedge\boldsymbol{e_{v}} + B^{uz}\boldsymbol{e_{u}}\wedge\boldsymbol{e_{z}} + B^{vz}\boldsymbol{e_{v}}\wedge\boldsymbol{e_{z}}$$

$$\boldsymbol{\nabla} f = \frac{\partial_{u} f}{\sqrt{\sin^{2}{\left(v \right)}\cosh^{2}{\left(u \right)} + \cos^{2}{\left(v \right)}\sinh^{2}{\left(u \right)}}\left|{a}\right|}\boldsymbol{e_{u}} + \frac{\partial_{v} f}{\sqrt{\sin^{2}{\left(v \right)}\cosh^{2}{\left(u \right)} + \cos^{2}{\left(v \right)}\sinh^{2}{\left(u \right)}}\left|{a}\right|}\boldsymbol{e_{v}} + \partial_{z} f\boldsymbol{e_{z}}$$

$$\boldsymbol{\nabla}\cdot A = \frac{\left|{a}\right|}{2\left(\sin^{2}{\left(v \right)}\cosh^{2}{\left(u \right)} + \cos^{2}{\left(v \right)}\sinh^{2}{\left(u \right)}\right)^{\frac{5}{2}}\left(\left(\cos^{2}{\left(v \right)}\cos{\left(2v \right)} - \cos^{2}{\left(v \right)}\cosh{\left(2u \right)}\right)\sinh^{2}{\left(u \right)} + \sin^{2}{\left(v \right)}\cos{\left(2v \right)}\cosh^{2}{\left(u \right)} - \sin^{2}{\left(v \right)}\cosh^{2}{\left(u \right)}\cosh{\left(2u \right)}\right)}\Bigg(\left(A^{u}\sinh{\left(2u \right)} + A^{v}\sin{\left(2v \right)}\right)\left(\sin^{2}{\left(v \right)}\cosh^{2}{\left(u \right)} + \cos^{2}{\left(v \right)}\sinh^{2}{\left(u \right)}\right)(\sin$$

$$-I(\boldsymbol{\nabla}\wedge A) = \left(- \partial_{z} A^{v} + \frac{\partial_{v} A^{z}}{\sqrt{\sin^{2}{\left(v \right)} + \sinh^{2}{\left(u \right)}}\left|{a}\right|}\right)\boldsymbol{e_{u}} + \left(\partial_{z} A^{u} - \frac{\partial_{u} A^{z}}{\sqrt{\sin^{2}{\left(v \right)} + \sinh^{2}{\left(u \right)}}\left|{a}\right|}\right)\boldsymbol{e_{v}} - \frac{\sqrt{2}\left|{a}\right|}{2\left(- \cos{\left(2v \right)} + \cosh{\left(2u \right)}\right)^{\frac{5}{2}}}\Big(\left(\left(- 2\left(\cosh{\left(2u \right)} - 1\right)^{2} + 2\right)\partial_{u} A^{v} + \left(2\left(\cosh{\left(2u \right)} - 1\right)^{2} - 2\right)\partial_{v} A^{u} + \left(2\partial_{v} A^{u} - 2\partial_{u} A^{v}\right)\cos^{2}{\left(2v \right)} + (-$$

$$\boldsymbol{\nabla}\wedge B = - \frac{\left|{a}\right|}{2\left(\sin^{2}{\left(v \right)}\cosh^{2}{\left(u \right)} + \cos^{2}{\left(v \right)}\sinh^{2}{\left(u \right)}\right)^{\frac{5}{2}}\left(\left(\cos^{2}{\left(v \right)}\cos{\left(2v \right)} - \cos^{2}{\left(v \right)}\cosh{\left(2u \right)}\right)\sinh^{2}{\left(u \right)} + \sin^{2}{\left(v \right)}\cos{\left(2v \right)}\cosh^{2}{\left(u \right)} - \sin^{2}{\left(v \right)}\cosh^{2}{\left(u \right)}\cosh{\left(2u \right)}\right)}\Bigg(\left(B^{uz}\sin{\left(2v \right)} - B^{vz}\sinh{\left(2u \right)}\right)\left(\sin^{2}{\left(v \right)}\cosh^{2}{\left(u \right)} + \cos^{2}{\left(v \right)}\sinh^{2}{\left(u\right)}$$

$$f = f$$
$$A = A^{\xi}\boldsymbol{e_{\xi}} + A^{\eta}\boldsymbol{e_{\eta}} + A^{\phi}\boldsymbol{e_{\phi}}$$
$$B = B^{\xi\xi}\boldsymbol{e_{\xi}} \wedge \boldsymbol{e_{\eta}} + B^{\xi\xi}\boldsymbol{e_{\xi}} \wedge \boldsymbol{e_{\phi}} + B^{\phi\phi}\boldsymbol{e_{\eta}} \wedge \boldsymbol{e_{\phi}}$$
$$\boldsymbol{\nabla} f = \frac{\partial_{\xi} f}{\sqrt{\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}}\left|{a}\right|}\boldsymbol{e_{\xi}} + \frac{\partial_{\eta} f}{\sqrt{\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}}\left|{a}\right|}\boldsymbol{e_{\eta}} + \frac{\partial_{\phi} f}{a \sin{\left(\eta \right)} \sinh{\left(\xi \right)}}\boldsymbol{e_{\phi}}$$
$$\boldsymbol{\nabla} \cdot A = \frac{1}{a^{2} \left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}\right)^{3} \sin{\left(\eta \right)} \sinh{\left(\xi \right)}} \left(a \left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}\right)^{3} \partial_{\phi} A^{\phi} + \left(\frac{1}{2} \left(A^{\eta} \sin{\left(2 \eta \right)} + A^{\xi} \sinh{\left(2 \xi \right)}\right) \left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}\right)^{\frac{3}{2}} \sin{\left(\eta \right)} \sinh{\left(\xi \right)} + \left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}\right)^{\frac{5}{2}} \left(\partial_{\eta} A^{\eta} + \partial_{\xi} A^{\xi}\right) \sin{\left(\eta \right)} \sinh{\left(\xi \right)} + \left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi\right.\right.\right.$$
$$\begin{aligned} -I (\boldsymbol{\nabla} \wedge A) = & \frac{1}{a^{2} \sin{\left(\eta \right)}} \left(- \frac{a \partial_{\phi} A^{\eta}}{\sinh{\left(\xi \right)}} + \left(\frac{A^{\phi} \cos{\left(\eta \right)}}{\sqrt{\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}}} + \frac{\sin{\left(\eta \right)}\partial_{\eta} A^{\phi}}{\sqrt{\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}}}\right) \left|{a}\right|\right) \boldsymbol{e_{\xi}} \\ & - \frac{1}{a^{2} \sinh{\left(\xi \right)}} \left(- \frac{a \partial_{\phi} A^{\xi}}{\sin{\left(\eta \right)}} + \left(\frac{A^{\phi} \cosh{\left(\xi \right)}}{\sqrt{\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}}} + \frac{\sinh{\left(\xi \right)}\partial_{\xi} A^{\phi}}{\sqrt{\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}}}\right) \left|{a}\right|\right) \boldsymbol{e_{\eta}} \\ & + \frac{1}{2 \left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}\right)^{\frac{3}{2}} \left|{a}\right|} \left(\left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}\right) \left(2 \partial_{\xi} A^{\eta} - 2 \partial_{\eta} A^{\xi}\right) + A^{\eta} \sinh{\left(2 \xi \right)} - A^{\xi} \sin{\left(2 \eta \right)}\right) \boldsymbol{e_{\phi}} \end{aligned}$$
$$\boldsymbol{\nabla} \wedge B = \frac{1}{a^{2} \left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}\right)^{3} \sin{\left(\eta \right)} \sinh{\left(\xi \right)}} \left(a \left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}\right)^{3} \partial_{\phi} B^{\xi\xi} + \left(\frac{1}{2} \left(B^{\phi\phi} \sinh{\left(2 \xi \right)} - B^{\xi\xi} \sin{\left(2 \eta \right)}\right) \left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}\right)^{\frac{3}{2}} \sin{\left(\eta \right)} \sinh{\left(\xi \right)} + \left(\sin^{2}{\left(\eta \right)} + \sinh^{2}{\left(\xi \right)}\right)^{\frac{5}{2}} \left(\partial_{\xi} B^{\phi\phi} - \partial_{\eta} B^{\xi\xi}\right) \sin{\left(\eta \right)} \sinh{\left(\xi \right)} + \left(\sin^{2}{\left(\eta \right)} + \mathrm{s}\right.\right.$$