Program:

```
import sys
from sympy import symbols,exp,I,Matrix,solve,simplify
from printer import Format,xpdf,Get_Program,Print_Function
from ga import Ga
from metric import linear_expand

Format()
X = (t,x,y,z) = symbols('t x y z',real=True)
(st4d,g0,g1,g2,g3) = Ga.build('gamma*t|x|y|z',g=[1,-1,-1,-1],coords=X)

i = st4d.i

B = st4d.mv('B','vector')
E = st4d.mv('E','vector')
B.set_coef(1,0,0)
E.set_coef(1,0,0)
B *= g0
E *= g0
F = E+i*B

kx, ky, kz, w = symbols('k_x k_y k_z omega',real=True)
kv = kx*g1+ky*g2+kz*g3
xv = x*g1+y*g2+z*g3
KX = ((w*g0+kv)|(t*g0+xv)).scalar()

Ixyz = g1*g2*g3

F = F*exp(I*KX)

print r'\text{Pseudo Scalar\;\;}I =',i
print r'%I_{xyz} =',Ixyz
F.Fmt(3,'\\text{Electromagnetic Field Bi-Vector\\;\\;} F')
gradF = st4d.grad*F

print '#Geom Derivative of Electomagnetic Field Bi-Vector'
gradF.Fmt(3,'grad*F = 0')

gradF = gradF / (I * exp(I*KX))
gradF.Fmt(3,r'%\lp\bm{\nabla}F\rp /\lp i e^{iK\cdot X}\rp = 0')

g = '1 # 0 0,# 1 0 0,0 0 1 0,0 0 0 -1'
X = (xE,xB,xk,t) = symbols('x_E x_B x_k t',real=True)
(EBkst,eE,eB,ek,et) = Ga.build('e_E e_B e_k t',g=g,coords=X)
```

```
i = EBkst.i

E,B,k,w = symbols('E B k omega',real=True)

F = E*eE*et+i*B*eB*et
kv = k*ek+w*et
xv = xE*eE+xB*eB+xk*ek+t*et
KX = (kv|xv).scalar()
F = F*exp(I*KX)

print r'%\mbox{set } e_{E}\cdot e_{k} = e_{B}\cdot e_{k} = 0'+\
      r'\mbox{ and } e_{E}\cdot e_{E} = e_{B}\cdot e_{B} = '+\
      r'e_{k}\cdot e_{k} = -e_{t}\cdot e_{t} = 1'

print 'g =', EBkst.g

print 'K|X =',KX
print 'F =',F
(EBkst.grad*F).Fmt(3,'grad*F = 0')

gradF_reduced = (EBkst.grad*F)/(I*exp(I*KX))

gradF_reduced.Fmt(3,r'%\lp\bm{\nabla}F\rp/\lp ie^{iK\cdot X} \rp = 0')

print r'%\mbox{Previous equation requires that: }e_{E}\cdot e_{B} = 0'+\
      r'\mbox{ if }B\ne 0\mbox{ and }k\ne 0'

gradF_reduced = gradF_reduced.subs({EBkst.g[0,1]:0})
gradF_reduced.Fmt(3,r'%\lp\bm{\nabla}F\rp/\lp ie^{iK\cdot X} \rp = 0')

(coefs,bases) = linear_expand(gradF_reduced.obj)

eq1 = coefs[0]
eq2 = coefs[1]

B1 = solve(eq1,B)[0]
B2 = solve(eq2,B)[0]

print r'\mbox{eq1: }B =',B1
print r'\mbox{eq2: }B =',B2

eq3 = B1-B2

print r'\mbox{eq3 = eq1-eq2: }0 =',eq3
eq3 = simplify(eq3 / E)
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```
print r'\mbox{eq3 = (eq1-eq2)/E: }0 =',eq3
print '#Solutions for $k$ and $B$ in terms of $\omega$ and $E$:'
print 'k =',Matrix(solve(eq3,k))
print 'B =',Matrix([B1.subs(w,k),B1.subs(-w,k)])
xpdf(paper='landscape',prog=True)
```

Code Output:

Pseudo Scalar $I = \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}$

$I_{xyz} = \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}$

$$\begin{aligned}
\text{Electromagnetic Field Bi-Vector } F = &- E^{x} e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \\
&- E^{y} e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \\
&- E^{z} e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_z} \\
&- B^{z} e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \\
&+ B^{y} e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} \\
&- B^{x} e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}
\end{aligned}$$

Geom Derivative of Electomagnetic Field Bi-Vector

$$\begin{aligned}
\boldsymbol{\nabla} F = 0 = &- i \left(E^{x} k_{x} + E^{y} k_{y} + E^{z} k_{z}\right) e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma_t} \\
&+ i \left(B^{y} k_{z} - B^{z} k_{y} - E^{x} \omega\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma_x} \\
&+ i \left(-B^{x} k_{z} + B^{z} k_{x} - E^{y} \omega\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma_y} \\
&+ i \left(B^{x} k_{y} - B^{y} k_{x} - E^{z} \omega\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma_z} \\
&+ i \left(-B^{z} \omega - E^{x} k_{y} + E^{y} k_{x}\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \\
&+ i \left(B^{y} \omega - E^{x} k_{z} + E^{z} k_{x}\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} \\
&+ i \left(-B^{x} \omega - E^{y} k_{z} + E^{z} k_{y}\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \\
&- i \left(B^{x} k_{x} + B^{y} k_{y} + B^{z} k_{z}\right) e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}
\end{aligned}$$

$$\begin{aligned}
\left(\boldsymbol{\nabla} F\right) / \left(i e^{iK\cdot X}\right) = 0 = &\left(-E^{x} k_{x} - E^{y} k_{y} - E^{z} k_{z}\right) \boldsymbol{\gamma_t} \\
&+ \left(B^{y} k_{z} - B^{z} k_{y} - E^{x} \omega\right) \boldsymbol{\gamma_x} \\
&+ \left(-B^{x} k_{z} + B^{z} k_{x} - E^{y} \omega\right) \boldsymbol{\gamma_y} \\
&+ \left(B^{x} k_{y} - B^{y} k_{x} - E^{z} \omega\right) \boldsymbol{\gamma_z} \\
&+ \left(-B^{z} \omega - E^{x} k_{y} + E^{y} k_{x}\right) \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \\
&+ \left(B^{y} \omega - E^{x} k_{z} + E^{z} k_{x}\right) \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} \\
&+ \left(-B^{x} \omega - E^{y} k_{z} + E^{z} k_{y}\right) \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \\
&+ \left(-B^{x} k_{x} - B^{y} k_{y} - B^{z} k_{z}\right) \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}
\end{aligned}$$

set $e_E \cdot e_k = e_B \cdot e_k = 0$ and $e_E \cdot e_E = e_B \cdot e_B = e_k \cdot e_k = -e_t \cdot e_t = 1$

$$g = \begin{bmatrix} 1 & (e_E \cdot e_B) & 0 & 0 \\ (e_E \cdot e_B) & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$K \cdot X = -\omega t + k x_k$$

$$F = -B e^{-i(\omega t - k x_k)} \boldsymbol{e_E} \wedge \boldsymbol{e_k} + E e^{i(-\omega t + k x_k)} \boldsymbol{e_E} \wedge \boldsymbol{t} + (e_E \cdot e_B) B e^{i(-\omega t + k x_k)} \boldsymbol{e_B} \wedge \boldsymbol{e_k}$$

$$\begin{aligned} \boldsymbol{\nabla} F = 0 = & i (Bk + E\omega) e^{i(-\omega t + k x_k)} \boldsymbol{e_E} \\ & - i (e_E \cdot e_B) B k e^{-i(\omega t - k x_k)} \boldsymbol{e_B} \\ & - i (B\omega + Ek) e^{-i(\omega t - k x_k)} \boldsymbol{e_E} \wedge \boldsymbol{e_k} \wedge \boldsymbol{t} \\ & + i (e_E \cdot e_B) B\omega e^{i(-\omega t + k x_k)} \boldsymbol{e_B} \wedge \boldsymbol{e_k} \wedge \boldsymbol{t} \end{aligned}$$

$$\begin{aligned} (\boldsymbol{\nabla} F) / \left(i e^{iK \cdot X}\right) = 0 = & (Bk + E\omega) \boldsymbol{e_E} \\ & - (e_E \cdot e_B) B k \boldsymbol{e_B} \\ & + (-B\omega - Ek) \boldsymbol{e_E} \wedge \boldsymbol{e_k} \wedge \boldsymbol{t} \\ & + (e_E \cdot e_B) B\omega \boldsymbol{e_B} \wedge \boldsymbol{e_k} \wedge \boldsymbol{t} \end{aligned}$$

Previous equation requires that: $e_E \cdot e_B = 0$ if $B \neq 0$ and $k \neq 0$

$$\begin{aligned} (\boldsymbol{\nabla} F) / \left(i e^{iK \cdot X}\right) = 0 = & (Bk + E\omega) \boldsymbol{e_E} \\ & + (-B\omega - Ek) \boldsymbol{e_E} \wedge \boldsymbol{e_k} \wedge \boldsymbol{t} \end{aligned}$$

eq1: $B = -\dfrac{E\omega}{k}$

eq2: $B = -\dfrac{Ek}{\omega}$

eq3 = eq1-eq2: $0 = -\dfrac{E\omega}{k} + \dfrac{Ek}{\omega}$

eq3 = (eq1-eq2)/E: $0 = -\dfrac{\omega}{k} + \dfrac{k}{\omega}$

Solutions for k and B in terms of ω and E:

$$k = \begin{bmatrix} -\omega \\ \omega \end{bmatrix}$$

$$B = \begin{bmatrix} -E \\ E \end{bmatrix}$$