```python
def main():
    Print_Function()
    (x, y, z) = xyz = symbols('x,y,z',real=True)
    (o3d, ex, ey, ez) = Ga.build('e_x e_y e_z', g=[1, 1, 1], coords=xyz)
    grad = o3d.grad
    (u, v) = uv = symbols('u,v',real=True)
    (g2d, eu, ev) = Ga.build('e_u e_v', coords=uv)
    grad_uv = g2d.grad
    v_xyz = o3d.mv('v','vector')
    A_xyz = o3d.mv('A','vector',f=True)
    A_uv = g2d.mv('A','vector',f=True)
    print '#3d orthogonal ($A$ is vector function)'
    print 'A =', A_xyz
    print '%A^{2} =', A_xyz * A_xyz
    print 'grad|A =', grad | A_xyz
    print 'grad*A =', grad * A_xyz
    print 'v|(grad*A) =',v_xyz|(grad*A_xyz)
    print '#2d general ($A$ is vector function)'
    print 'A =', A_uv
    print '%A^{2} =', A_uv * A_uv
    print 'grad|A =', grad_uv | A_uv
    print 'grad*A =', grad_uv * A_uv
    A = o3d.lt('A')
    print '#3d orthogonal ($A,\\;B$ are linear transformations)'
    print 'A =', A
    print r'\f{mat}{A} =', A.matrix()
    print '\\f{\\det}{A} =', A.det()
    print '\\overline{A} =', A.adj()
    print '\\f{\\Tr}{A} =', A.tr()
    print '\\f{A}{e_x^e_y} =', A(ex^ey)
    print '\\f{A}{e_x}^\\f{A}{e_y} =', A(ex)^A(ey)
    B = o3d.lt('B')
    print 'g =', o3d.g
    print '%g^{-1} =', o3d.g_inv

    print 'A + B =', A + B
    print 'AB =', A * B
    print 'A - B =', A - B
    print 'General Symmetric Linear Transformation'
    Asym = o3d.lt('A',mode='s')
    print 'A =', Asym
    print 'General Antisymmetric Linear Transformation'
    Aasym = o3d.lt('A',mode='a')
    print 'A =', Aasym
    print '#2d general ($A,\\;B$ are linear transformations)'
    A2d = g2d.lt('A')
    print 'g =', g2d.g
    print '%g^{-1} =', g2d.g_inv
    print '%gg^{-1} =', simplify(g2d.g * g2d.g_inv)
    print 'A =', A2d
    print r'\f{mat}{A} =', A2d.matrix()
    print '\\f{\\det}{A} =', A2d.det()
    A2d_adj = A2d.adj()
    print '\\overline{A} =', A2d_adj
    print '\\f{mat}{\\overline{A}} =', simplify(A2d_adj.matrix())
    print '\\f{\\Tr}{A} =', A2d.tr()
    print '\\f{A}{e_u^e_v} =', A2d(eu^ev)
    print '\\f{A}{e_u}^\\f{A}{e_v} =', A2d(eu)^A2d(ev)
    B2d = g2d.lt('B')
```

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    print 'B =', B2d
    print 'A + B =', A2d + B2d
    print 'AB =', A2d * B2d
    print 'A - B =', A2d - B2d
    a = g2d.mv('a','vector')
    b = g2d.mv('b','vector')
    print r'a|\f{\overline{A}}{b}-b|\f{\underline{A}}{a} =',((a|A2d.adj()(b))-(b|A2d(a))).simplify()
    m4d = Ga('e_t e_x e_y e_z', g=[1, -1, -1, -1],coords=symbols('t,x,y,z',real=True))
    T = m4d.lt('T')
    print 'g =', m4d.g
    print r'\underline{T} =',T
    print r'\overline{T} =',T.adj()
    print r'\f{\det}{\underline{T}} =',T.det()
    print r'\f{\mbox{tr}}{\underline{T}} =',T.tr()
    a = m4d.mv('a','vector')
    b = m4d.mv('b','vector')
    print r'a|\f{\overline{T}}{b}-b|\f{\underline{T}}{a} =',((a|T.adj()(b))-(b|T(a))).simplify()
    coords = (r, th, phi) = symbols('r,theta,phi', real=True)
    (sp3d, er, eth, ephi) = Ga.build('e_r e_th e_ph', g=[1, r**2, r**2*sin(th)**2], coords=coords)
    grad = sp3d.grad
    sm_coords = (u, v) = symbols('u,v', real=True)
    smap = [1, u, v]  # Coordinate map for sphere of r = 1
    sph2d = sp3d.sm(smap,sm_coords,norm=True)
    (eu, ev) = sph2d.mv()
    grad_uv = sph2d.grad
    F = sph2d.mv('F','vector',f=True)
    f = sph2d.mv('f','scalar',f=True)
    print 'f =',f
    print 'grad*f =',grad_uv * f
    print 'F =',F
    print 'grad*F =',grad_uv * F
    tp = (th,phi) = symbols('theta,phi',real=True)
    smap = [sin(th)*cos(phi),sin(th)*sin(phi),cos(th)]
    sph2dr = o3d.sm(smap,tp,norm=True)
    (eth, ephi) = sph2dr.mv()
    grad_tp = sph2dr.grad
    F = sph2dr.mv('F','vector',f=True)
    f = sph2dr.mv('f','scalar',f=True)
    print 'f =',f
    print 'grad*f =',grad_tp * f
    print 'F =',F
    print 'grad*F =',grad_tp * F
    return
```

Code Output: 3d orthogonal (A is vector function)

$$A = A^{x}\boldsymbol{e}_{x} + A^{y}\boldsymbol{e}_{y} + A^{z}\boldsymbol{e}_{z}$$

$$A^{2} = \left(A^{x}\right)^{2} + \left(A^{y}\right)^{2} + \left(A^{z}\right)^{2}$$

$$\boldsymbol{\nabla} \cdot A = \partial_{x} A^{x} + \partial_{y} A^{y} + \partial_{z} A^{z}$$

$$\boldsymbol{\nabla} A = \left(\partial_{x} A^{x} + \partial_{y} A^{y} + \partial_{z} A^{z}\right) + \left(-\partial_{y} A^{x} + \partial_{x} A^{y}\right)\boldsymbol{e}_{x}\wedge \boldsymbol{e}_{y} + \left(-\partial_{z} A^{x} + \partial_{x} A^{z}\right)\boldsymbol{e}_{x}\wedge \boldsymbol{e}_{z} + \left(-\partial_{z} A^{y} + \partial_{y} A^{z}\right)\boldsymbol{e}_{y}\wedge \boldsymbol{e}_{z}$$

$$v \cdot (\boldsymbol{\nabla} A) = \left(v^{y}\partial_{y} A^{x} - v^{y}\partial_{x} A^{y} + v^{z}\partial_{z} A^{x} - v^{z}\partial_{x} A^{z}\right)\boldsymbol{e}_{x} + \left(-v^{x}\partial_{y} A^{x} + v^{x}\partial_{x} A^{y} + v^{z}\partial_{z} A^{y} - v^{z}\partial_{y} A^{z}\right)\boldsymbol{e}_{y} + \left(-v^{x}\partial_{z} A^{x} + v^{x}\partial_{x} A^{z} - v^{y}\partial_{z} A^{y} + v^{y}\partial_{y} A^{z}\right)\boldsymbol{e}_{z}$$

2d general (A is vector function)

$$A = A^{u}\boldsymbol{e}_{u} + A^{v}\boldsymbol{e}_{v}$$

$$A^{2} = \left(e_{u}\cdot e_{u}\right)\left(A^{u}\right)^{2} + 2\left(e_{u}\cdot e_{v}\right) A^{u} A^{v} + \left(e_{v}\cdot e_{v}\right)\left(A^{v}\right)^{2}$$

$$\boldsymbol{\nabla} \cdot A = \partial_{u} A^{u} + \partial_{v} A^{v}$$

$$\boldsymbol{\nabla} A = \left (\partial_{u} A^{u} + \partial_{v} A^{v}\right) + \frac{ - \left (e_{u}\cdot e_{u}\right) \partial_{v} A^{u} + \left (e_{u}\cdot e_{v}\right) \partial_{u} A^{u} - \left (e_{u}\cdot e_{v}\right) \partial_{v} A^{v} + \left (e_{v}\cdot e_{v}\right) \partial_{u} A^{v}}{\left (e_{u}\cdot e_{u}\right) \left (e_{v}\cdot e_{v}\right) - \left (e_{u}\cdot e_{v}\right) ^{2}} \boldsymbol{e}_{u}\wedge \boldsymbol{e}_{v}$$

3d orthogonal ($A,\ B$ are linear transformations)

$$A = \left \{ \begin{array}{ll} L \left (\boldsymbol{e}_{x}\right) = & A^{xx} \boldsymbol{e}_{x} + A^{xy} \boldsymbol{e}_{y} + A^{xz} \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{y}\right) = & A^{yx} \boldsymbol{e}_{x} + A^{yy} \boldsymbol{e}_{y} + A^{yz} \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{z}\right) = & A^{zx} \boldsymbol{e}_{x} + A^{zy} \boldsymbol{e}_{y} + A^{zz} \boldsymbol{e}_{z} \end{array} \right \}$$

$$mat\left (A\right) = \left [\begin{array}{ccc} A^{xx} & A^{xy} & A^{xz} \\ A^{yx} & A^{yy} & A^{yz} \\ A^{zx} & A^{zy} & A^{zz} \end{array}\right]$$

$$\det\left (A\right) = A^{zx} \left (A^{xy} A^{yz} - A^{xz} A^{yy}\right) - A^{zy} \left (A^{xx} A^{yz} - A^{xz} A^{yx}\right) + A^{zz} \left (A^{xx} A^{yy} - A^{xy} A^{yx}\right)$$

$$\overline{A} = \left \{ \begin{array}{ll} L \left (\boldsymbol{e}_{x}\right) = & A^{xx} \boldsymbol{e}_{x} + A^{yx} \boldsymbol{e}_{y} + A^{zx} \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{y}\right) = & A^{xy} \boldsymbol{e}_{x} + A^{yy} \boldsymbol{e}_{y} + A^{zy} \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{z}\right) = & A^{xz} \boldsymbol{e}_{x} + A^{yz} \boldsymbol{e}_{y} + A^{zz} \boldsymbol{e}_{z} \end{array} \right \}$$

$$\mathrm{Tr}\left (A\right) = A^{xx} + A^{yy} + A^{zz}$$

$$A\left (e_{x}\wedge e_{y}\right) = \left (A^{xx} A^{yy} - A^{xy} A^{yx}\right) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{y} + \left (A^{xx} A^{yz} - A^{xz} A^{yx}\right) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{z} + \left (A^{xy} A^{yz} - A^{xz} A^{yy}\right) \boldsymbol{e}_{y}\wedge \boldsymbol{e}_{z}$$

$$A\left (e_{x}\right) \wedge A\left (e_{y}\right) = \left (A^{xx} A^{yy} - A^{xy} A^{yx}\right) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{y} + \left (A^{xx} A^{yz} - A^{xz} A^{yx}\right) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{z} + \left (A^{xy} A^{yz} - A^{xz} A^{yy}\right) \boldsymbol{e}_{y}\wedge \boldsymbol{e}_{z}$$

$$g = \left [\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

$$g^{-1} = \left [\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

$$A + B = \left \{ \begin{array}{ll} L \left (\boldsymbol{e}_{x}\right) = & \left (A^{xx} + B^{xx}\right) \boldsymbol{e}_{x} + \left (A^{xy} + B^{xy}\right) \boldsymbol{e}_{y} + \left (A^{xz} + B^{xz}\right) \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{y}\right) = & \left (A^{yx} + B^{yx}\right) \boldsymbol{e}_{x} + \left (A^{yy} + B^{yy}\right) \boldsymbol{e}_{y} + \left (A^{yz} + B^{yz}\right) \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{z}\right) = & \left (A^{zx} + B^{zx}\right) \boldsymbol{e}_{x} + \left (A^{zy} + B^{zy}\right) \boldsymbol{e}_{y} + \left (A^{zz} + B^{zz}\right) \boldsymbol{e}_{z} \end{array} \right \}$$

$$AB = \left \{ \begin{array}{ll} L \left (\boldsymbol{e}_{x}\right) = & \left (A^{xx} B^{xx} + A^{yx} B^{xy} + A^{zx} B^{xz}\right) \boldsymbol{e}_{x} + \left (A^{xy} B^{xx} + A^{yy} B^{xy} + A^{zy} B^{xz}\right) \boldsymbol{e}_{y} + \left (A^{xz} B^{xx} + A^{yz} B^{xy} + A^{zz} B^{xz}\right) \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{y}\right) = & \left (A^{xx} B^{yx} + A^{yx} B^{yy} + A^{zx} B^{yz}\right) \boldsymbol{e}_{x} + \left (A^{xy} B^{yx} + A^{yy} B^{yy} + A^{zy} B^{yz}\right) \boldsymbol{e}_{y} + \left (A^{xz} B^{yx} + A^{yz} B^{yy} + A^{zz} B^{yz}\right) \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{z}\right) = & \left (A^{xx} B^{zx} + A^{yx} B^{zy} + A^{zx} B^{zz}\right) \boldsymbol{e}_{x} + \left (A^{xy} B^{zx} + A^{yy} B^{zy} + A^{zy} B^{zz}\right) \boldsymbol{e}_{y} + \left (A^{xz} B^{zx} + A^{yz} B^{zy} + A^{zz} B^{zz}\right) \boldsymbol{e}_{z} \end{array} \right \}$$

$$A - B = \left \{ \begin{array}{ll} L \left (\boldsymbol{e}_{x}\right) = & \left (A^{xx} - B^{xx}\right) \boldsymbol{e}_{x} + \left (A^{xy} - B^{xy}\right) \boldsymbol{e}_{y} + \left (A^{xz} - B^{xz}\right) \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{y}\right) = & \left (A^{yx} - B^{yx}\right) \boldsymbol{e}_{x} + \left (A^{yy} - B^{yy}\right) \boldsymbol{e}_{y} + \left (A^{yz} - B^{yz}\right) \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{z}\right) = & \left (A^{zx} - B^{zx}\right) \boldsymbol{e}_{x} + \left (A^{zy} - B^{zy}\right) \boldsymbol{e}_{y} + \left (A^{zz} - B^{zz}\right) \boldsymbol{e}_{z} \end{array} \right \}$$

$$General Symmetric Linear Transformation$$

$$A = \left \{ \begin{array}{ll} L \left (\boldsymbol{e}_{x}\right) = & A^{xx} \boldsymbol{e}_{x} + A^{xy} \boldsymbol{e}_{y} + A^{xz} \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{y}\right) = & A^{xy} \boldsymbol{e}_{x} + A^{yy} \boldsymbol{e}_{y} + A^{yz} \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{z}\right) = & A^{xz} \boldsymbol{e}_{x} + A^{yz} \boldsymbol{e}_{y} + A^{zz} \boldsymbol{e}_{z} \end{array} \right \}$$

$$General Antisymmetric Linear Transformation$$

$$A = \left \{ \begin{array}{ll} L \left (\boldsymbol{e}_{x}\right) = & A^{xy} \boldsymbol{e}_{y} + A^{xz} \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{y}\right) = & - A^{xy} \boldsymbol{e}_{x} + A^{yz} \boldsymbol{e}_{z} \\ L \left (\boldsymbol{e}_{z}\right) = & - A^{xz} \boldsymbol{e}_{x} - A^{yz} \boldsymbol{e}_{y} \end{array} \right \}$$

2d general ($A,\ B$ are linear transformations)

$$g = \left [\begin{array}{cc} \left (e_{u}\cdot e_{u}\right) & \left (e_{u}\cdot e_{v}\right) \\ \left (e_{u}\cdot e_{v}\right) & \left (e_{v}\cdot e_{v}\right) \end{array}\right]$$

$$g^{-1} = \left [\begin{array}{cc} \frac{\left (e_{u}\cdot e_{u}\right) \left (e_{v}\cdot e_{v}\right) ^{2} - \left (e_{u}\cdot e_{v}\right) ^{2} \left (e_{v}\cdot e_{v}\right) }{\left (e_{u}\cdot e_{u}\right) ^{2} \left (e_{v}\cdot e_{v}\right) ^{2} - 2 \left (e_{u}\cdot e_{u}\right) \left (e_{u}\cdot e_{v}\right) ^{2} \left (e_{v}\cdot e_{v}\right) + \left (e_{u}\cdot e_{v}\right) ^{4}} & \frac{- \left (e_{u}\cdot e_{u}\right) \left (e_{u}\cdot e_{v}\right) \left (e_{v}\cdot e_{v}\right) + \left (e_{u}\cdot e_{v}\right) ^{3}}{\left (e_{u}\cdot e_{u}\right) ^{2} \left (e_{v}\cdot e_{v}\right) ^{2} - 2 \left (e_{u}\cdot e_{u}\right) \left (e_{u}\cdot e_{v}\right) ^{2} \left (e_{v}\cdot e_{v}\right) + \left (e_{u}\cdot e_{v}\right) ^{4}} \\ \frac{- \left (e_{u}\cdot e_{u}\right) \left (e_{u}\cdot e_{v}\right) \left (e_{v}\cdot e_{v}\right) + \left (e_{u}\cdot e_{v}\right) ^{3}}{\left (e_{u}\cdot e_{u}\right) ^{2} \left (e_{v}\cdot e_{v}\right) ^{2} - 2 \left (e_{u}\cdot e_{u}\right) \left (e_{u}\cdot e_{v}\right) ^{2} \left (e_{v}\cdot e_{v}\right) + \left (e_{u}\cdot e_{v}\right) ^{4}} & \frac{\left (e_{u}\cdot e_{u}\right) ^{2} \left (e_{v}\cdot e_{v}\right) - \left (e_{u}\cdot e_{u}\right) \left (e_{u}\cdot e_{v}\right) ^{2}}{\left (e_{u}\cdot e_{u}\right) ^{2} \left (e_{v}\cdot e_{v}\right) ^{2} - 2 \left (e_{u}\cdot e_{u}\right) \left (e_{u}\cdot e_{v}\right) ^{2} \left (e_{v}\cdot e_{v}\right) + \left (e_{u}\cdot e_{v}\right) ^{4}} \end{array}\right]$$

$$g g^{-1} = \left[\begin{matrix}1 & 0\\0 & 1\end{matrix}\right]$$

$$A = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_v)A^{uv}+(e_v\cdot e_v)A^{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A^{uv}-(e_u\cdot e_v)A^{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{-(e_u\cdot e_v)A^{vv}+(e_v\cdot e_v)A^{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A^{vv}-(e_u\cdot e_v)A^{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \end{array} \right\}$$

$$mat\left(A\right) = \left[\begin{array}{cc} A^{uu} & A^{uv} \\ A^{vu} & A^{vv} \end{array}\right]$$

$$\det\left(A\right) = \frac{1}{\sqrt{\left(e_u\cdot e_u\right)\left(e_v\cdot e_v\right) - \left(e_u\cdot e_v\right)^2}} \Bigg(- \Bigg(\frac{\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)A^{uv}}{\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2 - 2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right) + \left(e_u\cdot e_v\right)^4} - \frac{\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2 A^{uv}}{\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2 - 2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right) + \left(e_u\cdot e_v\right)^4} - \frac{\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A^{uu}}{\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2 - 2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right) +}$$

$$\overline{A} = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_v)A^{vu}+(e_v\cdot e_v)A^{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A^{vu}-(e_u\cdot e_v)A^{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{-(e_u\cdot e_v)A^{vv}+(e_v\cdot e_v)A^{uv}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A^{vv}-(e_u\cdot e_v)A^{uv}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \end{array} \right\}$$

$$mat\left(\overline{A}\right) = \left[\begin{array}{cc} \frac{1}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\left(\left(e_u\cdot e_u\right)^2 A^{vv} - \left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)A^{uv} - \left(e_u\cdot e_v\right)^2 A^{vv} + \left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A^{uv}\right) & \frac{1}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\left(\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)A^{vv} - \left(e_u\cdot e_v\right)^2 A^{uv} - \left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A^{vv} + \left(e_v\cdot e_v\right)^2 A^{uv}\right) \\ \frac{1}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\left(\left(e_u\cdot e_u\right)^2 A^{vu} - \left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)A^{uu} - \left(e_u\cdot e_v\right)^2 A^{vu} + \left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A^{uu}\right) & \frac{1}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\left(\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)A^{vu} - \left(e_u\cdot e_v\right)^2 A^{uu} - \left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A^{vu} + \left(e_v\cdot e_v\right)^2 A^{uu}\right) \end{array}\right]$$

$$\mathrm{Tr}\left(A\right) = -\frac{\left(e_v\cdot e_v\right)^2 A^{vv}\left(e_u\cdot e_u\right)^3}{-\left(e_u\cdot e_u\right)^3\left(e_v\cdot e_v\right)^3 + 3\left(e_u\cdot e_u\right)^2\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)^2 - 3\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^4\left(e_v\cdot e_v\right) + \left(e_u\cdot e_v\right)^6} + \frac{2\left(e_u\cdot e_u\right)^2\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)A^{vv}}{-\left(e_u\cdot e_u\right)^3\left(e_v\cdot e_v\right)^3 + 3\left(e_u\cdot e_u\right)^2\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)^2 - 3\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^4\left(e_v\cdot e_v\right) + \left(e_u\cdot e_v\right)^6} + \frac{}{-\left(e_u\cdot e_u\right)^3\left(e_v\cdot e_v\right)^3}$$

$$A\left(e_u\wedge e_v\right) = \frac{A^{uu}A^{vv} - A^{uv}A^{vu}}{\left(e_u\cdot e_u\right)\left(e_v\cdot e_v\right) - \left(e_u\cdot e_v\right)^2}\boldsymbol{e}_u\wedge\boldsymbol{e}_v$$

$$A\left(e_u\right)\wedge A\left(e_v\right) = \frac{A^{uu}A^{vv} - A^{uv}A^{vu}}{\left(e_u\cdot e_u\right)\left(e_v\cdot e_v\right) - \left(e_u\cdot e_v\right)^2}\boldsymbol{e}_u\wedge\boldsymbol{e}_v$$

$$B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_v)B^{uv}+(e_v\cdot e_v)B^{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)B^{uv}-(e_u\cdot e_v)B^{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{-(e_u\cdot e_v)B^{vv}+(e_v\cdot e_v)B^{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)B^{vv}-(e_u\cdot e_v)B^{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \end{array} \right\}$$

$$A+B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_v)A^{uv}-(e_u\cdot e_v)B^{uv}+(e_v\cdot e_v)A^{uu}+(e_v\cdot e_v)B^{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A^{uv}+(e_u\cdot e_u)B^{uv}-(e_u\cdot e_v)A^{uu}-(e_u\cdot e_v)B^{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{-(e_u\cdot e_v)A^{vv}-(e_u\cdot e_v)B^{vv}+(e_v\cdot e_v)A^{vu}+(e_v\cdot e_v)B^{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A^{vv}+(e_u\cdot e_u)B^{vv}-(e_u\cdot e_v)A^{vu}-(e_u\cdot e_v)B^{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \end{array} \right\}$$

$$AB = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{1}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4}\Big(-\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)A^{vv}B^{uv} + \left(e_u\cdot e_u\right)\left(e_v\cdot e_v\right)A^{vu}B^{uv} + \left(e_u\cdot e_v\right)^2 A^{uv}B^{uv} + \left(e_u\cdot e_v\right)^2 A^{vv}B^{uu} - \left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A^{uu}B^{uv} - \left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A^{uv}B^{uu} - \left(e_u\cdot e_v\right) \\ L\left(\boldsymbol{e}_v\right) = & \frac{1}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4}\Big(-\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)A^{vv}B^{vv} + \left(e_u\cdot e_u\right)\left(e_v\cdot e_v\right)A^{vu}B^{vv} + \left(e_u\cdot e_v\right)^2 A^{uv}B^{vv} + \left(e_u\cdot e_v\right)^2 A^{vv}B^{vu} - \left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A^{uu}B^{vv} - \left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A^{uv}B^{vu} - \left(e_u\cdot e_v\right) \end{array} \right.$$

$$A-B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_v)A^{uv}+(e_u\cdot e_v)B^{uv}+(e_v\cdot e_v)A^{uu}-(e_v\cdot e_v)B^{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A^{uv}-(e_u\cdot e_u)B^{uv}-(e_u\cdot e_v)A^{uu}+(e_u\cdot e_v)B^{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{-(e_u\cdot e_v)A^{vv}+(e_u\cdot e_v)B^{vv}+(e_v\cdot e_v)A^{vu}-(e_v\cdot e_v)B^{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A^{vv}-(e_u\cdot e_u)B^{vv}-(e_u\cdot e_v)A^{vu}+(e_u\cdot e_v)B^{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \end{array} \right\}$$

$$a\cdot\overline{A}\left(b\right) - b\cdot\underline{A}\left(a\right) = 0$$

$$g = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

$$\underline{T} = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_t\right) = & T^{tt}\boldsymbol{e}_t - T^{tx}\boldsymbol{e}_x - T^{ty}\boldsymbol{e}_y - T^{tz}\boldsymbol{e}_z \\ L\left(\boldsymbol{e}_x\right) = & T^{xt}\boldsymbol{e}_t - T^{xx}\boldsymbol{e}_x - T^{xy}\boldsymbol{e}_y - T^{xz}\boldsymbol{e}_z \\ L\left(\boldsymbol{e}_y\right) = & T^{yt}\boldsymbol{e}_t - T^{yx}\boldsymbol{e}_x - T^{yy}\boldsymbol{e}_y - T^{yz}\boldsymbol{e}_z \\ L\left(\boldsymbol{e}_z\right) = & T^{zt}\boldsymbol{e}_t - T^{zx}\boldsymbol{e}_x - T^{zy}\boldsymbol{e}_y - T^{zz}\boldsymbol{e}_z \end{array} \right\}$$

$$\overline{T} = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_t\right) = & T^{tt}\boldsymbol{e}_t - T^{xt}\boldsymbol{e}_x - T^{yt}\boldsymbol{e}_y - T^{zt}\boldsymbol{e}_z \\ L\left(\boldsymbol{e}_x\right) = & T^{tx}\boldsymbol{e}_t - T^{xx}\boldsymbol{e}_x - T^{yx}\boldsymbol{e}_y - T^{zx}\boldsymbol{e}_z \\ L\left(\boldsymbol{e}_y\right) = & T^{ty}\boldsymbol{e}_t - T^{xy}\boldsymbol{e}_x - T^{yy}\boldsymbol{e}_y - T^{zy}\boldsymbol{e}_z \\ L\left(\boldsymbol{e}_z\right) = & T^{tz}\boldsymbol{e}_t - T^{xz}\boldsymbol{e}_x - T^{yz}\boldsymbol{e}_y - T^{zz}\boldsymbol{e}_z \end{array} \right\}$$

$$\det\left(\underline{T}\right) = -T^{zt}\left(-T^{tx}T^{xy}T^{yz} + T^{tx}T^{xz}T^{yy} + T^{ty}T^{xx}T^{yz} - T^{ty}T^{xz}T^{yx} - T^{tz}T^{xx}T^{yy} + T^{tz}T^{xy}T^{yx}\right) - T^{zx}\left(T^{tt}T^{xy}T^{yz} - T^{tt}T^{xz}T^{yy} - T^{ty}T^{xt}T^{yz} + T^{ty}T^{xz}T^{yt} + T^{tz}T^{xt}T^{yy} - T^{tz}T^{xy}T^{yt}\right) + T^{zy}\Big(T^{tt}T^{xx}T^{yz} - T^{tt}T^{xz}T^{yx} - T^{tx}T$$

$$\mathrm{tr}\left(\underline{T}\right) = T^{tt} - T^{xx} - T^{yy} - T^{zz}$$

$$a\cdot\overline{T}\left(b\right) - b\cdot\underline{T}\left(a\right) = 0$$

$$f = f$$

$$\boldsymbol{\nabla} f = \partial_u f\boldsymbol{e}_u + \frac{\partial_v f}{\sin\left(u\right)}\boldsymbol{e}_v$$

$$F = F^{u}\boldsymbol{e}_{u} + F^{v}\boldsymbol{e}_{v}$$

$$\boldsymbol{\nabla} F = \left (\frac{F^{u}}{\tan{\left (u \right)}} + \partial_{u} F^{u} + \frac{\partial_{v} F^{v}}{\sin{\left (u \right)}} \right) + \left (\frac{F^{v}}{\tan{\left (u \right)}} + \partial_{u} F^{v} - \frac{\partial_{v} F^{u}}{\sin{\left (u \right)}} \right) \boldsymbol{e}_{u}\wedge \boldsymbol{e}_{v}$$

$$f = f$$

$$\boldsymbol{\nabla} f = \partial_{\theta} f \boldsymbol{e}_{\theta} + \frac{\partial_{\phi} f }{\sin{\left (\theta \right)}} \boldsymbol{e}_{\phi}$$

$$F = F^{\theta}\boldsymbol{e}_{\theta} + F^{\phi}\boldsymbol{e}_{\phi}$$

$$\boldsymbol{\nabla} F = \left (\frac{F^{\theta}}{\tan{\left (\theta \right)}} + \partial_{\theta} F^{\theta} + \frac{\partial_{\phi} F^{\phi}}{\sin{\left (\theta \right)}} \right) + \left (\frac{F^{\phi}}{\tan{\left (\theta \right)}} + \partial_{\theta} F^{\phi} - \frac{\partial_{\phi} F^{\theta}}{\sin{\left (\theta \right)}} \right) \boldsymbol{e}_{\theta}\wedge \boldsymbol{e}_{\phi}$$