$$\boldsymbol{A} = A + A^x\boldsymbol{e}_x + A^y\boldsymbol{e}_y + A^z\boldsymbol{e}_z + A^{xy}\boldsymbol{e}_x\wedge\boldsymbol{e}_y + A^{xz}\boldsymbol{e}_x\wedge\boldsymbol{e}_z + A^{yz}\boldsymbol{e}_y\wedge\boldsymbol{e}_z + A^{xyz}\boldsymbol{e}_x\wedge\boldsymbol{e}_y\wedge\boldsymbol{e}_z$$

$$\begin{aligned}\boldsymbol{A} =& A^x\boldsymbol{e}_x \\ &+ A^y\boldsymbol{e}_y \\ &+ A^z\boldsymbol{e}_z\end{aligned}$$

$$\begin{aligned}\boldsymbol{B} =& B^{xy}\boldsymbol{e}_x\wedge\boldsymbol{e}_y \\ &+ B^{xz}\boldsymbol{e}_x\wedge\boldsymbol{e}_z \\ &+ B^{yz}\boldsymbol{e}_y\wedge\boldsymbol{e}_z\end{aligned}$$

$$\begin{aligned}\boldsymbol{\nabla} f =& \partial_x f\boldsymbol{e}_x \\ &+ \partial_y f\boldsymbol{e}_y \\ &+ \partial_z f\boldsymbol{e}_z\end{aligned}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} = \partial_x A^x + \partial_y A^y + \partial_z A^z$$

$$\begin{aligned}\boldsymbol{\nabla}\boldsymbol{A} =& \left(\partial_x A^x + \partial_y A^y + \partial_z A^z\right) \\ &+ \left(-\partial_y A^x + \partial_x A^y\right)\boldsymbol{e}_x\wedge\boldsymbol{e}_y \\ &+ \left(-\partial_z A^x + \partial_x A^z\right)\boldsymbol{e}_x\wedge\boldsymbol{e}_z \\ &+ \left(-\partial_z A^y + \partial_y A^z\right)\boldsymbol{e}_y\wedge\boldsymbol{e}_z\end{aligned}$$

$$\begin{aligned}-I(\boldsymbol{\nabla}\wedge\boldsymbol{A}) =& \left(-\partial_z A^y + \partial_y A^z\right)\boldsymbol{e}_x \\ &+ \left(\partial_z A^x - \partial_x A^z\right)\boldsymbol{e}_y \\ &+ \left(-\partial_y A^x + \partial_x A^y\right)\boldsymbol{e}_z\end{aligned}$$

$$\begin{aligned}\boldsymbol{\nabla}\boldsymbol{B} =& \left(-\partial_y B^{xy} - \partial_z B^{xz}\right)\boldsymbol{e}_x \\ &+ \left(\partial_x B^{xy} - \partial_z B^{yz}\right)\boldsymbol{e}_y \\ &+ \left(\partial_x B^{xz} + \partial_y B^{yz}\right)\boldsymbol{e}_z \\ &+ \left(\partial_z B^{xy} - \partial_y B^{xz} + \partial_x B^{yz}\right)\boldsymbol{e}_x\wedge\boldsymbol{e}_y\wedge\boldsymbol{e}_z\end{aligned}$$

$$\boldsymbol{\nabla}\wedge\boldsymbol{B} = \left(\partial_z B^{xy} - \partial_y B^{xz} + \partial_x B^{yz}\right)\boldsymbol{e}_x\wedge\boldsymbol{e}_y\wedge\boldsymbol{e}_z$$

$$\begin{aligned}\boldsymbol{\nabla}\cdot\boldsymbol{B} =& \left(-\partial_y B^{xy} - \partial_z B^{xz}\right)\boldsymbol{e}_x \\ &+ \left(\partial_x B^{xy} - \partial_z B^{yz}\right)\boldsymbol{e}_y \\ &+ \left(\partial_x B^{xz} + \partial_y B^{yz}\right)\boldsymbol{e}_z\end{aligned}$$

$$g_{ij} = \begin{bmatrix} (a\cdot a) & (a\cdot b) & (a\cdot c) & (a\cdot d) \\ (a\cdot b) & (b\cdot b) & (b\cdot c) & (b\cdot d) \\ (a\cdot c) & (b\cdot c) & (c\cdot c) & (c\cdot d) \\ (a\cdot d) & (b\cdot d) & (c\cdot d) & (d\cdot d)\end{bmatrix}$$

$$\begin{aligned}\boldsymbol{a}\cdot(\boldsymbol{b}\boldsymbol{c}) =& -(a\cdot c)\,\boldsymbol{b} \\ &+ (a\cdot b)\,\boldsymbol{c}\end{aligned}$$

$$\begin{aligned}\boldsymbol{a}\cdot(\boldsymbol{b}\wedge\boldsymbol{c}) =& -(a\cdot c)\,\boldsymbol{b} \\ &+ (a\cdot b)\,\boldsymbol{c}\end{aligned}$$

$$\begin{aligned}\boldsymbol{a}\cdot(\boldsymbol{b}\wedge\boldsymbol{c}\wedge\boldsymbol{d}) =& (a\cdot d)\,\boldsymbol{b}\wedge\boldsymbol{c} \\ &- (a\cdot c)\,\boldsymbol{b}\wedge\boldsymbol{d} \\ &+ (a\cdot b)\,\boldsymbol{c}\wedge\boldsymbol{d}\end{aligned}$$

$$\boldsymbol{a}\cdot(\boldsymbol{b}\wedge\boldsymbol{c})+\boldsymbol{c}\cdot(\boldsymbol{a}\wedge\boldsymbol{b})+\boldsymbol{b}\cdot(\boldsymbol{c}\wedge\boldsymbol{a})=0$$

$$\boldsymbol{a}(\boldsymbol{b}\wedge\boldsymbol{c})-\boldsymbol{b}(\boldsymbol{a}\wedge\boldsymbol{c})+\boldsymbol{c}(\boldsymbol{a}\wedge\boldsymbol{b})=3\boldsymbol{a}\wedge\boldsymbol{b}\wedge\boldsymbol{c}$$

$$\boldsymbol{a}(\boldsymbol{b}\wedge\boldsymbol{c}\wedge\boldsymbol{d})-\boldsymbol{b}(\boldsymbol{a}\wedge\boldsymbol{c}\wedge\boldsymbol{d})+\boldsymbol{c}(\boldsymbol{a}\wedge\boldsymbol{b}\wedge\boldsymbol{d})-\boldsymbol{d}(\boldsymbol{a}\wedge\boldsymbol{b}\wedge\boldsymbol{c})=4\boldsymbol{a}\wedge\boldsymbol{b}\wedge\boldsymbol{c}\wedge\boldsymbol{d}$$

$$(\boldsymbol{a}\wedge\boldsymbol{b})\cdot(\boldsymbol{c}\wedge\boldsymbol{d})=-\left(a\cdot c\right)\left(b\cdot d\right)+\left(a\cdot d\right)\left(b\cdot c\right)$$

$$((\boldsymbol{a}\wedge\boldsymbol{b})\cdot\boldsymbol{c})\cdot\boldsymbol{d}=-\left(a\cdot c\right)\left(b\cdot d\right)+\left(a\cdot d\right)\left(b\cdot c\right)$$

$$\begin{aligned}(\boldsymbol{a}\wedge\boldsymbol{b})\times(\boldsymbol{c}\wedge\boldsymbol{d})=&-\left(b\cdot d\right)\boldsymbol{a}\wedge\boldsymbol{c}\\&+\left(b\cdot c\right)\boldsymbol{a}\wedge\boldsymbol{d}\\&+\left(a\cdot d\right)\boldsymbol{b}\wedge\boldsymbol{c}\\&-\left(a\cdot c\right)\boldsymbol{b}\wedge\boldsymbol{d}\end{aligned}$$

$$E=\boldsymbol{e}_1\wedge\boldsymbol{e}_2\wedge\boldsymbol{e}_3$$

$$E^2=\left(e_1\cdot e_2\right)^2-2\left(e_1\cdot e_2\right)\left(e_1\cdot e_3\right)\left(e_2\cdot e_3\right)+\left(e_1\cdot e_3\right)^2+\left(e_2\cdot e_3\right)^2-1$$

$$\begin{aligned}E1=(e2\wedge e3)E=&\left(\left(e_2\cdot e_3\right)^2-1\right)\boldsymbol{e}_1\\&+\left(\left(e_1\cdot e_2\right)-\left(e_1\cdot e_3\right)\left(e_2\cdot e_3\right)\right)\boldsymbol{e}_2\\&+\left(-\left(e_1\cdot e_2\right)\left(e_2\cdot e_3\right)+\left(e_1\cdot e_3\right)\right)\boldsymbol{e}_3\end{aligned}$$

$$\begin{aligned}E2=-(e1\wedge e3)E=&\left(\left(e_1\cdot e_2\right)-\left(e_1\cdot e_3\right)\left(e_2\cdot e_3\right)\right)\boldsymbol{e}_1\\&+\left(\left(e_1\cdot e_3\right)^2-1\right)\boldsymbol{e}_2\\&+\left(-\left(e_1\cdot e_2\right)\left(e_1\cdot e_3\right)+\left(e_2\cdot e_3\right)\right)\boldsymbol{e}_3\end{aligned}$$

$$\begin{aligned}E3=(e1\wedge e2)E=&\left(-\left(e_1\cdot e_2\right)\left(e_2\cdot e_3\right)+\left(e_1\cdot e_3\right)\right)\boldsymbol{e}_1\\&+\left(-\left(e_1\cdot e_2\right)\left(e_1\cdot e_3\right)+\left(e_2\cdot e_3\right)\right)\boldsymbol{e}_2\\&+\left(\left(e_1\cdot e_2\right)^2-1\right)\boldsymbol{e}_3\end{aligned}$$

$$E1\cdot e2=0$$

$$E1\cdot e3=0$$

$$E2\cdot e1=0$$

$$E2\cdot e3=0$$

$$E3\cdot e1=0$$

$$E3\cdot e2=0$$

$$(E1\cdot e1)/E^2=1$$

$$(E2\cdot e2)/E^2=1$$

$$(E3\cdot e3)/E^2=1$$

$$\begin{aligned}A=&A^r\boldsymbol{e}_r\\&+A^\theta\boldsymbol{e}_\theta\\&+A^\phi\boldsymbol{e}_\phi\end{aligned}$$

$$\begin{aligned}B=&B^{r\theta}\boldsymbol{e}_r\wedge\boldsymbol{e}_\theta\\&+B^{r\phi}\boldsymbol{e}_r\wedge\boldsymbol{e}_\phi\\&+B^{\phi\phi}\boldsymbol{e}_\theta\wedge\boldsymbol{e}_\phi\end{aligned}$$

$$\begin{aligned}\boldsymbol{\nabla} f =&\partial_{r} f \boldsymbol{e}_{r}\\ &+ \frac{1}{r}\partial_{\theta} f \boldsymbol{e}_{\theta}\\ &+ \frac{\partial_{\phi} f}{r \sin{\left(\theta \right)}} \boldsymbol{e}_{\phi}\end{aligned}$$

$$\boldsymbol{\nabla} \cdot A = \frac{1}{r} \left(r \partial_{r} A^{r} + 2 A^{r} + \frac{A^{\theta}}{\tan{\left(\theta \right)}} + \partial_{\theta} A^{\theta} + \frac{\partial_{\phi} A^{\phi}}{\sin{\left(\theta \right)}}\right)$$

$$\begin{aligned}-I (\boldsymbol{\nabla} \wedge A) =&\frac{1}{r} \left(\frac{A^{\phi}}{\tan{\left(\theta \right)}} + \partial_{\theta} A^{\phi} - \frac{\partial_{\phi} A^{\theta}}{\sin{\left(\theta \right)}}\right) \boldsymbol{e}_{r}\\ &+ \frac{1}{r} \left(- r \partial_{r} A^{\phi} - A^{\phi} + \frac{\partial_{\phi} A^{r}}{\sin{\left(\theta \right)}}\right) \boldsymbol{e}_{\theta}\\ &+ \frac{1}{r} \left(r \partial_{r} A^{\theta} + A^{\theta} - \partial_{\theta} A^{r}\right) \boldsymbol{e}_{\phi}\end{aligned}$$

$$\boldsymbol{\nabla} \wedge B = \frac{1}{r} \left(r \partial_{r} B^{\phi \phi} - \frac{B^{r \phi}}{\tan{\left(\theta \right)}} + 2 B^{\phi \phi} - \partial_{\theta} B^{r \phi} + \frac{\partial_{\phi} B^{r \theta}}{\sin{\left(\theta \right)}}\right) \boldsymbol{e}_{r}\wedge \boldsymbol{e}_{\theta}\wedge \boldsymbol{e}_{\phi}$$

$$\begin{aligned}B = \boldsymbol{B}\boldsymbol{\gamma_{t}} =&- B^{x} \boldsymbol{\gamma}_{t}\wedge \boldsymbol{\gamma}_{x}\\ &- B^{y} \boldsymbol{\gamma}_{t}\wedge \boldsymbol{\gamma}_{y}\\ &- B^{z} \boldsymbol{\gamma}_{t}\wedge \boldsymbol{\gamma}_{z}\end{aligned}$$

$$\begin{aligned}E = \boldsymbol{E}\boldsymbol{\gamma_{t}} =&- E^{x} \boldsymbol{\gamma}_{t}\wedge \boldsymbol{\gamma}_{x}\\ &- E^{y} \boldsymbol{\gamma}_{t}\wedge \boldsymbol{\gamma}_{y}\\ &- E^{z} \boldsymbol{\gamma}_{t}\wedge \boldsymbol{\gamma}_{z}\end{aligned}$$

$$\begin{aligned}F = E+IB =&- E^{x} \boldsymbol{\gamma}_{t}\wedge \boldsymbol{\gamma}_{x}\\ &- E^{y} \boldsymbol{\gamma}_{t}\wedge \boldsymbol{\gamma}_{y}\\ &- E^{z} \boldsymbol{\gamma}_{t}\wedge \boldsymbol{\gamma}_{z}\\ &- B^{z} \boldsymbol{\gamma}_{x}\wedge \boldsymbol{\gamma}_{y}\\ &+ B^{y} \boldsymbol{\gamma}_{x}\wedge \boldsymbol{\gamma}_{z}\\ &- B^{x} \boldsymbol{\gamma}_{y}\wedge \boldsymbol{\gamma}_{z}\end{aligned}$$

$$\begin{aligned}J =&J^{t} \boldsymbol{\gamma}_{t}\\ &+ J^{x} \boldsymbol{\gamma}_{x}\\ &+ J^{y} \boldsymbol{\gamma}_{y}\\ &+ J^{z} \boldsymbol{\gamma}_{z}\end{aligned}$$

$$\boldsymbol{\nabla} F = J$$

$$\begin{aligned}R =&\cosh{\left(\frac{\alpha}{2} \right)}\\ &+ \sinh{\left(\frac{\alpha}{2} \right)} \boldsymbol{\gamma}_{t}\wedge \boldsymbol{\gamma}_{x}\end{aligned}$$

$$t\boldsymbol{\gamma_{t}}+x\boldsymbol{\gamma_{x}} = t'\boldsymbol{\gamma'_{t}}+x'\boldsymbol{\gamma'_{x}} = R\left(t'\boldsymbol{\gamma_{t}}+x'\boldsymbol{\gamma_{x}}\right)R^{\dagger}$$

$$\begin{aligned}t\boldsymbol{\gamma_{t}}+x\boldsymbol{\gamma_{x}} =&\left(t' \cosh{\left(\alpha \right)} - x' \sinh{\left(\alpha \right)}\right) \boldsymbol{\gamma}_{t}\\ &+ \left(- t' \sinh{\left(\alpha \right)} + x' \cosh{\left(\alpha \right)}\right) \boldsymbol{\gamma}_{x}\end{aligned}$$

$$\sinh{\left(\alpha \right)} = \gamma\beta$$

$$\cosh{\left(\alpha \right)} = \gamma$$

$$
\begin{aligned}
t\boldsymbol{\gamma_t} + x\boldsymbol{\gamma_x} =& \gamma\left(-\beta x' + t'\right)\boldsymbol{\gamma}_t \\
&+ \gamma\left(-\beta t' + x'\right)\boldsymbol{\gamma}_x
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{A} =& A^t\boldsymbol{\gamma}_t \\
&+ A^x\boldsymbol{\gamma}_x \\
&+ A^y\boldsymbol{\gamma}_y \\
&+ A^z\boldsymbol{\gamma}_z
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{\psi} =& \psi \\
&+ \psi^{tx}\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \\
&+ \psi^{ty}\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y \\
&+ \psi^{tz}\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z \\
&+ \psi^{xy}\boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \\
&+ \psi^{xz}\boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_z \\
&+ \psi^{yz}\boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z \\
&+ \psi^{txyz}\boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z
\end{aligned}
$$