Base manifold (three dimensional)

Metric tensor (cartesian coordinates - norm = False)

$$g = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

Two dimensioanal submanifold - Unit sphere

Basis not normalised

$(\theta,\phi) \rightarrow (r,\theta,\phi) = [1, \quad \theta, \quad \phi]$

$e_{\theta} \cdot e_{\theta} = 1$

$e_{\phi} \cdot e_{\phi} = \sin{\left (\theta \right)}^{2}$

$$g = \left[\begin{array}{cc} 1 & 0 \\ 0 & \sin{\left (\theta \right)}^{2} \end{array}\right]$$

$$\text{g\_inv} = \left[\begin{matrix}1 & 0\\0 & \frac{1}{\sin^{2}{\left (\theta \right)}}\end{matrix}\right]$$

Christoffel symbols of the first kind:

$$\Gamma_{1, \alpha, \beta} = \left[\begin{matrix}0 & 0\\0 & - \frac{\sin{\left (2 \theta \right)}}{2}\end{matrix}\right] \quad \Gamma_{2, \alpha, \beta} = \left[\begin{matrix}0 & \frac{\sin{\left (2 \theta \right)}}{2}\\\frac{\sin{\left (2 \theta \right)}}{2} & 0\end{matrix}\right]$$

Christoffel symbols of the second kind:

$$\Gamma^{1}_{\alpha, \beta} = \left[\begin{matrix}0 & 0\\0 & - \frac{\sin{\left (2 \theta \right)}}{2}\end{matrix}\right] \quad \Gamma^{2}_{\alpha, \beta} = \left[\begin{matrix}0 & \frac{1}{\tan{\left (\theta \right)}}\\\frac{1}{\tan{\left (\theta \right)}} & 0\end{matrix}\right]$$

$$\boldsymbol{\nabla} = \boldsymbol{e}_{\theta} \frac{\partial}{\partial \theta} + \boldsymbol{e}_{\phi} \frac{1}{\sin{\left (\theta \right)}^{2}} \frac{\partial}{\partial \phi}$$

$$\boldsymbol{\nabla} f = \partial_{\theta} f \boldsymbol{e}_{\theta} + \frac{\partial_{\phi} f}{\sin{\left (\theta \right)}^{2}} \boldsymbol{e}_{\phi}$$

$$F = F^{\theta} \boldsymbol{e}_{\theta} + F^{\phi} \boldsymbol{e}_{\phi}$$

$$\boldsymbol{\nabla} F = \left (\frac{F^{\theta}}{\tan{\left (\theta \right)}} + \partial_{\theta} F^{\theta} + \partial_{\phi} F^{\phi} \right) + \left (\frac{2 F^{\phi}}{\tan{\left (\theta \right)}} + \partial_{\theta} F^{\phi} - \frac{\partial_{\phi} F^{\theta}}{\sin{\left (\theta \right)}^{2}} \right) \boldsymbol{e}_{\theta}\wedge \boldsymbol{e}_{\phi}$$

One dimensioanal submanifold

Basis not normalised

$(\phi) \rightarrow (\theta,\phi) = \left [\frac{\pi}{8}, \quad \phi\right]$

$$e_{\phi} \cdot e_{\phi} = - \frac{\sqrt{2}}{4} + \frac{1}{2}$$

$$g = \left[\begin{array}{c} - \frac{\sqrt{2}}{4} + \frac{1}{2} \end{array}\right]$$

$$\boldsymbol{\nabla} = \boldsymbol{e}_{\phi} \left(2 \sqrt{2} + 4\right) \frac{\partial}{\partial \phi}$$

$$\nabla h = \left(2 \sqrt{2} + 4\right) \partial_{\phi} h \boldsymbol{e}_{\phi}$$

$$H = H^{\phi} \boldsymbol{e}_{\phi}$$

$$\nabla H = \partial_{\phi} H^{\phi}$$