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Artificial Gravity in Interstellar Travel

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Abstract. Gravity-induced contact with the Earth's surface has been constant throughout the evolution of the human species, and human health depends on it. Providing "artificial gravity" and a firm contact surface to an interstellar crew is conceptually simple, due to the equivalence of gravitational and inertial mass. The physiological benefits of gravity are preserved through mechanical acceleration, which may be linear, centripetal, or some combination. Centripetal acceleration requires far less energy to maintain. Though it provides a distorted gravitational experience when the rotational radius is small, the minimum size of an interstellar spacecraft will almost certainly derive from the size of its population and other aspects of human life support, not from any dimensional limit for comfortable rotation. There must nevertheless be some linear acceleration if the spacecraft is ever to reach another star system. This may be either in-plane or on-axis relative to the centripetal. The optimal choice depends on the magnitude of the linear component relative to the centripetal.

1 Introduction

Throughout the entire evolution of life on this planet, across millions of millennia, species, and cultures, Earth gravity has been a shared constant – except for temporary forays into apparent "micro gravity" by a few hundred individuals (humans and other species) during the past few decades. When we speak of humans going to the stars, do we expect the humans that arrive to resemble the ones that left? Do we expect them to resemble *us*? If so, then gravity or an adequate substitute will be a requirement, even if traveling at near light speed. This paper attempts a rational extrapolation of future possibilities from current knowledge.

Section 2 reviews the significance of gravity and weight for human health "as we know it," but also admits possibilities for other conceptions of human health in space. The history of spaceflight has produced many measurements of the deleterious effects of prolonged weightlessness and the effectiveness of attempted countermeasures. There is also literature – both technical and philosophical – on countermeasures not yet attempted, and even questioning whether permanent weightlessness is ultimately "deleterious" or something humans should evolve for. Section 3 examines the theoretical basis of gravity and "artificial gravity" and their relevance to healthy weight. A cornerstone of this is the Equivalence Principle that unites gravity and acceleration in both Newton's and Einstein's theories. In light of these theories, observations in spaceflight – both actual and hypothetical – indicate that planetary gravity is practically irrelevant to weight. The key to weight is mechanical acceleration. Section 4 delves into acceleration, with subsections on the applicability of sustained linear, centripetal, and combined accelerations over the duration of an interstellar voyage. This includes a digression on the relationship between accel-

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eration, spacecraft size, and other aspects of life support that convolve with that. Section 5 offers some closing observations, but specifically refrains from "conclusions."

2 Healthy Weight

Fifty-eight years of human spaceflight have taught two great lessons about weightless living: 1) Despite early concerns, it does not bring sudden death; it is survivable for days, weeks, months, and even a year or more. 2) The effects of long-term weight deprivation are chronic, pernicious, and pervasive, damaging from the nanoscale of molecules and cells up through organ systems and ultimately the entire human organism.

A detailed review of undesirable adaptations to weightlessness is beyond the scope of this paper. Summaries appear in Clément, Charles, Norsk, and Paloski [1], Hall [2, 3], and the many sources that they cite.

A very brief list includes: fluid redistribution (from the feet and legs toward the torso and head); fluid loss; electrolyte imbalances; cardiovascular changes (the heart itself expands and shrinks dramatically with the fluid shifts and losses); red blood cell loss; muscle damage; bone damage; eye damage; hypercalcemia; and immune suppression [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

Even after nearly six decades of spaceflight, new detriments continue to come to light. For example, there may be cognitive declines as well. The organ and tissue systems interact with each other as well as with the apparent gravitational field (or its absence) in innumerable ways, to seek a new equilibrium in weightlessness that is ultimately contrary to return to a normal weighted life.

If all of those detriments are attributable to weightlessness, then preserving weight in spaceflight should prevent them and preserve health. By treating the underlying cause, rather than the innumerable symptoms, we would not need to enumerate them, nor develop myriad imperfect countermeasures with their own unknown and possibly adverse interactions. If weight is due to gravity, then artificial gravity in spaceflight should preserve weight and health.

Though humans have returned to Earth after a year or more in weightlessness, they have not returned in fully working condition [6, 16]. They have been met by ground crews to assist them from their capsules,

physicians to monitor their health, and physiotherapists to guide them through weeks or months of rehabilitation to a normal weighted life. It's clear that the current piecemeal countermeasures to weightlessness in orbit are only partially effective, and though they slow the decline of human health, they don't stop it.

Our nearest stellar neighbor, Proxima Centauri, is 4.244 lightyears away [17]. Considering that travel over such distances will take not mere months, or years, but rather centuries and generations, there can be little doubt that artificial gravity will be essential to the endeavor – whether the ultimate goal is to colonize a planet in the new neighborhood, or to park the ark among the asteroids as an O'Neill-style orbital colony [18, 19].

Alternatives?

"Suspended animation" is a recurring feature in the fiction of long-duration spaceflight. If all biological functions could be brought to a near standstill, practically death without the decay, and then somehow reanimated at the appropriate time, that would avoid the adverse effects of weightlessness. This would be much more profound than a mere medically induced coma, which itself is still an extreme measure of last resort for certain types of brain injury or infection. And, as Cohen and Brody [20] point out, the fate of an entire branch of humanity "would depend on a machine, an 'ultra-reliable' computer to reawaken the crew members upon arrival at the destination" after years or centuries in the harsh environment of interstellar space. "The crew would have no control over the potential single-point failure . . . because they would be 'asleep.'"

Another alternative is to accept weightlessness as a new life condition, without looking back. J. D. Bernal [21], for whom the O'Neillian "Bernal Sphere" colony concept is named [22], did not see gravity or weight as essential or even desirable. Bernal's concept of the orbital space colony was quite different from O'Neill's. Bernal foresaw that humans would adapt to a permanent "three-dimensional, gravitationless way of living." He acknowledged that the human body did not evolve to thrive in such an environment, but asserted that its current limitations would become irrelevant. Gradually, during a "larval, unspecialized existence" lasting sixty to one hundred twenty years, inadequate body parts would be replaced, and new sensory and motor mechanisms would be grafted on. Ultimately, a person "would emerge as a completely effective,

mentally-directed mechanism, and set about the tasks appropriate to his new capacities."

Given the rapid advance of microsensors, actuators, prosthetics, real-time computer processing, deep learning, and artificial intelligence – in contrast to suspended animation and reanimation, exotic means of propulsion, or "warping" space-time to shorten interstellar voyages – Bernal's vision might not be so far-fetched. The philosophy, ethics, and aesthetics of such a life are subjects for other papers – beginning with Bernal's.

Declined

The remainder of this paper declines those alternative visions, and focuses on the means to preserve a weighted way of life. The next section explores the nature of gravity – natural and artificial – and its association with weight.

3   Gravity, in Theory

Much of the material in this section is textbook physics, though bits of it are perhaps advanced, and often spread among disjoint chapters. Davies [23] provides a collected overview. Wikipedia also has articles on "Fundamental Interaction", "Standard Model", "Electromagnetism", "Electroweak Interaction", "Grand Unified Theory", "Gravity", "General Relativity", and "Theory of Everything". A brief review here lays the groundwork for later sections of this paper.

Contemporary physics posits Four Fundamental Forces or "Interactions" that drive everything in the universe: strong nuclear, weak nuclear, electromagnetic, and gravitational. Among these, the gravitational is the weakest, and also the only one that doesn't conform to the Standard Model of particle physics. Each of the other three operates through some mediating particle: the electromagnetic interaction through photons; the weak nuclear interaction through bosons; and the strong nuclear interaction through gluons. Moreover, the electromagnetic and weak interactions have been theoretically unified as two aspects of a more fundamental electroweak interaction that operates at extremely high temperatures (approximately 10^{15} K); they diverged shortly after the Big Bang into the interactions we see today. The search continues for a Grand Unified Theory that would incorporate the strong nuclear interaction as well, but even that might exclude gravity. Some physi-

cists hypothesize a "graviton" to bring gravity into conformance with the others and arrive at an even more elusive Theory of Everything, but to date there's no compelling evidence for it.

Another enduring curiosity of gravity, since Isaac Newton formulated his Laws of Motion and Gravitation more than 300 years ago, is the equivalence of gravitational and inertial mass. That equivalence is the reason that two stones (for example) of different masses fall to Earth at the same rate (disregarding other influences such as air resistance): the force required to accelerate the stone, and the gravitational force that compels it to accelerate, are both directly proportional to the same quality of mass.

Applying the Second Law of Motion to the stone s and the Earth e, relating force F, mass m, and acceleration A (using uppercase for motion relative to an inertial, non-accelerated, reference):

$$F_s = m_s \cdot A_s$$
$$F_e = m_e \cdot A_e$$

Applying the Law of Gravitation between them, where G is the universal gravitational constant and R is the distance between the masses – in the special case of uniform spheres, the distance between their centers:

$$F_s = F_e = \mathrm{G} \cdot \frac{m_s \cdot m_e}{R^2}$$

Setting the accelerating force equal to the gravitational force and solving for accelerations shows that the stone's acceleration toward the Earth is independent of the stone's own mass, but is proportional to the Earth's, whereas the infinitesimal acceleration of the Earth toward the stone is proportional to the stone's mass:

$$m_s \cdot A_s = m_e \cdot A_e = \mathrm{G} \cdot \frac{m_s \cdot m_e}{R^2}$$
$$A_s = \mathrm{G} \cdot \frac{m_e}{R^2}$$
$$A_e = \mathrm{G} \cdot \frac{m_s}{R^2}$$

The equivalence of inertial and gravitational mass is central to Albert Einstein's theories [24]. In a chapter titled "The Equality of Inertial and Gravitational Mass as an Argument for the General Postulate of Relativity," Einstein describes a thought experiment: In a large region of empty space, far removed from any appreciable mass, a large chest containing an observer accelerates "upward." Every experiment the

observer can perform within the confines of the chest runs exactly as if the chest were suspended motionless in a uniform gravitational field. Einstein concludes that "a gravitational field exists for the man in the chest, despite the fact that there was no such field for the coordinate system first chosen."

The equivalence of inertial and gravitational mass is the essence of artificial gravity. If "artificial gravity" is a misnomer, it's not because it's not gravity, but rather because it's not artificial.

This stance is in direct contradiction to notable experts on artificial gravity. Young, Yajima, and Paloski [25] state flatly: "Artificial gravity is not gravity at all; it is an inertial force." The reluctance to equate acceleration with "real" gravity may stem from the "distortions" that arise in some forms of acceleration, to be discussed later. However, not all gravitational fields are equal, and some are less uniform than others.

I prefer to take Einstein at his word. He doesn't mince words when he asserts that "a gravitational field exists for the man in the chest." He doesn't say an "artificial" gravity field, or "something like" a gravitational field. Moreover, the mapping of "inertial force" onto the Four Fundamental Forces as anything other than gravity is unclear.

This debate is of more academic or philosophical rather than practical significance, since gravity *per se* is practically irrelevant to human health – as illustrated by Einstein's thought experiment as well as the real-fife experience of astronauts in orbit.

Einstein doesn't elaborate on what force accelerates the chest, other than to rule out gravity by placing it far away from any other mass. But, the strong and weak nuclear interactions operate only on the scale of atomic nuclei and are incapable of accelerating the chest. That leaves only the electromagnetic interaction, and more specifically, the chemical and mechanical interaction between electron shells of adjacent atoms. It is this electromagnetic force, *not* the gravitational force, that we experience as weight. All chemical and mechanical interactions, including *bio*-chemical and *bio*-mechanical, are ultimately due to the electromagnetic interaction.

Unfortunately, our language and vocabulary are not always accurate expressions of reality. At the altitude of the International Space Station, the intensity of Earth's gravitational field is about 89% of Earth's surface value, and yet we commonly call it *micro* gravity. The astronauts' lack of weight is not due to

a lack of planetary *gravity*, but rather the lack of an "upward" *mechanical* accelerating force that contact with a planetary surface provides.

The apparent zero acceleration while standing on the planetary surface may be better understood as the sum of two equal and opposite accelerations: gravitational downward, and mechanical upward. Only the mechanical component contributes to our experience of weight. The gravitational component is relevant only to the extent that it draws atoms together until electromagnetic-mechanical interaction dominates; otherwise, it's neither necessary nor sufficient, as demonstrated by Einstein's man in the chest and the near-Earth astronauts.

Moreover, mechanical *acceleration* is key, as it propagates forces throughout every tissue of the body according to $F = m \cdot A$. If the "upward" mechanical force is opposed by an equal and opposite "downward" mechanical force, yielding zero net force and acceleration, that vice-like compression loads the skeleton but leaves fluid columns, otolith organs, and other soft tissues unaffected. Only *acceleration* from unbalanced mechanical force provides the "vertical" force gradient, otolith loading, and "upward" orientation that we associate with weight.

In Einstein's General Theory of Relativity, the apparent gravitational force is a consequence of the curvature of four-dimensional space-time, and so is fundamentally different than the other three fundamental interactions.

On the other hand, Puthoff [26] (building on developments in quantum mechanical theory by Dirac, Schrödinger, and Sakharov) has developed a detailed mathematical analysis of gravitational mass and force arising from the *Zitterbewegung* (trembling motion) of charged particles in the zero point field. Haisch, Rueda, and Puthoff [27] have extended that analysis to cover inertial mass and its equivalence to gravitational mass. If this line of reasoning holds, it may be a giant leap toward the Theory of Everything, uniting gravity with electromagnetism – though (quoting Puthoff) "more akin to the induced van der Waals and Casimir forces, than to the fundamental Coulomb force."

Gilster [28] goes on to speculate: "if we accept the idea that both inertia and gravity are the product of electromagnetic interactions, then manipulating either of them to create exotic modes of propulsion becomes a possibility." He further quotes Arthur C. Clarke: "An 'inertialess drive,' which would act exactly like a controllable gravity field, had never been discussed

seriously outside the pages of science fiction until very recently ...lf HR&P's theory can be proved, it opens up the prospect – however remote – of anti-gravity 'space drives,' and the even more fantastic possibility of controlling inertia."

If such a drive is ever devised, it had better get the crew to the new star system quickly, because removing inertia will also preclude any possibility of weight; the crew will necessarily be weightless for the duration.

4 ΔV in Interstellar Travel

Acceleration is the rate of change of velocity over time. Velocity is the rate of change of position. Velocity is a vector, characterized by both a magnitude and a direction. Likewise, acceleration is a vector, and may account for a change in the velocity's magnitude, or its direction, or both.

Acceleration components parallel to the velocity change only the velocity's magnitude, or speed, whereas perpendicular components change only its direction. These components are designated as linear and centripetal, respectively.

- Purely linear acceleration yields linear motion of ever increasing speed.

- Purely centripetal acceleration yields circular motion of constant speed; the acceleration necessarily changes direction along with the velocity, and the locus of acceleration vectors over time are directed toward the center of the circle.

Either linear or centripetal acceleration, or even some combination, may be employed to provide "artificial" gravity during spaceflight. Their implications, applications, and consequences are diverse.

4.1   Linear Acceleration

Continuous linear acceleration is not well suited for any spacecraft that needs to remain in any particular vicinity, such as orbit around a planet or even a star. However, continuous 1-g linear acceleration would be ideal on the scale of interstellar travel (or beyond), as it would minimize the transit time while also maintaining healthy weight for the crew.

Unfortunately, not only the kinetic energy but even the power – the *rate* of change of energy over time –

approaches infinity as long as this acceleration is maintained. For power P, energy E, work W, force $\mathbf{F}$, mass m, acceleration $\mathbf{A}$, velocity $\mathbf{V}$, position $\mathbf{R}$, and time t (using boldface for vectors):

$$P = \frac{dE}{dt} = \frac{dW}{dt}$$
$$= \mathbf{F} \cdot \frac{d\mathbf{R}}{dt}$$
$$= m \cdot \mathbf{A} \cdot \frac{d\mathbf{R}}{dt}$$
$$= m \cdot \mathbf{A} \cdot \mathbf{V}$$

In linear acceleration, $\mathbf{A}$ is aligned with $\mathbf{V}$, the cosine between the vectors is 1, and the vector dot product reduces to the simple scalar product $A \cdot V$, so $P = m \cdot A \cdot V$. In other words, at each instant, the power is proportional to the speed, which is ever increasing as long as the acceleration is maintained.

Lasers could beam propulsive power to the spacecraft, freeing it from the necessity to carry rockets, propellant, and tanks, reducing the spacecraft mass by orders of magnitude. However, to maintain constant linear acceleration, the power received by the spacecraft must increase with distance, whereas its tendency is to decrease. Power beamed from Earth would have to increase substantially to deliver increasing power over increasing distance.

Assuming that the remote star is a destination and not merely a flyby, the ideal solution for nearly constant gravity (unconstrained by power limits) would be to accelerate toward the destination to the halfway point, then reverse thrust and decelerate for the remainder of the journey.

To set a lower bound on the problem, the following calculations aim for our nearest stellar neighbor: Proxima Centauri, at 4.244 lightyears. Gilster [28] notes that Epsilon Eridani may be a better target, even though more distant at 10.47 lightyears, because its alignment with Earth's orbital plane allows for gravity assists from our local solar system planets. However, those would pale in comparison to the continuous 1-g acceleration contemplated here.

These calculations assume the following constants. The units of measure are defined by or derived from the *Bureau International des Poids et Mesures* [29] and the International Astronomical Union [30], and are exact values except where noted otherwise. The distance to Proxima Centauri is estimated from the ESA Gaia data archive (768.5 milliarcseconds of parallax) [17]:

c speed of light
   = 299,792,458 m/s
   = 1 ly/yr
yr Julian year
   = 365.25 days
   = 31,557,600 s
ly lightyear
   = c · yr
   = 9,460,730,472,580,800 m
g standard gravity
   = 9.80665 m/s^2
   = 1.03230 ly/yr^2 (rounded definition)
d distance to Proxima Centauri
   = 4.244 ly (rounded estimate)

Naive Non-Relativistic Calculations

A simple, naive, non-relativistic calculation is a useful starting point, if for no other reason than to explore the magnitude of the problem and the necessity for the relativistic calculations that follow.

Starting with the definitions of distance R, velocity V and acceleration A, solve for the time t to reach a given distance at a given constant acceleration:

$$V = \int A \cdot dt = A \cdot t$$
$$R = \int V \cdot dt = \int A \cdot t \cdot dt = A \cdot t^2/2$$
$$t = \sqrt{2 \cdot R/A}$$

Our plan is to accelerate at 1 g = 1.0323 ly/yr^2 halfway to Proxima Centauri, then reverse thrust and decelerate for the second half of the journey. The total elapsed time will be twice the time to accelerate through half the distance. The maximum velocity will occur at the halfway point. Substituting R = d/2, and A = 1 g (using units of ly and yr, so d=4.244 and g=1.0323), we have:

$$t_{half} = \sqrt{d/g} = 2.028 \text{ yr}$$
$$t_{total} = 2 \cdot t_{half} = 4.055 \text{ yr}$$
$$V_{max} = g \cdot t_{half} = \sqrt{d \cdot g} = 2.093 \text{ c}$$

Our naive plan requires the spacecraft to accelerate to more than twice the speed of light, which the laws of physics (as we currently understand them) prohibit. However, this does not mean that we cannot provide the crew with constant 1-g linear acceleration in *their* frame of reference, at least in theory; it merely means that the spacecraft will reach a velocity relative to Earth so great that we need to apply relativistic calculations.

Relativistic Calculations

A discourse on relativity is beyond the scope of this paper and frankly the expertise of this author. Fortunately, Geffen [31] provides an on-line "space travel calculator" that does exactly what we want; Gibbs, Woods, and Koks [32], and Oesper [33], outline the underlying math for just such a scenario – factored differently, but a bit of basic algebra shows them to be equivalent.

Plugging our distance and acceleration into the calculator (again using units of ly and yr, so d=4.244, g=1.0323, and c=1), we find the total elapsed times measured by observers in the spacecraft and on the Earth, and the maximum velocity of the spacecraft relative to Earth:

$$t_{total\_craft} = 2 \cdot \frac{c}{g} \cdot \cosh^{-1}\left(\frac{g \cdot d}{2 \cdot c^2} + 1\right)$$
$$= 3.541 \text{ yr}$$
$$t_{total\_earth} = 2 \cdot \frac{c}{g} \cdot \sqrt{\left(\frac{g \cdot d}{2 \cdot c^2} + 1\right)^2 - 1}$$
$$= 5.870 \text{ yr}$$
$$V_{max\_earth} = c \left/ \sqrt{\left(\frac{2 \cdot c}{g \cdot t_{total\_earth}}\right)^2 + 1}\right.$$
$$= 0.9496 \text{ c}$$

For the sake of the crew, the constant 1-g acceleration of the spacecraft is accounted at each instant relative to an inertial (non-accelerated) reference frame moving at the same instantaneous velocity as the spacecraft. When the spacecraft has attained a substantial fraction of the speed of light, that reference frame has a length contraction and a time dilation relative to Earth's: its meters are shorter and its seconds are longer.

Meanwhile, back on Earth, the spacecraft acceleration appears to decrease as its velocity approaches the speed of light as an asymptote.

In both reference frames, $F = m \cdot A$ and $P = m \cdot A \cdot V$. But, where Newtonian physics

regards the mass m as an immutable universal property of a particle, Einsteinian physics sees it as relative to the frame of reference.

Observers on Earth see the mass of the spacecraft increase as its acceleration decreases. They may question the value of pouring ever increasing power into it only to increase its mass and not its velocity – whether that power is being beamed continually from Earth or was somehow invested in the spacecraft before it left. It's the very definition of diminishing returns. Forward [34] asserts that a "properly optimized interstellar mission" would stop investing power after the craft had attained some lower fraction of the speed of light. From Earth's point of view, it would take only moderately longer to arrive at its destination and reduce overall energy requirements by orders of magnitude.

The crew on the spacecraft see the situation differently. Cutting the power off early would leave them in an unhealthy weightless state and could greatly increase the length of the journey as they experience it. So, even the value proposition of continual 1-g linear acceleration is relativistic.

But what if some phenomenal new power technology is developed that makes this a viable strategy? Why not increase the power and decrease the transit time even more? N times huge is still just huge. The answer is that this would subject the crew to hypergravity greater than 1 g, which is also not desirable to any great extent over the long term.

It may be that the exoplanet Proxima Centauri b is, or could be, a habitable "super Earth" with a surface gravity slightly exceeding Earth's. In that case, it might be beneficial for the spacecraft to exceed 1 g acceleration to prepare the crew for colonization, but that would add significantly to the required power.

In any case, the acceleration cannot *greatly* exceed 1 g, and this upper limit on acceleration sets a lower limit on the transit time even if unimaginably (but calculably) huge power is available. Nevertheless, Gibbs [32] calculates that even at *only* 1 g, the crew could cross the entire galaxy in only 12 of their years, though 113,243 Earth years.

Forward [34] maintains that interstellar travel is feasible (not merely a topic for idle mathematical speculation), but asserts flatly that continual 1-g linear acceleration is *not* feasible – even to the nearest star. The ultimate system he envisions would accelerate an 80,000 tonne spacecraft at 0.3 g to Epsilon Eridani and back by solar powered lasers orbiting the planet Mercury beaming 43,000 times Earth's total electric power production (as of 1986). The spacecraft velocity would reach about 0.5 c in about 1.6 years. The one-way transit time would be about 20 years for Earth and 17 years for the spacecraft and its crew.

Rather more conservatively, Lubin [35, 36] estimates that Earth-based laser arrays could accelerate a 100 tonne spacecraft to 0.0026 c, which is still 46 times faster than Voyager and exceeds the galactic escape velocity.

Others have suggested that 5% light-speed, or 1 milligee acceleration, or a 100-year transit to Proxima Centauri, are feasible goals [37, 38, 20, 39, 40]. Some sources cite two or three of these criteria together. Strong [40] for example writes: "we should brace ourselves for a century-long flight – but not a year longer. Our vehicle must accordingly be capable of a speed of the order of five psol" (percent speed of light). *If* we assume constant absolute $\pm$ acceleration with thrust reversal at the midpoint, those criteria don't coincide. Table 1 below compares them under that assumption. (Geffen's "space travel calculator" [31] rounds 1 g to 9.8 m/s^2, and the distance to Proxima Centauri to 4.2 ly, which yields slightly different results.)

To meet Strong's non-negotiable criterion of 1 century to Proxima Centauri, the acceleration must exceed 1 milligee early in the flight to achieve a speed of 5% c earlier, and then coast longer at that maximum speed before slowing down again. The exact profile remains to be determined.

Table 1: One-way journeys to Proxima Centauri based on "feasible" values for: maximum velocity in Earth's frame V_{max_earth}; continuous $\pm$ acceleration in the spacecraft's frame A_{craft} (reversed at midpoint); and elapsed time in the spacecraft's frame t_{total_craft}. In each row, the boldface value is the controlling criterion; the others derive from it.

V_{max_earth}	A_{craft}	t_{total_craft}
0.05000 c	0.000572 g	169.6 yr
0.06608 c	**0.001000 g**	128.2 yr
0.08461 c	0.001643 g	**100.0 yr**

Since much of the time will be spent with little or no linear acceleration, it will simplify design if the acceleration never exceeds some threshold – perhaps 0.01 g – so that it remains insignificant relative to a 1 g centripetal acceleration.

4.2 Centripetal Acceleration

Using currently foreseeable propulsion technology, centripetal acceleration is the only viable means of providing the interstellar crew with a significant fraction of Earth-surface weight for the duration of the journey. We still have $P = m \cdot \mathbf{A} \cdot \mathbf{V}$, but the acceleration vector $\mathbf{A}$ is perpendicular to the velocity vector $\mathbf{V}$, the cosine between the vectors is 0, and the vector dot product is 0. The centripetal force arises from structural tension in hoops and spokes. With a constant radius, the force acts over zero distance, performs zero work, and consumes zero energy and power. Power is required to start or stop the rotation, but steady-state rotation is self-sustaining through conservation of energy and momentum. This hugely reduces the energy required to sustain the crew in healthy weight.

Four rotation parameters are commonly used to specify rotating artificial-gravity environments: angular velocity or "spin rate" $\mathbf{\Omega}$ (radians per unit time), radius $\mathbf{R}$, tangential velocity $\mathbf{V}$, and centripetal acceleration $\mathbf{A}_{cent}$. Each of these are vectors, and their directions with respect to one another are significant. They're related by the following formulas:

$$\mathbf{V} = \mathbf{\Omega} \times \mathbf{R}$$
$$\mathbf{A}_{cent} = \mathbf{\Omega} \times \mathbf{V}$$
$$= \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{R})$$

The vector cross product of two vectors is another vector, perpendicular to both of the operands, with a magnitude equal to the product of their magnitudes times the sine of the angle between them.

The angular velocity $\mathbf{\Omega}$ aligns with the axis of rotation; the radius $\mathbf{R}$ is perpendicular to that; the tangential velocity $\mathbf{V}$ is perpendicular to both of those; and the centripetal acceleration $\mathbf{A}_{cent}$ is perpendicular to $\mathbf{\Omega}$ and $\mathbf{V}$ (and parallel but opposite to $\mathbf{R}$). The sine of the angle between perpendicular vectors is ± 1, so the magnitudes of these vectors have simple scalar relationships:

$$V = \Omega \cdot R$$
$$A_{cent} = \Omega \cdot V$$
$$= \Omega^2 \cdot R$$
$$= \frac{V^2}{R}$$

Another important consideration is Coriolis acceleration $\mathbf{A}_{Cor}$ that arises in proportion to relative linear velocity $\mathbf{v}$ within the rotating habitat (using lowercase for motion relative to a non-inertial reference); this is a transient "distortion" of the apparent gravity:

$$\mathbf{A}_{Cor} = 2 \cdot \mathbf{\Omega} \times \mathbf{v}$$

The relative velocity has components parallel and perpendicular to the rotation axis, either of which may be zero. Only the perpendicular component $\mathbf{v}_{perp}$ contributes to the Coriolis acceleration, as when walking around the circumference or climbing a ladder. The magnitude is:

$$A_{Cor} = 2 \cdot \Omega \cdot v_{perp}$$

and the Coriolis / centripetal ratio is:

$$\frac{A_{Cor}}{A_{cent}} = \frac{2 \cdot \Omega \cdot v_{perp}}{\Omega \cdot V}$$
$$= 2 \cdot \frac{v_{perp}}{V}$$

Hall [2, 41] provides the mathematical derivation of the formulas for centripetal and Coriolis accelerations from first-principle definitions of rotation, position, velocity, and acceleration.

The total acceleration of a person or object moving within a rotating habitat is the vector sum of three components:

$$\mathbf{A}_{tot} = \mathbf{A}_{cent} + \mathbf{A}_{Cor} + \mathbf{a}$$

where the final term $\mathbf{a}$ is acceleration – that is, the rate of change of $\mathbf{v}$ – within the rotating habitat.

For motion around the circumference, $\mathbf{a}$ is another centripetal acceleration. The Coriolis acceleration in this case, being perpendicular to both the rotation axis and the relative velocity (which is tangential), also aligns with the radius, but either adds or subtracts from the centripetal accelerations depending on the direction of motion. The total apparent gravity during circumferential motion at relative speed v_{circ} is:

$$A_{tot\_circ} = A_{cent} \pm A_{Cor} + a$$
$$= \frac{V^2}{R} \pm 2 \cdot \Omega \cdot v_{circ} + \frac{v_{circ}^2}{R}$$
$$= \frac{V^2}{R} \pm 2 \cdot \frac{V}{R} \cdot v_{circ} + \frac{v_{circ}^2}{R}$$
$$= \frac{(V \pm v_{circ})^2}{R}$$

Yet another consideration is relative *rotational* motion within a rotating habitat. If the rotation axes aren't aligned, the rotations "cross couple," yielding non-intuitive "gyroscopic" effects. These conform to Euler's equations of motion for rigid body dynamics, which involve concepts of angular momentum and moment of inertia that digress from the topic of this paper. Suffice to say: for any object with a non-aligned relative rotation, there's a torque around an axis perpendicular to both the object and habitat rotation axes. In the case of a person's head, this causes an angular acceleration of the fluid in the semicircular canals of the inner ear that may provoke a vestibular illusion of rotation about that third axis. For example, yawing the head left-right around the vertical axis, in a habitat that's rotating (pitching) around the north-south axis, may provoke a vestibular illusion of rolling around the east-west axis. This may cause dizziness or a loss of balance.

So, each of the four rotation parameters impacts habitability, and should be constrained to improve habitability, as follows:

Ω Small: The angular velocity of the habitat factors into Coriolis accelerations and cross-coupled angular accelerations. To keep these effects small, the angular velocity should be small.

R Large: The centripetal acceleration and "up" vector is aligned with and opposite to the radius – directed toward the center of rotation. Because the magnitude of the acceleration is proportional to the radial distance, there will be a gravity gradient in a standing person from the head (less) to the feet (more). To keep this gradient small, the radius should be large in proportion to a person's height.

V Large: When a person moves in the habitat perpendicular to its rotation axis, the ratio of Coriolis to centripetal acceleration is twice the ratio of the person's relative speed to the habitat's tangential speed (at that radius). To keep this ratio small, the tangential velocity should be large in proportion to a person's relative velocity.

A_{cent} 1 g: Over the long term, a centripetal acceleration that varies far from 1 g in either direction isn't healthy, though we don't yet know what the healthy range is. This paper maintains 1 g as a target.

The algebraic relationships between these parameters are such that a designer can choose any two of them independently and calculate the other two by rearranging and substituting terms. There are six combinations of independent parameters, and two equations for the dependent parameters per combination, so twelve equations overall. The algebra is straightforward and need not be expanded here.

Many studies during the past fifty years have investigated the boundaries of the "comfort zone" for rotating habitats [42, 43, 44, 45, 46, 47]. Hall [2, 48, 49] provides an overview of these, and an on-line calculator for computing the rotational parameters and comparing them with the hypothetical comfort boundaries.

In summary: most estimates of the maximum "comfortable" angular velocity range between 3 and 6 rotations per minute (rpm). For a 1-g centripetal acceleration, these correspond to radii of about 99 m down to 25 m. On the other hand, Lackner and DiZio [50] report that "adaptation to 10 rpm can be achieved relatively easily and quickly" through a deliberate training regime. That corresponds to a 1-g radius of only 9 m.

In spacecraft with small rotational radii, even though the apparent gravity may be "comfortable," it might not seem "normal" in comparison to Earth-surface gravity or linear acceleration. This will be especially apparent in any rapid non-axis-aligned motion within the spacecraft: translation in the plane of rotation (radial and circumferential), and rotation around a non-aligned axis. On the other hand, if the spacecraft is "large," then people will probably want the ability to transit through it "quickly." The higher the relative velocity in the rotational plane, the more apparent the Coriolis effects will be – either a Coriolis force to constrain the motion (for example, to a straight line along a "vertical" spoke), or a deviation from Earth-normal free-fall motion in the absence of such a force. Hall [2, 51, 48] diagrams some of these deviations, using basketball and ladder-climbing as examples. Figures 1 and 2 replicate some of those here.
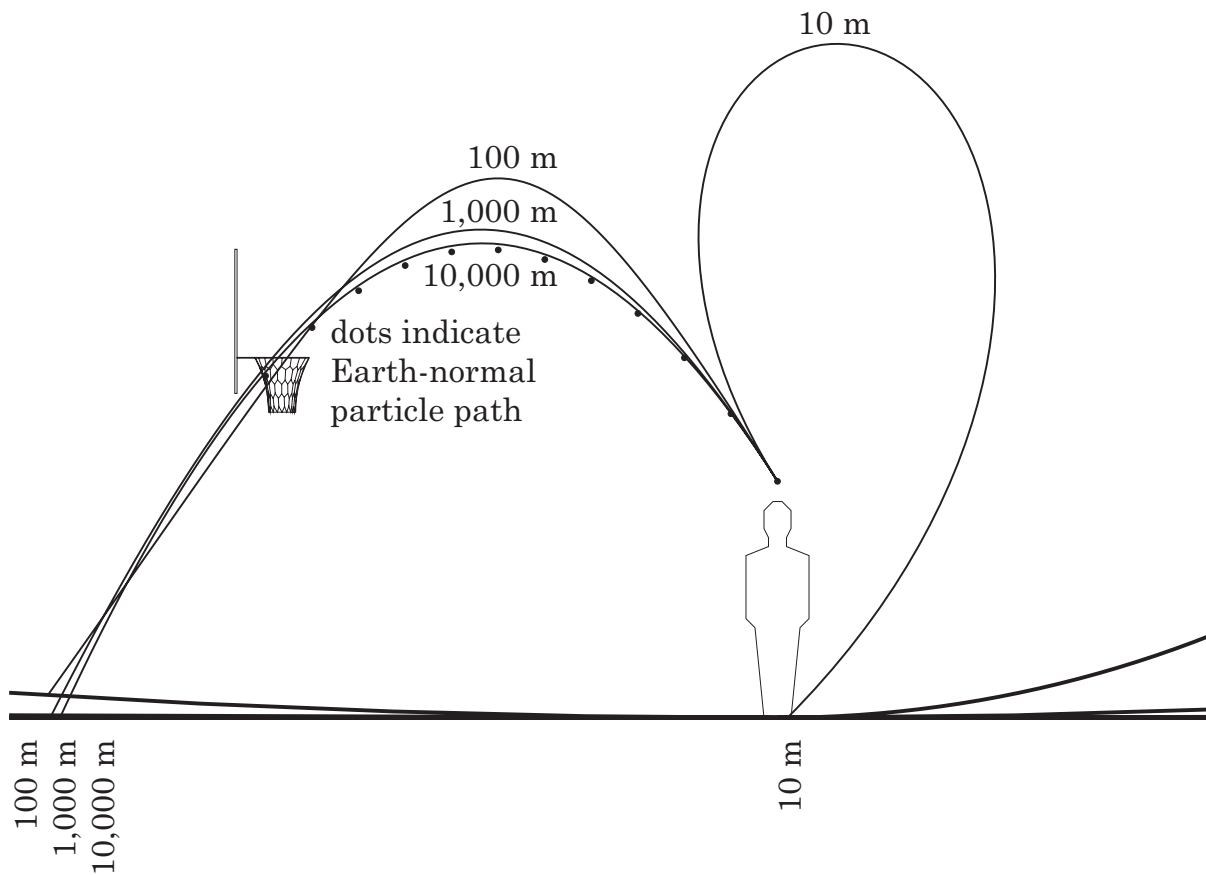
FIGURE 1. *Basketball in 1-g centripetal acceleration, shooting from 2 m above the free-throw line; from Hall [48]. The curves indicate particle paths in rotating habitats with floor radii of 10 m, 100 m, 1,000 m, and 10,000 m. (Closer under the net with a more "vertical" shot, the path is apt to curve backward even in a 100-m radius habitat. See Hall [48].)*

These deviations may compel some researchers to deny the equivalence of centripetal acceleration and "real" gravity, despite the Equivalence Principle uniting gravity and acceleration. Rather than reject the principle, I note that: not all gravitational fields are equal; none are perfectly uniform; and centripetal acceleration yields increasingly uniform gravity with increasing radius. In any case, Schmidt, Goodwin, and Pelligra [52] affirm that, "in accordance with Einstein's 'Theory of Equivalence,' the human body cannot distinguish between the effects of accelerations generated by gravitation or by centrifugation (though effects of the Coriolis force must be considered). It responds identically to both, at the cellular, systemic, and behavioral levels."

Unlike linear acceleration, centripetal acceleration is inextricably tied to the size of the spacecraft –

especially the radial dimension – and convolves with everything else related to spacecraft size – especially crew size, mission duration, and thus all aspects of life support.

If we assume that the maximum velocity is an insignificant fraction of 1 c – because either the linear acceleration is small or it's not sustainable for long – then these will be multi-generational journeys, and centripetal acceleration will be not merely a transient state but a culture-forming lifetime condition.

A Little Digression on Size

What size of habitat is necessary, not only to provide a sufficient rotational radius, but also to maintain a sane and healthy society over a century of confinement? The remote star system may provide resources for
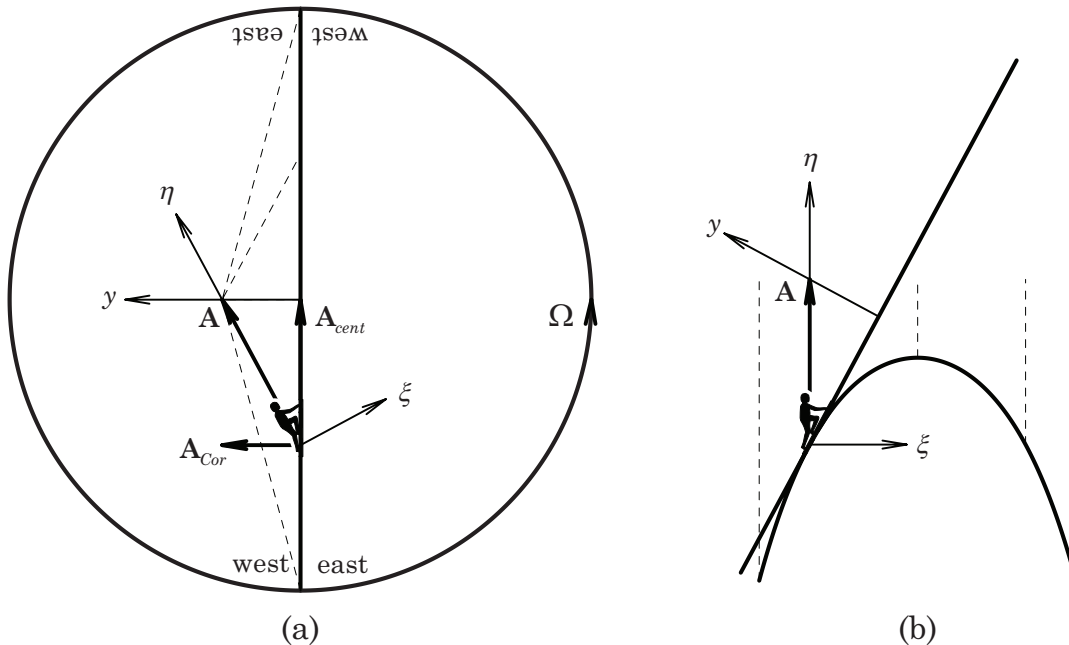
FIGURE 2. *Climbing a ladder in centripetal acceleration; from Hall [2, 51, 48]. (a) The x and y axes rotate with the structure; x is aligned with the ladder. The total acceleration* **A** *is the sum of centripetal* **A**$_{cent}$ *and Coriolis* **A**$_{Cor}$ *components.* **A**$_{cent}$ *depends on position and is independent of climbing speed.* **A**$_{Cor}$ *depends on climbing speed and is independent of position. The dotted lines show how the locus of total acceleration vectors* **A** *for all positions on the ladder converge on a common point. The climber's perceived "up" axis η aligns with the total acceleration; the perceived horizontal axis ξ is perpendicular to that. (b) In the climber's ξ, η axes, the apparent slope of the ladder follows a catenary arch. The straight ladder teeter-totters across the arch, tangent at the climber's position. The dotted lines show how the magnitude of the acceleration also follows the arch.*

expansion, as our home system does, but the space between might be an ocean of emptiness.

The point of including this discussion here is that the minimum size of the spacecraft will almost certainly be determined by societal requirements other than artificial gravity [53, 54].

Estimates of the minimum viable population for a multi-generational interstellar spacecraft vary by several orders of magnitude, based on requirements for diversity not only in the gene pool but also in the skill set and intelligent understanding (versus mere computer memory) necessary to bootstrap a new technological civilization at the destination star system.

- Birdsell [55]: 10 growing to 100s over four or five generations.

- Hodges [56]: as low as 10 per spacecraft, if there are at least several spacecraft to seed a viable independent society at the destination star system.

- Cohen et al. [38, 20]: 100 growing to 500.

- Wachter [57]: 100s for the lower bound.

- Marotta and Globus [54]: 1,000 to 8,000 for a Kalpana-class space settlement.

- Smith [58]: 40,000 with a reproductive subset of 23,400.

- Hein et al. [59]: 100,000 for a "world ship."

There is some diametric disagreement regarding the essential character of the vessels that may take our descendants to the stars. Hein et al. [59] assert that interstellar travel will be a "technological leap," not merely an incremental step. Therefore, "world ships will be vessels that are custom-built for their specific purpose and not simply space colonies with a propulsion system attached to them." In contrast, Marotta and Globus [54] hypothesize that the resources

of the Oort Cloud may extend halfway to the nearest star. If so, then expansion outward from the sun will continue smoothly inward to Alpha Centauri. Interstellar migration will not require "some heroic journey by dedicated adventurers," but rather "simply living in the comfort of free space settlements we've inhabited for hundreds of generations."

We're all free to speculate. Personally, I find it difficult to conceive of a population limited to a few 10s or 100s of people through multiple generations, remaining healthy in mind and body and establishing a viable society at another star system. As a point of reference: my university campus employs more than 28,000 faculty and staff and enrolls more than 44,000 students to convey the breadth and depth of knowledge required to sustain an advanced civilization – and this does not include the many manual skills conveyed by apprenticeship outside of academia. Academics sometimes fail to see or appreciate the multitude of non-academics who support them. It's also doubtful that anyone will learn brain surgery, air-conditioning repair, or many other essential skills from merely watching library videos, without an internship person-ally supervised by an accomplished specialist. If all of these things are entrusted to intelligent robots, then how long will it be before they deduce that maintenance of human life is no longer in their best interest? Or do we trust that intelligent creations would never go against the wishes of their creator?

Certainly, a radically constrained society that arrived at another star system would bear little resemblance to the one it left in our native Solar system a century earlier. What would motivate such an exodus? How bad would conditions here have to be? Consider that, until about 1/4 of the spacecraft's propulsive energy is expended, the crew – perhaps a regretful first generation or a resentful and mutinous second – might have the option of reversing thrust early and returning to Sol.

Back to the Point

Any spacecraft large enough to accommodate a viable independent population of humans over four generations will be large enough to provide "comfort-able" centripetal acceleration. Increasing the radius R while maintaining 1 g centripetal acceleration A_{cent} also drives the angular and tangential velocities Ω and V to increasingly comfortable levels: decreasing $\Omega = \sqrt{A_{cent}/R}$ and increasing $V = \sqrt{A_{cent} \cdot R}$.

With a sufficiently large radius (and accordingly super strength-per-weight materials and abundant energy for spin-up), the apparent gravitational field induced by centripetal acceleration could be as uniform as Earth's surface field – which is also not perfectly uniform.

In the mathematical limit, as R approaches infinity, neglecting aerodynamic effects, the trajectory of a free-falling particle approaches a parabola, just as it does on an infinite-radius flat earth; Hall [60, 2] provides a derivation. The tendency is apparent in the basketball trajectories traced in Figure 1. (On a finite spherical earth more like the one we inhabit, the trajectory of a sub-escape-velocity particle is not actually a parabola, but an ellipse with one focus at the earth's center.)

4.3 Combined Acceleration

It seems likely that an interstellar spacecraft will have to rely primarily on centripetal acceleration to provide the travelers with healthy weight. There must also be some linear acceleration to depart this star and arrive at another, but if that's a tiny fraction of 1 g, it will be insignificant to the travelers' experience of gravity and the gravitational design of their habitat.

On the other hand, a greater linear acceleration will shorten the journey, and this may be critical to success if the endeavor entails extreme confinement, restraint, and austerity. As mentioned previously, Forward [34] describes a system that would achieve 0.3 g linear accel-eration. Though it vastly reduces the transit time, it in-troduces other complications – unless it's great enough to provide healthy weight on its own, without the addi-tion of a centripetal component. (With purely linear ac-celeration, the floor would be a non-spinning flat plane perpendicular to the acceleration.)

By definition, linear acceleration operates in a con-stant direction, and centripetal acceleration does not, so their combination cannot be constant. It must involve some periodic change due to the rotation of the cen-tripetal vector relative to the linear, and a gravitational slope according to the ratio of linear to centripetal.

There are two orientations to consider for the lin-ear acceleration in relation to the centripetal: *in-plane* (perpendicular to the rotation axis), depicted in Fig-ure 3 (a); and *on-axis* (parallel to the axis), depicted in Figure 3 (b). Other oblique orientations are conceivable but offer no obvious advantages.

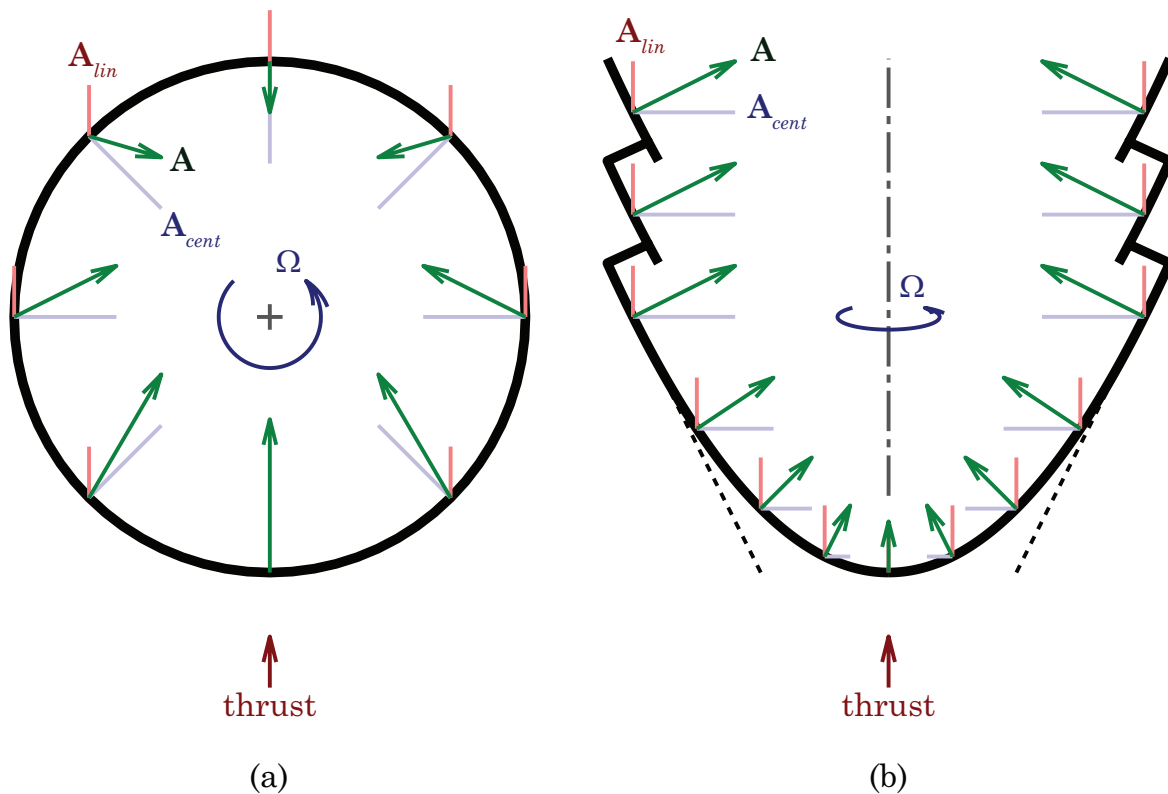The main advantage of the *in-plane* orientation is the ease of redirecting it – especially for the midcourse

FIGURE 3. *Combined linear $\mathbf{A}_{lin}$ and centripetal $\mathbf{A}_{cent}$ accelerations. The total acceleration $\mathbf{A}$ and perceived "up" vector for a stationary inhabitant is the sum of the linear and centripetal components. (a) Linear in-plane relative to centripetal. (b) Linear on-axis relative to centripetal. The dotted lines indicate how a steep paraboloid might be approximated by a cone. The floor might be terraced to build an adequate width while constraining the variance in total acceleration.*

thrust reversal – simply by pivoting the thrusters around the same axis as the spinning habitat: a nudge to start the thruster rotation and a nudge to stop it. However, if the linear acceleration is a significant fraction of the centripetal, then the main disadvantage is that inhabitants will experience a continuous waviness in their gravity. The centripetal vector periodically aligns with, crosses, and opposes the linear vector, with each rotation of the habitat.

The main advantage of the *on-axis* orientation is that the linear acceleration has a consistent relationship with the centripetal: always perpendicular. In purely centripetal acceleration, the ideal floor shape is a cylinder, with longitudes parallel to the axis. With the addition of axial linear acceleration, the longitudes of the ideal floor are no longer parallel to the axis, but sloped to remain perpendicular to the total acceleration. This then entails a change of radius across the width

of the floor and a proportional change of centripetal acceleration, whereas the linear acceleration remains uniform in magnitude and direction. Consequently, the slope of the total acceleration is inversely proportional to the radius, and the mathematically ideal shape, always perpendicular to that acceleration, is a paraboloid. If the change of slope across the width of the floor is small, the paraboloid might be approximated by a section of a cone, indicated by dotted lines in Figure 3 (b). Whether a perfect paraboloid or a cone, there will be a change of gravity across the width of the floor; but, the gravity at any particular location will be constant, without the waviness that might arise with the *in-plane* orientation with a significant linear component. To construct an adequate total floor width while constraining the variance of radius and gravity, the floor might be "terraced" as shown in the upper part of Figure 3 (b).

A major disadvantage of the *on-axis* orientation is the difficulty of redirecting it. The spinning habitat has immense angular momentum, and redirecting that momentum requires a continuous torquing thrust for the duration of the maneuver. Reversing the thrust e.g. from the "north pole" to the "south pole" of the spinning habitat is acceptable only if the linear acceleration is an insignificant fraction of the centripetal – so that the habitat floor is a cylinder and not a paraboloid or cone. But in that case, the *in-plane* orientation might offer a simpler design for reversing the thrust.

At what point does slope become significant? Terrestrial architecture offers some references. On the one hand: a 1% slope is sufficient to cause water run-off but is otherwise generally not perceptible [61, 62, 63, 64]. On the other hand: a 10% slope feels steep; a 5% slope is already considered to be a "ramp"; the slope of a means-of-egress ramp should not exceed 8%; the slope of a "landing" and the "wash" of a stair tread should not exceed 2% [65, 64]. Though a static 1% slope might be imperceptible on Earth, that might not be the case for a dynamic, oscillating slope of comparable magnitude. But, humans are highly adaptable, and perception is moderated by familiarity and expectation. The comfort limit for waviness in the perceived gravity remains an open question. Naval architecture might be a better analogue than the landlocked variety.

Relative motion within the rotating structure may incur a Coriolis component as well, which would add to the linear and centripetal to further modify the apparent gravity. But, with a large radius and a low rate of rotation, this would be small for unmechanized human motion.

5 Closing Thoughts

The subject of interstellar travel is still so speculative and far from realization that any claim of "Conclusions" would be sheer vanity. The best we can do is try to describe the view from this point in history.

Among those who have attempted a scholarly study, there seems to be a consensus that interstellar travel is possible, but a technology that could sustain 1 g linear acceleration for a significant fraction of the journey is not foreseeable. Continuous acceleration far less than 1 g, or briefer bursts of higher acceleration near the endpoints with coasting between, are foreseeable. In either case, the linear acceleration will not be sufficient in du-

ration and intensity to sustain the healthy weight of the crew.

Centripetal acceleration will be a requirement for human health "as we know it." The size of spacecraft necessary to accommodate a viable population for the duration of the journey to another star will certainly accommodate an adequate radius for comfortable centripetal acceleration.

Alternatively, as the situation now appears: either human physiology will need to reach a new equilibrium for permanent weightlessness (with a corresponding redefinition of human health); or some vast suite of non-gravitational countermeasures will need to be developed to preserve the health of active humans; or humans will need to be deactivated to prevent their deterioration, and reactivated by machines at a remote time and place.

Perhaps other alternatives will become apparent before we depart for the stars.

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