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Procedia Structural Integrity 25 (2020) 324-333

Structural Integrity
Procedia

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# 1st Virtual Conference on Structural Integrity - VCSI1

# Damage Detection in Four Point Bending Test on Benchmark RC Structure Using Feature based Fusion

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### Abstract

Sensor fusion has attracted significant attention in recent years due to its usability for many applications in real life. This encourages to use the fusion technique for damage detection in concrete structures. The major difficulty in a concrete damage detection lies in the fact of early crack detection and take proper action before the propagation of cracks. Therefore, different techniques were performed on a benchmark RC beam, which was subjected to four point loading. Then the features from multiple sensors were fused for early crack detection. In this framework, we first represent each measurement technique separately, in which coefficient of peak to peak amplitude from ultrasonic measurement, and the strain measured by strain gauges serve as the features indicator for damage detection. Canonical correlation analysis (CCA) is then applied to both features to construct a combination of features of peak to peak and strain matrices. The result indicates the possibility of using a feature-based fusion algorithm more robustly, and it increases the damage detection probability by reducing false alarm ratio.

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Keywords: Damage detection; Reinforced concrete; Embedded sensors; Diffuse ultrasonic signal; Feature based fusion; Canonical correlation analysis

# 1. Introduction

Nowadays, most of the civil infrastructures (such as bridges) is made of reinforced concrete and the idea of combining two different materials (concrete and steel) to use in the structure emerged because the concrete's tensile strength is much lower than its compressive strength. Steel reinforcements complement these deficiencies. The reinforcement reduces the amount and the width of cracks while improving the strength of the entire element. However, it may not

2452-3216  $\ensuremath{\mathbb{C}}$  2020 The Authors. Published by Elsevier B.V.

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Peer-review under responsibility of the VCSI1 organizers

<sup>10.1016/</sup>j.prostr.2020.04.037

provide full crack resistance and has specific strength limits depending on the concrete grade and degree of reinforcement (Wight (2015)).

Cracks in concrete can be a result of excessive loads or internal sources such as shrinkage or corrosion of the reinforcement. Therefore during infrastructure inspection, damage/crack detection is one of the most critical tasks when evaluating the health of the structure in terms of the life cycle. However, the problem is not in the appearance of crack, but problem is detecting the emergence of cracks or propagation of cracks. If the structure is properly maintained and controlled, then when the crack appears, the process of assessment of the identified crack begins and further propagation is monitored. If the structure can appropriately diagnosed and the reason for crack appearance or propagation can be understood in the first instance, then taking a proper action can be done. For this reason, structural health monitoring (SHM) plays a critical role (Fu (2005)).

SHM is a process of evaluating structural integrity and the level of damage to the structure during its service life. The SHM is based on non-destructive evaluation (NDE) procedures and its continuous monitoring of the structural parameters (such as strains, deflections, impulse loading) to determine the strength and location of the damage. An SHM system includes sensors, data acquisition system and signal processing tools (Chapuis (2018)).

The interpretations of the cracks can be distorted in the reinforced concrete structure, since reinforced concrete is a composite material. Therefore, the monitoring sensors that can be embedded with concrete can be beneficial in this sense. In SHM, there are two groups of sensors usually used: the fist group consists of traditional sensors (such as strain gauges), while the second one the more robust sensors (such as ultrasonic, fiber optic and other sensors). The conventional sensors usually used to measure the stress or strain in the structure due to traffic load, and the robust ones can measure strain as well as detect cracks (Ko and Ni (2005); Lu and Michaels (2005); Kozicki and Tejchman (2007)).

There are different types of sensors available in the market for SHM. Ultrasonic and acoustic emission is the most attractive from all of them due to the capability of crack detection (Scruby and Moss (1993); Li et al. (2019); Hoła (1999); Krautkraamer and Krautkraamer (1990)). Most of the sensors need to be placed on the surface of the structure, which creates an external influence on the measurements. As a surface of concrete structures is used for traffic, and due to its direct exposition to the sunlight, the measurements are biased by the influence of temperature. Acoustic emission technique especially faces these challenges (like noises and temperature effect). The commercially used ultrasonic technique also needs a trained operator and constant coupling, which is difficult in practical measurements. Therefore, Bundesanstalt für Materialforschung und -prüfung (BAM) developed new ultrasonic sensors that can be embedded with concrete inside the structure during or after manufacturing of structure. This new sensor has 62 kHz of center frequency and the bandwidth of 100 kHz, which is suitable for most of the concrete structures as the size of aggregates is less than the wavelength. This is also durable inside the structure (Niederleithinger et al. (2015); Wang et al. (2019)). There are also vibrating wire strain gauge and fiber optic sensors that can be embedded with concrete or inside the structure (Neild et al. (2005); Miller et al. (2017)). This embedding methodology can be more beneficial for the diagnosis of a structure. However, the two mentioned sensors measure different parameters (velocity of a signal and strain) of the structure. Therefore, these two features can be combined in a way that they can be more informative than a single feature. Feature-based fusion can play important role in combining these two features from two different sensors.

The idea of multisensor data fusion and integration has been applied to different applications for many years. However, recently NDT community of concrete structures shown more interest on this topic. Several research can be found on this topic (Heideklang and Shokouhi (2013); Chakraborty et al. (2019a); Völker et al. (2019)). There are different fusion algorithms can be found which was used in several applications (Luo (2012)). In (Li et al.), the authors applied wavelet transform to fuse signal acquired from two eddy current sensors and shown that proposed fusion methodology enhance inner flaw interpretation by reducing signal-to-noise ratio. In (He et al. (2012)), the authors proposed a fuzzy neural network for information fusion on structural damage decision. On the other hand, Liu (2009) covers fuzzy techniques for machinery fault diagnosis. The authors have shown the selection of features based on the fuzzy technique as well as feature-level to decision-level fusion using fuzzy integrals. In (Chakraborty et al. (2019a)), the authors proposed a voting scheme for feature level fusion obtained from ultrasonic measurements. In (Luo (2012)), three-dimensional canonical correlation analysis (CCA) was applied to feature extracted using Gabor wavelet for image recognition. Another example found in (Gong et al. (2013)), where image fusion using a hierarchical decomposition technique was shown. Also in (Correa et al. (2010)), the authors proposed the CCA algorithm to fuse

features obtained from different biomedical techniques and found that a fused image was more informative than single ones.

The research result found in literature shown the benefit of fusion technique in many applications. One of the main challenges for SHM in civil structures is to transfer the information acquired from all the sensors to a single platform for comprehensive results. The feature-based fusion technique can handle this challenge. Therefore, this research paper focuses on combining features coming from different types of sensors. The technique uses the CCA algorithm for feature-based fusion, which was indicated as the most effective in the literature. For this purpose, benchmark RC structure with initiating cracks till failure was considered and experiments were carried out. This fusion technique is used to fuse damage/change related features acquired from multiple sensors and provide suitability of this algorithm for real structures.

#### 2. Methodology

The idea behind the feature fusion is to combine features from single/multiple sensors in order to combine the information and improve the overall performance of damage detection and quantification. Techniques for processing of synchronized information from various sensors located in the same area of the structure that does not show the same accuracy (have different uncertainties) are rarely used in the SHM system. Multi-sensor feature level fusion techniques seek to address these challenges. In our study, we will fuse the two extracted features from ultrasonic and vibrating strain gauge sensors that allow for representation of single feature for damage detection.

#### 2.1. Canonical Correlation for multi-feature analysis

CCA has been fundamentally used to analyze correlations between two sets of source data and maximize the correlation between two random elements (Correa et al. (2010)). Suppose the features have a dimension of  $D \times N$ , where D is a dimension and N is an observation. The two feature matrices  $X_t = X_1, X_2, \ldots X_N$  and  $Y_t = Y_1, Y_2, \ldots Y_N$  have the same dimension. Then the CCA seeks for one pair of vectors  $\vec{X}_t \in \mathbb{R}^m$  and  $\vec{Y}_t \in \mathbb{R}^m$  such that the correlation between two features can be maximized, where  $\vec{X}_t$  and  $\vec{Y}_t$  can be obtained from  $\vec{X}_t = X_t - \vec{X}$  and  $\vec{Y}_t = Y_t - \vec{Y}$  respectively, then  $\vec{X}$  and  $\vec{Y}$  denotes the mean values respectively. In our study,  $\vec{X}_t$  and  $\vec{Y}_t$  can be seen as two views of one observation (e.g. features collected from two sensors located in similar position). Therefore, these two features are somehow correlated as both measuring a similar response. CCA is used for the feature fusion to find a pair of projection of U and V, where  $\vec{X}_t = U_1^T \vec{X}$  and  $\vec{Y}_t = V_1^T \vec{Y}$  are the first pair of canonical variables. The canonical variables has maximum correlation coefficient  $\rho$  (Gong et al. (2013)):

$$\rho = \frac{cov(\sum_{xy} U_1^T \vec{X}, V_1^T \vec{Y})}{\sqrt{var(U_1^T \sum_{xx} \vec{X})var(V_1^T \sum_{yy} \vec{Y})}},$$
(1)

where  $\sum_{xx}$  and  $\sum_{yy}$  denote the feature and training matrices, respectively. In our study, training matrices are considered to be features extracted before the test to measure changes in the structure due to the environmental effects. The reason behind choosing these matrices as a training data set (normally the value changes due to loading or microcracks) is more significant than environmental changes. The optimal correlation can be obtain, if  $U_1^T$  and  $V_1^T$  is the eigenvector with the maximum eigenvalue of the matrix:

$$\sum_{xx}^{-1} \sum_{xy} \sum_{yy}^{-1} \sum_{yx} \sum_{yx}^{-1} \sum_{xx}$$
(2)

$$\sum_{yy} \sum_{yx} \sum_{xx} \sum_{xy} \sum_{xy}$$
(3)

# 2.2. Feature extraction from raw ultrasonic signals

In the diffuse ultrasonic measurement, measured raw signals changes due to environmental or operational loading. These variables can be calculated using peak-to-peak amplitude (de Vera and Güemes (1998); Chakraborty and



Fig. 1. Benchmark RC structure (A) and location of sensors inside the beam.

Katunin (2019)). A research study (Moughty and Casas (2017)) evaluated a number of vibration features such as the maximum peak amplitude, minimum peak amplitude, standard deviation, and sum of squared differences between baseline acceleration and the acceleration for a damaged structure. Li et al. (2019), investigates the peak to peak amplitude as a damage sensitive feature for an acoustic wave and the results were verified through application of the acoustic emission. Casas and Rodrigues (Casas and Rodrigues (2015)) investigated vibration-based criteria and showed a correlation between peak acceleration amplitude and the existence of damage for bridge structures. The results from the previous research study show that peak amplitude has strong damage sensitivity and the potential for damage localization and quantification.

The peak-to-peak amplitudes can be defined as damage/change sensitive features for diffuse ultrasonic signals. The feature extracted as a difference of the peak amplitude in each window template for various change levels normalized by the reference undamaged condition. This peak-to-peak amplitude can be written as:

$$P_a = \frac{[PA_{measured} - PA_{reference}]}{PA_{reference}},\tag{4}$$

where  $PA_{measured}$  denotes the acquired signals during testing periods and  $PA_{reference}$  is the signal acquired during preparation of testing.

### 3. Experimental program and results

The goal of this experiment was to acquire measurements from multiple sensors and to verify the proposed fusion based methodology that gives comprehensive results related to early damage detection. For this purpose, the  $290 \times 40 \times 20$  cm reinforced concrete beam has been casted in the Department of Mechanics and Bridges lab at the Silesian University of Technology, which is presented in Figure 1 (more information about the preparation of the beam can be found in Chakraborty et al. (2019a)). During the casting period, four ultrasonic sensors and two vibrating wire strain gauge sensors were embedded with concrete inside the beam. The four ultrasonic sensors were placed in the bottom and top of the beam and one vibrating wire sensor was placed in the middle of the beam below its neutral axis (see Figure 1). The benchmark RC structure was subjected to tensile loading, and, applied in middle of the beam. The tensile test was carried out with the help of a controlling machine. The loading procedure was performed continuously with a rate of 1 kN/min till 120 kN, and then 5 kN/min till 170 kN. The loading procedure and ultrasonic measurements can be seen in Figure 2. One can see the lack of symmetry in the loading due to manual control of the loading machine during the experiment.

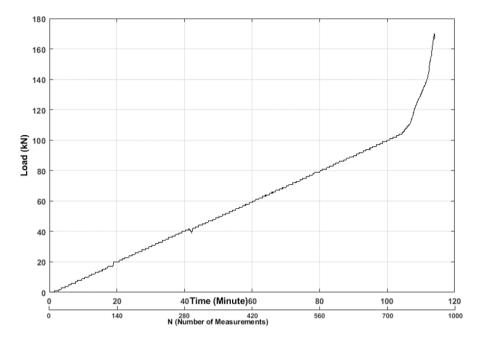


Fig. 2. Loading procedure and number of ultrasonic measurements.

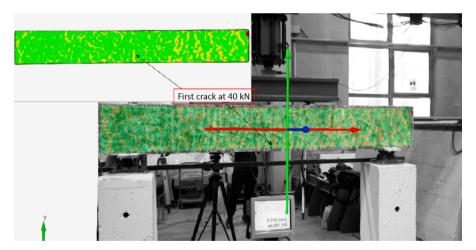


Fig. 3. First Crack propagation along with strain map.

During the loading procedure, the Digital image correlation (DIC) technique was used to record optical images. GOM correlate <sup>®</sup> software was used acquired optical images. The acquired strain map help to compute deflection, crack width and location of the cracks. Figure 4 shows the deflection and middle crack opening displacement values computed from DIC measurements. The deflection and crack opening displacement (COD) show that the first crack was appeared at around 40 kN, and it can be confirmed from the visual graphic (see Figure 3). The first crack was formed in the middle of the beam. In DIC, the location of cracks also computed from the strain map. The curve was placed from support to support of the beam which was 2.5 m long. Then, the surface curve was calculated from the major strain. Surface curve peaks started changing at certain amount when the crack starts to form and propagated along the beam. One can find the position of cracks in the beam from Figure 5.

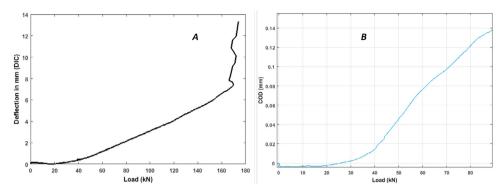


Fig. 4. Deflection (A) and COD (B) from DIC measurement.

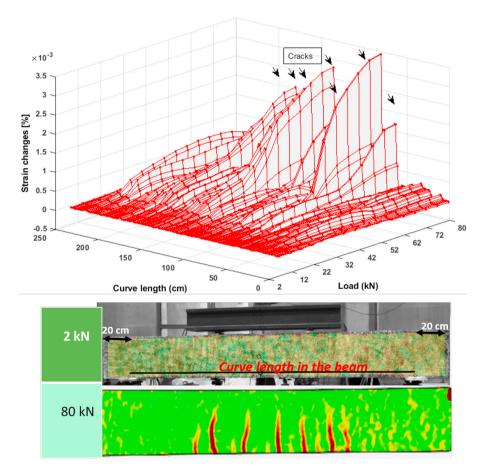


Fig. 5. Location of the cracks in the beam.

#### 3.1. Analysis of ultrasonic feature and strain gauge

During the experiment, to acquire ultrasonic signals the new data acquisition system specially designed for this novel sensors was used (more details about data acquisition system can be found in (Chakraborty et al. (2019b)). The acquisition system acquired seven raw signals for each pair of sensors per minute. In this study, the pair of ultrasonic sensors located in the bottom of the beam and vibrating wire strain gauge sensor located in the middle were chosen because both sensors were located in the same area. Therefore, they measured the same response from the beam.

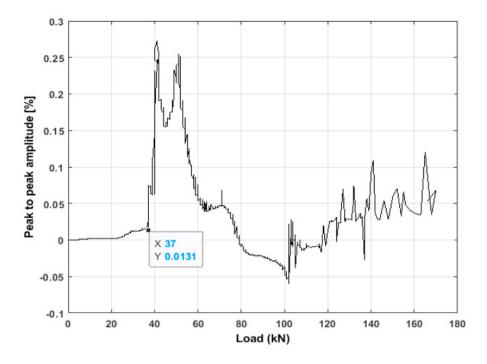


Fig. 6. Peak to peak amplitude coefficient changes with function of load.

During post-processing, the peak to peak amplitude (described in detail in section 2.2) feature was extracted from raw ultrasonic signals. The coefficient (peak to peak amplitude) changed slowly when the load increased. It is obvious that the increasing load has more influence on the lateral part of the amplitude of ultrasonic signals (as this part of signal samples resulted from scattering). Therefore, the acquired ultrasonic signal amplitude was changed with changing the velocity of the signal, which was caused by the tension of the beam. One can see from Figure 6 that, after 32 kN of loading there were sudden small changes in the coefficient between 32–38 kN (coefficient changes from 0.01 to 0.03 %)of the load is applied. However, when the stress lies between the crack initiation stress, limit value the peak to peak amplitude shows a larger change and more intensive fluctuations (starting from 38 kN to 42 kN). Then, after 52 kN of loading cracks started to be more saturated that started to decrease the coefficient, therefore the phase of the signals started to changing, which decrease the difference between the amplitude changes with respect to the initial signal. The peak to peak amplitude coefficient started to increase again at 95 kN of loading, due to signals were started to forced to failure.

The vibrating wire strain gauges placeded in the middle of the beam, registered a steady increase in strain from 1 up to 1000  $\mu\epsilon$ , during loading. The increase is caused by the elastic bending of the beam (see Figure 7). The growing strain in the bending tensile state from 43  $\mu\epsilon$  onwards, indicated an inelastic change as the surface cracks.

#### 3.2. Feature-based fusion

As one can see from Figures 6 and 7, the trend of changes for both features was different, but both types of sensors were located in the same area. In this situation, the feature-based fusion can play an important role. The strain data was normalized to find rate of strain changes. The proposed fusion technique was applied to combine both features. Then, we selected two training data sets from feature values which was extracted before the experiment took place. For each of the generated training sets, the applied Principal Component Analysis (PCA) was applied first to remove the redundant information. Then, the CCA projection matrix was extracted with the algorithm described in Section 2.1, which serves as a combined feature. In our experiments, training data set was the class of values related to environmental changes remaining matrices, which were used for testing. The fused result (Figure 8) shows that using

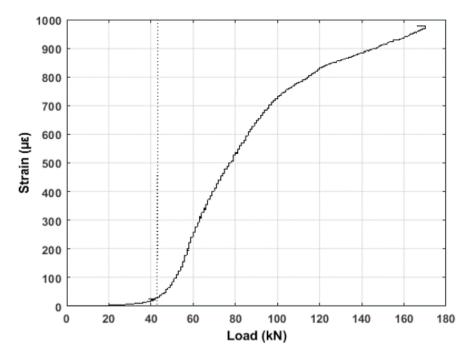


Fig. 7. Strain (middle vibrating wire) vs Load.

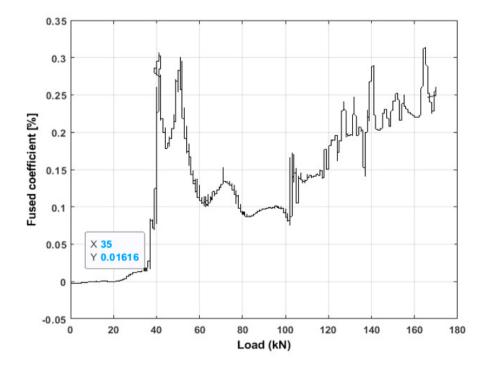


Fig. 8. Fused coefficient (middle vibrating wire) vs Load

peak to peak amplitude and strain features, can keep the original matrices structure, and the detection rate is higher than the single feature.

One can see that the fused coefficient first fluctuates at 35 kN of loading where the peak to peak amplitude was 38 kN and the strain gauge was 42 kN. Then the fused coefficient followed the same trend. However, there were no negative values in coefficient during crack formation to propagation (35 to 95 kN). On the other hand, the peak to peak amplitude values go to negative during crack propagation along the beam. These negative values of the coefficient can miss leading the detection between undamaged state and damage state of the structure, especially when the system will set up a threshold for warning of crack initiation. For example, in the study (Chakraborty et al. (2019a)), the CWT coefficient detected crack earlier than all other techniques, but still, the probability of detection was less compared to other features. It was caused by negative values of the feature and gave a false alarm for crack detection. Therefore, the proposed fusion methodology increases not only early crack detection, but also removes the negative class that reduces false alarm ratio. We concluded that in CCA-based feature fusion algorithm, the crack detection capability slightly increased compared to a single feature at higher index values.

# 4. Conclusions

The peak to peak amplitude and strain features from ultrasonic and vibrating wire strain gauges, which were the indicators for damage in a reinforced concrete beam had been evaluated. The CCA algorithm was used to fuse both features and get a comprehensive indication. Apart from DIC measurement, the primary attention was paid to embedded sensors. The ability of diffuse ultrasonic technique in monitoring the cracking behavior of the tested benchmark structure was verified. The strain map from vibrating wire strain gauges detect the crack as well as follow propagation of crack till failure. The crack initiation, propagation, and location were verified through DIC measurements.

It has been shown that the feature-based fusion display remarkably and improved sensitivity for damage detection. In this study, a new algorithm called CCA for damage detection in the RC structure was verified. The proposed fusion algorithm applied to combine both features was able to detect crack earlier than a single feature. It is important to mention that both sensors were embedded inside the benchmark structure. Therefore they should record crack related responses earlier than techniques/sensors installed on the surface of the structure. Further studies are necessary to compare the sensitivity of the proposed feature-based fusion on real structures. Also, comparative results can be expected to use multiple techniques.

#### Acknowledgement

The authors wish to acknowledge the help of Dr. Ernst Niederleithinger and Xin Wang at BAM for sharing the sensors. The project INFRASTAR (infrastar.eu) has received funding from European Union Horizon 2020 research and innovation programme under the Marie Curie-Sklodowska grant agreement number 676139. The grant is gratefully acknowledged.

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