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The Neutrosophic Triplet of BI-algebras

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Abstract: In this paper, the concepts of a Neutro-BI-algebra and Anti-BI-algebra are introduced, and some related properties are investigated. We show that the class of Neutro-BI-algebra is an alternative of the class of BI-algebras.

Keywords: BI-algebra; Neutro-BI-algebra; sub-Neutro-BI-algebra; Anti-BI-algebra; sub-Anti-BI-algebra; Neutrosophic Triplet of BI-algebra.

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1. Introduction

1.1. BI-algebras

In 2017, A. Borumand Saeid et al. introduced BI-algebras as an extension of both a (dual) implication algebras and an implicative BCK-algebra, and they investigated some ideals and congruence relations [1]. They showed that every implicative BCK-algebra is a BI-algebra, but the converse is not valid in general. Recently, A. Rezaei et al. introduced the concept of a (branchwise) commutative BI-algebra and showed that commutative BI-algebras form a class of lower semilattices and showed that every commutative BI-algebra is a commutative BH-algebra [2].

1.2 Neutrosophy

Neutrosophy is a new branch of philosophy that generalized the dialectics and took into consideration not only the dynamics of opposites, but the dynamics of opposites and their neutrals introduced by Smarandache in 1998 [5]. Neutrosophic Logic / Set / Probability / Statistics etc. are all based on it.

One of the most striking trends in the neutrosophic theory is the hybridization of neutrosophic set with other potential sets such as rough set, bipolar set, soft set, vague set, etc. The different hybrid structures such as rough neutrosophic set, single valued neutrosophic rough set, bipolar neutrosophic set, single valued neutrosophic vague set, etc. are proposed in the literature in a short period of time. Neutrosophic set has been a very important tool in all various areas of data mining, decision making, e-learning, engineering, computer science, graph theory, medical diagnosis, probability theory, topology, social science, etc.

1.3 NeutroLaw, NeutroOperation, NeutroAxiom, and NeutroAlgebra

In this section, we review the basic definitions and some elementary aspects that are necessary for this paper.

The Neutrosophy's Triplet is (<A>, <neutroA>, <antiA>), where <A> may be an item (concept, idea, proposition, theory, structure, algebra, etc.), <antiA> the opposite of <A>, while <neutroA> {also the notation <neutA> was employed before} the neutral between these opposites.

Based on the above triplet the following Neutrosophic Principle one has: a law of composition defined on a given set may be true (T) for some set's elements, indeterminate (I) for other set's elements, and false (F) for the remainder of the set's elements; we call it NeutroLaw.

A law of composition defined on a given sets, such that the law is false (F) for set's elements is called AntiLaw.

Similarly, an operation defined on a given set may be well-defined for some set's elements, indeterminate for other set's elements, and outer-defined for the remainder of the set's elements; we call it NeutroOperation.

While, an operation defined on a given set that is outer-defined for all set's elements is called AntiOperation.

In classical algebraic structures, the laws of compositions or operations defined on a given set are automatically well-defined [i.e. true (T) for all set's elements], but this is idealistic.

Consequently, an axiom (let's say Commutativity, or Associativity, etc.) defined on a given set, may be true (T) for some set's elements, indeterminate (I) for other set's elements, and false (F) for the remainder of the set's elements; we call it NeutroAxiom.

In classical algebraic structures, similarly an axiom defined on a given set is automatically true (T) for all set's elements, but this is idealistic too.

A NeutroAlgebra is a set endowed with some NeutroLaw (NeutroOperation) or some NeutroAxiom.

The NeutroLaw, NeutroOperation, NeutroAxiom, NeutroAlgebra and respectively AntiLaw, AntiOperation, AntiAxiom and AntiAlgebra were introduced by Smarandache in 2019 [4] and afterwards he recalled, improved and extended them in 2020 [5].

2. Neutro-BI-algebras, Anti-BI-Algebras

In this section, we apply Neutrosophic theory to generalize the concept of a $\boldsymbol{BI}$-algebra. Some new concepts as, Neutro-sub-$\boldsymbol{BI}$-algebra, Anti-sub-$\boldsymbol{BI}$-algebra, Neutro-$\boldsymbol{BI}$-algebra, sub-Neutro-$\boldsymbol{BI}$-algebra, NutroLow-sub-Neutro-$\boldsymbol{BI}$-algebra, AntiLow-sub-Neutro-$\boldsymbol{BI}$-algebra, Anti-$\boldsymbol{BI}$-algebra, sub-Anti-$\boldsymbol{BI}$-algebra, NeutroLow-sub-Anti-$\boldsymbol{BI}$-algebra and AntiLow-sub-Anti-$\boldsymbol{BI}$-algebra are proposed.

Definition 2.1. (Definition of classical BI-algebras [1])

An algebra $(X,*,0)$ of type $(2,0)$ (i.e. X is a nonempty set, $*$ is a binary operation and 0 is a constant element of X) is said to be a *BI-algebra* if it satisfies the following axioms:

(B) $(\forall x \in X)(x * x = 0)$,

(BI) $(\forall x, y \in X)(x * (y * x) = x)$.

Example 2.2.

([1]) (i). Let X be a set with $0 \in X$. Define a binary operation $*$ on X By

$$x * y = \begin{cases} 0 & if\ x = y \\ x & if\ x \neq y \end{cases}$$

Then $(X,*,0)$ is a BI-algebra.

(ii). Let S be a nonempty set and $\mathcal{P}(S)$ be the power set of S. Then $(\mathcal{P}(S), -, \emptyset)$ is a BI-algebra. Since $A - A = \emptyset$ and for every $A \in \mathcal{P}(S)$. Also, $A - (B - A) = A \cap (B \cap A^c)^c = A \cap (B^c \cup A^{cc}) = A$, for every $A, B \in \mathcal{P}(S)$. Thus, (B) and (BI) hold.

Definition 2.3. (Definition of classical sub-BI-algebras)

Let $(X,*,0)$ be a BI-algebra. A nonempty set S of X is said to be a *sub-BI-algebra* of X if

$(\forall x, y \in S)(x * y \in S)$.

We note that X and $\{0\}$ are sub-BI-algebra.

Example 2.4. Let $X := \{0, a, b, c\}$ be a set with the following table.

Table 1

$*$	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

Then $(X,*,0)$ is a BI-algebra. We can see that $S = \{0, a, b\}$ is a sub-algebra of X, $T = \{0, a, c\}$ is not a sub-algebra, since, $a, c \in T$, but $c * a = b \notin T$.

Definition 2.5. (Definition of Neutro-sub-BI-algebras)

Let $(X,*,0)$ be a BI-Algebra. A nonempty set NS of X is said to be a *Neutro-sub-BI-algebra* of X if $(\exists x, y \in NS)(x * y \in NS)$and $(\exists x, y \in NS)$ such that $x * y \notin NS$ or $x * y$ = indeterminate.

We note that X and $\{0\}$ are not Neutro-sub-BI-algebras. Since $*$ is a binary operation, and so $x * y \in X$, for all $x, y \in X$. Also, there are no $x, y \in \{0\}$ such that $x * y \notin \{0\}$.

Example 2.6. Consider the BI-algebra $(X,*,0)$ given in Example 2.4. $S = \{0, a, c\}$ is a Neutro-sub-BI-algebra, since $0 * a = 0 \in S$, $a * 0 = a \in S$ and $c * 0 = c \in S$, but $c * a = b \notin S$.

Definition 2.7. (Definition of Anti-sub-BI-algebras)

Let $(X,*,0)$ be a BI-algebra. A nonempty set AS of X is said to be an *Anti-sub-BI-algebra of X* if $(\forall x, y \in AS)(x * y \notin AS)$.

We note that X and $\{0\}$ are not Anti-sub-BI-algebra. Since $*$ is a binary operation, and so $x * y \in X$, for all $x, y \in X$. Also, $(\forall x, y \in \{0\})(x * y \in \{0\})$.

Example 2.8. Consider the BI-algebra $(X,*,0)$ given in Example 2.4. $S = \{c\}$ is an Anti-sub-BI-algebra, since $c * c = 0 \notin S$.

In classical algebraic structures, a Law (Operation) defined on a given set is automatically well-defined (i.e. true for all set's elements), but this is idealistic; in reality we have many more cases where the law (or operation) are not true for all set's elements. In NeutroAlgebra, a law (operation) may be well-defined (T) for some set's elements, indeterminate (I) for other set's elements, and outer-defined (F) for the other set's elements. We call it NeutroLaw (NeutroOperation).

In classical algebraic structures, an Axiom defined on a given set is automatically true for all set's elements, but this is idealistic too. In NeutroAlgebra, an axiom may be true for some of the set's elements, indeterminate (I) for other set's elements, and false (F) for other set's elements. We call it NeutroAxiom.

A NeutroAlgebra is a set endowed with some NeutroLaw (NeutroOperation) or NeutroAxiom. NeutroAlgebra better reflects our imperfect, partial, indeterminate reality.

There are several NeutroAxioms that can be defined on a BI-algebra. We neutrosophically convert its first two classical axioms: (B) into (NB), and (BI) into (NBI). Afterwards, the classical axiom (BI) is completed negated in two different ways $(ABI1)$ and $(ABI2)$ respectively.

- (NB) $(\exists x \in NX)(x *_N x = 0)$ and $(\exists x \in NX)(x *_N x \neq 0)$,
- (NBI) $(\exists x, y \in NX)(x *_N (y *_N x) = x)$ and $(\exists x, y \in NX)(x *_N (y *_N x) \neq x)$,
- $(ABI1)$ $(\forall x \in NX, \exists y \in NX)(x *_N (y *_N x) \neq x)$,
- $(ABI2)$ $(\exists x \in NX, \forall y \in NX)(x *_N (y *_N x) \neq x)$.

In this paper we consider the following:

Definition 2.9. (Definition of Neutro-BI-algebras)

An algebra $(NX, *_N, 0_N)$ of type $(2, 0)$ (i.e. NX is a nonempty set, $*_N$ is a binary operation and 0_N is a constant element of X) is said to be a *Neutro-BI-algebra* if it satisfies the following NeutroAxioms:

(NB) $(\exists x \in NX)(x *_N x = 0_N)$ and $(\exists x \in NX)(x *_N x \neq 0_N$ or indeterminate),

(NBI) $(\exists x, y \in NX)(x *_N (y *_N x) = x)$ and $(\exists x, y \in NX)(x *_N (y *_N x) \neq x$ or indeterminate).

Example 2.10.

(i) Let $NX := \{0_N, a, b, c\}$ be a set with the following table.

Table 2

$*_N$	0_N	a	b	c
0_N	0_N	0_N	0_N	0_N
a	a	0_N	a	b
b	b	b	a	b
c	c	b	b	0_N

Then $(NX, *_N, 0_N)$ is a Neutro-BI-algebra. Since $a *_N a = 0_N$ and $b *_N b = a \neq 0_N$. Also, $a *_N (b *_N a) = a *_N b = a$ and $c *_N (b *_N c) = c *_N b = b \neq c$.

(ii). Let $\mathbb{R}$ be the set of real numbers. Define a binary operation $*_N$ on $\mathbb{R}$ by $x *_N y = x + y + 1$. Then $(\mathbb{R}, *_N, 0)$ is a Neutro-BI-algebra. Since if $x = 0$, then $0 *_N 0 = 0 + 0 + 1 = 1 \neq 0$, and if $x = -0.5$, then $x *_N x = x + x + 1 = 2x + 1 = -1 + 1 = 0$, so (NB) holds. For (NBI), let $x \in \mathbb{R}$. If $y = -x - 2$, then $x *_N (y *_N x) = x$, and if $y \neq -x - 2$, then $x *_N (y *_N x) \neq x$.

(iii). Consider the BI-algebra given in Example 2.2 (ii), it is not a Neutro-BI-algebra. Since (NB) and (NBI) are not valid.

(iv). Let S be a nonempty set and $\mathcal{P}(S)$ be the power set of S. Then $(\mathcal{P}(S), \cap, \emptyset)$ is a Neutro-BI-algebra. Since $\emptyset \cap \emptyset = \emptyset$, and for every $A \neq \emptyset$, $A \cap A = A \neq \emptyset$. Further, if $A \subseteq B$, then $A \cap (B \cap A) = A \cap A = A$. Also, since $A, A^c \in \mathcal{P}(S)$, we get $A \cap (A^c \cap A) = A \cap \emptyset = \emptyset \neq A$. Thus, (NB) and (NBI) hold. Moreover, by a similar argument $(\mathcal{P}(S), \cup, \emptyset)$, is not a BI-algebra, but is a Neutro-BI-algebra.

(v). Similarly, $(\mathcal{P}(S), \cap, S)$ and $(\mathcal{P}(S), \cup, S)$ are Neutro-BI-algebras.

(vi). Let $\mathbb{R}$ be the set of real numbers. Define a binary operation $*_N$ on $\mathbb{R}$ by $x *_N y = x^2 - y$. Then $(\mathbb{R}, *_N, 0)$ is not a BI-algebra. Since $3 *_N 3 = 3^2 - 3 = 6 \neq 0$, so (B) is not valid. If $x \in \{0,1\}$, then $x *_N x = 0$. If $x \notin \{0,1\}$, $x *_N x \neq 0$. Hence (NB) holds. If $x \in \{-y, y\}$, then $x *_N (y *_N x) = x$. If $x \notin \{-y, y\}$, then $x *_N (y *_N x) \neq x$. Thus, (NBI) is valid. Therefore, $(\mathbb{R}, *_N, 0)$ is a Neutro-BI-algebra.

(vii). Let $\mathbb{R}$ be the set of real numbers. Define a binary operation $*_N$ on $\mathbb{R}$ by $x *_N y = x^3 - y$. Then $(\mathbb{R}, *_N, 0)$ is not a BI-algebra. Since $3 *_N 3 = 3^3 - 3 = 24 \neq 0$, so (B) is not valid. If $x \in \{-1,0,1\}$, then $x *_N x = 0$. If $x \notin \{-1,0,1\}$, $x *_N x \neq 0$. Hence (NB) holds. If $x = y$, then $x *_N (y *_N x) = x$. If $x \neq y$, then $x *_N (y *_N x) \neq x$. Thus, (NBI) is valid. Therefore, $(\mathbb{R}, *_N, 0)$ is a Neutro-BI-algebra.

Definition 2.11. (Definition of sub-Neutro-BI-algebras)

Let $(NX, *_N, 0)$ be a Neutro-BI-algebra. A nonempty set NS of NX is said to be a *sub-Neutro-BI-algebra* of NX if $(\forall x, y \in NS)(x *_N y \in NS)$ and NS is itself a Neutro-BI-algebras.

We note that NX is a sub-Neutro-BI-algebra, because $*_N$ is a binary operation, and so it is close. $\{0_N\}$ is not a sub-Neutro-BI-algebra, since it is not a Neutro-BI-algebra because $0_N = 0_N *_N 0_N \in \{0_N\}$.

Example 2.12. Consider the Neutro-BI-algebra $(NX, *_N, 0_N)$ given in Example 2.10 (i). $NS = \{0_N, a, b\}$ is a sub-Neutro-BI-algebra of NX, but $NT = \{0_N, b, c\}$ is not a sub-Neutro-BI-algebra, since $b \in NT$, $b *_N b = a \notin NT$.

Definition 2.13. (Definition of NeutroLaw-sub-Neutro-BI-algebras)

Let $(NX, *_N, 0_N)$ be a Neutro-BI-algebra. A nonempty set NS of NX is said to be a *NeutroLaw-sub-Neutro-BI-algebra* of NX if $(\exists x, y \in NS)(x *_N y \in NS)$and $(\exists x, y \in NS)(x *_N y \notin NS)$.

{As a parenthesis, we recall that NS had to be itself a Neutro-BI-algebra, and this could occur by NS satisfying one or more of the following: the (NB) NeutroAxiom, the (NBI) NeutroAxiom, or the NeutroLaw. We chose, as a particular definition, the NeutroLaw.}

We note that neither NX nor $\{0\}$ are NeutroLaw-sub-Neutro-algebra.

Example 2.14. From Example 2.12, $NT = \{0_N, b, c\}$ is a NeutroLaw-sub-Neutro-BI-algebra. Since $b *_N c = b \in NT$ and $b *_N b = a \notin NT$.

Definition 2.15. (Definition of AntiLaw-sub-Neutro-BI-algebras)

Let $(NX, *_N, 0_N)$ be a Neutro-BI-algebra. A nonempty set AS of NX is said to be an *AntiLaw-sub-Neutro-BI-algebra* of X if $(\forall x, y \in AS)(x *_N y \notin AS)$.

{Similarly, as a parenthesis, we recall that AS had to be itself an Anti-BI-algebra, and this could occur by AS satisfying one or more of the following: the (AB) AntiAxiom, the (NBI) AntiAxiom, or the AntiLaw. We chose, as a particular definition, the AntiLaw.}

In this case NX is not an AntiLaw-sub-Neutro-BI-algebra, but $\{0_N\}$ may or may not be an AntiLaw-sub-Neutro-algebra. If $0_N *_N 0_N \in \{0_N\}$, then it is not an AntiLaw-sub-Neutro-algebra. If $0_N *_N 0_N \notin \{0_N\}$, then it is.

Example 2.16. Let $NX := \{0_N, a, b, c\}$ be a set with the following table.

Table 3

$*_N$	0_N	a	b	c
0_N	0_N	0_N	0_N	0_N
a	a	0_N	a	b
b	b	b	a	a
c	c	b	a	a

Then $(NX, *_N, 0_N)$ is a Neutro-BI-algebra. $AS = \{b, c\}$ is an AntiLaw-sub-Neutro-BI-algebra, because $b *_N b = b *_N c = c *_N b = c *_N c = a \notin AS$.

Definition 2.17. (Definition of Anti-BI-algebras)

An algebra $(AX, *_A, 0_A)$ of type $(2, 0)$ (i.e. AX is a nonempty set, $*_A$ is a binary operation and 0_A is a constant element of AX) is said to be an *Anti-BI-algebra* if it satisfies the following AntiAxioms,

(AB) $(\forall x \in AX)(x *_A x \neq 0_A)$,

(ABI) $(\forall x, y \in AX)(x *_A (y *_A x) \neq x)$.

Example 2.18.

(i). Let $\mathbb{N}$ be the natural number and $AX := \mathbb{N} \cup \{0\}$. Define a binary operation $*$ on AX by $x *_A y = x + y + 1$. Then $(AX, *_A, 0)$ is an Anti-BI-algebra. Since $x *_A x = x + y + 1 \neq 0$, for all $x \in AX$, and $x *_A (y *_A x) = x *_A (y + x + 1) = x + (x + y + 1) + 1 = 2x + y + 2 \neq 0$, for all $x, y \in AX$.

(ii). Let S be a nonempty set and $\mathcal{P}(S)$ be the power set of S. Define the binary operation Δ (i.e. symmetric difference) by $A\Delta B = (A \cup B) - (A \cap B)$ for every $A, B \in \mathcal{P}(S)$. Then $(\mathcal{P}(S), \Delta, S)$ is not a BI-algebra neither Neutro-BI-algebra nor Anti-BI-algebra. Since $A\Delta A = \emptyset \neq S$ for every $A \in \mathcal{P}(S)$ we get (AB) hold, and so (B) and (NB) are not valid. Also, for every $A, B \in \mathcal{P}(S) - \{\emptyset\}$, we have $A\Delta(B\Delta A) = B \neq A$, and since $\emptyset \in \mathcal{P}(S)$, we get $\emptyset\Delta(\emptyset\Delta\emptyset) = \emptyset$. Thus, (ABI) is not valid.

(iii). Similarly, $(\mathcal{P}(S), \Delta, \emptyset)$ is not a BI-algebra neither Neutro-BI-algebra nor Anti-BI-algebra.

(iv). Let S be a nonempty set and $\mathcal{P}(S)$ be the power set of S. Define the binary operation ∇ as $A\nabla B = (A \cup B) \cup C$, for every $A, B \in \mathcal{P}(S)$, where C is a given set of $P(S)$ and $C \notin \{\emptyset, A, B\}$. Then $(\mathcal{P}(S) - \{S\}, \nabla, \emptyset)$ is an Anti-BI-algebra. Since $A\nabla A = (A \cup A) \cup C = \mathrm{A} \cup C$, which can never be equal to $\emptyset$ since $\mathrm{C} \neq \emptyset$. Hence (AB) holds. Also, $A\nabla(B\nabla \mathrm{A}) \neq A$ and so (ABI) holds.

(iv). Let $\mathbb{R}$ be the set of real numbers. Define a binary operation $*_A$ on $\mathbb{R}$ by $x *_A y = x^2 + 1$. Then $(\mathbb{R}, *_A, 0)$ is not a BI-algebra. Since $3 *_A 3 = 3^2 + 1 = 10 \neq 0$, so (B) is not valid. Let $x, y \in \mathbb{R}$, then $x *_A x = x^2 + 1 \neq 0$ and $x *_A (y *_A x) = x *_A (y^2 + 1) = x^2 + 1 \neq 0$. Thus, $(\mathbb{R}, *_A, 0)$ is an Anti-BI-algebra.

(v). Let $\mathbb{R}$ be the set of real numbers. Define a binary operation $*_A$ on $\mathbb{R}$ by $x *_A y = x^2 + 1$. Then $(\mathbb{R}, *_A, 0)$ is not a BI-algebra. Since $3 *_A 3 = 3^2 + 1 = 10 \neq 0$, so (B) is not valid. Let $x, y \in \mathbb{R}$, then $x *_A x = x^2 + 1 \neq 0$, thus one has (AB), and $x *_A (y *_A x) = x *_A (y^2 + 1) = x^2 + 1 \neq 0$, or one has (ABI). Therefore, $(\mathbb{R}, *_A, 0)$ is an Anti-BI-algebra.

Definition 2.19. (Definition of sub-Anti-BI-algebras)

Let $(AX, *_A, 0_A)$ be an Anti-BI-algebra. A nonempty set AS of AX is said to be a *sub-Anti-BI-algebra* of X if $(\forall x, y \in AS)(x *_A y \in AS)$.

We note that AX is a sub-Anti-BI-algebra, but $\{0_A\}$ is not a sub-Anti-BI-algebra, since $0_A *_A 0_A \notin \{0_A\}$.

Example 2.20. Consider the Anti-BI-algebra $(AX, *_A, 0)$ given in Example 2.18 (i). $\mathbb{N}$ is a sub-Anti-BI-algebra of AX. Since $x *_A y = x + y + 1 \in \mathbb{N}$, for all $x, y \in \mathbb{N}$.

Definition 2.21. (Definition of NeutroLaw-sub-Anti-BI-algebras)

Let $(AX, *_A, 0_A)$ be an Anti-BI-algebra. A nonempty set AS of AX is said to be a *NeutroLaw-sub-Anti-BI-algebra* of X if $(\exists x, y \in AS)(x *_A y \in AS)$ and $(\exists x, y \in AS)(x *_A y \notin AS)$.

In this case AX and $\{0_A\}$ are not NeutroLaw-sub-Anti-BI-algebras. Since $\nexists x, y \in AX$ such that $x *_A y \notin AX$, and similarly for $\{0_A\}$.

Example 2.22. Let $AX := \{0_A, a, b, c\}$ be a set with the following table.

Table 4

$*_A$	0_A	a	b	c
0_A	b	a	c	a
a	a	c	b	b
b	b	c	a	a
c	c	b	a	a

Then $(AX,*_A,0_A)$ is an Anti-BI-algebra. $NS=\{a,b\}$ is a NeutroLaw-sub-Anti-BI-algebra, since $a *_A b = b \in NS$ and $b *_A a = c \notin NS$.

Definition 2.23. (Definition of AntiLaw-sub-Anti-BI-algebras)

Let $(AX,*_A,0)$ be an Anti-BI-algebra. A nonempty set AS of AX is said to be an *AntiLaw-sub-Anti-BI-algebra* of X if $(\forall x,y \in AS)(x *_A y \notin AS)$.

In this case AX is not an AntiLaw-sub-Anti-BI-algebra, but $\{0_A\}$ may or may not be an AntiLaw-sub-Anti-BI-algebra. If $0_A *_A 0_A \in \{0_A\}$, then it is not an AntiLaw-sub-Anti-algebra. If $0_A *_A 0_A \notin \{0_A\}$, then it is.

Example 2.24. Consider the Anti-BI-algebra $(AX,*_A *,0_A)$ given in Example 2.22. $AS=\{0_A\}$ is an AntiLaw-sub-Anti-BI-algebra of AX, since $0_A *_A 0_A = b \notin AS$.

Note. It is obvious that the concepts of BI-algebra and Anti-BI-algebra are different. In the following example we show that the concept of Neutro-BI-algebra is different from the concepts of BI-algebra and Anti-BI-algebra.

Example 2.25. Let $X=\mathbb{R}-\{0\}$, endowed with the real division $\div$ of numbers. $(X,\div)$ is well defined, since there is no division by zero. Put $x \coloneqq 3$ and $y \coloneqq 2$, we obtain $2 \div (3 \div 2) = \frac{4}{3} \neq 2$, and so (BI) is not valid. Then $(X,\div,-1)$ is not a BI-algebra, but it is a Neutro-BI-algebra, since if $x = y \coloneqq \pm 1$, then $x \div y = (\pm 1) \div (\pm 1) = 1 \neq -1$. If $x \coloneqq 3$ and $y \coloneqq -3$, then $x \div y = 3 \div (-3) = -1$, and so (NB) holds. For (NBI), again $x = y \coloneqq -1$, we get $(-1) \div \big((-1) \div (-1)\big) = -1$, and if $x \coloneqq 4$ and $y \coloneqq 7$, we have $4 \div (7 \div 4) = \frac{16}{7} \neq 4$, so (NBI) holds. Also, we can see that $(X,\div,-1)$ is not an Anti-BI-algebra, since (AB) and (ABI) are not valid.

3. The Neutrosophic Triplet of BI-algebra

In 2020, F. Smarandache defined a novel definition of Neutrosophic Triplet of (*Algebra, NeutroAlgebra, AntiAlgebra*) [4]. In this section we give a particular example, when the Algebra is replaced by a BI-algebra, and we get (*BI-algebra, Neutro-BI-algebra, Anti-BI-algebra*) as below.

Definition 3.1. Let $\mathcal{U}$ be a nonempty universe of discourse, and X, NX and AX be nonempty sets of $\mathcal{U}$, and an operation $*$ defined on the set X, and the same operation restrained to the set NX (denoted as $*_N$) and to the set AX (denoted as $*_A$) respectively. A triplet (X,NX,AX) endowed with a triplet of binary operations $(*,*_N,*_A)$ and a triplet of constants $(0,0_N,0_A)$ is said to be The ***Neutrosophic Triplet of BI-algebra*** for briefly ***NT-BI-algebra*** if it satisfies the following **Axioms** $\{(B),(BI)\}$,

NeutroAxioms $\{(NB), (NBI)\}$, or **AntiAxioms** $\{(AB), (ABI)\}$ respectively:

(B) $(\forall x \in X)(x * x = 0)$,

(BI) $(\forall x, y \in X)(x * (y * x) = x)$,

(NB) $(\exists x \in NX)(x *_N x = 0_N)$ and $(\exists x \in NX)(x *_N x \neq 0_N$ or is $indeterminate)$,

(NBI) $(\exists x, y \in NX)(x *_N (y *_N x) = x)$

and $(\exists x, y \in NX)(x *_N (y *_N x) \neq x$ or is $indeterminate)$,

(AB) $(\forall x \in AX)(x *_A x \neq 0_A)$,

(ABI) $(\forall x, y \in AX)(x *_A (y *_A x) \neq x)$.

Definition 3.2. A triplet $((S,*,0), (NS,*_N, 0_N), (AS,*_A,*_A))$, where $S \subseteq X$, $NS \subseteq NX$ and $AS \subseteq AX$ is said to be a *sub-NT-BI-algebra* of NT-BI-algebra $((X,*,0), (NX,*_N, 0_N), (AX,*_A,*_A))$ if:

(i) $(S,*,0)$ is a sub-BI-algebra of $(X,*,0)$,

(ii) $(NS,*_N, 0_N)$ is a sub-Neutro-BI-algebra of $(NX,*_N, 0_N)$,

(iii) $(NS,*_A, 0_A)$ is an sub-Anti-BI-algebra of $(AX,*_A, 0_A)$.

4. Conclusions

In this paper, we introduced the notions of new types of sub-BI-algebras. Also, Neutro-BI-algebras, sub-Neutro-BI-algebras, NeutroLow-sub-Neutro-BI-algebras, AntiLow-sub-Neutro-BI-algebras, Anti-BI-algebras, sub-Anti-BI-algebras, NeutroLow-sub-Anti-BI-algebras, AntiLow-sub-Anti-BI-algebras are studied and by several examples showed that the notions are different. Finally, the concept of a Neutrosophic Triplet of BI-algebra is defined. For future work we would define some types of NeutroFilters, NeutroIdeals, AntiFilters, AntiIdeals in the Neutrosophic Triplet of BI-algebras.

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