



# Neutrosophic Triplet Partial $g$ - Metric Spaces

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**Abstract:** In this chapter, neutrosophic triplet partial  $g$  - metric spaces are obtained. Then, some definitions and examples are given for neutrosophic triplet partial  $g$  - metric space. Based on these definitions, new theorems are given and proved. In addition, it is shown that neutrosophic triplet partial  $g$  - metric spaces are different from the classical  $g$  - metric spaces, neutrosophic triplet metric spaces. Thus, we add a new structure in neutrosophic triplet theory. Also, thanks to neutrosophic triplet partial  $g$  - metric space, researchers can obtain new fixed point theorems for neutrosophic triplet theory.

**Keywords:**  $g$  - metric space, neutrosophic triplet set, neutrosophic triplet metric space, neutrosophic triplet  $g$  - metric space, neutrosophic triplet partial  $g$  - metric spaces.

## 1 Introduction

There are many uncertainties in daily life. The logic of classical mathematics is often insufficient to explain these uncertainties. Because it is not always possible to call a situation or event absolutely right or wrong. For example, we cannot always call the weather cold or hot. It can be hot for some, cold for some and cool for others. Similar situations in which we remain indecisive may appear in the professional proficiency assessment. It is often difficult to determine whether a work done or a product produced is always definite good or definite bad. Such a situation reduces the reliability of evaluating professional proficiencies. In order to cope with these uncertainties, Smarandache (1998) defined the concept of neutrosophic logic and neutrosophic set. In the concept of neutrosophic logic and neutrosophic sets, there is  $T$  degree of membership,  $I$  degree of indeterminacy and  $F$  degree of non-membership. These degrees are defined independently of each other. A neutrosophic value is shown by  $(T, I, F)$ . In other words, a condition is handled according to both its accuracy and its inaccuracy and its uncertainty. Therefore, neutrosophic logic and neutrosophic set help us to explain many uncertainties in our lives. In addition, many researchers have made studies on this theory [2-27, 50 - 56]. Recently, Baset et al. studied TOPSIS-CRITIC model for sustainable supply chain risk management [51]; Baset et al. obtained resource levelling problem in construction projects under neutrosophic environment [52].

In fact, in the concept of fuzzy logic and fuzzy sets [28] there is only a degree of membership. In addition, the concept of intuitionistic fuzzy logic and intuitionistic fuzzy set [29] includes membership degree, degree of indeterminacy and degree of non-membership. But these degrees are defined dependently of each other. Therefore, neutrosophic set is a generalized state of fuzzy and intuitionistic fuzzy set.

Also, Smarandache and Ali obtained neutrosophic triplet set (NTS) and neutrosophic triplet groups (NTG) [30]. For every element “ $x$ ” in NTS  $A$ , there exist a neutral of “ $x$ ” and an opposite of “ $x$ ”. Also, neutral of “ $x$ ” must different from the classical neutral element. Therefore, the NTS is different from the classical set. Furthermore, a neutrosophic triplet (NT) “ $x$ ” is showed by  $\langle x, \text{neut}(x), \text{anti}(x) \rangle$ . Also, many researchers have introduced NT structures [31-44]. Recently, Şahin, Kargın, Yücel and Özkartepe obtain neutrosophic triplet  $g$  – metric spaces [45].

Furthermore, Mustafa and Sims introduced  $g$  - metric spaces [46] in 2006.  $g$  - metric space is generalized form of metric space. The  $g$  - metric spaces have an important role in fixed point theory. Recently, researchers studied  $g$  - metric space [46-48]. Also, Salimi and Vetro introduced partial  $g$  – metric spaces [49].

In this chapter, we introduce neutrosophic triplet partial  $g$  - metric space (NTpgMS). In Section 2, we give definitions and properties for partial  $g$  - metric space (pgMS) [49], neutrosophic triplet sets (NTS) [30], neutrosophic triplet metric spaces (NTMS) [32] and neutrosophic triplet  $g$  – metric space (NTgMS) [45]. In Section 3, we define NTpgMS and we give some properties for NTpgMS. Also, we show that NTpgMSs are different from the pgMSs, NTMSs and NTgMSs, because the triangle inequality in the NTgMS, NTMS and pgMS differ from the triangle inequality in the NTpgMS. Then, we examine relationship between NTpgMS and NTgMS. In Section 4, we give conclusions.

## 2 Preliminaries

**Definition 2.1:** [30] Let  $\#$  be a binary operation. A NTS  $(X, \#)$  is a set such that for  $x \in X$ ,

- i) There exists neutral of “ $x$ ” such that  $x \# \text{neut}(x) = \text{neut}(x) \# x = x$ .
- ii) There exists anti of “ $x$ ” such that  $x \# \text{anti}(x) = \text{anti}(x) \# x = \text{neut}(x)$ .

Also, a neutrosophic triplet “ $x$ ” is denoted by  $(x, \text{neut}(x), \text{anti}(x))$ .

**Definition 2.2:** [32] Let  $(N, *)$  be a NTS and  $d_N: N \times N \rightarrow \mathbb{R}^+ \cup \{0\}$  be a function. If  $d_N: N \times N \rightarrow \mathbb{R}^+ \cup \{0\}$  and  $(N, *)$  satisfies the following conditions, then  $d_N$  is called NTM.

- a)  $x * y \in N$ ;
- b)  $d_N(x, y) \geq 0$ ;
- c) If  $x = y$ , then  $d_N(x, y) = 0$ ;
- d)  $d_N(x, y) = d_N(y, x)$ ;

e) If there exists at least a  $y \in N$  for each  $x, z \in N$  such that  $d_N(x, z) \leq d_N(x, z^* \text{neut}(y))$ , then  $d_N(x, z^* \text{neut}(y)) \leq d_N(x, y) + d_N(y, z)$ .

In this case,  $((N, *), d_N)$  is called a NTMS.

**Definition 2.3:** [36] Let  $(N, *)$  be a NTS. If  $d_p: N \times N \rightarrow \mathbb{R}^+ \cup \{0\}$  function satisfies the following conditions, then  $d_p$  is a NTpM. For all  $x, y, z \in N$ ,

- a)  $x * y \in N$ ,
- b)  $d_p(x, y) \geq d_p(x, x) \geq 0$ ,
- c) If  $d_p(x, y) = d_p(x, x) = d_p(y, y) = 0$ , then there exists at least one pair of elements  $x, y \in N$  such that  $x = y$ ,
- d)  $d_p(x, y) = d_p(y, x)$ ,
- e) If for each pair of  $x, z \in N$ , there exists at least one  $y \in N$  such that  $d_p(x, z) \leq d_p(x, z^* \text{neut}(y))$ , then  $d_p(x, z^* \text{neut}(y)) \leq d_p(x, y) + d_p(y, z) - d_p(y, y)$ .

In this case,  $((N, *), d_p)$  is called a NTpMS.

**Definition 2.4:** [45] Let  $(X, *)$  be a NTS. If the following conditions hold, then  $g: X \times X \times X \rightarrow \mathbb{R}^+ \cup \{0\}$  is an NTgM.

- a)  $\forall x, y \in X ; x * y \in X$ ,
- b) If  $x = y = z$ , then  $g(x, y, z) = 0$ ,
- c) If  $x \neq y$ , then  $g(x, y, z) > 0$ ,
- d) If  $z \neq y$ , then  $g(x, x, y) \leq g(x, y, z)$ ,
- e)  $g(x, y, z) = g(x, z, y) = g(y, x, z) = g(y, z, x) = g(z, x, y) = g(z, y, x)$ , for every  $x, y, z \in X$ ,
- f) If there exists at least an  $a \in X$  for each  $x, y, z \in X$  such that  $g(x, y, z) \leq g(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a))$ , then  $g(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq G(x, a, a) + G(a, y, z)$ .

In this case,  $(X, *, g)$  is called NTgMS.

**Definition 2.5:** [45] Let  $(X, *, g)$  be a NTgMS and  $\{x_n\}$  be a sequence in this space. A point  $x \in X$  is said to be limit of the sequence  $\{x_n\}$ , if  $\lim_{n, m \rightarrow \infty} g(x, x_n, x_m) = 0$  and  $\{x_n\}$  is called NT g - convergent to  $x$ .

**Definition 2.6:** [45] Let  $(X, *, g)$  be a NTgMS and  $\{x_n\}$  be a sequence in this space.  $\{x_n\}$  is called NT g - Cauchy sequence if  $\lim_{n, m, l \rightarrow \infty} g(x_n, x_m, x_l) = 0$ .

**Definition 2.7:** [45] Let  $(X, *, g)$  be a NTgMS. If every  $\{x_n\}$  NT g - Cauchy sequence is NT g - convergent, then  $(X, *, g)$  is called NT complete NTgMS.

**Definition 2.8:** [49] Let  $X$  be a neutrosophic triplet set. If the following conditions hold, then  $g: X \times X \times X \rightarrow \mathbb{R}^+ \cup \{0\}$  is a pgM. For all  $a, x, y, z \in X$ ;

- a) If  $x = y = z$ , then  $g(x, y, z) = g(x, x, x) = g(y, y, y) = g(z, z, z)$ ,

- b)  $g(x, x, x) + g(y, y, y) + g(z, z, z) \leq 3 g(x, y, z)$ ,
- c) If  $x \neq y$ , then  $\frac{1}{3} g(x, x, x) + \frac{2}{3} g(x, x, x) < g(x, y, y)$ ,
- d) If  $y \neq z$ , then  $g(x, x, y) - \frac{1}{3} g(x, x, x) \leq g(x, y, z) - \frac{1}{3} g(x, x, x)$ ,
- e)  $g(x, y, z) = g(x, z, y) = g(y, x, z) = g(y, z, x) = g(z, x, y) = g(z, y, x)$ ,
- f)  $g(x, y, z) \leq g(x, a, a) + g(a, y, z) - g(a, a, a)$ .

**Definition 2.9:** [49] Let  $(X, g)$  be a pgMS and  $\{x_n\}$  be a sequence in this space. A point  $x \in X$  is said to be limit of the sequence  $\{x_n\}$ , if  $\lim_{n,m \rightarrow \infty} g(x, x_n, x_m) = g(x, x, x)$  and  $\{x_n\}$  is called NT p- g - convergent to  $x$ .

### 3 Neutrosophic Triplet Partial g - Metric Space

**Definition 3.1:** Let  $(A, *)$  be a NTS. If the function  $d_{NG}: A \times A \times A \rightarrow R^+ \cup \{0\}$  satisfies the below conditions, then  $p_{NG}$  is called a NTpgMS. For  $\forall x, y, z \in A$ ;

- a)  $x * y \in A$ ,
- b)  $0 \leq p_{NG}(x, x, x) \leq p_{NG}(x, y, z)$ ,
- c) If  $p_{NG}(x, x, x) = p_{NG}(y, y, y) = p_{NG}(z, z, z) = p_{NG}(x, y, z) = 0$ , then  $x = y = z$ ,
- d) If  $z \neq y$ , then  $p_{NG}(x, x, y) \leq p_{NG}(x, y, z)$ ,
- e)  $p_{NG}(x, y, z) = p_{NG}(x, z, y) = p_{NG}(y, x, z) = p_{NG}(y, z, x) = p_{NG}(z, x, y) = p_{NG}(z, y, x)$
- f) If there exists at least an  $a \in X$  for each  $x, y, z \in X$  such that  $p_{NG}(x, y, z) \leq p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a))$ , then  $p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z) - p_{NG}(a, a, a)$ .

In this case,  $((A, *), p_{NG})$  is called NTpgMS.

**Example 3.2:** Let  $X = \{0,3,4,6,9\}$  be a set. We show that  $(X, *)$  is a NTS on  $Z_{12}$ . Also, we obtain that  $\text{neut}(0) = 0$ ,  $\text{anti}(0) = 0$ ;  $\text{neut}(3) = 9$ ,  $\text{anti}(3) = 6$ ;  $\text{neut}(4) = 4$ ,  $\text{anti}(4) = 4$ ;  $\text{neut}(6) = 6$ ,  $\text{anti}(6) = 6$ ;  $\text{neut}(9) = 9$ ,  $\text{anti}(9) = 9$ .

Thus,  $(X, .)$  is a NTS and NTs are  $(0,0,0)$ ,  $(3,6,9)$ ,  $(4,4,4)$ ,  $(6,6,6)$  and  $(9,9,9)$ .

Now, we define the function  $p_{NG}: X \times X \times X \rightarrow R^+ \cup \{0\}$  such that

$p_{NG}(x, y, z) = 1 + |4^x - 4^y| + |4^x - 4^z| + |4^y - 4^z|$ . We show that  $p_{NG}$  is a NTpgM.

a) From Table 1, it is clear that  $\forall x, y \in X; x * y \in X$

*	0	3	4	6	9
0	0	0	0	0	0
3	0	9	0	6	3
4	0	0	4	0	0
6	0	6	0	0	6
9	0	3	0	6	9

**Table 1:** "\*" binary operator under  $Z_{12}$

b) It is clear that  $0 \leq p_{NG}(x, x, x) = 1 \leq p_{NG}(x, y, z)$ .

c)  $p_{NG}(x, y, z) = 1 + |4^x - 4^y| + |4^x - 4^z| + |4^y - 4^z| \geq 0$ .

d) If  $y \neq z$ , it is clear that

$$p_{NG}(x, x, y) = 1 + |4^x - 4^x| + |4^x - 4^y| + |4^x - 4^y| \leq p_{NG}(x, y, z) = 1 + |4^x - 4^y| + |4^x - 4^z| + |4^y - 4^z|.$$

e) By absolute value function, it is clear that

$$p_{NG}(x, y, z) = p_{NG}(x, z, y) = p_{NG}(y, x, z) = p_{NG}(y, z, x) = p_{NG}(z, x, y) = p_{NG}(z, y, x), \text{ for every } x, y, z \in X.$$

f)

For  $x = 0, y = 6, z = 3, a = 3, neut(a) = 6$ ;

since  $p_{NG}(0, 6, 3) \leq p_{NG}(0 * 6, 6 * 6, 3 * 6) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 3, 3) + p_{NG}(3, 6, 6) - p_{NG}(3, 3, 3).$$

For  $x = 0, y = 3, z = 9, a = 3, neut(a) = 6$ ;

since  $p_{NG}(0, 3, 9) \leq p_{NG}(0 * 6, 3 * 6, 9 * 6) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 3, 3) + p_{NG}(3, 6, 6) - p_{NG}(3, 3, 3).$$

For  $x = 0, y = 9, z = 3, a = 6, neut(a) = 6$ ;

since  $p_{NG}(0, 9, 3) \leq p_{NG}(0 * 6, 9 * 6, 3 * 6) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For  $x = 0, y = 6, z = 9, a = 3, neut(a) = 6$ ;

since  $p_{NG}(0, 6, 9) \leq p_{NG}(0 * 6, 6 * 6, 9 * 6) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 3, 3) + p_{NG}(3, 6, 6) - p_{NG}(3, 3, 3).$$

For  $x = 0, y = 9, z = 6, a = 3, neut(a) = 6$ ;

since  $p_{NG}(0, 9, 6) \leq p_{NG}(0 * 6, 9 * 6, 6 * 6) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 3, 3) + p_{NG}(3, 9, 6) - p_{NG}(3, 3, 3).$$

For  $x = 3, y = 6, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 6, 9) \leq p_{NG}(3 * 9, 6 * 9, 9 * 9) = p_{NG}(3, 6, 9)$ , we obtain that

$$p_{NG}(9, 6, 9) \leq p_{NG}(3, 6, 9) + p_{NG}(9, 6, 9) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 9, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 9, 6) \leq p_{NG}(3 * 9, 9 * 9, 6 * 9) = p_{NG}(3, 9, 6)$ , we obtain that

$$p_{NG}(3, 9, 6) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 9, 6) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 0, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 0, 6) \leq p_{NG}(3 * 9, 0 * 9, 6 * 9) = p_{NG}(3, 0, 6)$ , we obtain that

$$p_{NG}(3, 0, 6) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 6, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 6, 0) \leq p_{NG}(3 * 9, 6 * 9, 0 * 9) = p_{NG}(3, 6, 0)$ , we obtain that

$$p_{NG}(3, 6, 0) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 9, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 9, 0) \leq p_{NG}(3 * 9, 9 * 9, 0 * 9) = p_{NG}(3, 9, 0)$ , we obtain that

$$p_{NG}(3, 9, 0) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 0, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 0, 9) \leq p_{NG}(3 * 9, 0 * 9, 9 * 9) = p_{NG}(3, 0, 9)$ , we obtain that

$p_{NG}(3, 0, 9) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9)$ .  
 For  $x = 6, y = 0, z = 3, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(6, 0, 3) \leq p_{NG}(6 * 9, 0 * 9, 3 * 9) = p_{NG}(6, 0, 3)$ , we obtain that  
 $p_{NG}(6, 0, 3) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 3) - p_{NG}(9, 9, 9)$ .  
 For  $x = 6, y = 3, z = 0, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(6, 3, 0) \leq p_{NG}(6 * 9, 3 * 9, 0 * 9) = p_{NG}(6, 3, 0)$ , we obtain that  
 $p_{NG}(6, 3, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 3, 0) - p_{NG}(3, 3, 3)$ .  
 For  $x = 6, y = 3, z = 9, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(6, 3, 9) \leq p_{NG}(6 * 9, 3 * 9, 9 * 9) = p_{NG}(6, 3, 9)$ , we obtain that  
 $p_{NG}(6, 3, 9) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 3, 9) - p_{NG}(9, 9, 9)$ .  
 For  $x = 6, y = 9, z = 3, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(6, 9, 3) \leq p_{NG}(6 * 9, 9 * 9, 3 * 9) = p_{NG}(6, 9, 3)$ , we obtain that  
 $p_{NG}(6, 9, 3) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 9, 3) - p_{NG}(9, 9, 9)$ .  
 For  $x = 6, y = 0, z = 9, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(6, 0, 9) \leq p_{NG}(6 * 9, 0 * 9, 9 * 9) = p_{NG}(6, 0, 9)$ , we obtain that  
 $p_{NG}(6, 0, 9) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9)$ .  
 For  $x = 6, y = 9, z = 0, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(6, 9, 0) \leq p_{NG}(6 * 9, 9 * 9, 0 * 9) = p_{NG}(6, 9, 0)$ , we obtain that  
 $p_{NG}(6, 9, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9)$ .  
 For  $x = 9, y = 0, z = 3, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(9, 0, 3) \leq p_{NG}(9 * 9, 0 * 9, 3 * 9) = p_{NG}(9, 0, 3)$ , we obtain that  
 $p_{NG}(9, 0, 3) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 3) - p_{NG}(9, 9, 9)$ .  
 For  $x = 9, y = 3, z = 0, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(9, 3, 0) \leq p_{NG}(9 * 9, 3 * 9, 0 * 9) = p_{NG}(9, 3, 0)$ , we obtain that  
 $p_{NG}(9, 3, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 3, 0) - p_{NG}(9, 9, 9)$ .  
 For  $x = 9, y = 3, z = 6, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(9, 3, 6) \leq p_{NG}(9 * 9, 3 * 9, 6 * 9) = p_{NG}(9, 3, 6)$ , we obtain that  
 $p_{NG}(9, 3, 6) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 3, 6) - p_{NG}(9, 9, 9)$ .  
 For  $x = 9, y = 6, z = 3, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(9, 6, 3) \leq p_{NG}(9 * 9, 6 * 9, 3 * 9) = p_{NG}(9, 6, 3)$ , we obtain that  
 $p_{NG}(9, 6, 3) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 6, 3) - p_{NG}(9, 9, 9)$ .  
 For  $x = 9, y = 0, z = 6, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(9, 0, 6) \leq p_{NG}(9 * 9, 0 * 9, 6 * 9) = p_{NG}(9, 0, 6)$ , we obtain that  
 $p_{NG}(9, 0, 6) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9)$ .  
 For  $x = 9, y = 6, z = 0, a = 9, neut(a) = 9$ ;  
 since  $p_{NG}(9, 6, 0) \leq p_{NG}(9 * 9, 6 * 9, 0 * 9) = p_{NG}(9, 6, 0)$ , we obtain that  
 $p_{NG}(9, 6, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9)$ .  
 For  $x = 0, y = 0, z = 3, a = 6, neut(a) = 6$ ;  
 since  $p_{NG}(0, 0, 3) \leq p_{NG}(0 * 6, 0 * 6, 3 * 6) = p_{NG}(0, 0, 6)$ , we obtain that  
 $p_{NG}(0, 0, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 0, 6) - p_{NG}(6, 6, 6)$ .

For  $x = 0, y = 3, z = 0, a = 6, neut(a) = 6$ ;

since  $p_{NG}(0, 3, 0) \leq p_{NG}(0 * 6, 3 * 6, 0 * 6) = p_{NG}(0, 6, 0)$ , we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For  $x = 3, y = 0, z = 0, a = 6, neut(a) = 6$ ;

since  $p_{NG}(3, 0, 0) \leq p_{NG}(3 * 6, 0 * 6, 0 * 6) = p_{NG}(6, 0, 0)$ , we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 0) - p_{NG}(6, 6, 6).$$

For  $x = 0, y = 0, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(0, 0, 6) \leq p_{NG}(0 * 9, 0 * 9, 6 * 9) = p_{NG}(0, 0, 6)$ , we obtain that

$$p_{NG}(0, 0, 6) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For  $x = 0, y = 6, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(0, 6, 0) \leq p_{NG}(0 * 9, 6 * 9, 0 * 9) = p_{NG}(0, 6, 0)$ , we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 0, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 0, 0) \leq p_{NG}(6 * 9, 0 * 9, 0 * 9) = p_{NG}(6, 0, 0)$ , we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For  $x = 0, y = 0, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(0, 0, 9) \leq p_{NG}(0 * 9, 0 * 9, 9 * 9) = p_{NG}(0, 0, 9)$ , we obtain that

$$p_{NG}(0, 0, 9) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

For  $x = 0, y = 9, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(0, 9, 0) \leq p_{NG}(0 * 9, 9 * 9, 0 * 9) = p_{NG}(0, 9, 0)$ , we obtain that

$$p_{NG}(0, 9, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For  $x = 9, y = 0, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 0, 0) \leq p_{NG}(9 * 9, 0 * 9, 0 * 9) = p_{NG}(9, 0, 0)$ , we obtain that

$$p_{NG}(9, 0, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 3, z = 0, a = 6, neut(a) = 6$ ;

since  $p_{NG}(3, 3, 0) \leq p_{NG}(3 * 6, 3 * 6, 0 * 6) = p_{NG}(6, 6, 0)$ , we obtain that

$$p_{NG}(6, 6, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For  $x = 3, y = 0, z = 3, a = 6, neut(a) = 6$ ;

since  $p_{NG}(3, 0, 3) \leq p_{NG}(3 * 6, 0 * 6, 3 * 6) = p_{NG}(6, 0, 6)$ , we obtain that

$$p_{NG}(6, 0, 6) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 6) - p_{NG}(6, 6, 6).$$

For  $x = 0, y = 3, z = 3, a = 6, neut(a) = 6$ ;

since  $p_{NG}(0, 3, 3) \leq p_{NG}(0 * 6, 3 * 6, 3 * 6) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For  $x = 3, y = 3, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 3, 6) \leq p_{NG}(3 * 9, 3 * 9, 6 * 9) = p_{NG}(3, 3, 6)$ , we obtain that

$$p_{NG}(3, 3, 6) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 3, 6) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 6, z = 3, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 6, 3) \leq p_{NG}(3 * 9, 6 * 9, 3 * 9) = p_{NG}(3, 6, 3)$ , we obtain that

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For  $x = 3, y = 9, z = 3, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 9, 3) \leq p_{NG}(3 * 9, 9 * 9, 3 * 9) = p_{NG}(3, 9, 3)$ , we obtain that

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For  $x = 9, y = 3, z = 3, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 3, 3) \leq p_{NG}(9 * 9, 3 * 9, 3 * 9) = p_{NG}(9, 3, 3)$ , we obtain that

$$p_{NG}(9, 3, 3) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 3, 3) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 6, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 6, 0) \leq p_{NG}(6 * 9, 6 * 9, 0 * 9) = p_{NG}(6, 6, 0)$ , we obtain that

$$p_{NG}(6, 6, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 0, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 0, 6) \leq p_{NG}(6 * 9, 0 * 9, 6 * 9) = p_{NG}(6, 0, 6)$ , we obtain that

$$p_{NG}(6, 0, 6) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For  $x = 0, y = 6, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(0, 6, 6) \leq p_{NG}(0 * 9, 6 * 9, 6 * 9) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 6) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 6, z = 3, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 6, 3) \leq p_{NG}(6 * 9, 6 * 9, 3 * 9) = p_{NG}(6, 6, 3)$ , we obtain that

$$p_{NG}(6, 6, 3) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 6, 3) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 3, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 3, 6) \leq p_{NG}(6 * 9, 3 * 9, 6 * 9) = p_{NG}(6, 3, 6)$ , we obtain that

$$p_{NG}(6, 3, 6) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 3, 6) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 6, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 6, 6) \leq p_{NG}(3 * 9, 6 * 9, 6 * 9) = p_{NG}(3, 6, 6)$ , we obtain that

$$p_{NG}(3, 6, 6) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 6, 6) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 6, z = 3, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 6, 3) \leq p_{NG}(6 * 9, 6 * 9, 3 * 9) = p_{NG}(6, 6, 3)$ , we obtain that

$$p_{NG}(6, 6, 3) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 6, 3) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 6, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 6, 9) \leq p_{NG}(6 * 9, 6 * 9, 9 * 9) = p_{NG}(6, 6, 9)$ , we obtain that

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For  $x = 6, y = 9, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 9, 6) \leq p_{NG}(6 * 9, 9 * 9, 6 * 9) = p_{NG}(6, 9, 6)$ , we obtain that

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For  $x = 3, y = 6, z = 6, a = 9, neut(a) = 9$ ;

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For  $x = 9, y = 9, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 9, 0) \leq p_{NG}(9 * 9, 9 * 9, 0 * 9) = p_{NG}(9, 9, 0)$ , we obtain that

$$p_{NG}(9, 9, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For  $x = 9, y = 0, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 0, 9) \leq p_{NG}(9 * 9, 0 * 9, 9 * 9) = p_{NG}(9, 0, 9)$ , we obtain that

$$p_{NG}(9, 0, 9) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

For  $x = 0, y = 9, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(0, 9, 9) \leq p_{NG}(0 * 9, 9 * 9, 9 * 9) = p_{NG}(0, 9, 9)$ , we obtain that

$$p_{NG}(0, 9, 9) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 9, 9) - p_{NG}(9, 9, 9).$$

For  $x = 9, y = 9, z = 3, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 9, 3) \leq p_{NG}(9 * 9, 9 * 9, 3 * 9) = p_{NG}(9, 9, 3)$ , we obtain that

$$p_{NG}(9, 9, 3) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 9, 3) - p_{NG}(9, 9, 9).$$

For  $x = 9, y = 3, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 3, 9) \leq p_{NG}(9 * 9, 3 * 9, 9 * 9) = p_{NG}(9, 3, 9)$ , we obtain that

$$p_{NG}(9, 3, 9) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 3, 9) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 9, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 9, 9) \leq p_{NG}(3 * 9, 9 * 9, 9 * 9) = p_{NG}(3, 9, 9)$ , we obtain that

$$p_{NG}(3, 9, 9) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 9, 9) - p_{NG}(9, 9, 9).$$

For  $x = 9, y = 9, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 9, 6) \leq p_{NG}(9 * 9, 9 * 9, 6 * 9) = p_{NG}(9, 9, 6)$ , we obtain that

$$p_{NG}(9, 9, 6) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 9, 6) - p_{NG}(9, 9, 9).$$

For  $x = 9, y = 6, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 6, 9) \leq p_{NG}(9 * 9, 6 * 9, 9 * 9) = p_{NG}(9, 6, 9)$ , we obtain that

$$p_{NG}(9, 6, 9) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 6, 9) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 9, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 9, 9) \leq p_{NG}(6 * 9, 9 * 9, 9 * 9) = p_{NG}(6, 9, 9)$ , we obtain that

$$p_{NG}(6, 9, 9) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 9, 9) - p_{NG}(9, 9, 9).$$

For  $x = 0, y = 0, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(0, 0, 0) \leq p_{NG}(0 * 9, 0 * 9, 0 * 9) = p_{NG}(0, 0, 0)$ , we obtain that

$$p_{NG}(0, 0, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 3, z = 3, a = 6, neut(a) = 6$ ;

since  $p_{NG}(3, 3, 3) \leq p_{NG}(3 * 6, 3 * 6, 3 * 6) = p_{NG}(6, 6, 6)$ , we obtain that

$$p_{NG}(6, 6, 6) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For  $x = 6, y = 6, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 6, 6) \leq p_{NG}(6 * 9, 6 * 9, 6 * 9) = p_{NG}(6, 6, 6)$ , we obtain that

$$p_{NG}(6, 6, 6) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 6, 6) - p_{NG}(9, 9, 9).$$

For  $x = 9, y = 9, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 9, 9) \leq p_{NG}(9 * 9, 9 * 9, 9 * 9) = p_{NG}(9, 9, 9)$ , we obtain that

$$p_{NG}(9, 9, 9) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 9, 9) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 0, z = 0, a = 4, neut(a) = 4$ ;

since  $p_{NG}(4, 0, 0) \leq p_{NG}(4 * 4, 0 * 4, 0 * 4) = p_{NG}(4, 0, 0)$ , we obtain that

$$p_{NG}(4, 0, 0) \leq p_{NG}(4, 4, 4) + p_{NG}(4, 0, 0) - p_{NG}(4, 4, 4).$$

For  $x = 4, y = 0, z = 3, a = 6, neut(a) = 6$ ;

since  $p_{NG}(4, 0, 3) \leq p_{NG}h_{dg}(4 * 6, 0 * 6, 3 * 6) = p_{NG}(0, 0, 6)$ , we obtain that

$$p_{NG}(4, 0, 3) \leq p_{NG}(4, 6, 6) + p_{NG}(6, 0, 3) - p_{NG}(6, 6, 6).$$

For  $x = 4, y = 3, z = 0, a = 6, neut(a) = 6$ ;

since  $p_{NG}(4, 3, 0) \leq p_{NG}(4 * 6, 3 * 6, 0 * 6) = p_{NG}(0, 6, 0)$ , we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For  $x = 3, y = 0, z = 4, a = 6, neut(a) = 6$ ;

since  $p_{NG}(3, 0, 4) \leq p_{NG}(3 * 6, 0 * 6, 4 * 6) = p_{NG}(6, 0, 0)$ , we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 0) - p_{NG}(6, 6, 6).$$

For  $x = 3, y = 4, z = 0, a = 6, neut(a) = 6$ ;

since  $p_{NG}(3, 4, 0) \leq p_{NG}(3 * 6, 4 * 6, 0 * 6) = p_{NG}(6, 0, 0)$ , we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 0) - p_{NG}(6, 6, 6).$$

For  $x = 4, y = 0, z = 4, a = 4, neut(a) = 4$ ;

since  $p_{NG}(4, 0, 4) \leq p_{NG}(4 * 4, 0 * 4, 4 * 4) = p_{NG}(4, 0, 4)$ , we obtain that

$$p_{NG}(4, 0, 4) \leq p_{NG}(4, 4, 4) + p_{NG}(4, 0, 4) - p_{NG}(4, 4, 4).$$

For  $x = 4, y = 4, z = 0, a = 4, neut(a) = 4$ ;

since  $p_{NG}(4, 4, 0) \leq p_{NG}(4 * 4, 4 * 4, 0 * 4) = p_{NG}(4, 4, 0)$ , we obtain that

$$p_{NG}(4, 4, 0) \leq p_{NG}(4, 4, 4) + p_{NG}(4, 4, 0) - p_{NG}(4, 4, 4).$$

For  $x = 0, y = 4, z = 4, a = 4, neut(a) = 4$ ;

since  $p_{NG}(0, 4, 4) \leq p_{NG}(0 * 4, 4 * 4, 4 * 4) = p_{NG}(0, 4, 4)$ , we obtain that

$$p_{NG}(0, 4, 4) \leq p_{NG}(0, 4, 4) + p_{NG}(4, 4, 4) - p_{NG}(4, 4, 4).$$

For  $x = 4, y = 0, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 0, 6) \leq p_{NG}(4 * 9, 0 * 9, 6 * 9) = p_{NG}(0, 0, 6)$ , we obtain that

$$p_{NG}(0, 0, 6) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 6, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 6, 0) \leq p_{NG}(4 * 9, 6 * 9, 0 * 9) = p_{NG}(0, 6, 0)$ , we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 4, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 4, 0) \leq p_{NG}(6 * 9, 4 * 9, 0 * 9) = p_{NG}(6, 0, 0)$ , we obtain that

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For  $x = 6, y = 0, z = 4, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 0, 4) \leq p_{NG}(6 * 9, 0 * 9, 4 * 9) = p_{NG}(6, 0, 0)$ , we obtain that

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For  $x = 4, y = 0, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 0, 9) \leq p_{NG}(4 * 9, 0 * 9, 9 * 9) = p_{NG}(0, 0, 9)$ , we obtain that

$$p_{NG}(0, 0, 9) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 9, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 9, 0) \leq p_{NG}(4 * 9, 9 * 9, 0 * 9) = p_{NG}(0, 9, 0)$ , we obtain that

$$p_{NG}(0, 9, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For  $x = 9, y = 0, z = 4, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 0, 4) \leq p_{NG}(9 * 9, 0 * 9, 4 * 9) = p_{NG}(9, 0, 0)$ , we obtain that

$$p_{NG}(9, 0, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For  $x = 9, y = 4, z = 0, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 4, 0) \leq p_{NG}(9 * 9, 4 * 9, 0 * 9) = p_{NG}(9, 0, 0)$ , we obtain that

$$p_{NG}(9, 0, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 3, z = 3, a = 3, neut(a) = 6$ ;

since  $p_{NG}(4, 3, 3) \leq p_{NG}(4 * 6, 3 * 6, 3 * 6) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For  $x = 4, y = 3, z = 4, a = 3, neut(a) = 6$ ;

since  $p_{NG}(4, 3, 4) \leq p_{NG}(4 * 6, 3 * 6, 4 * 6) = p_{NG}(0, 6, 0)$ , we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For  $x = 4, y = 4, z = 3, a = 3, neut(a) = 6$ ;

since  $p_{NG}(4, 4, 3) \leq p_{NG}(4 * 6, 4 * 6, 3 * 6) = p_{NG}(0, 0, 6)$ , we obtain that

$$p_{NG}(0, 0, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 0, 6) - p_{NG}(6, 6, 6).$$

For  $x = 3, y = 4, z = 4, a = 3, neut(a) = 6$ ;

since  $p_{NG}(3, 4, 4) \leq p_{NG}(3 * 6, 4 * 6, 4 * 6) = p_{NG}(6, 0, 0)$ , we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 0) - p_{NG}(6, 6, 6).$$

For  $x = 4, y = 3, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 3, 6) \leq p_{NG}(4 * 9, 3 * 9, 6 * 9) = p_{NG}(0, 3, 6)$ , we obtain that

$$p_{NG}(0, 3, 6) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 3, 6) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 6, z = 3, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 6, 3) \leq p_{NG}(4 * 9, 6 * 9, 3 * 9) = p_{NG}(0, 6, 3)$ , we obtain that

$$p_{NG}(0, 6, 3) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 3) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 4, z = 3, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 4, 3) \leq p_{NG}(6 * 9, 4 * 9, 3 * 9) = p_{NG}(6, 0, 3)$ , we obtain that

$$p_{NG}(6, 0, 3) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 3) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 3, z = 4, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 3, 4) \leq p_{NG}(6 * 9, 3 * 9, 4 * 9) = p_{NG}(6, 3, 0)$ , we obtain that

$$p_{NG}(6, 3, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 3, 0) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 6, z = 4, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 6, 4) \leq p_{NG}(3 * 9, 6 * 9, 4 * 9) = p_{NG}(3, 6, 0)$ , we obtain that

$$p_{NG}(3, 6, 0) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For  $x = 3, y = 4, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(3, 4, 6) \leq p_{NG}(3 * 9, 4 * 9, 6 * 9) = p_{NG}(3, 0, 6)$ , we obtain that

$$p_{NG}(3, 0, 6) \leq p_{NG}(3, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 3, z = 9, a = 6, neut(a) = 6$ ;

since  $p_{NG}(4, 3, 9) \leq p_{NG}(4 * 6, 3 * 6, 9 * 6) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For  $x = 4, y = 9, z = 3, a = 6, neut(a) = 6$ ;

since  $p_{NG}(4, 9, 3) \leq p_{NG}(4 * 6, 9 * 6, 3 * 6) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 6, 6) + p_{NG}(6, 6, 6) - p_{NG}(6, 6, 6).$$

For  $x = 9, y = 4, z = 3, a = 6, neut(a) = 6$ ;

since  $p_{NG}(9, 4, 3) \leq p_{NG}(9 * 6, 4 * 6, 3 * 6) = p_{NG}(6, 0, 6)$ , we obtain that

$$p_{NG}(6, 0, 6) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 6) - p_{NG}(6, 6, 6).$$

For  $x = 9, y = 3, z = 4, a = 6, neut(a) = 6$ ;

since  $p_{NG}(9, 3, 4) \leq p_{NG}(9 * 6, 3 * 6, 4 * 6) = p_{NG}(6, 6, 0)$ , we obtain that

$$p_{NG}(6, 6, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For  $x = 3, y = 9, z = 4, a = 6, neut(a) = 6$ ;

since  $p_{NG}(3, 9, 4) \leq p_{NG}(3 * 6, 9 * 6, 4 * 6) = p_{NG}(6, 6, 0)$ , we obtain that

$$p_{NG}(6, 6, 0) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 6, 0) - p_{NG}(6, 6, 6).$$

For  $x = 3, y = 4, z = 9, a = 6, neut(a) = 6$ ;

since  $p_{NG}(3, 4, 9) \leq p_{NG}(3 * 6, 4 * 6, 9 * 6) = p_{NG}(6, 0, 6)$ , we obtain that

$$p_{NG}(6, 0, 6) \leq p_{NG}(6, 6, 6) + p_{NG}(6, 0, 6) - p_{NG}(6, 6, 6).$$

For  $x = 4, y = 4, z = 4, a = 4, neut(a) = 4$ ;

since  $p_{NG}(4, 4, 4) \leq p_{NG}(4 * 4, 4 * 4, 4 * 4) = p_{NG}(4, 4, 4)$ , we obtain that

$$p_{NG}(4, 4, 4) \leq p_{NG}(4, 4, 4) + p_{NG}(4, 4, 4) - p_{NG}(4, 4, 4).$$

For  $x = 4, y = 4, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 4, 6) \leq p_{NG}(4 * 9, 4 * 9, 6 * 9) = p_{NG}(0, 0, 6)$ , we obtain that

$$p_{NG}(0, 0, 6) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 6, z = 4, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 6, 4) \leq p_{NG}(4 * 9, 6 * 9, 4 * 9) = p_{NG}(0, 6, 0)$ , we obtain that

$$p_{NG}(0, 6, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 4, z = 4, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 4, 4) \leq p_{NG}(6 * 9, 4 * 9, 4 * 9) = p_{NG}(6, 0, 0)$ , we obtain that

$$p_{NG}(6, 0, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 4, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 4, 9) \leq p_{NG}(4 * 9, 4 * 9, 9 * 9) = p_{NG}(0, 0, 9)$ , we obtain that

$$p_{NG}(0, 0, 9) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 9, z = 4, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 9, 4) \leq p_{NG}(4 * 9, 9 * 9, 4 * 9) = p_{NG}(0, 9, 0)$ , we obtain that

$$p_{NG}(0, 9, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For  $x = 9, y = 4, z = 4, a = 9, neut(a) = 9$ ;

since  $p_{NG}(9, 4, 4) \leq p_{NG}(9 * 9, 4 * 9, 4 * 9) = p_{NG}(9, 0, 0)$ , we obtain that

$$p_{NG}(9, 0, 0) \leq p_{NG}(9, 9, 9) + p_{NG}(9, 0, 0) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 6, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 6, 6) \leq p_{NG}(4 * 9, 6 * 9, 6 * 9) = p_{NG}(0, 6, 6)$ , we obtain that

$$p_{NG}(0, 6, 6) \leq p_{NG}(0, 9, 9) + h_{p_{NG}}(9, 6, 6) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 4, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 4, 6) \leq p_{NG}(6 * 9, 4 * 9, 6 * 9) = p_{NG}(6, 0, 6)$ , we obtain that

$$p_{NG}(6, 0, 6) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 6) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 6, z = 4, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 6, 4) \leq p_{NG}(6 * 9, 6 * 9, 4 * 9) = p_{NG}(6, 6, 0)$ , we obtain that

$$p_{NG}(6, 6, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 6, 0) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 6, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 6, 9) \leq p_{NG}(4 * 9, 6 * 9, 9 * 9) = p_{NG}(0, 6, 9)$ , we obtain that

$$p_{NG}(0, 6, 9) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 6, 9) - p_{NG}(9, 9, 9).$$

For  $x = 4, y = 9, z = 6, a = 9, neut(a) = 9$ ;

since  $p_{NG}(4, 9, 6) \leq p_{NG}(4 * 9, 9 * 9, 6 * 9) = p_{NG}(0, 9, 0)$ , we obtain that

$$p_{NG}(0, 9, 0) \leq p_{NG}(0, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 9, z = 4, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 9, 4) \leq p_{NG}(6 * 9, 9 * 9, 4 * 9) = p_{NG}(6, 9, 0)$ , we obtain that

$$p_{NG}(6, 9, 0) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 9, 0) - p_{NG}(9, 9, 9).$$

For  $x = 6, y = 4, z = 9, a = 9, neut(a) = 9$ ;

since  $p_{NG}(6, 4, 9) \leq p_{NG}(6 * 9, 4 * 9, 9 * 9) = p_{NG}(6, 0, 9)$ , we obtain that

$$p_{NG}(6, 0, 9) \leq p_{NG}(6, 9, 9) + p_{NG}(9, 0, 9) - p_{NG}(9, 9, 9).$$

Therefore,  $p_{NG}$  is a NTpgM.

### Corollary 3.3:

1) The NTpgMS differs from the pgMS. Because, there is not a  $*$  binary operation in pgMS. Also, triangle inequalities are different in this spaces.

2) The NTpgMS differs from the NTMS due to triangle inequalities.

3) The NTpgMS differs from the NTgMS. Because the triangle inequality in the NTgMS differs from the triangle inequality in the NTpgMS. Also, in a NTpgMS, it can be that  $p_{NG}(x, x) \neq 0$ .

**Theorem 3.4:** Let  $((X, *), p_{NG})$  be a NTpgMS and  $d_p: X \times X \rightarrow R^+ \cup \{0\}$  be a function such that

$$d_p(x, y) = p_{NG}(x, y, y) + p_{NG}(x, x, y). \text{ Then, } d_p \text{ is a NTpM.}$$

### Proof:

i) Since  $((X, *), p_{NG})$  is a NTpgMS, it is clear that for  $\forall x, y \in X; x * y \in X$ .

ii) Since  $p_{NG}$  is a NTpgMS,  $0 \leq d_p(x, x) \leq d_p(x, y)$  implies that

$$0 \leq p_{NG}(x, x, x) + p_{NG}(x, x, x) \leq p_{NG}(x, y, y) + p_{NG}(x, x, y).$$

iii) Since  $p_{NG}$  is a NTpgMS, if  $d_p(x, x) = d_p(y, y) = d_p(x, y) = 0$ , then we obtain  $x = y$ .

iv) Since  $p_{NG}$  is a NTpgMS, we obtain

$$d_p(x, y) = p_{NG}(x, y, y) + p_{NG}(x, x, y) = p_{NG}(y, x, x) + p_{NG}(y, y, x) = d_p(y, x).$$

v) We assume that there exists at least an element  $a \in X$  for each  $x, y$  and  $z$  such that  $p_{NG}(x, y, z) \leq p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a))$ . Thus, if we assume  $a = x$ , It is clear that  $d_p(x, y) \leq d_p(x, y * \text{neut}(a))$ .

Also, since  $((X, *), p_{NG})$  is a NTpgMS, it is obvious that

$$p_{NG}(x, y, z) \leq p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z) - p_{NG}(a, a, a).$$

Hence, we obtain

$$p_{NG}(x, y, y) + p_{NG}(x, x, y) \leq$$

$$p_{NG}(x, a, a) + p_{NG}(x, x, a) + p_{NG}(x, x, a) + p_{NG}(a, y, y) + p_{NG}(a, a, y) - p_{NG}(a, a, a) - p_{NG}(a, a, a).$$

Thus, we have  $d_p(x, y) \leq d_p(x, a) + d_p(a, y) - d_p(a, a)$ .

**Theorem 3.5:** Let  $((X, *), p_{NG})$  be a NTpgMS. If for all  $x \in X$ ,  $p_{NG}(x, x, x) = 0$ , then  $((X, *), p_{NG})$  is a NTgMS.

**Proof:** We suppose that  $(X, *)$  is a NTS and  $((X, *), p_{NG})$  is a NTpgMS.

i) Since  $((X, *), p_{NG})$  is a NTpgMS; then for all  $x, y \in X$ ;  $x * y \in X$ .

ii) Since  $p_{NG}(x, x, x) = 0$ , it is clear that  $0 \leq p_{NG}(x, x, x) = 0 \leq p_{NG}(x, y, z)$ .

iii) Since  $((X, *), p_{NG})$  is a NTpgMS, it is clear that if  $x \neq y$ , then  $p_{NG}(x, y, z) > 0$ .

iv) Since  $((X, *), p_{NG})$  is a NTpgMS, it is clear that if  $y \neq z$ , then  $p_{NG}(x, x, y) \leq p_{NG}(x, y, z)$ .

v) Since  $((X, *), p_{NG})$  is a NTpgMS, it is clear that

$$p_{NG}(x, y, z) = p_{NG}(x, z, y) = p_{NG}(y, x, z) = p_{NG}(y, z, x) = p_{NG}(z, x, y) = p_{NG}(z, y, x).$$

vi) We assume that there exists at least an element  $a \in X$  for each  $x, y, z$  such that

$$p_{NG}(x, y, z) = p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)), \text{ then}$$

$$p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z) - p_{NG}(a, a, a).$$

Since  $p_{NG}(x, x, x) = 0$ , we obtain that

$$p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z).$$

Thus,  $((X, *), p_{NG})$  is a NTgMS.

**Theorem 3.6:** Let  $((X, *), d_{NG})$  be a NTgM. Then, the function  $p_{NG}(x, y, z) = d_{NG}(x, y, z) + k$ ,  $k \in \mathbb{R}^+$  is an NTpgMS.

**Proof:**

i) Since  $d_{NG}$  is a NTgMS, for all  $x, y \in X$ ;  $x * y \in X$ .

ii) Since  $d_{NG}$  is a NTgMS, we obtain  $d_{NG}(x, x, x) \leq d_{NG}(x, y, z)$ . Thus, it is clear that

$$p_{NG}(x, x, x) = d_{NG}(x, x, x) + k \leq p_{NG}(x, y, z) = d_{NG}(x, y, z) + k.$$

iii)  $p_{NG}(x, y, z) = d_{NG}(x, y, z) + k > 0$ .

iv) Since  $d_{NG}$  is a NTgMS, if  $y \neq z$ , then  $d_{NG}(x, x, y) \leq d_{NG}(x, y, z)$ . Thus, it is clear that

$$p_{NG}(x, x, y) = d_{NG}(x, x, y) + k \leq p_{NG}(x, y, z) = d_{NG}(x, y, z) + k$$

v) Since  $d_{NG}$  is a NTgMS, we obtain

$$d_{NG}(x, y, z) = d_{NG}(x, z, y) = d_{NG}(y, x, z) = d_{NG}(y, z, x) = d_{NG}(z, x, y) = d_{NG}(z, y, x) . \text{ Thus, it is clear that}$$

$$d_{NG}(x, y, z) + k = d_{NG}(x, z, y) + k = d_{NG}(y, x, z) + k = d_{NG}(y, z, x) + k = d_{NG}(z, x, y) + k = d_{NG}(z, y, x) + k.$$

Therefore,

$$p_{NG}(x, y, z) = p_{NG}(x, z, y) = p_{NG}(y, x, z) = p_{NG}(y, z, x) = p_{NG}(z, x, y) = p_{NG}(z, y, x) .$$

vi) We assume that there exists at least an  $a \in X$  for each  $x, y, z \in X$  such that

$$d_{NG}(x, y, z) \leq d_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)). \text{ Thus, we obtain}$$

$$p_{NG}(x, y, z) = d_{NG}(x, y, z) + k \leq$$

$$p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) = d_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) + k. \tag{1}$$

Also, since  $d_{NG}$  is a NTgMS, we obtain

$$d_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq d_{NG}(x, a, a) + d_{NG}(a, y, z). \text{ Therefore, we obtain}$$

$$d_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) + k \leq d_{NG}(x, a, a) + k + d_{NG}(a, y, z) + k - k. \text{ Thus,}$$

$$p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z) - k =$$

$$p_{NG}(x * \text{neut}(a), y * \text{neut}(a), z * \text{neut}(a)) \leq p_{NG}(x, a, a) + p_{NG}(a, y, z) - p_{NG}(a, a, a). \tag{2}$$

From (1) and (2), this condition is hold.

In this case,  $((X, *), p_{NG})$  is called a NTpgMS.

**Corollary 3.7:** A NTpgMS can be obtained from a NTgMS.

**Definition 3.8:** Let  $((X, *), p_{NG})$  be a NTpgMS and  $\{x_n\}$  be a sequence in this space. A point  $x \in X$  is said to be the limit of the sequence  $\{x_n\}$ , if  $\lim_{n,m \rightarrow \infty} p_{NG}(x, x_n, x_m) - p_{NG}(x, x, x) = 0$  and  $\{x_n\}$  is called a NT pg - convergent to  $x$ .

**Definition 3.9:** Let  $((X, *), p_{NG})$  be a NTpgMS and  $\{x_n\}$  be a sequence in this space.  $\{x_n\}$  is called a NT pg - Cauchy sequence if there exists at least a  $x \in X$  such that  $\lim_{n,m,l \rightarrow \infty} p_{NG}(x_n, x_m, x_l) - p_{NG}(x, x, x) = 0$ .

**Definition 3.10:** Let  $((X, *), p_{NG})$  be a NTpgMS. If every  $\{x_n\}$  NT pg - Cauchy sequence is a NT pg - convergent, then  $((X, *), p_{NG})$  is called a NT complete NTpgMS.

### Conclusion

In this study we first obtained NTpgMS. We show that NTpgMS is different from pgMS, NTgMS and NTMS. Also, we show that a NTpgMS will provide the properties of a NTgMS under which conditions are met. Thus, we added a new structure to neutrosophic triple structures. Also, thanks to neutrosophic triplet partial g - metric space, researchers can obtain new fixed point theorems for neutrosophic triplet theory and neutrosophic triplet partial g - normed space, neutrosophic triplet partial g - inner product space.

### Abbreviations

gM: g - metric  
 gMS: g - metric space  
 NT: Neutrosophic triplet  
 NTS: Neutrosophic triplet set  
 NTM: Neutrosophic triplet metric  
 NTMS: Neutrosophic triplet metric space  
 NTpM: Neutrosophic triplet partial metric  
 NTpMS: Neutrosophic triplet partial metric space  
 NTgM: Neutrosophic triplet g - metric  
 NTgMS: Neutrosophic triplet g - metric space  
 NTpgM: Neutrosophic triplet partial g - metric  
 NTpgMS: Neutrosophic triplet partial g - metric space

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