Tipping point analysis of geophysical data

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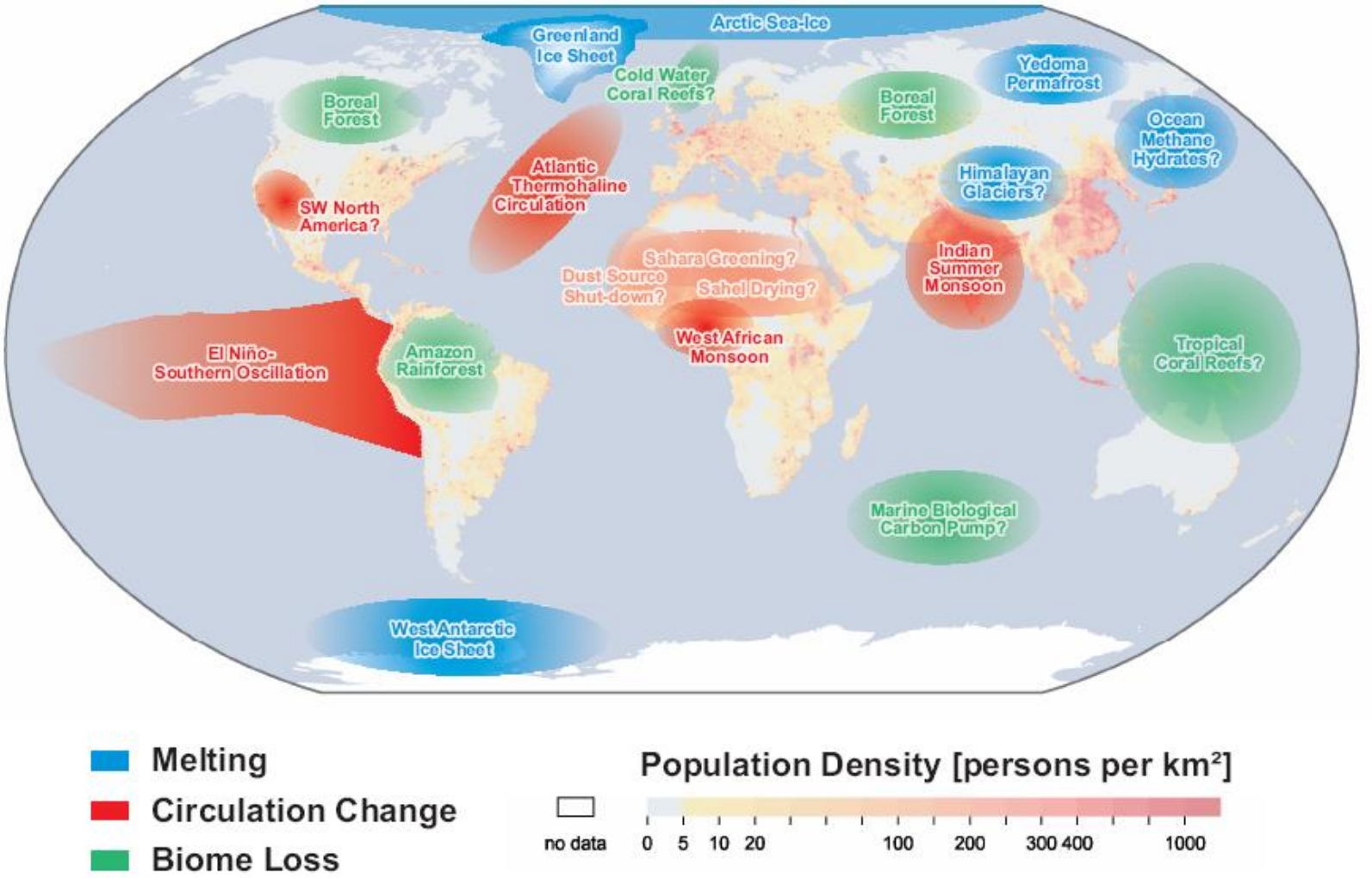
Tipping elements in the climate system



Lenton et al. PNAS, 2008

Tipping point toolbox (being developed since 2007)

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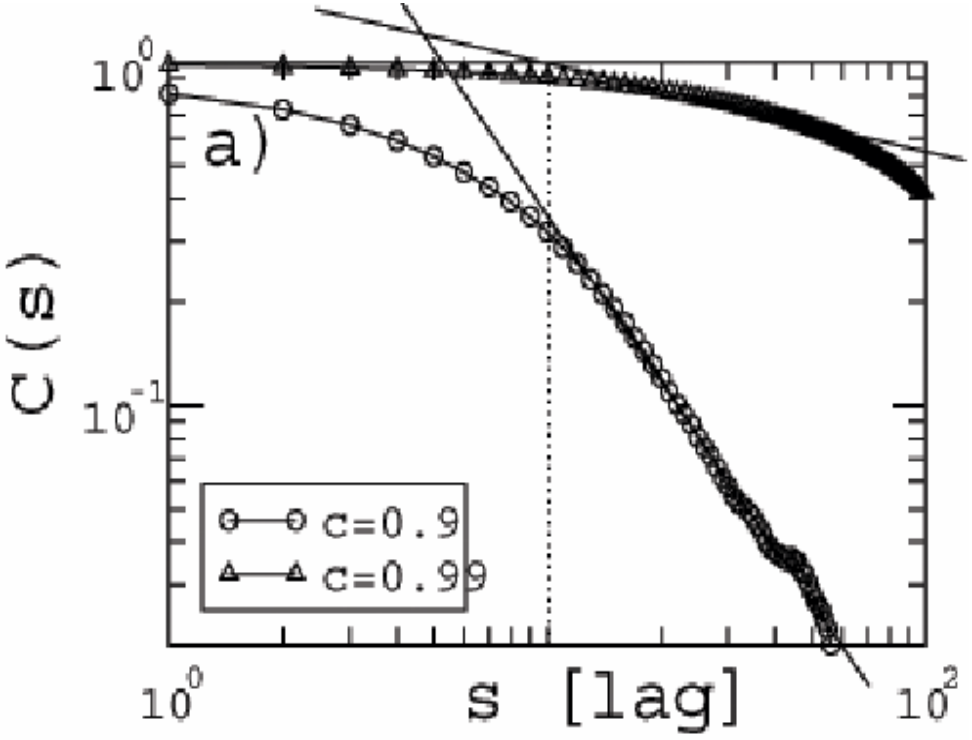
- Anticipating: early warning of climate tipping points (pre-tipping)
- Detecting: potential analysis (tipping)
- Forecasting: PDF & potential analysis, recently added bayesian techniques (post-tipping)

Early warning signals

Early warning signal model



Livina & Lenton, GRL 2007

Series is approximated by an AR(1) process, and exponential decay of the auto-correlation function (ACF) is estimated. Thus ACF-indicator c is defined; its gradual trend towards value 1 indicates critical behaviour.

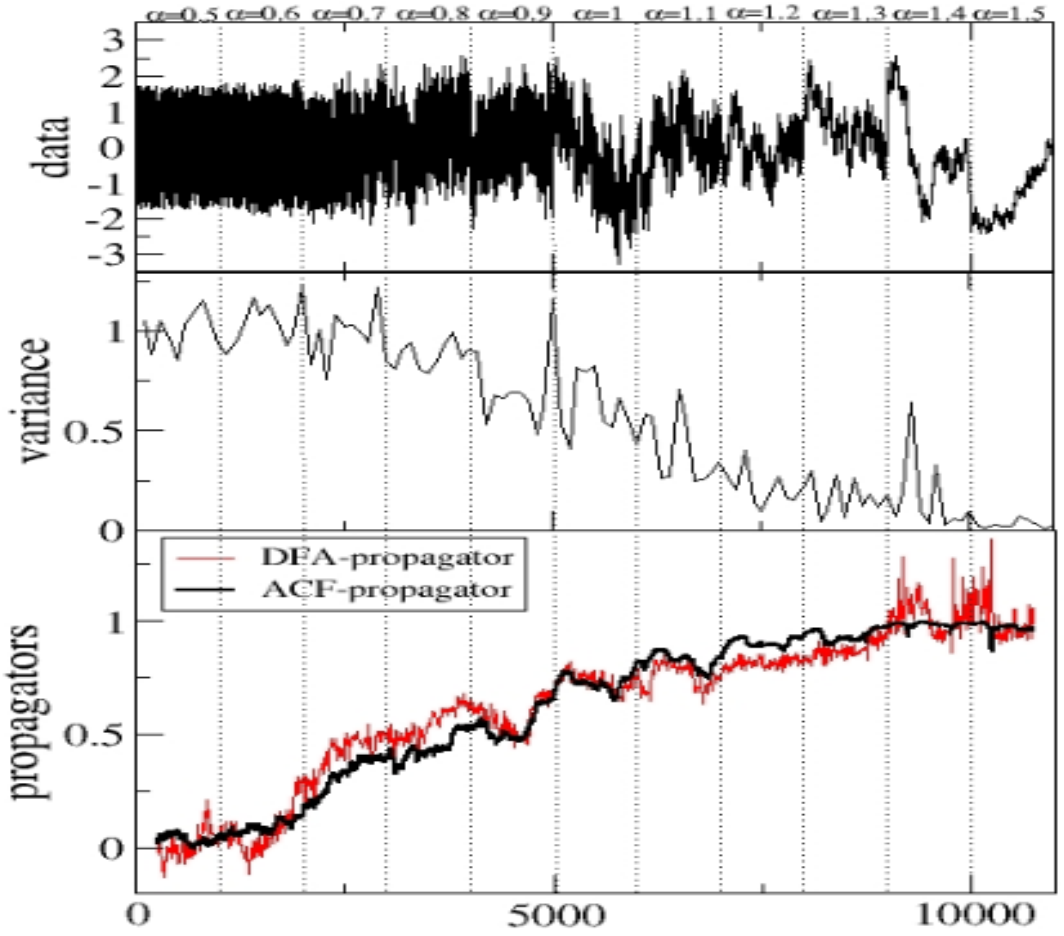
$$y_{n+1} = cy_n + \sigma\eta_n,$$

$c = \exp(-\kappa\Delta t)$, κ is decay rate

($\kappa = 0$ when $c = 1$)

Artificial data with increasing memory

Livina et al, Physica A, 2012



When ACF-indicator reaches critical value 1, DFA-indicator is still capable to reflect the variability in the variance

Detecting tipping

Potential analysis model

$$\dot{z}(t) = -U'(z) + \sigma\eta$$

$$U(z) = a_4 z^4 + a_3 z^3 + a_2 z^2 + a_1 z$$

double-well potential

Kwasniok & Lohmann, Phys Rev E, 2009

Livina et al, Climate of the Past 2010

Potential & Probability Density Function (PDF)

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Fokker-Planck equation

$$\partial_t p(z,t) = \partial_z [U'(z)\, p(z,t)] + \frac{1}{2}\sigma^2 \partial_z^2 p(z,t)$$

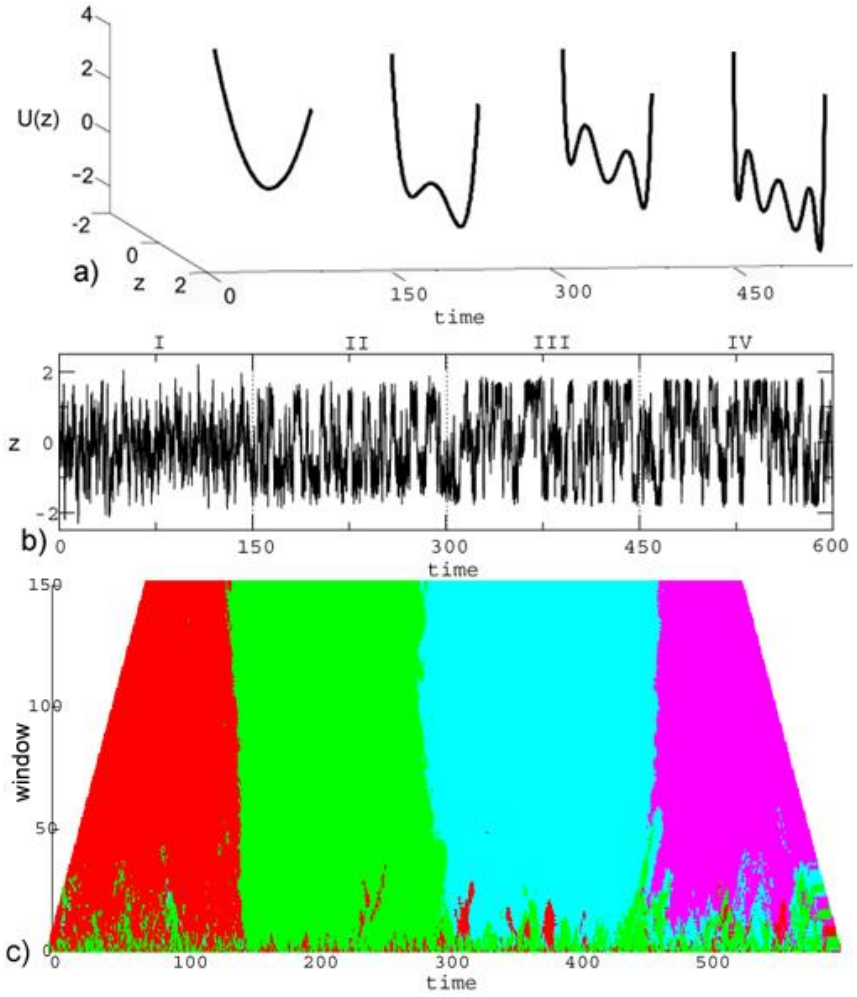
$$p(z) \approx \exp[-2U(z)/\sigma^2]$$

If we assume that the considered subset of data is stationary, then

$$U = -\frac{\sigma^2}{2} \log p_d$$

bimodal histogram $\Leftrightarrow$ double-well potential

AD with four potentials



Potential contour plot at different time scales

We generate artificial data using Euler scheme

$$x_{t+\Delta t} \approx x_t - \left.\frac{dU}{dx}\right|_t \cdot \Delta t + (W_{t+\Delta t} - W_t)$$

W is a Wiener process

Potentials:

$$U(z) = z^2$$

$$U(z) = z^4 - 2z^2$$

$$U(z) = z^6 - 4.5z^4 + 5z^2$$

$$U(z) = z^8 - 6.5z^6 + 13z^4 - 8z^2$$

GRIP & NGRIP temperature proxies

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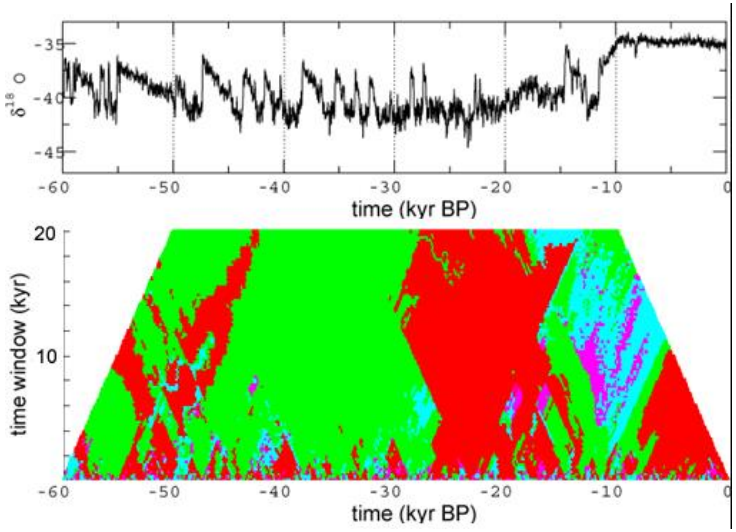
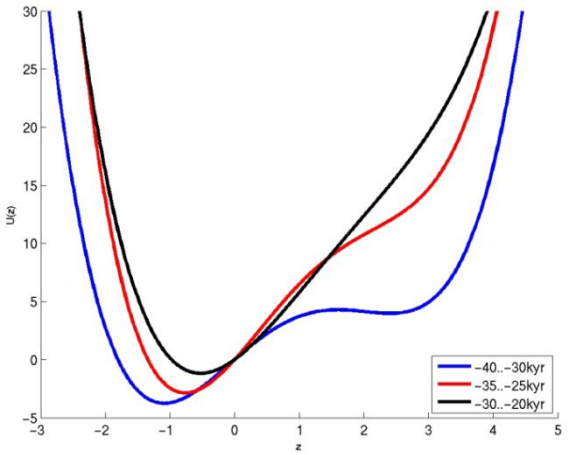
$\delta^{18}O$ data: bifurcation at 25-28 kyr BP

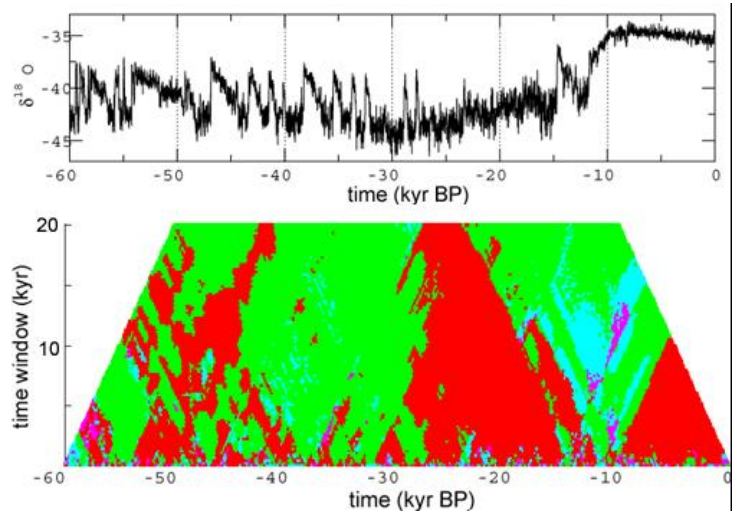
(Livina et al. Climate of the past, 2010)

GRIP



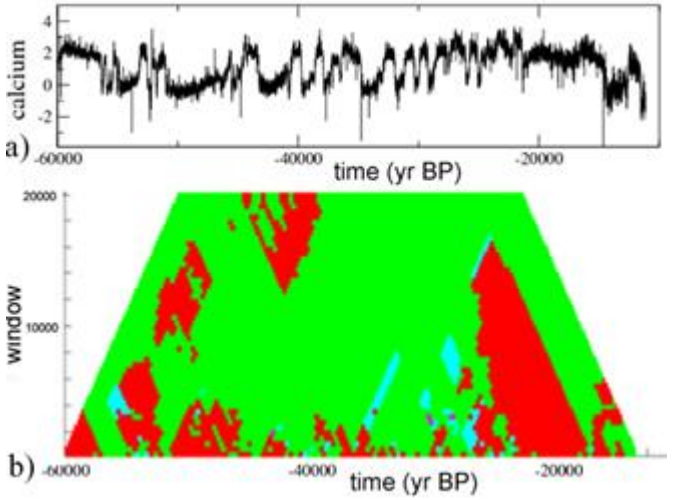


NGRIP



Calcium data: bifurcation at 27-28 kyr BP

GRIP



GICC05 time scale, resolution 20yr

Annual resolution

Forecasting

Potential forecast algorithm

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Livina et al, Physica A 2013

- Collect coefficients of Chebyshev approximation of PDF in sliding windows
- Extrapolate series of the coefficients
- Reconstruct forecast PDF
- Simulate time series from the obtained PDF (rejection sampling)
- Sort the series according to historic data (taking into account seasonality)

Potential forecast of bifurcating artificial data

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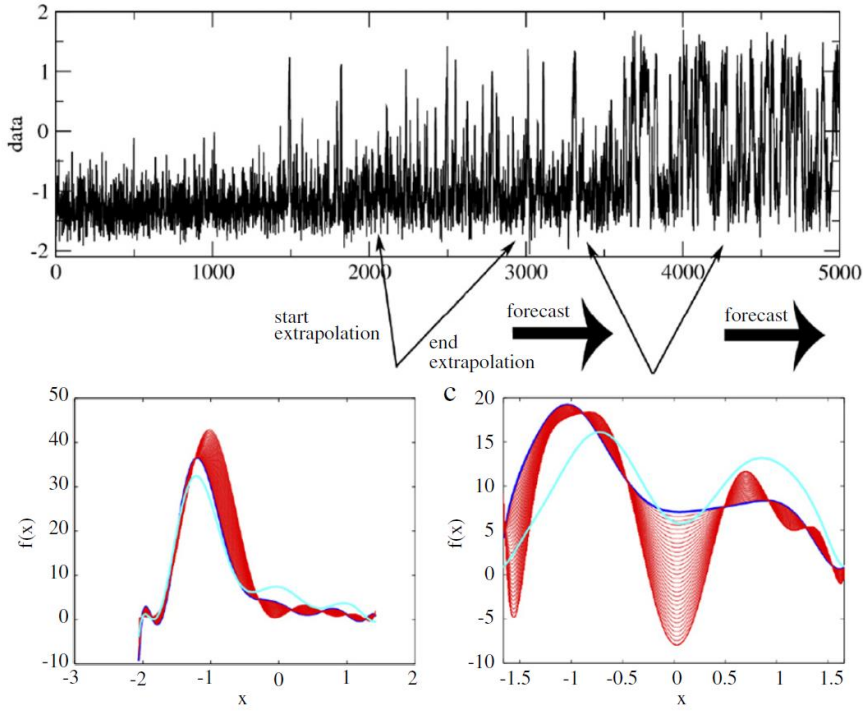
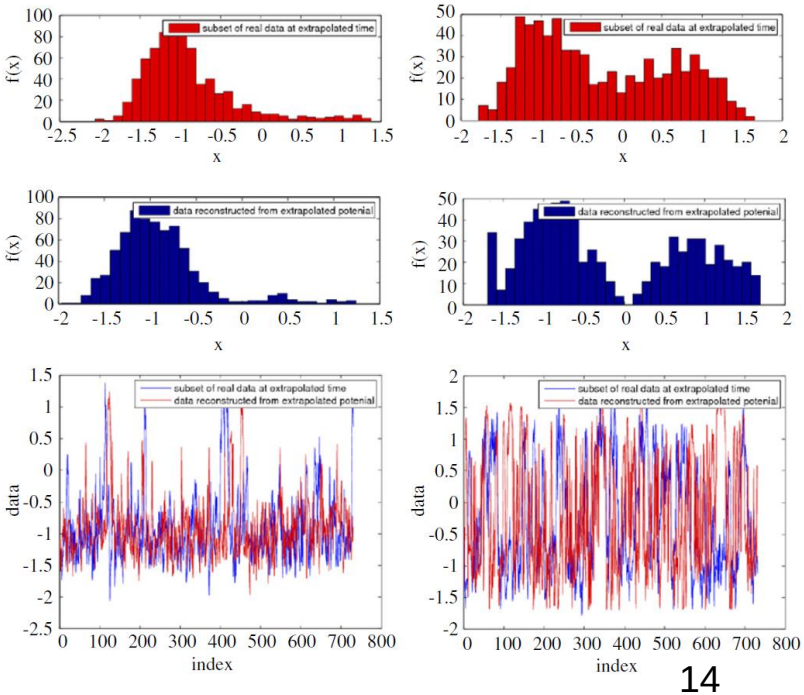


Two hindcasts shown: first collecting coefficients, then extrapolating and generating forecast time series



Extrapolated probability densities, histograms after extrapolation (actual and forecast) and time series for comparison (actual and forecast)

Summary

- Three modules of the tipping point toolbox: anticipating of tipping points using early warning signal indicators; detecting tipping points using potential analysis; potential forecasting of tipping points
- Analysed various artificial datasets and real/reconstructed data in paleoclimate, current climate, ecology, structure health monitoring, sensor data failure detection, etc
- Publications on tipping point analysis:

1) Livina & Lenton, GRL 2007
2) Lenton et al, PhilTrans RoyalSoc 2009
3) Livina et al, Climate of the Past 2010
4) Vaz Martins et al, Phys Rev E 2010
5) Livina et al, Climate Dynamics 2011
6) Lenton et al, PhilTrans RoyalSoc 2012
7) Lenton et al, CoP 2012
8) Livina et al, Physica A 2012
9) Dakos et al, PLOS ONE 2012
10) Livina and Lenton, Cryosphere 2013
11) Cimatoribus, CoP 2013
12) Drijfhout et al, PNAS 2013
13) Livina et al, Physica A, 2013
14) Livina et al, JCSHM 2014
15) Kefi et al, PLoS ONE 2014
16) Livina et al, Chaos 2015
17) Perry et al, SMS 2016
18) Livina et al, submitted
19) Prettyman et al, submitted

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Thank you