Ministry of Basic and Senior Secondary Education

Lesson Plans for

Senior Secondary *Mathematics Revision*

PART II

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Please see final page for further information.

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.

Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, <u>DO NOT WRITE IN THE BOOKS</u>.

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Introduction

to the Lesson Plans

These lesson plans are based on the National Curriculum and the West Africa Examination Council syllabus guidelines, and meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1 The lesson plans will not take the whole term, so use spare time to review material or prepare for examinations.

2 Teachers can use other textbooks alongside or instead of these lesson plans.

3 Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.

4 Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.

Learning outcomes

Preparation

5 If there is time, quickly review what you taught last time before starting each lesson.

6 Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.

7 Lesson plans have a mix of activities for the whole class and for individuals or in pairs.

8 Use the board and other visual aids as you teach.

9 Interact with all pupils in the class – including the quiet ones.

10 Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiners' Reports, as well as input from their examiners and Sierra Leonean teachers.

Using this book

The purpose of this SSS4 Lesson Plans is to review the material that pupils had learned in previous years of schooling, and prepare them for the West African Senior Secondary Certificate Examination (WASSCE).

This book has enough materials for 2 full terms. Depending on your school schedule, you may not have time to teach each lesson and complete each mock exam. Plan your time accordingly, and teach the topics that your pupils need the most review of. It is helpful to assess your pupils at the beginning of the academic year to understand which topics they need to review. This can be done by giving them a short exam similar to the WASSCE exam, with various topics from the syllabus.

There are 8 mock exams provided at the end of this book, in lessons 89 through 96. These are designed to be used in a 40-minute lesson. The exams are shortened so that pupils will have enough time to complete each problem that is similar to the time they will have in the exam. Each lesson plan includes tips for administering the mock exam and preparing pupils to sit the WASSCE exam. It is not necessary to administer the mock exams consecutively, or to wait until the end of the academic year to administer them. You may choose to administer mock exams throughout the year. If your school has additional time for mock exams, design your own exams similar in style to the mock exams in this book, using topics from across the curriculum.

Using the lesson plans

At the SSS level, it is generally better to keep explanations of content brief. Pupils have an overview of each topic in the Pupil Handbook. Spend most of the class time allowing pupils to solve problems in the Teaching and Learning and Practice sections of the lesson plans. If pupils have a good understanding of the topic, ask them to work independently. You may also ask them to solve some problems with seatmates before working independently to solve the rest.

Preparing pupils for the exam

It is important that candidates understand what to expect on the day of the WASSCE exam. Details of the exam are given below. Make sure this is clear to your pupils, and that they are well prepared.

Content of the WASSCE Exam

The WASSCE Mathematics exam consists of 3 sections as described below:

Paper 1 – Multiple Choice

- Paper 1 is 1.5 hours, consists of 50 multiple choice questions, and is worth 50 marks. This gives 1.8 minutes per problem, so time must be planned accordingly.
- The questions are drawn from all topics on the WASSCE syllabus.

Paper 2 – Essay Questions

- Paper 2 consists of 13 essay questions in 2 sections – 2A and 2B.
- Paper 2 is worth 100 marks in total.

- Pupils will be required to answer 10 essay questions in all, across the 2 sections.
- This is an average of 15 minutes per question. However, keep in mind that section 2B is more complicated, so plan your time accordingly.

Paper 2A – Compulsory Questions
- Paper 2A is worth 40 marks.
- There are 5 **compulsory** essay questions in paper 2A.
- Compulsory questions often have multiple parts (a, b, c, …). The questions may not be related to each other. Each part of the question should be completed.
- The questions in 2A are simpler than in 2B, generally requiring fewer steps.
- The questions in 2A are drawn from the common area of the WASSCE syllabus (they are not on topics that are specific to certain countries).

Paper 2B – Advanced Questions
- Paper 2B is worth 60 marks. There are 8 essay questions in paper 2B, and candidates are expected to answer 5 of them.
- Questions in section 2B are of a greater length and difficulty than section 2A.
- A maximum of 2 questions (from among the 8) may be drawn from parts of the WASSCE syllabus that are not meant for Sierra Leone. These topics are not in the Sierra Leone national curriculum for secondary schools. Candidates from Sierra Leone may choose to answer such questions, but it is not required.
- Choose 5 questions on topics that you are more comfortable with.

Exam Day
- Candidates should bring a pencil, geometry set, and scientific calculator to the WASSCE exam.
- Candidates are allowed to use log books (logarithm and trigonometry tables), which are provided in the exam room.

Exam-taking skills and strategies
- Candidates should read and follow the instructions carefully. For example, it may be stated that a trigonometry table should be used. In this case, it is important that a table is used and not a calculator.
- Plan your time. Do not spend too much time on one problem.
- For essay questions, show all of your working on the exam paper. Examiners can give some credit for rough working. Do not cross out working.
- If you complete the exam, take the time to check your solutions. If you notice an incorrect answer, double check it before changing it.
- For section 2B, it is a good idea to skim read the questions quickly before getting started. If you see a topic that you are familiar and comfortable with, work on those problems. Try not to spend a lot of time deciding which problems to solve, or thinking about problems you will not solve.

FACILITATION STRATEGIES

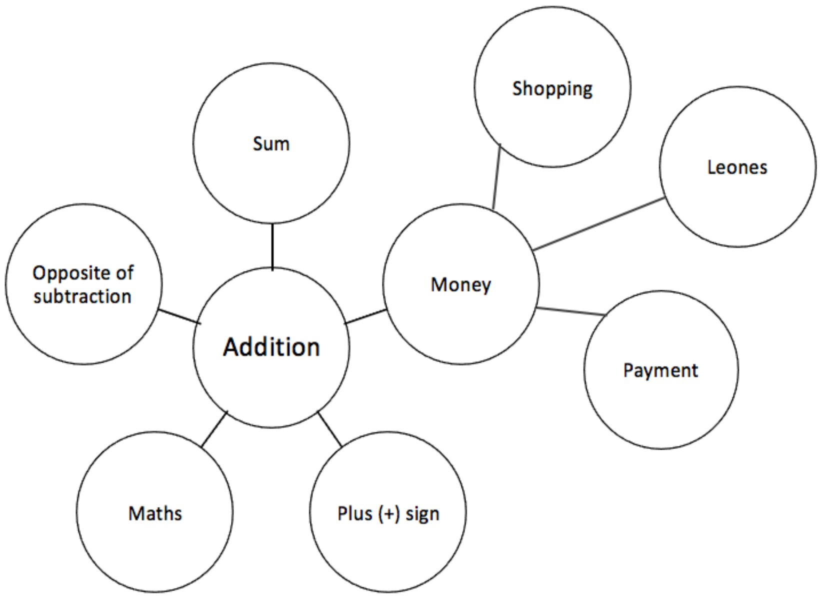
This section includes a list of suggested strategies for facilitating specific classroom and evaluation activities. These strategies were developed with input from national experts and international consultants during the materials development process for the Lesson Plans and Pupils' Handbooks for Senior Secondary Schools in Sierra Leone.

Strategies for introducing a new concept

- **Unpack prior knowledge:** Find out what pupils know about the topic before introducing new concepts, through questions and discussion. This will activate the relevant information in pupils' minds and give the teacher a good starting point for teaching, based on pupils' knowledge of the topic.
- **Relate to real-life experiences:** Ask questions or discuss real-life situations where the topic of the lesson can be applied. This will make the lesson relevant for pupils.
- **K-W-L:** Briefly tell pupils about the topic of the lesson, and ask them to discuss 'What **I** **k**now' and 'What **I** **w**ant to know' about the topic. At the end of the lesson have pupils share 'What **I** **l**earned' about the topic. This strategy activates prior knowledge, gives the teacher a sense of what pupils already know and gets pupils to think about how the lesson is relevant to what they want to learn.
- **Use teaching aids from the environment:** Use everyday objects available in the classroom or home as examples or tools to explain a concept. Being able to relate concepts to tangible examples will aid pupils' understanding and retention.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, can be used to activate prior knowledge and engage pupils in the content which is going to be taught in the lesson.

Strategies for reviewing a concept in 3-5 minutes

- **Mind-mapping:** Write the name of the topic on the board. Ask pupils to identify words or phrases related to the topic. Draw lines from the topic to other related words. This will create a 'mind-map', showing pupils how the topic of the lesson can be mapped out to relate to other themes. Example below:



- **Ask questions:** Ask short questions to review key concepts. Questions that ask pupils to summarise the main idea or recall what was taught is an effective way to review a concept quickly. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, is an effective way to review concepts as a whole group.
- **Matching:** Write the main concepts in one column and a word or a phrase related to each concept in the second column, in a jumbled order. Ask pupils to match the concept in the first column with the words or phrases that relate to in the second column.

Strategies for assessing learning without writing

- **Raise your hand:** Ask a question with multiple-choice answers. Give pupils time to think about the answer and then go through the multiple-choice options one by one, asking pupils to raise their hand if they agree with the option being presented. Then give the correct answer and explain why the other answers are incorrect.
- **Ask questions:** Ask short questions about the core concepts. Questions which require pupils to recall concepts and key information from the lesson are an effective way to assess understanding. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Think-pair-share:** Give pupils a question or topic and ask them to turn to seatmates to discuss it. Then, have pupils volunteer to share their ideas with the rest of the class.
- **Oral evaluation:** Invite volunteers to share their answers with the class to assess their work.

Strategies for assessing learning with writing

- **Exit ticket:** At the end of the lesson, assign a short 2-3 minute task to assess how much pupils have understood from the lesson. Pupils must hand in their answers on a sheet of paper before the end of the lesson.
- **Answer on the board:** Ask pupils to volunteer to come up to the board and answer a question. In order to keep all pupils engaged, the rest of the class can also answer the question in their exercise books. Check the answers together. If needed, correct the answer on the board and ask pupils to correct their own work.
- **Continuous assessment of written work:** Collect a set number of exercise books per day/per week to review pupils' written work in order to get a sense of their level of understanding. This is a useful way to review all the exercise books in a class which may have a large number of pupils.
- **Write and share:** Have pupils answer a question in their exercise books and then invite volunteers to read their answers aloud. Answer the question on the board at the end for the benefit of all pupils.
- **Paired check:** After pupils have completed a given activity, ask them to exchange their exercise books with someone sitting near them. Provide the answers, and ask pupils to check their partner's work.
- **Move around:** If there is enough space, move around the classroom and check pupils' work as they are working on a given task or after they have completed a given task and are working on a different activity.

Strategies for engaging different kinds of learners

- For pupils who progress faster than others:
  - Plan extension activities in the lesson.
  - Plan a small writing project which they can work on independently.
  - Plan more challenging tasks than the ones assigned to the rest of the class.
  - Pair them with pupils who need more support.

- For pupils who need more time or support:
  - Pair them with pupils who are progressing faster, and have the latter support the former.
  - Set aside time to revise previously taught concepts while other pupils are working independently.
  - Organise extra lessons or private meetings to learn more about their progress and provide support.
  - Plan revision activities to be completed in the class or for homework.
  - Pay special attention to them in class, to observe their participation and engagement.

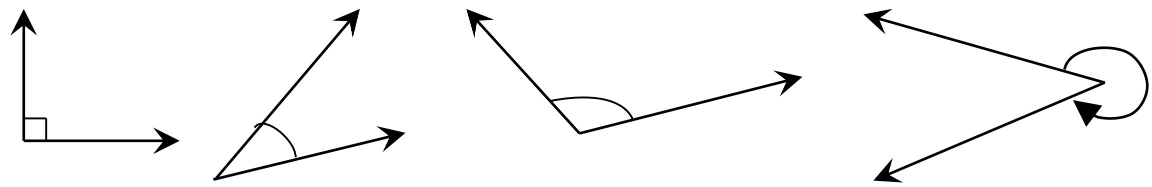
Lesson Title: Measuring angles	**Theme:** Geometry	
Lesson Number: M4-L049	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify various types of angles (acute, obtuse, right, reflex, straight). 2. Measure angles using a protractor.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Bring a protractor and ask pupils to bring protractors (either real or paper). You may make a large protractor from paper and use it for demonstrations on the board.	

Opening *(1 minute)*

1. Discuss: What types of angles do you know of? What are their characteristics? (Example answer: Acute angle, which is less than 90°.)
2. Explain that today's lesson is on solving for angles in triangles.

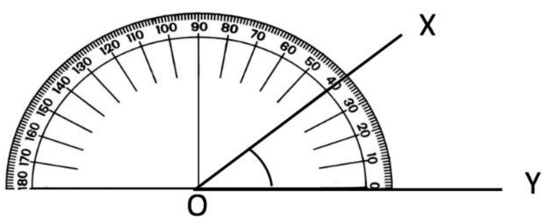
Teaching and Learning *(19 minutes)*

1. Write the following on the board and explain:

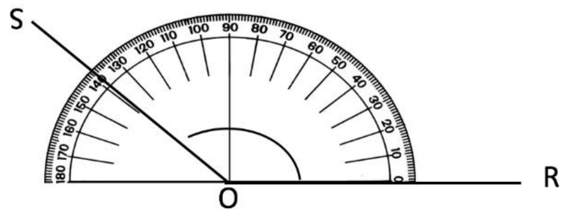


Right angle	**Acute angle**	**Obtuse angle**	**Reflex angle**
90°	less than 90°	greater than 90° and less than 180°	greater than 180° and less than 360°

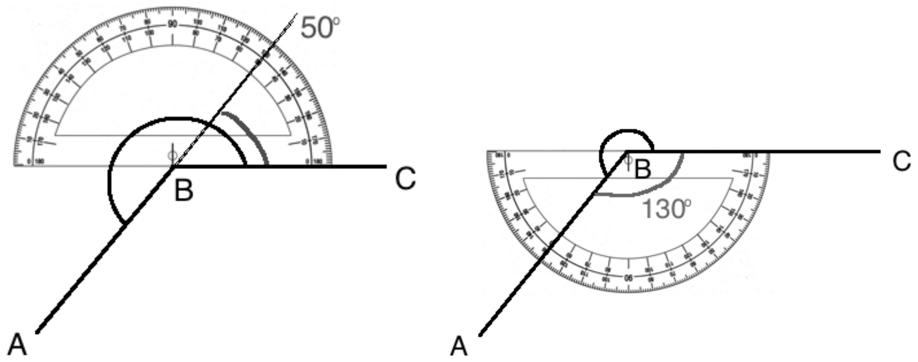
2. Discuss:
   - How many degrees are there in a full rotation? (Answer: 360°)
   - What is the name for to tool used to measure angles? (Answer: protractor)
3. Draw any acute angle on the board, and label it XOY. Demonstrate how to use the protractor to measure XOY (note that the diagram may be a different angle than your own) :
   - Place the centre of the protractor at O. One line of the angle is along the base line of the protractor, at 0°.
   - The other line of the angle gives the angle measure. Read the measure of your angle.
   - Label your angle on the board.



4. Draw an obtuse angle and ask a volunteer to come to the board and measure it. Ask them to label it with its measure. Example:



5. Discuss and allow pupils to share ideas: How would you measure a reflex angle?
6. Explain:
    - One way to measure a reflex angle is to extend one of the lines. We know that a straight line forms 180°. Measure the rest of the angle, and add it to 180°.
    - Another way is to measure the corresponding acute or obtuse angle, and subtract it from the full rotation, 360°.
7. Demonstrate both methods on the board. $\angle ABC$ is shown below as an example.
    - $\angle ABC$ is formed by 180° and 50°. Therefore, $\angle ABC = 180° + 50° = 230°$
    - $\angle ABC$ forms a full rotation with 130°. Therefore, $\angle ABC = 360° - 130° = 230°$.

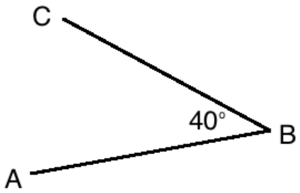


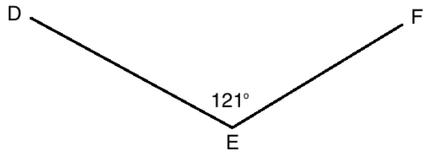
8. Write the problems on the board:
    a. Draw an acute angle and label it ABC. Use a protractor to measure it, then label it with its measure.
    b. Draw an obtuse angle and label it DEF. Use a protractor to measure it, then label it with its measure.
    c. Draw a reflex angle and label it XYZ. Use a protractor to measure it, then label it with its measure.
    d. Use your protractor to draw a right angle.
9. Ask pupils to work with seatmates to solve the problems. Make sure all groups of seatmates have at least 1 protractor.
10. Walk around to check for understanding and clear misconceptions.
11. Invite volunteers to write the solutions on the board and label the angles.

Solutions:

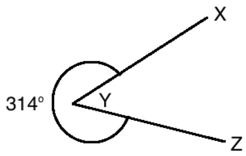
a. Example answer:

b. Example answer:





c. Example answer:



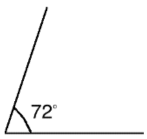
d. Example answer:

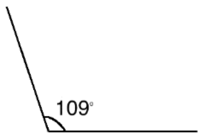
Practice *(19 minutes)*

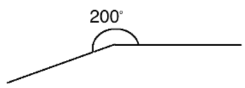
1. Write on the board:
   a. Use your protractor to draw the following angles:
      i. 72°   ii. 109°   iii. 200°
   b. Classify the following angles as acute, obtuse, or reflex:
      i. 181°   ii. 45°   iii. 91°
   c. Draw a triangle with all of its angles acute. Measure and label its angles.
   d. Draw a triangle with an obtuse angle. Measure and label its angles.
2. Ask pupils to work independently or with seatmates. If there are not enough protractors, encourage them to share and work together.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions. Parts c. and d. have many possible answers. If there is time, allow multiple pupils to write their answers.

   Solutions:

   a. Angles:

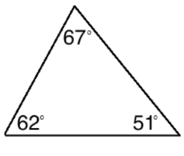
   i. 

   ii. 

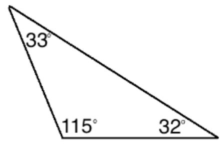
   iii. 

   b. i. reflex; ii. acute; iii obtuse

   c. Example answer:

   d. Example answer:

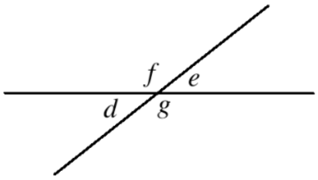
Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L049 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L050 in the Pupil Handbook before the next class.

Lesson Title: Solving for angles – Part 1	**Theme:** Geometry	
Lesson Number: M4-L050	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve for angles given intersecting lines, including parallel lines with a transversal.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

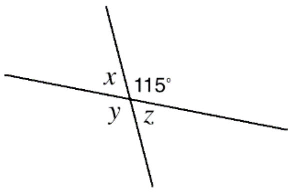
1. Draw the intersecting lines on the board:



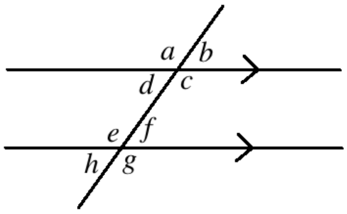
2. Ask pupils what they notice about angles d, e, f and g. Allow discussion.
3. Explain that today's lesson is on solving for angles in intersecting lines.

Teaching and Learning *(22 minutes)*

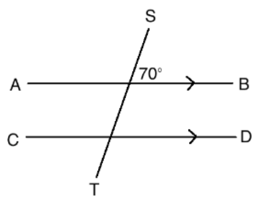
1. Explain:
    - When 2 lines intersect, the opposite angles are equal. In the diagram on the board, $d = e$ and $f = g$.
    - Adjacent angles are supplementary, which means they sum to $180°$. In the diagram on the board, the following are supplementary: f and e, e and g, d and g, d and f.
2. Write the following problem on the board: Find the measures of x, y and z in the diagram:



3. Ask volunteers to explain how to find the missing angles. As they explain, write the solution on the board.
   Solution:
    - x and $115°$ form a straight line, and thus are supplementary angles. Subtract from $115°$ to find x: $x = 180° - 115° = 65°$
    - y and $115°$ are opposite angles, and are thus equal. $y = 115°$
    - z is opposite x, so $z = x = 65°$. z can also be calculated using the fact that it is supplementary to y and $115°$.
4. Draw the diagram on the board:

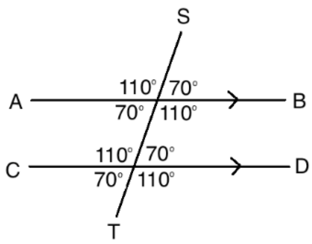


5. Ask pupils what they know about the angles in the diagram. Allow discussion.
6. Explain:
   - This is a set of parallel lines with a transversal. A transversal is a line that intersects both parallel lines.
   - The rules from the previous problem apply to each intersection. Additionally, there is a relationship between the 2 intersections.
7. Explain **alternate angles:**
   - Alternate angle are on opposite sides of the transversal, inside of the parallel lines. In the diagram, alternate angles are: d and f; c and e.
   - Alternate angles are **equal.**
8. Explain **corresponding angles:**
   - Corresponding angles are in the same position in the 2 intersections. In the diagram, corresponding angles are: a and e; b and f; c and g; d and h.
   - Corresponding angles are **equal.**
9. Explain **co-interior angles:**
   - Co-interior angles are on the same side of the transversal line, inside of the parallel lines. In the diagram, co-interior angles are: c and f; d and e.
   - Co-interior angles are **supplementary.**
10. Write a problem on the board: Find the measure of each angle in the diagram, and label them:

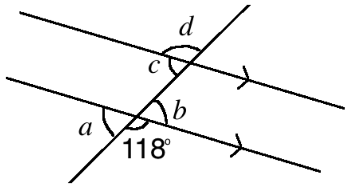


11. Ask volunteers to explain how to find each angle. As they explain, write the solution on the board and label the angles with their measures.
    Solution:
    Note that the following angles are equal to the labeled angle, $70°$: the opposite angle, and corresponding angles in the other intersection.

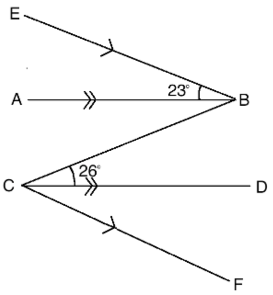
    Find the angles that are supplementary to the labeled angle by subtracting from $180°$: $180° - 70° = 110°$. Label the supplementary angles $110°$. Note that the corresponding angles in the other intersection are also $110°$.



12. Write the following problems on the board:
    e. Find the measures of a, b, c and d in the diagram below.



    f. In the diagram below, $AB||CD$ and $EB||CF$. Find the measure of $\angle BCF$.

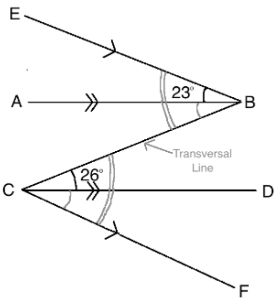


13. Ask pupils to work with seatmates to solve the problems. Remind them to look at the solved examples in the Pupil Handbook for guidance.
14. Walk around to check for understanding and clear misconceptions.
15. Invite volunteers to write the solutions on the board.

Solutions:

a. a and b are both supplementary to $118°$. Therefore, $a = b = 180° - 118° = 62°$.
   c and a are corresponding angles. Therefore, $c = a = 62°$
   d and c are supplementary angles. Therefore, $d = 180° - 62° = 118°$.

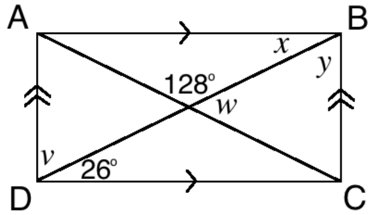
b. Note that BC is a transversal for both sets of parallel lines. Thus, we can label the alternate angles as equal. We have $\angle EBC = \angle BCF$ and $\angle ABC = \angle BCD$. This is shown in the diagram:



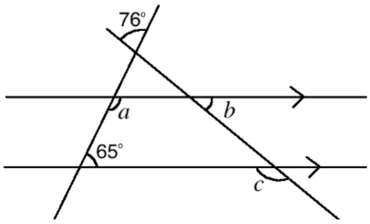
   Since we are given $\angle BCD = 26°$, we also have $\angle ABC = 26°$. Add $\angle EBA$ and $\angle ABC$ to find $\angle EBC$: $\angle EBC = \angle EBA + \angle ABC = 23° + 26° = 49°$.
   Now, since $\angle EBC = \angle BCF$, we have $\angle BCF = \angle EBC = 49°$.

Practice *(16 minutes)*

1. Write on the board:
   a. In the diagram, ABCD is a rectangle. Find the measures of v, w, x and y:



   b. Find the measures of a, b, and c in the diagram:



2. Ask pupils to work independently. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.
   Solutions:
   a. Note that the diagonals of the rectangle are transversal lines to each set of parallel lines. From this, $x = 26°$ because they are alternate angles. The other angles can be found using supplementary and complementary angles:
   $v = 90° - 26° = 64°$
   $w = 180° - 128° = 62°$
   $y = 90° - 26° = 64°$
   b. Note that a and $65°$ are co-interior angles, and are thus supplementary.
   $a = 180° - 65° = 125°$.
   Find the interior angles of the triangle. These angles can be used to find b. The angle adjacent to a corresponds to $65°$, and the top interior angle of the triangle is $76°$ because that is the measure of the opposite angle.
   Therefore, the angle opposite b is: $180° - 65° - 76° = 39°$.
   We also have $b = 39°$ because they are opposite angles.
   c corresponds to an angle that is supplementary to b, so we have $c = 180° - b = 180° - 39° = 141°$.
   Answer: $a = 125°$, $b = 39°$, $c = 141°$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L050 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L051 in the Pupil Handbook before the next class.

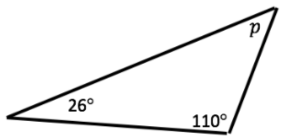
Lesson Title: Solving for angles – Part 2	Theme: Geometry	
Lesson Number: M4-L051	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve for angles in triangles.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

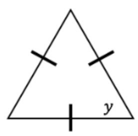
1. Discuss: What do you know about triangles? (Example answers: The 3 angles sum to 180°; for a right-angled triangle, Pythagoras' theorem can be used to find the lengths of the sides.)
2. Explain that today's lesson is on solving for angles in triangles.

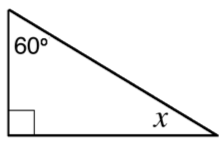
Teaching and Learning *(19 minutes)*

1. Draw the triangle below on the board:



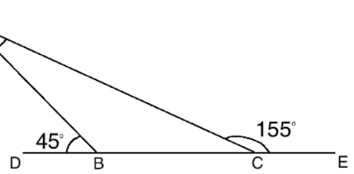
2. Discuss: How can you find the measure of p?
3. Allow pupils to share their ideas, then explain: The angles of a triangle add up to 180°. Missing angles are found by subtracting known angles from 180°.
4. Ask pupils to work with seatmates to find the measure of p.
5. Invite a volunteer to write the solution on the board. (Answer: $x = 180° - 110° - 26° = 44°$)
6. Draw the following triangles on the board:

a. 

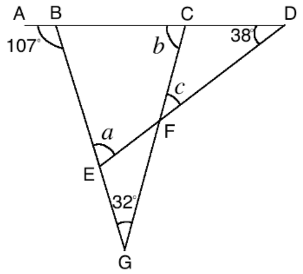
b. 

7. Discuss: What types of triangles are these? What do you know about them? (Answers: a. is equilateral and has 3 equal angles; b is a right-angled triangle and the other 2 angles sum to 90°.)
8. Ask volunteers to explain to the class how to solve for y and x. Guide the discussion and explain:
   - For the equilateral triangle, divide the total 180° by 3 to find y.
   - For the right-angled triangle, subtract 60° from 90° to find x.
9. Ask pupils to solve the problems with seatmates.
10. Ask volunteers to write the solutions on the board. (Answers: a. $y = 180° \div 3 = 60°$; b. $x = 90° - 60° = 30°$)
11. Write the following problems on the board:

a. Find $\angle BAC$ in the diagram below:



b. In the diagram below, $ABCD$ is a straight line. $\angle ABE = 107°$, $\angle CDF = 38°$, and $\angle EGF = 32°$. Find the measures of the angles labeled a, b and c.



12. Ask pupils to work with seatmates to solve the problems.
13. Walk around to check for understanding and clear misconceptions.
14. Invite volunteers to write the solutions on the board and label the angles.

Solutions:

a. Find $\angle ABC$ and $\angle ACB$ using their supplementary angles:

$\angle ABC = 180° - 45° = 135°$
$\angle ACB = 180° - 155° = 25°$

Find $\angle BAC$ by subtracting $\angle ABC$ and $\angle ACB$ from $180°$:

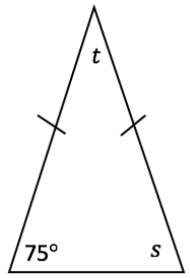
$\angle BAC = 180° - 135° - 25° = 20°$

b. Find $\angle CBG$ using its supplementary angle: $\angle CBG = 180° - 107° = 73°$
Find b using the interior angles of ΔBCG: $b = 180° - 73° - 32° = 75°$
Find a using the interior angles of ΔBDE: $a = 180° - 73° - 38° = 69°$
Find c using the interior angles of ΔCDF. This requires first finding the angle supplementary to b, $\angle DCF = 180° - 75° = 105°$
Therefore, $c = 180° - 105° - 38° = 37°$

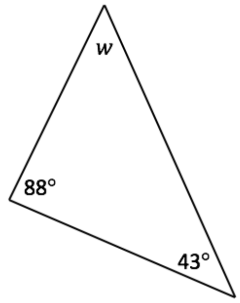
Practice *(19 minutes)*

1. Write on the board: Find the missing angles marked with a letter in each diagram:

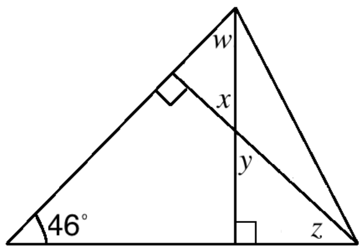
a.



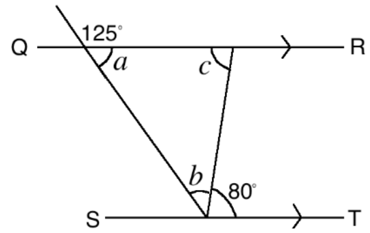
b.



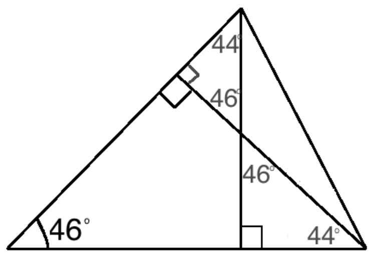
c.



d. $QR||ST$:



2. Remind them to refer to the example problems in the Pupil Handbook if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.
   Solutions:
   a. $s = 75°$ because the triangle is isosceles; $t = 180° - 75° - 75° = 30°$
   b. $w = 180° - 88° - 43° = 49°$
   c. Find w, x, y and z by solving the right-angled triangles, using the given angle $46°$ and the $90°$ angles. Label them as they are found:



      Answer: $w = 44°, x = 46°, y = 46°$ and $z = 44°$
   d. Find a using its supplementary angle: $a = 180° - 125° = 55°$.
      Note that b and $80°$ form a $125°$ angle because they correspond to the angle above. Therefore: $b = 125° - 80° = 45°$
      Subtract a and b from $180°$ to find c: $c = 180° - 55° - 45° = 80°$.
      Alternately, note that c and the given $80°$ angle are alternate angles, and are therefore equal.

Closing *(1 minute)*

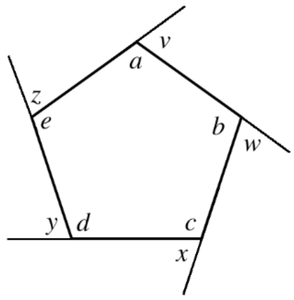
1. For homework, have pupils do the practice activity of PHM4-L051 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L052 in the Pupil Handbook before the next class.

Lesson Title: Solving for angles – Part 3	Theme: Mensuration	
Lesson Number: M4-L052	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve for angles in quadrilaterals and other polygons.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(2 minutes)*

1. Discuss:
   - What do you know about the angles in a triangle? (Example answer: They add up to 180°.)
   - What do you know about the angles in other shapes? (Example answers: Each shape's interior angles sum to a given amount; shapes have interior and exterior angles.)
2. Explain that this lesson is on solving problems on the angles of polygons, including quadrilaterals.

Teaching and Learning *(18 minutes)*



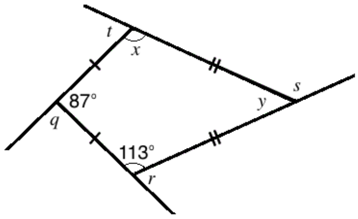
1. Draw the pentagon diagram on the board:
2. Explain:
   - Interior angles are inside of a shape, and in this example are a, b, c, d, e.
   - Exterior angles lie outside of the shape, and in this example are v, w, x, y, z.
3. Ask pupils to look at the table in the Pupil Handbook, which is also given below.
4. Explain
   - The interior angles of the given polygons will always sum to these amounts.
   - These are used to solve various problems. There are also some related equations.

Sides	Name	Sum of Interior Angles
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	900°
8	Octagon	1,080°
9	Nonagon	1,260°
10	Decagon	1,440°

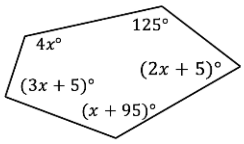
5. Write on the board:
   - Sum of the interior angles in a polygon: $(n-2) \times 180°$ where n is the number of sides.
   - Measure of each interior angle of a **regular** polygon: $\frac{(n-2)\times 180°}{n}$ where n is the number of sides.
6. Explain:
   - A regular polygon has all of its sides equal.

- For polygons that are not regular, missing angles can be found by subtracting known angles from the sum of the angles for that type of polygon.

7. Write the following problems on the board:
   a. Calculate the sum of the interior angles of a polygon with 21 sides.
   b. Find the missing interior and exterior angles of the kite:



   c. In the pentagon, solve for x:



8. Solve the problems as a class on the board:
   a. Substitute $n = 21$ in the formula and solve:

$$\begin{aligned} \text{Sum of angles} &= (n-2)\times 180° \\ &= (21-2)\times 180° \\ &= 19 \times 180° \\ &= 3{,}420° \end{aligned}$$

   b. Find the interior angles first. $x = 113°$ because these 2 opposite angles of the kite are equal. Subtract from $360°$ (the angles of a quadrilateral) to find y:

$y = 360° - 113° - 113° - 87° = 47°$

Find the exterior angles by subtracting the interior angles from $180°$:

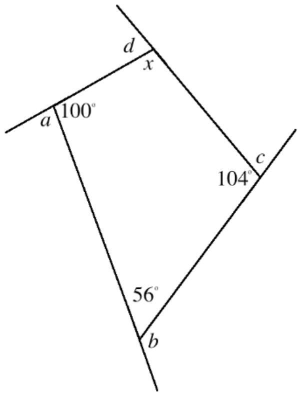
$q = 180° - 87° = 93°$
$r = 180° - 113° = 67°$
$s = 180° - 47° = 133°$
$t = 180° - 113° = 67°$

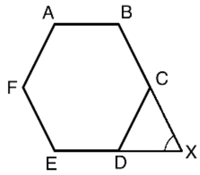
   c. Use the fact that the angles of a pentagon add up to 540°.

$$\begin{aligned} 540° &= 125° + (2x+5)° + (x+95)° + (3x+5)° + 4x° && \text{Add the angles} \\ &= (125° + 5° + 95° + 5°) + (2x + x + 3x + 4x)° && \text{Combine like terms} \\ &= 230° + 10x° \\ 540° - 230° &= 10x° \\ 310° &= 10x° && \text{Transpose } 230° \\ 31° &= x && \text{Divide by } 10° \end{aligned}$$

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. Find the missing angles a, b, c, d and x in the diagram:
   b. In the diagram, $ABCDEF$ is a regular polygon. When they are extended, sides BC and ED meet at point X. Find the measure of $\angle X$.





   c. A pentagon has one exterior angle of 70°. Two other angles are $(90-x)^{\circ}$, while the remaining angles are each $(40+2x)^{\circ}$. Find the value of x.
   d. The interior angle of a regular polygon is 140°. How many sides does it have?
2. Ask pupils to work independently or with seatmates to solve the problems. Remind them to refer to the example problems in the Pupil Handbook if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.
   Solutions:
   - $x=360^{\circ}-100^{\circ}-104^{\circ}-56^{\circ}=100^{\circ}$; $a=180^{\circ}-100^{\circ}=80^{\circ}$; $b=180^{\circ}-56^{\circ}=124^{\circ}$; $c=180^{\circ}-104^{\circ}=76^{\circ}$; $d=180^{\circ}-100^{\circ}=80^{\circ}$
   - Note that the triangle is made up of 2 external angles of a hexagon, and X.
     Exterior angle of a hexagon $=\frac{360^{\circ}}{n}=\frac{360^{\circ}}{6}=60^{\circ}$
     Subtract from to find X. $X=180^{\circ}-60^{\circ}-60^{\circ}=60^{\circ}$
   - Set the sum of the angles equal to 360°, which is always the sum of the exterior angles. Solve for x.

$$\begin{aligned}
360^{\circ} &= 70^{\circ}+2(90-x)^{\circ}+2(40+2x)^{\circ} && \\
&= 70^{\circ}+180^{\circ}-2x^{\circ}+80^{\circ}+4x^{\circ} && \text{Remove brackets}\\
&= (70^{\circ}+180^{\circ}+80^{\circ})+(-2x^{\circ}+4x^{\circ}) && \text{Combine like terms}\\
&= 330^{\circ}+2x^{\circ} && \text{Simplify}\\
360^{\circ}-330^{\circ} &= 2x^{\circ} && \text{Transpose } 330^{\circ}\\
30^{\circ} &= 2x^{\circ} && \text{Divide by } 2^{\circ}\\
15^{\circ} &= x &&
\end{aligned}$$

   - Use the formula for interior angle to find the number of sides, n:

$$\begin{aligned}
140^{\circ} &= \frac{(n-2)\times 180^{\circ}}{n} && \\
140^{\circ}n &= (n-2)\times 180^{\circ} && \text{Multiply throughout by } n\\
140^{\circ}n &= 180^{\circ}n-360^{\circ} && \text{Distribute the right-hand side}\\
140^{\circ}n-180^{\circ}n &= -360^{\circ} && \text{Transpose } 180^{\circ}n\\
-40^{\circ}n &= -360^{\circ} && \text{Divide throughout by } -40^{\circ}\\
n &= 9 &&
\end{aligned}$$

   Answer: The polygon has 9 sides.

Closing *(1 minute)*

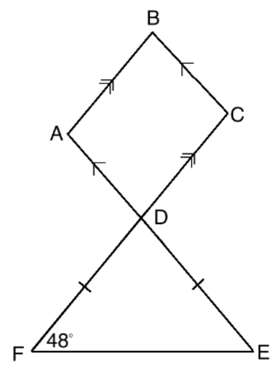
1. For homework, have pupils do the practice activity of PHM4-L052 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L053 in the Pupil Handbook before the next class.

Lesson Title: Solving for angles – Part 4	Theme: Geometry	
Lesson Number: M4-L053	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve for angles in compound and complex shapes.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem at the start of Teaching and Learning on the board.	

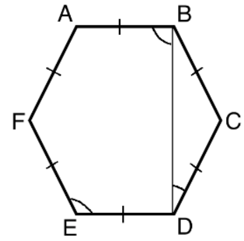
Opening *(2 minutes)*

1. Explain that this lesson is on solving for angles in compound and complex shapes. Pupils will use information from previous lessons.

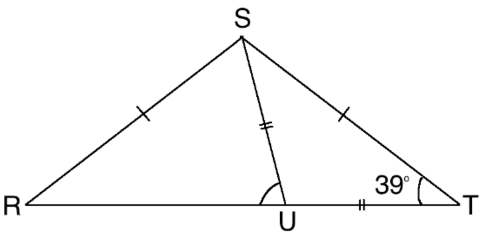
Teaching and Learning *(18 minutes)*



1. Write the following problem on the board: In the given shape, ΔFED is an isosceles triangle. $AB||DC$, and $BC||AE$. Find:
    a. $\angle ADF$
    b. $\angle BAD$
    c. $\angle BCD$
2. Discuss and allow pupils to share their ideas: How would you solve this problem? What steps would you take?
3. Explain: When you encounter a complex shape, break it down to its parts. Look for a strategy for finding the angles the problem asks you to solve.
4. Solve the problem on the board as a class.
   Solutions:
    a. Note that $\angle ADF$ is supplementary to an angle in ΔDEF. To find $\angle ADF$, first solve ΔDEF. Since ΔDEF is isosceles, $\angle DFE = \angle DEF = 48°$.
    Therefore, $\angle EDF = 180° - 2(48°) = 84°$
    Using the fact that $\angle ADF$ and $\angle EDF$ are supplementary, we have:
    $$\angle ADF = 180° - \angle EDF = 180° - 84° = 96°$$
    b. Note that $\angle BAD$ and $\angle ADF$ are alternate angles, which means they are equal. Therefore, $\angle BAD = \angle BAD = 96°$
    c. Note that the opposite angles in a parallelogram are equal. Therefore, $\angle BCD = \angle BAD = 96°$.



5. Write the following problems on the board:
    a. In the diagram, ABCDEF is a regular hexagon. Find the measures of: i. $\angle DEF$ ii. $\angle BDC$ iii. $\angle ABD$
    b. In the diagram below, |RS|=|ST| and |SU|=|TU|. $\angle STU = 39°$. Find the size of $\angle SUR$.

c. 

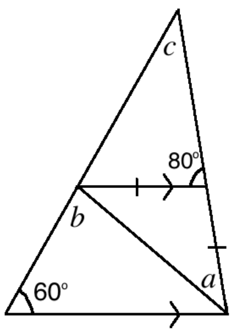
6. Ask pupils to solve the problems with seatmates.
7. Walk around to check for understanding and clear misconceptions. Remind pupils to look at the examples in the Pupil Handbook for guidance.
8. Invite volunteers to write the solutions on the board.
   a. i. To find $\angle DEF$, apply the formula for the interior angle of a regular polygon:

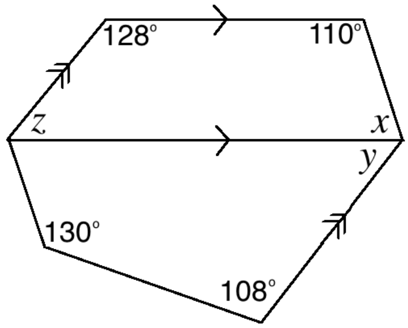
   $$\angle DEF = \frac{(n-2)\times 180°}{n} = \frac{(6-2)\times 180°}{6} = \frac{4\times 180°}{6} = 120°$$

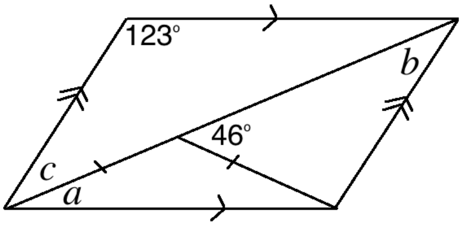
   ii. Note that ΔBCD is an isosceles triangle, and $\angle BCD = 120°$ because it is an angle of the regular pentagon.
   Subtract $\angle BCD$ from $180°$: $180° - \angle BCD = 180° - 120° = 60°$
   Divide by 2 to find $\angle BDC$: $60° \div 2 = 30°$
   iii. Subtract $\angle CBD = 30°$ from $\angle ABC = 120°$ to find $\angle ABD$:
   $\angle ABD = 120° - 30° = 90°$
   b. Note that $\angle UST = \angle STU = 39°$. Using isosceles triangle STU, $\angle SUT = 180° - 2(39°) = 180° - 78° = 102°$.
   $\angle SUT$ is supplementary to $\angle SUR$. Therefore:
   $\angle SUR = 180° - \angle SUT = 180° - 102° = 78°$

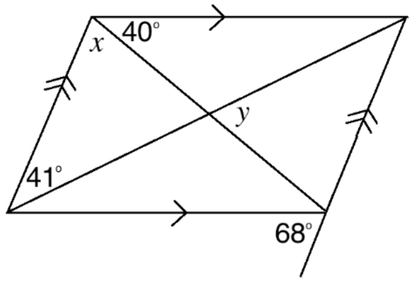
Practice *(19 minutes)*

1. Write the following problems on the board: Find the measures of the marked angles in the diagrams below:

a. 

b. 

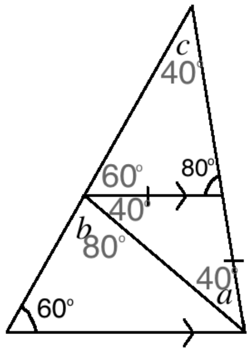
c. 

d. 

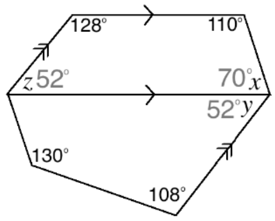
2. Ask pupils to work independently or with seatmates to solve the problems. Remind them to refer to the example problems in the Pupil Handbook if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions and explain. Ask them to label the angles of the diagrams as they solve.

   Solutions:

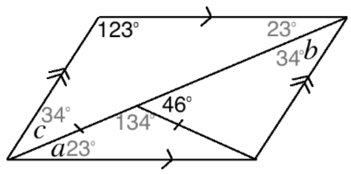
   a. Use the angle supplementary to 80 to solve the small isosceles triangle. This gives $a = \frac{180°-100°}{2} = 40°$.
   Use the side of the large triangle (straight line) to find b. The top angle is a corresponding angle to $60°$, and the other angle is equal to a (isosceles triangle). Therefore, $b = 180° - 60° - 40° = 80°$.
   To find c, use the top triangle. 2 angles are known, $60°$ and $80°$. This gives: $c = 180° - 60° - 80° = 40°$.



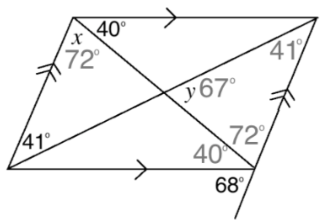
   b. The co-interior angles of x and z are given. Therefore, $x = 180° - 110° = 70°$ and $z = 180° - 128° = 52°$.
   z and y are alternate angles; therefore, $y = z = 52°$.



   c. Use the isosceles triangle to find a. The top interior angle is $180° - 46° = 134°$. This gives $a = \frac{180°-134°}{2} = 23°$.
   Use the triangle containing c to find its measure. The other missing angle is alternate to a, therefore is $23°$.
   This gives $c = 180° - 123° - 23° = 34°$.
   b and c are alternate angles; this gives $b = c = 34°$.



   d. Label the alternate angles to $40°$ and $41°$ inside the parallelogram. Solve for the angles in the triangle containing y. The bottom angle is $180° - 68° - 40° = 72°$. This is an alternate angle to x; therefore $x = 72$.
   Solve for y: $y = 180° - 72° - 41° = 67°$.



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L053 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L054 in the Pupil Handbook before the next class.

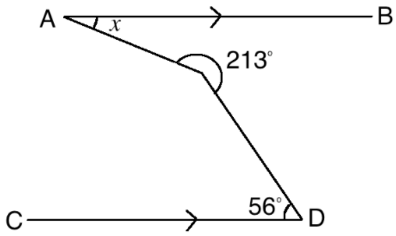
Lesson Title: Angle problem solving	Theme: Geometry	
Lesson Number: M4-L054	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply angle theorems and properties to solve problems.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem at the start of Teaching and Learning on the board.	

Opening *(2 minutes)*

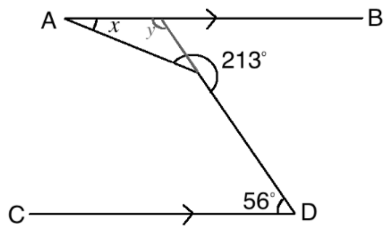
1. Explain that this lesson is on solving for angles in compound and complex shapes. Pupils will use information from previous lessons.

Teaching and Learning *(18 minutes)*

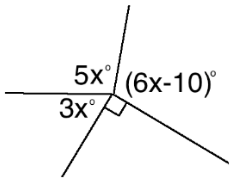
1. Write the following problem on the board: In the diagram below, $AB||CD$. Find the measure of angle x.



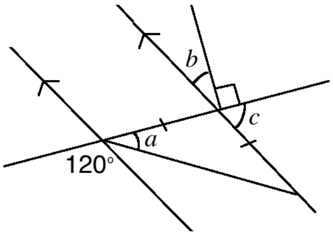
2. Discuss and allow pupils to share their ideas: How would you solve this problem? What steps would you take?
3. Solve the problem on the board as a class.
   Solution:
   Note that the line from D can be extended to become a transversal of the parallel lines (see below). This makes a triangle with interior angle x that can be solved.



   The newly formed angle y in the diagram is a co-interior angle with $56°$, which means they are supplementary. Therefore, $y = 180° - 56° = 124°$
   Note that the other interior angle of the triangle can be found by subtracting $180°$ (the straight line) from $213°$ (the given angle): $213° - 180° = 33°$
   Now that 2 interior angles of the triangle are known, subtract from $180°$ to find x:
   $$x = 180° - 124° - 33° = 23°$$
4. Write the following problems on the board:
   c. Find the measure of x in the diagram:



d. Find the measures of angles a, b and c in the diagram:



5. Ask pupils to solve the problems with seatmates.
6. Walk around to check for understanding and clear misconceptions.
7. Invite volunteers to write the solutions on the board.

Solutions:

e. Set the sum of the angles equal to $360°$ (a full rotation) and solve for x:

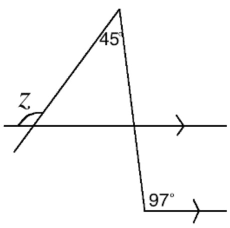
$$\begin{aligned} 90° + 5x + 3x + 6x - 10° &= 360° \\ 14x + 80° &= 360° \\ 14x &= 360° - 80° \\ 14x &= 280° \\ x &= \tfrac{280°}{14} = 20° \end{aligned}$$

f. Note that the obtuse angle of the isosceles triangle is equal to 120 (corresponding angles). This gives $a = \frac{180°-120°}{2} = 30°$.
Note that the angle made up of b and $90°$ is also $120°$ (it is opposite the $120°$ angle in the isosceles triangle). Therefore, $b = 120° - 90° = 30°$.
Note that c is supplementary to $120°$; therefore, $c = 180° - 120° = 60°$.

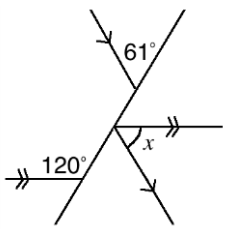
Practice *(19 minutes)*

1. Write the following problems on the board: Find the measures of the marked angles in the diagrams below:

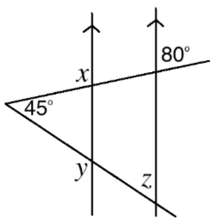
a.



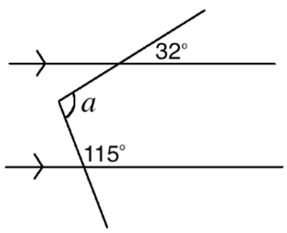
b.



c.



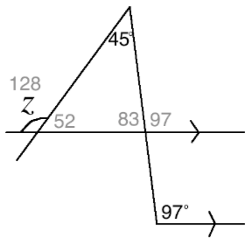
d.



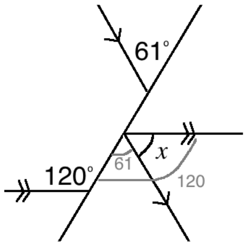
2. Ask pupils to work independently or with seatmates to solve the problems. Remind them to refer to the example problems in the Pupil Handbook if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions and explain. Ask them to label the angles of the diagrams as they solve.

   Solutions:

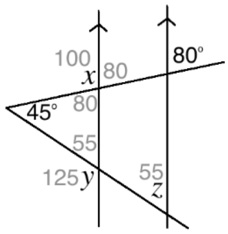
   a. Note that $97°$ has a cooresponding angle above it, which is also supplementary to an interior angle of the triangle. Label it as $97°$.
   Solve for the interior angles of the triangle: $180° - 97° = 83°$; $180° - 83° - 45° = 52°$.
   Solve for z, using the fact that it is supplementary to the second interior angle of the triangle: $z = 180° - 52° = 128°$



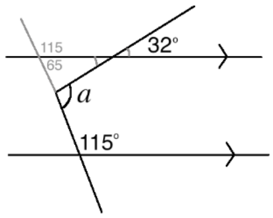
   b. Use the parallel lines to find the angle adjacent to x. The lower adjacent angle is $61°$. The angle that contains this $61°$ angle and x is $120°$; therefore, $x = 120° - 61° = 59°$



   c. x is supplementary to the angle that corresponds to $80°$. Therefore, $x = 180° - 80° = 100°$.
   Solve the small triangle. The angles are $180 - 100° = 80°$ and $180° - 80° - 45° = 55°$. y is supplementary to this $55°$ angle; therefore, $y = 180° - 55° = 125°$.
   z corresponds to the $55°$ angle in the triangle; therefore, $z = 55°$.



   d. Extend either transversal lines and use it to solve for a. By extending the transversal with 115, we form a triangle with an angle supplementary to a. Solve the triangle to find that the angle adjacent a is $83°$. This gives $a = 180° - 83° = 97°$



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L054 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L055 in the Pupil Handbook before the next class.

Lesson Title: Conversion of units of measurement	**Theme:** Mensuration	
Lesson Number: M4-L055	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Convert from large units to smaller units of measurement. 2. Convert from smaller units to larger units of measurement.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(2 minutes)*

1. Discuss: Ask questions to review common measurements. For example:
    - Which is longer: 1 metre or 1 kilometre? (Answer: 1 kilometre)
    - How many centimetres are in a metre? (Answer: 100 centimetres)
    - What are some units we use to measure volume or capacity? (Example answers: litres, millilitres, cm^3)
2. Explain that today's lesson is on conversion of units. A list of some common relationships between units of measurement can be found in the Pupil Handbook activity for this lesson.

Teaching and Learning *(18 minutes)*

1. Write the following problem on the board: Fatu walked a total of 3,000 metres in one day. How much did she walk in kilometres?
2. Discuss: How can we calculate the kilometres she walked?
3. Allow pupils to share ideas, then explain: Metres are smaller than kilometres. To convert from a smaller unit to a larger unit, **divide** by the conversion factor.
4. Write on the board: $1{,}000 \text{ m} = 1 \text{ km}$
5. Solve the word problem on the board: $3{,}000 \div 1{,}000 = 3$ km
6. Write the following problem on the board: Foday travels 1.5 kilometres to school each day. How much is that in metres?
7. Discuss: How can we calculate the metres he walked?
8. Allow pupils to share ideas, then explain: Kilometres are larger than metres. To convert from a larger unit to a smaller unit, **multiply** by the conversion factor.
9. Solve the word problem on the board: $1.5 \times 1{,}000 = 1{,}500$ m
10. Write another problem on the board: Sia owns land with area 600,000 m^2. What is the size of her land in square kilometres?
11. Explain:
    - You will see a power on certain units. Area is measured in square units, and volume/capacity can be measured in cubic units.
    - The conversion factor must be squared or cubed as well. This is the same as applying the factor 2 or 3 times.

12. Solve the problem on the board: $600{,}000\ \text{m}^2 \rightarrow 600{,}000 \div 1{,}000 = 600 \rightarrow 600 \div 1{,}000 = 0.6$ km^2.
13. Write the solution another way, and make sure pupils understand: $600{,}000\ \text{m}^2 = \frac{600{,}000}{1000^2} = \frac{600{,}000}{1{,}000{,}000} = 0.6\ \text{km}^2$
14. Write the following problem on the board: Bintu wants to make a dress with some fabric she has. She has a piece that is 2 metres, and another piece that is 80 cm. How many metres does she have in total?
15. Discuss: What steps do we need to take to solve this problem? (Answer: We should **add** to find the **total**, but the **units should be the same** before adding.)
16. Write the solution on the board:
    Step 1. Convert centimetres to metres: $80 \div 100 = 0.8$ m
    Step 2. Add: $2 + 0.8 = 2.8$ m
17. Write the following problems on the board:
    a. Convert 7,625 mg to g. Give your answer to 2 decimal places.
    b. Convert 32,000 cm to km.
    c. Convert 1.254 litres to ml.
    d. Convert 0.25 kg to mg.
    e. A tailor had 2.8 metres of fabric. If she used 150 cm to make a skirt, how much does she have left? Give your answer in centimetres.
18. Ask pupils to work with seatmates to solve the problems.
19. Invite volunteers to write the solutions on the board.
    Solutions:
    - $7{,}625 \div 1{,}000 = 7.625 = 7.63$ grammes
    - This problem is best done in 2 steps:
      Centimetres to metres: $32{,}000 \div 100 = 320$ metres
      Metres to kilometres: $320 \div 1{,}000 = 0.32$ kilometres
    - $1.254 \times 1{,}000 = 1{,}254$ ml.
    - This problem is best done in 2 steps:
      Kilogrammes to grammes: $0.25 \times 1{,}000 = 250$ g
      Grammes to milligrammes: $250 \times 1{,}000 = 250{,}000$ mg
    - Convert to centimetres: $2.8 \times 100 = 280$ cm
      Subtract: $280 - 150 = 130$ cm

Practice *(19 minutes)*

1. Write the following problems on the board:
    a. Convert the following: i. 600 m to km ii. 6,500 g to kg
    b. Convert the following: i. 0.5 km to m ii. 3 l to ml
    c. A carpenter has a piece of wood 75 cm long, and another piece of wood 2.2 metres long. How much wood does he have all together? Give your answer in metres.

d. Aminata's doctor told her to drink 3 litres of water each day. Today she drank 1.8 litres in the morning, and 1,500 ml in the evening. Did she drink enough water?
e. There are 50,000 litres of water in a tank. Find the volume of water in cubic metres (m^3).

2. Ask pupils to solve the problems either independently.
3. Walk around to check for understanding and clear misconceptions.
4. Ask volunteers to come to the board simultaneously to write the solutions.
   Solutions:
   a. i. $600 \div 1{,}000 = 0.6$ km; ii. $6{,}500 \div 1{,}000 = 6.5$ kg
   b. i. $0.5 \times 1{,}000 = 500$ m; $3 \times 1{,}000 = 3{,}000$ ml
   c. Convert 75 cm to metres: $75 \div 100 = 0.75$ m; Add: $0.75 + 2.2 = 2.95$ m
   d. Convert l to ml: $1.8 \times 1{,}000 = 1{,}800$ ml; Add the measurements: $1{,}800 + 1{,}500 = 3{,}300$ ml. This is more than 3,000 ml; therefore, she did drink enough water.
   e. Convert litres to cm^3 using the conversion factor 1 litre = 1,000 cm^3:
   $50{,}000\,\mathrm{l} \times 1{,}000 = 50{,}000{,}000\ \mathrm{cm}^3$
   Convert cm^3 to m^3 by applying the conversion factor 1 m = 100 cm three times: $50{,}000{,}000\ \mathrm{cm}^3 = \frac{50{,}000{,}000}{100^3} = \frac{50{,}000{,}000}{1{,}000{,}000} = 50\ \mathrm{m}^3$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L055 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L056 in the Pupil Handbook before the next class.

Lesson Title:	Theme:	
Lesson Title: Area and perimeter of triangles and quadrilaterals	**Theme:** Mensuration	
Lesson Number: M4-L056	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the area and perimeter of triangles and quadrilaterals.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Draw the table in Opening on the board.	

Opening *(4 minutes)*

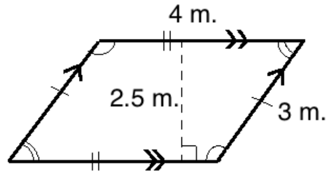
1. Write the table below on the board, with the perimeter and area columns empty.
2. Invite volunteers to come to the board and write the formulae for the area and perimeter for each shape. They are given this with diagrams in the Pupil Handbook.
   Solution:

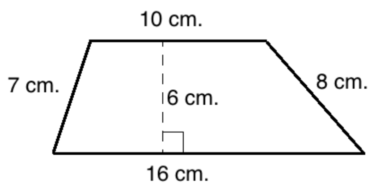
Shape	Perimeter	Area
Square	$P = l + l + l + l = 4l$	$A = l \times l = l^2$
Rectangle	$P = 2l + 2w$	$A = l \times w$
Parallelogram	$P = 2a + 2b$	$A = b \times h$
Trapezium	$P = a + b + c + d$	$A = \frac{1}{2}(a + b)h$
Rhombus	$P = l + l + l + l = 4l$	$A = \frac{1}{2}d_1 \times d_2$
Kite	$P = a + a + b + b = 2a + 2b$	$A = \frac{1}{2}d_1 \times d_2$
Triangle	$P = a + b + c$	$A = \frac{1}{2}b \times h$

3. Explain: This lesson is on calculating perimeter and area of shapes. You will use these formulae throughout the lesson.

Teaching and Learning *(18 minutes)*

1. Write the following problems on the board: Calculate the area and perimeter of the quadrilaterals:

a. 

b. 

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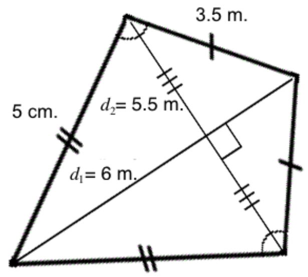

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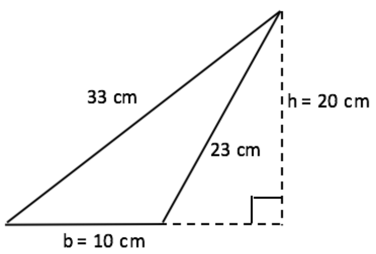


2. Solve the problems as a class. Ask volunteers to describe the steps or work them on the board.

   Solutions:

   a. $P = 2a + 2b = 2 \times 4\text{ m} + 2 \times 3\text{ m} = 14\text{ m}$; $A = b \times h = 4\text{ m} \times 2.5\text{ m} = 10\text{ m}^2$

   b. $P = a + b + c + d = 10 + 16 + 8 + 7 = 41\text{ cm}$; $A = \frac{1}{2}(a + b)h = \frac{1}{2}(10 + 16)6 = \frac{1}{2}(26)6 = 78\text{ cm}^2$

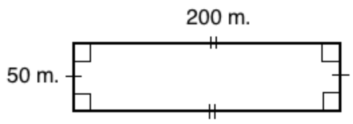
   c. $P = 2a + 2b = 2(5) + 2(3.5) = 10 + 7 = 17\text{ m}$; $A = \frac{1}{2}d_1 \times d_2 = \frac{1}{2}(6 \times 5.5) = \frac{1}{2}(33) = 16.5\text{ m}^2$

   d. $P = a + b + c = 33 + 10 + 23 = 66\text{ cm}$; $A = \frac{1}{2}b \times h = \frac{1}{2}(10 \times 20) = \frac{1}{2}(200) = 100\text{ cm}^2$

3. Write the following problem on the board: Mr. Bah has a rectangular farm. It is 200 metres on one side, and 50 metres on the other side.
   a. Draw the shape of Mr. Bah's farm.
   b. If he wants to fence his farm, how long will his fence be?
   c. He wants to find the area of his farm to know how much fertiliser to buy. What is the area?
   d. If he needs 1 bottle of fertiliser for each 100 m^2, how many bottles should he buy?
   e. If he plants half of his farm with corn, what is the area of the corn?
4. Ask pupils to solve the problems with seatmates.
5. Invite volunteers to write the solution on the board and explain.

   Solution:

   a. Diagram →

   b. $\text{P} = 2l + 2w = 2 \times 200\text{ m} + 2 \times 50\text{ m} = 400\text{ m} + 100\text{ m} = 500\text{ m}$

   c. $\text{A} = l \times w = 200\text{ m} \times 50\text{ m} = 10{,}000\text{ m}^2$

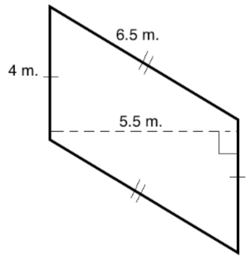
   d. Divide the area by 100: $10{,}000\text{ m}^2 \div 100\text{ m}^2 = 100$ bottles

   e. Calculate half of the area: $\frac{1}{2}\text{A} = \frac{1}{2}(10{,}000\text{ m}^2) = 5{,}000\text{ m}^2$

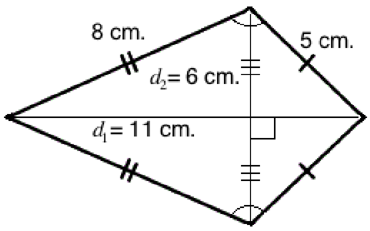
Practice *(17 minutes)*

1. Write the following problems on the board: Find the area of the following shapes:

a.



b.



c. Mrs. Jalloh has a perfectly square farm that measures 120 metres on each side. She will plant $\frac{1}{4}$ of the farm with cassava.
   i. Draw the shape of Mrs. Jalloh's farm.
   ii. What is the total area of Mrs. Jalloh's farm?
   iii. What is the area that she will plant with cassava?
   iv. If each square metre produces 2 pieces of cassava, how many pieces of cassava will she have in total?

d. A trapezium has parallel lines that are 10 m and 15 m. If its area is 100 m^2, what is its height?

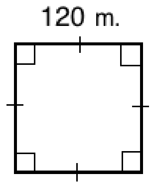
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.

Solutions:

a. $\mathrm{P} = 2\ \times\ 4\ \mathrm{m} + 2\ \times\ 6.5\ \mathrm{m} = 8\ \mathrm{m} + 13\ \mathrm{m} = 21\ \mathrm{m}; \mathrm{A} = 4\ \mathrm{cm} \times\ 5.5\ \mathrm{cm}\ = 22\ \mathrm{cm}^2$

b. $P = 2a + 2b = 2(8) + 2(5) = 16 + 10 = 26$ cm.; $A = \frac{1}{2}d_1 \times d_2 = \frac{1}{2}(11 \times 6) = \frac{1}{2}(66) = 33\ \mathrm{cm}^2$

c. i. Diagram →



ii. $\mathrm{A} = l^2 = 120\ \mathrm{m} \times\ 120\ \mathrm{m}\ = 14{,}400\ \mathrm{m}^2$

iii. Calculate $\frac{1}{4}$ of the area: $\frac{1}{4}A = \frac{1}{4}(14{,}400\ \mathrm{m}^2) = 3{,}600\ \mathrm{m}^2$

iv. $3{,}600\ \mathrm{m}^2 \times 2\,\frac{\text{pieces}}{\mathrm{m}^2} = 7{,}200$ pieces of cassava total.

d.

$A = \frac{1}{2}(a+b)h$	
$100\ \mathrm{m}^2 = \frac{1}{2}(10\ \mathrm{m} + 15\ \mathrm{m})h$	Substitute values
$2 \times 100\ \mathrm{m}^2 = (25\ m)h$	Multiply throughout by 2
$\frac{200\ \mathrm{m}^2}{25\ \mathrm{m}} = h$	Divide throughout by 25
$8\ \mathrm{m} = h$	

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L056 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L057 in the Pupil Handbook before the next class.

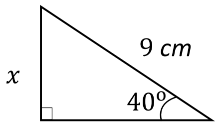
Lesson Title: Trigonometric ratios	**Theme:** Trigonometry	
Lesson Number: M4-L057	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to identify trigonometric and inverse trigonometric ratios and use them to solve for sides and angles of a triangle.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Bring trigonometric tables ("log books") to class if available, and ask pupils to bring them.	

Opening *(2 minutes)*

1. Write on the board: SOHCAHTOA. Discuss:
   - What does this stand for? (Answer: Sine is Opposite over Hypotenuse; Cosine is Adjacent over Hypotenuse; Tangent is Opposite over Adjacent.)
   - What is this used for? (Answer: It is used to remember the trigonometric ratios, which are used to find missing sides in a triangle.)
2. Explain: This lesson is trigonometry. We will use trigonometric and inverse trigonometric ratios to find the missing sides and angles of triangles.

Teaching and Learning *(21 minutes)*

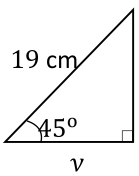
1. Explain:
   - Trigonometric ratios are used to find missing sides in right-angled triangles.
   - Inverse trigonometric ratios are used to find the missing angles.
2. Write the following problem on the board: Find the measure of missing side x:



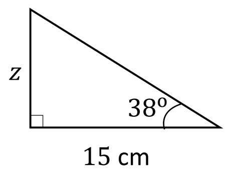
3. Discuss: Which trigonometric ratio can we use to solve this problem? Why? (Answer: Sine, because it is the ratio for opposite side and hypotenuse.)
4. Solve on the board, involving pupils by asking them to give the steps:

$$\sin \theta = \frac{\text{O}}{\text{H}}$$
$$\sin 40^{\text{o}} = \frac{x}{9}$$
$$9 \times \sin 40^{\text{o}} = x$$
$$9 \times 0.6428 = x \qquad \text{Find } \sin 40^{\text{o}} = 0.6428 \text{ in the sine table}$$
$$x = 5.7\text{cm to 1 d.p}$$

5. Write 2 additional problems on the board:
   a. Find the measure of y:

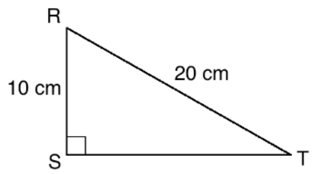


   b. Find the measure of z:



6. Ask pupils to solve the problems with seatmates.
7. Invite volunteers to write the solutions on the board and explain.
   Solutions:
   a. Find the measure of y:
   $$\cos\theta = \frac{A}{H}$$
   $$\cos 45^o = \frac{y}{19}$$
   $$19 \times \cos 45^o = y$$
   $$9 \times 0.7071 = y$$
   $$y = 13.4349$$
   $y =$ 13.4 cm to 1 d.p.

   b. Find the measure of z:
   $$\tan\theta = \frac{O}{A}$$
   $$\tan 38^o = \frac{z}{15}$$
   $$15 \times \tan 38^o = z$$
   $$15 \times 0.7813 = z$$
   $$z = 11.7195$$
   $z =$ *11.7*cm to 1 d.p.
8. Explain: The inverse of a function is its opposite. It's another function that can undo the given function.
9. Write the 3 inverse trigonometric functions on the board: $\cos^{-1} x$, $\tan^{-1} x$, $\sin^{-1} x$
10. Write an example showing how an inverse function "undoes" a function on the board: $\sin^{-1}(\sin\theta) = \theta$
11. Explain:
    - You can use inverse trigonometric functions to find the degree measure of an angle.
    - Using trigonometric tables, you will work backwards. Find the decimal number in the chart, and identify the angle that it corresponds to.
12. Write the following example on the board: Calculate the following: $\sin^{-1}(0.5015)$
13. Show how to solve this problem using a sine table.
    - Find 0.5015 in the trigonometric table for sine.
    - It is in row 31, under the first column (.0). This means that the angle has measure 31.0°.
14. Write the following problem on the board: Find the measures of angles R and T. →

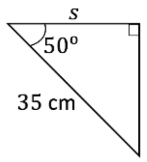


15. Solve for angle R on the board as a class.
    Step 1. Apply the ratio to find $\cos R$: $\cos R = \frac{10}{20} = \frac{1}{2} = 0.5$
    Step 2. Apply inverse trigonometry:
    $$\begin{aligned} \cos R &= 0.5 \\ \cos^{-1}(\cos R) &= \cos^{-1}(0.5) \\ R &= 60° \end{aligned}$$
16. Ask pupils to find the measure of angle T with seatmates, using inverse trigonometry.
17. Invite a volunteer to write the solution on the board.
    Step 1. Apply the ratio to find $\sin T$: $\sin T = \frac{10}{20} = \frac{1}{2} = 0.5$
    Step 2. Apply inverse trigonometry:
    $$\sin T = 0.5$$

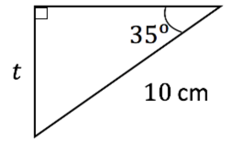
$$\begin{aligned} \sin^{-1}(\sin T) &= \sin^{-1}(0.5) \\ T &= 30° \end{aligned}$$

Practice *(16 minutes)*

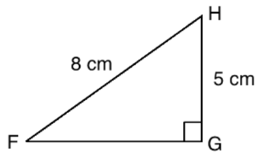
1. Write the following problems on the board:
   a. Find the measure of s:



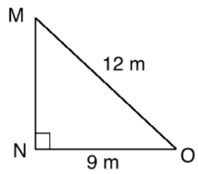
   b. Find the measure of t:



   c. Find the measures of F and H:



   d. Find the measures of M and O:



2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board at the same time to write the solutions.
   Solutions:
   a. Find the measure of s:

$$\begin{aligned} \cos\theta &= \frac{\mathrm{A}}{\mathrm{H}} \\ \cos 50° &= \frac{s}{35} \\ 35 \times \cos 50° &= s \\ 35 \times 0.6428 &= s \\ s &= 22.498 \\ s &= 22.5 \text{ cm to 1 d.p.} \end{aligned}$$

   b. Find the measure of t:

$$\begin{aligned} \sin\theta &= \frac{\mathrm{O}}{\mathrm{H}} \\ \sin 35° &= \frac{t}{10} \\ 10 \times \sin 35° &= t \\ 10 \times 0.5736 &= t \\ t &= 5.736 \\ t &= 5.7 \text{ cm to 1 d.p.} \end{aligned}$$

   c. Calculate $\angle F$:

$$\begin{aligned} \sin F &= \frac{5}{8} = 0.625 \\ \sin^{-1}(\sin F) &= \sin^{-1}(0.625) \\ F &= 38.68° \end{aligned}$$

   Calculate $\angle H$:
   $\angle H = 180° - 90° - 38.68° = 51.32°$

   d. Calculate $\angle M$:

$$\begin{aligned} \sin M &= \frac{9}{12} = 0.75 \\ \sin^{-1}(\sin M) &= \sin^{-1}(0.75) \\ M &= 48.59° \end{aligned}$$

   Calculate $\angle O$:
   $180° - 90° - 48.59° = 41.41°$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L057 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L058 in the Pupil Handbook before the next class.

Lesson Title: Solving right-angled triangles	Theme: Trigonometry	
Lesson Number: M4-L058	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply the Pythagorean theorem and trigonometric ratios to solve for sides and angles of right-angled triangles, including word problems.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Bring trigonometric tables ("log books") to class if available, and ask pupils to bring them.	

Opening *(2 minutes)*

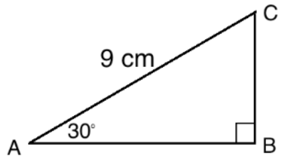
1. Discuss:
   - What does it mean to "solve" a triangle? (Answer: To "solve" means to find any missing side or angle measures.)
   - What methods do you know for solving triangles? (Answer: trigonometric and inverse trigonometric functions; Pythagoras' theorem; finding angle measures by subtracting from 180°.)

Explain that today's lesson is on solving right-angled triangles using trigonometric ratios and Pythagoras' Theorem.

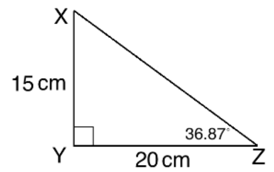
Teaching and Learning *(23 minutes)*

1. Write on the board: Find the missing sides and angles of the triangles:

   a.



   b.



2. Discuss the best way to solve each problem. Allow pupils to share their ideas.
   - Problem a.:
     - How can we find the missing sides? (Answer: Trigonometric ratios)
     - How can we find the missing angle C? (Answer: Subtract the known angles from 180°.)
   - Problem b.:
     - How can we find the missing side? (Answer: Pythagoras' theorem or trigonometric ratios)
     - How can we find the missing angle X? (Answer: Subtract the known angles from 180°.)
3. Solve the problems as a class. Ask pupils to give the steps, and solve on the board as they explain.

   Solutions:

   a. Calculate $|AB|$:

   $\cos 30^{o} = \frac{|AB|}{8}$ Apply the cosine ratio

$$8 \times \cos 30^{\circ} = |AB| \quad \text{Multiply throughout by 8}$$

$$8 \times \frac{\sqrt{3}}{2} = |AB| \quad \text{Use the special angle ratio}$$

$$|AB| = 4\sqrt{3} \text{ cm}$$

Calculate $|BC|$:

$$\sin 30^{\circ} = \frac{|BC|}{8} \quad \text{Apply the sine ratio}$$

$$8 \times \sin 30^{\circ} = |BC| \quad \text{Multiply throughout by 8}$$

$$8 \times \frac{1}{2} = |BC| \quad \text{Use the special angle ratio}$$

$$|BC| = 4 \text{ cm}$$

Calculate $\angle C$: $180° - 90° - 30° = 60°$

b. Calculate $|XZ|$:

$$15^2 + 20^2 = |XZ|^2 \quad \text{Substitute the sides into the formula}$$

$$225 + 400 = |XZ|^2 \quad \text{Simplify}$$

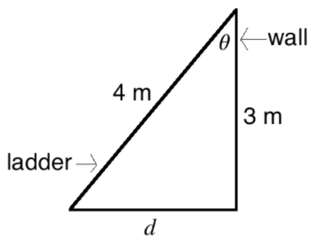
$$625 = |XZ|^2$$

$$\sqrt{625} = \sqrt{|XZ|^2} \quad \text{Take the square root of both sides}$$

$$25 \text{ cm} = |XZ|$$

Calculate $\angle X$: $180° - 90° - 36.87° = 53.13°$

4. Write the following problem on the board: A ladder 4 metres in length is leaning against the side of a building. The height from the ground to the point where the ladder touches the building is 3 metres. Find correct to 1 decimal place:
   a. The distance d is the base of the ladder from the building.
   b. The angle θ is where the ladder meets the building.
5. Ask pupils to work with seatmates to draw a diagram of the problem.
6. Invite a volunteer to draw the diagram on the board. →
7. Ask pupils to work with seatmates to solve problems a. and b.
8. Invite volunteers to write the solutions on the board.



Solutions:

a. Find d using Pythagoras' theorem:

$$\begin{aligned} 3^2 + d^2 &= 4^2 \\ 9 + d^2 &= 16 \\ d^2 &= 16 - 9 \\ \sqrt{d^2} &= \sqrt{7} \\ d &= \sqrt{7} = 2.6 \text{ m.} \end{aligned}$$

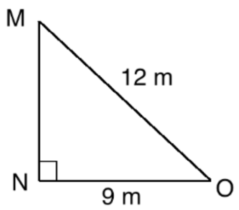
b. Find θ using inverse trigonometry:

$$\begin{aligned} \cos\theta &= \tfrac{3}{4} = 0.75 \\ \cos^{-1}(\cos\theta) &= \cos^{-1}(0.75) \\ \theta &= 41.4° \end{aligned}$$

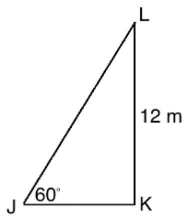
Practice *(14 minutes)*

1. Write on the board: Find the missing sides and angles of the triangles:

a.



b.



c. A ladder leans against a vertical wall at an angle of $30°$ to the wall. If the foot of the ladder is 3 metres away from the wall, calculate the length of the ladder.

2. Ask pupils to work with independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

Solutions:

a. Calculate $\angle M$:

$$\begin{aligned} \sin M &= \tfrac{9}{12} = 0.75 \\ \sin^{-1}(\sin M) &= \sin^{-1}(0.75) \\ M &= 48.59° \end{aligned}$$

Calculate $\angle O$:

$$180° - 90° - 48.59° = 41.41°$$

Calculate $|MN|$:

$$\begin{aligned} |MN|^2 + 9^2 &= 12^2 \\ |MN|^2 + 81 &= 144 \\ |MN|^2 &= 63 \\ |MN| &= \sqrt{63} \\ |MN| &= 3\sqrt{7} \end{aligned}$$

b. Calculate $|JK|$:

$$\begin{aligned} \tan 60° &= \tfrac{12}{|JK|} \\ |JK| &= \tfrac{12}{\tan 60°} \\ |JK| &= \tfrac{12}{\sqrt{3}} \end{aligned}$$

Calculate $|JL|$:

$$\begin{aligned} \sin 60° &= \tfrac{12}{|JL|} \\ |JL| &= \tfrac{12}{\sin 60°} \\ |JL| &= \tfrac{12}{\frac{\sqrt{3}}{2}} \\ |JL| &= \tfrac{24}{\sqrt{3}} \end{aligned}$$

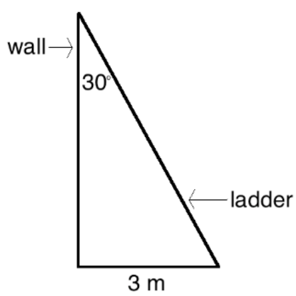
Pythagoras' theorem may also be used to find $|JL|$.

Calculate $\angle L$:

$$180° - 90° - 60° = 30°$$

c. Draw a diagram, and use the sine ratio to solve for the ladder's length (l):

$$\begin{aligned} \sin 30° &= \tfrac{3}{l} \\ 0.5 &= \tfrac{3}{l} \\ 0.5l &= 3 \\ l &= \tfrac{3}{0.5} = 6 \text{ m.} \end{aligned}$$



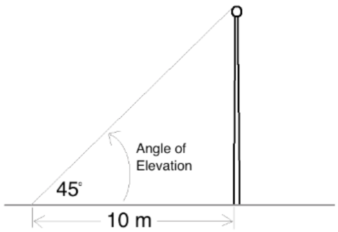
Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L058 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L059 in the Pupil Handbook before the next class.

Lesson Title: Angles of elevation and depression	**Theme:** Trigonometry	
Lesson Number: M4-L059	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve practical problems related to angles of elevation and depression.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(2 minutes)*

1. Write the following problem on the board: At a point 10 metres away from a flag pole, the angle of elevation of the top of the pole is 45°. What is the height of the pole?
2. Ask pupils to work with seatmates to draw a diagram for the problem.
3. Invite a group of pupils with a correct diagram to draw it on the board.
4. Explain that today's lesson is angles of elevation and depression.

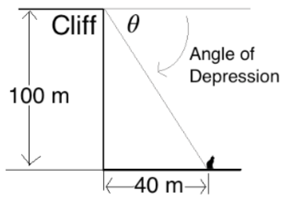


Teaching and Learning *(18 minutes)*

1. Explain:
    - "Elevation" is related to height. Problems on angles of elevation handle the angle that is associated with the height of an object.
    - Angle of elevation problems generally deal with 3 measures: the angle, the distance from the object, and the height of the object.
2. Discuss: Looking at the diagram, how would you find the height of the flag pole? (Answer: Apply trigonometry; we can use the tangent ratio.)
3. Solve the problem on the board, involving pupils in each step:

$$\tan 45° = \frac{h}{10} \quad \text{Set up the equation}$$
$$1 = \frac{h}{10} \quad \text{Substitute } \tan 45° = 1$$
$$10 \text{ m} = h$$

5. Write the following problem on the board: A cliff is 100 metres tall. At a distance of 40 metres from the base of the cliff, there is a cat sitting on the ground. What is the angle of depression of the cat from the cliff?
6. Invite pupils to work with seatmates to draw a diagram for the problem.
7. Ask a group of pupils with a correct diagram to draw it on the board.



4. Explain:
    - "Depression" is the opposite of elevation. An angle of depression is an angle in the downward direction.

- The angle of depression is the angle made with the **horizontal** line. In this example, the horizontal line is at the height of the cliff.
- Angle of depression problems generally deal with 3 measures: the angle, the horizontal distance, and the depth of the object.
- Depth is the opposite of height. It is the distance downward.

5. Solve the problem on the board, explaining each step:

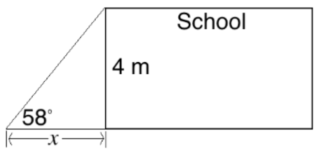
$$\begin{aligned} \tan\theta &= \tfrac{100}{40} = 2.5 && \text{Set up the equation} \\ \tan^{-1}(\tan\theta) &= \tan^{-1}(2.5) && \text{Take the inverse tangent} \\ \theta &= 68.2 && \text{Use the tangent tables} \end{aligned}$$

The angle of depression is 68.2°.

6. Write the following problems on the board:
    a. A school building is 4 metres tall. At a point x metres away from the building, the angle of elevation is 58°. Find x.
    b. A hospital is 5 metres tall. A point is x metres away from the building, and the angle of depression is 17.35°. Find x.
7. Ask pupils to work with seatmates to draw diagrams and solve each problem.
8. Invite volunteers to write the solutions and diagrams on the board.

Solutions:

a. Diagram:

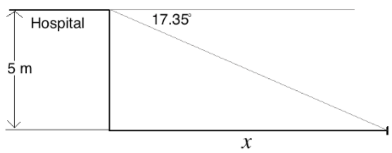


Solution:

$$\begin{aligned} \tan 58° &= \tfrac{4}{x} \\ 1.6 &= \tfrac{4}{x} \\ x &= \tfrac{4}{1.6} \\ x &= 2.5 \text{ m} \end{aligned}$$

The point is 2.5 metres away.

b. Diagram:



Solution:

$$\begin{aligned} \tan 17.35° &= \tfrac{5}{x} \\ 0.3125 &= \tfrac{5}{x} \\ x &= \tfrac{5}{0.3125} \\ x &= 16 \text{ m} \end{aligned}$$

The point is 16 metres away.

Practice *(19 minutes)*

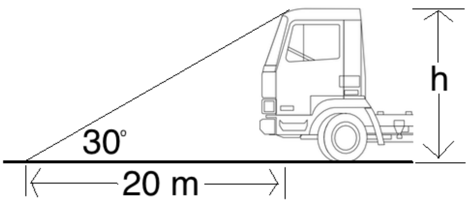
1. Write the following problems on the board:
    a. At a point 20 metres away from a truck, the angle of elevation of the top of the truck is 30°. What is the height of the truck?
    b. A house is 2 metres tall. At a distance d metres away from the house, the angle of elevation is 50.2°. Find d.
    c. A child kicked a football off the top of a tower that is 3 metres tall. The ball landed on the ground. The angle of depression of the ball from the top of the tower is 7.12°. How far is the ball from the tower?

d. A point X is on the same horizontal level as the base of a building. If the distance from X to the building is 10 m and the height of the building is 23 m, calculate the angle of depression of X from the top of the building. Give your answer to the nearest degree.

2. Ask pupils to work with independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

Solutions:

a. Diagram:



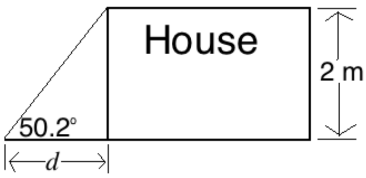
Solution:

Using special angle 30:

$$\tan 30° = \frac{h}{20}$$
$$\frac{\sqrt{3}}{3} = \frac{h}{20}$$
$$h = \frac{20\sqrt{3}}{3} \text{ m}$$

Alternatively, pupils can use $\tan 30° = 0.5774$, and find $h = 11.548$ m.

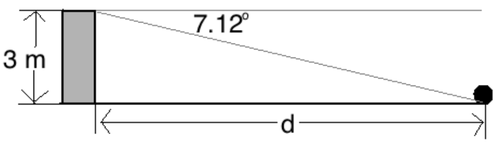
b. Diagram:



Solution:

$$\tan 50.2° = \frac{2}{d}$$
$$1.2 = \frac{2}{x}$$
$$x = \frac{2}{1.2}$$
$$x = 1.7 \text{ m}$$

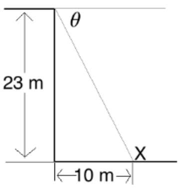
c. Diagram:



Solution:

$$\tan 7.12° = \frac{3}{d}$$
$$0.125 = \frac{3}{d}$$
$$d = \frac{3}{0.125}$$
$$d = 24 \text{ m}$$

d. Diagram:



Solution:

$$\tan\theta = \frac{23}{10} = 2.3$$
$$\tan^{-1}(\tan\theta) = \tan^{-1}(2.3)$$
$$\theta = 66.5$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L059 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L060 in the Pupil Handbook before the next class.

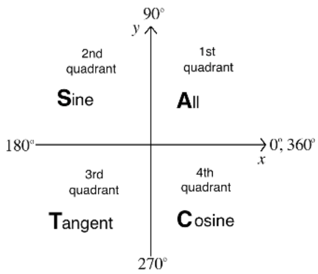
Lesson Title: The unit circle and trigonometric functions of larger angles	**Theme:** Trigonometry	
Lesson Number: M4-L060	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Define $\sin\theta$ and $\cos\theta$ as ratios within a unit circle. 2. Solve problems involving trigonometric functions of obtuse and reflex angles.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Prepare the Unit Circle on vanguard, or draw it on the board before class.	

Opening *(2 minutes)*

1. Write the following on the board: $\sin 100°$ $\cos 180°$ $\tan 240°$
2. Discuss: Can you find the trigonometric functions of these angles?
3. Encourage pupils to see that trigonometry tables cannot be used for obtuse and reflex angles. If they recall how to solve these from a previous class, allow them to explain.
4. Explain that today's class is on finding the trigonometric functions of angles greater than 90°, which are obtuse and reflex angles.

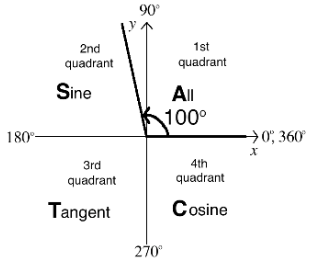
Teaching and Learning *(25 minutes)*

1. Draw the diagram at right on the board:

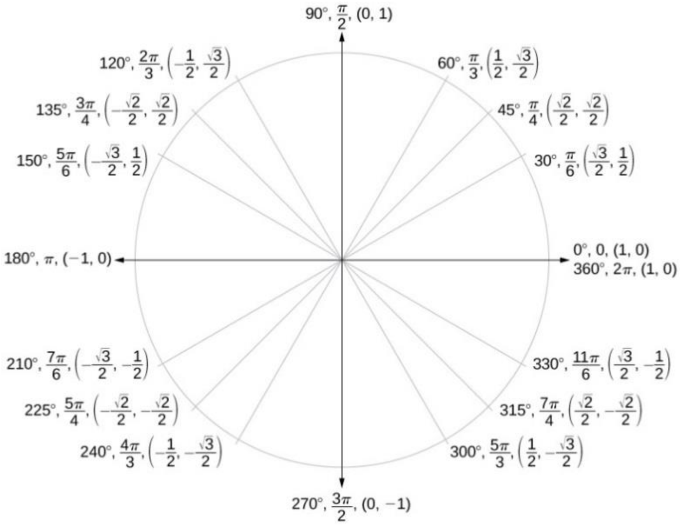


2. Explain:
   - This chart is used to determine the sign (positive or negative) of the trigonometric function of an angle.
   - Angles are centered at the origin and open in the counterclockwise direction.
   - An angle in the first quadrant is acute, an angle in the second quadrant is obtuse, and an angle in the third or fourth quadrant is a reflex angle.
3. Explain how to determine the sign of a ratio:
   - We use the word "ACTS" to remember which trigonometric functions are **positive** in which quadrant. The word ACTS starts in the **first** quadrant and goes in a **clockwise** direction.
4. Explain how to determine the value of a ratio:
   - Each obtuse or reflex angle has an "associated acute angle". This is the acute angle that it forms with the x-axis when it is laid on the 4 quadrants.
   - To find the trigonometric ratio of an obtuse or reflex angle, find the ratio of the associated acute angle. Then, apply the correct sign for that quadrant.
5. Call pupils' attention to the first example you wrote on the board: $\sin 100°$

6. Draw 100° on the ACTS diagram:



7. Discuss: What is the associated acute angle for 100°? (Answer: 80°, the angle formed by 100° and the x-axis.)
8. Ask a volunteer to find $\sin 80°$ in the sine table. (Answer: $\sin 80° = 0.9848$)
9. Write on the board: $\sin 80° = \sin 100° = 0.9848$
10. Explain: The result is positive because the angle is in the second quadrant, S.
11. Draw the unit circle on the board or prepare it on vanguard:



Unit Circle[2]

12. Explain:
    - This is a unit circle. It is drawn on the Cartesian plane so that the length of its radius is 1 unit.
    - Each point P on the circle has coordinates that are an ordered pair.
    - The x-value of the ordered pair is the cosine of the angle formed by P.
    - The y-value of the ordered pair is the sine of the angle formed by P.
13. Write on the board: $x = \cos\theta$, $y = \sin\theta$
14. Ask pupils to look at the second problem on the board: $\cos 180°$
15. Ask pupils to find the answer on the unit circle, and allow them to discuss until they find the answer. (Answer: $\cos 180° = -1$)
16. Ask pupils to look at the third problem on the board: $\tan 240°$

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17. Explain: We can solve this by finding the corresponding angle formed by 240° and the x-axis, or we can solve using the unit circle and $\tan 240° = \frac{\sin 240°}{\cos 240°}$.
18. Solve on the board using both methods. Make sure pupils understand both.

Method 1: Use the corresponding angle	**Method 2:** Use the unit circle
• The corresponding acute angle is $240° - 180° = 60°$. • Find $\tan 60°$ in the tangent table, which is 1.732. • Tangent is positive, because $240°$ falls in the 3rd quadrant. • Therefore, $\tan 240° = 1.732$ Alternatively, use $\tan 60° = \sqrt{3}$, which is a special angle. This gives $\tan 240° = \sqrt{3}$.	• At $240°$ on the unit circle, we have $\cos 240° = -\frac{1}{2}$ and $\sin 240° = -\frac{\sqrt{3}}{2}$. • Therefore, we have: $\tan 240° = \frac{\sin 240°}{\cos 240°} = -\frac{\sqrt{3}}{2} \div -\frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$

19. Explain: The answer can be given as $\sqrt{3}$ or 1.732. These are equal.
20. Write the following on the board: Find: a. $\sin 120°$ b. $\cos 225°$ c. $\tan 300°$
21. Ask pupils to work with seatmates to solve the problems. They may use either method they prefer, but they should **not** use a calculator.
22. Ask volunteers to give their answers and explain how they found them. (Answers: a. $\frac{\sqrt{3}}{2}$ or 0.8660; b. $-\frac{\sqrt{2}}{2}$ or -0.7071; c. $-\sqrt{3}$ or -1.732)

Practice *(12 minutes)*

1. Write the following problems on the board: Find the trigonometric functions of the angles:
   a. $\tan 110°$ b. $\sin 300°$ c. $\sin 150°$ d. $\tan 210°$ e. $\cos 330°$
2. Ask pupils to work with independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.
   Solutions:
   a. $\tan 110° = -2.747$; b. $\sin 300° = -\frac{\sqrt{3}}{2} = -0.8660$; c. $\sin 150° = \frac{1}{2} = 0.5$; d. $\tan 210° = \frac{1}{\sqrt{3}} = 0.5774$; e. $\cos 330° = \frac{\sqrt{3}}{2} = 0.8660$

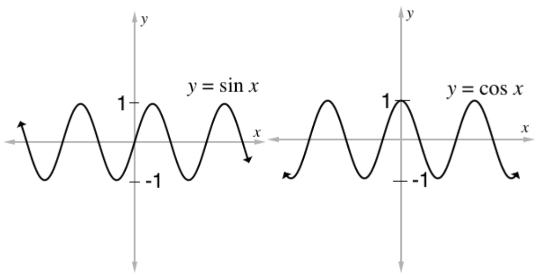
Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L060 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L061 in the Pupil Handbook before the next class.

Lesson Title: Graphs of trigonometric functions	**Theme:** Trigonometry	
Lesson Number: M4-L061	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to draw the graph of $\sin\theta$, $\cos\theta$, and functions of the form $y = a\sin\theta + b\cos\theta$.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Draw the graphs in Opening on the board.	

Opening *(2 minutes)*

1. Sketch the graphs below on the board:



2. Discuss: What do you notice about the graphs of sine and cosine?
3. Allow pupils to share ideas, and encourage them to observe the following:
   - The curves have the same shape.
   - They both go on forever in both x-directions, and remain between $y = -1$ and $y = 1$.
   - They have different starting points. $y = \sin x$ intersects the origin. $y = \cos x$ intersects the y-axis at $y = 1$.
4. Explain that this lesson is on graphing sine and cosine functions.

Teaching and Learning *(19 minutes)*

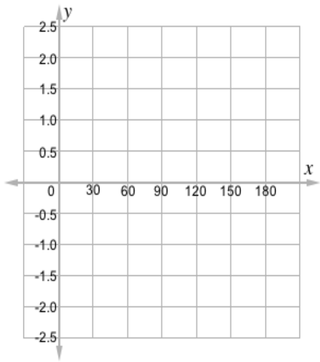
1. Explain:
   - We can graph the sine and cosine curves using values from the trigonometric tables, from the unit circle, or from a calculator.
   - In this lesson we will work with examples that include both sine and cosine functions, because that type of problem is often on the WASSCE exam.
2. Write the following problem on the board: Draw the graph of $y = 2\sin x + \cos x$ for values of x from 0° to 180°, using intervals of 30°.
3. Discuss and let pupils share their ideas: What steps would you take to graph this function on the Cartesian plane?
4. Draw the empty table of values on the board:

x	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$
$\sin x$							
$2\cos x$							
$\sin x + 2\cos x$							

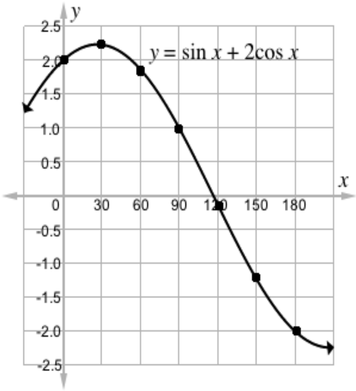
5. Explain:
    - To help us stay organised while doing calculations, we have a row for each trigonometric function.
    - In the last row, we will add the sine and cosine terms together and get the value of our function y.
6. Ask volunteers to give the values of $\sin x$ and $2\cos x$ for each angle in the table, correct to **1 decimal place**. Encourage them to use trigonometric tables.
7. As they give the values, write them in the table:

x	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$
$\sin x$	0	0.5	0.9	1	0.9	0.5	0
$2\cos x$	2.0	1.7	1.0	0	-1.0	-1.7	-2.0
$\sin x + 2\cos x$	2.0	2.2	1.9	1.0	-1.9	-2.2	-2.0

8. Draw an empty Cartesian plane on the board, labeling the axes as shown below:



9. Invite volunteers to come to the board and plot the points. Support the pupils as needed.
10. Connect all of the points in a curve, and label it as shown:



11. Write the following on the board: Use the graph to solve $\sin x + 2\cos x = 0$
12. Ask pupils to give the solution and explain how they found it. Accept approximate answers. (Answer: $x = 116°$, because that is where the curve crosses the x-axis.)

Practice *(18 minutes)*

1. Write the following on the board:
    a. Copy and complete the table of values, correct to one decimal place, for the relation $y = 3\sin x - \cos x$.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$3\sin x - \cos x$	-1.0	0.6		3.0				-0.6	-2.1	-3.0		-2.4	

b. Using scales of 2 cm to 30° on the x-axis and 2 cm to 1 unit on the y-axis, draw the graph of the relation $y = 3\sin x - \cos x$ for $0° \leq x \leq 360°$.

c. Use the graph to solve $3\sin x - \cos x = 0$

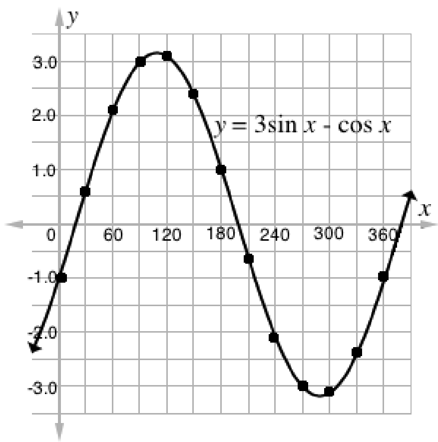
2. Explain: If you have a ruler, use it to make the marks on your x- and y-axes 2 centimetres apart. If you do not have a ruler, estimate 2 cm. What is important is that the tick marks on your axes are the same distance apart.
3. Work as a class if needed to complete a few values in the table and plot the first few points on the graph.
4. Ask pupils to work independently or with seatmates to complete the problem.
5. Walk around to check for understanding and clear misconceptions.
6. Invite a few volunteers to come to the board to write the solutions.

Solutions:

a. Completed table:

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$3\sin x - \cos x$	-1.0	0.6	2.1	3.0	3.1	2.4	1.0	-0.6	-2.1	-3.0	-3.1	-2.4	-1.0

b. Graph:



c. Identify the points where the function intersects the x-axis. Approximate values are $x = 20°, 200°$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L061 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L062 in the Pupil Handbook before the next class.

<table>
<tr><td>Lesson Title: Sine and cosine rules</td><td colspan="2">Theme: Trigonometry</td></tr>
<tr><td>Lesson Number: M4-L062</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to use the sine and cosine rules to calculate lengths and angles in triangles.</td><td colspan="2">Preparation
1. Review the content of this lesson and be prepared to explain the solutions.
2. Write the problem in Opening on the board.</td></tr>
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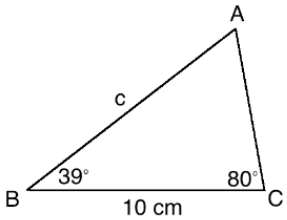
Opening *(2 minutes)*

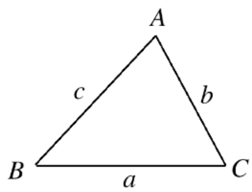
1. Write the following problem on the board: Find the length of missing side c:



2. Discuss and let pupils share their ideas:
   - Is it possible to find the length of c with the information given?
   - What steps would you take to solve this?
3. Explain that this lesson is on the sine and cosine rules. These allow us to solve for missing angles and sides in a triangle.

Teaching and Learning *(15 minutes)*

1. Discuss: What information do we have in this triangle? (Answer: 2 angles and the side between them.)
2. Explain:
   - We will use the **sine rule** to solve this problem.
   - Using the sine rule, we can solve a triangle if we are **given 2 angles and 1 side**, or if we are given **2 sides and the angle opposite 1 of them**.
3. Write on the board: Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ for a triangle:



4. Solve the problem in Opening on the board, involving pupils in each step:

   Use two fractions from the sine rule: $\frac{a}{\sin A} = \frac{c}{\sin C}$

   Substitute known values (a and C) into the formula:

   $$\frac{10}{\sin A} = \frac{c}{\sin 80°}$$

   There are 2 unknowns. Find A by subtracting the known angles of the triangle from 180: $A = 180° - (39° + 80°) = 61°$

   Substitute $A = 61°$ into the formula:

$$\frac{10}{\sin 61°} = \frac{c}{\sin 80°}$$

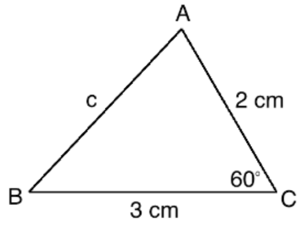
$$10 \times \sin 80° = c \times \sin 61°$$

$$c = \frac{10 \times \sin 80°}{\sin 61°}$$ Solve for c

$$c = \frac{10 \times 0.9848}{0.8746}$$ Substitute values from the sine table

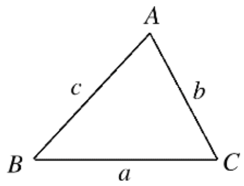
$$c = 11.26 \text{ cm}$$ Simplify

5. Write the following problem on the board: Find the length of missing side c:



6. Explain:
   - We cannot use the sine rule, because we do not have enough information.
   - We can use the **cosine rule** if **two sides and the angle between them** are given, as in the problem on the board.
7. Write on the board: Cosine rule: The following are true for the triangle:

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$



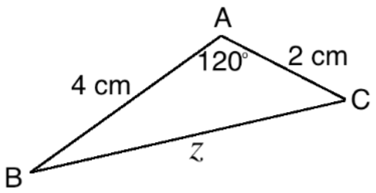
8. Solve the problem on the board, involving pupils in each step:

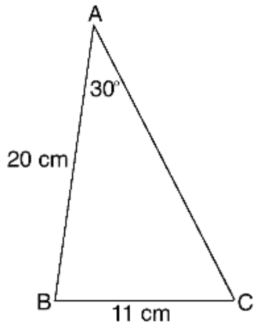
c^2	$=$	$a^2 + b^2 - 2ab \cos C$	Formula
	$=$	$3^2 + 2^2 - 2(3)(2) \cos 60°$	Substitute values from triangle
	$=$	$3^2 + 2^2 - 2(3)(2)(0.5)$	Substitute $\cos 60° = 0.5$
	$=$	$9 + 4 - 12\ (0.5)$	Simplify
	$=$	$13 - 6$	
c^2	$=$	7	
c	$=$	$\sqrt{7} = 2.65$ cm to 2 d.p.	Take the square root of both sides

9. Discuss:
   - When can you use the sine rule? (Answer: When we are given 2 angles and 1 side, or 2 sides and the angle opposite 1 of them.)
   - When can you use the cosine rule? (Answer: When two sides and the angle between them are given.)

Practice *(22 minutes)*

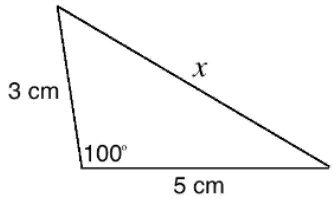
1. Write the following problems on the board:
   a. Find the length of z:
   b. Find angles B and C in the triangle below:





c. Find the remaining angles of ΔABC if $a = 8$ cm, $b = 9.2$ cm, and $\angle B = 60°$.

d. Find the length of x in the triangle below:



2. Ask pupils to work with independently or with their seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

Solutions:

a. Use cosine rule:

$$\begin{aligned} z^2 &= 4^2 + 2^2 - 2(4)(2)\cos 120° \\ &= 4^2 + 2^2 - 2(4)(2)(-0.5) \\ &= 16 + 4 + 8 \\ z^2 &= 28 \\ z &= \sqrt{28} \\ z &= 5.29 \quad \text{cm to 2 d.p.} \end{aligned}$$

b. Use sine rule:

$$\begin{aligned} \frac{11}{\sin 30°} &= \frac{20}{\sin C} \\ 11 \times \sin C &= 20 \times \sin 30° \\ \sin C &= \frac{20 \times \sin 30°}{11} \\ \sin C &= \frac{20 \times 0.5}{11} = \frac{10}{11} \\ \sin C &= 0.9091 \\ C &= \sin^{-1} 0.9091 \\ C &= 65.38° \end{aligned}$$

c. Use sine rule:

Step 1. Find the measure of A:

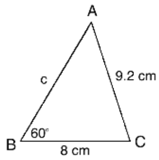
$$\begin{aligned} \frac{a}{\sin A} &= \frac{b}{\sin B} \\ \frac{8}{\sin A} &= \frac{9.2}{\sin 60°} \\ 9.2 \times \sin A &= 8 \times \sin 60° \\ \sin A &= \frac{8 \times \sin 60°}{9.2} \\ \sin A &= \frac{8 \times 0.8660}{11} = \frac{6.928}{11} \\ \sin A &= 0.6298 \\ A &= \sin^{-1} 0.6298 \\ A &= 39.03° \end{aligned}$$

d. Use cosine rule:

$$\begin{aligned} x^2 &= 3^2 + 5^2 - 2(3)(5)\cos 100° \\ &= 3^2 + 5^2 - 2(3)(5)(-0.1736) \\ &= 9 + 25 + 5.208 \\ x^2 &= 39.208 \\ x &= \sqrt{39.208} \\ x &= 6.26 \quad \text{cm to 2 d.p.} \end{aligned}$$

Diagram for problem c:

Step 2. Find the measure of C.
$C = 180° - (60° + 39.03°) = 80.97°$



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L062 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L063 in the Pupil Handbook before the next class.

Lesson Title: Three-figure bearings	Theme: Bearings	
Lesson Number: M4-L063	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify angles measured clockwise from the geographic north. 2. Represent bearings as angles in three digits. 3. Find the reverse bearing of a given bearing. 4. Solve simple problems involving three figure bearings.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Bring a protractor to class, and ask pupils to bring protractors. If possible, make a large protractor from paper that you can use on the board.	

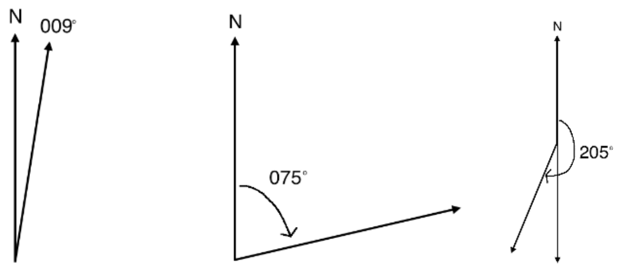
Opening *(1 minute)*

1. Discuss: What are bearings used for? (Example answers: navigation, determining direction and distance)
2. Explain that this lesson is on three-figure bearings.

Teaching and Learning *(18 minutes)*

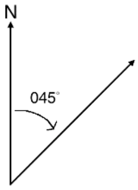
1. Explain: Three-figure bearings are bearings given in 3 digits. These 3 digits give the angle of the bearing from the geographic north.
2. Draw an arrow pointing north on the board. →

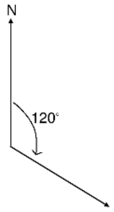
3. Explain:
   - Three-figure bearings give the angle in the clockwise direction.
   - The angles range from 000° to 360°. They must always have 3 digits, even when they're actually less than 100 degrees.
4. Use a protractor to draw and label 009°, 075° and 205° on the board. Make sure pupils understand:

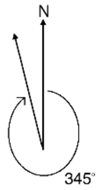


5. Write the following on the board: Draw diagrams for the following three point bearings:
   a. 045°   b. 120°   c. 345°
6. Ask pupils to work with seatmates to draw the diagrams.
7. Invite volunteers to share their drawings, or draw sketches on the board. For the sake of time, they do not need to do the work again with a protractor.

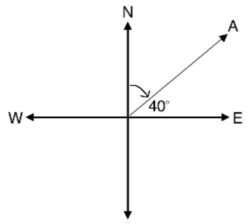
Answers:

a. 

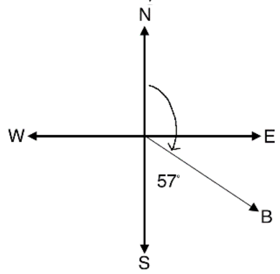
b. 

c. 

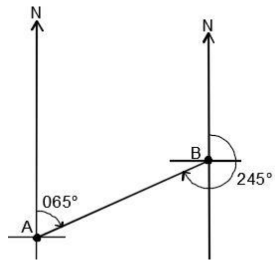
8. Write the following problem on the board, and draw the diagram at right: Find the three-point bearing of A. →



9. Discuss: How can we find the bearing of A? (Answer: The 4 directions are given, and we know east is 90° from north. Subtract the given angle from 90°.)
10. Solve on the board: $A = 90° - 40° = 50°$; $A = 050°$
11. Write the following problem on the board and draw the diagram at right: Find the three-point bearing of B. →



12. Ask pupils to solve the problem with seatmates.
13. Invite a volunteer to write the solution on the board. (Answer: $B = 180° - 57° = 123°$)
14. Discuss: What is the meaning of "reverse"? (Example answers: Opposite, to go backwards or in the opposite direction.)
15. Explain:
    - When we talk about "reverse" bearings, we must have 2 points.
    - The bearing from B to A is the reverse of the bearing from A to B.
16. Draw the diagram on the board. →

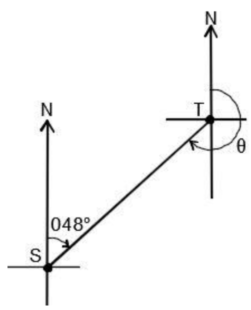


17. Explain:
    - The bearing from A to B is 065°.
    - The bearing from B to A is 245°.
    - To find each bearing, we use the line that joins them and the north direction.
18. Write on the board and explain:

    Reverse bearing = $\theta + 180°$ if θ is less than $180°$

    Reverse bearing = $\theta - 180°$ if θ is more than $180°$
19. Show that this is true for the example given on the board. Add 180 to 65 to find the reverse bearing: $65° + 180° = 245°$
20. Write the following problem on the board: If the bearing of T from S is 048°, find the bearing of S from T.
21. Ask pupils to work with seatmates to draw a diagram and solve the problem.
22. Invite volunteers to share their diagram and solution with the class.

    Solution: $\theta = 48° + 180° = 228°$; Diagram →



Practice *(20 minutes)*

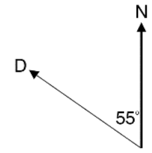
1. Write the following problems on the board:

a. Draw points with the following bearings from north (N) on one diagram:

A: 136°, B: 200°, C: 301°

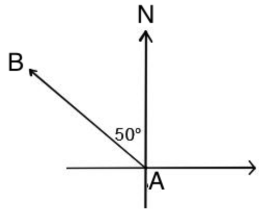
b. A ship at sea is on a bearing of 068° from your current location. Draw a diagram for this.

c. Find the three-point bearing of D in the diagram:



d. The bearing of X from Y is 072°. Draw a diagram and find the bearing of Y from X.

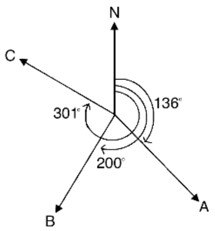
e. In the diagram, find:
   i. The bearing of B from A
   ii. The bearing of A from B



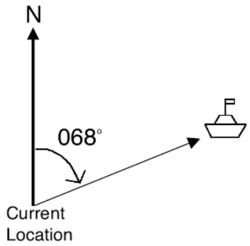
2. Ask pupils to work with independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board.

Solutions:

a.

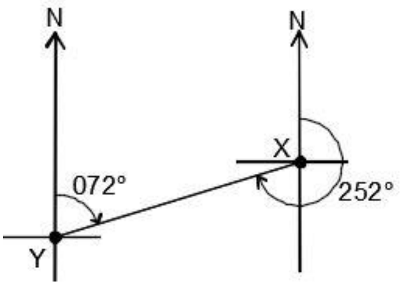


b.



c. Subtract the given angle from 360° to find the angle that D makes when the line rotates clockwise from N: $360° - 55° = 305°$

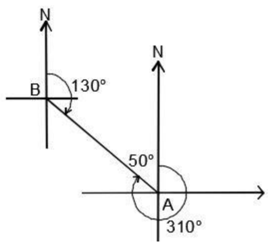
d.



Solution: $72° + 180° = 252°$

e. i. Find the bearing from north. Subtract the given angle (50°) from 360°: $360° - 50° = 310°$.

ii. Find the reverse bearing using the result from part i: $310° - 180° = 130°$.



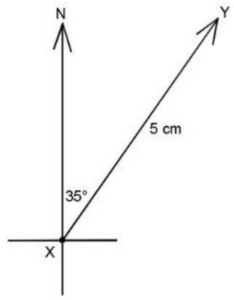
Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L063 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L064 in the Pupil Handbook before the next class.

Lesson Title: Distance-bearing form	Theme: Bearings	
Lesson Number: M4-L064	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Write the distance and bearing of one point from another as (r, θ). 2. Interpret a distance-bearing problem and draw a corresponding diagram.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Draw the diagram in Opening on the board.	

Opening *(2 minutes)*

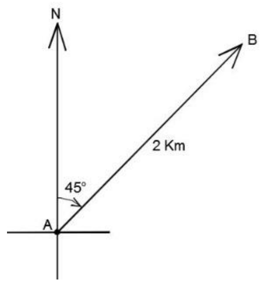
1. Draw the diagram at right on the board:



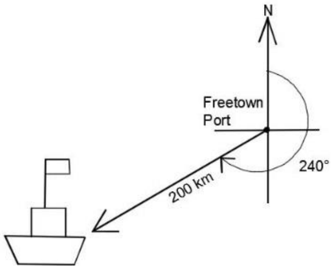
2. Discuss:
   - What is the three-point bearing from X to Y? (Answer: 035°)
   - How would you write the bearing from X to Y?
3. Allow pupils to write their answers on the board. They may recall bearings from a previous class and write the distance-bearing form.
4. Explain that this lesson is on distance-bearing form.

Teaching and Learning *(21 minutes)*

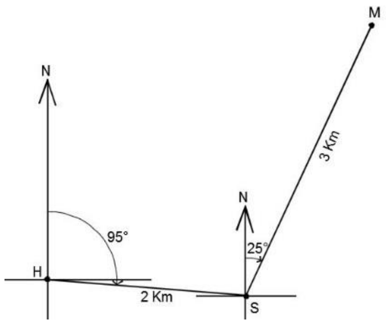
1. Write on the board: $\overrightarrow{XY} = (5 \text{ cm}, 035°)$
2. Explain:
   - The position of point Y from point X is described by these 2 numbers.
   - To describe the relationship between two points, give the distance and then the three-point bearing in brackets.
3. Write on the board: The position of a point Q from another point P can be represented by $\overrightarrow{PQ} = (r, \theta)$, where r is the distance between the 2 points, and θ is the three-point bearing from P to Q.
4. Write a problem on the board: A hunter starts at point A and travels through the bush 2 km in the direction 045° to point B. Give the bearing and draw a diagram.
5. Ask a volunteer to give the bearing for this problem. (Answer: $\overrightarrow{AB} = (2 \text{ km}, 045°)$)
6. Ask pupils to describe what the diagram will look like in their own words. Encourage discussion.
7. Draw the diagram on the board:



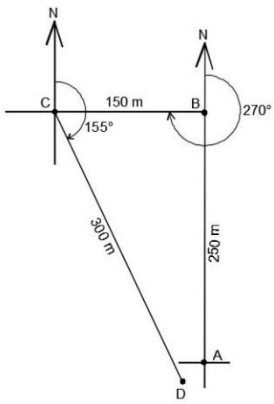
8. Write the following problem on the board: A boat sailed from Freetown port at a bearing of 240°. It is now 200 km from Freetown. Write the ship's bearing and draw a diagram.
9. Ask pupils to work with seatmates to draw the diagram.
10. Walk around to check for understanding and clear misconceptions.
11. Invite a volunteer to write the bearing on the board. (Answer: $(200 \text{ km}, 240°)$)
12. Ask volunteers to share their drawings with the class. Accept accurate diagrams.
    Diagram:



13. Write the following problem on the board: A pupil walked 2 km in the 095° direction from home (point H) to school (point S). She then walked 3 km in the 025° direction from school to the market (point M).
    a. Give the bearing from H to S.
    b. Give the bearing from S to M.
    c. Draw the diagram.
14. Ask pupils to give the answers to a. and b., then write them on the board. (Answers: a. $\overrightarrow{HS} = (2 \text{ km}, 095°)$; b. : $\overrightarrow{SM} = (3 \text{ km}, 025°)$)
15. Ask pupils to work with seatmates to draw the diagram.
16. Invite a volunteer to draw the diagram on the board:

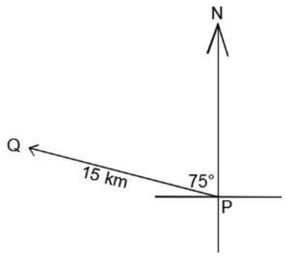


17. Write the following problem on the board: Sia walked 250 metres due north, then 150 metres due west. She then walked 300 metres on a bearing of 155°.
    a. Write the bearings for each of her 3 walks.
    b. Draw a diagram of her movement.
18. Ask pupils to work with seatmates to solve the problem.
19. Walk around to check for understanding and clear misconceptions.
20. Invite volunteers to write the solution on the board.
    Solutions:
    1. $(250 \text{ m}, 000°)$, $(150 \text{ m}, 270°)$, $(300 \text{ m}, 155°)$
    2. Diagram →



Practice *(16 minutes)*

1. Write the following problems on the board:
   a. Write the bearing from point P to Q from the diagram:

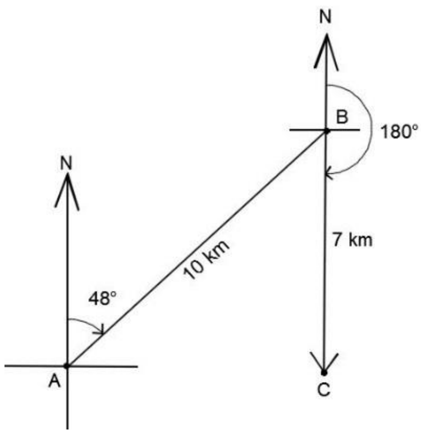


   b. A driver starts at point A and travels 10 km in the direction 048° to point B. He then travels 7 km south to point C.
      i. Write the bearing from A to B.
      ii. Write the bearing from B to C.
      iii. Draw a diagram.
   c. A ship travels 60 km from point R in the direction 300° to point S. It then travels 75 km from point S in the direction 160° to point T.
      i. Write the bearing from R to S.
      ii. Write the bearing form S to T.
      iii. Draw a diagram.
2. Ask pupils to work individually to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

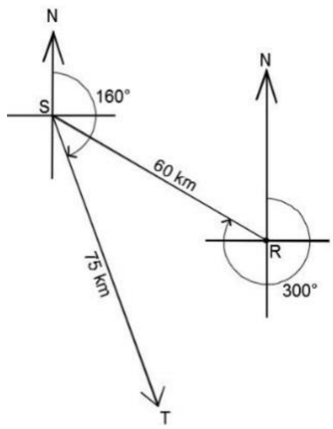
   Solutions:

   a. Find the three point bearing: $360° - 75° = 285°$;
   Bearing: $\overrightarrow{PQ} = (15\text{ km}, 285°)$

   b. **i.** $\overrightarrow{AB} = (10\text{ km}, 048°)$
   ii. $\overrightarrow{BC} = (7\text{ km}, 180°)$
   iii. Diagram:



   c. **i.** $\overrightarrow{RS} = (60\text{ km}, 300°)$
   ii. $\overrightarrow{ST} = (75\text{ km}, 160°)$
   iii. Diagram:

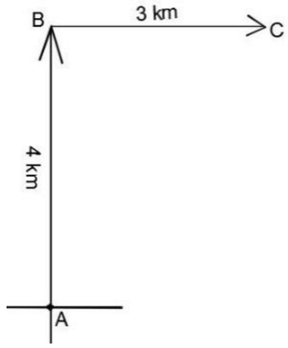


Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L064 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L065 in the Pupil Handbook before the next class.

Lesson Title: Bearing problem solving – Part 1	**Theme:** Bearings	
Lesson Number: M4-L065	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Solve bearings problems with right triangles. 2. Apply Pythagoras' theorem and trigonometric ratios to calculate distance and direction.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

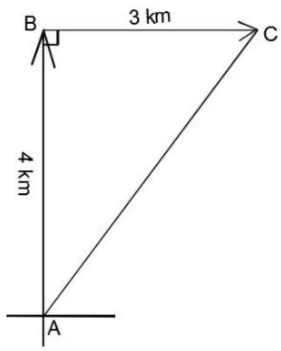
Opening *(2 minutes)*



1. Write on the board: Hawa walked 4 km from point A to B in the north direction, then 3 km from point B to C in the east direction. Draw a diagram.
2. Ask pupils to work with seatmates to draw the diagram.
3. Invite a volunteer to draw the diagram on the board. Diagram →
4. Explain that this lesson is on solving bearing problems. Pupils will use Pythagoras' theorem and trigonometric ratios to solve for distance and direction.

Teaching and Learning *(21 minutes)*

1. Discuss: How far is point C from point A? How can you find the distance?
2. Allow discussion, then explain: The points A, B and C form a right-angled triangle. We can use Pythagoras' theorem to find the distance from C to A.
3. Draw a line connecting A to C, and the lines to show that B is a right angle. →



4. Solve on the board, involving pupils in each step:

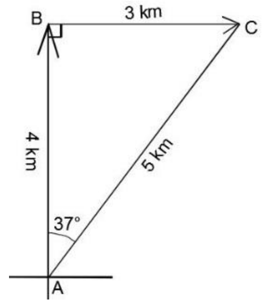
$$\begin{aligned} |AB|^2+|BC|^2 &= |AC|^2 && \text{Apply Pythagoras' theorem} \\ 4^2+3^2 &= |AC|^2 && \text{Substitute known lengths} \\ 16+9 &= |AC|^2 && \text{Simplify} \\ 25 &= |AC|^2 && \\ \sqrt{25} &= \sqrt{|AC|^2} && \text{Take the square root of both sides} \\ 5\text{ km} &= |AC| && \end{aligned}$$

5. Discuss: What is the bearing of C from A? How can you find it?
6. Allow discussion, then explain: We can use trigonometry to find the angle of the triangle at point A.

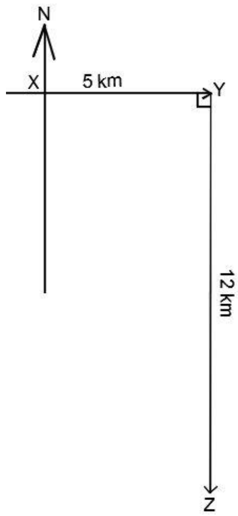
7. Solve on the board, involving pupils in each step:

$\tan A$	$=$	$\frac{3}{4} = 0.75$	Apply tangent ratio
$\tan^{-1}(\tan\ A)$	$=$	$\tan^{-1}(0.75)$	Take inverse tangent of both sides
A	$=$	$\tan^{-1}(0.75)$	
A	$=$	$36.87°$	From the tangent table

8. Label the diagram with the length and angle you have calculated:



9. Write in distance-bearing form: $\overrightarrow{AC} = (5\text{ km}, 037°)$
10. Write the following problem on the board: A ship traveled 5 km due east from point X to point Y, then 12 km due south from point Y to point Z.
    a. Draw a diagram for the problem.
    b. Find the distance from point X to point Z.
    c. Find the bearing from point X to point Z.
11. Ask pupils to work with seatmates to draw the diagram.
12. Walk around to check for understanding and clear misconceptions.
13. Invite a volunteer to draw the diagram on the board. Diagram →
14. Ask pupils to work with seatmates to solve b and c.
15. Invite volunteers to write the solutions on the board.

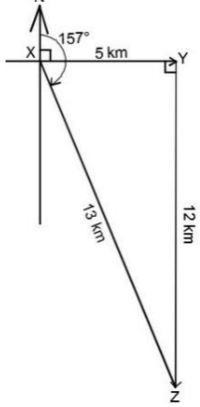


Solutions:

b. Use Pythagoras' theorem:

$\lvert XY\rvert^2 + \lvert YZ\rvert^2$	$=$	$\lvert XZ\rvert^2$	Apply Pythagoras' theorem
$5^2 + 12^2$	$=$	$\lvert XZ\rvert^2$	Substitute known lengths
$25 + 144$	$=$	$\lvert XZ\rvert^2$	Simplify
169	$=$	$\lvert XZ\rvert^2$	
$\sqrt{169}$	$=$	$\sqrt{\lvert XZ\rvert^2}$	Take the square root of both sides
13 km	$=$	$\lvert XZ\rvert$	

c. The angle of the bearing from X to Z is more than 90°. Find the angle of X in the triangle XYZ, and add this to 90°.



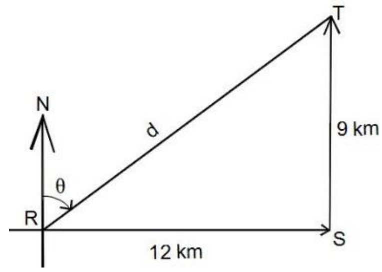
$\tan X$	$=$	$\frac{12}{5} = 2.4$	Apply tangent ratio
$\tan^{-1}(\tan\ X)$	$=$	$\tan^{-1}(2.4)$	Take inverse tangent of both sides
X	$=$	$\tan^{-1}(2.4)$	
X	$=$	$67.38°$	From the tangent table

Round to the nearest degree, and add to 90°: $90° + 67° = 157°$

The bearing from X to Z is $\overrightarrow{XZ} = (13\text{ km}, 157°))$

Practice *(15 minutes)*

1. Write the following problems on the board:
   a. Find the bearing from R to T in the diagram. →
   b. A farmer travels 10 km due north to reach his land. He then travels 24 km due east to bring his harvest to a market.
      ii. Draw a diagram for the problem.
      iii. Find the distance from his starting point to the market.
      iv. Find the bearing from his starting point to the market.



2. Ask pupils to work individually or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

Solutions:

a. Use Pythagoras' theorem to find RT:

$$\begin{aligned} |RS|^2 + |ST|^2 &= |RT|^2 \\ 12^2 + 9^2 &= |RT|^2 \\ 144 + 81 &= |RT|^2 \\ 225 &= |RT|^2 \\ \sqrt{225} &= \sqrt{|RT|^2} \\ 15 \text{ km} &= |RT| \end{aligned}$$

Find angle R inside the triangle:

$$\begin{aligned} \tan R &= \frac{9}{12} = 0.75 \\ \tan^{-1}(\tan\ R) &= \tan^{-1}(0.75) \\ R &= \tan^{-1}(0.75) \\ R &= 36.87° \end{aligned}$$

Subtract from 90°: $90° - 37° = 53°$

Bearing: $\overrightarrow{RT} = (15 \text{ km}, 053°)$

b. i. Diagram: See below. Points may be labeled with any letter of the pupil's choice. In the example diagram, O, F and M are used.

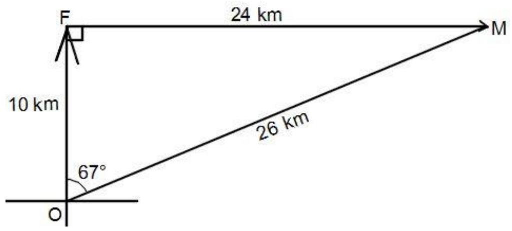
Find OM:

$$\begin{aligned} |OF|^2 + |FM|^2 &= |OM|^2 \\ 10^2 + 24^2 &= |OM|^2 \\ 100 + 576 &= |OM|^2 \\ 676 &= |OM|^2 \\ \sqrt{676} &= \sqrt{|OM|^2} \\ 26 \text{ km} &= |OM| \end{aligned}$$

Bearing: $\overrightarrow{OM} = (26 \text{ km}, 021°)$

Find angle O inside the triangle:

$$\begin{aligned} \tan O &= \frac{10}{26} = 0.3846 \\ \tan^{-1}(\tan\ O) &= \tan^{-1}(0.3846) \\ O &= \tan^{-1}(0.3846) \\ O &= 21.04° \end{aligned}$$



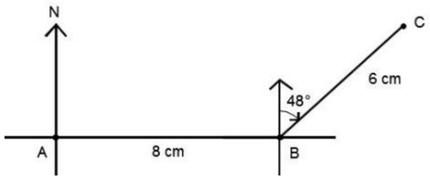
Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L065 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L066 in the Pupil Handbook before the next class.

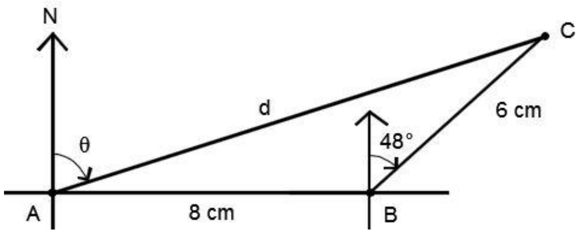
Lesson Title: Bearing problem solving – Part 2	**Theme:** Bearings	
Lesson Number: M4-L066	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Solve bearings problems with acute and obtuse triangles. 2. Apply the sine and cosine rules to calculate distance and direction.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(2 minutes)*

1. Write on the board: A woman walks due east from point A to point B, a distance of 8 kilometres. She then changes direction and walks 6 km to point C on a bearing of 048°.
2. Ask pupils to work with seatmates to draw a diagram for this story.
3. Invite a volunteer to draw the diagram on the board.
4. Explain that this lesson is on solving bearing problems. Pupils will use the sine and cosine rules to calculate distance and direction.



Teaching and Learning *(22 minutes)*



1. Write on the board:
   a. What is the distance from A to C?
   b. What is the bearing of C from A?
2. Label the diagram on the board as shown.
3. Discuss:
   - What steps would you take to solve question a.? Why? (Answer: Use the cosine rule, because we know 2 sides and the angle between them.)
   - What steps would you take to find the bearing of C from A? (Answer: Find angle A in the triangle using the sine rule, and subtract from 90°)
4. Allow discussion, then explain: When you draw a bearings diagram and find a triangle that is not a right-angled triangle, you can use the sine and/or cosine rule.
5. Solve on the board, involving pupils in each step:
   a. Use cosine rule to find $|AC|$:

$\|AC\|^2$	$=$	$\|AB\|^2+\|BC\|^2-2\|AB\|\|BC\|\cos B$	Formula
	$=$	$8^2+6^2-2(8)(6)\ \cos(90+48)°$	Substitute values from triangle
	$=$	$64+36-96\ \cos 138°$	
	$=$	$100-96\ (-0.7431)$	Substitute $\cos 138° = -0.7431$
	$=$	$100+71.3376$	
$\|AC\|^2$	$=$	171.3376	
$\|AC\|$	$=$	$\sqrt{171.3376}=13.09$ km to 2 d.p.	Take the square root of both sides

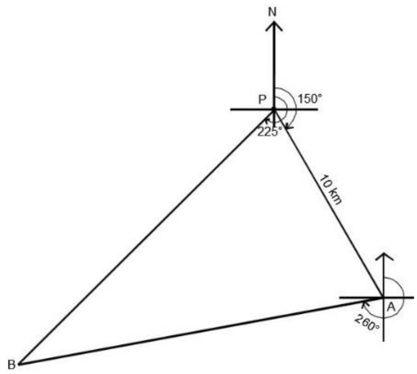
b. Use the sine rule to find the angle inside the triangle at A:

$$\begin{array}{rcll}
\frac{6}{\sin A} & = & \frac{13.09}{\sin 138°} & \text{Substitute in the formula} \\
\sin A & = & \frac{6 \sin 138°}{13.09} & \text{Solve for } A \\
\sin A & = & \frac{6 \times 0.6691}{13.09} & \\
\sin A & = & 0.3067 & \\
A & = & \sin^{-1} 0.3067 & \text{Take the inverse sine of both sides} \\
A & = & 17.86° & \text{Use the sine table}
\end{array}$$

Round to $18°$, and subtract from $90°$ to find the bearing: $90° - 18° = 72°$

The bearing of C from A is $\overrightarrow{AC} = (13.09 \text{ km}, 72°)$.

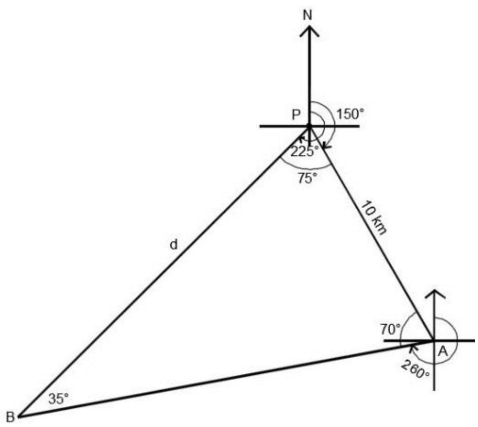
6. Write the following problem on the board: Two ships A and B left a port P at the same time. Ship A travels on a bearing of 150°, and ship B travels on a bearing of 225°. After some time, ship A is 10 km from the port and the bearing of B from A is 260°.
    a. Draw a diagram for the problem.
    b. Find the distance of ship B from the port.



7. Ask pupils to work with seatmates to draw the diagram.
8. Walk around to check for understanding and clear misconceptions.
9. Invite a volunteer to draw the diagram on the board.
10. Discuss: How can we find the distance of ship B from the port? What steps would you take? (Answer: The angles of the triangle can all be found using the properties of triangles and subtraction. We can then apply the sine rule to find the side of the triangle, PB.)
11. Solve the problem on the board. Involve pupils in each step.

Step 1. Solve for missing angles. Label them on the diagram as you find them (see below):

- Find angle P in the triangle using subtraction: $P = 225° - 150° = 75°$
- To find the angle of A in the triangle, first find the other missing angle at point A. It is an opposite interior angle with an angle at point P. The angle at P can be found using subtraction: $180° - 150° = 30°$. Subtract the known angles at A from $360°$: $A = 360° - 260° - 30° = 70°$.
- Find angle B in the triangle by subtracting angles P and A from $180°$: $B = 180° - 75° - 70° = 35°$.



Step 2. Apply the sine rule:

With angle B, there is enough information to apply the sine rule.

$$\frac{10}{\sin 35°} = \frac{d}{\sin 70°} \quad \text{Substitute in the formula}$$
$$d = \frac{10 \sin 70°}{\sin 35°} \quad \text{Solve for } d$$
$$d = \frac{10 \times 0.9397}{0.5736} \quad \text{Use the sine table}$$
$$d = 16.38 \text{ km}$$

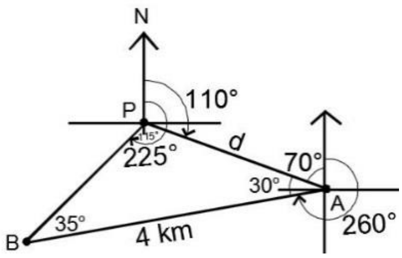
Practice *(15 minutes)*

1. Write the following problem on the board: The bearings of ships A and B from a port P are 225° and 110°, respectively. Ship A is 4 km from ship B on a bearing of 260°. Calculate the distance of ship A from the port.
2. Ask pupils to work independently to solve the problem.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to the board to write the solutions and label the diagram.
   Solution:
   Step 1. Solve for missing angles. Label them on the diagram as you find them (see below):
   - Find angle P in the triangle using subtraction: $P = 225° - 110° = 115°$
   - Find the opposite interior angles of A and P: $180° - 110° = 70°$. Subtract the known angles at A from $360°$: $A = 360° - 260° - 70° = 30°$.
   - Find angle B in the triangle by subtracting angles P and A from $180°$: $B = 180° - 115° - 30° = 35°$.

   Step 2. Apply the sine rule: With angle B, there is enough information to apply the sine rule.

$$\frac{d}{\sin 35°} = \frac{4}{\sin 115°} \quad \text{Substitute in the formula}$$
$$d = \frac{4 \sin 35°}{\sin 115°} \quad \text{Solve for } d$$
$$d = \frac{10 \times 0.5736}{0.9063} \quad \text{Use the sine table}$$
$$d = 6.33 \text{ km}$$

Labelled diagram:



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L066 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L067 in the Pupil Handbook before the next class.

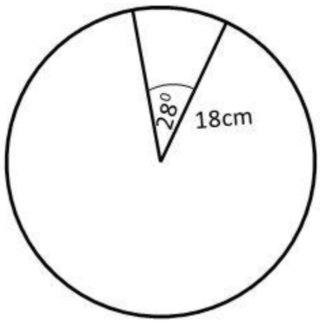
<table>
<tr><td>Lesson Title: Circles</td><td colspan="2">Theme: Geometry</td></tr>
<tr><td>Lesson Number: M4-L067</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, you will be able to:
1. Calculate the circumference and area of a circle.
2. Calculate the length of an arc and area of a sector of a circle.</td><td colspan="2">Preparation
Review the content of this lesson and be prepared to explain the solutions.</td></tr>
</table>

Opening *(2 minutes)*

1. Ask volunteers to write the formulae for circumference and area of a circle on the board. (Answers: $C = 2\pi r$; $A = \pi r^2$)
2. Remind pupils that circumference is the perimeter, or distance around a circle.
3. Explain that this lesson is on solving problems related to area and circumference of a circle, including finding the length of an arc and area of a sector of a circle.

Teaching and Learning *(21 minutes)*

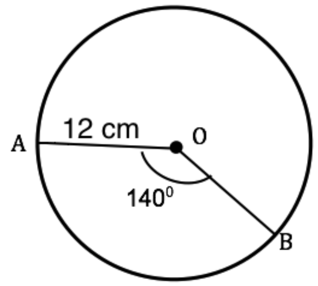
1. Write on the board: For a circle with radius 14 metres, find: a. The circumference; b. The area. Use $\pi = \frac{22}{7}$.
2. Ask pupils to give the steps to solve the problem. As they give them, solve on the board:
   a. $C = 2\pi r = 2 \times \frac{22}{7} \times 14 = 2 \times 22 \times 2 = 88$ m
   b. $A = \pi r^2 = \frac{22}{7} \times 14^2 = \frac{22}{7} \times 196 = 616$ m^2
3. Write the following problem on the board: An arc subtends an angle of 28° at the centre of a circle with radius 18 cm. Find the length of the arc. Use $\pi = \frac{22}{7}$.



4. Explain:
   - An **arc** is a part of the circumference of a circle.
   - The length of an arc of a circle is in proportion to the angle it subtends.
   - To find the length of an arc, multiply the circumference by θ as a fraction of $360°$.
5. Write on the board: Length of arc $= \frac{\theta}{360°} \times C = \frac{\theta}{360°} \times 2\pi r$
6. Solve the problem on the board, explaining each step:

$$\begin{aligned} \text{length of arc} &= \frac{\theta}{360} \times 2\pi r && \text{Formula} \\ &= \frac{28}{360} \times 2 \times \frac{22}{7} \times 18 && \text{Substitute values} \\ & \frac{28\times2\times22\times18}{360\times7} && \text{Simplify} \\ &= 8.8 \text{ cm} \end{aligned}$$

7. Write the following problem on the board: The radius of a circle is 12 cm. Find the area of a sector AOB which has an angle of 140˚. Use $\pi = \frac{22}{7}$.



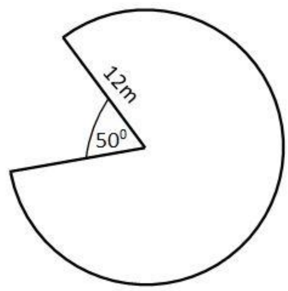
8. Explain:
   - A **sector** is part of the area of a circle.
   - As with an arc, the area of a sector is in proportion to the angle it subtends.
   - To find the area of a sector, multiply the circumference by θ as a fraction of $360°$.
9. Write on the board: Area of sector $= \frac{\theta}{360°} \times A = \frac{\theta}{360°} \times \pi r^2$
10. Solve the problem on the board, explaining each step:

A	$=$	$\frac{140}{360} \times \pi r^2$	Formula
	$=$	$\frac{140}{360} \times \frac{22}{7} \times 12^2$	Substitute values
	$=$	$\frac{140 \times 22 \times 144}{360 \times 7}$	Simplify
A	$=$	176 cm^2	

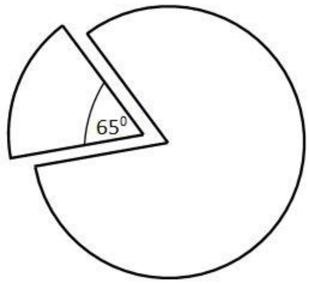
11. Write the following problem on the board: An arc subtends an angle of $125°$ at the centre of a circle of radius 9 cm. Find, correct to 2 decimal places: a. The length of the arc; b. The area of the sector. Use $\pi = 3.14$.
12. Ask pupils to work with seatmates to solve.
13. Walk around to check for understanding and clear misconceptions.
14. Invite volunteers to write the solutions on the board.
    Solutions:
    a. $C = \frac{\theta}{360} \times 2\pi r = \frac{125}{360} \times 2 \times 3.14 \times 9 = 19.63$ cm
    b. $A = \frac{\theta}{360} \times \pi r^2 = \frac{125}{360} \times 3.14 \times 9^2 = 88.31$ cm^2

Practice *(16 minutes)*

1. Write the following problems on the board:
   a. Find the radius of a circle whose area is 66 m^2. Give your answer to 2 decimal places.
   b. A circle with a radius of 7 cm has an arc length of 11 cm. Find its angle. Use $\pi = \frac{22}{7}$
   c. Calculate the arc length of the given shape, which a segment of 50 has been removed from. Give your answer to 3 significant figures. Use $\pi . = 3.14$.
   d. The area of a sector is 690 cm^2. If the radius of the circle is 0.45 m, find the angle of the sector to the nearest degree.



e. A sector of $65°$ was removed from a circle of radius 15 cm. What is the area of the circle left?



2. Ask pupils to work independently to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Ask volunteers to come to the board to write the solutions.

Solutions:

a.
$$\begin{aligned} A &= \pi r^2 \\ r^2 &= \frac{A}{\pi} \\ &= \frac{66}{3.14} \qquad \text{Use } \pi = 3.14 \\ &= 21.019 \\ r &= \sqrt{21.019} = 4.58 \end{aligned}$$

b.
$$\begin{aligned} \text{length of arc} &= \frac{\theta}{360} \times 2\pi r \\ \theta &= \frac{\text{length of arc} \times 360}{2\pi r} \\ &= \frac{11\times 360}{2\times\frac{22}{7}\times 7} \\ &= \frac{11\times 360\times 7}{2\times 22\times 7} \\ \theta &= 90° \end{aligned}$$

c.
$$\begin{aligned} \text{length of arc} &= \frac{\theta}{360} \times 2\pi r \\ &= \frac{310}{360} \times 2 \times 3.14 \times 12 \qquad \text{Since } \theta = 360 - 50 = 310 \\ &= \frac{310\times 2\times 3.14\times 12}{360} \\ \text{length of arc} &= 64.9 \text{ m} \end{aligned}$$

d.
$$\begin{aligned} A &= \frac{\theta}{360} \times \pi r^2 \\ 690 &= \frac{\theta}{360} \times 3.14 \times 45^2 \qquad 0.45 \text{ m} = 45 \text{ cm} \\ \theta &= \frac{690\times 360}{3.14\times 45^2} \\ \theta &= 39° \end{aligned}$$

e.
$$\begin{aligned} \text{From the diagram, } \theta &= 360 - 65 = 295° \\ \text{Area of sector} &= \frac{\theta}{360} \times \pi r^2 \\ \text{Area of sector} &= \frac{295}{360} \times 3.14 \times 15^2 = 579.4 \text{ cm}^2 \end{aligned}$$

Closing *(1 minute)*

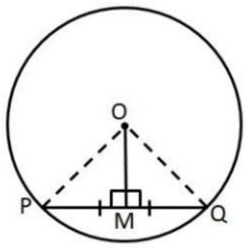
1. For homework, have pupils do the practice activity of PHM4-L067 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L068 in the Pupil Handbook before the next class.

Lesson Title: Circle Theorems 1 and 2	Theme: Geometry	
Lesson Number: M4-L068	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Solve problems related to the perpendicular bisector of a chord. 2. Solve problems related to angles subtended at the centre or circumference of a circle.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Explain that this is the first lesson on circle theorems. This lesson covers 2 circle theorems that can be used to solve various problems.

Teaching and Learning *(21 minutes)*

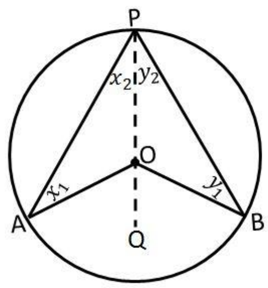


1. Draw the diagram at right on the board.
2. Explain **Circle theorem 1:**
   - A straight line from the centre of a circle that bisects a chord is at right angles to the chord.
   - In the diagram, we have circle with centre O and line OM to mid-point M on chord PQ such that $|\mathrm{PM}| = |\mathrm{QM}|$. Note that OM ⊥ PQ.
3. Write the following problem on the board: The radius of a circle is 12 cm. The length of a chord of the circle is 18 cm. Calculate the distance of the mid-point of the chord from the centre of the circle. Give your answer to the nearest cm.
4. Solve the problem on the board and explain.

M	=	mid-point of PQ	
|MQ|	=	9 cm	$\frac{1}{2}\times$|PQ| $= \frac{18}{2} = 9$
∠OMQ	=	$90°$	line from centre to mid-point ⊥
$\|OQ\|^2$	=	$\|OM\|^2 + \|MQ\|^2$	Pythagoras' Theorem
12^2	=	$\|OM\|^2 + 9^2$	substitute|OQ| $= 12$ cm, |MQ| $= 9$ cm
$\|OM\|^2$	=	$12^2 - 9^2$	
	=	$144 - 81$	
|OM|	=	$\sqrt{63} = 7.94$	
|OM|	=	8 cm	

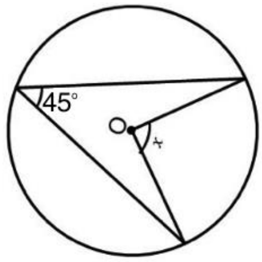
The distance from the mid-point of the chord to the centre of the circle is 8 cm to the nearest cm.

5. Make sure pupils understand circle theorem 1.
6. Draw the diagram on the board →
7. Explain **Circle theorem 2:**
   - The angle subtended at the centre of a circle is twice that subtended at the remaining part of the circumference.



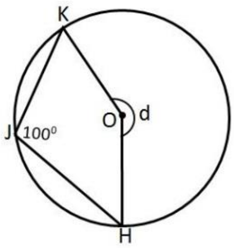
- In the diagram, ∠AOB is subtended at the centre of the circle, and ∠APB at the circumference. Thus, ∠AOB is twice ∠APB.

8. Write on the board: ∠AOB=2 × ∠APB
9. Write the following problem on the board: Find the value of $\angle x$ in the diagram:

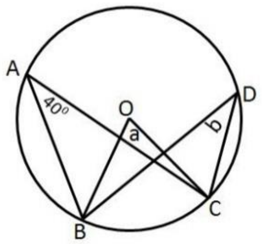


10. Solve the problem on the board, explaining each step:
    Multiply the angle at the circumference by 2 to find x**:**
    $$x = 2 \times 45^{\circ}$$
    $$x = 90^{\circ}$$
11. Make sure pupils understand circle theorem 2.
12. Write the following problems on the board:
    a. The distance of the chord of a circle from the centre of the circle is 4 cm. If the radius of the circle is 8 cm, calculate the length of the chord.
    b. In the diagrams below, O is the centre of each circle. Find the measures of the marked angles.

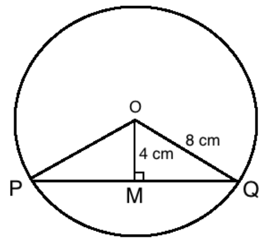
i.



ii.



13. Ask pupils to work with seatmates to solve the problems.
14. Walk around to check for understanding and clear misconceptions.
15. Invite volunteers to write the solutions on the board.
    Solutions:
    a. Draw a diagram (right).



M	=	mid-point of |PQ|	
∠OMQ	=	90°	line from centre to mid-point ⊥
$\lvert OQ\rvert^2$	=	$\lvert OM\rvert^2 + \lvert MQ\rvert^2$	Pythagoras' Theorem
8^2	=	$4^2 + \lvert MQ\rvert^2$	
$\lvert MQ\rvert^2$	=	$8^2 - 4^2$	substitute $\lvert OB\rvert = 8$ cm, $\lvert OM\rvert = 4$ cm
	=	$64 - 16$	
|MQ|	=	$\sqrt{48}$	
	=	6.928	
|MQ|	=	7 cm	
|MQ|	=	|PM|	equal radii
|PQ|	=	|PM| + |MQ|	
	=	$7 + 7$	
|PQ|	=	14 cm	

The length of the chord is 14 cm to the nearest cm.

b. i. d subtends the same arc as 100. It is subtended at the centre while 100 is subtended at the circumference. Therefore:

ii. a subtends the arc at the centre, and b subtends the arc at the circumference.

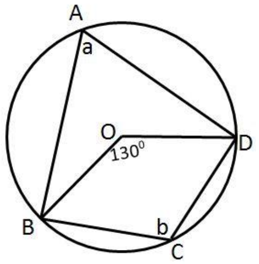
$$\begin{aligned} d &= 2 \times 100 \\ d &= 200° \end{aligned}$$

$$\begin{aligned} a &= 2 \times 40 \\ a &= 80° \\ b &= \tfrac{1}{2} \times 80 \\ b &= 40° \end{aligned}$$

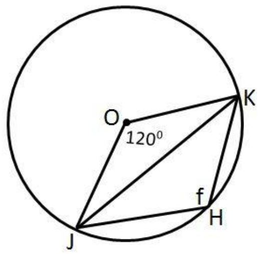
Practice *(17 minutes)*

1. Write the problems on the board:
   a. A chord PQ of length 6 cm is 15 cm from the centre of the circle. Calculate the radius of the circle.
   b. In the diagrams below, the centre of each circle is given by O. Find the measure of each marked angle.

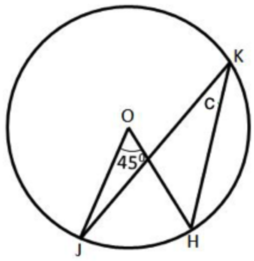
i.



ii.



iii.



2. Ask pupils to work independently to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.
   Solutions:

a.

$$\begin{aligned} N &= \text{mid-point of } |PQ| \\ \angle ONQ &= 90° && \text{line from centre to mid-point } \perp \\ |OQ|^2 &= |ON|^2 + |NQ|^2 && \text{Pythagoras' Theorem} \\ |OQ|^2 &= |15|^2 + 6^2 \\ |OQ|^2 &= 225 + 36 = 261 \\ |OQ| &= \sqrt{261} \\ &= 16.155 \\ |OQ| &= 16 \text{ cm} \end{aligned}$$

b. i. $a = \frac{1}{2} \times 130 = 65°$

reflex $\angle \text{BOD} = 360 - 130 = 230°$ → $b = \frac{1}{2} \times 230 = 115°$

ii. reflex $\angle \text{JOK} = 360 - 120 = 240°$ → $f = \frac{1}{2} \times 240° = 120°$

iii. $c = \frac{1}{2} \times 45° = 22.5°$

Closing *(1 minute)*

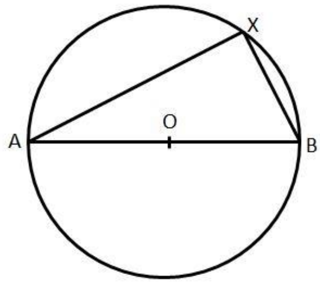
1. For homework, have pupils do the practice activity of PHM4-L068 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L069 in the Pupil Handbook before the next class.

Lesson Title: Circle Theorems 3, 4 and 5	Theme: Geometry	
Lesson Number: M4-L069	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Solve problems related to the angle in a semi-circle. 2. Solve problems related to angles in the same segment. 3. Solve problems related to opposite angles of a cyclic quadrilateral.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

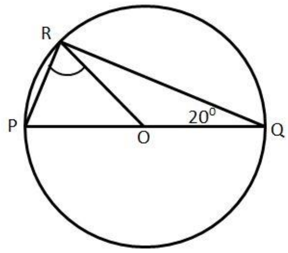
Opening *(1 minutes)*

1. Explain that this is the second lesson on circle theorems. This lesson covers 3 circle theorems that can be used to solve various problems.

Teaching and Learning *(19 minutes)*

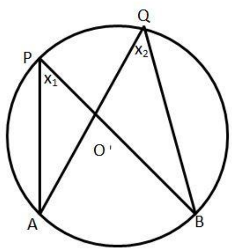


1. Draw the diagram on the board →
2. Explain **Circle theorem 3:**
   - The angle in a semi-circle is a right angle.
   - In the diagram, we have a circle with centre O and diameter AB. X is any point on the circumference of the circle. For any such point X, $\angle AXB$=90°. This theorem shows that the angle of the diameter of a circle subtends a right angle at the circumference.

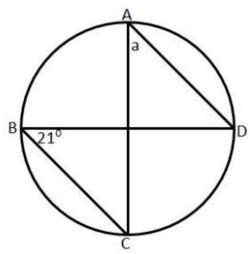


3. Write the following problem and diagram on the board: P, Q and R are points on a circle, centre O. If $\angle RQO$=20°, what is the size of $\angle PRO$?
4. Solve the problem on the board and explain.

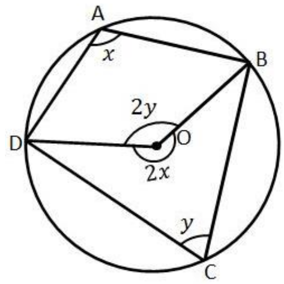
$$
\begin{aligned}
\angle QRO &= \angle RQO = 20° && \text{base } \angle\text{s of isosceles } \Delta \\
\angle PRO &= \angle PRQ - \angle QRO \\
\angle PRO &= 90 - 20 && \angle \text{ in the semi-circle} \\
\angle PRO &= 70°
\end{aligned}
$$



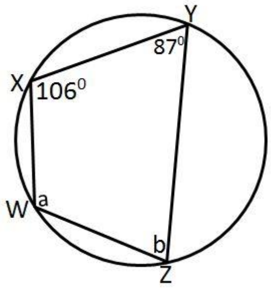
5. Draw the diagram on the board →
6. Explain **Circle theorem 4:**
   - Angles in the same segment are equal.
   - In the diagram, we have circle with centre O with points P and Q on the circumference of the circle. Arc AB subtends $\angle APB$ and $\angle AQB$ in the same segment of the circle. Two angles subtended by the same arc are equal**:** $\angle APB$=$\angle AQB$.
7. Write the following problem on the board: Find the measure of a:



8. Solve the problem on the board: $a = 21°$ because they are angles in the same segment.
9. Draw the diagram on the board →



10. Explain **Circle theorem 5:**
    - The opposite angles of a cyclic quadrilateral are supplementary.
    - A cyclic quadrilateral has all 4 vertices on the circumference of the circle. Both sets of opposite angles are supplementary (they sum to 180°). In the diagram, $\angle BAD + \angle BCD = 180°$ and $\angle ABC + \angle ADC = 180°$.
11. Write the following problem on the board: Find angles a and b in the diagram:



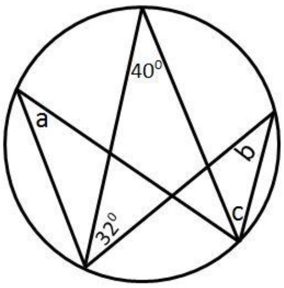
12. Solve the problem on the board, explaining each step:

$$\begin{aligned} a + 87 &= 180 && \text{opposite } \angle\text{s of cyclic quadrilateral} \\ a &= 180 - 87 \\ a &= 93° \\ b + 106 &= 180 && \text{opposite } \angle\text{s of cyclic quadrilateral} \\ b &= 180 - 106 \\ b &= 74° \end{aligned}$$

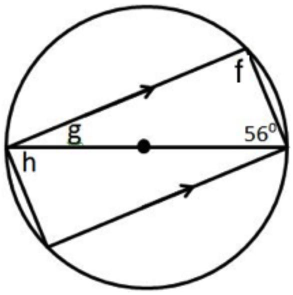
Practice *(19 minutes)*

1. Write the following problems on the board: Find the unknown angles for each of the circles shown below. Point O is the centre of the circle. Give reasons for your answers.

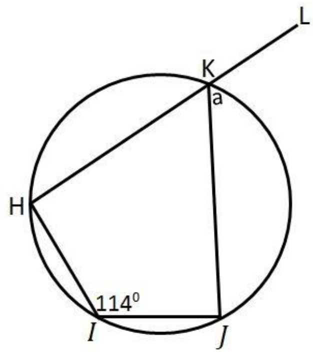
a.



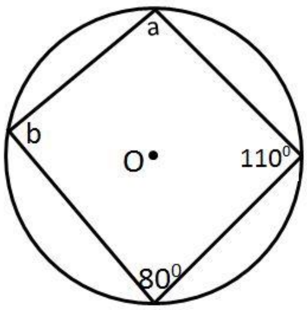
b.



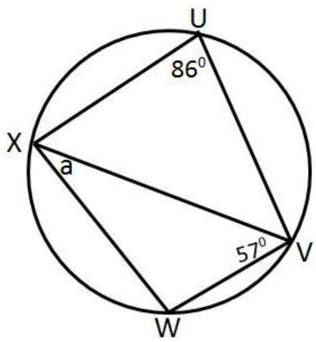
c.



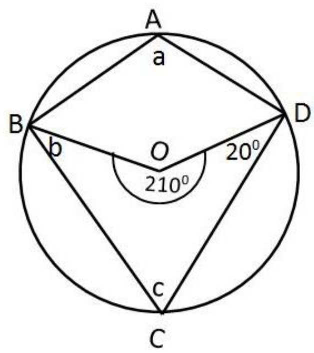
d.



e.



f.



2. Ask pupils to work independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

Solutions:

a.
$$\begin{aligned} a &= 40° && \angle\text{s in the same segment} \\ a &= b && \angle\text{s in the same segment} \\ b &= 40° \\ c &= 32° && \angle\text{s in the same segment} \end{aligned}$$

b.
$$\begin{aligned} f &= 90° && \angle \text{ in a semi-circle} \\ g &= 180 - 90 - 56 && \angle \text{ in a triangle} \\ g &= 34° \\ h &= 56° && \text{alternate } \angle\text{s} \end{aligned}$$

c.
$$\begin{aligned} a &= \angle HIJ \\ a &= 114° \end{aligned}$$

d.
$$\begin{aligned} a &= 180 - 80 && \angle\text{s in a cyclic quadrilateral} \\ a &= 100° \\ b &= 180 - 110 && \angle\text{s in a cyclic quadrilateral} \\ b &= 70° \end{aligned}$$

e.
$$\begin{aligned} \angle VWX + 86 &= 180 && \text{opposite } \angle\text{s of a cyclic quadrilateral} \\ \angle VWX &= 180 - 86 \\ \angle VWX &= 94° \\ a + \angle VWX + 57 &= 180° && \angle\text{s in a triangle} \\ a + 94 + 57 &= 180° \\ a &= 180 - 94 - 57 \\ a &= 29° \end{aligned}$$

f.
$$\begin{aligned} 2a &= 210 && \angle \text{ at the centre} = 2\angle \text{ at the circumference} \\ a &= \tfrac{1}{2} \times 210 \\ a &= 105° \\ c + 105 &= 180 && \text{opposite } \angle\text{s of a cyclic quadrilateral} \\ c &= 180 - 105 \\ c &= 75° \\ b + 20 + 75 + 210 &= 360 && \text{sum of } \angle\text{s in a quadrilateral} \\ b &= 360 - 20 - 75 - 210 \\ b &= 55° \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L069 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L070 in the Pupil Handbook before the next class.

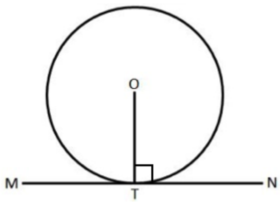
Lesson Title: Circle Theorems 6 and 7	Theme: Geometry	
Lesson Number: M4-L070	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify and draw the tangent line to a circle. 2. Solve problems related to the tangent to a circle.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

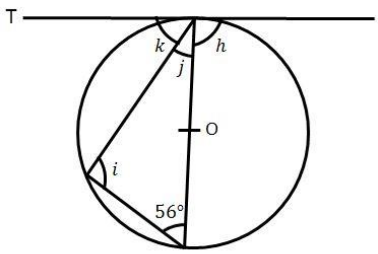
1. Explain that this is the third lesson on circle theorems. This lesson covers 2 circle theorems related to tangent lines.

Teaching and Learning *(22 minutes)*

1. Explain: A tangent is a line which touches a circle at one point without cutting across the circle. It makes contact with a circle at only one point on the circumference.
2. Draw the diagram on the board →

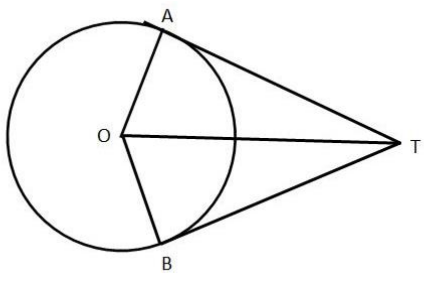


3. Explain **Circle theorem 6:**
   - The angle between a tangent and a radius is equal to $90°$.
   - The shortest line from the centre of a circle to a tangent is a line that is perpendicular to the tangent. In the diagram, $\angle OTN = 90°$, or OT ⊥ MN.
4. Write a problem on the board: In the given figure, a line drawn through T is a tangent and O is the centre of the circle. Find the lettered angles.
5. Solve the problem on the board and explain.



$$\begin{aligned} i &= 90^\circ && \angle \text{ in a semicircle} \\ i + j + 56^\circ &= 180^\circ && \text{sum of interior } \angle\text{s in a } \Delta \\ 90^\circ + j + 56^\circ &= 180^\circ \\ 146^\circ + j &= 180^\circ \\ \\ j &= 180^\circ - 146^\circ \\ j &= 34^\circ \\ \\ h &= 90^\circ && \text{radius} \perp \text{tangent} \\ k + j &= 90^\circ && \text{radius} \perp \text{tangent} \\ k + 34^\circ &= 90^\circ \\ k &= 90^\circ - 34^\circ \\ k &= 56^\circ \end{aligned}$$

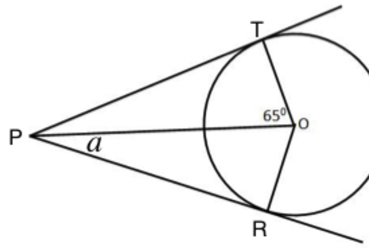
6. Draw the diagram on the board →



7. Explain **Circle theorem 7:**
   - The lengths of the two tangents from a point to a circle are equal.

- For a point T outside a circle with centre O, TA and TB are tangents to the circle at A and B respectively. The lengths of TA and TB are equal. |TA| = |TB|
- Since $\angle$AOT=$\angle$BOT and $\angle$ATO=$\angle$BTO, line TO bisects the angles at O and T. TO is the line of symmetry for the diagram.

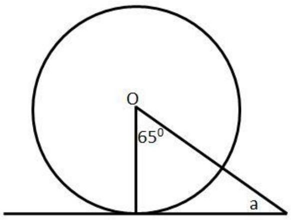
8. Write the following problem on the board: Find the missing angle a in the given circle with centre O.
9. Solve the problem on the board:



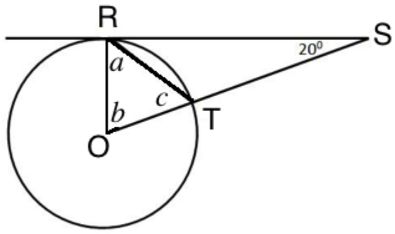
$$\begin{aligned} \angle PTO &= 90° && \text{radius } \perp \text{ tangent} \\ a &= \angle TPO && \text{symmetry (equal tangents)} \\ a + 65 + 90 &= 180 && \angle\text{s in a triangle} \\ a &= 180 - 65 - 90 \\ a &= 25° \end{aligned}$$

10. Write the following problems on the board: Find the measures of the lettered angles:

a.



b.



11. Ask pupils to work with seatmates to solve the problems.
12. Walk around to check for understanding and clear misconceptions.
13. Invite volunteers to write the solutions on the board.

Solutions:

a. Use the interior angles of the triangle to find a; use theorem 6 to identify the right angle.

$$\begin{aligned} a + 90° + 65° &= 180° && \angle\text{s in a triangle} \\ a &= 180° - 90° - 65° \\ a &= 25° \end{aligned}$$

b. Use triangle ROS to find the measure of b; use theorem 6 to identify the right angle. Use the fact that ROT is an isosceles triangle to find a and c. Note that it is isosceles because 2 sides are radii of the circle.

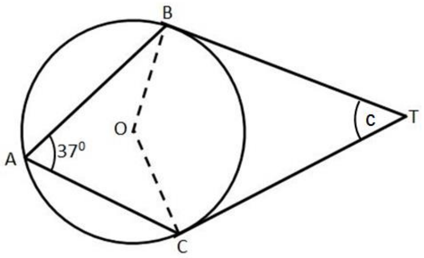
$$\begin{aligned} \angle OSR &= 20° && \text{given} \\ \angle ORS &= 90° && \text{radius } \perp \text{ tangent} \\ 90 + 20 + b &= 180 && \angle\text{s in a triangle} \\ b &= 180 - 110 \\ b &= 70° \\ b &= c && \text{base } \angle\text{s of isosceles } \Delta \\ 2a + 70 &= 180 \\ 2a &= 180 - 70 = 110 \end{aligned}$$

$$a = \frac{110}{2}$$
$$a = c = 55°$$

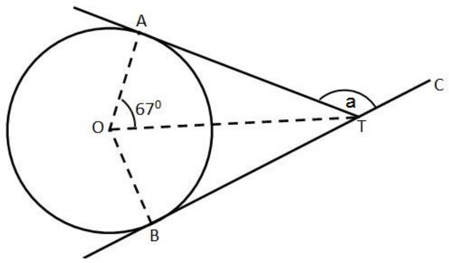
Practice *(16 minutes)*

1. Write the following problems on the board: Find the unknown angles for each of the circles shown below. Point O is the centre of the circle. Give reasons for your answers.

g.



h.



2. Ask pupils to work independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.
   Solutions:

a.

$\angle$BAC	=	$37°$	given
$\angle$BOC	=	$2 \times 37 = 74°$	$\angle$ at the centre=2$\angle$ at the circum.
$\angle$TOC	=	$\angle$TOB	symmetry
$\angle$TOC	=	$\frac{74}{2} = 37°$	
$\angle$CTO $+ 90 + 37$	=	180	$\angle$s in a triangle
$\angle$CTO	=	$180 - 90 - 37 = 53°$	
$\angle$BTC	=	$53 + 53$	
$\angle$BTC	=	$106°$	

b.

$\angle$AOT	=	$67°$	given
$\angle$ATO $+ 90 + 67$	=	180	$\angle$s in a triangle
$\angle$ATO	=	$180 - 90 - 67 = 23°$	
$\angle$ATO	=	$\angle$BTO	symmetry
$\angle$ATB	=	$23 + 23 = 46°$	
$\angle$ATB $+$ $\angle$ATC	=	180	
$46 +$ $\angle$ATC	=	180	$\angle$s in a straight line
$\angle$ATC	=	$180 - 46 = 134°$	

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L070 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L071 in the Pupil Handbook before the next class.

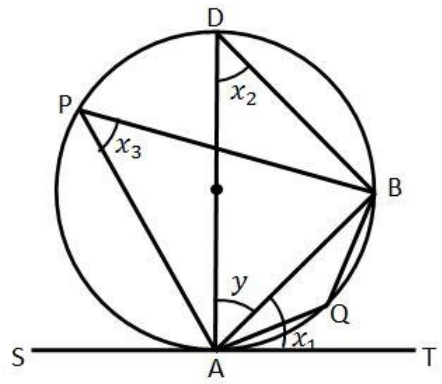
Lesson Title: Circle Theorem 8	**Theme:** Geometry	
Lesson Number: M4-L071	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify the alternate segment theorem. 2. Solve for missing angles using the alternate segment theorem.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

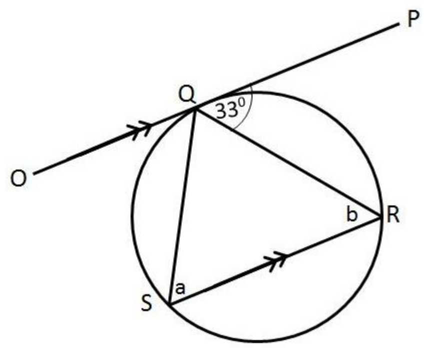
1. Explain that this is the fourth lesson on circle theorems. This lesson covers the eighth and last circle theorem.

Teaching and Learning *(20 minutes)*

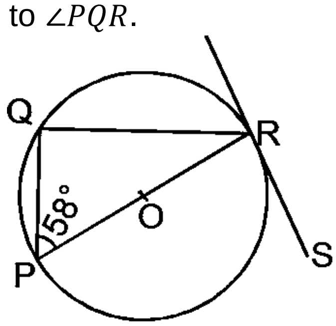
1. Draw the diagram on the board →
2. Explain **Circle theorem 8:**
   - The angle between a chord and a tangent at the end of the chord equals the angle in the alternate segment (i.e. the angle in the other segment, not the one in which the first angle lies).
   - In the diagram, this means that:
     ∠TAB = ∠APB
     ∠SAB = ∠AQB
   - This is known as the alternate segment theorem.



3. Write the following problem on the board: Find the missing angles a and b in the given circle.



4. Solve the problem on the board and explain.
   From circle theorem 8, angle b is equal to $\angle PQR$.
   $$a = \angle PQR = 33°$$
   Note that $\angle PQR$ and b are alternate angles because lines OP and SR are parallel. Therefore, angle b is also equal to $\angle PQR$.
   $$b = \angle PQR = 33°$$
5. Write the following problem on the board: In the diagram, PR is a diameter of the circle centre O. RS is a tangent at R and QPR = 58˚. Find ∠QRS.
6. Solve the problem on the board and explain:

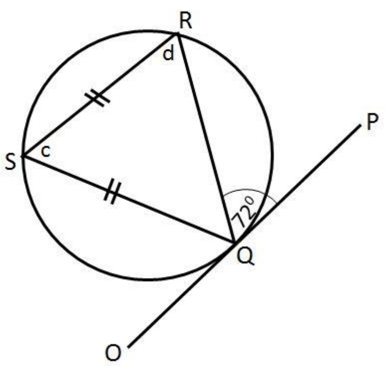


∠PQR	=	90°	∠ in a semicircle
∠PRS	=	∠PQR	∠s in alternate segment

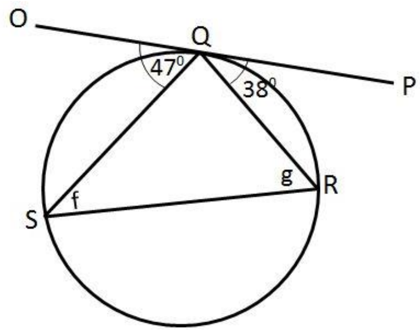
$$\begin{aligned} \angle PRS &= 90° \\ \angle PQR + \angle QPR + \angle PRQ &= 180 && \text{sum of } \angle s \text{ in a } \Delta \\ 90 + 58 + \angle PRQ &= 180 \\ \angle PRQ + 148 &= 180 \\ \angle PRQ &= 180 - 148 \\ \angle PRQ &= 32 \\ \angle QRS &= \angle PRQ + \angle PRS \\ \angle QRS &= 32 + 90 \\ \angle QRS &= 122° \end{aligned}$$

7. Write the following problems on the board: Find the measures of the lettered angles:

   a.



   b.



8. Ask pupils to work with seatmates to solve the problems.
9. Walk around to check for understanding and clear misconceptions.
10. Invite volunteers to write the solutions on the board.

    Solutions:

    a. Use theorem 8 to identify c, then use the isosceles triangle to find d.

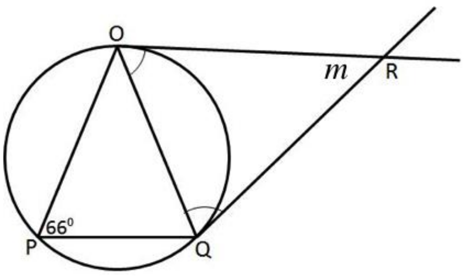
$$\begin{aligned} \angle PQR &= 72° && \text{given} \\ c &= 72° && \angle\text{s in alternate segment} \\ d &= \frac{180-72}{2} && \text{isosceles triangle} \\ d &= 54° \end{aligned}$$

    b. Apply theorem 8 to find f and g. $g = 47°$ and $f = 38°$.

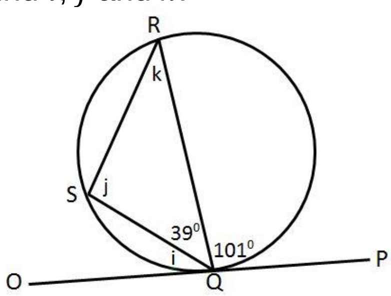
Practice *(18 minutes)*

1. Write the following problems on the board: Find the unknown angles for each of the circles shown below. Give reasons for your answers.

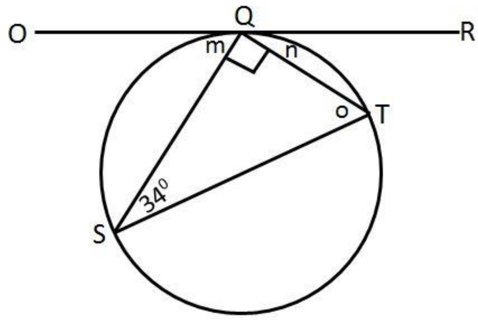
   i. Find m:



   j. Find i, j and k:



k. Find m, n and o:



2. Ask pupils to work independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

Solutions:

a.	$\angle$OPQ	$=$	$66°$	given
	$\angle$ROQ = $\angle$RQO	$=$	$66°$	isosceles triangle, $\angle s$ in alternate segment
	$66 + 66 + m$	$=$	180	$\angle$s in a triangle
	m	$=$	$180 - 66 - 66$	
	m	$=$	$48°$	
b.	$101 + 39 + i$	$=$	$= 180$	$\angle$s in a straight line
	i	$=$	$180 - 101 - 39$	
	i	$=$	$40°$	
	j	$=$	$101°$	$\angle s$ in alternate segment
	k	$=$	i	$\angle s$ in alternate segment
	k	$=$	$40°$	
c.	n	$=$	$34°$	$\angle s$ in alternate segment
	$90 + 34 + o$	$=$	180	$\angle$s in a triangle
	o	$=$	$180 - 90 - 34$	
	o	$=$	$56°$	
	o	$=$	m	$\angle s$ in alternate segment
	m	$=$	$56°$	

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L071 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L072 in the Pupil Handbook before the next class.

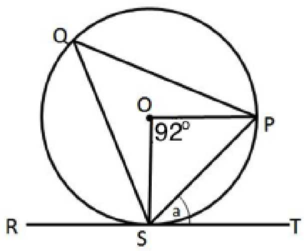
Lesson Title: Circle problem solving	**Theme:** Geometry
Lesson Number: M4-L072	**Class:** SSS 4 **Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply circle theorems and other properties to find missing angles in various circle diagrams.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.

Opening *(1 minute)*

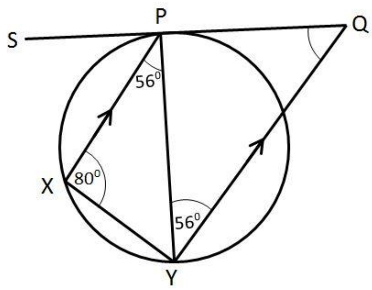
1. Explain that this lesson is on solving problems using the circle theorems and other properties of shapes and angles. These types of problems are often featured on the WASSCE exam.

Teaching and Learning *(19 minutes)*

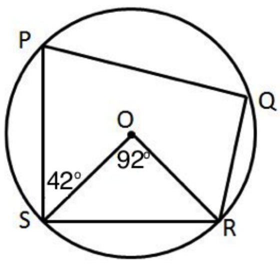
1. Write the following problems on the board:
   a. PQS is a circle with centre O. RST is a tangent at S and $\angle$SOP = $92°$. find $\angle$PST



   b. In the diagram, SQ is a tangent to the circle at P, XP ∥ YQ, $\angle$XPY = $56°$ and $\angle$PXY = $80°$. Find angle PQY.



   c. A circle has centre at O. If $\angle$SOR =$92°$ and $\angle$PSO =$42°$, calculate $\angle$PQR.



2. Ask pupils to work with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain.
   Solutions:
   a. Given: RST is a tangent at S; $\angle$SOP = $96°$)
   Method 1

$\angle PST = \angle SOP$ $\angle$s in alternate segment

$\angle SOP = \frac{1}{2} \times \angle SQP$ $\angle$ at the centre=2$\angle$ at the circumference

$\angle SOP = \frac{1}{2} \times 92 = 46°$

$\therefore \angle PST = 46°$

b. $\angle XPY = 56°$ and $\angle PXY = 80°$ Given

$\angle YPQ = 80°$ alternate $\angle$

$\angle PQY = 180 - 80 - 56$ $\angle s$ in a triangle

$\angle PQY = 46°$

c. $\angle SOR = 92°$ $\angle PSO = 42°$, Given

$\angle OSR = \frac{180-92}{2}$ isosceles triangle

$= 44°$

$\angle PSR = 44 + 42$

$= 86°$

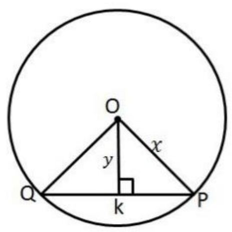
$\angle PQR + 86 \quad 180$ $\angle$s in a cyclic quadrilateral

$\angle PQR = 180 - 86$

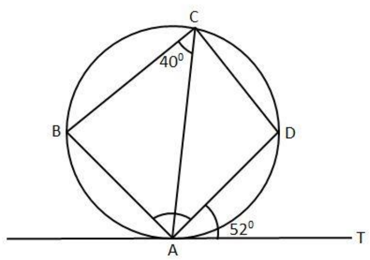
$\angle PQR = 94°$

Practice *(19 minutes)*

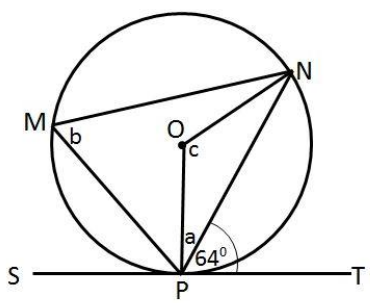
1. Write the following problems on the board:
   a. In the diagram, O is the centre of the circle with radius x. $|PQ| = z$, $|OK| = y$ and $\angle PQR = 90°$. Find the value of z in terms of x and y.



   b. TA is a tangent to the given circle at A. If: $\angle BCA = 40°$ and $\angle DAT = 52°$, find $\angle BAD$.



   c. O is the centre of the circle. ST is a tangent line, and $\angle NPT = 64°$. Find the measures of the angles marked a, b and c.



2. Ask pupils to work independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.

4. Invite volunteers to come to the board to write the solutions.
   Solutions:
   a.
   $$|PQ| = z \quad \text{Given}$$
   $$|KP| = \frac{1}{2}z$$
   $$x^2 = y^2 + \left(\frac{1}{2}z\right)^2 \quad \text{Pythagoras' Theorem}$$
   $$\left(\frac{1}{2}z\right)^2 = x^2 - y^2$$
   $$\frac{1}{2}z = \sqrt{x^2 - y^2}$$
   $$z = 2\sqrt{x^2 - y^2}$$

   b. $\angle$ BCA = $40°$ and $\angle$ DAT = $52°$ given
   $$\angle ACD = 52° \quad \angle s \text{ in alternate segment}$$
   $$\angle BCD = 40 + 52$$
   $$= 92°$$
   $$\angle BAD = 180 - 92 \quad \angle\text{s in a cyclic quadrilateral}$$
   $$\angle BAD = 88°$$
   c. given angle $= 64°$
   $$\angle OPT = 90 \quad \text{radius} \perp \text{tangent}$$
   $$90 = a + 64$$
   $$a = 90 - 64$$
   $$a = 26°$$
   $$b = 64° \quad \angle s \text{ in alternate segment}$$
   $$c = 180 - 2a$$
   $$c = 180 - 2 \times 26 \quad \text{isosceles triangle}$$
   $$c = 128°$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L072 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L073 in the Pupil Handbook before the next class.

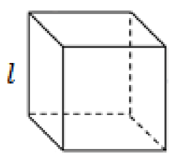
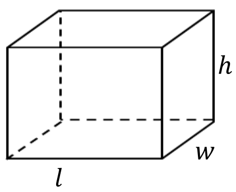
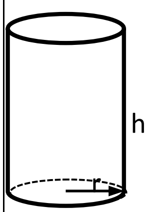
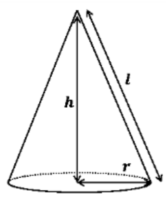
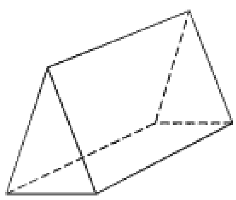
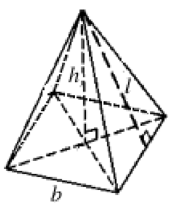
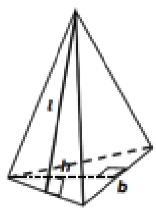
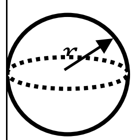
Lesson Title: Surface area	Theme: Mensuration	
Lesson Number: M4-L073	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify the formulae for surface area. 2. Find the surface area of cubes, cuboids, prisms, cylinders, cones, pyramids, spheres and composite solids.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Explain: This lesson is on volume of 8 different types of solids. There is not enough time in this lesson to solve problems on each. However, if you can identify the formula for the volume of a solid, you can solve the related problems.

Teaching and Learning *(18 minutes)*

1. Briefly review the surface area formulae in the Pupil Handbook (in the table below). For the sake of time, it is not necessary to draw them all on the board. Identify the formulae for each. Make sure pupils understand what the diagrams look like, and what the variables stand for.

Cube	Cuboid	Cylinder	Cone
 $SA = 6 \times l^2$	 $SA = 2(lh + hw + lw)$	 $SA = 2\pi r(r + h)$	 $SA = \pi r(l + r)$
Triangular Prism	**Pyramid with a rectangular base**	**Pyramid with a triangular base**	**Sphere**
 No formula	 $SA = b^2 + 2bl$	 $SA = \frac{1}{2}b(h + 3l)$	 $SA = 4\pi r^2$

2. Write the following problem on the board: Find the surface area of sphere with a radius of 0.5 metres. (Use $\pi = 3.14$)

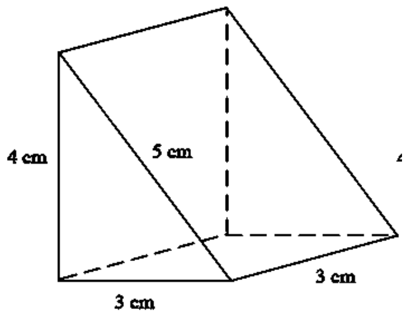
3. Ask pupils to work with seatmates to find the surface area using the correct formula.
4. Invite a volunteer to write the solution on the board and explain.
   Solution:

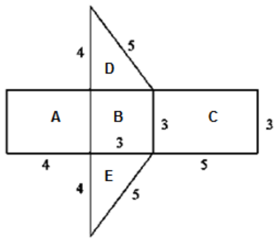
SA	$=$	$4\pi r^2$	Apply the formula
	$=$	$4(3.14)(0.5)^2$	Substitute the known values
	$=$	$4(3.14)(0.25)$	Subtract 14 cm from both sides
	$=$	3.14 m^2	

5. Write the following problem on the board: A cube has surface area 216 cm^2. Find the lengths of its sides.
6. Discuss: How would you solve this problem? (Answer: Apply the formula $SA = 6 \times l^2$ and solve for l.)
7. Solve the problem on the board as a class.
   Solution:

SA	$=$	$6 \times l^2$	Apply the formula
216	$=$	$6 \times l^2$	Substitute the known values
$\frac{216}{6}$	$=$	l^2	Subtract 14 cm from both sides
36	$=$	l^2	
6 cm	$=$	l	

8. Write the following problem on the board: Find the surface area of the triangular prism.
9. Discuss: How would you solve this problem? (Answer: There is no formula, so find the area of each side and add them.)
10. Draw and label a net on the board, and make sure pupils understand.





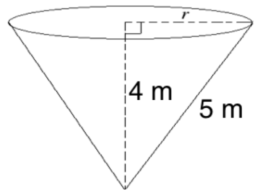
11. Explain: When calculating the surface area, a net can be drawn to show each face in one diagram.
12. Solve the problem as a class:
    Find the area of each face. Note that D and E have the same shape, so only 1 needs to be calculated.

$$\text{A} = 4 \times 3 = 12\text{ cm}^2$$
$$\text{B} = 3 \times 3 = 9\text{ cm}^2$$
$$\text{C} = 5 \times 3 = 15\text{ cm}^2$$
$$2 \times \text{D} = 3 \times 4 = 12\text{ cm}^2$$
$$\text{surface area} = 12 + 9 + 15 + 12 = 48$$

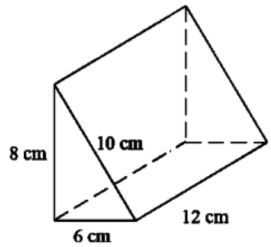
The surface area of the triangular prism $= 48$ cm^2.

Practice *(20 minutes)*

1. Write the following problems on the board:
    a. The diagram shows a cone with height of 4 m and a slant height of 5 m. Find: i. The base radius, r; ii. The surface area of the cone, correct to 3 significant figures. Use $\pi = \frac{22}{7}$.



    b. The surface area of a cylinder is 440 cm^2. If the radius of its base is 7 cm, find its height. Use $\pi = \frac{22}{7}$.
    c. Find the surface area of the triangular prism at right:

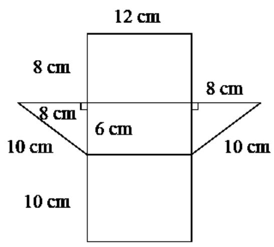


2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.
   Solutions:
   a. i. Use Pythagoras' theorem to find the radius: $r = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3 \text{ m}$
      ii. Calculate the surface area: $SA = \left(\frac{22}{7}\right)(3)(5 + 3) = \frac{22}{7}(24) = 75.4 \text{ cm}^2$
   b. Apply the formula for the surface area of a cylinder. Substitute the given values and solve for h:

SA	$=$	$2\left(\frac{22}{7}\right)(7)(7 + h)$	Apply the formula
440	$=$	$2(22)(7 + h)$	Substitute the known values
440	$=$	$44(7 + h)$	Subtract 14 cm from both sides
$\frac{440}{44}$	$=$	$7 + h$	
$10 - 7$	$=$	h	
3 cm	$=$	h	

   c. Draw a net and note that it has 1 large rectangle and 2 triangles. Surface area = area of large rectangle + 2 × area of triangle

$$\begin{aligned} \text{SA} &= ((8 + 6 + 10) \times 12) + (6 \times 8) \\ &= (24 \times 12) + 48 = 336 \end{aligned}$$



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L073 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L074 in the Pupil Handbook before the next class.

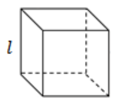
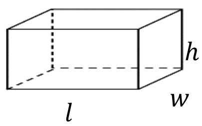
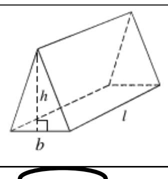
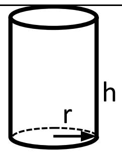
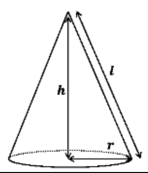
Lesson Title: Volume	**Theme:** Mensuration	
Lesson Number: M4-L074	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify the formulae for volume. 2. Find the volume of cubes, cuboids, prisms, cylinders, cones, pyramids, spheres and composite solids.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

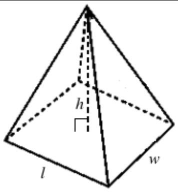
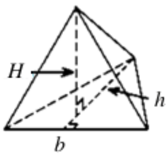
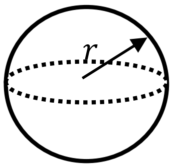
Opening *(1 minute)*

1. Explain: This lesson is on the volume of 8 different types of solids. There is not enough time in this lesson to solve problems for each type. However, if you can identify the formula for the volume of a solid, you can solve related problems.

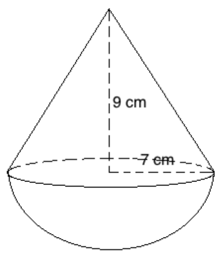
Teaching and Learning *(18 minutes)*

1. Briefly review the volume formulae in the Pupil Handbook (in the table below). For the sake of time, it is not necessary to draw them all on the board. Identify the formulae for each. Make sure pupils understand what the diagrams look like, and what the variables stand for.

Solid	Diagram	Formula
Cube		$V = l^3$, where l is the length of a side.
Cuboid		$V = lwh$, where l and w are the length and width of the base, and h is the height.
Triangular Prism		$V = \frac{1}{2}bhl$, where b and h are the base and height of the triangular face, and l is the length.
Cylinder		$V = \pi r^2 \times h$, where r is the radius of the circular face and h is the height.
Cone		$V = \frac{1}{3}\pi r^2 h,$ where r is the radius and h the height.

Pyramid with a rectangular base		$V = \frac{1}{3}lwh$, where l and w are the length and width of the base rectangle, and h is the height.
Pyramid with a triangular base		$V = \frac{1}{3}AH$, where A is the area of the triangular base, and H is the height.
Sphere		$V = \frac{4}{3}\pi r^3$, where r is the radius.

2. Write the following problem on the board: A container of petrol is in the shape of a cone mounted on a hemisphere. It is built such that the plane face of the hemisphere fits exactly on the base of the cone. The radius of the plane face is 7 cm, and the height of the cone is 9 cm.
   a. Illustrate this information in a diagram
   b. Calculate the: i. Volume of the hemisphere; ii. Volume of the entire solid. (Use $\pi = \frac{22}{7}$)
3. Discuss:
   - What is a hemisphere? (Answer: Half of a sphere.)
   - What does this solid look like? (Answer: A cone sitting on top of a hemisphere.
4. Ask pupils to work with seatmates to draw the diagram. Remind them to label the lengths.
5. Invite a volunteer to draw the diagram on the board. →



6. Discuss: How would we find the volume of a hemisphere? (Answer: Since it is half of a sphere, find the volume of an entire sphere and multiply it by half.)
7. Solve the problem on the board as a class. Involve pupils.
   b. i. Volume of the hemisphere:

V	$=$	$\frac{1}{2}\times\frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$	Formula for hemisphere
	$=$	$\frac{2}{3}\left(\frac{22}{7}\right)7^3$	Substitute $\pi = \frac{22}{7}$, $r = 7$
	$=$	$\frac{44}{21}7^3$	Simplify
	$=$	718.7 cm^3	

   ii. Volume of the solid:

   Step 1. Find the volume of the cone:

V	$=$	$\frac{1}{3}\pi r^2 h$	Formula for hemisphere

$$= \frac{1}{3}\left(\frac{22}{7}\right)(7^2)(9) \qquad \text{Substitute } \pi = \frac{22}{7},\ r = 7,\ h = 9$$

$$= 22(7)(3) \qquad \text{Simplify}$$

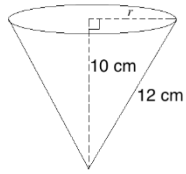
$$= 462 \text{ cm}^3$$

Step 2. Add the volume of the cone and hemisphere:

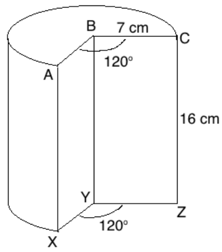
$$V = 718.7 + 462 = 1180.7 \text{ cm}^3$$

Practice *(20 minutes)*

1. Write the following problems on the board:
    a. A cylindrical container has a base radius of 14 cm and a height 20 cm. How many litres, correct to the nearest litre, of liquid can it hold? (Use $\pi = \frac{22}{7}$)
    b. The diagram shows a cone with height 10 cm and slant height 12 cm. Find: i. the base radius, r; ii. the volume of the cone. (Use $\pi = \frac{22}{7}$)



    c. The diagram shows part of a solid cylinder with radius 7 cm and height 16 cm. The missing piece is formed by 2 radii and an angle of 120°. Calculate the volume, correct to 1 decimal place.



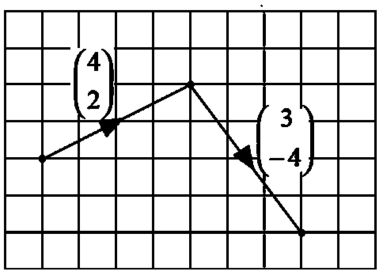
2. Explain problem a: A volume of $1{,}000 \text{ cm}^3$ holds 1 litre ($1\ l = 1{,}000 \text{ cm}^3$). This fact may be needed to solve WASSCE problems.
3. Ask pupils to solve the problems either independently or with seatmates. Solve problems as a class if they do not understand.
4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to come to the board simultaneously to write the solutions.
Solutions:
    a. **Step 1.** Calculate volume: $V = \pi r^2 \times h = \left(\frac{22}{7}\right) 14^2 \times 20 = 12{,}320 \text{ cm}^3$
    Step 2. Divide by $1{,}000\ cm^3$ to find litres: $\frac{12{,}320 \text{ cm}^3}{1{,}000 \text{ cm}^3} = 12\ l$ to the nearest litre
    b. i. Find r with Pythagoras' theorem: $r = \sqrt{12^2 - 10^2} = \sqrt{144 - 100} = \sqrt{44}$ cm
    ii. Calculate volume: $V = \frac{1}{3}\left(\frac{22}{7}\right)\left(\sqrt{44}\right)^2(10) = \frac{22}{21}(44)(10) = 460.95 \text{ cm}^3$
    c. **Step 1.** Find the angle of the rotation in the circular face: $360° - 120° = 240°$
    Step 2. Multiply the volume formula by the fraction of a full rotation that is in the solid $\left(\frac{240}{360} = \frac{2}{3}\right)$: $V = \frac{2}{3}\pi r^2 \times h = \frac{2}{3}\left(\frac{22}{7}\right) 7^2 \times 16 = 1{,}642.7 \text{ cm}^3$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L074 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L075 in the Pupil Handbook before the next class.

Lesson Title: Operations on vectors	**Theme:** Vectors and Transformation	
Lesson Number: M4-L075	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Add and subtract vectors. 2. Multiply a vector by a scalar.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

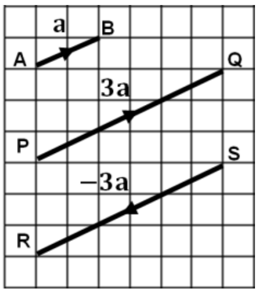
Opening *(1 minute)*



1. Draw the diagram at right on the board.
2. Ask volunteers to describe the diagram. Allow discussion. (Example answer: There are 2 vectors on a plane, $\binom{4}{2}$ and $\binom{3}{-4}$.)
3. Explain that this lesson is on the operations on vectors.

Teaching and Learning *(19 minutes)*

1. Explain:
   - A **vector** is any quantity which has both magnitude and direction.
   - There are 2 vectors in the diagram, $\binom{4}{2}$ and $\binom{3}{-4}$.
   - In general, any vector $\binom{a}{b}$ has 2 components: the horizontal component a measured along the x-axis, and the vertical component, b measured along the y-axis.
2. Write the following problem on the board: If $\mathbf{a} = \binom{4}{2}$ and $\mathbf{b} = \binom{3}{-4}$, what is $\mathbf{a} + \mathbf{b}$?
3. Ask volunteers to explain how to solve this problem.
4. Join the ends of the lines on the board. Count to find the x and y- components of the vector and label it $\binom{7}{-2}$.
5. Explain: The vectors can be added by drawing a triangle. This can also be calculated by adding the x-components and y-components of the 2 vectors.
6. Add the vectors on the board: $\mathbf{a} + \mathbf{b} = \binom{4+3}{2+(-4)} = \binom{7}{-2}$
7. Explain that subtraction follows the same process – the y-components and the x-components are each subtracted.
8. Draw the diagram on the board:



9. Ask pupils to write down the vectors $\overrightarrow{AB}$, $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ as shown on the board.

10. Invite volunteers to give their answer. (Answer: $\overrightarrow{AB} = \binom{2}{1}, \overrightarrow{PQ} = \binom{6}{3}, \overrightarrow{RS} = \binom{-6}{-3}$)
11. Invite volunteers to describe what they notice about the components of the vectors. (Example answers: The absolute value of each component in $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ is three times that of corresponding components in $\overrightarrow{AB}$. $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ are in the same direction but opposite to $\overrightarrow{RS}$.)
12. Explain:
    - Consider the vector $\overrightarrow{AB} = \mathbf{a} = \binom{2}{1}$ shown in the diagram.
    - It can be seen that $\overrightarrow{PQ}$ is **three times** $\overrightarrow{AB}$, and has the **same** direction.
    - It can be seen that $\overrightarrow{RS}$ is **three times** $\overrightarrow{AB}$, and has the **opposite** direction.
13. Write on the board and explain:

$$\begin{aligned} \overrightarrow{PQ} &= \binom{6}{3} \\ &= 3\binom{2}{1} \\ &= 3\mathbf{a} && \text{3 times vector } \mathbf{a} \text{ in the same direction} \\ \overrightarrow{RS} &= \binom{-6}{-3} \\ &= -3\binom{2}{1} \\ &= -3\mathbf{a} && \text{3 times vector } \mathbf{a} \text{ in the opposite direction} \end{aligned}$$

14. Explain:
    - In scalar multiplication, each component of the vector is multiplied by the scalar amount. It has the effect of "scaling" the vector up or down by the factor of the scalar quantity.
    - If the scalar is positive, the resulting vector is in the same direction as the original vector. If the scalar is negative, the resulting vector is in the opposite direction as the original vector.
15. Write the following problems on the board:
    a. If $\mathbf{a} = \binom{3}{2}$, $\mathbf{b} = \binom{4}{-2}$, and $\mathbf{c} = \binom{-3}{0}$, find: i. $\mathbf{a} + \mathbf{b}$ ii. $\mathbf{c} - \mathbf{a}$ iii. $\mathbf{a} + \mathbf{c} - \mathbf{b}$
    b. If $\mathbf{a} = \binom{3}{1}$, $\mathbf{b} = \binom{4}{-5}$ and $\mathbf{c} = \binom{-1}{2}$ find: i. $2\mathbf{b}$ ii. $6\mathbf{a}$ iii. $-4\mathbf{c}$
    c. If $\mathbf{p} = \binom{0}{-3}$, $\mathbf{q} = \binom{2}{3}$ and $\mathbf{r} = \binom{-2}{-5}$ find: i. $\mathbf{p} + 2\mathbf{r}$ ii. $4\mathbf{q} - 3\mathbf{p}$
16. Ask pupils to work with seatmates to solve the problems.
17. Walk around to check for understanding and clear misconceptions.
18. Invite volunteers to write the solutions on the board.

Solutions:

a. i. $\mathbf{a} + \mathbf{b} = \binom{3}{2} + \binom{4}{-2} = \binom{3+4}{2+(-2)} = \binom{7}{0}$
   ii. $\mathbf{c} - \mathbf{a} = \binom{-3}{0} - \binom{3}{2} = \binom{-3-3}{0-2} = \binom{-6}{-2}$
   iii. $\mathbf{a} + \mathbf{c} - \mathbf{b} = \binom{3}{2} + \binom{-3}{0} - \binom{4}{-2} = \binom{3+(-3)-4}{2+0-(-2)} = \binom{-4}{4}$

b. i. $2\mathbf{b} = 2\binom{4}{-5} = \binom{8}{-10}$
   ii. $6\mathbf{a} = 6\binom{3}{1} = \binom{18}{6}$
   iii. $-4\mathbf{c} = -4\binom{-1}{2} = \binom{4}{-8}$

c. i. $\mathbf{p} + 2\mathbf{r} = \binom{0}{-3} + 2\binom{-2}{-5} = \binom{0}{-3} + \binom{-4}{-10} = \binom{0+(-4)}{-3+(-10)} = \binom{-4}{-13}$
   ii. $4\mathbf{q} - 3\mathbf{p} = 4\binom{2}{3} - 3\binom{0}{-3} = \binom{8}{12} - \binom{0}{-9} = \binom{8-0}{12-(-9)} = \binom{8}{21}$

Practice *(19 minutes)*

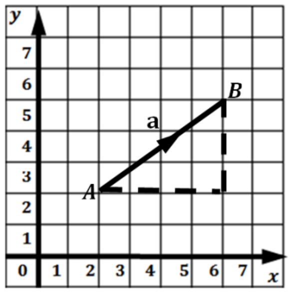
1. Write the following problems on the board:
   a. If $\mathbf{p} = \binom{-3}{-1}$, $\mathbf{q} = \binom{2}{6}$, and $\mathbf{r} = \binom{1}{-9}$, find: i. $\mathbf{p} + \mathbf{r}$ ii. $\mathbf{r} - \mathbf{p}$ iii. $\mathbf{p} - \mathbf{r} - \mathbf{q}$
   b. If $\mathbf{a} = \binom{0}{5}$, $\mathbf{b} = \binom{-8}{-1}$, and $\mathbf{c} = \binom{-1}{1}$, solve the equations below to find column vector **x**:
      i. $\mathbf{a} + \mathbf{x} = \mathbf{b}$ ii. $\mathbf{x} - \mathbf{c} = \mathbf{a}$ iii. $\mathbf{x} + \mathbf{b} = \mathbf{c}$
   c. If $\mathbf{a} = \binom{2}{1}$, $\mathbf{b} = \binom{0}{-9}$, and $\mathbf{c} = \binom{-1}{12}$, find: i. $5\mathbf{a} + 3\mathbf{c}$ ii. $\mathbf{a} - 2\mathbf{b} + \mathbf{c}$
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.
   Solutions:
   a. i. $\mathbf{p} + \mathbf{r} = \binom{-3}{-1} + \binom{1}{-9} = \binom{-3+1}{-1+(-9)} = \binom{-2}{-10}$
      ii. $\mathbf{r} - \mathbf{p} = \binom{1}{-9} - \binom{-3}{-1} = \binom{1-(-3)}{-9-(-1)} = \binom{4}{-8}$
      iii. $\mathbf{p} - \mathbf{r} - \mathbf{q} = \binom{-3}{-1} - \binom{1}{-9} - \binom{2}{6} = \binom{-3-1-2}{-1-(-9)-6} = \binom{-6}{2}$
   b. Substitute the given vectors into each equation and solve for both components of **x**:
      i.
      $$\begin{aligned} \mathbf{a} + \mathbf{x} &= \mathbf{b} \\ \binom{0}{5} + \mathbf{x} &= \binom{-8}{-1} \\ \mathbf{x} &= \binom{-8}{-1} - \binom{0}{5} \\ &= \binom{-8-0}{-1-5} \\ &= \binom{-8}{-6} \end{aligned}$$
      ii.
      $$\begin{aligned} \mathbf{x} - \mathbf{c} &= \mathbf{a} \\ \mathbf{x} - \binom{-1}{1} &= \binom{0}{5} \\ \mathbf{x} &= \binom{0}{5} + \binom{-1}{1} \\ &= \binom{0+(-1)}{5+1} \\ &= \binom{-1}{6} \end{aligned}$$
      iii.
      $$\begin{aligned} \mathbf{x} + \mathbf{b} &= \mathbf{c} \\ \mathbf{x} + \binom{-8}{-1} &= \binom{-1}{1} \\ \mathbf{x} &= \binom{-1}{1} - \binom{-8}{-1} \\ &= \binom{-1-(-8)}{1-(-1)} \\ &= \binom{7}{2} \end{aligned}$$
   c. i. $5\mathbf{a} + 3\mathbf{c} = 5\binom{2}{1} + 3\binom{-1}{12} = \binom{10}{5} + \binom{-3}{36} = \binom{10+(-3)}{5+36} = \binom{7}{41}$
      ii. $\mathbf{a} - 2\mathbf{b} + \mathbf{c} = \binom{2}{1} - 2\binom{0}{-9} + \binom{-1}{12} = \binom{2}{1} - \binom{0}{-18} + \binom{-1}{12} = \binom{2-0+(-1)}{1-(-18)+12} = \binom{1}{31}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L075 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L076 in the Pupil Handbook before the next class.

Lesson Title: Magnitude and direction of vectors	**Theme:** Vectors and Transformation	
Lesson Number: M4-L076	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Find the magnitude or length of a column vector. 2. M Find the direction of a vector.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Draw the problem and diagram in Opening on the board.	

Opening *(3 minute)*



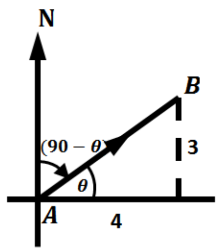
1. Write the following problem on the board: Find the vector joining the points A and B in the diagram.
2. Ask pupils to solve the problem in their exercise books.
3. Invite a volunteer to give their answer. (Answer: $A(2,2), B(6,5)$; $\overrightarrow{AB} = \binom{6-2}{5-2} = \binom{4}{3}$)
4. Explain that this lesson is on finding the magnitude and direction of vectors.

Teaching and Learning *(20 minutes)*

1. Explain:
   - Consider the 2 points $A(x_1, y_1)$ and $B(x_2, y_2)$. We can use Pythagoras' Theorem to find the magnitude $|\overrightarrow{AB}|$, of the vector $\overrightarrow{AB}$.
   - Since $\overrightarrow{AB} = \mathbf{a} = \binom{x}{y}$ then $|\overrightarrow{AB}|^2 = \sqrt{x^2 + y^2}$
   - Alternatively, we can find the magnitude of $\overrightarrow{AB}$ by substituting the co-ordinates of the given points: $|\overrightarrow{AB}|^2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. Write on the board: Find the magnitude of $\overrightarrow{AB}$ in the diagram on the board.
3. Solve the problem on the board, explaining each step:

   We know from the first problem that: $\overrightarrow{AB} = \binom{4}{3}$. Therefore, use the x- and y-values of $\overrightarrow{AB}$ in Pythagoras' theorem:

$$
\begin{aligned}
|\overrightarrow{AB}|^2 &= \sqrt{x^2 + y^2} \\
&= \sqrt{4^2 + 3^2} \\
&= \sqrt{16 + 9} \\
|\overrightarrow{AB}| &= \sqrt{25} = 5
\end{aligned}
$$



4. Draw the diagram of vector $\overrightarrow{AB}$ on the board as shown. →
5. Explain:
   - The direction of the vector is given by the angle it makes when measured from the north in a clockwise direction.
   - We find this angle by first finding the acute angle θ, the vector makes with the x-axis.

- This angle is given by $\tan\theta = \frac{y}{x}$, where x, y are the components of the resultant vector.
- From our sketch, we can then deduce the angle the vector makes when measured from the north in a clockwise direction.
- In our example, this angle is given by $(90 - \theta)°$.
- This is the same as finding the bearing of B from A.

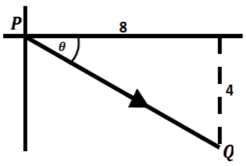
6. Write on the board: Find the direction of the vector joining A and B in the diagram.
7. Solve the problem, explaining each step. Use the diagram above.

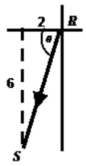
$\tan\theta = \frac{3}{4} = 0.75$ from diagram, use tan ratio

$\theta = \tan^{-1}(0.75) = 36.87$

The direction of $\overrightarrow{AB}$ measured from the north: $90 - 36.87 = 53°$ to the nearest degree

8. Write the following problems on the board: Find the magnitude and direction of the given vectors to the nearest whole number: a. $\overrightarrow{PQ} = \binom{8}{-4}$ b. $\overrightarrow{RS} = \binom{-2}{-6}$
9. Invite volunteers to sketch the diagrams on the board. It is not important that the sketches are to scale for the purpose of solving these problems. Diagrams:





10. Ask pupils to work with seatmates to solve the problems.
11. Invite volunteers to write the solutions on the board.

Solutions:

a. $\overrightarrow{PQ} = \binom{8}{-4}$:

Magnitude:

$$\begin{aligned} |\overrightarrow{PQ}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{8^2 + (-4)^2} \\ &= \sqrt{64 + 16} \\ &= \sqrt{80} = 8.94 \\ |\overrightarrow{PQ}| &= 9 \end{aligned}$$

Direction

$$\begin{aligned} \tan\theta &= \frac{4}{8} = 0.5 \\ \theta &= \tan^{-1}(0.5) \\ &= 26.57° = 27° \end{aligned}$$

Direction of $\overrightarrow{PQ}$ measured from north:

$$90 + 27 = 117°$$

b. $\overrightarrow{RS} = \binom{-2}{-6}$:

Magnitude:

$$\begin{aligned} |\overrightarrow{RS}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-2)^2 + (-6)^2} \\ &= \sqrt{4 + 36} \\ &= \sqrt{40} = 6.32 \\ |\overrightarrow{RS}| &= 6 \end{aligned}$$

Direction

$$\begin{aligned} \tan\theta &= \frac{6}{2} = 3 \\ \theta &= \tan^{-1}(3) \\ &= 71.57° = 72° \end{aligned}$$

Direction of $\overrightarrow{RS}$ measured from north:

$$270 - 72 = 198°$$

Practice *(16 minutes)*

1. Write the following problems on the board:
   a. Find the magnitude and direction of the vectors to the nearest whole number: i. $\mathbf{p} = \binom{-4}{6}$ ii. $\overrightarrow{AC}$ which connects $A(0,1)$ and $C(7,0)$
   b. A column vector $\binom{x}{6}$ has a magnitude of 10. Find x.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions at the same time.

a.i. Magnitude:

$$\begin{aligned} |\mathbf{p}| &= \sqrt{x^2+y^2} \\ &= \sqrt{4^2+(-6)^2} \\ &= \sqrt{16+36} \\ &= \sqrt{52} = 7.211 \\ |\mathbf{p}| &= 7 \text{ units} \end{aligned}$$

Direction:

$$\begin{aligned} \tan\theta &= \tfrac{6}{4} = 1.5 \\ \theta &= \tan^{-1}(1.5) \\ &= 56.3° = 56° \end{aligned}$$

Direction of $\mathbf{p}$ measured from north:

$$270 + 56 = 326°$$

ii. Vector: $\overrightarrow{AC} = \binom{7-0}{0-1} = \binom{7}{-1}$

Magnitude:

$$\begin{aligned} |\overrightarrow{AC}| &= \sqrt{x^2+y^2} \\ &= \sqrt{7^2+(-1)^2} \\ &= \sqrt{49+1} \\ &= \sqrt{50} = 7.07 \\ |\overrightarrow{AC}| &= 7 \end{aligned}$$

Direction:

$$\begin{aligned} \tan\theta &= \tfrac{1}{7} \\ \theta &= \tan^{-1}(\tfrac{1}{7}) \\ \theta &= 8.13° = 8° \end{aligned}$$

Direction of $\overrightarrow{AC}$ from north:

$$90 + 8 = 98°$$

b. Substitute the values into the magnitude formula and solve:

$$\begin{aligned} \sqrt{x^2+y^2} &= 10 \\ \sqrt{x^2+6^2} &= 10 \\ x^2+36 &= 100 \\ x^2 &= 100-36 \\ x^2 &= 64 \\ x &= \sqrt{64} = 8 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L076 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L077 in the Pupil Handbook before the next class.

Lesson Title: Transformation	Theme: Vectors and Transformation	
Lesson Number: M4-L077	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to perform transformations (reflection, rotation, translation, and enlargement).	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

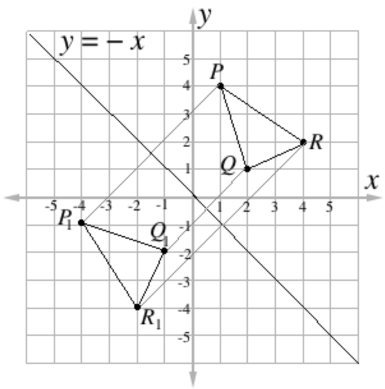
Opening *(1 minute)*

1. Discuss:
   - What is a transformation? (Answer: Transformation changes the position, shape, or size of an object.)
   - What transformations do you know? (Answers: Translation, reflection, rotation, enlargement)
2. Explain that this lesson is on solving problems involving transformations.

Teaching and Learning *(21 minutes)*

1. Revise each transformation with pupils. Allow discussion for each.
   - **Translation** – moves all the points of an object in the same direction and the same distance without changing its shape or size.
   - **Reflection** – an object is reflected in a line of symmetry. The direction that it faces changes, but not its size.
   - **Rotation** – an object rotates (or turns) around a point, which is called the centre of rotation.
   - **Enlargement** – the object is magnified (made larger) or diminished (made smaller). Its shape does not change, but its size does.
2. There are formulae for each type of transformation given in the Pupil Handbook.
3. Write the problems on the board:
   a. Triangle PQR has coordinates $P(1,4), Q(2,1)$ and $R(4,2)$. Find the co-ordinates, P_1, Q_1 and R_1 of the image of the triangle formed under reflection in the line $y = -x$.
   b. Use the appropriate formula to find the co-ordinates of the image point when point $X(-3,-2)$ is rotated $90°$ clockwise about the point $(0,-4)$.

4. Solve the problems as a class. Ask volunteers to give the steps, and work the problems on the board.
   Solutions:
   a. Reflection:
   Step 1. Assess and extract the given information from the problem. Given: points $P(1,4), Q(2,1)$ and $R(4,2)$, line $y = -x$
   Step 2. Draw the x- and y- axes. Locate the points $P,\ Q$ and R on the graph. Draw the lines joining the points.



Step 3. Draw the line $y = -x$.
Step 4. Draw a line at right angles from P to the mirror line ($y = -x$). Measure this distance.
Step 5. Measure the same distance on the opposite side of the mirror line ($y = x$) to locate the point P_1 on the graph.
Step 6. Write the new coordinates: $P_1(-4,-1)$, $Q_1(-1,-2)$ and $R_1(-2,-4)$

b. Rotation:
Apply the following formula for rotation:

$$\begin{pmatrix} x-a \\ y-b \end{pmatrix} \longrightarrow \begin{pmatrix} (y-b)+a \\ -(x-a)+b \end{pmatrix} = \begin{pmatrix} (y-(-4))+0 \\ -(x-0)+(-4) \end{pmatrix} = \begin{pmatrix} y+4 \\ -x-4 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ -2 \end{pmatrix} \longrightarrow \begin{pmatrix} -2+4 \\ -(-3)-4 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad \text{apply the formula}$$

$X(-3,-2)$ rotated $90°$ clockwise about the point $(0,-4)$ gives $(2,-1)$.

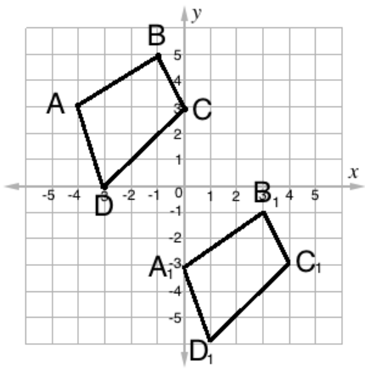
5. Write the following problem on the board: Quadrilateral $ABCD$ has coordinates $A(-4,3)$, $B(-1,5)$ and $C(0,3)$ and $D(-3,0)$.
   a. Draw quadrilateral $ABCD$ on the Cartesian plane.
   b. Draw quadrilateral $A_1B_1C_1D_1$, which is $ABCD$ translated by the vector $\binom{4}{-6}$.
6. Ask pupils to work with seatmates to solve.
7. Walk around to check for understanding and clear misconceptions.
8. Invite volunteers to write the solution on the board.

Solutions:

a. Identify points A, B, C, and D on the Cartesian plane, and connect them in a quadrilateral as shown below.
b. Identify point A_1 by translating $A(-4,3)$ by the vector $\binom{4}{-6}$:

$$\binom{-4}{3} \longrightarrow \binom{-4}{3} + \binom{4}{-6} = \binom{-4+4}{3-6} = \binom{0}{-3}$$

Draw the image of $A_1B_1C_1D_1$ using A_1 as a reference point. Note that it has the same shape and size as $ABCD$.



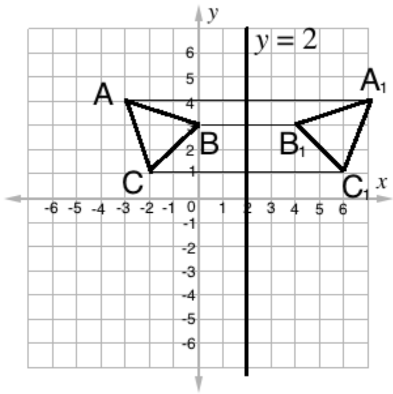
Practice *(17 minutes)*

1. Write the following problems on the board:
   a. Triangle ABC has coordinates $A(-3,4), B(0,3)$ and $C(-2,1)$. Find the co-ordinates, A_1, B_1 and C_1 of the image of the triangle formed under reflection in the line $y = 2$.
   b. Find the image of $(3,-2)$ under the enlargement with a scale factor of 2 from the point $(2,1)$.
2. Ask pupils to work independently to solve the problems. Allow discussion with seatmates if needed.

3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

   Solutions:

   a. **Step 1.** Draw the x- and y- axes and locate the points $A,\ B$ and C on the graph. Draw the lines joining the points. Draw the line $y = 2$.
   Step 2. Draw a line at right angles from each point (A, B and C) to the mirror line ($y = 2$).
   Step 3. Measure the same distance on the opposite side of the mirror line ($y = 2$) to locate and plot the points A_1, B_1 and C_1 on the graph.
   Step 4. Write the new coordinates: $A_1(7, 4)$, $B_1(4, 3)$ and $C_1(6, 1)$



   b. Apply the formula for enlargement:

$\binom{x-a}{y-b}$	$\longrightarrow$	$\binom{3-2}{-2-1}$	$= \binom{1}{-3}$	subtract components of the centre of rotation from the given point
$\binom{1}{-3}$	$\longrightarrow$	$2\binom{1}{-3}$	$= \binom{2}{-6}$	enlarge using the given scale factor
$\binom{2}{-6}$	$\longrightarrow$	$\binom{2+2}{-6+1}$	$= \binom{4}{-5}$	add back components of the centre of rotation

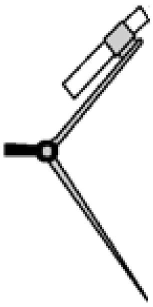
Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L077 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L078 in the Pupil Handbook before the next class.

Lesson Title: Bisection	Theme: Geometry	
Lesson Number: M4-L078	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to bisect a given line or angle.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Bring a pair of compasses and a protractor to class (purchased or handmade), and a ruler for drawing lines. Ask pupils to bring geometry sets if they already have them.	

Opening *(3 minutes)*

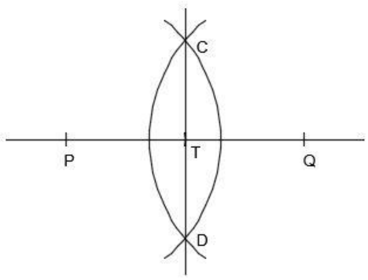
1. Hold up the pair of compasses if you have one. If you do not have one, sketch one on the board or show pupils this picture. →



2. Discuss:
   - What is this tool called? (Answer: A pair of compasses)
   - What do we use this tool for? (Answer: It is used in geometry construction. For example, it is used to draw circles or to bisect lines or angles.)
3. Explain:
   - This is the first lesson on geometry construction.
   - Today we will bisect lines and angles.
   - There are problems on the WASSCE exam on geometry construction. If you can find a geometry set, bring it to class and practice with it at home.

Teaching and Learning *(18 minutes)*

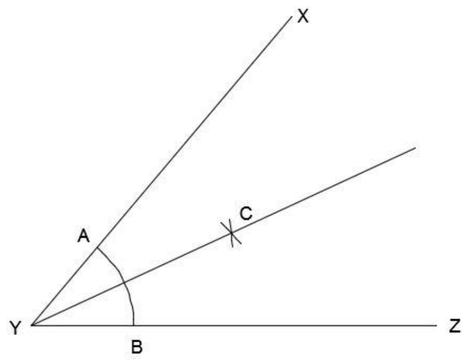
1. Discuss: What does it mean to bisect something? (Answer: to divide it into 2 equal parts.)
2. Draw a horizontal line segment across the board.
3. Ask a volunteer to choose any point around the middle of the line, and label it T.
4. Explain:
   - We will construct a perpendicular line at point T.
   - We will use T as a centre and choose any radius for our pair of compasses.
5. Take the following steps on the board, explaining each:
   - Draw arcs to cut the line segment at 2 points the same distance from T, and label these points P and Q (It is important that $\overline{PT} = \overline{TQ}$).
   - With point P as the centre, open your compass more than half way to point Q. Then draw an arc that intersects $\overline{PQ}$.



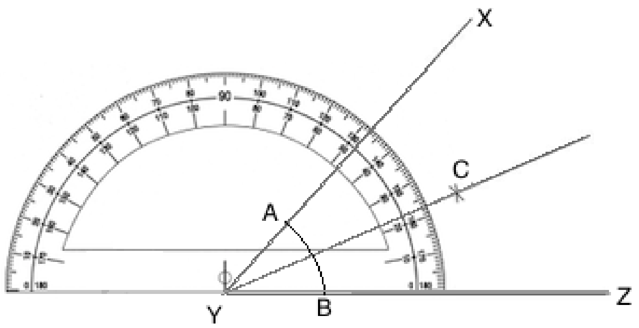
- Using the same radius and point Q as centre, draw an arc that intersects the first arc. Label the points where the 2 arcs intersect as C and D.
- Draw $\overline{CD}$.

6. Explain:

   $\overline{CD}$ is perpendicular to $\overline{PQ}$ at point T. Point T is the mid-point of $\overline{PQ}$.
   - $\overline{CD}$ is called the **perpendicular bisector** of line segment $\overline{PQ}$**.**
   - $\overline{CD}$ divides the line segment $\overline{PQ}$ into two parts that are equal. (If you have a ruler, demonstrate that $\overline{PT}$ and $\overline{TQ}$ are equal segments.)
   - A perpendicular bisector forms a 90° angle with the line.
7. Discuss: We can also bisect an angle. What do you think it means to bisect an angle?
8. Allow discussion, then explain: To bisect an angle means to divide it into 2 equal parts. For example, if an angle is 60°, we can bisect it to form two 30° angles.
9. Draw an angle with any measure on the board, and label it XYZ.
10. Explain: We will construct an **angle bisector** that divides XYZ into 2 equal parts.
11. Take the following steps on the board, explaining each one:
    - With point Y as the centre, open your pair of compasses to any convenient radius. Draw an arc AB to cut XY at A and YZ at B.
    - With point A as the centre, draw an arc using any convenient radius.
    - With the same radius as the step above, use point B as the centre and draw another arc to intersect the first one at C.
    - Label point C.
    - Join Y to C to get the angle bisector as shown.



12. Explain: $\overline{YC}$ is the angle bisector of $\angle XYZ$.
13. Explain: After drawing an angle bisector, we can check it. We will use a protractor to check the angles.
14. Demonstrate how to check the perpendicular bisection on the board: Hold a protractor to $\angle XYZ$, and measure the entire angle.



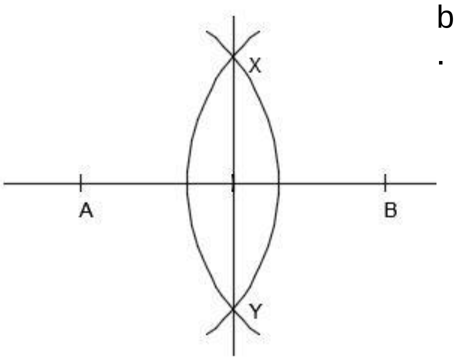
- Write the angle measure on the board. (For example: $\angle XYZ = 48°$)
- Hold the protractor up again, and measure angles CYZ and XYC.

- Write the measure of each bisection on the board. (For example: $\angle CYZ = 24°$ and $\angle XYC = 24°$)

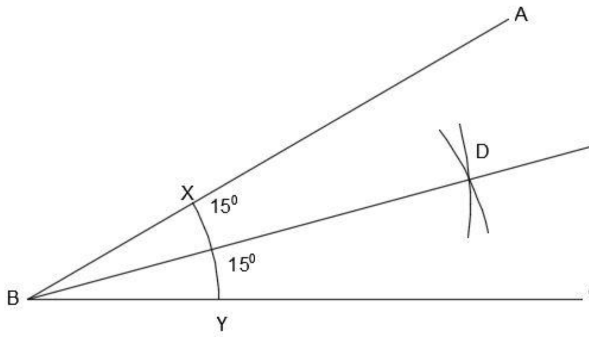
15. Write the following problems on the board:
    a. Draw line segment $\overline{AB}$. Construct its perpendicular bisector $\overline{XY}$.
    b. Draw an angle ABC. Construct its bisector using the letter D. Check your bisection using a protractor, and label each angle with its measurement.
16. Ask pupils to work with seatmates to draw the constructions.
17. Make sure each group of seatmates has a compass and protractor. They can make their own by following the instructions in the Pupil Handbook.
18. Invite volunteers to show the paper with their construction to the class. They should explain the steps they took to draw it. Allow other pupils to ask questions and discuss.

Solutions:

a.



b. With an example angle measure:



Practice *(18 minutes)*

1. Write the following problems on the board:
    a. Draw a line segment labelled with the initials of your name. Construct its perpendicular bisector and give it the initials of your best friend.
    b. Draw an angle labelled with your favourite 3 letters. Bisect the angle. Check your bisection using a protractor, and label each angle with its measurement.
2. Ask pupils to work independently to do the construction.
3. Walk around to check for understanding and clear misconceptions.
4. Ask 2-3 volunteers to show their paper and explain how they did their construction. Allow discussion. (Solutions should look the same as in the problems in Teaching and Learning, but with different letters.)

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L078 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L079 in the Pupil Handbook before the next class.

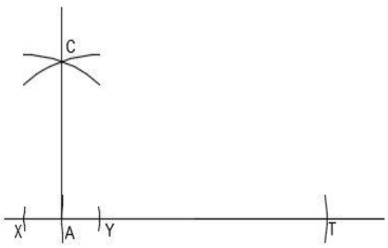
Lesson Title: Angle construction	Theme: Geometry	
Lesson Number: M4-L079	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use a pair of compasses to construct special angles and their combinations (90°, 45°, 60°, 120°, 30°, 75°, 105°, and 150°).	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Bring a pair of compasses and a protractor to class (purchased or handmade), and a ruler for drawing lines. Ask pupils to bring geometry sets if they already have them.	

Opening *(1 minute)*

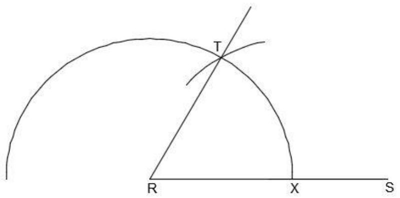
1. Explain: This lesson is on constructing 8 angles of various degrees. These are all based on the construction of the angles 90°, 60°, and 120°. From these angles, you can construct all of the others using bisection.

Teaching and Learning *(19 minutes)*

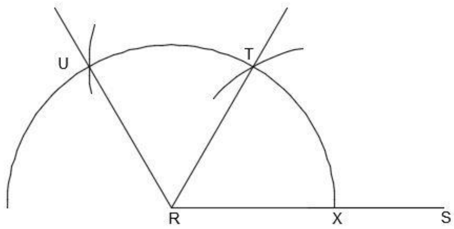
1. Start by constructing angles 90°, 60°, and 120°. Involve pupils as much as possible by asking them to give steps or do part of the construction.
2. Demonstrate how to construct a **90° angle** on the board:
    a. Draw a horizontal line and label it AT.
    b. Extend the straight line outwards from A.
    c. With A as the centre, open your compass to a convenient radius and draw a semi-circle that intersects the line at X and Y.
    d. Use X and Y as centres. Using any convenient radius, draw arcs to intersect at C.



3. Demonstrate how to construct a **60° angle** on the board:
    a. Draw the line RS.
    b. With centre R, open your compass to any convenient radius and draw a semi-circle that cuts RS at X.
    c. With centre X, use the **same radius** and mark another arc on the semi-circle. Label this point T.
    d. Draw a line from R to T.

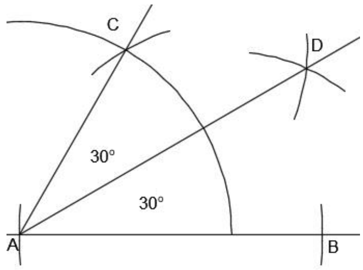


4. Continue with the 60° angle construction to construct a **120° angle:**
    a. Use the same radius that we used to create the semi-circle.
    b. Use T as the centre, and draw another arc on the semi-circle. Label this point U.

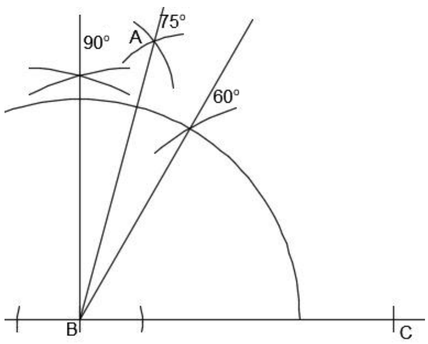


c. Draw a line from R to U.

5. Explain: The angles 45°, 30°, and 15° are constructed by simply bisecting angles 60° and 90°. For 15°, the 60° angle will need to be bisected twice. The first bisection gives a 30° angle. Bisection of the 30°angle gives a 15° angle.
6. Construct a **30° angle** on the board:
    a. Construct a 60° angle using steps from the previous lesson. Label it $\angle CAB$.
    b. Centre your pair of compasses at the points where the semi-circle intersects CA and AB. Draw arcs from each point, using a convenient radius.
    c. Label the point where the arcs intersect as D.
    d. Join A to D to get the angle bisector.



7. Explain: The angles 75°, 105°, and 150° can be constructed using bisection of other angles. They each require you to construct 2 angles in the same diagram and bisect them.
8. Discuss. Ask each question and give pupils a moment to think before responding:
    - Which angles would you construct and bisect to create a 75° angle? (Answer: 90° and 60° angles, because 75° is halfway between them.)
    - Which angles would you construct and bisect to create a 105° angle? (Answer: 90° and 120° angles, because 105° is halfway between them.)
    - Which angles would you construct and bisect to create 150°? (Answer: 120° and 180° angles, because 150° is halfway between them.)
9. Construct a **75° angle** on the board:
    a. Construct a 90-degree angle from base line BC.
    b. Using the same base line BC, construct a 60-degree angle.
    c. Bisect the angle between the 60 and 90 degree angles.
    d. Label the bisection line as A.

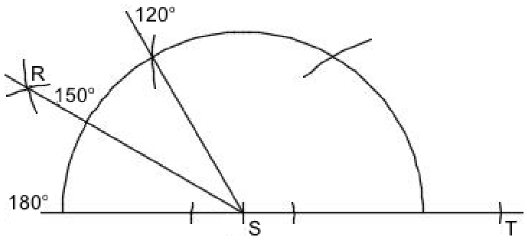


Practice *(19 minutes)*

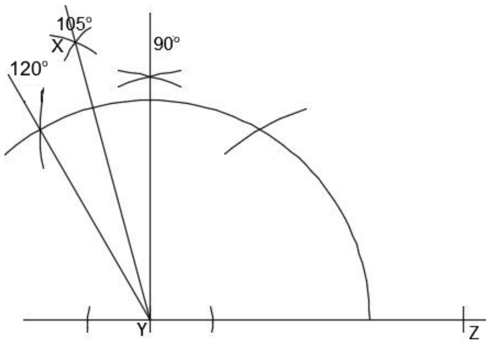
1. Write the following problems on the board:
    c. Construct a 150° angle and label it $\angle RST$.
    d. Construct a 105° angle and label it $\angle XYZ$.
    e. Create one construction that has all of the following angles:
        i. $\angle KIT = 15°$
        ii. $\angle WIT = 30°$
        iii. $\angle PIT = 75°$
        iv. $\angle MIT = 105°$
2. Ask pupils to work independently or with seatmates to do the constructions.

3. Walk around to check for understanding and clear misconceptions.
4. Ask volunteers to show their paper to the class and explain how they did their construction. Allow discussion.
   Solutions:

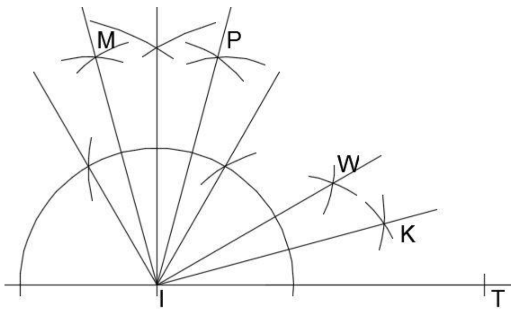
a.



b.



c.



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L079 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L080 in the Pupil Handbook before the next class.

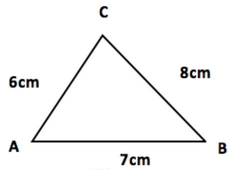
Lesson Title: Triangle construction	Theme: Geometry	
Lesson Number: M4-L080	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use a pair of compasses to construct a triangle from given side and angle lengths.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Bring a pair of compasses and a protractor to class (purchased or handmade), and a ruler for drawing lines. Ask pupils to bring geometry sets if they already have them.	

Opening *(1 minute)*

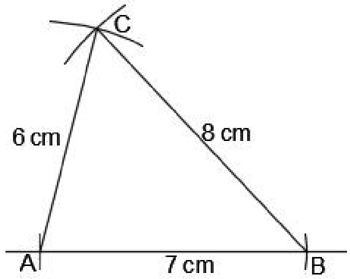
1. Explain:
   - There are three types of basic triangle construction problems, which will all be covered in this lesson.
   - You may be asked to construct a triangle given three sides (SSS), with two given sides and an angle (SAS), or with two given angles and a side (ASA).

Teaching and Learning *(19 minutes)*

1. Write the following problems on the board:
   a. Construct a triangle ABC with sides 6 cm, 7 cm, and 8 cm.
   b. Construct triangle ABC where $|AB|$ is 6 cm, $|BC|$ is 7 cm, and angle B is 60°.
   c. Construct triangle ANT where $\angle N = 45°$, $\angle T = 60°$ and $\overline{NT} = 7$ cm.
2. Explain:
   - We will construct each of these triangles as a class.
   - Before constructing a shape, it is best to draw a sketch first. This will help us determine the type of triangle (SSS, SAS, or ASA) and decide how to do the construction.
3. Draw a sketch of the triangle in problem a. on the board:

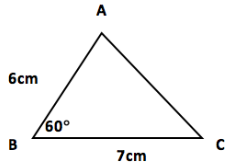


4. Construct the triangle for problem a. on the board:
   - Draw a line and label point A on one end.
   - Open your compass to the length of 7 cm. Use it to mark point B 7 cm from point A. This gives line segment $\overline{AB} = 7\text{ cm}$.
   - Open your compass to the length of 6 cm. Use A as the centre, and draw an arc of 6 cm above $\overline{AB}$.

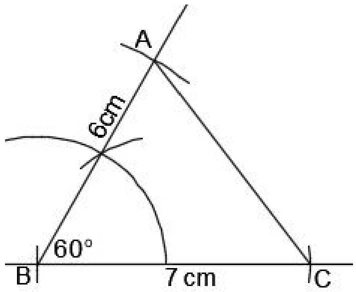


- Open your compass to the length of 8 cm. With the point B as the centre, draw an arc that intersects with the arc you drew from point A. Label the point of intersection C.
- Join $\overline{AC}$ and $\overline{BC}$. This is the required triangle ABC.

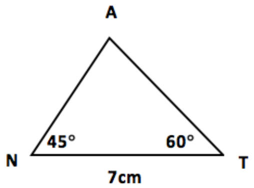
5. Ask pupils to draw a sketch of the triangle in problem b. in their exercise books.
6. Ask a volunteer to quickly draw the sketch on the board:



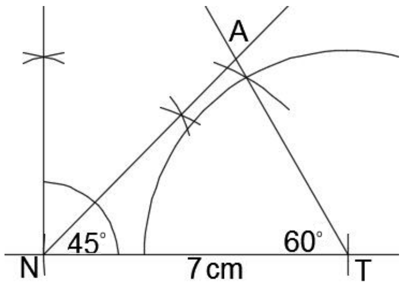
7. Ask a volunteer to tell the class what type of triangle construction this is. (Answer: SAS)
8. Construct the triangle for problem b. on the board:
   - Draw the side $|BC| = 7\text{ cm}$ and label it 7 cm.
   - From $\overline{BC}$, construct an angle of 60° at B, and label it 60°.
   - Open your compass to the length of 6 cm. Use B as centre, and draw an arc of 6 cm on the 60° line. Label this point A.
   - Join $\overline{AB}$ and $\overline{BC}$. This is the required triangle ABC.



9. Ask pupils to draw a sketch of the triangle in problem c. in their exercise books.
10. Invite a volunteer to quickly draw the sketch on the board:



11. Ask a volunteer to tell the class what type of triangle construction this is. (Answer: ASA).
12. Construct the triangle for problem c. on the board:
    - Draw the side $\overline{NT} = 7\text{ cm}$ and label it 7 cm.
    - From $\overline{NT}$, construct an angle of 45° at N, and label it 45°.
    - From $\overline{NT}$, construct an angle of 60° at T, and label it 60°.
    - Extend the 2 angle constructions until they meet. Label this point A. This is the required triangle ANT.



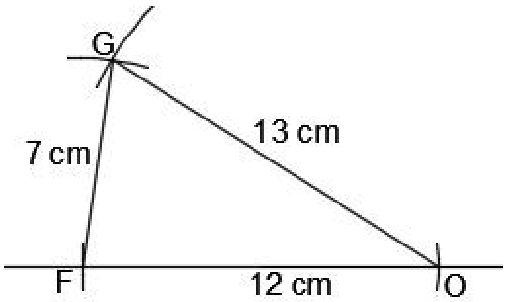
Practice *(19 minutes)*

1. Write the following problems on the board:
   a. Construct triangle BIG where $\angle I = 90°$, $\angle G = 30°$ and $\overline{IG} = 10$ cm.
   b. Construct triangle RST where $\overline{RS}$ is 5 cm, $\overline{ST}$ is 6 cm, and S is 90°.
   c. Construct triangle BIT where $\angle I = 120°$, $\angle T = 15°$ and $\overline{IT} = 12$ cm.
   d. Construct triangle CAT where $\overline{CA}$ is 5 cm, $\overline{AT}$ is 10 cm, and A is 120°.
2. Ask pupils to work independently or with seatmates to do the constructions.
3. Walk around to check for understanding and clear misconceptions.

4. Ask volunteers to show their papers to the class and explain how they did their construction. Allow discussion.
    e. Construct triangle FOG with sides 7 cm, 12 cm, and 13 cm.

Solutions:

a.



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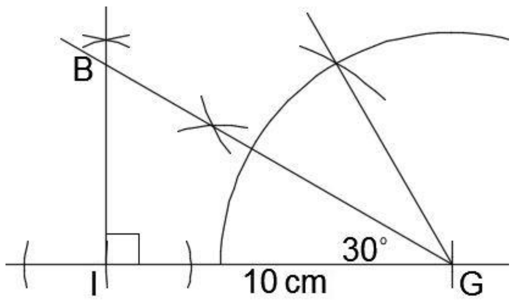
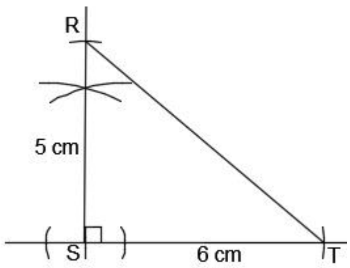


c.



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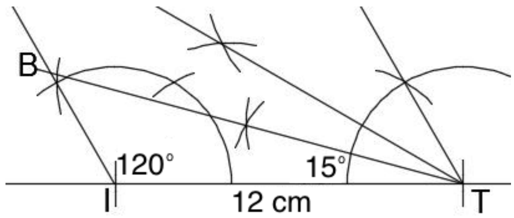
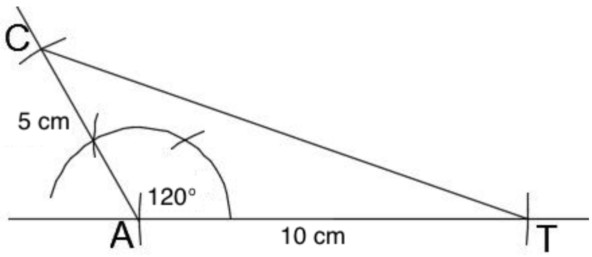


e.



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L080 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L081 in the Pupil Handbook before the next class.

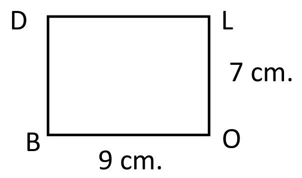
Lesson Title: Quadrilateral construction	Theme: Geometry	
Lesson Number: M4-L081	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use a pair of compasses to construct a quadrilateral from a given side and angle lengths.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Bring a pair of compasses and a protractor to class (purchased or handmade), and a ruler for drawing lines. Ask pupils to bring geometry sets if they already have them.	

Opening *(1 minute)*

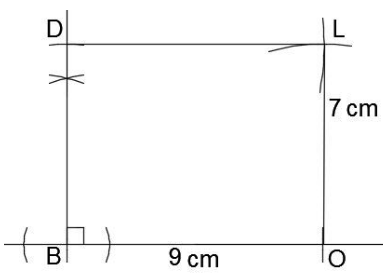
1. Explain: This lesson will cover construction of different kinds of quadrilaterals, including squares, rectangles, parallelograms and trapeziums.

Teaching and Learning *(19 minutes)*

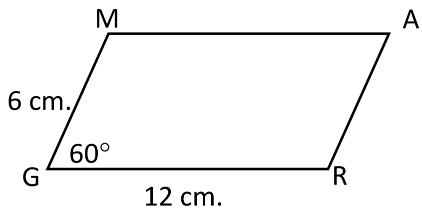
1. Write the following problems on the board:
   d. Construct rectangle $BOLD$ where $l = 9$ cm and $w = 7$ cm.
   e. Construct parallelogram $GRAM$ where $|GR| = 12 \text{ cm}$, $|GM| = 6 \text{ cm}$, and angle $G = 60°$.
   f. Construct a trapezium $QRST$ such that $|QR| = 10 \text{ cm}$, $|RS| = 6 \text{ cm}$, $|ST| = 6 \text{ cm}$, and $\angle QRS = 60°$ and line $\overline{QR}$ is parallel to line $\overline{ST}$.
2. Explain:
   - We will construct each of these quadrilaterals as a class.
   - Before constructing a shape, it is best to draw a sketch first. This will help us decide how to do the construction.
2. Draw a sketch of the rectangle in problem a. on the board:



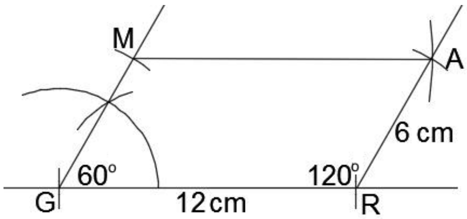
3. Construct rectangle $BOLD$ on the board:
   - Draw the side $\overline{BO} = 9 \text{ cm}$ and label it 9 cm.
   - From $\overline{BO}$, construct an angle of 90° at B.
   - Open your pair of compasses to 7 cm. With B as the centre, draw an arc on the 90° line. Label the intersection D.
   - Keep the radius of your pair of compasses at 7 cm. With O as the centre, draw an arc above the line BO.
   - Change the radius of your pair of compasses to 9 cm. With D as the centre, draw an arc to the right, above O. Label the intersection of these 2 arcs L.
   - Draw lines to connect D with L, and O with L.



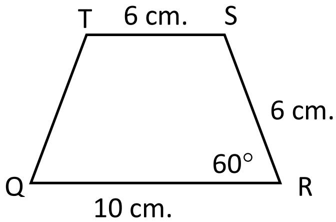
4. Ask pupils to draw a sketch of the parallelogram in problem b. in their exercise books.
5. Invite a volunteer to quickly draw the sketch on the board:



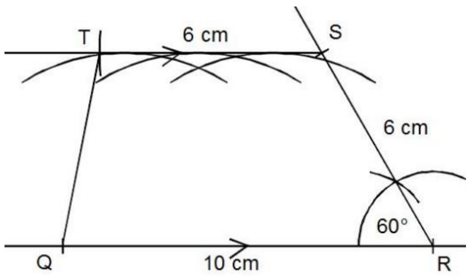
6. Construct parallelogram $GRAM$ on the board:
   - Draw the side $|GR| = 12\text{ cm}$ and label it 12 cm.
   - From $\overline{GR}$, construct an angle of 60° at G.
   - Open your pair of compasses to 6 cm. With G as the centre, draw an arc on the 60° line. Label the intersection M.
   - Keep the radius of your pair of compasses at 6 cm. With R as the centre, draw an arc above the line GR.
   - Change the radius of your pair of compasses to 12 cm. With M as the centre, draw an arc to the right, above R. Label the intersection of these 2 arcs A.
   - Draw lines to connect M with A, and A with R.



7. Ask pupils to draw a sketch of the trapezium in problem c in their exercise books.
8. Ask a volunteer to quickly draw the sketch on the board:



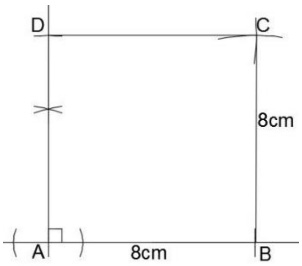
9. Construct trapezium $QRST$ on the board:
   - Draw the side $|QR| = 10\text{ cm}$ and label it 10 cm.
   - From $\overline{QR}$, construct an angle of 60° at R.
   - Open your pair of compasses to 6 cm. With R as the centre, draw an arc on the 60° line. Label the intersection S.
   - Construct a line parallel to QR:
     - Centre your pair of compasses at S, and open them to the distance between point S and the line QR.
     - Choose any 3 points on line QR. Keep your compass open to the distance between S and QR, and draw 3 arcs above QR.
     - Place your ruler on the highest points of these 3 arcs, and connect them to make a line parallel to QR.
   - Open your compass to 6 cm. With S as the centre, draw an arc through the parallel line you constructed. Label the intersection T.
   - Draw a line to connect T with Q.



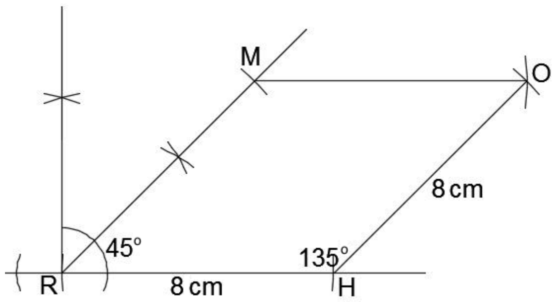
Practice *(19 minutes)*

1. Write the following problems on the board:
   a. Construct square $ABCD$ with sides 8 cm.
   b. Construct a rhombus $RHOM$ with sides of length 8 cm, and angle $R = 45°$.
   c. Construct trapezium $NOTE$ where $\overline{NO} = 11 \text{ cm}$, $\overline{OT} = 7 \text{ cm}$, $\overline{TE} = 5 \text{ cm}$, and $\angle O = 60°$ and line $\overline{NO}$ is parallel to line $\overline{TE}$.
   d. Construct quadrilateral $CRAB$ with $CR = 9 \text{ cm}$, $RA = 12 \text{ cm}$, $AB = 7 \text{ cm}$, $CB = 10 \text{ cm}$, and $\angle C = 60°$.
2. Ask pupils to work independently or with seatmates to do the constructions.
3. Walk around to check for understanding and clear misconceptions. There are more examples of quadrilateral construction in the Pupil Handbook that you may refer pupils to.
4. Ask volunteers to show their papers to the class and explain how they did their construction. Allow discussion.
   Solutions:

a.



b.



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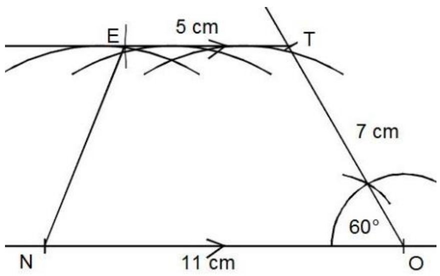

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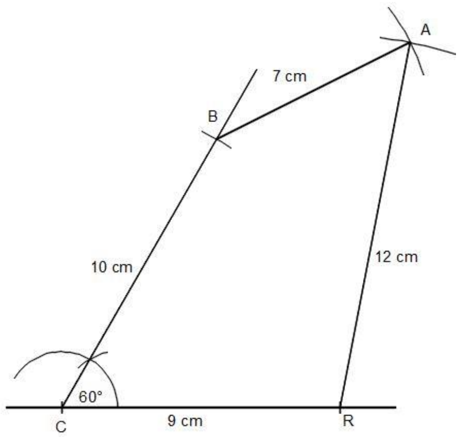


Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L081 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L082 in the Pupil Handbook before the next class.

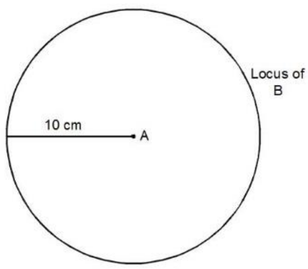
Lesson Title: Construction of loci	**Theme:** Geometry	
Lesson Number: M4-L082	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use a pair of compasses to construct various loci.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Bring a pair of compasses and a protractor to class (purchased or handmade), and a ruler for drawing lines. Ask pupils to bring geometry sets if they already have them.	

Opening *(2 minutes)*

1. Explain:
   - This lesson will cover construction of different kinds of loci, which is plural for locus.
   - A locus is a specific path that a point moves through. The point obeys certain rules as it moves through the locus.
   - These are the 4 types of loci that you will construct:
     - The locus of a point P a given distance from a point.
     - The locus of a point P equidistant from 2 given points.
     - The locus of a point P equidistant from 2 given lines.
     - The locus of a point P a given distance from a line segment or a line.

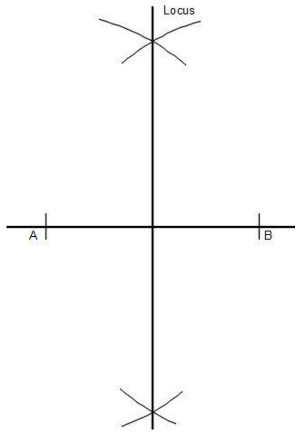
Teaching and Learning *(20 minutes)*

1. Write the following problems on the board:
   a. Construct the locus of point B if it is 10 cm from point A.
   b. Construct the locus of points equidistant from the 2 points A and B.
   c. Draw two intersecting lines AB and CD. Construct the locus of points equidistant from the two lines.
   d. Draw a line segment AB. Construct the locus of points 6 cm from AB.
   e. Draw a line q. Construct the locus of points 5 cm away from q.
3. Explain:
   - We will construct each of these loci as a class.
   - For problem a., the locus of points a given distance from a point is a circle where the radius is the given distance.
4. Construct the locus of B on the board, explaining each step:
   a. Mark point A anywhere.
   b. Open the compass to 10 cm.
   c. With A as the centre, construct a full circle.
   d. Label the circle "locus of B".

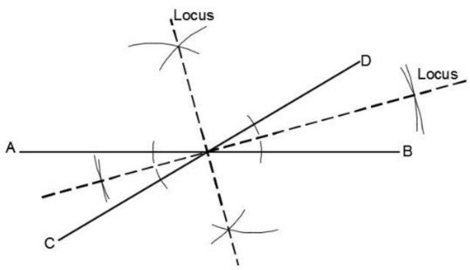


5. Explain problem b.:

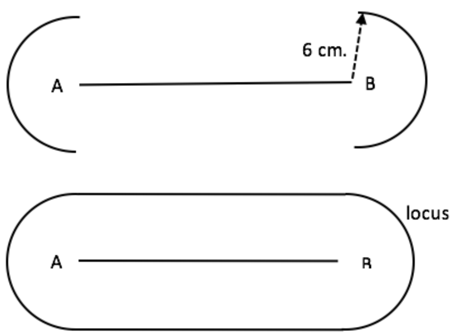
   a. The locus of points equidistant from 2 given points is the perpendicular bisector of the line that connects the 2 points.
6. Construct the locus for b.:
   a. Draw 2 points A and B horizontally from one another on the board
   b. With A as the centre, draw arcs above and below line AB.
   c. With B as the centre, draw arcs above and below line AB.
   d. Draw the perpendicular bisector through the two points where the arcs intersect.
   e. Label the line “locus”.



7. Explain problem c.:
   a. The points that are equidistant from 2 lines can be found by bisecting the angles formed by the lines.
   b. The points that are equidistant from AB and CD are all of the points on the angle bisectors.
8. Construct the locus for c. (see below):
   a. Draw 2 intersecting lines, AB and CD.
   b. With the intersection of the lines as the centre, draw an arc on each line. Use any radius, but use the same radius for each arc.
   c. Use the intersection of each arc with the lines as a centre. Adjust your compass to a convenient radius, and draw arcs on both sides of each line. The arcs should intersect within the angles between the lines.
   d. Connect the intersections of the arcs to create 2 new lines. These are the locus of the point that is equidistant from AB and CD.
   e. Label each new line “locus”.



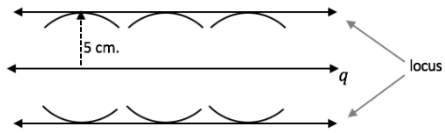
9. Explain problem d.: The locus of points equidistant from a line segment is an oblong shape.
10. Construct the locus for d.:
   a. Draw a line segment AB on the board.
   b. Open your pair of compasses to a radius of 6 cm.
   c. With A as the centre, draw a semi-circle to the left and right of the line:
   d. Use a straight edge to connect the semi-circles above and below the line segment.



11. Explain problem e.:
   a. A line can extend in 2 directions forever.

b. The locus of points a given distance from the line is 2 other parallel lines.

12. Construct the locus for e.:



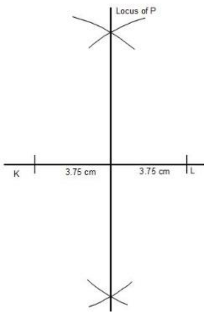
a. Draw a horizontal line q on the board.
b. Open your pair of compasses to a radius of 5 cm.
c. Choose several points on q, and centre your compass at each. From each point, draw an arc directly above and below line q.
d. Hold a straight edge along the points of the arcs farthest from line q. Connect these points.
e. Draw arrows to show that the locus extends forever in both directions.

Practice *(17 minutes)*

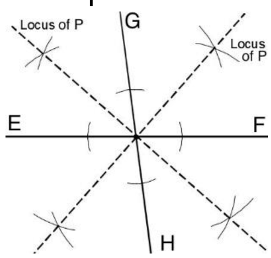
1. Write the following problems on the board:
   a. P is a point that is equidistant from 2 points K and L. Draw K and L a distance of 7.5 cm from each other, then construct the locus of P.
   b. Draw any 2 intersecting lines and label them EF and GH. Construct the locus of a point P that is equidistant from the 2 lines.
   c. Draw a line segment $\overline{MN} = 6\text{ cm}$. Construct the locus of points 2 cm from the line segment.
   d. Draw a vertical line y that extends forever in both directions. Construct the locus of points 3 cm from the line.
2. Ask pupils to work independently or with seatmates to do the constructions.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to show their papers to the class and explain. Allow discussion.

Solutions:

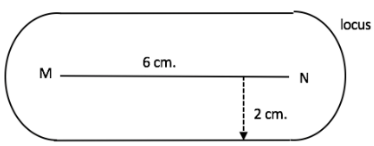
a.



b. Example Answer:



c.



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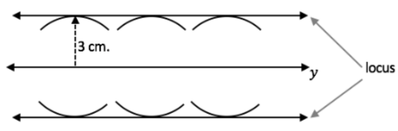


Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L082 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L083 in the Pupil Handbook before the next class.

<table>
<tr><td>Lesson Title: Construction word problems</td><td colspan="2">Theme: Geometry</td></tr>
<tr><td>Lesson Number: M4-L083</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to construct shapes based on information given in word problems.</td><td colspan="2">Preparation
1. Review the content of this lesson and be prepared to explain the solutions.
2. Write the word problems in this lesson on the board.</td></tr>
</table>

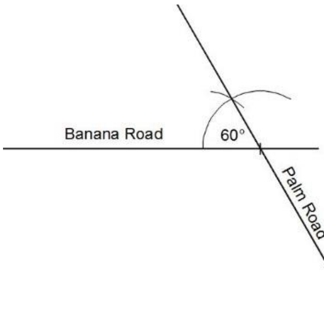
Opening *(2 minutes)*

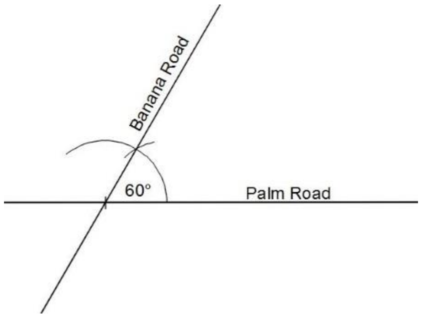
1. Discuss:
   - In what other situations is geometry construction useful?
   - What professionals could use geometry construction in their jobs? (Example: Carpenters, architects, builders, engineers, city planners)
2. Explain that this is lesson is on construction techniques that pupils already know. They will apply the techniques to everyday problems.

Teaching and Learning *(18 minutes)*

1. Write the following problems on the board:
   a. Hawa is drawing a map of her community. She knows that 2 roads intersect at a 60° angle, Banana Road and Palm Road. Draw the intersection of these 2 roads.
   b. Mr. Bangura is a carpenter. He wants to construct a table with a triangular top. He wants each angle to be equal, and each side to be 1 metre. Construct a triangle that gives the shape of his table top. Construct it to a smaller scale, with sides of 10 cm.
   c. Foday wants to draw a map of his land, which is in the shape of a rhombus. It borders 2 roads that form a 45° angle in the corner. He knows that one side is 70 m long. Help him draw the map. Use 1 cm for each 10 m.
   d. Two sides of a fence form a 90° angle. Bintu wants to plant a tree equidistant from the 2 fence sides.
      i. Construct the 2 sides of the fence (a right angle).
      ii. Construct the locus of points where she could plant the tree.
2. Ask pupils to work with seatmates to draw the construction for each story. Remind them to draw a sketch of each shape first, before starting their construction.
3. Walk around to check for understanding. If needed, discuss the stories as a class.
4. Invite volunteers to show each construction to the class. Ask them to explain the steps they took to draw it. If another set of seatmates drew theirs differently, allow them to also share.
   Solutions:

a. Example solutions. Note that other answers may also be accurate. Check for the 60° angle and road labels.





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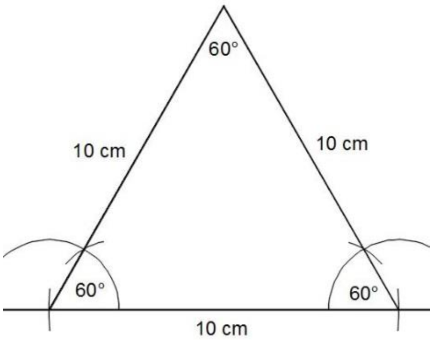

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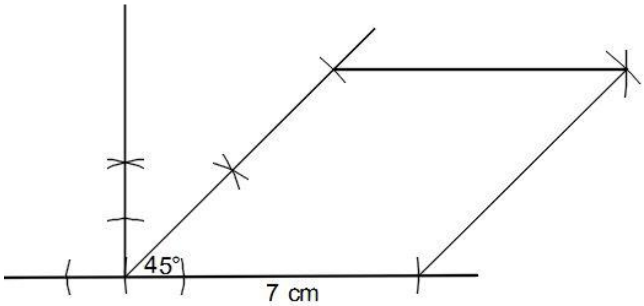

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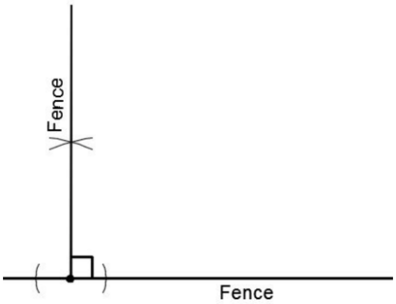

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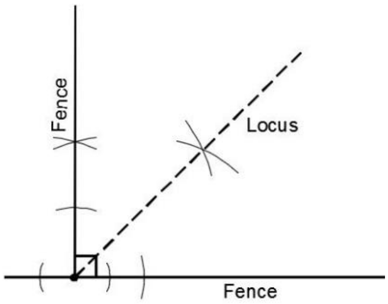
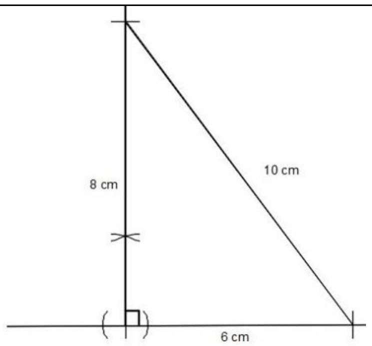
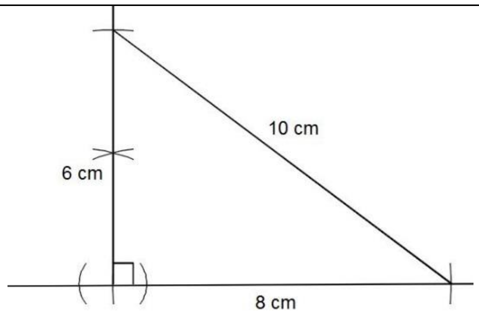
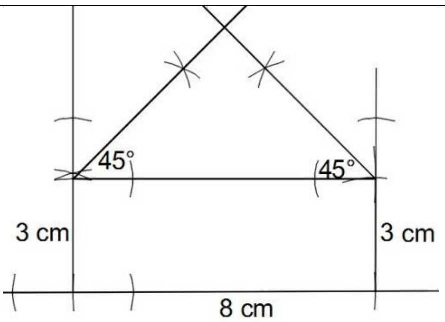
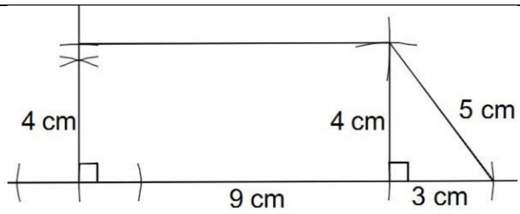


Practice *(19 minutes)*

1. Write the following problems on the board:
   - Fatu has saved her money and she will build an interesting house. The walls will be in the shape of a right-angled triangle. The outside walls will be 6 metres, 8 metres, and 10 metres. Draw the walls of her house using 1 cm for each metre. With the centre at O and radius $\overline{OP}$, construct a circle.
   - Aminata dreams of becoming an architect. She practises by drawing the front of her parents' house. The front wall forms a rectangle that is 8 metres long and 3 metres tall. The roof forms a 45° angle with each wall. Construct the front of her parents' house. Use 1 cm for each m.
   - Fatu and Hawa own land next to each other. They decided to make a farm on both pieces of land. Fatu's land is a rectangle that is 90 metres long by 40 metres wide. It shares one 40 metre side with Hawa's land. Hawa's land is a right triangle with sides of length 40 metres, 50 metres, and 30 metres. Construct the shape of the land using 1 cm for each 10 m.

2. Ask pupils to construct the diagrams independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to show their papers to the class and explain how they did the construction. Construct the solutions on the board if needed.

Solutions:

a.	Example solutions:		
			
b.		c.	

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L083 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L084 in the Pupil Handbook before the next class.

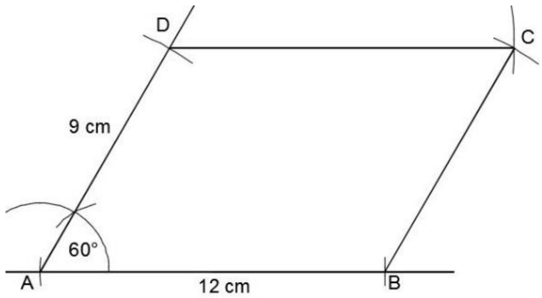
Lesson Title: Construction of complex shapes	**Theme:** Geometry	
Lesson Number: M4-L084	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use a pair of compasses to construct various complex shapes.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(2 minutes)*

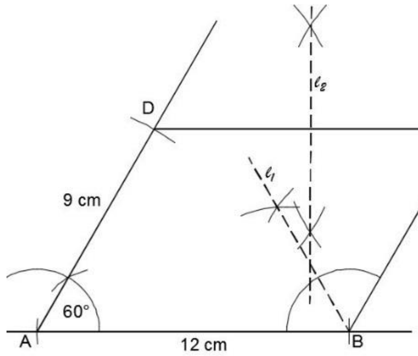
1. Discuss: What are some things you know how to construct using a pair of compasses and a straight edge? (Example answers: angles, triangles, perpendicular lines, angle and line bisectors, quadrilaterals, loci.)
2. Explain that this is the last lesson on construction. Pupils will combine construction techniques from different lessons to construct various figures.

Teaching and Learning *(18 minutes)*

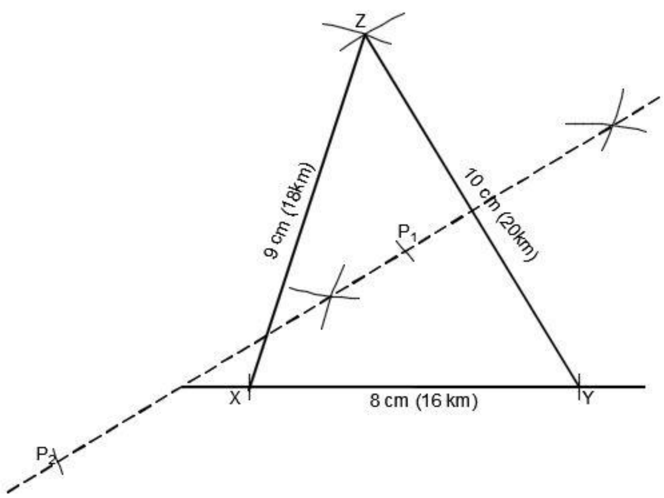
1. Write the following problem on the board: Using a ruler and pair of compasses only, construct:
    a. Parallelogram $ABCD$ such that $A = 60°$, $AB = 12$ cm, and $AD = 9$ cm.
    b. The locus l_1 of points equidistant from AB and BC.
    c. The locus l_2 of points equidistant from C and D.
2. Explain: There are often questions on the WASSCE exam in this style. They ask you to construct a shape before constructing loci on the same shape.
3. Ask pupils to work with seatmates to construct parallelogram $ABCD$. As they are working, construct the parallelogram on the board.



4. After pupils have drawn the parallelograms at their seats, discuss:
    - How can we find locus l_1? (Answer: It is the locus of points equidistant from 2 **lines**, so we construct the **angle bisector**.)
    - How can we find locus l_2? (Answer: It is the locus of points equidistant from 2 **points**, so we construct the **perpendicular bisector** of the line that connects them.)
5. Ask pupils to work with seatmates to construct l_1 and l_2.
6. Invite volunteers to construct the loci on the board.



7. Write the following problem on the board: Three towns, X, Y and Z are such that Y is 16 km from X and 20 km from Z. X is 18 km from Z. The government wants to build a secondary school so that pupils in towns Y and Z will travel the same distance to reach it, while pupils from town X will travel 10 km to reach it.
   a. Draw a map showing the 3 towns. Use a scale of 1 cm to 2 km.
   b. Identify the possible locations where the school could be built.
   c. Measure and record the distances of the possible locations from Z and Y.
   d. Which location would be most convenient for all 3 towns?
8. Discuss the problem with pupils and make sure they understand:
   - How will we draw a map of these villages? (Answer: They form a triangle, and we are given the 3 lengths.)
   - How will we find the possible locations of the school? (Answer: It will be equidistant from points Y and Z, so we construct the loci of such points. We also know it's 10 km from X, so we open our compass to the correct radius and find the points on the locus that are 5 cm from X.)
9. Ask pupils to work with seatmates to do the construction (parts a. and b.)
10. Invite volunteers to construct the triangle and loci on the board (see below).
11. Label the 2 points on the locus that are 5 cm from X as P_1 and P_2.
12. Ask pupils to complete part c. with seatmates.
13. Ask volunteers to share the answers. (Answers: P_1 is around 5.5 cm from Y and Z, which is 11 km. P_2 is around 12.5 cm from Y and Z, which is 25 km.)
14. Discuss part d. of the question as a class: Which location is most convenient? (Answer: P_1 is most convenient because it is near all 3 towns.)

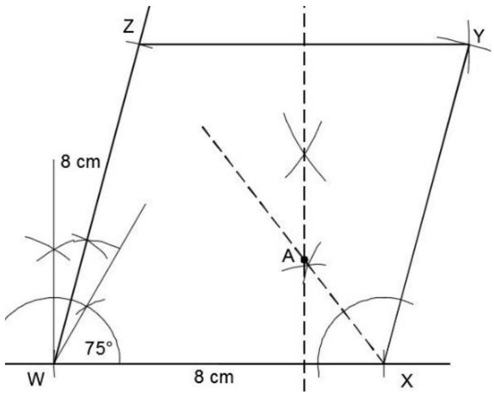


Practice *(19 minutes)*

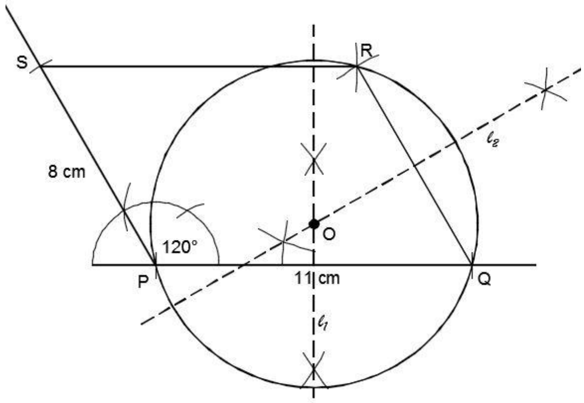
1. Write the following problems on the board:
   - Using a ruler and pair of compasses only:
     i. Construct rhombus $WXYZ$ with sides 8 cm such that $W = 75°$.
     ii. Locate point A such that A lies on the locus of points equidistant from lines WX and XY, and is also equidistant from Z and Y.
   - Using a ruler and pair of compasses only:
     i. Construct a parallelogram $PQRS$, such that $\angle P = 120°$, $\overline{PQ} = 11$ cm, and $\overline{PS} = 8$ cm.
     ii. Construct locus l_1 of points equidistant from P and Q.
     iii. Construct locus l_2 of points equidistant from Q and R.
     iv. Label the point where l_1 and l_2 intersect as O.
     v. Construct a circle with the centre at O and radius $\overline{OP}$.
2. Ask pupils to construct the diagrams independently or with seatmates.
3. Ask volunteers to show their papers to the class and explain how they did the construction. Construct the solutions on the board if needed.

   Solutions:

   a.



   b.



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L084 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L085 in the Pupil Handbook before the next class.

Lesson Title: Addition law of probability	Theme: Probability and Statistics	
Lesson Number: M4-L085	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply the addition law to find the probability of mutually exclusive events.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problem in Opening on the board.	

Opening *(4 minutes)*

1. Revise probability. Write the following problem on the board: A fair die is rolled. What is the probability of obtaining: a. 3 b. An even number
2. Ask pupils to work with seatmates to find the answers.
3. Invite volunteers to write the solutions on the board and explain.
   a. Probability of obtaining 3: $P(3) = \frac{1}{6}$
   Explanation: One of 6 numbers on the die is 3.
   b. Probability of obtaining an even number: $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$
   Explanation: 3 of 6 numbers on the die are even (2, 4, 6).
4. Explain that this lesson is on the addition law of probability.

Teaching and Learning *(18 minutes)*

1. Ask pupils to consider and discuss with seatmates the 2 events:
   A: Attending school on Monday.
   B: Being late for school.
   Can they both happen at the same time?
2. Invite volunteers to give their answer and state the reason why. (Example answer: Yes, you can attend school on Monday and be late, early or on time.)
3. Ask pupils now to consider and discuss with seatmates the 2 events:
   A: Winning a football match
   B: Losing a football match
   Can they both happen at the same time?
4. Invite volunteers to give their answer and state the reason why. (Example answer: No, you cannot win and lose the same football match.)
5. Explain:
   - If two events cannot happen at the same time, then they are called **mutually exclusive events**.
   - The events are connected by the word "or".
6. Write on the board: If two events A and B are mutually exclusive events, then the probability of A or B is given by:
$$\begin{aligned} P(A \text{ or } B) &= P(A \cup B) \\ &= P(A) + P(B) \end{aligned}$$
7. Explain:

- This is the Addition Law for mutually exclusive events.
- In probability, the word "or" or the symbol ∪ indicates addition.
- For two mutually exclusive events which cover all possible outcomes, all the individual probabilities add up to 1. ($P(A) + P(B) = 1$)
- There may be more than 2 events; the additional law applies to cases of more than 2 events as well.

8. Write the following problem on the board: A card is taken at random from an ordinary pack of cards. What is the probability that it will be an Ace or the 10 of Clubs?
9. Solve the problem on the board and explain:
   Step 1. Find the individual probabilities.

   the total number of possible outcomes $n(S) = 52$
   probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

   Let A be the event of choosing an ace, B the event of 10 of clubs

   A = {ace of clubs, ace of spades, ace of diamonds, ace of hearts}

   $n(A) = 4$ $\qquad P(A) = \frac{4}{52} = \frac{1}{13}$

   B = {10 of clubs}

   $n(B) = 1$ $\qquad P(B) = \frac{1}{52}$

   Step 2. Find the probability of Ace or 10 of Clubs.

   $P(A \text{ or } B) = P(A) + P(B) = \frac{1}{13} + \frac{1}{52} = \frac{4}{52} + \frac{1}{52} = \frac{5}{52}$
10. Write the following problem on the board: The word P A R A L L E L O G R A M was written on identical pieces of paper and put in a bag. One of the pieces of paper is selected at random. What is the probability of getting:

    a. A b. L c. O
    d. A or L e. A or O f. A or L or O
11. Ask pupils to work with seatmates to solve the problem.
12. Walk around to check for understanding and clear misconceptions.
13. Invite a volunteer to write the solution on the board and explain.

    Solutions:

    Note the total number of possible outcomes: $n(S) = 13$

    a. Since $n(A) = 3$, the probability of getting A is $P(A) = \frac{3}{13}$
    b. Since $n(L) = 3$, the probability of getting L is $P(L) = \frac{3}{13}$
    c. Since $n(O) = 1$, the probability of getting O is $P(O) = \frac{1}{13}$
    d. Add $P(A)$ and $P(L)$:
       $P(A \text{ or } L) = P(A) + P(L) = \frac{3}{13} + \frac{3}{13} = \frac{6}{13}$
    e. Add $P(A)$ and $P(O)$:
       $P(A \text{ or } O) = P(A) + P(O) = \frac{3}{13} + \frac{1}{13} = \frac{4}{13}$
    f. Add $P(A)$, $P(L)$ and $P(O)$:
       $P(A \text{ or } L \text{ or } O) = P(A) + P(L) + P(O) = \frac{3}{13} + \frac{3}{13} + \frac{1}{13} = \frac{7}{13}$

Practice *(17 minutes)*

1. Write the following problems on the board:
   a. The table gives the probability of getting 1, 2, 3 or 4 on a biased 4-sided spinner.

Number	1	2	3	4
Probability	0.2	0.35	0.15	0.3

   What is the probability of getting:
   i. 1 or 4 ii. 2 or 3 iii. 2 or 4 iv. 1 or 2 or 3

   b. A letter is chosen at random from the word M A G N I T U D E. What is the probability that it is:
      i. Either in the word M U G or in the word I D E A
      ii. Neither in the word A G E N T nor in the word M I D
2. Ask pupils to solve the problems independently or with seatmates.
3. Invite volunteers to write the solutions on the board and explain.

Solutions:

a. Note the total number of possible outcomes**:** $n(S) = 4$
   i. $P(1 \text{ or } 4) = P(1) + P(4) = 0.2 + 0.3 = 0.5$
   ii. $P(2 \text{ or } 3) = P(2) + P(3) = 0.35 + 0.15 = 0.5$
   iii. $P(2 \text{ or } 4) = P(2) + P(4) = 0.35 + 0.3 = 0.65$
   iv. $P(1 \text{ or } 2 \text{ or } 3) = P(1) + P(2) + P(3) = 0.2 + 0.35 + 0.15 = 0.7$

b. Note the total number of possible outcomes: $n(S) = 9$
   i. $M = \{M, U, G\}$, $n(M) = 3$ and $I = \{I, D, E, A\}$, $n(I) = 4$. Therefore, $P(M \text{ or } I) = P(M) + P(I) = \frac{3}{9} + \frac{4}{9} = \frac{7}{9}$.

   The probability that the letter is in M U G or I D E A is $\frac{7}{9}$.

   ii. $A = \{A, G, E, N, T\}$, $n(A) = 5$ and $D = \{M, I, D\}$, $n(D) = 3$. Since we want to find the probability that the letter is not in these, we want to find the probability of their complements occurring.

   Since the probabilities of all mutually exclusive events must sum to 1, we have $P(\overline{A} \text{ or } \overline{D}) = 1 - (P(A) + P(D))$.

   $P(A) + P(D) = \frac{5}{9} + \frac{3}{9} = \frac{8}{9}$

   Thus, $P(\overline{A} \text{ or } \overline{D}) = 1 - \frac{8}{9} = \frac{1}{9}$

   The probability the letter is neither in A G E N T nor in M I D is $\frac{1}{9}$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L085 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L086 in the Pupil Handbook before the next class.

Lesson Title: Multiplication law of probability	**Theme:** Probability and Statistics	
Lesson Number: M4-L086	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply the multiplication law to find the probability of independent events.	**Preparation** Review the content of this lesson and be prepared to explain the solutions.	

Opening *(1 minute)*

1. Explain that this lesson is on using the multiplication law of probability to find the probability of independent events.

Teaching and Learning *(20 minutes)*

1. Ask pupils to consider and discuss with seatmates the 2 events:
   A: You travel in a poda-poda to school which breaks down.
   B: You are late for school.
   Does one event have any effect on the other?
2. Invite volunteers to give their answer and state the reason why. (Example answer: Yes, the poda-poda breaking down caused you to be late.)
3. Ask pupils now to consider and discuss with seatmates the 2 events:
   A: Being a girl
   B: Being left-handed
   Does one event have any effect on the other?
4. Invite volunteers to give their answer and state the reason why. (Example answer: No, being a girl does not have any effect on which hand is used to write and being left-handed does not have an effect on being a girl)
5. Explain:
   - If one event happening has no effect on another event happening they are called **independent events.**
   - In the example above, the event “being a girl” and the event “being left-handed” do not affect each other, so they are independent events.
   - Independent events are examples of compound or combination events.
   - The events are connected by the word “and”.
6. Write on the board: If two events A and B are independent events, then the probability of A and B is given by: $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$.
7. Explain:
   - This is the Multiplication Law for independent events.
   - In probability, the word “and” or the symbol ∩ indicates multiplication.
   - There may be more than 2 events; the multiplication law applies to cases of more than 2 events as well.
   - As before: $P(\text{not } A) = 1 - P(A)$

8. Write a problem on the board: A fair die is rolled twice. What is the probability that it will land on a 6 in the first roll and land on an odd number in the second roll?
9. Solve the problem on the board and explain.

   Solution:

   Step 1. Find the possible outcomes.

   The possible outcomes are $S = \{1, 2, 3, 4, 5, 6\}$, so the total number of possible outcomes is $n(S) = 6$.

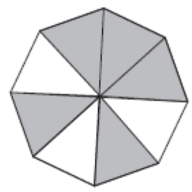
   Step 2. Find the probability of each independent event:

   Probability of rolling a 6: $P(6) = \frac{1}{6}$

   Probability of rolling an odd number: $P(\text{odd number}) = \frac{3}{6} = \frac{1}{2}$

   Step 3. Find the probability of rolling a 6 **and** rolling an odd number:

   $P(6 \text{ and odd number}) = P(6) \times P(\text{odd number}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
10. Write another problem on the board: The spinner shown has eight sections of equal size. Each one is coloured white or black. The events B and W are:

    B: the spinner lands on black.

    W: the spinner lands on white.

    Find the following probabilities:

    a. $P(B)$ b. $P(W)$ c. $P(B \text{ and } B)$

    d. $P(W \text{ and } W)$ e. $P(B \text{ and } W)$ f. $P(W \text{ and } B)$

    If the spinner is spun twice, find the probabilities of the following outcomes:

    g. White is obtained both times.

    h. A different colour is obtained on each spin.

    i. The same colour is obtained on each spin.
11. Solve parts a. through d. as a class. Ask volunteers to give the steps, and write the solution on the board.

    Solutions:

    First, note the possible outcomes: $S = \{B, B, B, B, B, W, W, W\}$. Therefore, $n(S) = 8$.

    a. $P(B) = \frac{5}{8}$

    b. $P(W) = 1 - \frac{5}{8} = \frac{3}{8}$

    c. $P(B \text{ and } B) = P(B) \times P(B) = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$
12. Ask pupils to work with seatmates to solve parts d. through i.
13. Walk around to check for understanding and clear misconceptions.
14. Invite volunteers to write the solutions on the board

    Solutions:

    d. $P(W \text{ and } W) = P(W) \times P(W) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

    e. $P(B \text{ and } W) = P(B) \times P(W) = \frac{5}{8} \times \frac{3}{8} = \frac{5}{64}$

    f. $P(W \text{ and } B) = P(W) \times P(B) = \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$

    g. $P(\text{White is obtained both times}) = P(W \text{ and } W) = P(W) \times P(W) = \frac{9}{64}$

    h. $P(\text{different colour obtained each time}) = P(B \text{ and } W)$ or $P(W \text{ and } B)$

$= (P(B) \times P(W)) + (P(W) \times P(B))$
$= \frac{15}{64} + \frac{15}{64}$
$= \frac{30}{64}$
$= \frac{15}{32}$

i. $P(\text{same colour}) = 1 - P(\text{different colour}) = 1 - \frac{15}{32} = \frac{17}{32}$

15. Make sure pupils understand how to apply the addition and multiplication rules in the same problem, as in question h.

Practice *(18 minutes)*

1. Write the following problems on the board:
   a. A coin is tossed and a die is rolled. What is the probability of getting a tail on the coin and a 3 on the die?
   b. The probability that Jamil will forget his ruler for his Maths examination is 0.35. The probability that he will forget his calculator for the examination is 0.15. What is the probability that he will:
      i. Not forget his ruler
      ii. Not forget his calculator
      iii. Not forget his ruler and not forget his calculator
2. Ask pupils to solve the problems independently or with seatmates.
3. Ask volunteers to write the solutions on the board and explain.
   Solutions:
   a. The possible outcomes of the coin are: $S_{\text{coin}} = \{H, T\}$ so that $n(S_{\text{coin}}) = 2$.
      The possible outcomes of the die are: $S_{\text{die}} = \{1, 2, 3, 4, 5, 6\}$ so $n(S_{\text{die}}) = 6$.
      Probability of getting a tail: $P(T) = \frac{1}{2}$
      Probability of getting 3: $P(3) = \frac{1}{6}$
      Probability of getting a tail and 3: $P(T \text{ and } 3) = P(T) \times P(3) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
   b. **Note that** $P(\text{Jamil forgets ruler}) = 0.35$ and $P(\text{Jamil forgets calculator}) = 0.15$
      i. $P(\text{Jamil does not forget ruler}) = 1 - 0.35 = 0.65$
      ii. $P(\text{Jamil does not forget calculator}) = 1 - 0.15 = 0.85$
      iii. $P(\text{Jamil does not forget calculator and ruler}) = 0.65 \times 0.85 = 0.5525$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L086 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L087 in the Pupil Handbook before the next class.

<table>
<tr><td>Lesson Title: Illustration of probabilities</td><td colspan="2">Theme: Probability and Statistics</td></tr>
<tr><td>Lesson Number: M4-L087</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to use outcome tables and tree diagrams to illustrate probability and solve problems.</td><td colspan="2">Preparation
Review the content of this lesson and be prepared to explain the solutions.</td></tr>
</table>

Opening *(1 minute)*

1. Explain that this lesson is on creating and using illustrations to solve probability problems. This lesson covers two methods of illustrating probabilities: outcome tables and tree diagrams. Venn diagrams are illustrated in the next lesson.

Teaching and Learning *(19 minutes)*

1. Ask pupils to write down the sample space S for throwing an unbiased die.
2. Invite a volunteer to give their answer. (Answer: $S = \{1, 2, 3, 4, 5, 6\}$)
3. Explain:
    - When dealing with the probability of an event occurring, it is very important to identify all the outcomes of the experiment.
    - For **outcomes which are all equally likely**, we can use an outcome or 2-way table to identify all the outcomes.
    - Drawing a table means we do not have to calculate the required probabilities.
4. Write the problems on the board:
    a. Use 2-way tables to list all the outcomes for tossing 2 fair coins
    b. Using the table obtained in question a., find the probability that:
        i. Both coins show heads.
        ii. Only one coin shows a tail.
        iii. Both coins land the same way up.
5. Solve the problems on the board, explaining each step:
    a. 2-way table showing possible coin outcomes →
    b. Calculate the probabilities by identifying the appropriate information in the tables:
        i. $P(\text{both coins show head}) = \frac{1}{4}$
        ii. $P(\text{only one coin show a tail}) = \frac{2}{4} = \frac{1}{2}$
        iii. $P(\text{both coins land the same way up}) = \frac{2}{4} = \frac{1}{2}$

		Second coin	
		H	**T**
First coin	**H**	HH	HT
	T	TH	TT

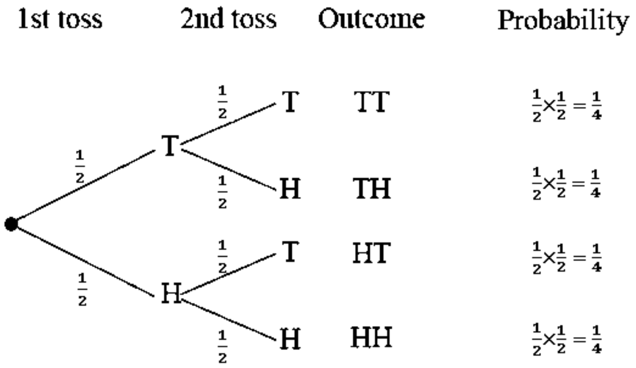
6. Explain:
    - Outcome tables cannot be used when the events are not equally likely to occur or when we have more than 2 events.
    - In such situations, we use a tree diagram where every branch represents an event together with its probability of occurring.

7. Write a problem on the board: A fair coin is tossed twice. What is the probability of getting: a. Two heads b. No heads c. Only one head
8. Solve the problem using a tree diagram:

Step 1. Draw the tree diagram showing all the outcomes.

Step 2. Find the probability of each outcome.

The probability of each outcome is obtained by multiplying the probabilities of the branches leading to that outcome.



Step 3. Find and write the required probabilities.

a. The probability of 2 heads is given by the bottom branch and is $\frac{1}{4}$.

b. The probability of no heads is given by the top branch and is $\frac{1}{4}$.

c. The probability of only head is given by the 2 middle branches – The combined probability is: $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$.

The probabilities should all add up to 1. Check this: $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1$

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. Create an outcome table for when 2 unbiased dice are rolled, and the outcomes are added together. Using the table obtained, find the probability of getting:
      i. A score of 7
      ii. A score of 5
      iii. A score that is an even number
      iv. A score of more than 8
      v. A score of less than 6
   b. The probability that Jane is late for school is 0.3. Using a tree diagram, find the probability that on two consecutive days, she is: i. Never late; ii. Late only once.
2. Ask pupils to solve the problems independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain.

Solutions:

a.

i. Table shown to the right.

From the table: $n(S) = 36$

ii. $P(\text{a score of } 7) = \frac{6}{36} = \frac{1}{6}$

iii. $P(\text{a score of } 5) = \frac{4}{36} = \frac{1}{9}$

iv. $P(\text{an even number score}) = \frac{18}{36} = \frac{1}{2}$

v. $P(\text{score of more than } 8) = \frac{10}{36} = \frac{5}{18}$

vi. $P(\text{score of less than } 6) = \frac{10}{36} = \frac{5}{18}$

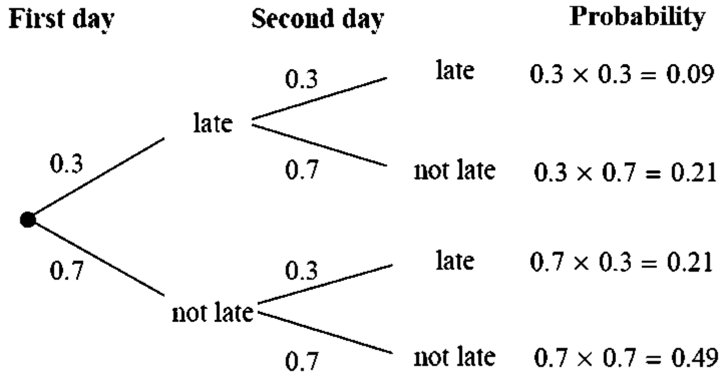
Second die (columns); First die (rows)

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b. probability of being late $= 0.3$

$P(\text{not being late}) = 1 - 0.3 = 0.7$

The probability of each outcome is obtained by multiplying the probabilities of the branches leading to that outcome.



Check that the probabilities add up to 1: $0.09 + 0.21 + 0.21 + 0.49 = 1$

From the tree diagram,

i. $P(\text{Jane is never late}) = 0.49$

ii. Let L = late, N = not late

$$\begin{aligned} P(\text{Jane is late only once}) &= P(LN) + P(NL) \\ &= 0.21 + 0.21 \\ &= 0.42 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L087 in the Pupil Handbook.
2. Ask pupils to read the overview of the next lesson, PHM4-L088 in the Pupil Handbook before the next class.

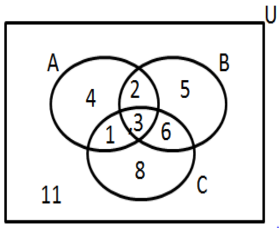
Lesson Title: Probability problem solving	Theme: Probability and Statistics	
Lesson Number: M4-L088	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems related to probability.	**Preparation** 1. Review the content of this lesson and be prepared to explain the solutions. 2. Write the problems at the start of Teaching and Learning on the board.	

Opening *(1 minute)*

1. Explain that this lesson uses the information in the previous 3 lessons to solve problems on probability.

Teaching and Learning *(22 minutes)*

1. Write the following problems on the board:
   a. Once a week, Kelfala checks his car. The probability that he needs to pump up a tyre is $\frac{1}{20}$. The probability that he has to add oil is $\frac{1}{10}$ and the probability he has to add water is $\frac{1}{5}$. If the events are independent, what is the probability that Kelfala:
      i. Does not need to do anything to his car.
      ii. Has to do one thing to his car.
      iii. Has to do at least one thing to his car.
   b. The Venn diagram shows the ways in which the events A, B and C can take place. Find:



      i. $P(A \text{ and } B \text{ and } C)$ ii. $P(A \text{ or } B \text{ or } C)$
      iii. $P(A \text{ or } B)$ iv. $P(A \text{ and } B)$
      v. $P(A \text{ or } C)$ vi. $P(A)$
   c. A white die, a blue die and a yellow die are rolled. What is the probability that:
      i. The score on all 3 dice is odd.
      ii. The score on the white die is 2 or 3, and the yellow die is greater than 3.
      iii. The score on the white die is a prime number, the blue die is 1 or 4 and the yellow die is a multiple of 3.
2. Ask pupils to work with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain.
   Solutions:
   a. First, identify the probability of each event:

$$\text{Let } A = \text{Kelfala pumps tyre} \quad P(A) = \frac{1}{20} \quad P(\bar{A}) = \frac{19}{20}$$

$$B = \text{Kelfala adds oil} \quad P(B) = \frac{1}{10} \quad P(\bar{B}) = \frac{9}{10}$$

C = Kelfala adds water $\quad P(C) = \frac{1}{5} \quad P(\bar{C}) = \frac{4}{5}$

Calculate the required probabilities.

i. $P(\text{Kelfala does not do anything to his car}) = P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})$

$$= \frac{19}{20} \times \frac{9}{10} \times \frac{4}{5} = \frac{684}{1000} = \frac{171}{250}$$

ii. $P(\text{Kelfala does one thing to his car})$

$$= P(A) \times P(\bar{B}) \times P(\bar{C}) + P(\bar{A}) \times P(B) \times P(\bar{C}) + P(\bar{A}) \times P(\bar{B}) \times P(C)$$

$$= \left(\frac{1}{20} \times \frac{9}{10} \times \frac{4}{5}\right) + \left(\frac{19}{20} \times \frac{1}{10} \times \frac{4}{5}\right) + \left(\frac{19}{20} \times \frac{9}{10} \times \frac{1}{5}\right)$$

$$= \frac{36}{1000} + \frac{76}{1000} + \frac{171}{1000}$$

$$= \frac{283}{1000}$$

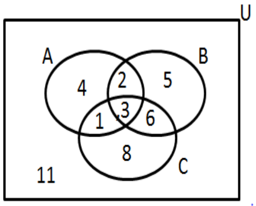
iii. $P(\text{Kelfala does at least one thing to his car})$

$$= 1 - P(\text{Kelfala does not do anything to his car})$$

$$= 1 - \frac{171}{250}$$

$$= \frac{79}{250}$$

b. Note that $n(U) = 40$. Find the required probabilities:



i. $P(A \text{ and } B \text{ and } C) = \frac{3}{40} \qquad P(A \cap B \cap C)$

ii. $P(A \text{ or } B \text{ or } C) = \frac{29}{40} \qquad P(A \cup B \cup C)$

iii. $P(A \text{ or } B) = \frac{21}{40} \qquad P(A \cup B)$

iv. $P(A \text{ and } B) = \frac{5}{40} = \frac{1}{8} \qquad P(A \cap B)$

v. $P(A \text{ or } C) = \frac{24}{40} = \frac{3}{5} \qquad P(A \cup C)$

vi. $P(A) = \frac{10}{40} = \frac{1}{4}$

c. Let A = score on white die, B = score on blue die, and C = score on yellow die. Possible outcomes for each die are $S = \{1, 2, 3, 4, 5, 6\}$ and $n(S) = 6$. Therefore:

i. $P(\text{the score on all 3 dice is odd}) = P(A \text{ odd}) \times P(B \text{ odd}) \times P(C \text{ odd}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

ii. $P(A \text{ is 2 or 3 and } B \text{ is greater than 3}) = P(A \text{ is 2 or 3}) \times P(B > 3) = \left(\frac{1}{6} + \frac{1}{6}\right) \times \frac{1}{2} = \frac{1}{6}$

iii. $P(A \text{ is prime number and B is 1 or 4 and C is a multiple of 3}) = \frac{1}{2} \times \left(\frac{1}{6} + \frac{1}{6}\right) \times \frac{1}{3} = \frac{1}{18}$

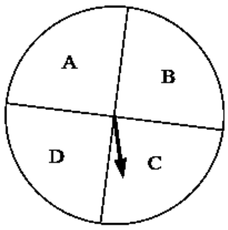
Practice *(16 minutes)*

1. Write the following problems on the board:
   a. A bag contains 2 red marbles, 1 blue marble and 1 yellow marble. A second bag contains 1 red marble, 2 blue marbles and 1 yellow marble. A marble is drawn from each bag.
      i. Complete the table showing all the possible pairs of colours.

Marble from second bag

Marble from first bag		R	B	B	Y
	R	RR	RB	RB	RY
	R	RR			
	B	BR			
	Y	YR			

   What is the probability that:
      ii. Both marbles are the same colour.
      iii. At least one marble is yellow.
      iv. No marble is yellow.
   b. A child's toy shown can point to one of 4 regions A, B, C or D when spun.



      i. Draw a tree diagram to show all the possible outcomes.
      ii. What is the probability that when it is spun twice it points to the same letter?
2. Ask pupils to construct the diagrams independently or with seatmates.
3. Invite volunteers to write the solutions on the board and explain.

   Solutions:

   a. i. Completed table is shown right.

Marble from second bag

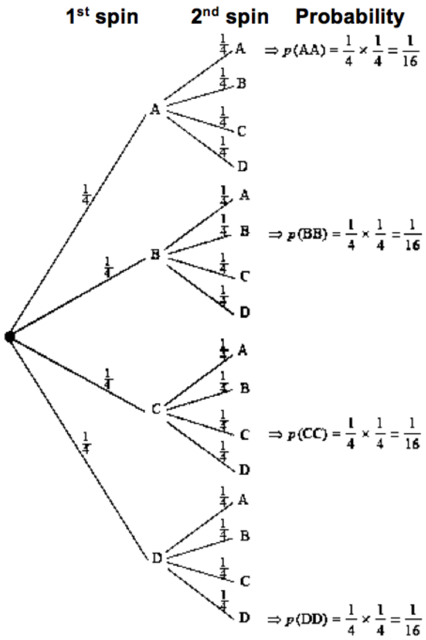
Marble from first bag		R	B	B	Y
	R	RR	RB	RB	RY
	R	RR	RB	RB	RY
	B	BR	BB	BB	BY
	Y	YR	YB	YB	YY

   ii. $P(\text{both marbles are same colour}) = \frac{5}{16}$

   iii. $P(\text{at least one marble is yellow}) = \frac{7}{16}$

   iii. $P(\text{no marble is yellow}) = 1 - P(\text{at least one marble is yellow}) = 1 - \frac{7}{16} = \frac{9}{16}$

b. i. Tree diagram:



ii. $P(\text{same letter}) = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16} = \frac{1}{4}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM4-L088 in the Pupil Handbook.
2. Inform pupils that this is the last lesson of instruction. The following lessons in the Teacher Guide and Pupil Handbook are mock exams. Make sure pupils are prepared for the mock exams.

Lesson Title: Mock Examination: Paper 1 – Multiple Choice	**Theme:** WASSCE Exam Preparation	
Lesson Number: M4-L089	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Complete a section of a mock WASSCE paper. 2. Answer multiple choice questions on various topics.	**Preparation** Read the note at the end of this lesson plan and prepare accordingly.	

Opening *(1 minute)*

1. Explain that the WASSCE exam has 3 sections. Today, pupils will practise section 1, which consists of multiply choice questions.

Teaching and Learning *(2 minutes)*

2. Explain Paper 1 – Multiple Choice
    - Paper 1 is 1.5 hours, and consists of 50 multiple choice questions. It is worth 50 marks.
    - This gives 1.8 minutes per problem, so time must be planned accordingly.
    - The questions are drawn from all topics on the WASSCE syllabus.

Practice *(35 minutes)*

1. Ask pupils to turn to PHM4-T2-W23-L089 in the Pupil Handbook. They are given 18 multiple-choice questions that are also attached to this lesson plan.
2. Pupils will have 34 minutes to complete the mock exam. This is approximately 1.8 minutes per question, as with the real exam.
3. Remind pupils that this is a mock examination. They should work independently, and **they should not look at the answer key during the exam**.
4. Move around the classroom and observe pupils. Provide guidance to pupils who need help.

Closing *(2 minutes)*

1. Encourage pupils if they did not complete the exam. Remind them that it is challenging to complete the exam within the timeframe, and this is why it is important to practice.
2. Answer any questions that pupils have about questions or topics on the mock exam.
3. Ask pupils to check their answers in the Answer Key at the back of the Pupil Handbook on their own time at home. Full solutions are given. Encourage pupils to review and study these solutions.

[NOTE ON ADMINISTERING MOCK EXAMINATIONS]

This is the first of 8 mock exams. These are shortened for a 40-minute lesson time, but are each based on a specific section of the WASSCE exam.

It is important to practice the WASSCE examination. Teachers should facilitate an environment that is as similar as possible to the real exam. Try to do the following in order to create such an environment:

- Arrange the classroom by spacing desks as much as possible, and seating pupils as far from each other as possible.
- Bring a watch or timer to keep time.
- Allow pupils to use calculators and log books.
- Ask pupils to work independently and quietly.

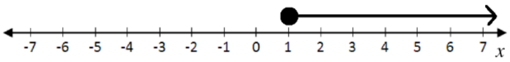
It is encouraged for teachers to mark the mock examination. It can make pupils feel good and prepared for the WASSCE exam if they have their mock exams checked by teachers. However, if you have large classes or many classes, it may be challenging to mark each mock exam. In that case, pupils can mark their own exams. The mock exam is important for the practice that it gives pupils in exam taking. They are able to check their answers and assess themselves using the Answer Key of the Pupil Handbook, which includes full solutions. Encourage them to note any topics they struggle with, and to focus their studies on these.

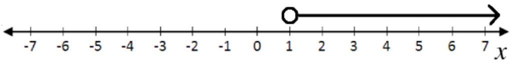
[MOCK EXAM 1 - MULTIPLE CHOICE QUESTIONS]

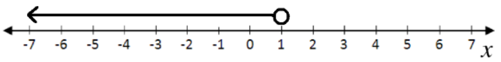
1. Find the 7^{th} term of the sequence: 4, 12, 36, …
   A. 22
   B. 60
   C. 2,916
   D. 8,748
2. Bintu draws the graphs of $y = x^2 + 2x - 3$ and $y = 3x - 1$ on the same axes. Which of these equations is she solving?
   A. $x^2 + 5x - 4 = 0$
   B. $x^2 - x - 2 = 0$
   C. $x^2 - x - 4 = 0$
   D. $x^2 - 5x + 2 = 0$
3. The population of students in a school is 625, of which 300 are girls. If this is represented on a pie chart, calculate the sectoral angle for girl students.
   A. $46°$
   B. $173°$
   C. $180°$
   D. $187°$
4. The table below shows the distribution of the scores of some students on a test. Calculate the mean score.

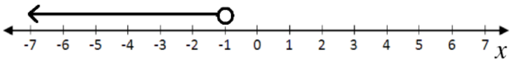
Scores	1-5	6-10	11-15	16-20
Frequency	1	2	5	2

   A. 10
   B. 11
   C. 12
   D. 13

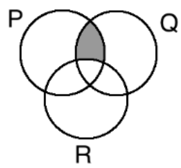
5. Illustrate graphically the solution of $\frac{2x}{3} - \frac{5}{6} > -\frac{x}{6}$.
   A. 

   B. 

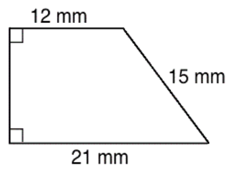
   C. 

   D. 

6. Make a the subject of the relation:
   $b = \sqrt{\frac{2a+5}{a-1}}$
   A. $a = \frac{b^2+5}{b^2-2}$
   B. $a = \frac{b^2-2}{b^2+5}$
   C. $a = \frac{b+5}{b-2}$
   D. $a = \frac{b-2}{b+5}$
7. Describe the shaded portion in the diagram.
   A. $R \cap (P \cap Q)'$
   B. $R \cup (P \cap Q)'$
   C. $R' \cup (P \cap$
   D. $R' \cap (P \cap Q)$

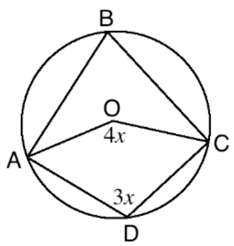


8. In a circle of radius r,
   a chord 24 cm long is 16 cm from the centre of the circle. Find the value of r, to the nearest cm.
   A. 16 cm
   B. 20 cm
   C. 29 cm
   D. 40 cm
9. Half of a number added to 3 times that number gives 77. Find the missing number.
   A. 7
   B. 11
   C. 22
   D. 38.5
10. A woman's eye level is 1.8 m above the horizontal ground and 12 m from a flag pole. If the pole is 5.4 m tall, calculate the angle of elevation of the top of the pole from her eyes. Give your answer to the nearest degree.
    A. 17°
    B. 24°
    C. 66°
    D. 73°
11. How many sides has a regular polygon with interior angles of 135°?
    A. 5
    B. 6
    C. 7
    D. 8
12. A bag of rice can feed 20 people for 12 days. How many days will it last for 80 people?
    A. 3 days
    B. 6 days
    C. 12 days
    D. 24 days
13. Simplify: $\frac{a}{2a+4b} + \frac{b}{a+2b} - \frac{1}{2}$
    A. 0
    B. $\frac{1}{2}$
    C. 1
    D. $\frac{2b}{a+2b}$
14. Calculate the area of the trapezium to the nearest square millimetre.
    A. 198 mm^2
    B. 248 mm^2
    C. 396 mm^2
    D. 495 mm^2



15. Simplify $36^{\frac{1}{2}} \times 8^{-\frac{2}{3}}$.
    A. $\frac{3}{4}$
    B. $1\frac{1}{2}$
    C. $4\frac{1}{2}$
    D. 24

16. The volume of a cuboid is 162 m^3. If the length, width, and height are in the ratio 2 : 1 : 3 respectively, find its total surface area.
    A. 36 m^2
    B. 99 m^2
    C. 198 m^2
    D. 324 m^2
17. Sia spent $\frac{1}{2}$ of her money on food, $\frac{1}{5}$ on school supplies and saved the rest. If she saved ₦9,000.00, how much did she spend on food?
    A. ₦2,700.00
    B. ₦9,000.00
    C. ₦15,000.00
    D. ₦ 30,000.00
18. The diagram is a circle with centre O. ABCD are points on the circle. Find the value of $\angle ABC$.



    A. 36°
    B. 72°
    C. 108°
    D. 144°

[SOLUTIONS]

1. Answer: C. 2,916

   Solution:

$$\begin{aligned} U_7 &= ar^{n-1} && \text{Formula for } n\text{th term of a GP} \\ &= 4(3^{7-1}) && \text{Substitute } a, n, \text{ and } r \\ &= 4(3^6) && \text{Simplify} \\ &= 2{,}916 \end{aligned}$$

2. Answer: B. $x^2 - x - 2 = 0$

   Solution:

$$\begin{aligned} x^2 + 2x - 3 &= 3x - 1 && \text{Set equations for } y \text{ equal} \\ x^2 + 2x - 3x - 3 + 1 &= 0 && \text{Write in standard form} \\ x^2 - x - 2 &= 0 && \text{Simplify} \end{aligned}$$

3. Answer: B. 173°

   Solution:

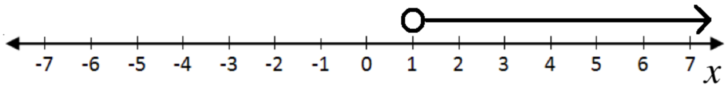
   Girls $= \frac{300}{625} \times 360° = 172.8° = 173°$ to the nearest degree.

4. Answer: C. 12

   Solution:

$$\begin{aligned} \bar{x} &= \frac{\sum fx}{\sum f} && \text{Formula for mean of grouped data} \\ &= \frac{3(1)+8(2)+13(5)+18(2)}{1+2+5+2} && \text{Substitute values} \\ &= \frac{120}{10} && \text{Simplify} \\ &= 12 \text{ marks} \end{aligned}$$

5. Answer: B.



   Solution:

$$\begin{aligned} \frac{2x}{3} - \frac{5}{6} &> -\frac{x}{6} \\ 4x - 5 &> -x && \text{Multiply by the LCM, 6} \\ -5 &> -x - 4x && \text{Transpose } 4x \\ -5 &> -5x \\ \frac{-5}{-5} &< \frac{-5x}{-5} && \text{Divide throughout by -5 (inequality changes)} \\ 1 &< x \end{aligned}$$

6. Answer: A. $a = \frac{b^2+5}{b^2-2}$

   Solution:

$$\begin{aligned} b &= \sqrt{\frac{2a+5}{a-1}} \\ b^2 &= \frac{2a+5}{a-1} && \text{Square both sides} \\ b^2(a-1) &= 2a+5 && \text{Multiply both sides by } (a-1) \\ ab^2 - b^2 &= 2a+5 \\ ab^2 - 2a &= b^2 + 5 && \text{Collect terms with } a \text{ on one side} \end{aligned}$$

$$a(b^2-2) = b^2+5 \quad \text{Factor out } a$$
$$a = \frac{b^2+5}{b^2-2} \quad \text{Divide both sides by } (b^2-2)$$

7. Answer: D. $R' \cap (P \cap Q)$

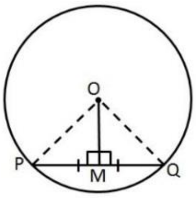
Solution:

$R' \cap (P \cap Q)$ gives the elements in the intersection of P and Q, which are not also in set R.

8. Answer: B. 20 cm

Solution:

Recall that the distance of the chord from the centre of the circle is the perpendicular bisector of the chord. Using the diagram on the right, we are given |OM|=16 cm and |PQ|=24 cm. Therefore, we have $|\mathrm{PM}| = \frac{1}{2}|\mathrm{PQ}| = \frac{1}{2}(24) = 12$. Apply Pythagoras' theorem to find the radius, OP.



$$\begin{aligned} OP^2 &= PM^2 + OM^2 \\ r^2 &= 12^2 + 16^2 \\ r^2 &= 144 + 256 \\ r^2 &= 400 \\ r &= \sqrt{400} = 20 \text{ cm} \end{aligned}$$

9. Answer: C. 22

Solution:

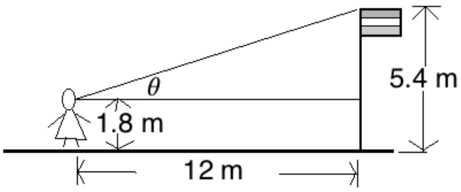
$\frac{1}{2}x + 3x$	$= 77$	Equation with x as the unknown number
$x + 6x$	$= 154$	Multiply throughout by 2
$7x$	$= 154$	Simplify
x	$= \frac{154}{7} = 22$	

10. Answer: A. 17°

Solution:

Using the diagram on the right,

$$\begin{aligned} \tan\theta &= \frac{5.4-1.8}{12} \\ \tan\theta &= \frac{3.6}{12} \\ \tan\theta &= 0.3 \\ \theta &= \tan^{-1} 0.3 \\ \theta &= 17° \text{ to the nearest degree} \end{aligned}$$



11. Answer: D. 8

Solution:

$135°$	$= \frac{(n-2)\times 180°}{n}$	Formula for the interior angle of a regular polygon
$135°n$	$= (n-2) \times 180°$	Solve for n
$135°n$	$= 180°n - 360°$	
$360°$	$= 180°n - 135°n$	
$360°$	$= 45°n$	
$\frac{360°}{45°}$	$= n$	
n	$= 8$	

12. Answer: A. 3 days

Solution:

If 20 people eat a bag of rice in 12 days, then 1 person eats the same bag of rice in $20 \times 12 = 240$ days.

Divide 240 days by 80 people: $240 \div 80 = 3$ days

13. Answer: A. 0

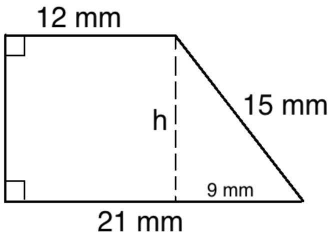
Solution:

$$\begin{aligned}
\frac{a}{2a+4b} + \frac{b}{a+2b} - \frac{1}{2} &= \frac{a}{2(a+2b)} + \frac{b}{a+2b} - \frac{1}{2} && \text{Factor the denominators} \\
&= \frac{a}{2(a+2b)} + \frac{2b}{2(a+2b)} - \frac{a+2b}{2(a+2b)} && \text{Change denominators to the LCM} \\
&= \frac{a+2b-(a+2b)}{2(a+2b)} && \text{Add/Subtract} \\
&= \frac{a+2b-a-2b}{2(a+2b)} && \text{Remove the brackets} \\
&= \frac{0}{2(a+2b)} = 0 && \text{Simplify}
\end{aligned}$$

14. Answer: A. 198 mm^2

Solution:

Use Pythagoras' theorem to find the height of the trapezium.



$$\begin{aligned}
h^2 + 9^2 &= 15^2 \\
h^2 + 81 &= 225 \\
h^2 &= 225 - 81 = 144 \\
h &= \sqrt{144} = 12 \text{ mm}
\end{aligned}$$

Apply the formula for area of a trapezium:

$$\begin{aligned}
A &= \frac{1}{2}(a+b)h \\
&= \frac{1}{2}(12+21)12 \\
&= 198 \text{ mm}^2
\end{aligned}$$

15. Answer: B. $1\frac{1}{2}$

Solution:

$$\begin{aligned}
36^{\frac{1}{2}} \times 8^{-\frac{2}{3}} &= \sqrt{36} \times \frac{1}{8^{\frac{2}{3}}}. && \text{Simplify} \\
&= 6 \times \frac{1}{\sqrt[3]{8^2}} \\
&= 6 \times \frac{1}{\sqrt[3]{64}} \\
&= 6 \times \frac{1}{4} \\
&= \frac{6}{4} = 1\frac{1}{2} && \text{Multiply}
\end{aligned}$$

16. Answer: C. 198 m^2

Using the ratio 2 : 1 : 3, let $l = 2x$, $w = x$ and $h = 3x$ for some value of x.

Solve for x using volume:

$$V = l \times w \times h$$

$$
\begin{aligned}
162 &= (2x)(x)(3x) \\
162 &= 6x^3 \\
\frac{162}{6} &= x^3 \\
27 &= x^3 \\
\sqrt{27} &= x \\
3 &= x
\end{aligned}
$$

Therefore, the dimensions are: $l = 2(3) = 6$ m, $w = 3$ m and $h = 3(3) = 9$ m.

Apply the formula for the surface area of a cuboid:

$$
\begin{aligned}
SA &= 2(lh + hw + lw) \\
&= 2(6 \times 9 + 9 \times 3 + 6 \times 3) \\
&= 2(54 + 27 + 18) \\
&= 198 \text{ m}^2
\end{aligned}
$$

17. Answer: C. ₦15,000.00

Fraction of money saved: $1 - \frac{1}{2} - \frac{1}{5} = \frac{10}{10} - \frac{5}{10} - \frac{2}{10} = \frac{3}{10}$

Set up an equation with the total amount, call it A:

$$
\begin{aligned}
\frac{3}{10}A &= ₦9{,}000 \\
3A &= ₦90{,}000 \\
A &= ₦\frac{90{,}000}{3} = ₦30{,}000.00
\end{aligned}
$$

Amount spent on food: $\frac{1}{2}(30{,}000) =$ ₦15,000.00

18. Answer: B. 72°

Solution:

Using circle theorems, we know that $\angle ABC + \angle ADC = 180°$ because they are opposite angles in a cyclical quadrilateral. We also know that $2\angle ABC = \angle AOC$, because the same chord subtends these 2 angles at the circumference and centre. Using these theorems, we can write 2 equations for $\angle ABC$ in x:

$$\angle ABC + \angle ADC = 180° \rightarrow \angle ABC = 180° - \angle ADC \rightarrow \angle ABC = 180° - 3x$$

$$2\angle ABC = \angle AOC \rightarrow \angle ABC = \frac{1}{2}\angle AOC \rightarrow \angle ABC = \frac{1}{2}(4x) = 2x$$

Set the 2 equations for $\angle ABC$ equal, and solve for x:

$$
\begin{aligned}
180° - 3x &= 2x \\
180° &= 2x + 3x \\
180° &= 5x \\
\frac{180°}{5} &= x \\
36° &= x
\end{aligned}
$$

Substitute $x = 36°$ into either formula for $\angle ABC$.

$$\angle ABC = 2x = 2(36°) = 72°$$

<table>
<tr><td>Lesson Title: Mock Examination: Paper 1 – Multiple Choice</td><td colspan="2">Theme: WASSCE Exam Preparation</td></tr>
<tr><td>Lesson Number: M4-L090</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Complete a section of a mock WASSCE paper.
2. Answer multiple choice questions on various topics.</td><td colspan="2">Preparation
Read the note at the end of this lesson plan and prepare accordingly.</td></tr>
</table>

Opening *(1 minute)*

1. Explain that the WASSCE exam has 3 sections. Today, pupils will practise section 1, which consists of multiply choice questions.

Teaching and Learning *(2 minutes)*

1. Discuss:
   - How did you feel about the previous mock exam for paper 1? What were your successes and challenges?
   - What exam-taking skills did you use during the previous exam?
2. Explain some exam-taking skills:
   - Plan your time. Do not spend too much time on one problem.
   - For the multiple-choice section, it is not necessary to show all of your work on the exam paper. Therefore, your work does not have to be as neat and clear as on the other sections of the exam.
   - If you complete the exam, take time to check your solutions. If you notice an incorrect answer, double check it before changing it.

Practice *(35 minutes)*

1. Ask pupils to turn to PHM4-T2-W23-L090 in the Pupil Handbook. They are given 18 multiple-choice questions that are also attached to this lesson plan.
2. Pupils will have 34 minutes to complete the mock exam. This is approximately 1.8 minutes per question, as with the real exam.
3. Remind pupils that this is a mock examination. They should work independently, and **they should not look at the answer key during the exam**.
4. Move around the classroom and observe pupils. Provide guidance to pupils who need help.

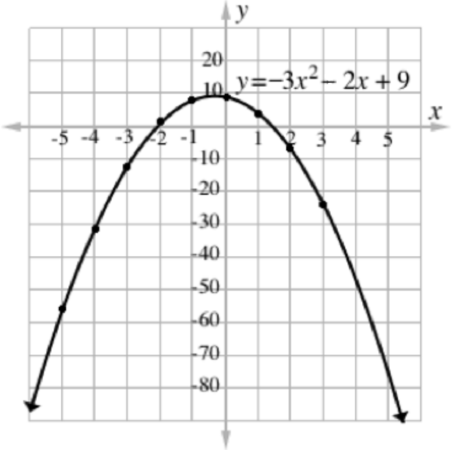
Closing *(2 minutes)*

1. Encourage pupils if they did not complete the exam. Remind them that it is challenging to complete the exam within the timeframe, and this is why it is important to practice.
2. Answer any questions that pupils have about questions or topics on the mock exam.
3. Ask pupils to check their answers in the Answer Key at the back of the Pupil Handbook on their own time at home. Full solutions are given. Encourage pupils to review and study these solutions.

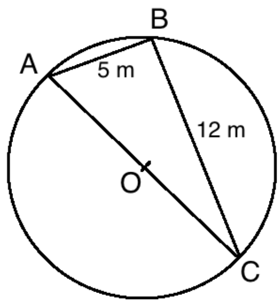
[MOCK EXAM 2 – MULTIPLE-CHOICE QUESTIONS]

1. If $U = \{t, u, v, w, x, y, z\}$, $A = \{t, v, x, z\}$ and $B = \{x, y, z\}$ find $(A \cap B)'$.
   A. $\{x, z\}$
   B. $\{t, v, y\}$
   C. $\{t, v, x, y, z\}$
   D. $\{t, u, v, w, y\}$
2. x varies directly as y and inversely as z. If $x = 2$ when $y = 8$ and $z = 12$, Find x in terms of y and z.
   A. $x = \frac{3y}{z}$
   B. $x = \frac{3z}{y}$
   C. $x = \frac{4z}{3y}$
   D. $x = \frac{yz}{48}$

Use the graph of $y = -3x^2 - 2x + 9$ to answer questions 3 and 4.

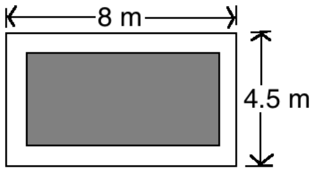


3. Which of the following is approximately equal to the smaller root of the equation?
   A. -3.0
   B. -2.1
   C. -0.2
   D. 1.5
4. Estimate the gradient at the point $x = 2$.
   A. -1.4
   B. -5
   C. -14
   D. 14
5. In the diagram, AC is a diameter of the circle with centre O. Find the radius of the circle.



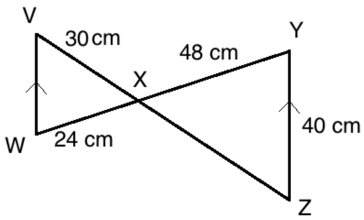
   A. 5 m
   B. 6.5 m
   C. 7 m
   D. 13 m

6. The diagram shows a carpet laid in a room that is 8 metres long by 4.5 metres wide. There is a 0.75-metre margin between each wall and the carpet. Find the area of the carpet, correct to 1 decimal place.



   A. 19.5 m²
   B. 27.2 m²
   C. 36.0 m²
   D. 57.0 m²

7. In the diagram, $VW||YZ$, $|VX| = 30$ cm, $|WX| = 24$ cm, $|XY| = 48$ cm, and $|YZ| = 40$ cm. Calculate $|VW|$.



   A. 15 cm
   B. 20 cm
   C. 25 cm
   D. 30 cm

8. Find the equation whose roots are -3 and $\frac{1}{2}$.
   A. $x^2 - 5x - 3 = 0$
   B. $2x^2 - 7x - 3 = 0$
   C. $2x^2 + 5x + 3 = 0$
   D. $2x^2 + 5x - 3 = 0$

9. Factorise completely: $2ax - 21by - 3bx + 14ay$.
   A. $3a(2x - 3y) + 4b(2x - 3y)$
   B. $2x(3a + 4b) - 3y(3a + 4b)$
   C. $2ax - 3b(7b + x) + 14ay$
   D. $(x + 7y)(2a - 3b)$

10. If $a = 4$, $b = 3$ and $x = -2$, evaluate $\frac{a}{b} + \frac{3x}{a} - 1\frac{1}{2}$.
   A. $-1\frac{2}{3}$
   B. $-1\frac{1}{3}$
   C. $-\frac{2}{3}$
   D. $-\frac{1}{3}$

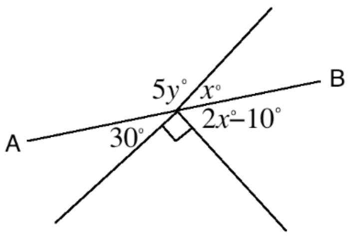
11. A sector of a circle with radius of 8 cm subtends an angle of 90° at the centre. Calculate its perimeter in terms of π.
   A. 4π
   B. $4(4 + \pi)$
   C. $4(2 + \pi)$
   D. $4(1 + \pi)$

12. If $\frac{8^x \times 2^{x+1}}{4^{3x}} = 1$, find the value of x.
   A. -1
   B. $-\frac{1}{2}$
   C. 0
   D. $\frac{1}{2}$

13. If $34_x = 10011_2$, find the value of x.
   A. 4
   B. 5
   C. 6
   D. 7

14. The area of a sector of a circle with radius 10 cm is 25π cm². If the sector is folded to form a cone, calculate the radius of the base of the cone.
   A. 2.5
   B. 2.5π
   C. 5
   D. 5π

15. Foday has 16 currency notes in his pocket, all of which are Le 1,000.00 or Le 5,000.00 notes. If he has a total of Le 52,000.00, how many Le 5,000.00 notes does he have?
    A. 7
    B. 8
    C. 9
    D. 10
16. In the diagram, AB is a straight line. Find the value of y.



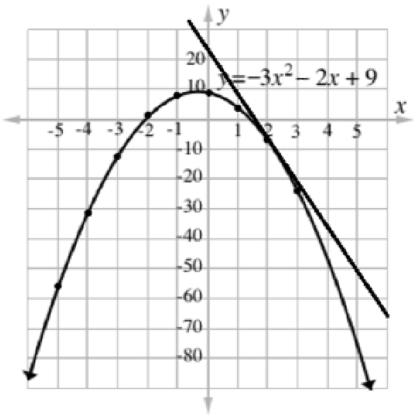
    A. 29°
    B. 35°
    C. 36°
    D. 145°
17. A salesperson gave change of \$18.00 instead of \$22.00. Calculate his percentage error.
    A. 3.3%
    B. 18.2%
    C. 22.2%
    D. 81.8%
18. Fatu traveled 240 km from Bo to Freetown in 4 hours. What was her speed in m/s? Give your answer to 3 significant figures.
    A. 864 m/s
    B. 216 m/s
    C. 66.7 m/s
    D. 16.7 m/s

Answer Key

1. Answer: D. $\{t, u, v, w, y\}$
   $(A \cap B)'$ is the complement of the intersection of A and B. In other words, it is the elements in the universal set U which are not in the intersection of A and B. First, find the intersection of A and B: $A \cap B = \{x, z\}$. Next, list the elements of U that are not in $A \cap B$. This gives: $(A \cap B)' = \{t, u, v, w, y\}$.
2. Answer: A. $x = \frac{3y}{z}$
   x varies directly as y and inversely as z is written in symbols as $x \propto \frac{y}{z}$. This can also be written as an equation $x = \frac{ky}{z}$, where k is a constant. Use the information in the problem to solve for k:

$$\begin{aligned} x &= \frac{ky}{z} && \text{Equation} \\ 2 &= \frac{k8}{12} && \text{Substitute } x = 2,\ y = 8 \text{ and } z = 12 \\ 2(12) &= 8k \\ 24 &= 8k \\ \frac{24}{8} &= k \\ 3 &= k \end{aligned}$$

   Therefore, the relationship between x, y and z is $x = \frac{3y}{z}$.
3. Answer: B. -2.1
   The roots are points at which the curve intersects the x-axis. The smaller root is the negative one, which intersects the curve near $x = -2$. Select the answer nearest to -2, which is -2.1.
4. Answer: C. -14
   Sketch a tangent to the curve at $x = 2$, and use it to estimate the slope. Note that the slope is negative, and the y-axis has a scale of 10 units. At this point, you could eliminate answers that are not feasible (such as positive 14).



   You can also apply the formula for the gradient. Choose any 2 points on the tangent line, and use whole or rounded numbers to save time. Points $(5, -50)$ and $(0, 20)$ are on or near the tangent line.

$$\begin{aligned} \text{Gradient: } m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{20-(-50)}{0-5} \\ &= -\frac{70}{5} \\ &= -14 \end{aligned}$$

5. Answer: B. 6.5 m.

From the circle theorems, recall that an angle subtended in a circle by the diameter is a right angle. Thus, ABC is a right-angled triangle. Apply Pythagoras' theorem to find the diameter:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ AC^2 &= 5^2 + 12^2 \\ AC^2 &= 25 + 144 \\ AC^2 &= 169 \\ AC &= \sqrt{169} = 13 \text{ m} \end{aligned}$$

Use the diameter to find the radius: $r = \frac{d}{2} = \frac{13}{2} = 6.5$ m

6. Answer: A. 19.5 m^2

   Note that there is a 0.75-metre margin on each side of the carpet. Thus, subtract twice that from each dimension before finding the area.

   Length $= 8 - 2(0.75) = 8 - 1.5 = 6.5$ m

   Width $= 4.5 - 2(0.75) = 4.5 - 1.5 = 3$

   Area $= l \times w = 6.5 \times 3 = 19.5$ m^2

7. Answer: B. 20 cm

   Note that XWV and XYZ are similar triangles. Therefore, their sides are proportional, and a ratio can be created with $|VW|$ and known side lengths:

   $\frac{24 \text{ cm}}{48 \text{ cm}} = \frac{|VW|}{40 \text{ cm}}$

   Simplifying this, we have $\frac{1 \text{ cm}}{2 \text{ cm}} = \frac{|VW|}{40 \text{ cm}}$. Solve for $|VW|$:

$$\begin{aligned} \frac{|VW|}{40 \text{ cm}} &= \frac{1 \text{ cm}}{2 \text{ cm}} \\ 2|VW| &= 40 \times 1 \\ 2|VW| &= 40 \\ |VW| &= \frac{40}{2} \\ |VW| &= 20 \text{ cm} \end{aligned}$$

8. Answer: D. $2x^2 + 5x - 3 = 0$

   To find a quadratic equation given its roots, find b and c of the quadratic equation in standard form. This can be done by finding the sum and product of the roots and substituting them in the following equation:

$$x^2 + bx + c = x^2 - (sum\ of\ roots)x + (product\ of\ roots) = 0$$

   Sum of the roots: $-3 + \frac{1}{2} = -2\frac{1}{2} = -\frac{5}{2}$

   Product of the roots: $-3 \times \frac{1}{2} = -\frac{3}{2}$

   Quadratic equation: $x^2 - \left(-\frac{5}{2}\right)x + \left(-\frac{3}{2}\right) = x^2 + \frac{5}{2}x - \frac{3}{2} = 0$

   Multiply throughout by 2 to eliminate fractions: $2x^2 + 5x - 3 = 0$

9. Answer: D. $(x + 7y)(2a - 3b)$

   Rearrange the terms and factorise as follows:

$2ax - 21by - 3bx + 14ay$	$=$	$2ax - 3bx - 21by + 14ay$	Rearrange
	$=$	$x(2a - 3b) + 7y(-3b + 2a)$	Factorize
	$=$	$x(2a - 3b) + 7y(2a - 3b)$	

$$= \quad (x+7y)(2a-3b)$$

10. Answer: A. $-1\frac{2}{3}$

    Substitute the given values into the formula and evaluate:

$$\begin{aligned} \frac{a}{b}+\frac{3x}{a}-1\frac{1}{2} &= \frac{4}{3}+\frac{3(-2)}{4}-1\frac{1}{2} \\ &= \frac{4}{3}-\frac{6}{4}-\frac{3}{2} && \text{Simplify} \\ &= \frac{16}{12}-\frac{18}{12}-\frac{18}{12} && \text{Change denominators to the LCM, 12} \\ &= \frac{16-18-18}{12} && \text{Subtract} \\ &= \frac{-20}{12} && \text{Simplify} \\ &= \frac{-5}{3} = -1\frac{2}{3} \end{aligned}$$

11. Answer: B. $4(4+\pi)$

    Note that the perimeter is composed of 2 radii and an arc. We already know the radius of the circle. Find the length of the arc:

$$\begin{aligned} \frac{90°}{360°}C &= \frac{1}{4}2\pi r = \frac{1}{2}\pi r && \text{Simplify the formula} \\ &= \frac{1}{2}\pi(8) && \text{Substitute } r = 8 \text{ cm} \\ &= 4\pi \end{aligned}$$

    Add to find the perimeter:

$$\begin{aligned} P &= 8+8+4\pi && \text{Simplify the formula} \\ &= 16+4\pi && \text{Substitute } r = 8 \text{ cm} \\ &= 4(4+\pi) && \text{Factorise} \end{aligned}$$

12. Answer: C. $\frac{1}{2}$

    Convert each term to an index with base 2, then apply the laws of logarithms to simplify.

$$\begin{aligned} \frac{8^x \times 2^{x+1}}{4^{3x}} &= 1 \\ \frac{2^{3x} \times 2^{x+1}}{2^{2\times 3x}} &= 1 && \text{Convert to base 2} \\ \frac{2^{3x} \times 2^{x+1}}{2^{6x}} &= 1 && \text{Simplify} \\ \frac{2^{3x+x+1}}{2^{6x}} &= 1 && \text{Apply law of multiplication of logarithms} \\ \frac{2^{4x+1}}{2^{6x}} &= 1 && \text{Simplify} \\ 2^{4x+1-6x} &= 1 \\ 2^{-2x+1} &= 1 && \text{Note that } a^0 = 1 \\ 2^{-2x+1} &= 2^0 \\ -2x+1 &= 0 && \text{Set the powers equal} \\ -2x &= -1 && \text{Solve for } x \end{aligned}$$

$$x = \frac{-1}{-2} = \frac{1}{2}$$

13. Answer: B. 5

Convert both sides to base 10, then set them equal and solve for x.

Convert the left-hand side from base x to base 10:

$$\begin{aligned} 34_x &= (3 \times x^1) + (4 \times x^0) \\ &= 3x + 4 \end{aligned}$$

Convert the right-hand side from base 2 to base 10:

$$\begin{aligned} 10{,}011_2 &= (1 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\ &= 16 + 0 + 0 + 2 + 1 \\ &= 19 \end{aligned}$$

Set the two sides equal and solve for x:

$$\begin{aligned} 3x + 4 &= 19 \\ 3x &= 19 - 4 && \text{Transpose 4} \\ 3x &= 15 \\ x &= \frac{15}{3} && \text{Divide throughout by 3} \\ x &= 5 \end{aligned}$$

14. Answer: A. 2.5 cm

Use the area of the sector to find the angle subtended by the arc of the circle. The length of the arc will be the circumference of the base of the cone. Use the formula for circumference to solve for the radius of the base.

$$\begin{aligned} A &= \frac{\theta}{360°}\pi r^2 && \text{Area of a segment} \\ 25\pi &= \frac{\theta}{360°}\pi 10^2 \\ 25 &= \frac{100}{360°}\theta && \text{Cancel } \pi \text{ and simplify} \\ 25 &= \frac{5}{18}\theta \\ 25 \times 18 &= 5\theta \\ 450 &= 5\theta \\ \frac{450}{5} &= \theta \\ 90° &= \theta \end{aligned}$$

Length of arc:

$$\begin{aligned} L &= \frac{\theta}{360°}2\pi r && \text{Length of an arc} \\ L &= \frac{90°}{360°}2\pi(10) && \text{Substitute } \theta = 90° \text{ and } r = 10 \\ L &= \frac{1}{4}20\pi && \text{Simplify} \\ L &= 5\pi \end{aligned}$$

Since the length of the arc is the circumference of the base, apply the formula for circumference:

$$C = 2\pi r \qquad \text{Circumference of a circle}$$

$$5\pi = 2\pi r \quad \text{Substitute } C = 5\pi$$

$$\frac{5\pi}{2\pi} = r \quad \text{Solve for } r$$

$$\frac{5}{2} = r$$

$$2.5 = r$$

15. Answer: C. 9

Set up simultaneous equations and solve using substitution. Let x be the number of Le 5,000 notes and y be the number of Le1,000 notes. Then we have:

$x + y$	$=$	16	Equation (1)
$5{,}000x + 1{,}000y$	$=$	$52{,}000$	Equation (2)

y	$=$	$16 - x$	Change the subject of (1)
$5{,}000x + 1{,}000(16 - x)$	$=$	$52{,}000$	Substitute (1) in (2)
$5{,}000x + 16{,}000 - 1{,}000x$	$=$	$52{,}000$	Solve for x
$4{,}000x + 16{,}000$	$=$	$52{,}000$	
$4{,}000x$	$=$	$52{,}000 - 16{,}000$	
$4{,}000x$	$=$	$36{,}000$	
x	$=$	9	

He has 9 Le 5,000.00 notes.

16. Answer: A. 29

Note that in order to solve for y, we must first solve for x. Use the angles below the line AB to find the measure of x.

$180°$	$=$	$30° + 90° + (2x° - 10°)$	Set equal to $180°$
$180°$	$=$	$30° + 90° + 2x° - 10°$	Solve for x
$180°$	$=$	$110° + 2x°$	
$180° - 110°$	$=$	$2x°$	
$70°$	$=$	$2x°$	
$\frac{70°}{2}$	$=$	$x°$	
$35°$	$=$	x	

Use the angles above the line AB to solve for y:

$180°$	$=$	$5y° + x°$	Set equal to $180°$
$180°$	$=$	$5y° + 35°$	Solve for y
$180° - 35°$	$=$	$5y°$	
$145°$	$=$	$5y°$	
$\frac{145°}{5}$	$=$	$y°$	
$29°$	$=$	y	

17. Answer: B. 18.2%

Calculate percentage error using the formula:

$$\text{Percentage error} = \frac{|\text{exact value} - \text{approximate value}|}{\text{exact value}} \times 100\%$$

$= \frac{|18-22|}{22} \times 100\%$
$= \frac{4}{22} \times 100\%$
$= 18.2\%$

18. Answer: D. 16.7 m/s
Use the information in the problem to find her spend in km/hour:

Speed $= \frac{240\text{ km}}{4\text{ hr}} = 60$ km/hr

Use the conversion factors 1 hour = 3,600 seconds, and 1 km = 1,000 metres to convert her speed to m/s.

Speed $= \frac{60\text{ km}}{1\text{ hr}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ hr}}{3600\text{ s}} = \frac{600}{36} = 16.7$ m/s

<table>
<tr><td>Lesson Title: Mock Examination: Paper 2A – Compulsory Questions</td><td colspan="2">Theme: WASSCE Exam Preparation</td></tr>
<tr><td>Lesson Number: M4-L091</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Complete a section of a mock WASSCE paper.
2. Answer essay questions on various topics.</td><td colspan="2">Preparation
Prepare your classroom to administer the mock exam.</td></tr>
</table>

Opening *(1 minute)*

1. Explain that the WASSCE exam has 3 sections. Today pupils will practise section 2A, which consists of compulsory essay questions.

Teaching and Learning *(2 minutes)*

1. Explain Paper 2 – Essay Questions
   - Paper 2 consists of 13 essay questions in 2 sections – 2A and 2B.
   - Paper 2 is worth 100 marks in total.
   - Pupils will be required to answer 10 essay questions in all, across the 2 sections.
   - This is an average of 15 minutes per question. However, keep in mind that section 2B is more complicated, therefore, plan your time accordingly.
2. Explain Paper 2A – Compulsory Questions
   - Paper 2A is worth 40 marks.
   - There are 5 **compulsory** essay questions in paper 2A.
   - Compulsory questions often have multiple parts (a, b, c, …). The questions may not be related to each other. Each part of the question should be completed.
   - The questions in paper 2A are simpler than those in 2B, generally requiring fewer steps.
   - The questions on paper 2A are drawn from the common area of the WASSCE syllabus (they are not on topics that are specific to certain countries).

Practice *(35 minutes)*

1. Ask pupils to turn to PHM4-T2-W23-L091 in the Pupil Handbook. They are given 3 essay questions that are also attached to this lesson plan.
2. Pupils will have 34 minutes to complete the mock exam. This is approximately 11 minutes per question.
3. Remind pupils that this is a mock examination. They should work independently, and **they should not look at the answer key during the exam**.

4. Move around the classroom and observe pupils. Provide guidance to pupils who need help.

Closing *(2 minutes)*

1. Answer any questions that pupils have about questions or topics on the mock exam.
2. Ask pupils to check their answers in the Answer Key at the back of the Pupil Handbook on their own time at home. Full solutions are given. Encourage pupils to review and study these solutions.

[MOCK EXAM 3 – COMPULSORY ESSAY QUESTIONS]

1. a. Simplify: $\frac{1\frac{1}{2}+2\frac{1}{3}}{2\frac{1}{2}-3\frac{3}{4}\times\frac{2}{5}}$

   b. Given that $(\sqrt{2}-3\sqrt{5})(\sqrt{2}+\sqrt{5}) = a + b\sqrt{10}$, find a and b.
2. The table shows the number of children of the families living in a certain community.

Children	0	1	2	3	4	5
Frequency	1	4	3	5	4	3

   i. Find the: i. Mode ii. Mean iii. Third quartile
   ii. If a pie chart were drawn for the data, what would be the angle of the sector showing families with 4 children?
3. a. In a class of 50 students, 35 offered chemistry (C), 23 offered French (F) and 13 offered neither of the 2 subjects. i. Draw the Venn diagram to represent the information; ii) How many students offered both chemistry and French? iii. What is $n(C \cup F)$?

   b. If $a = \frac{2x}{x+1}$ and $b = \frac{x-1}{x+1}$, express $\frac{a+b}{a-b}$ in terms of x.

[SOLUTIONS]

19. a. Apply the correct order of operations (BODMAS):

$$\frac{1\frac{1}{2}+2\frac{1}{3}}{2\frac{1}{2}-3\frac{3}{4}\times\frac{2}{5}} = \frac{\frac{3}{2}+\frac{7}{3}}{\frac{5}{2}-\frac{15}{4}\times\frac{2}{5}}$$ Convert to improper fractions

$$= \frac{\frac{3}{2}+\frac{7}{3}}{\frac{5}{2}-\frac{30}{20}}$$ Multiply

$$= \frac{\frac{9}{6}+\frac{14}{6}}{\frac{5}{2}-\frac{3}{2}}$$

$$= \frac{\frac{23}{6}}{\frac{2}{2}}$$ Add/Subtract

$$= \frac{3\frac{5}{6}}{1}$$ Simplify

$$= 3\frac{5}{6}$$

b. Multiply, then simplify:

$$\begin{aligned}(\sqrt{2}-3\sqrt{5})(\sqrt{2}+\sqrt{5}) &= \sqrt{2}(\sqrt{2}+\sqrt{5})-3\sqrt{5}(\sqrt{2}+\sqrt{5}) \\ &= \sqrt{2}\sqrt{2}+\sqrt{2}\sqrt{5}-3\sqrt{2}\sqrt{5}-3\sqrt{5}\sqrt{5} \\ &= 2+\sqrt{10}-3\sqrt{10}-3(5) \\ &= 2-15-2\sqrt{10} \\ &= -13-2\sqrt{10}\end{aligned}$$

Answer: $a=-13$, $b=-2$

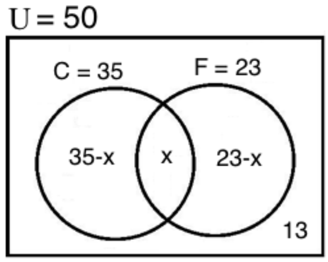
20. a. i. Mode: 3 children

ii. Mean $= \frac{\sum fx}{\sum f} = \frac{0(1)+1(4)+2(3)+3(5)+4(4)+5(3)}{1+4+3+5+4+3} = \frac{4+6+15+16+15}{20} = \frac{56}{20} =$ 2.8 children

iii. The position of the 3rd quartile is the $\frac{3}{4}(20) = 15$th family. The 15th family falls into the group with 4 children. Thus, the 3rd quartile is 4 children.

b. The segment representing 4 children is given by: $\frac{4}{20}(360°) = 72°$.

21. a. i. Venn diagram:



ii. Set the sum of the segments equal to 50, the total number of pupils, and solve for x.

$$\begin{aligned}50 &= (35-x)+x+(23-x)+13 \\ 50 &= 35+23+13-x \\ 50 &= 71-x \\ x &= 71-50 \\ x &= 21\end{aligned}$$

iii. To find the cardinality of the union, add the cardinality of each set and subtract the elements in their intersection.

$$\begin{aligned}n(C\cup F) &= n(C)+n(F)-n(C\cap F) \\ &= 35+23-21 \\ &= 37\end{aligned}$$

Alternatively, identify the cardinality of each section of the union of C and F, and add them:

$$\begin{aligned}n(C\cup F) &= 35-x+x+23-x \\ &= 58-x \\ &= 58-21 \\ &= 37\end{aligned}$$

b. Substitute a and b in the expression, and evaluate.

$$
\begin{aligned}
\frac{a+b}{a-b} &= \frac{\frac{2x}{x+1}+\frac{x-1}{x+1}}{\frac{2x}{x+1}-\frac{x-1}{x+1}} \\
&= \frac{\frac{2x+x-1}{x+1}}{\frac{2x-x+1}{x+1}} \\
&= \frac{\frac{3x-1}{x+1}}{\frac{x+1}{x+1}} \\
&= \frac{\frac{3x-1}{x+1}}{1} \\
&= \frac{3x-1}{x+1}
\end{aligned}
$$

Lesson Title: Mock Examination: Paper 2A – Compulsory Questions	**Theme:** WASSCE Exam Preparation	
Lesson Number: M4-L092	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Complete a section of a mock WASSCE paper. 2. Answer essay questions on various topics.	**Preparation** Prepare your classroom to administer the mock exam.	

Opening *(1 minute)*

1. Explain that the WASSCE exam has 3 sections. Today, pupils will practise section 2A, which consists of compulsory essay questions.

Teaching and Learning *(2 minutes)*

1. Discuss:
   - How did you feel about the previous mock exam for paper 2A? What were your successes and challenges?
   - What exam-taking skills did you use during the previous exam?
2. Explain some exam-taking skills:
   - Plan your time. Do not spend too much time on one problem.
   - Show all of your work on the exam paper. Examiners can give some credit for rough work. Do not cross out your work.
   - If you complete the exam, take time to check your solutions. If you notice an incorrect answer, double check it before changing it.

Practice *(35 minutes)*

1. Ask pupils to turn to PHM4-T2-W23-L092 in the Pupil Handbook. They are given 3 essay questions that are also attached to this lesson plan.
2. Pupils will have 34 minutes to complete the mock exam. This is approximately 11 minutes per question.
3. Remind pupils that this is a mock examination. They should work independently, and **they should not look at the answer key during the exam**.
4. Move around the classroom and observe pupils. Provide guidance to pupils who need help.

Closing *(2 minutes)*

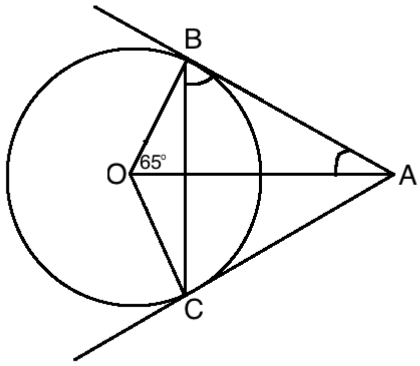
1. Answer any questions that pupils have about questions or topics on the exam.

2. Ask pupils to check their answers in the Answer Key at the back of the Pupil Handbook on their own time at home. Full solutions are given. Encourage pupils to review and study these solutions.

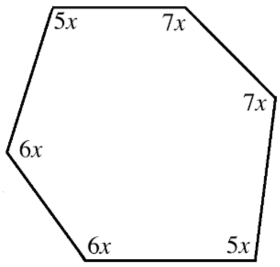
[MOCK EXAM 4 – COMPULSORY ESSAY QUESTIONS]

1. a. Make p the subject of the relation: $q = \sqrt{t^2p - \frac{rp}{t}}$

   b. If $2^{x+1} = 8^{3y}$ and $2x + y = 36$, find the value of $x - y$.
2. There are 45 students in a class. If the probability of selecting a female is $\frac{1}{3}$, calculate the number of:
   a. Male students.
   b. Female students who should be enrolled in the class such that the probability of picking a female student will be $\frac{1}{2}$.
3. a. In the given diagram, O is the centre of the circle. AB and AC are tangent lines, and $\angle AOB = 65°$. Calculate the measures of angles $\angle ABC$ and $\angle OAB$.



   b. Find the value of x in the diagram below, in degrees.



[SOLUTIONS]

1. a. Change the subject:

$$
\begin{aligned}
q &= \sqrt{t^2p - \frac{rp}{t}} \\
q^2 &= t^2p - \frac{rp}{t} && \text{Square both sides} \\
q^2 &= \frac{t^3p - rp}{t} && \text{Subtract the right-hand side} \\
q^2t &= t^3p - rp && \text{Multiply throughout by } t \\
q^2t &= p(t^3 - r) && \text{Factor } p \text{ from the right-hand side} \\
\frac{q^2t}{t^3 - r} &= p && \text{Divide throughout by } (t^3 - r)
\end{aligned}
$$

   b. Write the first equation with indices of base 2 on both sides of the equation:

$$2^{x+1} = 8^{3y} \rightarrow 2^{x+1} = 2^{3(3y)} \rightarrow 2^{x+1} = 2^{9y}$$

Set the exponents equal: $x + 1 = 9y$

Now we have simultaneous linear equations: $x + 1 = 9y$ and $2x + y = 36$.

Solve using substitution:

$x + 1$	$=$	$9y$	(1)	
x	$=$	$9y - 1$		Change the subject of equation (1)
$2(9y - 1) + y$	$=$	36	(1)	Substitute equation (1) into equation (2)
$18y - 2 + y$	$=$	36		Simplify the left-hand side
$19y - 2$	$=$	36		
$19y$	$=$	38		Add 2 throughout
$\frac{19y}{19}$	$=$	$\frac{38}{19}$		Divide throughout by 19
y	$=$	2		

Substitute y into either equation to find x: $x = 9y - 1 = 9(2) - 1 = 18 - 1 = 17$

$x = 17$, $y = 2$

Therefore, $x - y = 17 - 2 = 15$.

2. a. Note that the total number of students is 45, and the probability of selecting a male is $1 - \frac{1}{3} = \frac{2}{3}$. Multiply this by the number of students: $\frac{2}{3}(45) = 30$

   Answer: There are 30 male students.

   b. The probability of selecting a female student is $\frac{1}{3}$, so the number of female students currently is $\frac{1}{3} \times 45 = 15$. A certain number of females should be added to the total to create a probability of $\frac{1}{2}$. Let's call that number f. Then we have: $\frac{1}{2} = \frac{15+f}{45+f}$. This is based on the new probability, and adding f to both the total number, and the number of females.

$\frac{1}{2}$	$=$	$\frac{15+f}{45+f}$	
$45 + f$	$=$	$2(15 + f)$	Cross multiply
$45 + f$	$=$	$30 + 2f$	
$45 - 30$	$=$	$2f - f$	
15	$=$	f	

   Answer: 15 more female students should be enrolled.

3. a. Note that $\angle OBA = 90°$ because a tangent line is perpendicular to the radius. Therefore, we can subtract $\angle OBC$ from $90°$ to find $\angle ABC$. Find $\angle OBC$ using the triangle formed by the chord, which forms a perpendicular angle with AO.

$$\angle OBC = 180° - 90° - 65° = 25°$$

Therefore, $\angle ABC = 90° - \angle OBC = 90° - 25° = 65°$.

Note that $\angle OAB$ can be found using $\triangle\ OAB$. Subtract the known angles from $180°$: $\angle OAB = 180° - 90° - 65° = 25°$

Answers: $\angle ABC = 65°$, $\angle OAB = 25°$

b. Note that the interior angles of a hexagon sum to $720°$. Sum the interior angles and solve x:

$$\begin{aligned}
720° &= 5x + 5x + 6x + 6x + 7x + 7x \\
720° &= 36x \\
\frac{720°}{36} &= x \\
20° &= x
\end{aligned}$$

Lesson Title: Mock Examination: Paper 2A – Compulsory Questions	**Theme:** WASSCE Exam Preparation	
Lesson Number: M4-L093	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Complete a section of a mock WASSCE paper. 2. Answer essay questions on various topics.	**Preparation** Prepare your classroom to administer the mock exam.	

Opening *(1 minute)*

1. Explain that the WASSCE exam has 3 sections. Today, pupils will practise section 2A, which consists of compulsory essay questions.

Teaching and Learning *(2 minutes)*

1. Ask pupils to turn and discuss with seatmates for 2 minutes: What are some things to keep in mind during the exam? What is the most important advice you would give your classmates?

Practice *(35 minutes)*

1. Ask pupils to turn to PHM4-T2-W24-L093 in the Pupil Handbook. They are given 3 essay questions that are also attached to this lesson plan.
2. Pupils will have 34 minutes to complete the mock exam. This is approximately 11 minutes per question.
3. Remind pupils that this is a mock examination. They should work independently, and **they should not look at the answer key during the exam**.
4. Move around the classroom and observe pupils. Provide guidance to pupils who need help.

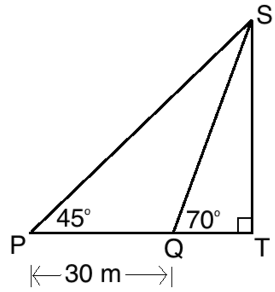
Closing *(2 minutes)*

1. Answer any questions that pupils have about questions or topics on the exam.
2. Ask pupils to check their answers in the Answer Key at the back of the Pupil Handbook on their own time at home. Full solutions are given. Encourage pupils to review and study these solutions.

[MOCK EXAM 5 – COMPULSORY ESSAY QUESTIONS]

1. a. Simplify: $(3x + y)^2 - (y - 2x)^2$
   b. Given that $\sin x = \frac{4}{5}$ and $0° \leq x \leq 90°$, find $\frac{2\cos x - 3\sin x}{\tan x}$.

2. Foday walked 3 kilometres from his house to school on a bearing of 45°. After school, he walked 4 kilometres to the market on a bearing of 135°. How far is he from his house?
3. In the diagram at right, two points P and Q are on the same horizontal as the base of a vertical pole, ST. P and Q are 30 metres from each other. Find, to 3 significant figures:
    a. The height of the pole.
    b. |PS|

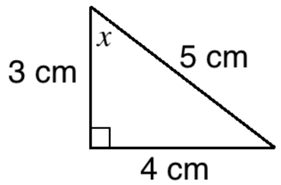


[SOLUTIONS]

1. a. Expand each part of the expression, then simplify:

$$\begin{aligned}(3x+y)^2-(y-2x)^2 &= 3x(3x+y)+y(3x+y)-y(y-2x)+2x(y-2x)\\ &= 9x^2+3xy+3xy+y^2-y^2+2xy+2xy-4x^2\\ &= 5x^2+10xy\\ &= 5x(x+2y)\end{aligned}$$

b. Draw a right-angled triangle using (see below). Use Pythagoras' theorem to find the third side, which is 3.

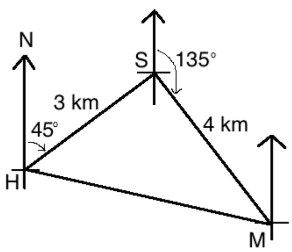


Use the triangle to find the values of $\cos x$ and $\tan x$, which are needed for the formula in the problem. $\cos x = \frac{3}{5}$; $\tan x = \frac{4}{3}$.

Substitute each trigonometric ratio into the formula and simplify:

$$\begin{aligned}\frac{2\cos x - 3\sin x}{\tan x} &= \frac{2\left(\frac{3}{5}\right)-3\left(\frac{4}{5}\right)}{\frac{4}{3}}\\ &= \frac{\frac{6}{5}-\frac{12}{5}}{\frac{4}{3}}\\ &= \frac{-\frac{6}{5}}{\frac{4}{3}}\\ &= \frac{-6}{5}\times\frac{3}{4}\\ &= \frac{-18}{20} = -\frac{9}{10}\end{aligned}$$

2. First, draw a diagram. In the diagram below, his house, school and market are represented by H, S, and M, respectively.



Notice that $\angle HSM = 90°$. The angle formed by SM and the north-south line is 45°. The angle formed by HS and the north-south line is also 45°, because it is an alternate interior angle of the 45° bearing of S from H. This gives:

$$\angle HSM = 45° + 45° = 90°$$

ΔHSM is a right-angled triangle. Apply Pythagoras' theorem to find |HM|:

$$\begin{aligned} |HM|^2 &= |HS|^2 + |SM|^2 \\ &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \\ |HM| &= \sqrt{25} = 5 \text{ km} \end{aligned}$$

Foday is 5 kilometres from his house.

3. a. Apply the tangent ratio to angles 45° and 70°. This will give simultaneous linear equations with 2 unknowns, |QT| and |ST|.

$$\begin{aligned} \tan 45° &= \frac{|ST|}{30+|QT|} \\ 1 &= \frac{|ST|}{30+|QT|} \\ 30 + |QT| &= |ST| ---(1) \end{aligned} \qquad \begin{aligned} \tan 70° &= \frac{|ST|}{|QT|} \\ 2.75 &= \frac{|ST|}{|QT|} \\ 2.75|QT| &= |ST| ---(2) \end{aligned}$$

Solve the system of equations using substitution:

$$\begin{aligned} 30 + |QT| &= 2.75|QT| && \text{Substituting equation (1) into equation (2)} \\ 30 &= 2.75|QT| - |QT| \\ 30 &= 1.75|QT| \\ \frac{30}{1.75} &= |QT| \\ |QT| &= 17.1 \end{aligned}$$

Substitute $|QT|$ into either linear equation to find $|ST|$:

$$|ST| = 30 + |QT| = 30 + 17.1 = 47.1 \text{ m}$$

b. Apply Pythagoras' theorem to find $|PS|$:

$$\begin{aligned} |PT|^2 + |ST|^2 &= |PS|^2 \\ 47.1^2 + 47.1^2 &= |PS|^2 \\ 4{,}436.82 &= |PS|^2 \\ \sqrt{4{,}436.82} &= |PS| \\ 66.6 \text{ m} &= |PS| \end{aligned}$$

Lesson Title: Mock Examination: Paper 2B – Advanced Questions	**Theme:** WASSCE Exam Preparation	
Lesson Number: M4-L094	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Complete a section of a mock WASSCE paper. 2. Select and solve advanced essay questions on various topics.	**Preparation** Prepare your classroom to administer the mock exam. Note that one problem on this section requires a geometry set. Ask pupils to bring geometry sets if they have them.	

Opening *(1 minute)*

1. Explain that the WASSCE exam has 3 sections. Today, pupils will practise section 2B, which consists of advanced essay questions.

Teaching and Learning *(2 minutes)*

1. Explain Paper 2B – Advanced Questions
   - Paper 2B is worth 60 marks.
   - There are 8 essay questions in paper 2A, and candidates are expected to answer 5 of them.
   - Questions on section 2B have a greater length and difficulty that section 2A.
   - A maximum of 2 questions (from among the 8) may be drawn from parts of the WASSCE syllabus that are not meant for Sierra Leone. These topics are not in the Sierra Leone national curriculum for secondary schools. Candidates from Sierra Leone may choose to answer such questions, but it is not required.
   - Choose 5 questions on topics that you are more comfortable with.

Practice *(35 minutes)*

1. Ask pupils to turn to PHM4-T2-W24-L094 in the Pupil Handbook. They are given 4 essay questions that are also attached to this lesson plan. They should choose any 2 questions to complete.
2. Pupils will have 34 minutes to complete the mock exam. This is approximately 17 minutes per question.
3. Remind pupils that this is a mock examination. They should work independently, and **they should not look at the answer key during the exam**.

Move around the classroom and observe pupils. Provide guidance to pupils who need help.

Closing *(2 minutes)*

1. Answer any questions that pupils have about questions or topics on the mock exam.

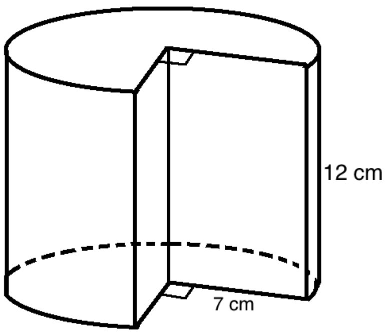
2. Ask pupils to check their answers in the Answer Key at the back of the Pupil Handbook on their own time at home. Full solutions are given. Encourage pupils to review and study these.
3. Encourage pupils to solve the additional 2 problems from the mock exam at home for practice.

[MOCK EXAM 6 – ADVANCED ESSAY QUESTIONS]

1. Y is 80 km away from X on a bearing of 120°. Z is 100 km away from X on a bearing of 225°. Find, correct to 3 significant figures: a. The distance of Z from Y; b. the bearing of Z from Y.
2. a. Copy and complete the table of values for $y = 5\cos x + 2\sin x$ to one decimal place.

x	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$	$210°$
y	5.0		4.2		-0.8			-5.3

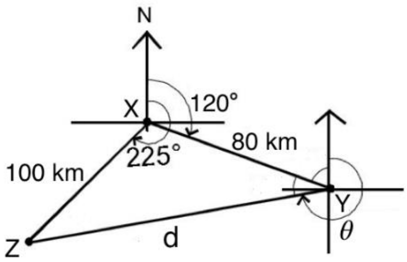
b. Using a scale of 2 cm to $30°$ on the x-axis and 2 cm to 1 unit on the y-axis, draw the graph of $5\cos x + 2\sin x = 0$ for $0° \leq x \leq 210°$.
c. Use your graph to solve the equation $5\cos x + 2\sin x = 0$, correct to the nearest degree.
d. Find the maximum value of y, correct to 1 decimal place.
3. The solid given below is a cylinder with a segment of 90° removed. Calculate the: a. Volume of the solid; b. Surface area of the solid. [Use $\pi = \frac{22}{7}$]



4. Using a ruler and a pair of compasses only, construct: a. Triangle ABC in which $|AB| = 6$ cm, $|BC| = 5$ cm and $\angle ABC = 60°$. Measure AC.
b. In a. above, locate by construction a point D such that CD is parallel to AB and D is equidistant from points A and C. Measure $\angle BAD$.

[SOLUTIONS]

22. Draw a diagram:



a. In the triangle, 2 sides and the angle between them are known. The cosine rule can be used. Note that the angle inside the triangle at X is $X = 225° - 120° = 105°$.

$\lvert YZ\rvert^2$	$=$	$\lvert XZ\rvert^2 + \lvert XY\rvert^2 - 2\lvert XZ\rvert\lvert XY\rvert \cos X$	Formula
d^2	$=$	$100^2 + 80^2 - 2(100)(80)\ \cos(105°)$	Substitute values from the triangle
	$=$	$10{,}000 + 6{,}400 - \ 16{,}000 \cos 105°$	
	$=$	$16{,}400 - 16{,}000\ (-0.2588)$	Substitute $\cos 105° = -0.2588$
	$=$	$16{,}400 + 4140.8$	
d^2	$=$	$20{,}540.8$	
d	$=$	$\sqrt{20{,}540.8} = 143$ km to 3 s.f.	Take the square root of both sides

b. To find the bearing of Z from Y, identify the other angles at Y and subtract them from 360°. The other angle outside of the triangle at Y is 60° because it is the alternate interior angle with the 60° angle formed by the bearing and north-south line at point X.

The angle inside the triangle (call it y) can be found using the sine rule:

$\frac{143}{\sin 105°}$	$=$	$\frac{100}{\sin y}$	Substitute in the formula
$\sin y$	$=$	$\frac{100 \sin 105°}{143}$	Solve for y
$\sin y$	$=$	$\frac{100(0.9659)}{143}$	Use the sine table
$\sin y$	$=$	0.6755	
y	$=$	$\sin^{-1}(0.6755)$	
y	$=$	$42.5°$	

Subtract the known angles from 360 to find the bearing:

$$\theta = 360° - 60° - 42.5° = 257.5° = 258° \text{ to 3 s.f.}$$

Bearing: $\overrightarrow{YZ} = (143 \text{ km}, 258°)$

23. a. Completed table (see calculations below):

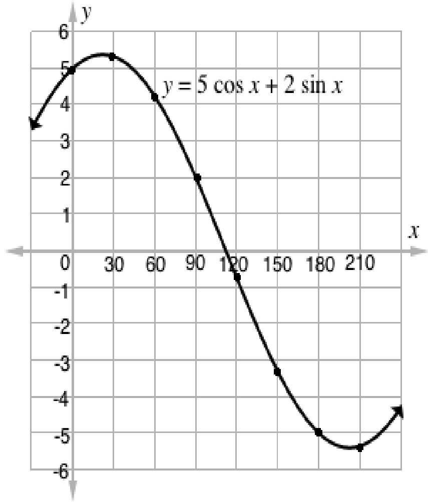
x	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$	$210°$
y	5	5.3	4.2	2	-0.8	-3.3	-5	-5.3

Calculations:

$$5 \cos 30° + 2 \sin 30° = 5(0.866) + 2(0.5)$$
$$= 5.3$$

$$
\begin{aligned}
5\cos 90^\circ + 2\sin 90^\circ &= 5(0) + 2(1) \\
&= 2 \\
5\cos 150^\circ + 2\sin 150^\circ &= 5(-0.866) + 2(0.5) \\
&= -3.3 \\
5\cos 180^\circ + 2\sin 180^\circ &= 5(-1) + 2(0) \\
&= -5
\end{aligned}
$$

b. Graph (not to scale; ensure that tick marks are 2 cm apart on your graph):

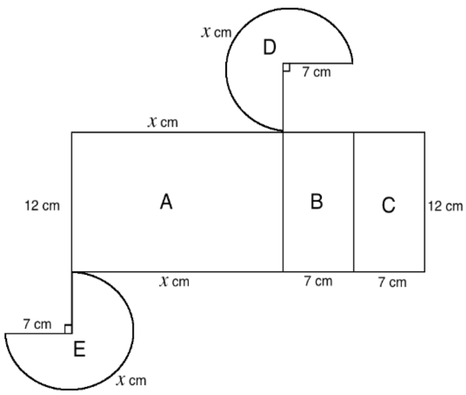


c. The solution to $5\cos x + 2\sin x = 0$ consists of the points where the curve intersects the x-axis. On the interval $0^\circ \le x \le 210^\circ$, this only occurs at one point. The solution is approximately $x = 110^\circ$.
d. The maximum value of y is approximately 5.3.

24. a. Since 90° was removed, the angle remaining in the solid is $360^\circ - 90^\circ = 270^\circ$. Use 270° as a fraction of 360° (one full rotation) to calculate the volume.

$$
\begin{aligned}
V &= \frac{270}{360}\pi r^2 h \qquad \frac{270}{360} \times \text{volume of a cylinder} \\
&= \frac{3}{4}\left(\frac{22}{7}\right)(7^2)(12) \\
&= \frac{3}{4}(22)(7)(12) \\
&= 1{,}386 \ \text{cm}^3
\end{aligned}
$$

b. To find the surface area, find the area of each of the 5 faces and add them. To facilitate this, draw a net:



Find the measure of unknown length x. Note that it is $\frac{270}{360}$ the circumference of the circle.

$$\begin{aligned} x &= \frac{270}{360}2\pi r \qquad \frac{270}{360} \times \text{circumference of a circle} \\ &= \frac{3}{4}(2)\left(\frac{22}{7}\right)(7) \\ &= \frac{3}{4}(2)(22) \\ &= 33 \text{ cm} \end{aligned}$$

Calculate the area of each shape:

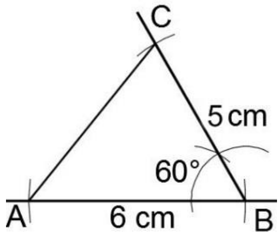
$$A = 33 \times 12 = 396 \text{ cm}^2$$

$$B = C = 7 \times 12 = 84 \text{ cm}^2$$

$$D = E = \frac{270}{360}\pi r^2 = \frac{3}{4}\left(\frac{22}{7}\right)7^2 = \frac{3}{4}(22)7 = 115.5 \text{ cm}^2$$

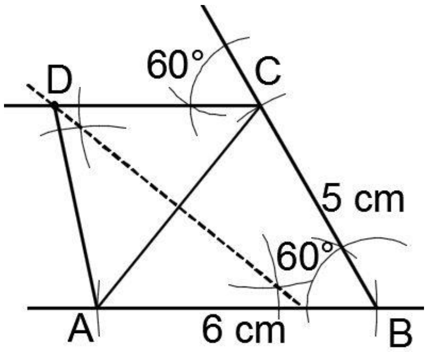
Surface area $= A + B + C + D + E = 396 + 2(84) + 2(115.5) = 795$ cm^2

25. a. Draw the triangle construction, as shown (note that the constructions are not to scale):



Measure |AC| with a ruler. $|AC| = 5.5$ cm.

b. On the same triangle construction, draw the locus of points equidistant to A and C. Also draw a line from C parallel to |AB|. This can be done in a number of ways. In the diagram below, this is done using a 60° angle at point C. D is the point where the locus and parallel line intersect.



Measure $\angle BAD$ with a protractor. $\angle BAD = 82°$.

Lesson Title: Mock Examination: Paper 2B – Advanced Questions	**Theme:** WASSCE Exam Preparation	
Lesson Number: M4-L095	**Class:** SSS 4	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Complete a section of a mock WASSCE paper. 2. Select and solve advanced essay questions on various topics.	**Preparation** Prepare your classroom to administer the mock exam.	

Opening *(1 minute)*

1. Explain that the WASSCE exam has 3 sections. Today, pupils will practise section 2B, which consists of advanced essay questions.

Teaching and Learning *(2 minutes)*

1. Discuss:
   - How did you feel about the previous mock exam for paper 2B? What were your successes and challenges?
   - What exam-taking skills did you use during the previous exam?
2. Explain:
   - Remember that section 3B has 8 questions, and you must choose 5 of them to solve.
   - It is a good idea to skim read the questions quickly before getting started. If you see a topic that you are familiar and comfortable with, work that problem.
   - Try not to spend a lot of time deciding, or thinking about problems you will not solve. This will take valuable time away from the exam.

Practice *(35 minutes)*

1. Ask pupils to turn to PHM4-T2-W24-L095 in the Pupil Handbook. They are given 4 essay questions that are also attached to this lesson plan. They should choose any 2 questions to complete.
2. Pupils will have 34 minutes to complete the mock exam. This is approximately 17 minutes per question.
3. Remind pupils that this is a mock examination. They should work independently, and **they should not look at the answer key during the exam**.
4. Move around the classroom and observe pupils. Provide guidance to pupils who need help.

Closing *(2 minutes)*

1. Answer any questions that pupils have about questions or topics on the mock exam.
2. Ask pupils to check their answers in the Answer Key at the back of the Pupil Handbook on their own time at home. Full solutions are given. Encourage pupils to review and study these solutions.
3. Encourage pupils to solve the additional 2 problems from the mock exam at home for practice.

[MOCK EXAM 7 – ADVANCED ESSAY QUESTIONS]

1. a. Use logarithm tables to evaluate $\frac{20.3\times\sqrt{1.568}}{2.34\times1.803}$.
   b. Mr. Bangura has 7 books on his shelf, 3 Mathematics books and 4 science books. Of these, he selects 2 at random, one after the other, with replacement. Find the probability that:
      i. Both were Mathematics books.
      ii. One was a Mathematics book and one was a science book.
2. The frequency distribution table shows the marks achieved by 100 pupils in a Mathematics test.

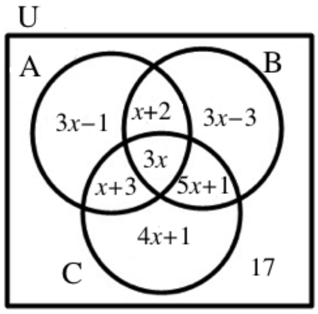
Marks (%)	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	2	4	3	6	8	12	20	24	14	7

   a. Draw a cumulative frequency curve for the distribution.
   b. Use the graph to find the:
      i. 60^{th} percentile.
      ii. Probability that a pupil passed the test if the pass mark was fixed at 55%.
3. The table is for the relation $y = px^2 + x + q$.

x	-4	-3	-2	-1	0	1	2	3
y		9			-6		4	15

   a. i. Use the table to find the values of p and q.
      ii. Copy and complete the table.
   b. Using scales of 2 cm to 1 unit on the x-axis, and 2 cm to 3 units on the y-axis, draw the graph of the relation for $-4 \leq x \leq 3$.
   c. Use the graph to find:
      i. y when $x = 2.5$
      ii. x when $y = -2$

4. a. In the Venn diagram, A, B and C are subsets of the universal set U. If $n(U) = 120$, find:
   i. The value of x
   ii. $n(A \cup B \cup C')$



   b. Given that $4\sin(x + 3.5) - 1 = 0$ and $0° \leq x \leq 90°$, calculate, correct to the nearest degree, the value of x.

[SOLUTIONS]

1. a. Use a table to organise your calculations. Convert each decimal number to a logarithm, and apply the appropriate operations, using BODMAS. Recall that for multiplication of numbers, logarithms are added; for division, they are subtracted. For a square root, the logarithm is divided by 2.

Number	**Logarithm**
20.3	1.3075
$\sqrt{1.568}$	$0.1953 \div 2 = 0.0977$
Product (Numerator)	$1.3075 + 0.0977 = 1.4052$
2.34	0.3692
1.803	0.2560
Product (Denominator)	$0.3692 + 0.2560 = 0.6252$
Division	$1.4052 - 0.6252 = 0.78$

Antilog $0.78 = 6.026$

Answer: $\frac{20.3\times\sqrt{1.568}}{2.34\times1.803} = 6.026$

b. Note that the probability of selecting a Maths book is $\frac{3}{7}$, and the probability of selecting a science book is $\frac{4}{7}$.

i. Multiply to find the probability that both were Maths books:

P(Both Maths books)$= \frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$

ii. Multiply to find the probability that one is a Maths book and one is a science book:

P(One Maths, one science))$= \frac{3}{7} \times \frac{4}{7} = \frac{12}{49}$

2. a. Organise a cumulative frequency table with upper class boundaries:

Marks	Upper boundary	Frequency	Cumulative Frequency
1-10	10.5	2	2
11-20	20.5	4	2+4=6

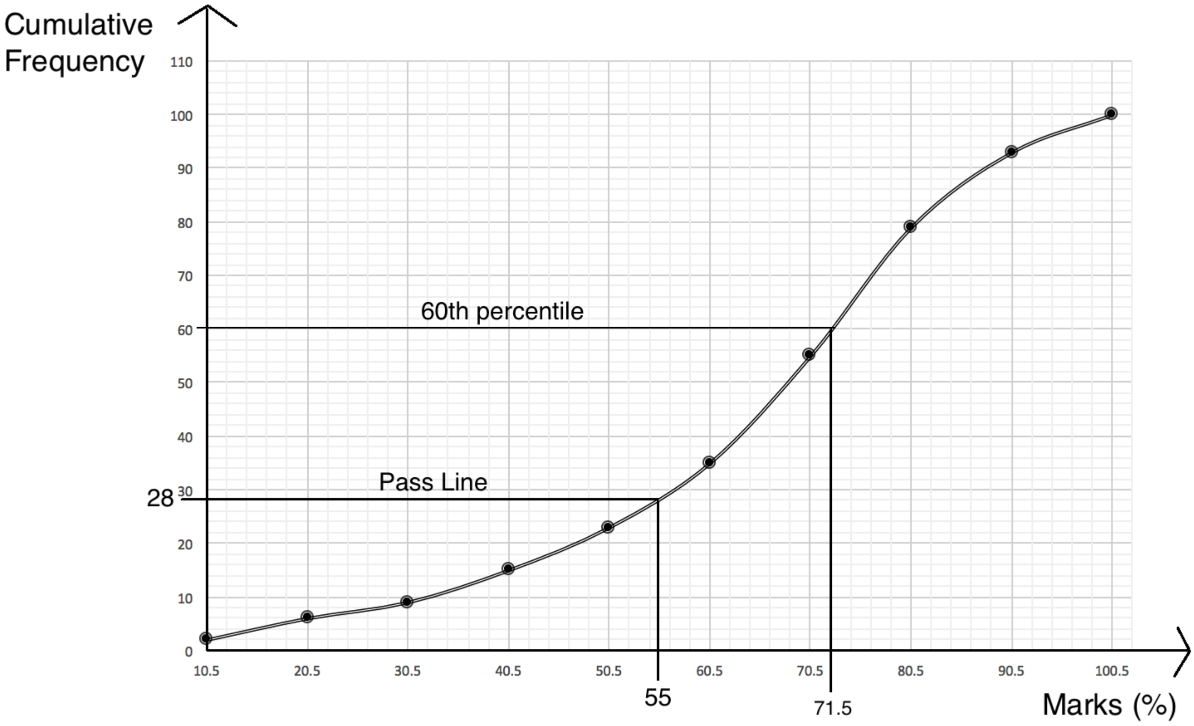
21-30	30.5	3	6+3=9
31-40	40.5	6	9+6=15
41-50	50.5	8	15+8=23
51-60	60.5	12	23+12=35
61-70	70.5	20	35+20=55
71-80	80.5	24	55+24=79
81-90	90.5	14	79+14=93
91-100	100.5	7	93+7=100

Use the table to plot a cumulative frequency curve, with marks on the x-axis and cumulative frequency on the y-axis (see below).

b. i. Find the position of the 60th percentile: $\frac{n}{100}\sum f = \frac{60}{100}(100) = 60$

Identify the 60th percentile using the c.f. curve. Identify 60 on the y-axis – the corresponding x-value gives the percentile. The 60th percentile is 71.5 marks (see curve below).

ii. Find the number of pupils scoring at least 55% using the curve. The corresponding cumulative frequency is 28. Therefore, the number of pupils who passed is $100 - 28 = 72$. Calculate the probability that a random pupil passed: PR(Pupil passed) $= \frac{72}{100} = \frac{18}{25}$.



3. a. i. Choose 2 sets of x- and y-values from the table. Substitute these into the quadratic equation $y = px^2 + x + q$. This will give simultateous equations that can be solved for p and q.

For example, substitute $(0, -6)$ and $(2, 4)$:

$$\begin{aligned} y &= px^2 + x + q \\ -6 &= p0^2 + 0 + q \\ -6 &= q \end{aligned} \qquad \begin{aligned} y &= px^2 + x + q \\ 4 &= p2^2 + 2 + q \\ 4 &= 4p + 2 + q \end{aligned}$$

$$\begin{aligned} 4-2 &= 4p+q \\ 2 &= 4p+q \end{aligned}$$

Substitute $q = -6$ into the second equation, and solve for p:

$$\begin{aligned} 2 &= 4p+q \\ 2 &= 4p-6 \\ 2+6 &= 4p \\ 8 &= 4p \\ 2 &= p \end{aligned}$$

We now have $p = 2$ and $q = -6$ the equation $y = 2x^2 + x - 6$.

ii. Complete the table using the function $y = 2x^2 + x - 6$:

x	-4	-3	-2	-1	0	1	2	3
y	22	9	0	-5	-6	-3	4	15

Working:

$$\begin{aligned} y &= 2x^2+x-6 \\ &= 2(-4)^2+(-4)-6 \\ &= 2(16)-4-6 \\ &= 22 \end{aligned}$$

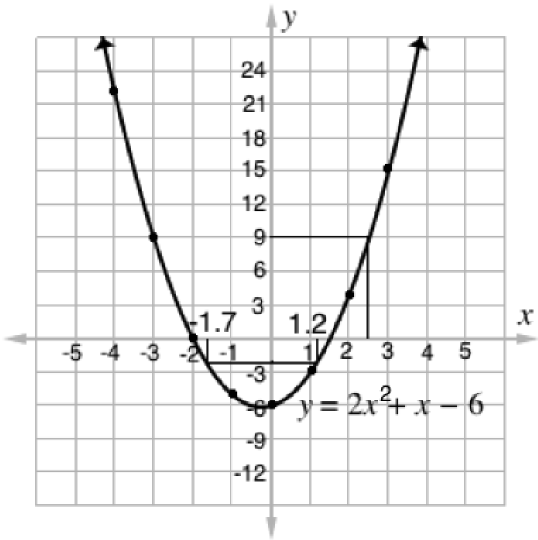
$$\begin{aligned} y &= 2x^2+x-6 \\ &= 2(-2)^2+(-2)-6 \\ &= 2(4)-2-6 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y &= 2x^2+x-6 \\ &= 2(-1)^2+(-1)-6 \\ &= 2-1-6 \\ &= -5 \end{aligned}$$

$$\begin{aligned} y &= 2x^2+x-6 \\ &= 2(1)^2+(1)-6 \\ &= 2(1)+1-6 \\ &= -3 \end{aligned}$$

a. See the graph below. Note that it is not to scale. Ensure that the tick marks on your x- and y-axes are 2 cm apart.

b. i. Identify $x = 2.5$ on the graph, which corresponds to $y = 9$. (see below).
ii. Identify $y = -2$ on the graph, which has 2 corresponding x-values, approximately -1.7 and 1.2 (see below).



Note that graphs are generally used to make approximations. On the WASSCE exam, examiners accept estimated scores within a certain range. Acceptable answers depend on the scale used. For example, for a 2 cm to 1 unit scale, the disparity allowed is 0.2. Therefore, if the exact values for part c. ii. are -1.7 and

1.2, then examiners should accept scores in the range of -1.9 to -1.5, and 1.0 to 1.4.

4. a. i. Add the expressions from all segments of the Venn diagram. Set them equal to 120, and solve for x.

$$\begin{aligned} n(U) = 120 = &\quad 3x - 1 + x + 2 + 3x - 3 + 3x + 5x + 1 + x + 3 + 4x + 1 + 17 \\ 120 = &\quad 20x + 20 \\ 120 - 20 = &\quad 20x \\ 100 = &\quad 20x \\ 5 = &\quad x \end{aligned}$$

ii. Note that $A \cup B \cup C'$ is the union of A and B, except for those in C. Find the sum of the sections that are in A or B, not including those that are also in C:

$$\begin{aligned} n(A \cup B \cup C') = &\quad 3x - 1 + x + 2 + 3x - 3 \\ = &\quad 3(5) - 1 + (5) + 2 + 3(5) - 3 \\ = &\quad 33 \end{aligned}$$

b. Make x the subject of the equation. Since there is a sine function in the equation, this will require inverse sine to eliminate it.

$4\sin(x + 3.5) - 1$	$=$	0	
$4\sin(x + 3.5)$	$=$	1	Transpose 1
$\sin(x + 3.5)$	$=$	$\frac{1}{4}$	Divide throughout by 4
$\sin(x + 3.5)$	$=$	0.25	Convert to decimal
$x + 3.5$	$=$	$\sin^{-1} 0.25$	Take the inverse sine of both sides
$x + 3.5$	$=$	14.5	Substitute $\sin^{-1} 0.25 = 14.48°$
x	$=$	$14.5 - 3.5$	Transpose 3.5
x	$=$	11	

<table>
<tr><td>Lesson Title: Mock Examination: Paper 2B – Advanced Questions</td><td colspan="2">Theme: WASSCE Exam Preparation</td></tr>
<tr><td>Lesson Number: M4-L096</td><td>Class: SSS 4</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Complete a section of a mock WASSCE paper.
2. Select and solve advanced essay questions on various topics.</td><td colspan="2">Preparation
Prepare your classroom to administer the mock exam.
Note that one problem on this section requires a geometry set. Ask pupils to bring geometry sets if they have them.</td></tr>
</table>

Opening *(1 minute)*

1. Explain that the WASSCE exam has 3 sections. Today, pupils will practise section 2B, which consists of advanced essay questions.

Teaching and Learning *(2 minutes)*

1. Ask pupils to turn and discuss with seatmates for 2 minutes: What are your strategies for completing section 2B of the exam? What advice do you have for your classmates?

Practice *(35 minutes)*

1. Ask pupils to turn to PHM4-T2-W24-L096 in the Pupil Handbook. They are given 4 essay questions that are also attached to this lesson plan. They should choose any 2 questions to complete.
2. Pupils will have 34 minutes to complete the mock exam. This is approximately 17 minutes per question.
3. Remind pupils that this is a mock examination. They should work independently, and **they should not look at the answer key during the exam**.
4. Move around the classroom and observe pupils. Provide guidance to pupils who need help.

Closing *(2 minutes)*

1. Answer any questions that pupils have about questions or topics on the mock exam.
2. Ask pupils to check their answers in the Answer Key at the back of the Pupil Handbook on their own time at home. Full solutions are given. Encourage pupils to review and study these solutions.
3. Encourage pupils to solve the additional 2 problems from the mock exam at home for practice.

[MOCK EXAM 8 – ADVANCED ESSAY QUESTIONS]

1. a. Copy and complete the following table for multiplication in modulo 11.

$\otimes$	1	2	4	6	8
1	1	2	4	6	8
2	2				
4	4				
6	6				
8	8				

Use the table to:

i. Evaluate $(8 \otimes 6) \otimes (4 \otimes 6)$.

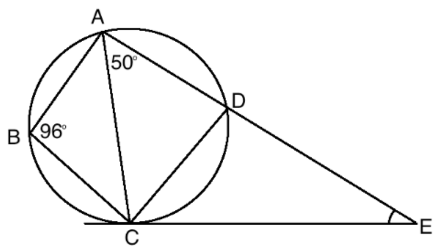
ii. Find the truth set of $8 \otimes m = 4$.

b. When a fraction is simplified to its lowest term, it is equal to $\frac{2}{3}$. The numerator of the fraction when doubled is 12 greater than the denominator. Find the fraction.

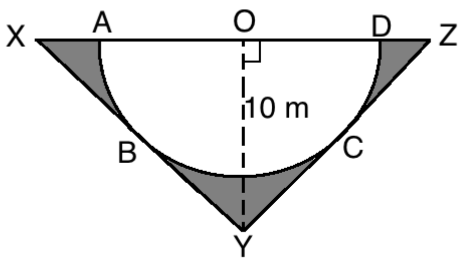
2. The table shows the distribution of outcomes when a die is thrown 50 times. Calculate the: a. Mean deviation; b. Probability that a score selected at random is at least 3.

Scores	1	2	3	4	5	6
Frequency (f)	8	7	10	11	5	9

3. a. In the diagram, CE is tangent to circle $ABCD$, $\angle ABC = 96°$ and $\angle CAD = 50°$. Find the measure of $\angle CED$.



b. In the diagram, semicircle $ABCD$ with centre O is inscribed in an isosceles triangle XYZ, where $|OY| = 10$ m and $\angle XYZ = 94°$. Find, correct to 3 significant figures: a. The area of semicircle $ABCD$. b. The area of the shaded portion. (Take $\pi = \frac{22}{7}$)



4. A school received $5,000.00 from a group of alumni to make improvements. A committee decided to spend 20% on new furniture, 30% on new books, 15% on teacher training, and 35% on scholarships for pupils.
   a. Represent this information on a pie chart.
   b. Calculate, correct to the nearest whole number, the percentage increase of the amount for scholarships over that for teacher training.

[SOLUTIONS]

1. Recall the rules for multiplying in modulo. Multiply the 2 numbers of concern, then divide them by the given modulo. The remainder is the answer. For example, consider $4 \otimes 4$. Multiply: $4 \times 4 = 16$. Divide the result by the module, 11: $16 \div 11 = 1r5$. The remainder (5) is written in the table where column 4 and row 4 meet.

   Completed table:

$\otimes$	1	2	4	6	8
1	1	2	4	6	8
2	2	4	8	1	5
4	4	8	5	2	10
6	6	1	2	3	4
8	8	5	10	4	9

i. To evaluate $(8 \otimes 6) \otimes (4 \otimes 6)$, apply the normal order of operations. Remove the brackets first.

$$(8 \otimes 6) \otimes (4 \otimes 6) = 4 \otimes 2 \quad \text{Remove the brackets}$$
$$= 8$$

ii. The truth set of $8 \otimes m = 4$ is the set of all m values that make this statement true. There is only one such value in the table, so $m = \{6\}$.

b. Use the information to create simultaneous equations. Let the numerator of the fraction be x, and the denominator be y. That is, $\frac{x}{y} = \frac{2}{3}$. Consider this equation 1.

From the problem, we have $2x = y + 12$. Solve this equation for y, we have $y = 2x - 12$. This is equation 2. Substitute equation 2 into equation 1, and solve for x:

$$\frac{x}{y} = \frac{2}{3} \quad \text{Equation 1}$$
$$\frac{x}{2x-12} = \frac{2}{3} \quad \text{Substitute Equation 2}$$
$$3x = 2(2x - 12) \quad \text{Cross multiply}$$
$$3x = 4x - 24 \quad \text{Solve for } x$$
$$x = 24$$

Substitute $x = 24$ into equation 2:

$$y = 2x - 12 \quad \text{Equation 2}$$
$$y = 2(24) - 12 \quad \text{Substitute } x = 24$$
$$y = 48 - 12 \quad \text{Solve for } y$$
$$y = 36$$

We have $x = 24$ and $y = 36$, which gives the fraction $\frac{24}{36}$

2. Complete the following table to calculate mean deviation. After filling the first 3 columns, calculate mean (shown below) and use it to fill the other columns.

x	f	fx	$x - \bar{x}$	$\lvert x - \bar{x}\rvert$	$f\lvert x - \bar{x}\rvert$
1	8	8	$1 - 3.5 = -2.5$	2.5	20
2	7	14	$2 - 3.5 = -1.5$	1.5	10.5
3	10	30	$3 - 3.5 = -0.5$	0.5	5
4	11	44	$4 - 3.5 = 0.5$	0.5	5.5
5	5	25	$5 - 3.5 = 1.5$	1.5	7.5
6	9	54	$6 - 3.5 = 2.5$	2.5	22.5
Totals:	$\sum f = 50$	$\sum fx = 175$			$\sum f\lvert x - \bar{x}\rvert = 71$

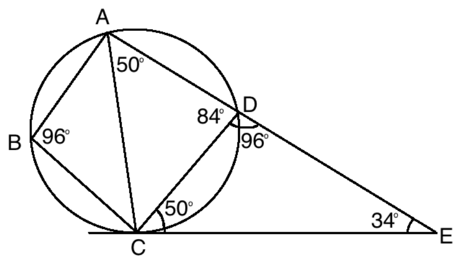
Mean: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{175}{50} = 3.5$ scores

Mean deviation: $\mathrm{MD} = \frac{\sum f|x-\bar{x}|}{\sum f} = \frac{71}{50} = 1.42$

b. Note that this is not the probability of rolling a 3 or higher. It is the probability that, among the rolls in the table, a 3 or higher is selected.

Pr(at least 3) $= \frac{10+11+5+9}{50} = \frac{35}{50} = 0.7$

3. Note that $\angle ABC = \angle CDE = 96°$, because $\angle CDE$ is the opposite exterior angle of a cyclic quadrilateral. Also, $\angle CAD = \angle DCE = 50°$, because these are angles in alternate segments. We now have 2 of the 3 angles of triangle CDE. Subtract from $180°$ to find $\angle CED$: $\angle CED = 180° - 96° - 50 = 34°$.



b.

i. Note that, because the triangle is isosceles, $\angle OYZ = \frac{1}{2}\angle XYZ = \frac{1}{2}94° = 47°$. Also note that the radius of the circle OC is perpendicular to the tangent line YZ, according to circle theorems. Therefore, use right-angled triangle OCY to find the radius of the circle, OC:

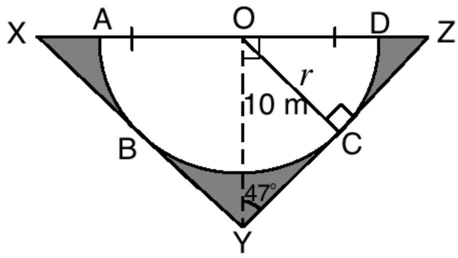
$$\begin{aligned} \sin 47° &= \frac{|OC|}{10} \\ 0.7314 &= \frac{|OC|}{10} \qquad \text{Substitute } \sin 47° = 0.7314 \text{ from sine table} \\ 0.7314 \times 10 &= |OC| \\ 7.314 &= |OC| \end{aligned}$$

Use $r = 7.314$ to find the area of the semicircle:

$$A = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2}\left(\frac{22}{7}\right)7.314^2$$
$$= 84.1 \text{ m}^2$$

ii. To find the area of the shaded portion, subtract the area of the semicircle from the area of the triangle. The height of the triangle is $|OY| = h = 10$ m. Find the base using right-angled triangle YOZ.



$$\tan 47° = \frac{|OZ|}{10}$$
$$1.072 = \frac{|OZ|}{10}$$ Substitute $\tan 47° = 1.072$ from sine table
$$1.072 \times 10 = |OZ|$$
$$10.72 = |OZ|$$

Base of the triangle $= |OX|+|OZ| = 2|OZ| = 2(10.72) = 21.44$

Area of the triangle:

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(21.44)10$$
$$= 107.2 \text{ m}^2$$

Area of the shaded portion: $107.2 - 84.1 = 23.1$ m^2

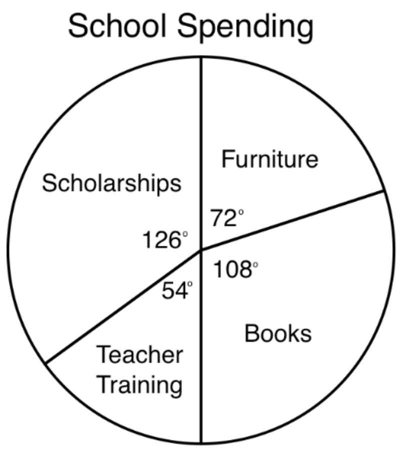
4. Write each percentage as a fraction, and use them to find the degree of each sector in the pie chart. Remember that there are 360° in a full rotation:

Furniture: $\frac{20}{100} \times 360° = 72°$

Books: $\frac{30}{100} \times 360° = 108°$

Teacher training: $\frac{15}{100} \times 360° = 54°$

Scholarships: $\frac{35}{100} \times 360° = 126°$



Check your calculations by adding them: $72° + 108° + 54° + 126° = 360°$

Use a protractor and these degree measures to draw a pie chart:

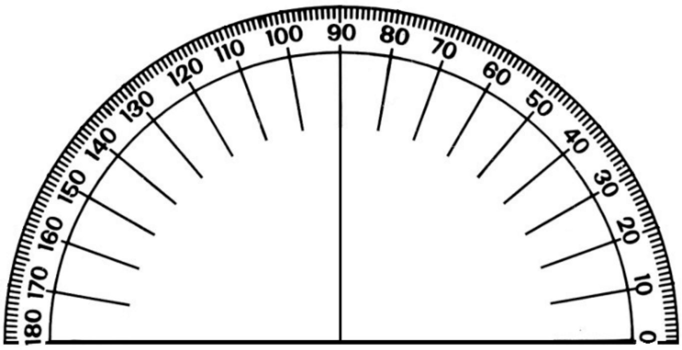
b. Amount spent on scholarships: $\frac{35}{100} \times 5{,}000 = 1{,}750$

Amount spent on teacher training: $\frac{15}{100} \times 5{,}000 = 750$

Percentage increase $= \frac{1{,}750 - 750}{750} \times 100 = \frac{1{,}000}{750} \times 100 = 133\%$

Appendix I: Protractor

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.



Appendix II: Sines of Angles

Sines of Angles (x in degrees)

$x \to \sin x$

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	2	3	5	7	9	10	12	14	16
1	0·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	2	3	5	7	9	10	12	14	16
2	0·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	2	3	5	7	9	10	12	14	16
3	0·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	2	3	5	7	9	10	12	14	16
4	0·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	2	3	5	7	9	10	12	14	16
5	0·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	2	3	5	7	9	10	12	14	16
6	0·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	2	3	5	7	9	10	12	14	16
7	0·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	2	3	5	7	9	10	12	14	16
8	0·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	2	3	5	7	9	10	12	14	16
9	0·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	2	3	5	7	9	10	12	14	15
10	0·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	2	3	5	7	9	10	12	14	15
11	0·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2	3	5	7	9	10	12	14	15
12	0·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2	3	5	7	9	10	12	14	15
13	0·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2	3	5	7	8	10	12	14	15
14	0·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2	3	5	7	8	10	12	14	15
15	0·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2	3	5	7	8	10	12	13	15
16	0·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2	3	5	7	8	10	12	13	15
17	0·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	2	3	5	7	8	10	12	13	15
18	0·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	2	3	5	7	8	10	12	13	15
19	0·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	2	3	5	7	8	10	12	13	15
20	0·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	2	3	5	7	8	10	11	13	15
21	0·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	2	3	5	6	8	10	11	13	15
22	0·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	2	3	5	6	8	10	11	13	15
23	0·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	2	3	5	6	8	10	11	13	14
24	0·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	2	3	5	6	8	10	11	13	14
25	0·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	2	3	5	6	8	9	11	13	14
26	0·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	2	3	5	5	8	9	11	12	14
27	0·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	2	3	5	6	8	9	11	12	14
28	0·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	2	3	5	6	8	9	11	12	14
29	0·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	2	3	5	6	8	9	11	12	14
30	0·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	2	3	5	6	8	9	11	12	14
31	0·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	1	3	4	6	7	9	10	12	13
32	0·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	1	3	4	6	7	9	10	12	13
33	0·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	1	3	4	6	7	9	10	12	13
34	0·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	1	3	4	6	7	9	10	12	13
35	0·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	1	3	4	6	7	9	10	11	13
36	0·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	1	3	4	6	7	8	10	11	13
37	0·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	1	3	4	6	7	8	10	11	12
38	0·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	1	3	4	5	7	8	10	11	12
39	0·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	1	3	4	5	7	8	9	11	12
40	0·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	1	3	4	5	7	8	9	11	12
41	0·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	1	3	4	5	7	8	9	10	12
42	0·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	1	3	4	5	6	8	9	10	12
43	0·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	1	3	4	5	6	8	9	10	11
44	0·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
45	0·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	1	2	4	5	6	7	9	10	11
46	0·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	1	2	4	5	6	7	8	10	11
47	0·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	1	2	4	5	6	7	8	9	11
48	0·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	1	2	3	5	6	7	8	9	10
49	0·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	1	2	3	5	6	7	8	9	10
50	0·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	1	2	3	4	6	7	8	9	10
51	0·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	1	2	3	4	5	7	8	9	10
52	0·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	1	2	3	4	5	6	7	8	10
53	0·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	1	2	3	4	5	6	7	8	9
54	0·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	1	2	3	4	5	6	7	8	9
55	0·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	1	2	3	4	5	6	7	8	9
56	0·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	1	2	3	4	5	6	7	8	9
57	0·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	1	2	3	4	5	6	7	8	8
58	0·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	1	2	3	4	5	5	6	7	8
59	0·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	2	3	4	4	5	6	7	8
60	0·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	2	3	3	4	5	6	7	8
61	0·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	2	2	3	4	5	6	7	7
62	0·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	2	2	3	4	5	6	6	7
63	0·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	2	2	3	4	5	5	6	7
64	0·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	2	2	3	4	5	5	6	7
65	0·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	1	2	3	4	4	5	6	7
66	0·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	1	2	3	3	4	5	6	6
67	0·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	1	2	3	3	4	5	5	6
68	0·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	1	2	3	3	4	4	5	6
69	0·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	1	2	2	3	4	4	5	6
70	0·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	1	2	2	3	3	4	5	5
71	0·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	1	2	2	3	3	4	4	5
72	0·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	1	2	2	3	3	4	4	5
73	0·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	0	1	1	2	2	3	3	4	4
74	0·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0	1	1	2	2	3	3	4	4
75	0·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	0	1	1	2	2	3	3	3	4
76	0·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	0	1	1	2	2	2	3	3	4
77	0·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	0	1	1	2	2	2	3	3	3
78	0·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	0	1	1	1	2	2	2	3	3
79	0·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0	1	1	1	2	2	2	3	3
80	0·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	1	1	2	2	2	3
81	0·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	1	1	2	2	2	2
82	0·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	0	1	1	1	1	2	2	2
83	0·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	0	1	1	1	1	1	2	2
84	0·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	0	1	1	1	1	1	1	2
85	0·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	0	1	1	1	1	1	1
86	0·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	0	0	1	1	1	1	1
87	0·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	0	0	0	1	1	1
88	0·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998									
89	0·9998	9999	9999	9999	9999	1·0000	1·0000	1·0000	1·0000	1·0000									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix III: Cosines of Angles

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	SUBTRACT Differences 1	2	3	4	5	6	7	8	9
0	1·0000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9999									
1	0·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995									
2	0·9994	·9993	·9993	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	0	0	0	1	1	1
3	0·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	0	0	1	1	1	1	1
4	0·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	0	1	1	1	1	1	1
5	0·9962	·9960	·9959	·9957	·9956	·9954	·9952	·9951	·9949	·9947	0	0	1	1	1	1	1	1	2
6	0·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	0	1	1	1	1	1	2	2
7	0·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	0	1	1	1	1	2	2	2
8	0·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	1	1	2	2	2	2
9	0·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	1	1	2	2	2	3
10	0·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	0	1	1	1	2	2	2	3	3
11	0·9816	·9813	·9810	·9806	·9803	·9799	·9796	·9792	·9789	·9785	0	1	1	1	2	2	2	3	3
12	0·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	0	1	1	2	2	2	3	3	3
13	0·9744	·9740	·9736	·9732	·9728	·9724	·9720	·9715	·9711	·9707	0	1	1	2	2	2	3	3	4
14	0·9703	·9699	·9694	·9690	·9686	·9681	·9677	·9673	·9668	·9664	0	1	1	2	2	3	3	3	4
15	0·9659	·9655	·9650	·9646	·9641	·9636	·9632	·9627	·9622	·9617	0	1	1	2	2	3	3	4	4
16	0·9613	·9608	·9603	·9598	·9593	·9588	·9583	·9578	·9573	·9568	0	1	1	2	2	3	3	4	4
17	0·9563	·9558	·9553	·9548	·9542	·9537	·9532	·9527	·9521	·9516	1	1	2	2	3	3	4	4	5
18	0·9511	·9505	·9500	·9494	·9489	·9483	·9478	·9472	·9466	·9461	1	1	2	2	3	3	4	4	5
19	0·9455	·9449	·9444	·9438	·9432	·9426	·9421	·9415	·9409	·9403	1	1	2	2	3	3	4	5	5
20	0·9397	·9391	·9385	·9379	·9373	·9367	·9361	·9354	·9348	·9342	1	1	2	2	3	4	4	5	6
21	0·9336	·9330	·9323	·9317	·9311	·9304	·9298	·9291	·9285	·9278	1	1	2	3	3	4	4	5	6
22	0·9272	·9265	·9259	·9252	·9245	·9239	·9232	·9225	·9219	·9212	1	1	2	3	3	4	5	5	6
23	0·9205	·9198	·9191	·9184	·9178	·9171	·9164	·9157	·9150	·9143	1	1	2	3	3	4	5	6	6
24	0·9135	·9128	·9121	·9114	·9107	·9100	·9092	·9085	·9078	·9070	1	1	2	3	4	4	5	6	7
25	0·9063	·9056	·9048	·9041	·9033	·9026	·9018	·9011	·9003	·8996	1	2	2	3	4	5	5	6	7
26	0·8988	·8980	·8973	·8965	·8957	·8949	·8942	·8934	·8926	·8918	1	2	2	3	4	5	5	6	7
27	0·8910	·8902	·8894	·8886	·8878	·8870	·8862	·8854	·8846	·8838	1	2	2	3	4	5	6	6	7
28	0·8829	·8821	·8813	·8805	·8796	·8788	·8780	·8771	·8763	·8755	1	2	2	3	4	5	6	7	7
29	0·8746	·8738	·8729	·8721	·8712	·8704	·8695	·8686	·8678	·8669	1	2	3	3	4	5	6	7	8
30	0·8660	·8652	·8643	·8634	·8625	·8616	·8607	·8599	·8590	·8581	1	2	3	4	4	5	6	7	8
31	0·8572	·8563	·8554	·8545	·8536	·8526	·8517	·8508	·8499	·8490	1	2	3	4	5	5	6	7	8
32	0·8480	·8471	·8462	·8453	·8443	·8434	·8425	·8415	·8406	·8396	1	2	3	4	5	6	7	8	8
33	0·8387	·8377	·8368	·8358	·8348	·8339	·8329	·8320	·8310	·8300	1	2	3	4	5	6	7	8	9
34	0·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	1	2	3	4	5	6	7	8	9
35	0·8192	·8181	·8171	·8161	·8151	·8141	·8131	·8121	·8111	·8100	1	2	3	4	5	6	7	8	9
36	0·8090	·8080	·8070	·8059	·8049	·8039	·8028	·8018	·8007	·7997	1	2	3	4	5	6	7	8	9
37	0·7986	·7976	·7965	·7955	·7944	·7934	·7923	·7912	·7902	·7891	1	2	3	4	5	6	7	8	10
38	0·7880	·7869	·7859	·7848	·7837	·7826	·7815	·7804	·7793	·7782	1	2	3	4	5	6	7	9	10
39	0·7771	·7760	·7749	·7738	·7727	·7716	·7705	·7694	·7683	·7672	1	2	3	4	6	7	8	9	10
40	0·7660	·7649	·7638	·7627	·7615	·7604	·7593	·7581	·7570	·7559	1	2	3	5	6	7	8	9	10
41	0·7547	·7536	·7524	·7513	·7501	·7490	·7478	·7466	·7455	·7443	1	2	3	5	6	7	8	9	10
42	0·7431	·7420	·7408	·7396	·7385	·7373	·7361	·7349	·7337	·7325	1	2	4	5	6	7	8	9	11
43	0·7314	·7302	·7290	·7278	·7266	·7254	·7242	·7230	·7218	·7206	1	2	4	5	6	7	8	10	11
44	0·7193	·7181	·7169	·7157	·7145	·7133	·7120	·7108	·7096	·7083	1	2	4	5	6	7	9	10	11
45	0·7071	·7059	·7046	·7034	·7022	·7009	·6997	·6984	·6972	·6959	1	2	4	5	6	7	9	10	11
46	0·6947	·6934	·6921	·6909	·6896	·6884	·6871	·6858	·6845	·6833	1	3	4	5	6	8	9	10	11
47	0·6820	·6807	·6794	·6782	·6769	·6756	·6743	·6730	·6717	·6704	1	3	4	5	6	8	9	10	12
48	0·6691	·6678	·6665	·6652	·6639	·6626	·6613	·6600	·6587	·6574	1	3	4	5	7	8	9	10	12
49	0·6561	·6547	·6534	·6521	·6508	·6494	·6481	·6468	·6455	·6441	1	3	4	5	7	8	9	11	12
50	0·6428	·6414	·6401	·6388	·6374	·6361	·6347	·6334	·6320	·6307	1	3	4	5	7	8	9	11	12
51	0·6293	·6280	·6266	·6252	·6239	·6225	·6211	·6198	·6184	·6170	1	3	4	5	7	8	10	11	12
52	0·6157	·6143	·6129	·6115	·6101	·6088	·6074	·6060	·6046	·6032	1	3	4	5	7	8	10	11	12
53	0·6018	·6004	·5990	·5976	·5962	·5948	·5934	·5920	·5906	·5892	1	3	4	6	7	8	10	11	13
54	0·5878	·5864	·5850	·5835	·5821	·5807	·5793	·5779	·5764	·5750	1	3	4	6	7	9	10	11	13
55	0·5736	·5721	·5707	·5693	·5678	·5664	·5650	·5635	·5621	·5606	1	3	4	6	7	9	10	12	13
56	0·5592	·5577	·5563	·5548	·5534	·5519	·5505	·5490	·5476	·5461	1	3	4	6	7	9	10	12	13
57	0·5446	·5432	·5417	·5402	·5388	·5373	·5358	·5344	·5329	·5314	1	3	4	6	7	9	10	12	13
58	0·5299	·5284	·5270	·5255	·5240	·5225	·5210	·5195	·5180	·5165	1	3	4	6	7	9	10	12	13
59	0·5150	·5135	·5120	·5105	·5090	·5075	·5060	·5045	·5030	·5015	2	3	5	6	8	9	11	12	14
60	0·5000	·4985	·4970	·4955	·4939	·4924	·4909	·4894	·4879	·4863	2	3	5	6	8	9	11	12	14
61	0·4848	·4833	·4818	·4802	·4787	·4772	·4756	·4741	·4726	·4710	2	3	5	6	8	9	11	12	14
62	0·4695	·4679	·4664	·4648	·4633	·4617	·4602	·4586	·4571	·4555	2	3	5	6	8	9	11	12	14
63	0·4540	·4524	·4509	·4493	·4478	·4462	·4446	·4431	·4415	·4399	2	3	5	6	8	9	11	12	14
64	0·4384	·4368	·4352	·4337	·4321	·4305	·4289	·4274	·4258	·4242	2	3	5	6	8	9	11	13	14
65	0·4226	·4210	·4195	·4179	·4163	·4147	·4131	·4115	·4099	·4083	2	3	5	6	8	10	11	13	14
66	0·4067	·4051	·4035	·4019	·4003	·3987	·3971	·3955	·3939	·3923	2	3	5	6	8	10	11	13	14
67	0·3907	·3891	·3875	·3859	·3843	·3827	·3811	·3795	·3778	·3762	2	3	5	6	8	10	11	13	15
68	0·3746	·3730	·3714	·3697	·3681	·3665	·3649	·3633	·3616	·3600	2	3	5	6	8	10	11	13	15
69	0·3584	·3567	·3551	·3535	·3518	·3502	·3486	·3469	·3453	·3437	2	3	5	6	8	10	11	13	15
70	0·3420	·3404	·3387	·3371	·3355	·3338	·3322	·3305	·3289	·3272	2	3	5	7	8	10	12	13	15
71	0·3256	·3239	·3223	·3206	·3190	·3173	·3156	·3140	·3123	·3107	2	3	5	7	8	10	12	13	15
72	0·3090	·3074	·3057	·3040	·3024	·3007	·2990	·2974	·2957	·2940	2	3	5	7	8	10	12	13	15
73	0·2924	·2907	·2890	·2874	·2857	·2840	·2823	·2807	·2790	·2773	2	3	5	7	8	10	12	13	15
74	0·2756	·2740	·2723	·2706	·2689	·2672	·2656	·2639	·2622	·2605	2	3	5	7	8	10	12	13	15
75	0·2588	·2571	·2554	·2538	·2521	·2504	·2487	·2470	·2453	·2436	2	3	5	7	8	10	12	14	15
76	0·2419	·2402	·2385	·2368	·2351	·2334	·2317	·2300	·2284	·2267	2	3	5	7	8	10	12	14	15
77	0·2250	·2233	·2215	·2198	·2181	·2164	·2147	·2130	·2113	·2096	2	3	5	7	9	10	12	13	15
78	0·2079	·2062	·2045	·2028	·2011	·1994	·1977	·1959	·1942	·1925	2	3	5	7	9	10	12	14	15
79	0·1908	·1891	·1874	·1857	·1840	·1822	·1805	·1788	·1771	·1754	2	3	5	7	9	10	12	14	15
80	0·1736	·1719	·1702	·1685	·1668	·1650	·1633	·1616	·1599	·1582	2	3	5	7	9	10	12	14	15
81	0·1564	·1547	·1530	·1513	·1495	·1478	·1461	·1444	·1426	·1409	2	3	5	7	9	10	12	14	16
82	0·1392	·1374	·1357	·1340	·1323	·1305	·1288	·1271	·1253	·1236	2	3	5	7	9	10	12	14	16
83	0·1219	·1201	·1184	·1167	·1149	·1132	·1115	·1097	·1080	·1063	2	3	5	7	9	10	12	14	16
84	0·1045	·1028	·1011	·0993	·0976	·0958	·0941	·0924	·0906	·0889	2	3	5	7	9	10	12	14	16
85	0·0872	·0854	·0837	·0819	·0802	·0785	·0767	·0750	·0732	·0715	2	3	5	7	9	10	12	14	16
86	0·0698	·0680	·0663	·0645	·0628	·0610	·0593	·0576	·0558	·0541	2	3	5	7	9	10	12	14	16
87	0·0523	·0506	·0488	·0471	·0454	·0436	·0419	·0401	·0384	·0366	2	3	5	7	9	10	12	14	16
88	0·0349	·0332	·0314	·0297	·0279	·0262	·0244	·0227	·0209	·0192	2	3	5	7	9	10	12	14	16
89	0·0175	·0157	·0140	·0122	·0105	·0087	·0070	·0052	·0035	·0017	2	3	5	7	9	10	12	14	16
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix IV: Tangents of Angles

Tangents of Angles (*x* in degrees)

$x \to \tan x$

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	0·0000	·0017	·0035	·0052	·0070	·0087	·0105	·0122	·0140	·0157	2	3	5	7	9	10	12	14	16
1	0·0175	·0192	·0209	·0227	·0244	·0262	·0279	·0297	·0314	·0332	2	3	5	7	9	10	12	14	16
2	0·0349	·0367	·0384	·0402	·0419	·0437	·0454	·0472	·0489	·0507	2	3	5	7	9	10	12	14	16
3	0·0524	·0542	·0559	·0577	·0594	·0612	·0629	·0647	·0664	·0682	2	4	5	7	9	11	12	14	16
4	0·0699	·0717	·0734	·0752	·0769	·0787	·0805	·0822	·0840	·0857	2	4	5	7	9	11	12	14	16
5	0·0875	·0892	·0910	·0928	·0945	·0963	·0981	·0998	·1016	·1033	2	4	5	7	9	11	12	14	16
6	0·1051	·1069	·1086	·1104	·1122	·1139	·1157	·1175	·1192	·1210	2	4	5	7	9	11	12	14	16
7	0·1228	·1246	·1263	·1281	·1299	·1317	·1334	·1352	·1370	·1380	2	4	5	7	9	11	12	14	16
8	0·1405	·1423	·1441	·1459	·1477	·1495	·1512	·1530	·1548	·1566	2	4	5	7	9	11	12	14	16
9	0·1584	·1602	·1620	·1638	·1655	·1673	·1691	·1709	·1727	·1745	2	4	5	7	9	11	13	14	16
10	0·1763	·1781	·1799	·1817	·1835	·1853	·1871	·1890	·1908	·1926	2	4	5	7	9	11	13	14	16
11	0·1944	·1962	·1980	·1998	·2016	·2035	·2053	·2071	·2089	·2107	2	4	5	7	9	11	13	15	16
12	0·2126	·2144	·2162	·2180	·2199	·2217	·2235	·2254	·2272	·2290	2	4	5	7	9	11	13	15	16
13	0·2309	·2327	·2345	·2364	·2382	·2401	·2419	·2438	·2456	·2475	2	4	6	7	9	11	13	15	17
14	0·2493	·2512	·2530	·2549	·2568	·2586	·2605	·2623	·2642	·2661	2	4	6	7	9	11	13	15	17
15	0·2679	·2698	·2717	·2736	·2754	·2773	·2792	·2811	·2830	·2849	2	4	6	8	9	11	13	15	17
16	0·2867	·2886	·2905	·2924	·2943	·2962	·2981	·3000	·3019	·3038	2	4	6	8	9	11	13	15	17
17	0·3057	·3076	·3096	·3115	·3134	·3153	·3172	·3191	·3211	·3230	2	4	6	8	10	12	13	15	17
18	0·3249	·3269	·3288	·3307	·3327	·3346	·3365	·3385	·3404	·3424	2	4	6	8	10	12	14	16	17
19	0·3443	·3463	·3482	·3502	·3522	·3541	·3561	·3581	·3600	·3620	2	4	6	8	10	12	14	16	18
20	0·3640	·3659	·3679	·3699	·3719	·3739	·3759	·3779	·3799	·3819	2	4	6	8	10	12	14	16	18
21	0·3839	·3859	·3879	·3899	·3919	·3939	·3959	·3979	·4000	·4020	2	4	6	8	10	12	14	16	18
22	0·4040	·4061	·4081	·4101	·4122	·4142	·4163	·4183	·4204	·4224	2	4	6	8	10	12	14	16	18
23	0·4245	·4265	·4286	·4307	·4327	·4348	·4369	·4390	·4411	·4431	2	4	6	8	10	12	15	17	19
24	0·4452	·4473	·4494	·4515	·4536	·4557	·4578	·4599	·4621	·4642	2	4	6	8	11	13	15	17	19
25	0·4663	·4684	·4706	·4727	·4748	·4770	·4791	·4813	·4834	·4856	2	4	6	9	11	13	15	17	19
26	0·4877	·4899	·4921	·4942	·4964	·4986	·5008	·5029	·5051	·5073	2	4	7	9	11	13	15	17	20
27	0·5095	·5117	·5139	·5161	·5184	·5206	·5228	·5250	·5272	·5295	2	4	7	9	11	13	16	18	20
28	0·5317	·5340	·5362	·5384	·5407	·5430	·5452	·5475	·5498	·5520	2	5	7	9	11	14	16	18	20
29	0·5543	·5566	·5589	·5612	·5635	·5658	·5681	·5704	·5727	·5750	2	5	7	9	12	14	16	18	21
30	0·5774	·5797	·5820	·5844	·5867	·5890	·5914	·5938	·5961	·5985	2	5	7	9	12	14	16	19	21
31	0·6009	·6032	·6056	·6080	·6104	·6128	·6152	·6176	·6200	·6224	2	5	7	10	12	14	17	19	22
32	0·6249	·6273	·6297	·6322	·6346	·6371	·6395	·6420	·6445	·6469	2	5	7	10	12	15	17	20	22
33	0·6494	·6519	·6544	·6569	·6594	·6619	·6644	·6669	·6694	·6720	3	5	8	10	13	15	18	20	23
34	0·6745	·6771	·6796	·6822	·6847	·6873	·6899	·6924	·6950	·6976	3	5	8	10	13	15	18	21	23
35	0·7002	·7028	·7054	·7080	·7107	·7133	·7159	·7186	·7212	·7239	3	5	8	11	13	16	18	21	24
36	0·7265	·7292	·7319	·7346	·7373	·7400	·7427	·7454	·7481	·7508	3	5	8	11	14	16	19	22	24
37	0·7536	·7563	·7590	·7618	·7646	·7673	·7701	·7729	·7757	·7785	3	6	8	11	14	17	19	22	25
38	0·7813	·7841	·7869	·7898	·7926	·7954	·7983	·8012	·8040	·8069	3	6	9	11	14	17	20	23	26
39	0·8098	·8127	·8156	·8185	·8214	·8243	·8273	·8302	·8332	·8361	3	6	9	12	15	18	21	23	26
40	0·8391	·8421	·8451	·8481	·8511	·8541	·8571	·8601	·8632	·8662	3	6	9	12	15	18	21	24	27
41	0·8693	·8724	·8754	·8785	·8816	·8847	·8878	·8910	·8941	·8972	3	6	9	12	16	19	22	25	28
42	0·9004	·9036	·9067	·9099	·9131	·9163	·9195	·9228	·9260	·9293	3	6	10	13	16	19	22	26	29
43	0·9325	·9358	·9391	·9424	·9457	·9490	·9523	·9556	·9590	·9623	3	7	10	13	17	20	23	27	30
44	0·9657	·9691	·9725	·9759	·9793	·9827	·9861	·9896	·9930	·9965	3	7	10	14	17	21	24	27	31
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
45	1·000	1·003	1·007	1·011	1·014	1·018	1·021	1·025	1·028	1·032	0	1	1	1	2	2	2	3	3
46	1·036	1·039	1·043	1·046	1·050	1·054	1·057	1·061	1·065	1·069	0	1	1	1	2	2	3	3	3
47	1·072	1·076	1·080	1·084	1·087	1·091	1·095	1·099	1·103	1·107	0	1	1	2	2	2	3	3	3
48	1·111	1·115	1·118	1·122	1·126	1·130	1·134	1·138	1·142	1·146	0	1	1	2	2	2	3	3	4
49	1·150	1·154	1·159	1·163	1·167	1·171	1·175	1·179	1·183	1·188	0	1	1	2	2	2	3	3	4
50	1·192	1·196	1·200	1·205	1·209	1·213	1·217	1·222	1·226	1·230	0	1	1	2	2	3	3	4	4
51	1·235	1·239	1·244	1·248	1·253	1·257	1·262	1·266	1·271	1·275	0	1	1	2	2	3	3	4	4
52	1·280	1·285	1·289	1·294	1·299	1·303	1·308	1·313	1·317	1·322	0	1	1	2	2	3	3	4	4
53	1·327	1·332	1·337	1·342	1·347	1·351	1·356	1·361	1·366	1·371	0	1	1	2	2	3	3	4	4
54	1·376	1·381	1·387	1·392	1·397	1·402	1·407	1·412	1·418	1·423	1	1	2	2	3	3	4	4	5
55	1·428	1·433	1·439	1·444	1·450	1·455	1·460	1·466	1·471	1·477	1	1	2	2	3	3	4	4	5
56	1·483	1·488	1·494	1·499	1·505	1·511	1·517	1·522	1·528	1·534	1	1	2	2	3	3	4	5	5
57	1·540	1·546	1·552	1·558	1·564	1·570	1·576	1·582	1·588	1·594	1	1	2	2	3	4	4	5	5
58	1·600	1·607	1·613	1·619	1·625	1·632	1·638	1·645	1·651	1·658	1	1	2	3	3	4	4	5	6
59	1·664	1·671	1·678	1·684	1·691	1·698	1·704	1·711	1·718	1·725	1	1	2	3	3	4	5	5	6
60	1·732	1·739	1·746	1·753	1·760	1·767	1·775	1·782	1·789	1·797	1	1	2	3	4	4	5	6	6
61	1·804	1·811	1·819	1·827	1·834	1·842	1·849	1·857	1·865	1·873	1	2	2	3	4	5	5	6	7
62	1·881	1·889	1·897	1·905	1·913	1·921	1·929	1·937	1·946	1·954	1	2	2	3	4	5	6	7	7
63	1·963	1·971	1·980	1·988	1·997	2·006	2·014	2·023	2·032	2·041	1	2	3	4	4	5	6	7	8
64	2·050	2·059	2·069	2·078	2·087	2·097	2·106	2·116	2·125	2·135	1	2	3	4	5	6	7	8	8
65	2·145	2·154	2·164	2·174	2·184	2·194	2·204	2·215	2·225	2·236	1	2	3	4	5	6	7	8	9
66	2·246	2·257	2·267	2·278	2·289	2·300	2·311	2·322	2·333	2·344	1	2	3	4	5	7	8	9	10
67	2·356	2·367	2·379	2·391	2·402	2·414	2·426	2·438	2·450	2·463	1	2	4	5	6	7	8	10	11
68	2·475	2·488	2·500	2·513	2·526	2·539	2·552	2·565	2·578	2·592	1	3	4	5	7	8	9	10	12
69	2·605	2·619	2·633	2·646	2·660	2·675	2·689	2·703	2·718	2·733	1	3	4	6	7	9	10	11	13
70	2·747	2·762	2·778	2·793	2·808	2·824	2·840	2·856	2·872	2·888	2	3	5	6	8	9	11	13	14
71	2·904	2·921	2·937	2·954	2·971	2·989	3·006	3·024	3·042	3·060	2	3	5	7	9	10	12	14	16
72	3·078	3·096	3·115	3·133	3·152	3·172	3·191	3·211	3·230	3·251	2	4	6	8	10	12	14	15	17
73	3·271	3·291	3·312	3·333	3·354	3·376	3·398	3·420	3·442	3·465	2	4	6	9	11	13	15	17	19
74	3·487	3·511	3·534	3·558	3·582	3·606	3·630	3·655	3·681	3·706	2	5	7	10	12	15	17	20	22
75	3·732	3·758	3·785	3·812	3·839	3·867	3·895	3·923	3·952	3·981	3	6	8	11	14	17	20	22	25
76	4·011	4·041	4·071	4·102	4·134	4·165	4·198	4·230	4·264	4·297	3	6	10	13	16	19	22	26	29
77	4·331	4·366	4·402	4·437	4·474	4·511	4·548	4·586	4·625	4·666									
78	4·705	4·745	4·787	4·829	4·872	4·915	4·959	5·005	5·050	5·097									
79	5·145	5·193	5·242	5·292	5·343	5·396	5·449	5·503	5·558	5·614									
80	5·671	5·730	5·789	5·850	5·912	5·976	6·041	6·107	6·174	6·243	differences unreliable:								
81	6·314	6·386	6·460	6·535	6·612	6·691	6·772	6·855	6·940	7·026									
82	7·115	7·207	7·300	7·396	7·495	7·596	7·700	7·806	7·916	8·028									
83	8·144	8·264	8·386	8·513	8·643	8·777	8·915	9·058	9·205	9·357									
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20									
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95									
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46									
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27									
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08									
89	57·29	63·66	71·62	81·85	95·46	114·6	143·2	191·0	286·5	573·0									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

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