Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' Handbook for

Senior Secondary *Mathematics*

SSS III

Term III

STRICTLY NOT FOR SALE

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Please see final page for further information.

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

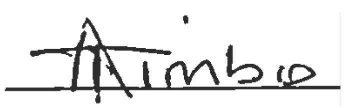
The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future



Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, DO NOT WRITE IN THE BOOKS.

Table of Contents

Introduction
to the Pupils' Handbook

These practice activities are aligned to the Lesson Plans, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.

Learning Outcomes

5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiner Reports, as well as input from WAEC examiners and Sierra Leonean teachers.

Lesson Title: Introduction to probability – Part 1	**Theme:** Statistics and Probability
Lesson Number: PHM3-L097	**Class:** SSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Define, use, and give examples of terms used in probability.
2. Use the language of probability to describe events in real life.

Overview

Probability is used to describe how likely or unlikely it is for something to happen. It is used in many different fields such as statistics, commerce, gambling, insurance, science and technology. Before we can study probability, we need to be able to define and use the terms used in probability. Because probability is based on uncertainty and chance, in everyday life we use words such as certain, impossible, likely, unlikely.

Other terms to explain and describe probability are used in order to solve problems. Some of the main ones are shown in the table below. New probability terms will be defined as we encounter them throughout the topic.

Term	Definition	Examples
Experiment	The act of conducting a test or investigation whose result cannot be predicted with certainty.	• tossing a fair coin • rolling an unbiased die • playing a football match • examples by pupils
Trial	A single performance of the experiment.	
Outcome	The result of an experiment	• getting a head when a fair coin is tossed • getting a 6 when an unbiased die is rolled • win a football match • examples by pupils
Event	A collection of outcomes from a specified sample space.	• getting a head and a tail when a fair coin is tossed twice • getting even numbers when a die is rolled • win, lose or draw a series of 4 matches • examples by pupils
Sample space	All possible outcomes of a trial of the experiment, denoted by S. Write using set notation.	• tossing a fair coin: $S = \{H, T\}$ • rolling an unbiased die: $S = \{1,2,3,4,5,6\}$ • football match: $S =$ {win, lose, draw} • examples by pupils
Number of outcomes	Total number of possible outcomes of a trial of the experiment denoted by $n(S)$	• tossing a fair coin: : $n(S)) = 2$ • rolling an unbiased die: $n(S) = 6$ • football match: $n(S) = 3\}$ • examples by pupils

Equally likely outcomes	Two or more events which have an equal chance of happening.	• getting a head or a tail when a fair coin is tossed. • getting any one of the six numbers when an unbiased die is rolled. • Note win/lose/draw not equally likely as outcome depends on factors such as form of team/opponent and so on. • examples by pupils
Random sampling	Choosing a sample from a population without being biased..	

Solved Examples

1. Use one of the following words to describe the given statements:

   certain   likely   unlikely   impossible

   a. It will rain tomorrow.
   b. The sun will rise in the east tomorrow.
   c. You are late for school tomorrow.
   d. You complete all your Maths practice correctly.
   e. A human being grows to 20 feet tall.
   f. Your favourite football team win its next match.

Solutions:

Statement	**Example answers**
a. It will rain tomorrow.	likely / unlikely depending on time of year
b. The sun will rise in the east tomorrow.	certain
c. You are late for school tomorrow.	likely / unlikely
d. You complete all your Maths practice correctly.	likely / unlikely
e. A human being grows to 20 feet tall.	impossible
f. Your favourite football team will win its next match.	likely / unlikely

- b. is a certainty – the sun always rises in the east.
- e. is impossible. No human being will grow to be 20 feet tall.
- The opposite of a certain event is an impossible event.
- The only impossible statement is iv. All the others have a degree of uncertainty depending on time of year, the person making the statement and similar factors.

2. Describe two events that are:

   a. certain   b. likely   c. unlikely   d. impossible

Solutions:

Given: describe two events for each given probability outcome

certain

It will rain in August.

Independence Day will fall on 27 April next year.

likely
You eat rice for lunch today.
You score 100% in your next Maths test.
unlikely
People will be able to live on Mars.
You will win first prize in the State lottery.
impossible
A human being will live to be 200 years old.
The sun will rise in the west tomorrow.

3. Give all the possible outcomes for the following experiment.
   Write the sample space using set notation.
   a. Guessing the gender of a new-born child.
   b. Guessing a prime number less than 20.
   c. Guessing a vowel.

Solutions:

Given: give all the possible outcomes of given experiments

a. Guessing the gender of a new-born child: $S = \{$girl, boy$\}$
b. Guessing the prime numbers less than 20: $S = \{2, 3, 5, 7, 11, 13, 17, 19\}$
c. Guessing a vowel $S = \{$a, e, i, o, u$\}$

Practice

1. Use one of the following words to describe the given statements:
   certain likely unlikely impossible
   a. You will use a mobile phone today.
   b. If this is 2017, the next leap year will be 2020.
   c. You roll an unbiased die and score 7.
   d. It will rain on Christmas day in Bo.
   e. Someone in your class has a birthday on February 29.

2. Think of 2 events that are:
   certain likely unlikely impossible

3. Write down the sample spaces of the following using set notation:
   a. Even numbers between 11 and 21.
   b. Throwing a fair die once.
   c. Tossing an unbiased coin.
   d. Odd numbers between 2 and 14.
   e. Multiples of 4 less than 41.
   f. Drawing a card from a pack of cards bearing numbers from 1 to 52.
   g. Choosing from integers 5 to 20 inclusive.

Lesson Title: Introduction to probability – Part 2	Theme: Statistics and Probability
Lesson Number: PHM3-L098	**Class:** SSS 3
Learning Outcome By the end of the lesson, pupils will be able to use probability notation to describe simple events.	

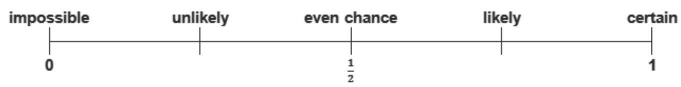
Overview

We use a probability scale to indicate the likelihood of something happening.

Probabilities are given values between 0 and 1.

A probability of 0 means that the event is impossible.

A probability of 1 means that it is certain.

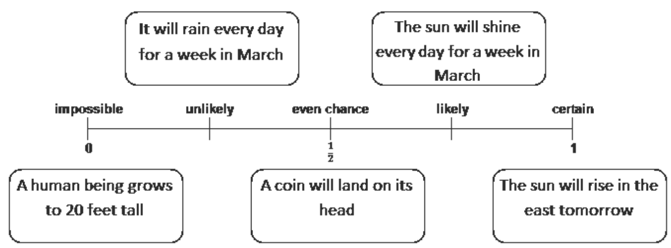


The closer the probability of an event is to 1, the more likely it is to happen. The closer the probability is to 0, the less likely it is to happen.

Probabilities cannot be greater than 1. (1.0 as decimal, 100% as percentage)

When we toss a coin, it is equally likely for it to fall on its head as on its tail. The event of getting a head has an even chance of happening and we mark that in the middle of the probability scale. Its probability is $\frac{1}{2}$, 0.5 or 50%.

The probabilities of some events have been marked on the probability scale shown at right.



Experiments have to be fairly conducted to ensure that the outcomes are indeed equally likely to happen. When doing experiments, we talk about tossing fair coins, rolling unbiased dice, picking cards or numbers at random, even choosing people at random.

Consider the experiment of tossing a fair coin.

There are two outcomes, each of which is equally likely to occur.

∴ probability of obtaining head $P(\text{head}) = \frac{1}{2}$

probability of obtaining tail: $P(\text{tail}) = \frac{1}{2}$

For equally likely outcomes, the probability that an event, E, will happen is:

$$P(E) = \frac{\text{number of ways of obtaining event } E}{\text{total number of possible outcomes}}$$

In set notation, this is given as: $= \frac{n(E)}{n(S)}$

where $n(E)$ is the number of elements in event, E

$n(S)$ is the number of elements in the sample space, S

Probabilities can be expressed **as a fraction, a decimal, or a percentage.**

fraction decimal percentage

$$\therefore P(\text{head}) = \frac{1}{2} \qquad 0.5 \qquad 50\%$$

From the outcomes for tossing a coin, we can clearly see that:

$$P(\text{head}) + P(\text{tail}) = 1 \qquad (1)$$

Since this gives all the possible outcomes for the experiment, we can conclude that the sum of all probabilities in an experiment is equal to 1.

From equation (1), obtaining a tail is the same as not obtaining a head.

$$\therefore P(\text{head}) + P(\text{not head}) = 1$$

$$P(\text{not head}) = 1 - P(\text{head}) \qquad (2)$$

This is called the **complement of the event.** If the probability of obtaining a head is denoted as $P(A)$, the complement is written as $P(\overline{A})$.

Solved Examples

1. An unbiased die is rolled. What is the probability of obtaining:

   a. A six   b. A five

   c. An even number   d. A multiple of 3

   Mark each probability on a copy of the probability scale in Figure 1.

   Solutions:

   Given: An unbiased die is rolled, find the required probabilities

   the sample space for the experiment is given by $S = \{1, 2, 3, 4, 5, 6\}$

   the number of the element in the sample space gives the total number of possible outcomes $n(S) = 6$

   probability of an event occurring $P(E) = \frac{n(E)}{n(S)}$

   a. the event of obtaining a 6, $E = \{6\}$ number of elements in the event $n(E) = 1$

   probability of obtaining a 6 $P(6) = \frac{1}{6}$

   b. the event of obtaining a 5, $E = \{5\}$ number of elements in the event $n(E) = 1$

   probability of obtaining a 5 $P(5) = \frac{1}{6}$

   Since the probability of obtaining any of the numbers is equally likely,

   $$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

   c. the event of obtaining even numbers $E = \{2, 4, 6\}$

   number of elements in the event $n(E) = 3$

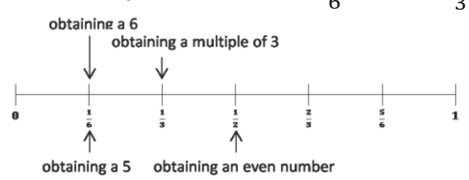
   probability of obtaining an even number $P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$

   d. the event of obtaining multiples of 3 $E = \{3, 6\}$

   number of elements in the event $n(E) = 2$

   probability of obtaining a multiple of 3 $P(\text{multiple of 3}) = \frac{2}{6} = \frac{1}{3}$

   Since there are 6 elements in the sample space we divide the probability scale into 6.



2. A card is taken at random from a full pack of cards (no jokers).

What is the probability that the card:

a. Is an ace
b. Is black
c. Is a heart
d. Has an even number on it

Solutions:

Given: A card is taken at random from a full pack of cards (no jokers);
There are 4 suits: clubs, spades, diamonds and hearts.
Clubs and spades are black. Diamonds and hearts are red. Each suit has 13 cards.
Ace (A) is taken as 1. Jack (J), queen (Q) and king (K) are called picture cards. The remaining cards are numbered 2 to 10. There are 52 cards in total.

the total number of possible outcomes $n(S) = 52$

probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

For this question, let us identify each set by a capital letter.

a. Event ace, $A = \{\text{ace of clubs, ace of spades, ace of diamonds, ace of hearts}\}$

$n(A) = 4 \qquad P(A) = \frac{4}{52} = \frac{1}{13}$

b. Event black, $B = \{2, 3, 4, 5, \ldots \text{J, Q. K, A of clubs}, 2, 3, 4, 5, \ldots \text{J, Q. K, A of spades}\}$

$n(B) = 26 \qquad P(B) = \frac{26}{52} = \frac{1}{2}$

c. Event hearts, $H = \{$2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q. K, A of hearts$\}$

$n(H) = 13 \qquad P(H) = \frac{13}{52} = \frac{1}{4}$

d. Event even number, $E = \{2, 4, 6, 8{,}10 \text{ of clubs}, \ldots \text{spades}, \ldots \text{hearts}, \ldots \text{diamonds}\}$

$n(E) = 20 \qquad P(E) = \frac{20}{52} = \frac{5}{13}$

Practice

1. A bag contains 3 black, 4 yellow and 7 red balls. A ball is picked from the bag at random, what is the probability that it is:
   a. Red b. Black c. Yellow d. White
2. The probability that you will pass your Maths exam is 0.7. What is the probability that you will fail? Write this as a fraction and as a percentage.
3. A bag contains 40 identical balls, 22 of which are orange and the rest mauve. Find the probability of picking at random:
   a. An orange ball b. A mauve ball
4. The probability that it will rain tomorrow is $\frac{3}{5}$.
   a. What is the probability that it will not rain tomorrow?
   b. Is it more likely to rain or not rain tomorrow?
5. The table below shows some lucky numbers in a raffle draw.
   If a number is picked at random in a draw, what is the probability that it is:
   a. Divisible by 2
   b. Even
   c. A perfect square
   d. More than 15

7	11	3	4	6
9	8	12	7	3
16	19	20	14	13
1	6	24	20	7

Lesson Title:	
Lesson Title: Addition law for mutually exclusive events – Part 1	**Theme:** Statistics and Probability
Lesson Number: TGM3-L099	**Class:** SSS 3
Learning Outcome By the end of the lesson, you will be able to apply the addition law to find the probability of mutually exclusive events.	

Overview

Consider the 2 events:

A: Attending school on Monday.

B: Being late for school.

They can both happen at the same time – you can attend school on Monday and be late, early or on time.

Now consider these 2 events:

A: Winning a football match

B: Losing a football match

They cannot both happen at the same time – a team cannot win and lose the same football match.

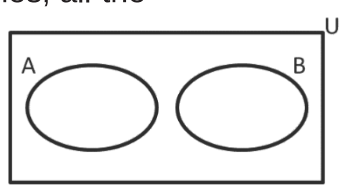
If two events cannot happen at the same time then they are called **mutually exclusive events**. Mutually exclusive events are examples of compound or combination events. The events are connected by the word "or".

If two events A and B are mutually exclusive events, then the probability of A or B is given by:

$$\begin{aligned} P(A \text{ or } B) &= P(A \cup B) \\ &= P(A) + P(B) \end{aligned}$$

This is the Addition Law for mutually exclusive events.
In probability, the word "or" or the symbol ∪ indicates addition.

A and B are disjoint sets as shown by the Venn diagram.
For two mutually exclusive events which cover all possible outcomes, all the individual probabilities add up to 1.



$$P(A) + P(B) = 1$$

If there are more than 2 events, then:

$$\begin{aligned} P(A \text{ or } B \text{ or } C \text{ or } D \text{ or } \ldots) &= P(A) + P(B) + P(C) + P(D) + \cdots \\ P(A) + P(B) + P(C) + P(D) + \cdots &= 1 \\ \text{Also} \quad P(\text{not } A) &= 1 - P(A) \end{aligned}$$

Solved Examples

1. A card is taken at random from an ordinary pack of cards. What is the probability that it will be any Ace or the 10 of Clubs?
   Solution:

Step 1. Assess and extract the given information from the problem.

Given: card taken at random from an ordinary pack of cards; find probability of Ace or 10 of Clubs

Step 2. Find the individual probabilities.

$n(S) = 52$

$P(E) = \frac{n(E)}{n(S)}$

Let A be the event of choosing an ace, B the event of 10 of clubs

$A = \{$ace of clubs, ace of spades, ace of diamonds, ace of hea

$n(A) = 4$ $\qquad P(A) = \frac{4}{52} = \frac{1}{13}$

$B = \{$10 of clubs$\}$

$n(B) = 1$ $\qquad P(B) = \frac{1}{52}$

Step 3. Find the probability of Ace or the 10 of Clubs.

$P(A \text{ or } B) = P(A) + P(B)$

$= \frac{1}{13} + \frac{1}{52} = \frac{4}{52} + \frac{1}{52} = \frac{5}{52}$

Step 4. Write the answer.

The probability of choosing an ace or the 10 of clubs is $\frac{5}{52}$.

2. The letters in the word P A R A L L E L O G R A M were written on identical pieces of paper and put in a bag. One piece of paper is selected at random. What is the probability of getting:

a. A $\qquad$ b. L $\qquad$ c. O

d. A or L $\qquad$ e. A or O $\qquad$ f. A or L or O

Solutions:

Given: P A R A L L E L O G R A M written on identical pieces of paper and put in a bag, find required probabilities

the total number of possible outcomes $\quad n(S) = 13$

probability of an event E occurring $\quad P(E) = \frac{n(E)}{n(S)}$

Let the letters represent their respective events.

a. $n(A) = 3$ $\qquad P(A) = \frac{3}{13}$

b. $n(L) = 3$ $\qquad P(B) = \frac{3}{13}$

c. $n(O) = 1$ $\qquad P(O) = \frac{1}{13}$

d. $P(A \text{ or } L) = P(A) + P(L)$

$= \frac{3}{13} + \frac{3}{13} = \frac{6}{13}$

e. $P(A \text{ or } O) = P(A) + P(O)$

$= \frac{3}{13} + \frac{1}{13} = \frac{4}{13}$

f. $P(A \text{ or } L \text{ or } O) = P(A) + P(L) + P(O)$

$= \frac{3}{13} + \frac{3}{13} + \frac{1}{13} = \frac{7}{13}$

3. The table gives the probability of getting 1, 2, 3 or 4 on a biased 4-sided spinner.

Number	1	2	3	4
Probability	0.2	0.35	0.15	0.3

What is the probability of getting:

a. 1 or 4 b. 2 or 3 c. 2 or 4 d. 1 or 2 or 3

Solutions:

Given: table of probability of obtaining 1, 2, 3, 4 on a 4-sided spinner.

The probabilities are given as decimal numbers. These can be added in the same way that we added probabilities as fractions.

Let the numbers represent their respective events.

a. $P(1 \text{ or } 4) = P(1) + P(4)$

$= 0.2 + 0.3 = 0.5$

b. $P(2 \text{ or } 3) = P(2) + P(3)$

$= 0.35 + 0.15 = 0.5$

c. $P(2 \text{ or } 4) = P(2) + P(4)$

$= 0.35 + 0.3 = 0.65$

d. $P(1 \text{ or } 2 \text{ or } 3) = P(1) + P(2) + P(3)$

$= 0.2 + 0.35 + 0.15 = 0.7$

Practice

1. Which of these pairs of events are mutually exclusive?
   a. Studying Mathematics and studying Geography.
   b. Choosing an even number and a prime number less than 10.
   c. Eating garri for breakfast and rice for lunch.
   d. Getting the right answer and the wrong answer for the same Maths problem.
2. A boy picked a number from the integers 10 to 25 inclusive. What is the probability that it is either a prime or an even number?
3. If a number is chosen at random from the integers 5 to 25 inclusive, find the probability that the number is:
   a. A multiple of 3 or 10.
   b. Even or prime number.
   c. Less than 12 or greater than 18.
4. A bag contains 10 balls. Six of the balls are red and the rest are equal numbers of white and blue. If a ball is picked at random. Find the probability that it is:
   a. Either white or red b. Not red
   c. Blue or red d. White
5. The table shows the probability of getting a particular colour of counters which have been put in a bag.

Colour	Yellow	Red	Green	Blue
Probability	0.5	0.2		0.1

   a. Complete the table to show the probability of getting green.
   b. A counter is taken at random from the bag, what is the probability of getting yellow or blue?

Lesson Title: Addition law for mutually exclusive events – Part 2	**Theme:** Statistics and Probability
Lesson Number: PHM3-L100	**Class:** SSS 3
Learning Outcome By the end of the lesson, you will be able to further apply the addition law to find the probability of mutually exclusive events.	

Overview

This lesson focuses on more complex uses of the addition law for mutually exclusive events.

Solved Examples

1. A bag contains a number of balls of different colours. The probability of obtaining a ball of a particular colour is given in the table below.

Colour	Probability
black	$\frac{3}{8}$
white	$\frac{1}{4}$
yellow	$\frac{1}{5}$

What is the probability that a ball taken at random from the bag is:

a. Black or white
b. Not yellow or white
c. Not one of the colours listed in the table

Solutions:

Step 1. Assess and extract the given information from the problem.

Given: probabilities of obtaining a ball of a particular colour in a bag

Step 2. Find the probabilities

the total number of possible outcomes $n(S) = 3$

probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

Let the initials of the colours represent their respective events.

a. $P(B \text{ or } W) = P(B) + P(W)$

$= \frac{3}{8} + \frac{1}{4} \qquad = \frac{5}{8}$

b. $P(\overline{W \text{ or } Y}) = P(\overline{W} \text{ or } \overline{Y}) = 1 - (P(W) + P(Y))$

$\overline{W}$ is the complement of W, $\overline{Y}$ is the complement of Y

$P(W) + P(Y)) = \frac{1}{4} + \frac{1}{5} \qquad = \frac{9}{20}$

$P(\overline{W} \text{ or } \overline{Y}) = 1 - \frac{9}{20} \qquad = \frac{11}{20}$

c. P(none of colours listed) $= 1 - P$(one of the colours listed)

$= 1 - (P(B) + P(W) + P(Y))$

$= 1 - \frac{3}{8} + \frac{1}{4} + \frac{1}{5}$

$= 1 - \frac{33}{40} \qquad = \frac{7}{40}$

Step 3. Write the answers.

The probability of black or white is $\frac{5}{8}$.

The probability of not yellow or white is $\frac{11}{20}$.

The probability of none of the colours listed in the table is $\frac{7}{40}$.

2. A letter is chosen at random from the word M A G N I T U D E. What is the probability that it is:
   a. Either in the word M U G or in the word I D E A.
   b. Neither in the word A G E N T nor in the word M I D.

Solutions:

Given: A letter is chosen at random from the word M A G N I T U D E.

Find the required probabilities

the total number of possible outcomes $n(S) = 9$

probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

a. $M = \{M, U, G\}$, $n(M) = 3$ $\qquad I = \{I, D, E, A\}$, $n(I) = 4$

$$P(M \text{ or } I) = P(M) + P(I)$$
$$= \frac{3}{9} + \frac{4}{9} = \frac{7}{9}$$

b. $A = \{A, G, E, N, T\}$, $n(I) = 5$ $\qquad D = \{M, I, D\}$, $n(D) = 3$

$$P(\overline{A} \text{ or } \overline{D}) = 1 - (P(A) + P(D))$$

$\overline{A}$ is the complement of A, D is the complement of D

$$P(A) + P(D)) = \frac{5}{9} + \frac{3}{9} = \frac{8}{9}$$
$$P(\overline{A} \text{ or } \overline{D}) = 1 - \frac{8}{9} = \frac{1}{9}$$

3. Pupils in SS3 classified the weather for a fortnight in October as very good, good, poor and very poor. They found out that probability the weather will be good or very good is 0.6. The probability that it will be poor is 0.3.
   a. What is the probability that the weather will be very poor?
   b. If the probability that the weather is good is twice the probability that it is very good, what is the probability that it will be good or poor?

Solutions:

Given: weather classified as very good, good, poor and very poor; probability of good or very good is 0.6, probability of poor is 0.3.

a. P(good or very good} $= 0.6$ $\qquad P$(poor} $= 0.3$

P(good} +P (very good)+P (poor)+P (very poor) $= 1$

$\Rightarrow$ P(very poor) $= 1 - P$(good} +P (very good)+P (poor)

$= 1 - (0.6 + 0.3) = 1 - 0.9$

$= 0.1$

The probability that the weather is very poor is 0.1.

b. $P(\text{good}) = 2 \times P$ (very good) given

$2 \times P$ (very good) $= 0.6 - P$ (very good) from part a.

$3 \times P$ (very good) $= 0.6$

P (very good) $= \frac{0.6}{3} = 0.2$

P (good)$+P$ (poor) $= 1 - (P$(very good) $+P$ (very poor))

$= 1 - (0.2 + 0.1) = 1 - 0.3$

$= 0.7$

The probability that it will be good or poor is 0.7.

Practice

1. A and B are two mutually exclusive events. P(A)$= 0.45$ and P(A or B)$= 0.8$. What is the value of P(B)?
2. A letter is chosen at random from the word CREATION. What is the probability that it is:
   a. Either in the word ONE or in the word ACT.
   b. Either in the word ART or in the word NICE.
   c. Neither in the word CONE nor in the word RAT.
3. A football competition has a number of different teams. The probability of teams winning the competition is given in the table below.

Teams	Probability
Blackpool	$\frac{3}{10}$
…East End Lions	$\frac{1}{4}$
Real Republicans	$\frac{1}{5}$

   What is the probability of a team selected at random winning the competition:
   a. Blackpool or East End Lions.
   b. East End Lions or Real Republicans.
   c. None of these teams winning.
4. A bag contains blue, yellow and white balls. The probability of selecting a ball at random and getting a white is $\frac{1}{7}$. The probability of getting a yellow ball is $\frac{3}{7}$.
   a. What is the probability of getting a blue ball?
   b. If the bag contains 4 white balls, how many yellow balls does it contain?
   c. If instead the bag contains 6 blue balls, how many balls does the bag contain in total?
5. A pack contains cards that are coloured pink, yellow or black. When a card is chosen at random, the probability of choosing a black or pink card is $\frac{5}{7}$ and the probability of choosing a black or yellow card is $\frac{3}{5}$.
   What is the probability of choosing a card of each colour?

Lesson Title: Multiplication law for independent events – Part 1	**Theme:** Statistics and Probability
Lesson Number: PHM3-L101	**Class:** SSS 3
Learning Outcome By the end of the lesson, you will be able to apply the multiplication law to find the probability of independent events occurring.	

Overview

Consider the 2 events:

A: You travel in a poda-poda to school which breaks down.
B: You are late for school.

Event A has an effect on event B – the poda-poda breaking down caused you to be late.

Now consider the following 2 events:

A: Being a girl
B: Being left-handed

Event A has no effect on event B and event B has no effect on event A.
Being a girl does not have any effect on which hand is used to write and being left-handed does not have an effect on being a girl.

If one event happening has no effect on another event happening they are called **independent events**. Independent events are examples of compound or combination events. The events are connected by the word "and".

Other examples of independent events include: raining on Monday this week, raining on Monday next week; a coin tossed twice lands on head then lands on tail; a die rolled twice shows a 6 then an odd number.
If two events A and B are independent events, then the probability of A and B is given by:

$$\begin{aligned} P(A \text{ and } B) &= P(A \cap B) \\ &= P(A) \times P(B) \end{aligned}$$

This is the Multiplication Law for independent events. In probability, the word "and" or the symbol ∩ indicates multiplication.

If there are more than 2 events, then:

$$\begin{aligned} P(A \text{ and } B \text{ and } C \text{ and } D \text{ and } \ldots) &= P(A \cap B \cap C \cap D \ \ldots) \\ &= P(A) \times P(B) \times P(C) \times P(D) + \cdots \end{aligned}$$

- As before:

$$P(\text{not } A) = 1 - P(A)$$

Solved Examples

1. A fair die is rolled twice. What is the probability that it will land on a 6 in the first roll and land on an odd number in the second roll?

   Solution:

   Step 1. Assess and extract the given information from the problem.

   Given: fair die rolled twice, find probability of 6 on the first roll and odd number on the second

   Step 2. Calculate the required probability.

   $S = \{1, 2, 3, 4, 5, 6\}$

   the total number of possible outcomes $n(S) = 6$

   probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

   $P(6) = \frac{1}{6}$ $\quad P(\text{odd number}) = \frac{3}{6} = \frac{1}{2}$

   $P(6 \text{ and odd number}) = P(6) \times P(\text{odd number})$

   $= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

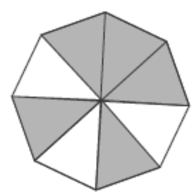
   Step 3. Write the answer.

   The probability of 6 in the first roll and odd number in the second is $\frac{1}{12}$.

2. The spinner shown at right has eight sections of equal size. Each one is coloured white or black. The events B and W are:

   B: The spinner lands on black.

   W: The spinner lands on white.



   Find the following probabilities:

   a. $P(B)$ b. $P(W)$ c. $P(B \text{ and } B)$

   d. $P(W \text{ and } W)$ e. $P(B \text{ and } W)$ f. $P(W \text{ and } B)$

   If the spinner is spun twice, find the probabilities of the following outcomes:

   g. White is obtained both times.

   h. A different colour is obtained on each spin.

   i. The same colour is obtained on each spin.

   Solutions:

   Given: spinner with eight sections of equal size, coloured white or black; find the required probabilities.

   $S = \{B, B, B, B, B, W, W, W\}$ $\quad n(S) = 8$

   probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

   a. $P(B) = \frac{5}{8}$

   b. $P(W) = 1 - \frac{5}{8} = \frac{3}{8}$

   c. $P(B \text{ and } B) = P(B) \times P(B)$
   $= \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

   d. $P(W \text{ and } W) = P(W) \times P(W)$
   $= \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

   e. $P(B \text{ and } W) = P(B) \times P(W)$

   f. $P(W \text{ and } B) = P(W) \times P(B)$

$= \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$ $\quad\quad$ $= \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$

g. P(White is obtained both times) $= P(W \text{ and } W) = P(W) \times P(W)$

$= \frac{9}{64}$

h. P(different colour obtained on each spin) $= P(B \text{ and } W)$ or $P(W \text{ and } B)$

$= (P(B) \times P(W)) + (P(W) \times P(B))$

$= \frac{15}{64} + \frac{15}{64} = \frac{30}{64}$

$= \frac{15}{32}$

i. $P(\text{same colour}) = 1 - P$(different colour)

$= 1 - \frac{15}{32}$

$= \frac{17}{32}$

Practice

1. The probability that it will rain tomorrow is $\frac{2}{3}$. The probability that Yasmin will forget her umbrella is $\frac{3}{4}$. What is the probability that it will rain tomorrow and Yasmin will forget her umbrella?
2. A bag contains 6 red balls and 5 black balls. A ball is picked from the bag, replaced and a second ball picked. Find the probability that:
   a. The first is black.
   b. Both are black.
   c. Both are the same colour.
3. The probability that a man travels by motorbike is 0.4 and the probability that he is late for his programme is 0.7. Find the probability that the man:
   a. Was late for his programme and travelled by motorbike.
   b. Was late for his programme and does not travel by motorbike.
   c. Was not late for his programme and travelled by motorbike.
   d. Was not late for his programme and does not travel by motorbike.
4. A die is thrown twice. What is the probability that:
   a. Two odd numbers are obtained.
   b. The same two numbers are obtained.
5. In a primary school, 80% of the boys and 65% of the girls walk to school. If a boy and a girl are chosen at random, what is the probability that:
   a. Both of them walk to school.
   b. Neither of them walk to school.

Lesson Title: Multiplication law for independent events – Part 2	**Theme:** Statistics and Probability
Lesson Number: PHM3-L102	**Class:** SSS 3
Learning Outcome By the end of the lesson, you will be able to further apply the multiplication law to find the probability of independent events occurring.	

Overview

This lesson focuses on more complex use of the multiplication law for independent events.

Solved Examples

1. The events $A = \{2, 4, 6\}$ and $B = \{6, 8\}$ are subsets of the sample space $S = \{2, 4, 6, 8, 10, 12\}$. Show that A and B are independent.

   Solution:

   Step 1. Assess and extract the given information from the problem.
   Given: $A = \{2, 4, 6\}$ and $B = \{6, 8\}$ are subsets of the sample space $S = \{2, 4, 6, 8, 10, 12\}$.

   Step 2. Calculate the individual probabilities
   If A and B are independent, then:

   $$P(A \cap B) = P(A) \times P(B) \qquad (1)$$
   $$(A \cap B) = \{6\}$$
   $$P(A \cap B) = \frac{1}{6}$$
   $$P(A) = \frac{3}{6} = \frac{1}{2}$$
   $$P(B) = \frac{2}{6} = \frac{1}{3}$$

   Step 3. Substitute into equation (1)

   $$\frac{1}{6} = \frac{1}{2} \times \frac{1}{3}$$
   $$\frac{1}{6} = \frac{1}{6}$$
   $$\text{LHS} = \text{RHS}$$

   Step 4. Write the answer.
   The events A and B are independent.

2. A card is taken at random from each of two ordinary packs of cards, pack A and pack B. What is the probability of getting:
   a. A red card from pack A and a red card from pack B.
   b. A diamond from pack A and a club from pack B.
   c. A king from pack A and a picture card (king, queen, jack) from pack B.
   d. A 10 from pack A and a 10 of clubs from pack B.
   e. An ace of hearts from each pack.

Solutions:

Given: A card is taken at random pack A and pack B; find the required probabilities

$$n(S) = 52$$

$$P(A) \text{ and } P(B) = P(A) \times P(B)$$

a. $P(\text{red}) = \frac{26}{52} = \frac{1}{2}$

$P(r\text{ed from } A) \text{ and } P(\text{red from } B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

b. $P(\text{diamond}) = P(\text{club}) = \frac{13}{52} = \frac{1}{4}$

$P(\text{diamond from } A) \text{ and } P(\text{club from } B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

c. $P(\text{king}) = \frac{4}{52} = \frac{1}{13}$

$P(\text{picture card}) = \frac{12}{52} = \frac{3}{13}$

$P(\text{king from } A) \text{ and } P(\text{picture card from } B) = \frac{1}{13} \times \frac{3}{13} = \frac{3}{169}$

d. $P(10) = \frac{4}{52} = \frac{1}{13}$

$P(10 \text{ of clubs}) = \frac{1}{52}$

$P(10 \text{ from } A) \text{ and } P(10 \text{ of clubs from } B) = \frac{1}{13} \times \frac{1}{52} = \frac{1}{676}$

e. $P(\text{ace of hearts}) = \frac{1}{52}$

$P(\text{ace of hearts from } A) \text{ and } P(\text{ace of hearts from } B) = \frac{1}{52} \times \frac{1}{52} = \frac{1}{2{,}704}$

3. The table below show the money Ahmed and Mommoh each have.

	Number of Leone notes			
	1,000	2,000	5,000	10,000
Ahmed	5	2	3	1
Mommoh	3	1	2	2

Ahmed and Mommoh each put their notes in a bag. Each pick one of their own notes from their bag at random. What is the probability that:

a. Ahmed picks Le 5,000.00 and Mommoh picks Le 5,000.00.
b. Ahmed picks Le 1,000.00 and Mommoh picks Le 2,000.00.
c. Ahmed does not pick Le 10,000.00 and Mommoh does not pick Le 10,000.00?

Solutions:

Given: probabilities given in table; find required probabilities

Let $A = \text{note Ahmed picks}$ $B = \text{note Mommoh picks}$

a. $P(A \text{ being Le } 5{,}000.00) = \frac{3}{11}$

$P(B \text{ being Le } 5{,}000.00) = \frac{2}{8} = \frac{1}{4}$

$P(A \text{ being Le } 5{,}000.00) \text{ and } P(B \text{ being Le } 5{,}000.00) = \frac{3}{11} \times \frac{1}{4} = \frac{3}{44}$

b. $P(A \text{ being Le } 1{,}000) = \frac{5}{11}$

$$P(B \text{ being Le } 2{,}000) = \frac{1}{8}$$

$$P(A \text{ being Le } 1{,}000.00) \text{ and } P(B \text{ being Le } 2{,}000.00) = \frac{5}{11} \times \frac{1}{8} = \frac{5}{88}$$

c. $$P(A \text{ not being Le } 10{,}000.00) = 1 - P(A \text{ being Le } 10{,}000.00)$$

$$= 1 - \frac{1}{11} = \frac{10}{11}$$

$$P(B \text{ not being Le } 10{,}000.00) \quad 1 - P(B \text{ being Le } 10{,}000.00)$$

$$= 1 - \frac{2}{8} = \frac{6}{8} = \frac{3}{4}$$

$$P(A \text{ not being Le } 10{,}000) \text{ and } P(B \text{ not being Le } 10{,}000) = \frac{10}{11} \times \frac{3}{4} = \frac{30}{44} = \frac{15}{22}$$

Practice

1. A company which makes mobile phones calculated the probability of any phone being defective as 0.025. Mr. Cole bought 2 phones. What is the probability that:
   a. Both phones are defective. b. Only one phone is defective.
   c. If the company found 3 defective phones in a batch they tested, how many phones were likely in the batch?
2. A die is biased so that the probability of obtaining a six is $\frac{7}{12}$. The other numbers have equal probabilities of being obtained.
   a. What is the probability of obtaining any one of the other numbers?
   b. If the die is thrown two times what is the probability of obtaining a six and an odd number?
3. A pupil sits for examinations in Mathematics, English and Geography. The probability that she passes Mathematics is 0.7. The probability that she passes English is 0.8. The probability that she passes Geography is 0.6. Given that her result in each subject is independent, what is the probability that she:
   a. Passes Mathematics and Geography.
   b. Passes English and Geography but not Mathematics.
   c. Passes all three subjects.
4. A coin has been tampered with so it lands on its head 3 out of every 4 throws. What is the probability it will land on its tail for 3 throws in a row?
5. To win a car in a competition, a player must throw 6 sixes in a row. What is the probability that the car is won?

Lesson Title: Application of the addition and multiplication laws	**Theme:** Statistics and Probability
Lesson Number: PHM3-L103	**Class:** SSS 3
Learning Outcome By the end of the lesson, you will be able to apply the addition and multiplication laws to a variety of probability questions.	

Overview

Consider two events – A and B.

If they are mutually exclusive then if A happens B cannot happen. This is pretty much the opposite of independent events which is that if A happens it has no effect on whether B happens.

Mutually exclusive events result from the outcomes of one experiment.

Independent events arise when considering the outcomes of either the same experiment several times such as rolling one dice twice or a single experiment such as rolling two dice at once.

We will also look at problems as in Question 2, where we are to find the probability of two events either both or neither occurring; or one, or at least one of them occurring.

Solved Examples

For all questions $P(A)$ or $P(B) = P(A) + P(B)$

$P(A)$ and $P(B) = P(A) \times P(B)$

1. A die is rolled twice. What is the probability that:
   a. The score on the first roll is 2 and the second roll is 5.
   b. The score on the first roll is 1 and the second roll is even.
   c. The score on the first roll is either 3 or 5.
   d. The score on the first roll is either 3 or 5 and the second roll is odd.
   e. The score on the first and second roll is either 3 or 5.

Solutions:

Step 1. Assess and extract the given information from the problem.
Given: a die is rolled twice; find required probabilities

Step 2. Calculate the required probabilities.

$S = \{1, 2, 3, 4, 5, 6\}$ $\quad n(S) = 6$

Let $A = \text{score on first roll}$ $\quad B = \text{score on second roll}$

a. $P(A \text{ is } 2)$ and $P(B \text{ is } 5) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

b. $P(A \text{ is } 1)$ and $P(B \text{ is even}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

c. $P(A \text{ is 3 or 5}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

d. $P(A \text{ is 3 or 5})$ and $P(B \text{ is odd}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

e. $P(A \text{ is 3 or 5})$ and $P(B \text{ is 3 or 5}) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

2. The probability that Ahmed gains admission into the university is $\frac{4}{5}$ and that of Brima is $\frac{2}{3}$. What is the probability that:

a. Both gain admission
b. None gain admission
c. One gains admission
d. At least one gain admission

Solutions:

Given: the probability that Ahmed gains admission into the university is $\frac{4}{5}$ and that of Brima is $\frac{2}{3}$; find required probabilities

Let $A = \text{Ahmed gains admission}$ $\quad P(A) = \frac{4}{5} \quad P(\overline{A}) = 1 - \frac{4}{5} = \frac{1}{5}$

$B = \text{Brima gains admission}$ $\quad P(B) = \frac{2}{3} \quad P(\overline{B}) = 1 - \frac{2}{3} = \frac{1}{3}$

i. P(both gain admission) $= P(A)$ and $P(B)$

$= \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$

ii. P(none gain admission) $= P(\ A)$ $P(\overline{A})$ and $P(\overline{B})$

$= \frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$

iii. P(one gains admission) $\Rightarrow$ Ahmed gains admission and Brima does not

or Ahmed does not gain admission and Brima does

$=$ $P(A)$ and $P(\overline{B})$ or $P(\overline{A})$ and $P(B)$

$= \left(\frac{4}{5} \times \frac{1}{3}\right) + \left(\frac{1}{5} \times \frac{2}{3}\right) = \frac{4}{15} + \frac{2}{15} = \frac{6}{15}$

$= \frac{2}{5}$

iv $P(\text{at least one gains admission}) = 1 - P$(none gain admission)

$= 1 - \frac{1}{15} = \frac{14}{15}$

3. When Mariama plays a game on her mobile phone, the probability that she gets to the top level is $\frac{1}{7}$. One day, Mariama plays two games on her mobile phone. What is the probability that:

a. She gets to the top level in both games.
b. She does not get to the top level in both games.
c. She gets to the top level in one of the two games.

If Mariama plays three games instead of two, what is the probability that

d. She will get to the top level in all three.

Solutions:

Given: probability that Mariama gets to the top level in her game is $\frac{1}{7}$; find required probabilities.

Let $A = \text{1st game}$ $\quad B = \text{2nd game}$

$P(A) = P(B) = \frac{1}{7} \quad P(\overline{A}) = P(\overline{B}) = 1 - \frac{1}{7} = \frac{6}{7}$

a. P(top level in *A* and *B*) $= P(A)$ and $P(B)$

$= \frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$

b. P(not top level in A and B) $= P(\overline{A})$ and $P(\overline{B})$

$= \frac{6}{7} \times \frac{6}{7} = \frac{36}{49}$

c. $P(\text{one of the two games}) \Longrightarrow$ top level in A and not top level in B

or not top level in A and top level in B

$= \left(P(A) \text{ and } P(\overline{B})\right) \text{ or } \left(P(\overline{A}) \text{ and } P\ P(B)\right)$

$= \left(\frac{1}{7} \times \frac{6}{7}\right) + \left(\frac{6}{7} \times \frac{1}{7}\right) = \frac{6}{49} + \frac{6}{49} = \frac{12}{49}$

d. P(top level in 3 games) $= P(A)$ and $P\ (B)\ and\ P(C)$

$= \frac{1}{7} \times \frac{1}{7} \times \frac{1}{7} = \frac{1}{343}$

Practice

1. There are two SSS3 classes in a school, Class A has 35 boys and 20 girls. Class B has 25 boys and 30 girls. One pupil from each class is selected at random to attend a debate. What is the probability that a boy and a girl are selected?
2. Mrs. Kamara meets 2 sets of traffic lights on her way to work. The probability of having to stop at the first is 0.4. The probability of having to stop at the second is 0.7. What is the probability that she will have to stop at:
    a. Both sets of traffic lights
    b. Only one set of traffic light
    c. At least one set of traffic lights
3. One out of every 6 civil servants owns a car.
    a. Write this probability as a fraction.

   Find the probability that:
    b. Two civil servants, X and Y, selected at random each owns a car.
    c. Out of two civil servants, P and Q, selected at random, only one owns a car.
    d. Out of three civil servants, A, B and C, selected at random, only one owns a car.
4. A fair die is thrown 3 times. What is the probability of:
    a. Getting 3 sixes.
    b. A six on the first and second throw but not on the third throw.
    c. Exactly 2 sixes in the three throws.
    d. At least 2 sixes in the three throws?

Lesson Title: Outcome Tables	**Theme:** Statistics and Probability
Lesson Number: PHM3-L104	**Class:** SSS 3

Learning Outcome

By the end of the lesson, you will be able to illustrate probability spaces with outcome tables and use them to solve probability problems.

Overview

When dealing with the probability of an event occurring, it is very important to identify all the outcomes of the experiment. One way in which this can be done for outcomes which are all equally likely is by systematically listing all of them in a sample space as shown for throwing a die. That is $S = \{1, 2, 3, 4, 5, 6\}$

However, when we have to identify the outcomes for two equally likely events occurring, listing can result in missing out some of the outcomes.

Instead, we use an outcome or 2-way table to identify all the outcomes. Drawing a table means we do not have to calculate the required probabilities. We can just count them off the table.

The 2-way table in Figure 1 shows all the outcomes when tossing 2 fair coins.

The 2-way table in Figure 2 shows all the outcomes for rolling 2 unbiased dice.

Second coin (columns); **First coin** (rows)

	H	T
H	HH	HT
T	TH	TT

Figure 1

Second die (columns); **First die** (rows)

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Figure 2

Solved Examples

1. Two fair coins are tossed. Using the table in Figure 1, find the probability that:
    a. Both coins show heads.
    b. Only one coin shows a tail.
    c. Both coins land the same way up.

Solutions:

Step 1. Assess and extract the given information from the problem.

Given: Figure 1, find the required probabilities

Step 2. Calculate the required probabilities.

From the table: $n(S) = 4$

a. $P(\text{both coins show head}) = \frac{1}{4}$

b. $P(\text{only one coin shows a tail}) = \frac{2}{4} = \frac{1}{2}$

c. $P(\text{both coins land the same way up}) = \frac{2}{4} = \frac{1}{2}$

2. Two unbiased dice are thrown. Using the table In Figure 2, find the probability that:
    a. Both dice show an even number.
    b. At least one die shows a 5.
    c. No die shows a 5.
    d. Both dice show the same number.

Solutions:

Given: Figure 2, find the required probabilities

From the table: $n(S) = 36$

a. $P(\text{both dice show even numbers}) = \frac{9}{36} = \frac{1}{4}$

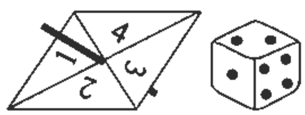
b. $P(\text{at least one die shows 5}) = \frac{11}{36}$

c. $P(\text{no die shows 5}) = 1 - P(\text{at least one die shows 5})$

$= 1 - \frac{11}{36} = \frac{25}{36}$

d. $P(\text{both dice show the same number}) = \frac{6}{36} = \frac{1}{6}$

3. A four-sided spinner is spun and a die is rolled. The two results are then multiplied to give a score.
    a. Draw a 2-way outcome table to show all the possible outcomes.



What is the probability of getting:

b. a score of 12?

c. a score of more than 15?

d. a score of less than 2?

Solutions:

Given: a four-sided spinner is spun and a die is rolled find the required probabilities

a. See the outcome table on the following page.

From the table: $n(S) = 24$

b. $P(\text{a score of 12}) = \frac{3}{24} = \frac{1}{8}$

c. $P(\text{a score of more than 15}) = \frac{4}{24}$

$$= \frac{1}{6}$$

d. $P(\text{a score of less than 2}) = \frac{1}{24}$

		Dice					
		1	2	3	4	5	6
Spinner	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18

Practice

1. Two fair 4-sided spinners are spun and the difference between the numbers is calculated.
    a. Copy and complete the outcome table showing all the possible outcomes.

		Spinner A			
		1	2	3	4
Spinner B	1	0	1		
	2	1			
	3				
	4				

    What is the probability of getting a difference of:
    b. 0   c. 3
    d. 4   e. 1 or 2.
2. Ibrahim has two packets of coloured pencils. In the first packet, there is a pink pencil, a blue pencil, a yellow pencil and a white pencil. In the second packet, there is a pink, a yellow and a blue pencil. Ibrahim takes a pencil at random from each packet.
    a. Draw an outcome table to show all the possible pairs of colours.
    Find the probability that the pencils will be:
    b. Both yellow   c. The same colour   d. Different colours
3. A school cook decides at random which of 3 sauces she will cook each day. She chooses from cassava leaves (C), potato leaves (P) and groundnut soup (G).
    a. Draw an outcome table showing all the possible outcomes from two consecutive days.
    What is the probability of:
    b. Cassava leaves on both days.   c. The same sauce on both days.
    d. Potato leaves or groundnut soup on both days.
    e. Kadija does not like groundnut soup. What is the probability she will not eat lunch in school for two consecutive days?
4. Ahmed, Brima, Charles, Dayo and Eku are in a competition. Each player must play each of the other player twice. If Ahmed wins the first game and Brima wins the second game, the outcome will be represented as AB.
    a. Draw an outcome table to show all the possible outcomes of the two games.
    What is the probability that:
    b. Brima wins both games.
    c. The same person wins both games.
    d. Charles wins at least one game.

Lesson Title: Tree diagrams – Part 1	**Theme:** Statistics and Probability
Lesson Number: PHM3-L105	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to illustrate probability spaces with tree diagrams and use them to solve probability problems.

Overview

Using 2-way outcome tables is a systematic method of listing all the outcomes from two equally likely events. However, they cannot be used when the events are not equally likely to occur or when we have more than 2 events. In such situations, we use a tree diagram where every branch represents an event together with its probability of occurring.

Solved Examples

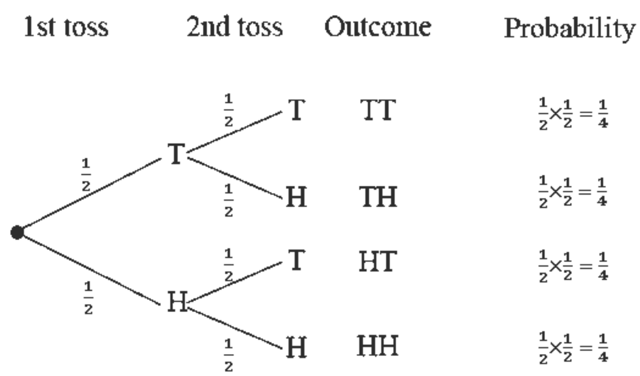
1. A fair coin is tossed twice. What is the probability of getting:
   a. Two heads b. No heads c. Only one head

   Solutions:

   Step 1. Assess and extract the given information from the problem.
   Given: toss a fair coin twice and find required probabilities.

   Step 2. Draw the tree diagram showing all the outcomes.

   Step 3. Find the probability of each outcome.
   The probability of each outcome is obtained by multiplying the probabilities of the branches leading to that outcome.



   Step 4. Find and write the required probabilities.
   a. The probability of 2 heads is given by the bottom branch and is $\frac{1}{4}$.
   b. The probability of no heads is given by the top branch and is $\frac{1}{4}$.
   c. The probability of only head is given by the 2 middle branches – The combined probability is: $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$.

   The probabilities should all add up to 1.

   $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4} = 1$ as expected

2. The probability that Jane is late for school is 0.3. What is the probability that on two consecutive days, she is:

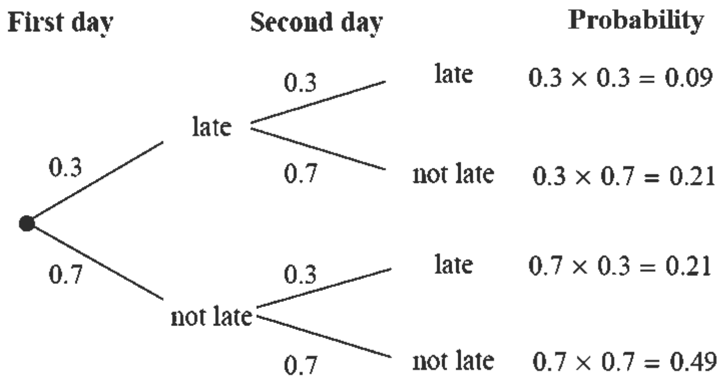
   a. Never late b. Late only once

   Solutions:

   Given: probability of rain $= 0.3$

   P(not being late) $= 1 - 0.3 = 0.7$

   The probability of each outcome is obtained by multiplying the probabilities of the branches leading to that outcome.



   From the tree diagram,

   a. $P(\text{Jane is never late}) = 0.49$

   b. Let $L =$ late, $N =$ not late

   $P(\text{Jane is late only once}) = P(LN) + P(NL)$

   $= 0.21 + 0.21$

   $= 0.42$

   The probabilities should all add up to 1.

   $0.09 + 0.21 + 0.21 + 0.49 = 1$ as expected

3. A die has 6 faces of which 3 are black, 2 white and 1 yellow. If the die is rolled twice, what is the probability of getting:

   a. Both faces are yellow
   b. Both faces the same colour
   c. At least one is black

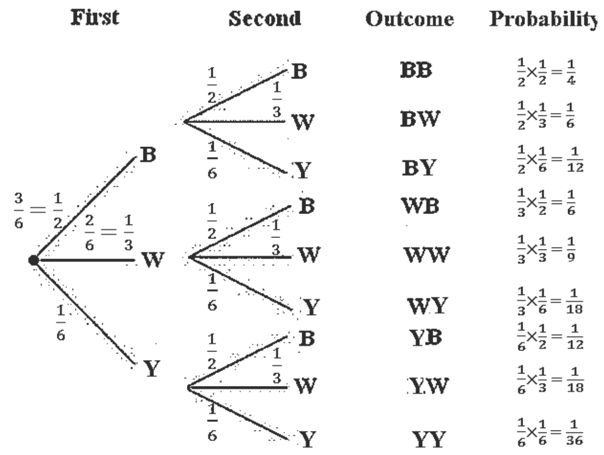
   Solutions:

   Given: die with 6 faces, 3 black, 2 white, 1 yellow; die rolled twice

   a. P(both yellow) $= P(YY) = \frac{1}{36}$

   b. P(same colour) $= P(BB) + P(WW) + P(YY)$

$= \frac{1}{4} + \frac{1}{9} + \frac{1}{36}$



$= \frac{7}{18}$

c. $P(\text{at least one is black})$

$= P(BB) + P(BW) + P(BY) + P(WB) + P(YB)$

$= \frac{1}{4} + \frac{1}{6} + \frac{1}{12} + \frac{1}{6} + \frac{1}{12}$

$= \frac{9}{12}$

$= \frac{3}{4}$

Practice

1. A bag contains 5 red balls and 3 green balls. A ball is chosen from the bag and then replaced. A second ball is chosen.
   a. Show the possible ways of selecting the balls on a tree diagram.
   What is the probability that:
   b. They are both red. c. One is red and one is green.
   d. At least one is red. e. At most one is red.
2. There are 10 pencils in a case. Three of the pencils are HB pencils. A pencil is taken at random from the pencil case and returned. A second pencil is now taken from the pencil case and then returned.
   a. Draw a tree diagram to show all the possible outcomes.
   b. What is the probability that only one of the pencils will be an HB pencil?
3. Victoria spins two spinners, A and B. The probability of getting a 6 on spinner A is 0.3. The probability of getting a six on spinner B is 0.45.
   a. Draw a tree diagram to show all the possible outcomes.
   Workout the probability of getting a 6 on:
   b. Neither spinner c. Only one spinner d. Spinner B only
4. The probability that Aruna burns the rice he is cooking is $\frac{1}{9}$. He cooks rice two days in a row.
   a. Draw a tree diagram to calculate the probability that he burns the rice at least once over the two days.
   b. Extend your tree diagram to include cooking rice three days in a row. What is the probability of burning the rice at least once over the three days?

Lesson Title: Tree diagrams – Part 2	Theme: Statistics and Probability
Lesson Number: PHM3-L106	**Class:** SSS 3
Learning Outcome By the end of the lesson, you will be able to use tree diagrams to further solve probability problems.	

Overview

So far, we have used tree diagrams to solve problems where the probabilities do not change between trials of an experiment. However, there are instances when the probabilities of certain events may change as a result of earlier events. For example, if it rains today the probability that it will rain tomorrow may be greater than if it was sunny today. This is called **conditional probability**. Tree diagrams help greatly in solving conditional probability problems as we can adjust the probabilities as we move along the branches.

Carefully consider the Solved Examples 1 and 2. In example 1, the counters are replaced after each trial before another selection is made. In example 2, the counter is not replaced before the second selection. Work through both examples carefully making sure you understand how the probabilities changed between the two trials.

Solved Examples

1. A bag contains 7 blue counters and 3 yellow counters. A counter is taken at random from the bag, replaced and a second counter taken. What is the probability that both counters are yellow?

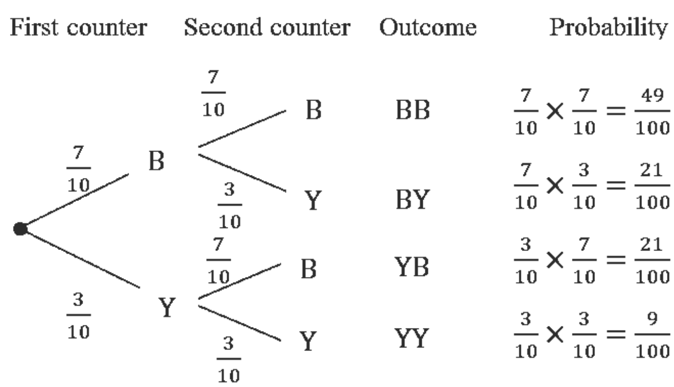
   Solution:

   Step 1. Assess and extract the given information from the problem.

   Given: bag with 7 blue counters and 3 yellow counters, 1st counter taken randomly, replaced, 2nd counter taken randomly, find required probability

   Step 2. Draw the tree diagram showing all the outcomes.

   total number of counters = 10



   Step 3. Find the probability of each outcome.

   From the tree diagram:

$P(\text{both counters are yellow}) \quad = \quad \frac{9}{100}$

Step 4. Write the answer.

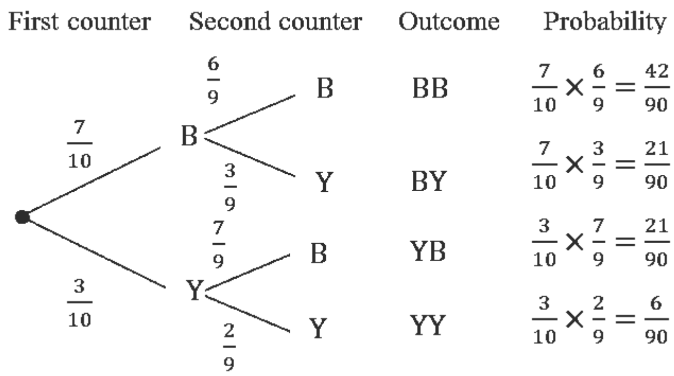
The probability that both counters are yellow is $\frac{9}{100}$.

2. A bag contains 7 blue counters and 3 yellow counters. Two counters are taken at random from the bag without replacement. What is the probability that both counters are yellow?

Solution:

Given: bag with 7 blue counters and 3 yellow counters, counter taken randomly, not replaced, find required probability

total number of $= \quad 10$



From the tree diagram: $P(\text{both counters are yellow}) = \frac{6}{90}$

3. A game with 2 outcomes, win or lose is played twice. The probability of a team winning the first game is 0.4. If the team wins the first game the probability of them winning the second game is 0.6. If the team loses the first game, the probability of losing the second is 0.8. What is the probability of the team:
   a. Winning both games
   b. Winning at least one game
   c. Losing both games

Solutions:

Given: game with 2 outcomes, win or lose is played twice; probability of winning the first game is 0.4.

Let $W =$ win, $L =$ lose $\quad$ 1st game: $P(\text{W}) = 0.4 \quad P(L) = 1 - 0.4 = 0.6$

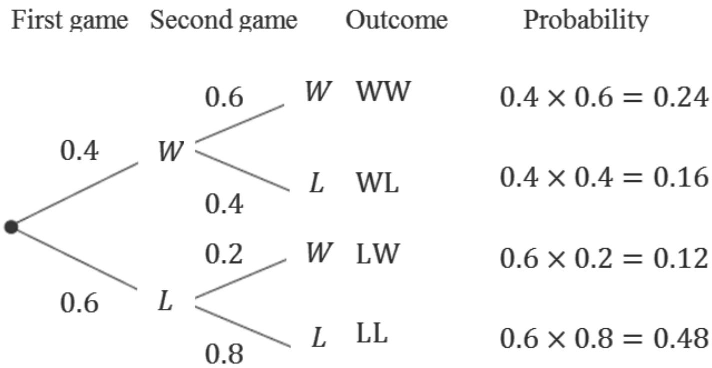
From the tree diagram (next page)

a. $\quad P(\text{winning both games}) \quad = \quad 0.24$

b. $\quad P(\text{winning at least 1}) \quad = \quad P(WW) + P(WL) + P(LW)$

$= \quad 0.24 + 0.16 + 0.12$

$= \quad 0.52$



c. $P(\text{losing both games}) = 0.48$ this is also $P(1 - \text{winning at least } 1)$

Practice

1. A bag contains 5 red balls and 3 green balls. A ball is chosen from the bag without replacement. A second ball is chosen.
   a. Show all the possible ways of selecting the balls on a tree diagram.
   What is the probability that:
   b. They are both red
   c. One is red and one is green
   d. At least one is red
   e. At most one is red
2. Seven girls and 5 boys want to be chosen to represent their class in a competition. Two of the pupils are chosen at random.
   a. Draw a tree diagram to show all the possible outcomes.
   What is the probability of getting:
   b. 2 girls
   c. 1 girl
   d. 1 or more girl
3. On any school day, the probability that Sia oversleeps is $\frac{1}{5}$. If she oversleeps, the probability that she will remember her homework is $\frac{2}{9}$. If she does not oversleep the probability that she will remember her homework is $\frac{5}{7}$. Use a tree diagram to work out the probability that Sia will not remember her homework tomorrow.
4. Issa either takes a poda-poda or walks to school. The probability that he takes a poda-poda is 0.45. If he takes a poda-poda, the probability that he will be late is 0.15. If he walks, the probability that he will be late is 0.35. What is the probability that he will not be late for school?
5. There are 8 counters in a box. Seven of the counters are yellow and 1 counter is blue. Jeneba selects counters at random. She stops when she gets the blue counter. Use a tree diagram to help you calculate the probability that Jeneba selects the blue counter in one of her first three attempts.

Lesson Title: Venn diagrams	**Theme:** Statistics and Probability
Lesson Number: PHM3-L107	**Class:** SSS 3
Learning Outcome By the end of the lesson, you will be able to illustrate probability spaces with Venn diagrams and use them to solve probability problems.	

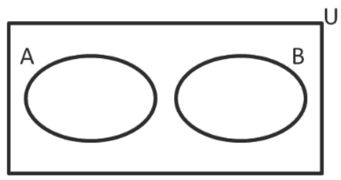
Overview

Consider two events – A and B.

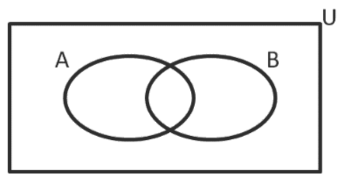
We know that if they are mutually exclusive then they cannot both occur at the same time. The probability of either one of them occurring is given by:

$$P(A \text{ or } B) = P(A \cup B)$$
$$= P(A) + P(B)$$

The first Venn diagram shows the two mutually exclusive events as disjoint sets with no common elements.
The complete set is denoted by U which is the Universal set.
Now consider events which cannot be classified as mutually exclusive events.



The second Venn diagram shows such events. It is clear from the diagram that if the events are not mutually exclusive then:



$$P(A \text{ or } B) = P(A \cup B)$$

Since $A \cup B = A + B - A \cap B$ where $A \cap B$ gives the common elements for both A and B

We can write:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This is the general Addition Law of Probability.
The law for mutually exclusive events is the special case for when $P(A \cap B) = 0$.

Solved Examples

1. In a class of 60 SS3 pupils, there are 40 who enjoy listening to music and 50 who enjoy watching films. Every pupil enjoys at least one of these activities.
   Draw a Venn diagram and find the probability that a pupil selected at random:
   a. Enjoys listening to music and watching films.
   b. Does not enjoy listening to music but enjoys watching films.
   c. Enjoys watching films but does not enjoy listening to music.

Solutions:

Step 1. Assess and extract the given information from the problem.
Given: in a class of 60 SS3 pupils, 40 enjoy listening to music, 50 enjoy watching films, every pupil enjoys at least one of these

Step 2. Draw the Venn diagram.
Let $A = \text{enjoy listening to music}$, $B = \text{enjoy watching films}$, $U = 60$

- Since every pupil enjoys at least one of the activities a 0 can be placed outside the ovals representing the events.

We know that:

$A \cup B \quad = \quad A + B - A \cap B$

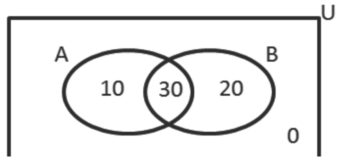
$A \cup B \quad = \quad 60 \qquad A \quad = \quad 40 \qquad B \quad = \quad 50$

$\therefore \quad 60 \quad = \quad 40 + 50 - P(A \cap B)$

$\quad = \quad 90 - (A \cap B)$

$A \cap B \quad = \quad 90 - 60$

$\quad = \quad 30$



- Complete the diagram as shown.
- The total number will always add up to the universal set.

Step 3. Find the required probabilities.

a. Let $C =$ enjoy listening to music and watching films

$P(C) = P(A \cap B) \quad = \quad \frac{30}{60} \quad = \quad \frac{1}{2}$

b. Let $D =$ does not enjoy listening to music but enjoys watching films

$P(D) \quad = \quad \frac{20}{60} \quad = \quad \frac{1}{3}$

c. Let $E =$ enjoy watching films but does not enjoy listening to music

$P(E) \quad = \quad \frac{10}{60} \quad = \quad \frac{1}{6}$

Check that the probabilities add up to 1: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} \quad = \quad 1$ as expected.

2. The Venn diagrams I, II and III show the ways two events A and B can take place. For each case find:

a. $P(A$ and $B)$ b. $P(A$ but not $B)$ c. $P(A$ or $B)$ d. $P(B$ but not $A)$

Diagram I

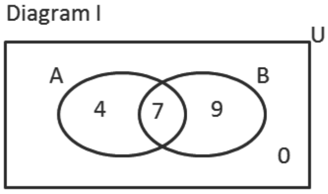


Diagram II

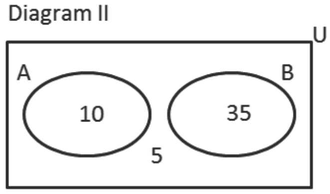
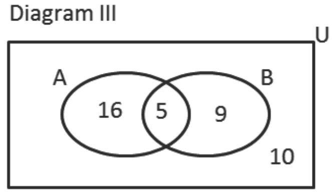


Diagram III



Solutions:

Given: Venn diagrams I, II and III show the ways two events A and B can take place; find required probabilities.

Diagram I $\quad U = 20$

a. $P(A$ and $B) \quad = \quad \frac{7}{20} \qquad P(A \cap B)$

b. $P(A$ but not $B) \quad = \quad \frac{4}{20}$

c. $P(A$ or $B) \quad = \quad 1 \qquad P(A \cup B)$

d. $P(B$ but not $A) \quad = \quad \frac{9}{20}$

Diagram I

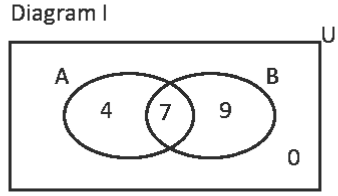


Diagram II $\quad U = 50$

a. $P(A$ and $B) \quad = \quad 0 \qquad P(A \cap B)$

b. $P(A$ but not $B) \quad = \quad \frac{10}{50} \quad = \quad \frac{1}{5}$

c. $P(A$ or $B) \quad = \quad \frac{45}{50} \quad = \quad \frac{9}{10} \qquad P(A \cup B)$

d. $P(B$ but not $A) \quad = \quad \frac{35}{50} \quad = \quad \frac{7}{10}$

Diagram II

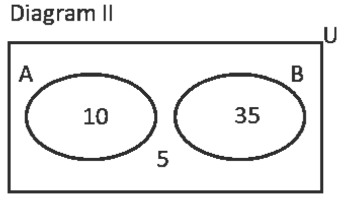
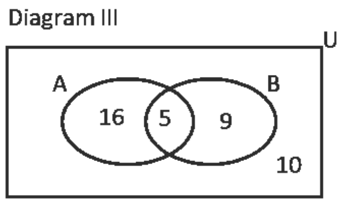


Diagram III $U = 40$



a. $P(A \text{ and } B) = \frac{5}{40} = \frac{1}{8}$ $P(A \cap B)$

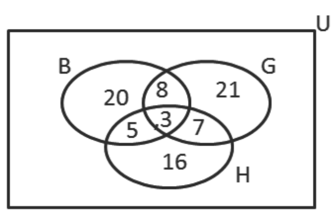
b. $P(A \text{ but not } B) = \frac{16}{40} = \frac{2}{5}$

c. $P(A \text{ or } B) = \frac{30}{40} = \frac{3}{4}$ $P(A \cup B)$

d. $P(B \text{ but not } A) = \frac{9}{40}$

Practice

1. When a card is selected at random from a pack of ordinary playing cards, the following 2 events are identified:
   A: an ace is selected S: a spade is selected
   Draw a Venn diagram to illustrate the probability space. What is the probability of selecting:
   a. An ace or spades b. The Ace of spades
2. In a class of 52, 24 study Mathematics and 26 study English. Two students study both English and Mathematics. A pupil is selected at random in the class. Find the probability that the pupil studies:
   a. Mathematics only b. English c. Mathematics
   d. Neither English nor Mathematics e. Either English or Mathematics
3. In a survey, 100 people were asked whether they played tennis or football - 20 played neither, 80 played football, and 40 played tennis. Draw a Venn diagram and find the probabilities that a person selected at random from the sample:
   a. Played both tennis and football b. Played tennis but not football
   c. Played football but not tennis
4. The Venn diagram shows the options of 80 pupils in a certain SSS3 class in Biology (B), Geography (G) and History (H). If a pupil is chosen at random from the class, what is the probability that:
   a. She studies geography b. She studies one optional subject only



Lesson Title:	Theme:
Lesson Title: Solve Probability Problems	**Theme:** Statistics and Probability
Lesson Number: PHM3-L108	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve a variety of probability problems.

Overview

Problems for this lesson will cover all the concepts learned during our study of probability.

Solved Examples

1. Once a week, Kelfala checks his car. The probability that he needs to pump up a tyre is $\frac{1}{20}$. The probability that he has to add oil is $\frac{1}{10}$ and the probability he has to add water is $\frac{1}{5}$. If the events are independent, what is the probability that Kelfala:
   a. Does not need to do anything to his car?
   b. Has to do one thing to his car?
   c. Has to do at least one thing to his car?

Solutions:

Step 1. Assess and extract the given information from the problem.

Given: probability Kelfala pumps tyre is $\frac{1}{20}$, probability that he adds oil is $\frac{1}{10}$ and the probability he adds water is $\frac{1}{5}$, find required probabilities.

Let $A = \text{Kelfala pumps tyre} \quad P(A) = \frac{1}{20} \quad P(\overline{A}) = \frac{19}{20}$

$B = \text{Kelfala adds oil} \quad P(B) = \frac{1}{10} \quad P(\overline{B}) = \frac{9}{10}$

$C = \text{Kelfala adds water} \quad P(C) = \frac{1}{5} \quad P(\overline{C}) = \frac{4}{5}$

Step 2. Calculate required probabilities.

a. $P(\text{Kelfala does not do anything to his car}) = P(\overline{A}) \times P(\overline{B}) \times P(\overline{C})$

$$= \frac{19}{20} \times \frac{9}{10} \times \frac{4}{5}$$
$$= \frac{684}{1{,}000}$$
$$= \frac{171}{250}$$

b. $P(\text{Kelfala does one thing to his car})$

$$= P(A) \times P(\overline{B}) \times P(\overline{C}) + P(\overline{A}) \times P(B) \times P(\overline{C}) + P(\overline{A}) \times P(\overline{B}) \times P(C)$$
$$= \left(\frac{1}{20} \times \frac{9}{10} \times \frac{4}{5}\right) + \left(\frac{19}{20} \times \frac{1}{10} \times \frac{4}{5}\right) + \left(\frac{19}{20} \times \frac{9}{10} \times \frac{1}{5}\right)$$
$$= \frac{36}{1{,}000} + \frac{76}{1{,}000} + \frac{171}{1{,}000}$$

$= \frac{283}{1,000}$

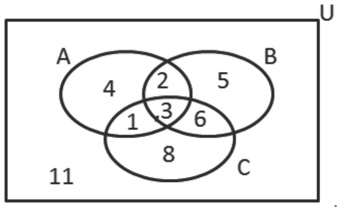
c. $P(\text{Kelfala does at least one thing to his car})$

$= 1 - P(\text{Kelfala does not do anything to his car})$

$= 1 - \frac{171}{250}$

$= \frac{79}{250}$

2. The Venn diagram shows the ways in which the events A, B and C can take place. Find:

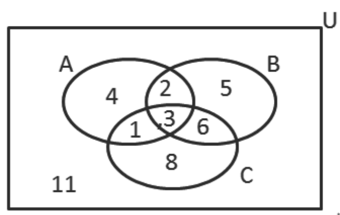


a. $P(A$ and B and $C)$ b. $P(A$ or B or $C)$
c. $P(A$ or $B)$ d. $P(A$ and $B)$
e. $P(A$ or $C)$ f. $P(A)$

Solutions:

Given: Venn diagram showing the ways in which the events A, B and C can take place; find required probabilities

$U = 40$



a. $P(A \text{ and } B \text{ and } C) = \frac{3}{40}$ $\quad P(A \cap B \cap C)$

b. $P(A \text{ or } B \text{ or } C) = \frac{29}{40}$ $\quad P(A \cup B \cup C)$

c. $P(A \text{ or } B) = \frac{21}{40}$ $\quad P(A \cup B)$

d. $P(A \text{ and } B) = \frac{5}{40} = \frac{1}{8}$ $\quad P(A \cap B)$

e. $P(A \text{ or } C) = \frac{24}{40} = \frac{3}{5}$ $\quad P(A \cup C)$

f. $P(A) = \frac{10}{40} = \frac{1}{4}$

3. A bag contains 2 red marbles, 1 blue marble and 1 yellow marble. A second bag contains 1 red marble, 2 blue marbles and 1 yellow marble. A marble is drawn from each bag.
   a. Complete the table showing all the possible pairs of colours.

   What is the probability:

   b. That both marbles are the same colour,
   c. At least one marble is yellow.
   d. No marble is yellow.

   Solutions:

		Marble from second bag			
		R	B	B	Y
Marble from first bag	R	RR	RB	RB	RY
	R	RR			
	B	BR			
	Y	YR			

Given: A bag contains 2 red marbles, 1 blue marble and 1 yellow marble; second bag contains 1 red, 2 blue and 1 yellow marble.

a. completed table shown right.

b. $P(\text{both marbles are same colour}) = \frac{5}{16}$

c. $P(\text{at least one marble is yellow}) = \frac{7}{16}$

d. $P(\text{no marble is yellow})$

$= 1 - P(\text{at least one marble is yellow})$

$= 1 - \frac{7}{16}$

$= \frac{9}{16}$

		Marble from second bag			
		R	B	B	Y
Marble from first bag	R	RR	RB	RB	RY
	R	RR	RB	RB	RY
	B	BR	BB	BB	BY
	Y	YR	YB	YB	YY

Practice

Use an appropriate diagram to answer the question where necessary.

1. The table below shows the number of pupils with their body weights.

Weight (kg)	40	30	25	20	15	12
No. of pupils	11	9	3	6	6	5

a. How many pupils are there?

What is the probability that if a pupil is chosen at random the body weight is:

b. 25 kg and above c. Less than 20 kg d. More than 30 kg

1. One hundred tickets are sold in a raffle to win a car.
   a. Ali buys one ticket. What is the probability that he wins the car?
   b. Eku buys five tickets. What is the probability that she wins the car?
2. The probability of an event X is $\frac{2}{3}$ while that of another event Y is $\frac{1}{7}$. If the probability of both X and Y is $\frac{5}{11}$, what is the probability of:
   a. Either X or Y b. Neither X nor Y
3. Two unbiased dice are rolled at the same time. The scores are then multiplied together to get a score. What is the probability of getting:
   a. A score of 12 b. A score of more than 20
   c. A score of less than 8
4. Nine slips of paper are numbered 1 to 9. A slip is drawn at random. This is replaced before a second slip is drawn. What is the probability that one is an odd number and the other an even number?
5. A bag contains identical stones of which 12 are painted red, 16 are painted white and 8 are painted blue. Three stones are drawn from the bag one after the other without replacement. Find the probability that:
   a. Three are red. b. The first is blue and the other two are red.
   c. They are of the same colour.

Lesson Title: Review of cumulative frequency curve	**Theme:** Statistics and Probability
Practice Activity: PHM3-L109	**Class:** SSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Construct a cumulative frequency curve and estimate quartiles.
2. Calculate inter-quartile range and semi interquartile range.

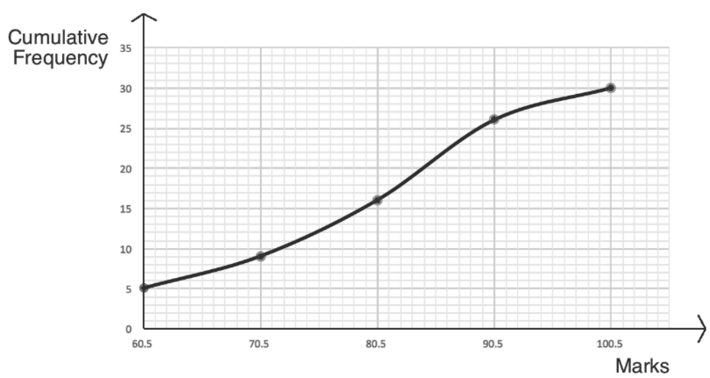
Overview

This lesson is on cumulative frequency curves, which are constructed based on cumulative frequency tables. A cumulative frequency table is one with class intervals, where the cumulative frequency for each row is calculated by adding that row's frequency to the cumulative frequency for rows above it. See the example below.

A **cumulative frequency (c.f.) curve** can be graphed in a similar way to a line graph. Cumulative frequency curves can also be called "**ogive**". A c.f. curve increases as it moves in the positive direction along the x-axis. For the x-values, we will plot the upper class boundary of each class interval. This is the highest data point in each class interval. In the table below, notice that there is a space of 1 unit between each interval. The first class interval ends at 60, and the second class interval begins at 61. For the purpose of graphing, we will take the point in the middle of the class intervals. For example, we will plot the value 60.5. For the y-value, we will plot the cumulative frequency from the table.

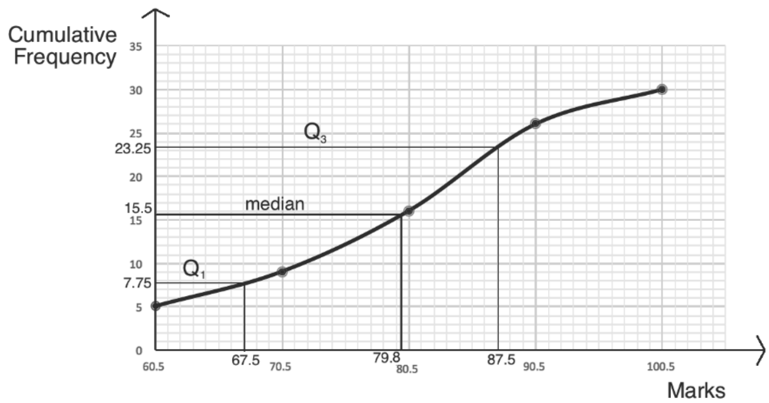
Pupils' Scores on a Maths Test			
Marks	**Frequency**	**Cumulative Frequency**	**Upper Class Boundary**
51 – 60	5	5	60.5
61 – 70	4	5 + 4 = 9	70.5
71 – 80	7	7 + 9 = 16	80.5
81 – 90	10	10 + 16 = 26	90.5
91 – 100	4	4 + 26 = 30	100.5
Total	30		

Plot the points and connect them with a smooth curve:



Quartiles divide a data set into 4 equal parts. The lower quartile (Q_1) is one-quarter of the way from the bottom of the data. The upper quartile (Q_3) is one-quarter of the way from the top of the data. The second quartile (Q_2) is the median, or the middle quartile. When we find quartiles from grouped data, the results are only estimates.

Estimate quartiles by first finding their placement in the dataset, then using the c.f. curve. The quartiles are located at $Q_1: \frac{1}{4}(n+1)$, $Q_2: \frac{1}{2}(n+1)$ and $Q_3: \frac{3}{4}(n+1)$, where n is the total frequency. The quartiles of the example c.f. curve are shown below. Their estimated values are $Q_1 = 67.5$ marks, $Q_2 = 79.8$ marks, $Q_3 = 87.5$ marks.



Just as we can calculate the range of a data set, we can calculate the interquartile range. The interquartile range represents how spread out the middle half of the data is. It is found by subtracting the lower quartile from the upper quartile ($Q_3 - Q_1$). For this dataset, the interquartile range is $87.5 - 67.5 = 20$ marks.

The semi-interquartile range tells us about one quarter of the data set ("semi" means half, so it is half of the interquartile range). The semi-interquartile range is given by the formula: $Q = \frac{Q_3 - Q_1}{2}$. For this data set, the interquartile range is $Q = \frac{87.5-67.5}{2} = \frac{20}{2} = 10$ marks. This tells us that about half of the pupils scored within 10 marks of the median score.

Solved Examples

1. The table below gives the marks of 25 pupils on a Maths test.
   a. Fill the empty columns.
   b. Draw the cumulative frequency curve.
   c. Use the curve to estimate the median mark.
   d. Use the curve to estimate the upper and lower quartiles.
   e. Calculate the interquartile range.

Pupils' Scores on a Maths Test			
Marks	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
51 – 60	3		
61 – 70	5		
71 – 80	7		
81 – 90	6		
91 – 100	4		
Total	25		

f. Calculate the semi-interquartile range.

Solutions:

a. Completed table:

Pupils' Scores on a Maths Test			
Marks	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
51 – 60	3	60.5	3
61 – 70	5	70.5	5 + 3 = 8
71 – 80	7	80.5	7 + 8 = 15
81 – 90	6	90.5	15 + 6 = 21
91 – 100	4	100.5	21 + 4 = 25
Total	25		

b. See below.

c. Estimated median: Pupil 13. This gives median mark $= 77.5$

d. Estimated Q_1:

$\frac{1}{4}(n+1) = \frac{1}{4}(25+1) =$

$\frac{1}{4}(26) = \frac{26}{4} = 6\frac{1}{2}$.

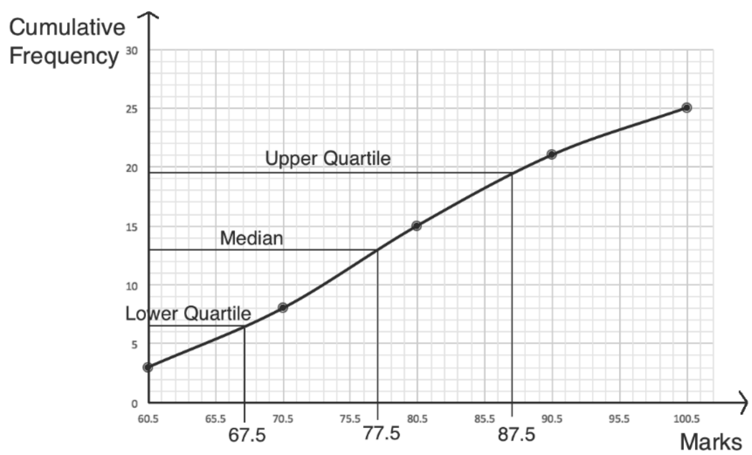
This gives $Q_1 = 67.5$.

Estimated Q_3:

$\frac{3}{4}(n+1) = \frac{3}{4}(25+1) =$

$\frac{3}{4}(26) = \frac{78}{4} = 19\frac{1}{2}$.

This gives $Q_3 = 87.5$.



e. Interquartile range:

$Q_3 - Q_1 = 87.5 - 67.5 = 20$ marks

f. $Q = \frac{Q_3 - Q_1}{2} = \frac{87.5-67.5}{2} = \frac{20}{2} = 10$ marks

Practice

1. a. Construct a cumulative frequency table for the distribution, which shows pupils' scores on a test:

Marks (%)	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	3	4	6	9	15	12	4

b. Use the cumulative frequency table to draw a cumulative frequency curve.
c. Use the curve to estimate the median.
d. Use the curve to estimate the interquartile range.

2. The cassava harvests of 25 farmers are shown in the frequency table:

Harvest (kg)	31-40	41-50	51-60	61-70	71-80
Frequency	4	8	6	4	3

a. Construct a cumulative frequency table for the distribution.
b. Use the table to draw a cumulative frequency curve.
c. Estimate the semi-interquartile range using the curve.

Lesson Title: Percentiles	**Theme:** Statistics and Probability
Practice Activity: PHM3-L110	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to estimate percentiles of data from the cumulative frequency curve.

Overview

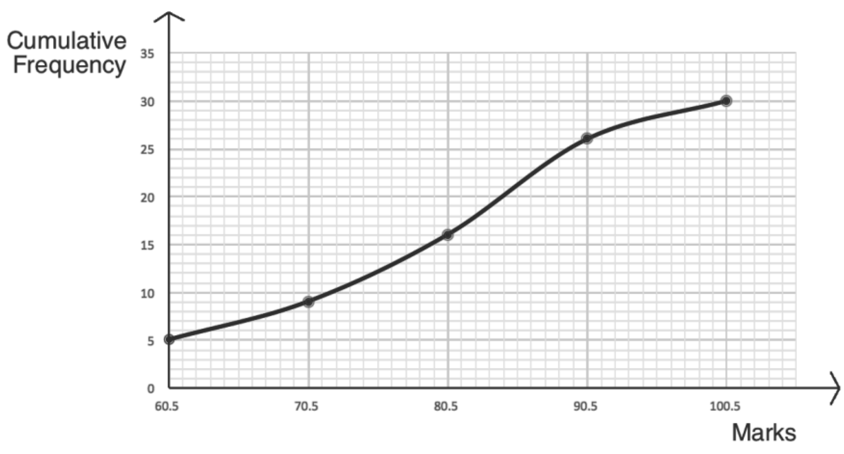
Percentiles are estimated using the cumulative frequency curve, in a similar way to quartiles. Percentiles divide the data set into 100 equal parts. For example, the 30th percentile divides off the lowest 30% of the data.

We use a formula to find the position of a quartile, then use the cumulative frequency curve to identify its value.

The nth percentile is the mark at $\frac{n}{100}\sum f$. This is the formula that gives a percentile's position. After using the formula, find the position on the y-axis of the curve. Draw horizontal and vertical lines to estimate the corresponding value on the x-axis. This is the estimated percentile.

Solved Examples

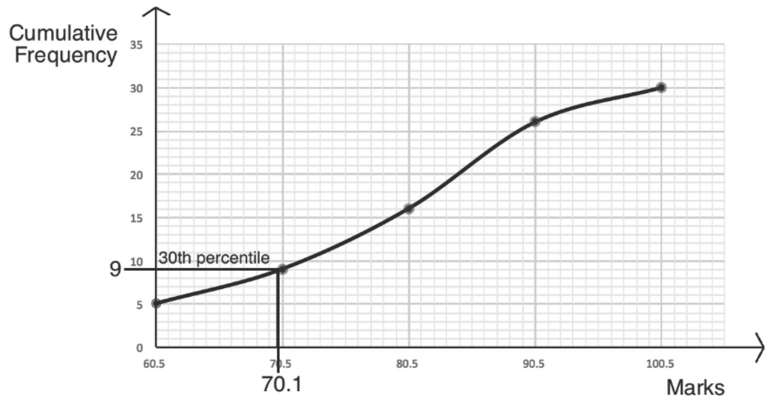
1. This cumulative frequency curve shows the marks that 30 pupils received on an exam. Find the value of the 30th percentile.



Solution:
Use the formula to find the position of the 30th percentile:

$$\frac{n}{100}\sum f = \frac{30}{100}(30) = \frac{900}{100} = 9$$

Estimate the 30th percentile using the curve, as shown



The 30th percentile is 70.1 marks.

2. The table below gives the weights of 100 individuals.

Weights (kg)	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64
Frequency	1	9	10	22	27	14	11	6

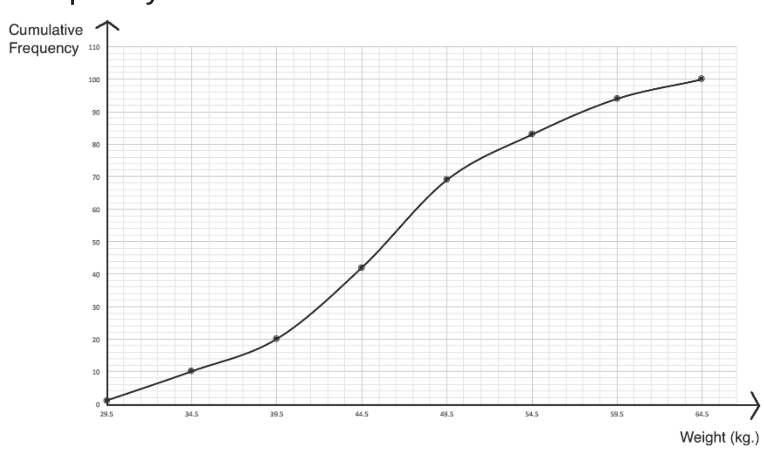
   a. Construct a cumulative frequency table.
   b. Use the table to draw the cumulative frequency curve.
   c. Use the curve to estimate the following: i. 15th percentile; ii. 60th percentile; iii. 95th percentile.

Solutions:

a. Cumulative frequency table:

Weights			
Weight (kg)	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
25 – 29	1	29.5	1
30 – 34	9	34.5	1+9=10
35 – 39	10	39.5	10+10=20
40 – 44	22	44.5	20+22=42
45 – 49	27	49.5	42+27=69
50 – 54	14	54.5	69+14=83
55 – 59	11	59.5	83+11=94
60 – 64	6	64.5	94+6=100
Total	100		

b. Cumulative frequency curve:

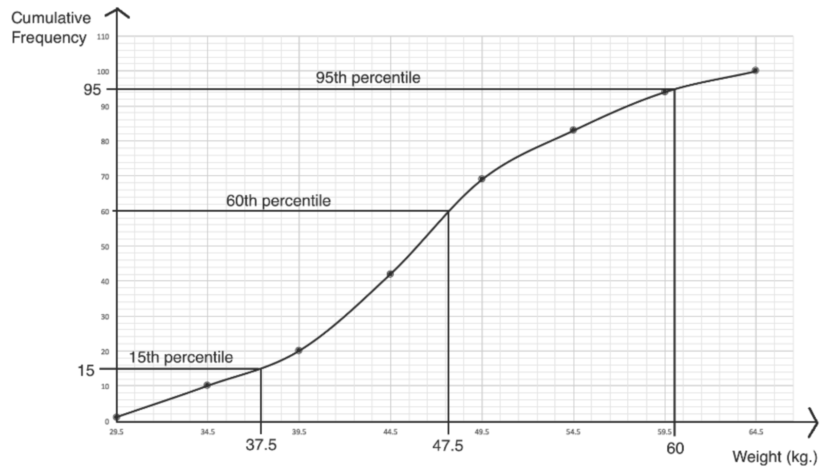


c. Estimations of percentiles (see graph below):

i. Position: $\frac{n}{100}\sum f = \frac{15}{100}(100) = \frac{1500}{100} = 15$; 15th percentile = 37.5 kg

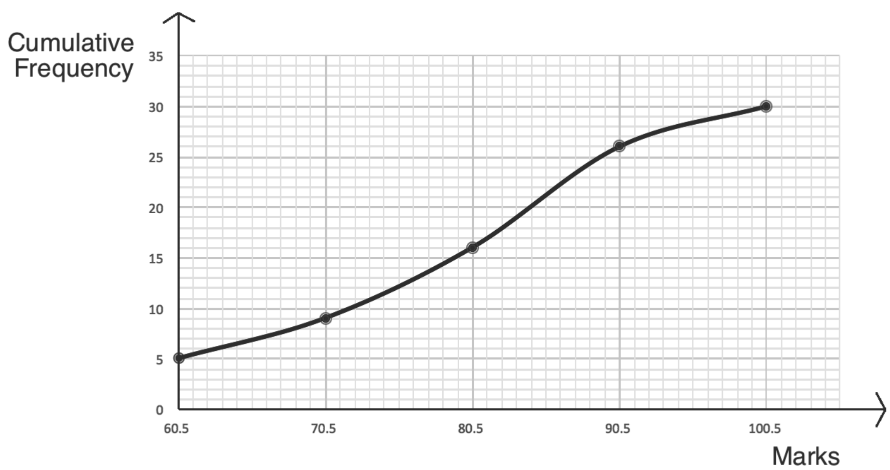
ii. Position: $\frac{n}{100}\sum f = \frac{60}{100}(100) = 60$; 60th percentile= 47.5 kg

iii. Position: $\frac{n}{100}\sum f = \frac{95}{100}(100) = 95$; 95th percentile= 60 kg



Practice

1. The cumulative frequency curve below gives the distribution of marks that 30 pupils scored on a Maths exam. Use the curve to estimate the following percentiles: a. 35th; b. 70th; c. 90th



2. The scores of 50 pupils on a test are shown in the frequency table below.

Marks (%)	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	3	4	3	6	7	10	12	5

Draw a cumulative frequency curve for the distribution, and use it to estimate the: a. 30th percentile; b. 45th percentile; c. 90th percentile.

Lesson Title: Applications of percentiles	**Theme:** Statistics and Probability
Practice Activity: PHM3-L111	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to apply percentiles to real-life problems.

Overview

Recall that percentiles divide the data set into 100 equal parts. For example, the 30th percentile divides off the lowest 30% of the data. This can be used to solve practical problems. To solve the problems in this lesson, you will use information from the previous lesson and your problem-solving skills.

Solved Examples

1. The scores of 50 pupils on a test are shown in the frequency table.

Marks (%)	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	2	3	5	4	9	13	11	3

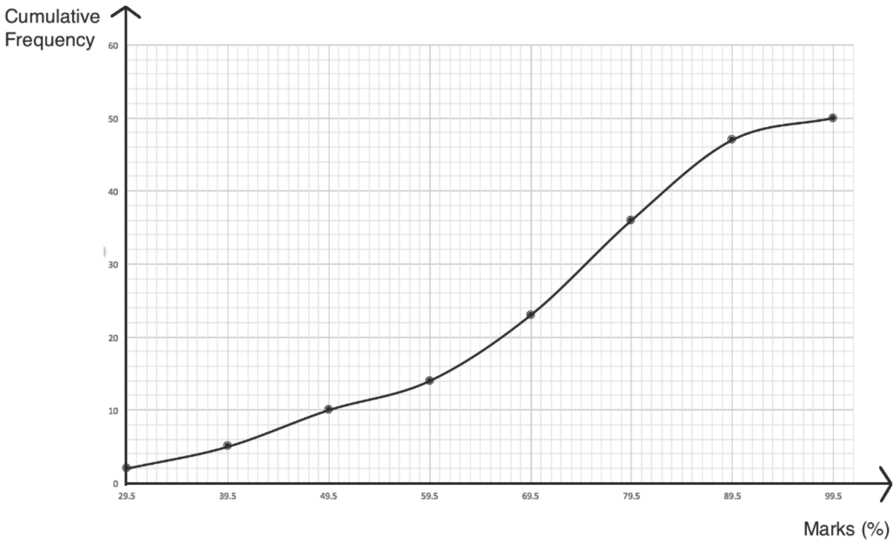
   a. Draw a cumulative frequency table.
   b. Use the table to draw the cumulative frequency curve.
   c. If 70% of pupils passed, find the pass mark.
   d. If 8% of candidates were awarded distinction, estimate the lowest mark for distinction.
   e. How many pupils were awarded distinction?

Solutions:

a. Cumulative frequency table:

Pupils' Scores on a Test			
Marks	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
20-29	2	29.5	2
30-39	3	39.5	2+3=5
40-49	5	49.5	5+5=10
50-59	4	59.5	10+4=14
60-69	9	69.5	14+9=23
70-79	13	79.5	23+13=36
80-89	11	89.5	36+11=47
90-99	3	99.5	47+3=50
Total	50		

b. Cumulative frequency curve:



c. If 70% of pupils passed, then 30% of pupils failed. If 30% of pupils scored below the passing mark, we can find the 30th percentile to find the passing mark. This question is just another way of asking you to find the 30th percentile.

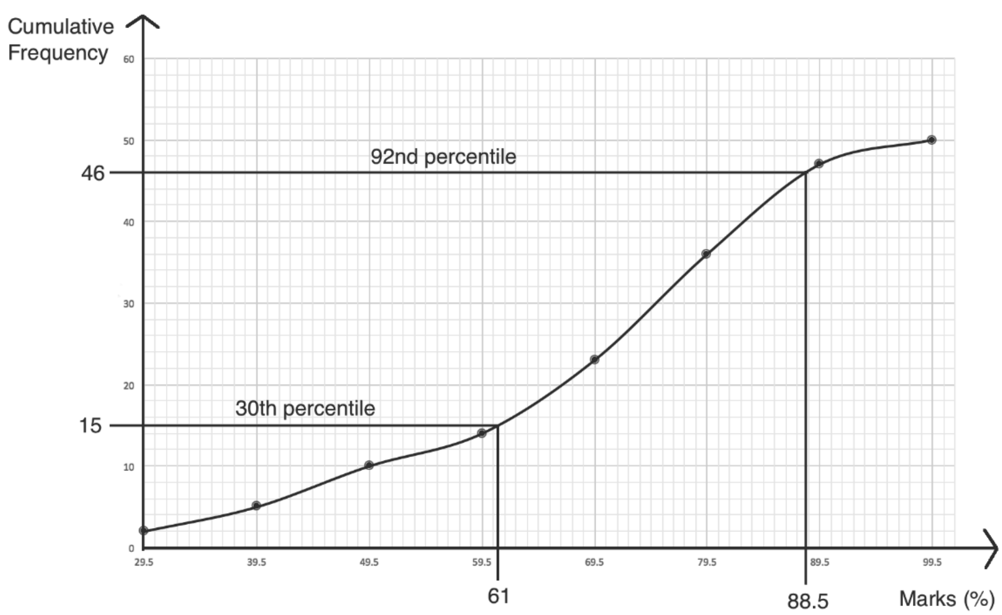
Find the position of the 30th percentile: $\frac{n}{100}\sum f = \frac{30}{100}(50) = \frac{1{,}500}{100} = 15$

Find the percentile on the c.f. curve (see below). The 30th percentile is 61%, which is the pass mark.

d. If 8% were awarded distinction, then 92% were not. Thus, 92% of pupils had scores under the mark for distinction, and we should find the 92nd percentile.

Find the position of the 92nd percentile: $\frac{n}{100}\sum f = \frac{92}{100}(50) = \frac{4{,}600}{100} = 46$

Find the percentile on the c.f. curve (see below). The 92nd percentile is 88.5%. This gives the lowest mark for distinction.



e. Four pupils were awarded distinction. The position of the 92nd percentile is 46, which means there are 4 pupils above distinction ($50 - 46 = 4$).

Practice

1. The table below gives the age distribution of all pupils enrolled in primary and secondary school in a village.

Age (years)	5-8	9-11	12-14	15-17	18-20
Frequency	20	32	18	14	6

   a. Construct a cumulative frequency table.
   b. Use the table to draw the cumulative frequency curve.
   c. Use the curve to estimate the 80th percentile.
   d. If the youngest 40% of the pupils are eligible for an early education programme, estimate how many pupils are eligible.
   e. Estimate the age of the oldest pupil eligible for the programme.
   f. If the oldest 30% of pupils are eligible for a technical training programme, estimate how many pupils are eligible.
   g. Estimate the age of the youngest pupil eligible for the technical training programme.

2. Bentu manages a chimpanzee sanctuary for rescued chimpanzees. The table below gives the weights of the 50 chimpanzees living at the sanctuary.

Weight (kg)	10-19	20-29	30-39	40-49	50-59	60-69
Frequency	4	9	12	15	8	2

   a. Construct a cumulative frequency table.
   b. Use the table to draw the cumulative frequency curve.
   c. Bentu designed one living space for the smallest 30% of chimpanzees. Estimate how many chimpanzees live in that space.
   d. The heaviest 40% of chimpanzees live in a very large space outdoors. Estimate how many chimpanzees live in that space.
   e. Estimate the weight of the smallest chimpanzee living in the large space outdoors.

Lesson Title: Measures of dispersion	**Theme:** Statistics and Probability
Practice Activity: PHM3-L112	**Class:** SSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Describe and interpret the dispersion or spread of values in a data set.
2. Calculate the range and variance of a set of ungrouped values.

Overview

This lesson is on dispersion, which is related to how spread out the data is. There are many different ways to measure the dispersion of a data set, but this lesson focuses on range, deviation, and variance.

Range is one of the most common ways to measure dispersion. Recall that range is the difference between the greatest and least values in a data set, which is found using subtraction. The larger the range, the more spread out the data set is. This is true for all measures of dispersion. If a measure of dispersion has greater value, the data is more spread out.

Deviation is another measure of dispersion, which is applied to an individual piece of data in the set. The deviation of a value gives its distance from the mean. To calculate deviation, subtract the mean from the given data point.

Variance is another measure of dispersion. Variance is calculated using the deviations of the data. The deviations of the data tell you information about each piece of data, and this information can be used to obtain a single number which indicates the overall dispersion of the data. To calculate variance, find the sum of the square of each deviation from the mean. Then, divide by the frequency. See Solved Example 2 for a calculation of variance.

Solved Examples

1. The ages of 20 university students are 18, 18,18, 19, 19, 19, 19, 20, 20, 20, 20, 21, 21, 21, 22, 22, 23, 23, 24, and 25.
   a. Calculate the range of the data.
   b. Calculate the mean.
   c. Calculate the deviation of a 24-year-old student.
   d. Calculate the deviation of an 18-year-old student.

   Solutions:

   a. Range = greatest value – least value $= 25 - 18 = 7$ years
   b. Mean= $\frac{\text{sum of ages}}{\text{number of students}} = \frac{3(18)+4(19)+4(20)+3(21)+2(22)+2(23)+24+25}{20} = \frac{412}{20} =$ 20.6 years old
   c. Deviation = $24 - \text{mean} = 24 - 20.6 = +3.4$
   d. Deviation = $18 - \text{mean} = 18 - 20.6 = -2.6$

2. Six children living in one household have weights of 20 kg, 24 kg, 51 kg, 30 kg, 16 kg and 21 kg. Calculate:
   a. The range.
   b. The mean.
   c. The variance.

   Solutions:

   a. Range $= 51 - 16 = 35$ kg

   b. Mean $= \frac{\text{sum of weights}}{\text{number of children}} = \frac{20+24+51+30+16+21}{6} = \frac{162}{6} = 27$ kg

   c. Variance:

   Step 1. Find the deviation of each value in the data set:

   - $20 - \text{mean} = 20 - 27 = -7$
   - $24 - \text{mean} = 24 - 27 = -3$
   - $51 - \text{mean} = 51 - 27 = +24$
   - $30 - \text{mean} = 30 - 27 = +3$
   - $16 - \text{mean} = 16 - 27 = -11$
   - $21 - \text{mean} = 21 - 27 = -6$

   Step 2. Calculate variance:

   $$\text{Variance} = \frac{(-7)^2+(-3)^2+(24)^2+(3)^2+(-11)^2+(-6)^2}{6}$$
   $$= \frac{49+9+576+9+121+36}{6}$$
   $$= \frac{800}{6}$$
   $$\approx 133 \text{ to 3 significant figures}$$

3. Ten players on a football team need new shoes. Their shoe sizes are 41, 39, 44, 41, 40, 44, 45, 40, 42, and 44. Calculate:
   a. The range in shoe sizes.
   b. The mean shoe size.
   c. The variance in shoe sizes.

   Solutions:

   a. Range $= 45 - 39 = 6$ sizes

   b. Mean $= \frac{\text{sum of sizes}}{\text{number of players}} = \frac{41+39+44+41+40+44+45+40+42+44}{10} = \frac{420}{10} = 42$

   c. Variance:

   Step 1. Find the deviation of each value in the data set:

   - $41 - \text{mean} = 41 - 42 = -1$
   - $39 - \text{mean} = 39 - 42 = -3$
   - $44 - \text{mean} = 44 - 42 = +2$
   - $41 - \text{mean} = 41 - 42 = -1$
   - $40 - \text{mean} = 40 - 42 = -2$
   - $44 - \text{mean} = 44 - 42 = +2$
   - $45 - \text{mean} = 45 - 42 = +3$

- $40 - \text{mean} = 40 - 42 = -2$
- $42 - \text{mean} = 42 - 42 = 0$
- $44 - \text{mean} = 44 - 42 = +2$

Step 2. Calculate variance:

$$\text{Variance} = \frac{(-1)^2+(-3)^2+(+2)^2+(-1)^2+(-2)^2+(+2)^2+(+3)^2+(-2)^2+(0)^2+(+2)^2}{10}$$

$$= \frac{1+9+4+1+4+4+9+4+0+4}{10}$$

$$= \frac{40}{10}$$

$$= 10$$

Practice

1. The heights of 6 pupils in centimetres are 120, 122, 124, 127, 127, and 130. Calculate:
   a. The range in height.
   b. The mean height.
   c. The variance of their heights.
2. The scores that 10 pupils achieved on an exam are 68, 83, 70, 72, 84, 69, 88, 75, 78, and 83. Calculate the range and variance of their scores.
3. The weights of 8 goats in kilogrammes are 20, 24, 30, 22, 27, 31, 27, and 35. Calculate:
   a. The range in weight.
   b. The mean weight.
   c. The variance of their weights.

Lesson Title:	Theme:
Lesson Title: Standard deviation of ungrouped data	**Theme:** Statistics and Probability
Practice Activity: PHM3-L113	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to calculate the standard deviation of a set of ungrouped values.

Overview

This lesson is the first lesson on standard deviation. This is another measure of dispersion that is very common and useful. Standard deviation is a measure of dispersion that tells us how close data points generally are to the mean. A low standard deviation indicates that points are generally close to the mean (the deviation is low). A high standard deviation indicates that points are spread out over a wide range of values, farther from the mean (the deviation is high).
Standard deviation is derived from variance. It is the square root of the variance of a set of data, $\text{standard deviation} = \sqrt{\text{variance}}$.

Standard deviation of ungrouped data is given by the formula $s = \sqrt{\frac{1}{n}\sum(x - \bar{x})^2}$, where n is the frequency, x is each value in the set, and $\bar{x}$ is the mean.

We need to take several steps to solve each problem. We will use a table to organise our calculations. When drawing a table, it is important to have a column for each term that is needed in the formula. To calculate standard deviation of ungrouped data, the table has columns x, $x - \bar{x}$, and $(x - \bar{x})^2$. These give us the information needed for the formula above.

Notice that the formula requires us to find the mean. The mean does not get a column in the table. We will calculate the mean of each data set separately.

Solved Examples

1. There are 20 pupils in a classroom. They measured their heights and found that the variance in their heights is 36. What is the standard deviation of their heights?
   Solution:
   Use the formula $\text{standard deviation} = \sqrt{\text{variance}}$:
   $$s = \sqrt{36} = 6$$

2. The ages of 10 university students are 18, 19, 19, 19, 20, 20, 21, 21, 21, 22. Calculate the standard deviation of the data.

Solution:

Step 1. Calculate mean: $\bar{x} = \frac{18+3(19)+2(20)+3(21)+22}{10} = \frac{200}{10} = 20$ years

Step 2. Fill the columns of the table using values from the problem (at right). The first column contains the list of data given. In the second column, subtract mean from each value of x. In the last column, find the square of the second column.

Step 3. Find the sum of the values in the $(x - \bar{x})^2$ column. Note that this is what the standard deviation formula requires.

Step 4. Calculate standard deviation:

$$s = \sqrt{\frac{1}{n}\sum(x-\bar{x})^2} = \sqrt{\frac{14}{10}} \approx 1.18$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
18	$18 - 20 = -2$	4
19	$19 - 20 = -1$	1
19	$19 - 20 = -1$	1
19	$19 - 20 = -1$	1
20	$20 - 20 = 0$	0
20	$20 - 20 = 0$	0
21	$21 - 20 = +1$	1
21	$21 - 20 = +1$	1
21	$21 - 20 = +1$	1
22	$22 - 20 = +2$	4
	Total	14

3. The ages of 7 children living in one house are 3, 5, 8, 12, 8, 4, and 9. Find the standard deviation of their ages.

Solution:

Step 1. Calculate mean: $\bar{x} = \frac{3+5+8+12+8+4+9}{7} = \frac{49}{7} = 7$ years old

Step 2. Fill the columns of the table.

Step 3. Find the sum of the values in the $(x - \bar{x})^2$ column.

Step 4. Calculate standard deviation:

$$s = \sqrt{\frac{1}{n}\sum(x-\bar{x})^2} = \sqrt{\frac{60}{7}} \approx 2.93$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
3	$3 - 7 = -4$	16
5	$5 - 7 = -2$	4
8	$8 - 7 = +1$	1
12	$12 - 7 = +5$	25
8	$8 - 7 = +1$	1
4	$4 - 7 = -3$	9
9	$9 - 7 = +2$	4
	Total	60

Practice

1. The ages of 6 Maths teachers are 28, 31, 34, 36, 42, and 45. Find the standard deviation of their ages.
2. Ten pupils scored the following marks on a Maths test: 84, 82, 78, 65, 69, 72, 70, 88, 67, 75. Find the standard deviation of their scores.
3. The heights of 5 football players in centimetres are 167, 175, 188, 170, and 185. Calculate the standard deviation of their heights.
4. The ages of 8 children are 3, 4, 4, 5, 7, 8, 12, 13. Calculate the standard deviation of their ages.

Lesson Title: Standard deviation of grouped data – Part 1	**Theme:** Statistics and Probability
Practice Activity: PHM3-L114	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to calculate the standard deviation of a set of grouped values **without** class intervals.

Overview

Recall that we have a different method for calculating the mean for grouped data, which can be described by the general formula $\bar{x} = \frac{\sum fx}{\sum f}$. See the first Solved Example for the use of this formula.

We also use a different formula when calculating standard deviation of grouped data. The formula is $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$, where f is frequency, x is each data point, and $\bar{x}$ is the mean.

Note that this formula can also be written as $s = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$. This is the same formula, but it is rearranged. The above formula is generally easier to use and will be used in this lesson. However, either formula may be used to solve these problems.

As in the previous lesson, we set up a table to organise our standard deviation calculations. However, this table does not have a row for each piece of data as in the previous lesson. It has a row for each value in the data, but these may have a frequency of more than 1. The columns of the table in this case will be x, f, fx, and fx^2. We will find the sums of the columns to facilitate our calculations.

Note that the sums of the columns can also be used in the formula for calculating the mean.

Solved Examples

1. The ages of 20 children are given in the table below. Calculate the mean and standard deviation of their ages.

Age (years)	1	2	3	4	5	6
Frequency (f)	3	4	2	3	6	2

Solution:

To find the mean, use the formula $\bar{x} = \frac{\sum fx}{\sum f}$. This is the sum of each score multiplied by its frequency, divided by the total frequency.

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{3(1)+4(2)+2(3)+3(4)+6(5)+2(6)}{20}$$
$$= \frac{3+8+6+12+30+12}{20}$$
$$= \frac{71}{20} = 3.55 \text{ years old}$$

To find the standard deviation, use the formula $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$. Set this up by forming the table shown below. Note that the last column is found by multiplying the values in the columns x and fx.

x	f	fx	fx^2
1	3	$1 \times 3 = 3$	$1 \times 3 = 3$
2	4	$2 \times 4 = 8$	$2 \times 8 = 16$
3	2	$3 \times 2 = 6$	$3 \times 6 = 18$
4	3	$4 \times 3 = 12$	$4 \times 12 = 48$
5	6	$5 \times 6 = 30$	$5 \times 30 = 150$
6	2	$6 \times 2 = 12$	$6 \times 12 = 72$
Totals	$\sum f = 20$	$\sum fx = 71$	$\sum fx^2 = 307$

Standard deviation:

$$s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$
$$= \sqrt{\frac{307}{20} - (3.55)^2}$$
$$= \sqrt{15.35 - 12.60}$$
$$= \sqrt{2.75}$$
$$\approx 1.66$$

2. The scores of 20 pupils on an exam are given in the table:

Score	82	83	84	85	86	87	88
Frequency (f)	1	3	5	4	3	1	3

   a. Calculate the mean score.
   b. Calculate the standard deviation of the distribution.

   Solutions:

   Prepare for the calculations by forming the table shown below.

x	f	fx	fx^2
82	1	82	$82(82) = 6{,}724$
83	3	$3(83) = 249$	$83(249) = 20{,}667$
84	5	$5(84) = 420$	$84(420) = 35{,}280$
85	4	$4(85) = 340$	$85(340) = 28{,}900$

86	3	$3(86) = 258$	$86(258) = 22{,}188$
87	1	87	$87(87) = 7{,}569$
88	3	$3(88) = 264$	$88(264) = 23{,}232$
Totals	$\sum f = 20$	$\sum fx = 1{,}700$	$\sum fx^2 = 144{,}560$

a. Calculate the mean using the formula:

$$\begin{aligned} \bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{1{,}700}{20} \\ &= 85 \text{ marks} \end{aligned}$$

b. Calculate the standard deviation using the formula:

$$\begin{aligned} s &= \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} \\ &= \sqrt{\frac{144{,}560}{20} - 85^2} \\ &= \sqrt{7{,}228 - 7{,}225} \\ &= \sqrt{3} \\ &\approx 1.73 \end{aligned}$$

Practice

1. The shoe sizes of 20 pupils are shown in the table below. Calculate the mean and standard deviation of their shoe sizes.

Shoe size	35	36	37	38	39	40
Frequency (f)	2	5	6	4	2	1

2. The ages of 30 secondary school pupils are shown in the table below. Calculate the standard deviation of their ages.

Age (years)	15	16	17	18	19
Frequency (f)	4	8	12	4	2

3. The table below gives the approximate height of 20 football players in metres. Calculate the mean and standard deviation.

Height (m)	1.5	1.6	1.7	1.8	1.9	2.0
Frequency (f)	2	3	4	5	3	3

Lesson Title: Standard deviation of grouped data – Part 2	Theme: Statistics and Probability
Practice Activity: PHM3-L115	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to calculate the standard deviation of a set of grouped values **with** class intervals.

Overview

This lesson is on calculating the standard deviation of grouped data with class intervals. We use the same formula that we used in the previous lesson for calculating standard deviation of grouped data. The formula is $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$, where f is frequency, x is each data point, and $\bar{x}$ is the mean.

When data is presented with class intervals, we do not have exact values for x. Take the mid-point of each class interval and use these in the formula.

We will set up tables for the calculations that are similar to those in the previous lesson. However, they will have 2 columns for class intervals and mid-points (x) in place of the single column for x that we used in the previous lesson.

Solved Examples

1. The weights of 25 children are given in the table below. Calculate the mean and standard deviation of their ages.

Weight (kg)	10-14	15-19	20-24	25-29	30-34	35-39
Frequency (f)	3	4	6	5	4	3

Solution:

Prepare for the calculations by forming the table shown below.

Interval	**Mid-point (x)**	f	fx	fx^2
10-14	12	3	36	432
15-19	17	4	68	1,156
20-24	22	6	132	2,904
25-29	27	5	135	3,645
30-34	32	4	128	4,096
35-39	37	3	111	4,107
	Totals	$\sum f = 25$	$\sum fx = 610$	$\sum fx^2 = 16{,}340$

Calculate the mean using the formula:

$$\bar{x} = \frac{\sum fx}{\sum f}$$
$$= \frac{610}{25}$$
$$= 24.4 \text{ kg}$$

Calculate the standard deviation using the formula:

$$s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$
$$= \sqrt{\frac{16{,}340}{25} - 24.4^2}$$
$$= \sqrt{653.6 - 595.36}$$
$$= \sqrt{58.24}$$
$$\approx 7.63$$

2. The scores of 15 pupils on an exam are given in the table.

Score	51-60	61-70	71-80	81-90	91-100
Frequency (f)	1	2	6	5	1

   c. Calculate the mean score.
   d. Calculate the standard deviation of the distribution.

Solutions:
Prepare for the calculations by forming the table shown below.

Interval	**Mid-point (x)**	f	fx	fx^2
51-60	55.5	1	55.5	3,080.25
61-70	65.5	2	131	8,580.5
71-80	75.5	6	453	34,201.5
81-90	85.5	5	427.5	36,551.25
91-100	95.5	1	95.5	9,120.25
	Totals	$\sum f = 15$	$\sum fx = 1162.5$	$\sum fx^2 = 91{,}533.75$

c. Calculate the mean using the formula:

$$\bar{x} = \frac{\sum fx}{\sum f}$$
$$= \frac{1{,}162.5}{15}$$
$$= 77.5 \text{ marks}$$

d. Calculate the standard deviation using the formula:

$$s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

$$= \sqrt{\frac{91{,}533.75}{15} - (77.5)^2}$$
$$= \sqrt{6{,}102.25 - 6{,}006.25}$$
$$= \sqrt{96}$$
$$\approx 9.80$$

Practice

1. Sia is a farmer. She measures the height of her pepper plants, which are given in the table below. Calculate the mean and standard deviation of the data, correct to 2 decimal places.

Height (cm)	20-24	25-29	30-34	35-39	40-44
Frequency (f)	2	6	7	3	2

2. The ages of 25 pupils are shown in the table below. Calculate the standard deviation of their ages, correct to 2 decimal places.

Age (years)	4-6	7-9	10-12	13-15	16-18
Frequency (f)	3	9	8	3	2

3. The scores of 12 pupils on an exam are given in the table.

Score	51-60	61-70	71-80	81-90	91-100
Frequency (f)	1	2	4	4	1

   a. Calculate the mean score, correct to 2 decimal places.
   b. Calculate the standard deviation of the distribution, correct to 2 decimal places.

Lesson Title: Standard deviation practice	**Theme:** Statistics and Probability
Practice Activity: PHM3-L116	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve standard deviation problems using the appropriate formulae.

Overview

This lesson is on calculating the standard deviation of ungrouped and grouped data using the formulae. Recall that the formulae are:

Ungrouped data: $s = \sqrt{\frac{1}{n}\sum(x - \bar{x})^2}$

Grouped data: $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$

where n is the total frequency of ungrouped data, x is each piece of data, $\bar{x}$ is the mean, and f is frequency of grouped data.

Solved Examples

1. A doctor treated 30 cases of malaria within one month. The age of his patients are in the table below. Calculate the standard deviation of the patients' ages.

Patients' age (years)	0-4	5-9	10-14	15-19	20-24
Frequency (f)	7	9	6	5	3

Solution:
Identify that this is a grouped data problem. Complete a table as shown:

Interval	**Mid-point (x)**	f	fx	fx^2
0-4	2	7	14	28
5-9	7	9	63	441
10-14	12	6	72	864
15-19	17	5	85	1,445
20-24	22	3	66	1,452
	Totals	$\sum f = 30$	$\sum fx = 300$	$\sum fx^2$ =4,230

Calculate mean: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{300}{30} = 10$ years old

Calculate standard deviation: $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{4230}{30} - 10^2} = \sqrt{141 - 100} = \sqrt{41} \approx 6.40$

2. Ten pupils ran a 1 kilometre race. Their finishing times (in minutes) were 7.0, 5.5, 7.0, 6.0, 6.5, 5.0, 8.0, 7.5, 9.0 and 8.5. Calculate:

a. The mean, and give your answer to 1 decimal place.
b. The standard deviation, correct to 2 decimal places.

Solutions:

Identify that this is an ungrouped data problem.

a. Calculate mean: $\bar{x} = \frac{7.0+5.5+7.0+6.0+6.5+5.0+8.0+7.5+9.0+8.5}{10} = \frac{70.0}{10} = 7.0$ minutes

Complete a table as shown:

x	$x - \bar{x}$	$(x - \bar{x})^2$
7.0	$7 - 7 = 0$	0
5.5	$5.5 - 7 = -1.5$	2.25
7.0	$7 - 7 = 0$	0
6.0	$6 - 7 = -1.0$	1
6.5	$6.5 - 7 = -0.5$	0.25
5.0	$5 - 7 = -2$	4
8.0	$8 - 7 = +1$	1
7.5	$7.5 - 7 = +0.5$	0.25
9.0	$9 - 7 = +2$	4
8.5	$8.5 - 7 = +1.5$	2.25
	Total	15

b. Calculate standard deviation:

$$s = \sqrt{\frac{1}{n}\sum(x - \bar{x})^2} = \sqrt{\frac{15}{10}} \approx 1.22$$

3. The ages of 20 football players on a secondary school team are in the table below. Calculate: i. The mean; ii. Standard deviation.

Age (years)	15	16	17	18	19
Frequency (f)	3	4	6	4	3

Solution:

Identify that this is a grouped data problem. Complete a table as shown:

x	f	fx	fx^2
15	3	45	675
16	4	64	1024
17	6	102	1734
18	4	72	1296
19	3	57	1083
	$\sum f = 20$	$\sum fx = 340$	$\sum fx^2 = 5812$

Calculate mean: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{340}{20} = 17$ years old

Calculate standard deviation: $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{5812}{20} - 17^2} = \sqrt{290.6 - 289} = \sqrt{1.6} \approx 1.26$

Practice

1. Eight children are aged 5, 6, 3, 8, 2, 7, 8, 9. Calculate: a. The range; b. The mean; c. Standard deviation, correct to 2 decimal places.

2. The table below gives the marks that 30 pupils achieved on an assignment, which was worth 15 marks in total. Calculate the mean and standard deviation, correct to 2 significant figures.

Marks	1-3	4-6	7-9	10-12	13-15
Frequency (f)	3	7	10	8	2

3. The table below gives the weights, to the nearest half kilogramme, of the babies born in a hospital during a one-month period. Estimate the mean and standard deviation of the data, correct to 2 decimal places.

Weight (kg)	2.5	3	3.5	4	4.5
Frequency (f)	3	5	7	4	1

Lesson Title: Mean deviation of ungrouped data	Theme: Statistics and Probability
Practice Activity: PHM3-L117	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to calculate the mean deviation of ungrouped data.

Overview

Mean deviation is similar to standard deviation. It is another measure of dispersion that tells us how close data points generally are to the mean. It is similar to standard deviation. A low mean deviation indicates that points are generally close to the mean (the deviation is low). A high mean deviation indicates that points are spread out over a wide range of values, farther from the mean (the deviation is high).

Note that mean and standard deviation measures are positive. When we calculate standard deviation, we square the deviations, which makes them positive. For mean deviation, we use absolute value to make the deviations positive.

As with standard deviation, we use different formulae to calculate mean deviation of ungrouped and grouped data.
For ungrouped data, use the formula $MD = \frac{\sum |x-\bar{x}|}{n}$, where x is each piece of data, $\bar{x}$ is the mean, and n is the frequency.

As with standard deviation, we will use a table to organise our calculations. When drawing a table, it is important to have a column for each term that is needed in the formula.

Notice that each formula requires us to find the mean. The mean does not get a column in the table. We will calculate the mean of each data set separately.

Solved Examples

1. The heights of 8 pupils in centimetres are 160, 165, 163, 159, 158, 157, 162, and 164. Calculate the mean deviation of their heights.
   Solution:
   Step 1. Calculate the mean: $\bar{x} = \frac{160+165+163+159+158+157+162+164}{8} = \frac{1288}{8} = 161$ centimetres.
   Step 2. Draw and fill a table as follows:

x	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$
160	$160 - 161 = -1$	1
165	$165 - 161 = +4$	4
163	$163 - 161 = +2$	2
159	$159 - 161 = -2$	2
158	$158 - 161 = -3$	3

157	$157 - 161 = -4$	4
162	$162 - 161 = +1$	1
164	$164 - 161 = +3$	3
	Total$= \sum \lvert \boldsymbol{x} - \overline{\boldsymbol{x}} \rvert =$	20

Step 3. Calculate the mean deviation:

$$MD = \frac{\sum|x-\bar{x}|}{n} = \frac{20}{8} = 2.5$$

2. The goals scored by 10 football players in a season are 3, 1, 4, 7, 3, 8, 12, 15, 8 and 9. Calculate the mean and mean deviation of the data.

   Solution:

   Step 1. Calculate the mean: $\bar{x} = \frac{3+1+4+7+3+8+12+15+8+9}{10} = \frac{70}{10} =$ 7 goals

   Step 2. Draw a fill a table as follows:

$\boldsymbol{x}$	$\boldsymbol{x} - \overline{\boldsymbol{x}}$	$\lvert \boldsymbol{x} - \overline{\boldsymbol{x}} \rvert$
3	$3 - 7 = -4$	4
1	$1 - 7 = -6$	6
4	$4 - 7 = -3$	3
7	$7 - 7 = 0$	0
3	$3 - 7 = -4$	4
8	$8 - 7 = +1$	1
12	$12 - 7 = +5$	5
15	$15 - 7 = +8$	8
8	$8 - 7 = +1$	1
9	$9 - 7 = +2$	2
	Total$= \sum \lvert \boldsymbol{x} - \overline{\boldsymbol{x}} \rvert =$	34

   Step 3. Calculate the mean deviation:

$$MD = \frac{\sum|x-\bar{x}|}{n} = \frac{34}{10} = 3.4$$

Practice

1. Eight children are aged 5, 6, 3, 8, 2, 7, 8, 9. Calculate the mean deviation.
2. The shoe sizes of 10 football players are 39, 41, 42, 40, 38, 37, 40, 41, 40, and 42. Calculate the mean and mean deviation.
3. The weights of the same 10 football players in kilogrammes are 67, 75, 72, 85, 80, 71, 83, 89, 76, and 72. Calculate: a. The range; b. The mean; c. The mean deviation.
4. Hawa raises chickens on her farm. The weights of 6 of her chickens in kilogrammes are 0.8, 1.5, 1.3, 1.0, 1.2 and 0.8. Find the mean and mean deviation of their weights, correct to 1 decimal place.

Lesson Title: Mean deviation of grouped data – Part 1	**Theme:** Statistics and Probability
Practice Activity: PHM3-L118	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to calculate the mean deviation of grouped data **without** class intervals.

Overview

We also use a different formula when calculating mean deviation of grouped data. The formula is $MD = \frac{\sum f|x-\bar{x}|}{\sum f}$, where x is each piece of data, $\bar{x}$ is the mean, and f is frequency.

As in the previous lesson, we set up a table to organise our mean deviation calculations. However, this table does not have a row for each piece of data as in the previous lesson. It has a row for each value in the data, but these may have a frequency of more than 1. The columns of the table in this case will be x, f, fx, $x - \bar{x}$, $|x - \bar{x}|$, $f|x - \bar{x}|$. We will find the sums of the columns to facilitate our calculations.

Recall that we have a different method of calculating mean for grouped data, which can be described by the general formula $\bar{x} = \frac{\sum fx}{\sum f}$.

Solved Examples

1. The ages of young children in one community are given in the table below. Calculate the mean and mean deviation of their ages.

Age (years)	1	2	3	4	5	6
Frequency (f)	5	5	8	2	2	3

Solution:
Create a table and fill it with the values needed to calculate the mean deviation (see below). Follow these steps:

a. Draw the empty table
b. Fill the first 3 columns (x, f, fx)
c. Use the first 3 columns to calculate the mean (see mean calculation below)
d. Use the mean to fill the rest of the table.
e. Use the table to calculate the mean deviation.

$\boldsymbol{x}$	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{x - \bar{x}}$	$\boldsymbol{\lvert x - \bar{x}\rvert}$	$\boldsymbol{f\lvert x - \bar{x}\rvert}$
1	5	5	$1 - 3 = -2$	2	10
2	5	10	$2 - 3 = -1$	1	5
3	8	24	$3 - 3 = 0$	0	0

4	2	8	$4-3=+1$	1	2
5	2	10	$5-3=+2$	2	4
6	3	18	$6-3=+3$	3	9
Totals:	$\sum f=25$	$\sum fx=75$			$\sum f\lvert x-\bar{x}\rvert=30$

Mean: $\bar{x}=\frac{\sum fx}{\sum f}=\frac{75}{25}=3$ years old

Mean deviation: $\text{MD}=\frac{\sum f|x-\bar{x}|}{\sum f}=\frac{30}{25}=1.2$

2. The shoe sizes of 15 football players are given in the table below. Calculate the mean deviation of their shoe sizes, correct to 2 decimal places.

Shoe size	38	39	40	41	42
Frequency (f)	2	4	4	2	3

Solution:

Create a table and fill it with the values needed to calculate the mean deviation. Calculate the mean and use it to complete the table.

$\boldsymbol{x}$	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{x-\bar{x}}$	$\boldsymbol{\lvert x-\bar{x}\rvert}$	$\boldsymbol{f\lvert x-\bar{x}\rvert}$
38	2	76	$38-40=-2$	2	4
39	4	156	$39-40=-1$	1	4
40	4	160	$40-40=0$	0	0
41	2	82	$41-40=+1$	1	2
42	3	126	$42-40=+2$	2	6
Totals:	$\sum f=15$	$\sum fx=600$			$\sum f\lvert x-\bar{x}\rvert=16$

Mean: $\bar{x}=\frac{\sum fx}{\sum f}=\frac{600}{15}=40$

Mean deviation: $\text{MD}=\frac{\sum f|x-\bar{x}|}{\sum f}=\frac{16}{15}=1.07$

Practice

1. The ages of 15 children are given in the table below. Calculate the mean deviation of their ages, correct to 2 decimal places.

Age (years)	2	3	4	5	6
Frequency (f)	1	1	5	3	5

2. The heights of 20 pupils in centimetres are given in the table. Calculate:
   a. The mean, correct to 3 significant figures.
   b. The mean deviation, correct to 1 decimal place.

Height (cm)	154	155	156	157	158	159
Frequency (f)	5	6	3	2	3	1

3. The weights in kilogrammes of 19 babies are given in the table. Calculate the mean and mean deviation.

Weight (kg)	2.5	3	3.5	4	4.5	5
Frequency (f)	2	5	6	4	1	1

4. Martin is a goat farmer. The weights of his 20 goats are given in the table below, to the nearest 5 kilogrammes. Find the mean to the nearest whole number and the mean deviation correct to 1 decimal place.

Weight (kg)	30	35	40	45	50
Frequency (f)	2	5	7	4	2

Lesson Title: Mean deviation of grouped data – Part 2	**Theme:** Statistics and Probability
Practice Activity: PHM3-L119	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to calculate the mean deviation of grouped data **with** class intervals.

Overview

To calculate the mean deviation of grouped data with class intervals, we will use the same formula from the previous lesson. The formula is $MD = \frac{\sum f|x-\bar{x}|}{\sum f}$, where x is each piece of data, $\bar{x}$ is the mean, and f is frequency.

When data is presented with class intervals, we do not have exact values for x. Take the mid-point of each class interval and use these in the formula.

We will set up tables for the calculations that are similar to those in the previous lesson. However, they will have 2 columns for class intervals and mid-points (x) in place of the single column for x that we used in the previous lesson.

Solved Examples

1. The weights of 15 children are given in the table below. Calculate the mean and mean deviation of their weights.

Weight (kg)	10-14	15-19	20-24	25-29	30-34
Frequency (f)	1	2	4	5	3

Solution:
Create a table and fill it with the values needed to calculate mean deviation. Calculate mean (shown below) and use it to complete the table.

Interval	**Mid-point (x)**	f	fx	$x-\bar{x}$	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
10-14	12	1	12	$12-24=-12$	12	12
15-19	17	2	34	$17-24=-7$	7	14
20-24	22	5	110	$22-24=-2$	2	10
25-29	27	4	108	$27-24=+3$	3	12
30-34	32	3	96	$32-24=+8$	8	24
	Totals:	$\sum f = 15$	$\sum fx = 360$			$\sum f\lvert x-\bar{x}\rvert = 72$

Mean: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{360}{15} = 24$ kg

Mean deviation: $\text{MD} = \frac{\sum f|x-\bar{x}|}{\sum f} = \frac{72}{15} = 4.8$

2. The ages of 15 pupils are given in the table below. Calculate the mean and mean deviation of their ages.

Ages (years)	5-7	8-10	11-13	14-16
Frequency (f)	1	5	7	2

Solution:

Create a table and fill it with the values needed to calculate mean deviation. Calculate the mean (shown below) and use it to complete the table.

Interval	**Mid-point (x)**	f	fx	$x - \bar{x}$	$\lvert x - \bar{x}\rvert$	$f\lvert x - \bar{x}\rvert$
5-7	6	1	6	$6 - 11 = -5$	5	5
8-10	9	5	45	$9 - 11 = -2$	2	10
11-13	12	7	84	$12 - 11 = +1$	1	7
14-16	15	2	30	$15 - 11 = +4$	4	8
	Totals:	$\sum f = 15$	$\sum fx = 165$			$\sum f\lvert x - \bar{x}\rvert = 30$

Mean: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{165}{15} = 11$ years old

Mean deviation: $\text{MD} = \frac{\sum f|x-\bar{x}|}{\sum f} = \frac{30}{15} = 2$

Practice

1. The heights of 20 children are given in the table below, to the nearest 5 cm. Find the mean and mean deviation of their heights.

Height (cm)	120-124	125-129	130-134	135-139
Frequency (f)	3	6	7	4

2. The ages of 15 Maths teachers are given in the table below. Find the mean deviation.

Age (years)	31-35	36-40	41-45	46-50	51-55
Frequency (f)	2	1	4	5	3

3. The weights of 12 football players are given in the table below. Find the mean and mean deviation.

Weight (kg)	65-69	70-74	75-79	80-84	85-89
Frequency (f)	1	3	4	3	1

Lesson Title: Statistics and probability	Theme: Statistics and Probability
Practice Activity: PHM3-L120	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve WASSCE-style statistics problems that include probability questions.

Overview

For this lesson, you will combine the information you learned in previous lessons on probability and statistics. The WASSCE exam often features statistics problems that include a question on probability.

Solved Examples

1. The ages of 20 pupils are given in the table below. Use the table to calculate: a. The mean; b. The mean deviation; c. The probability that a child chosen at random is at least 11 years old.

Ages (years)	5-7	8-10	11-13	14-16
Frequency (f)	2	5	4	9

Solutions:

a. Organise a table (see below) and apply the formula:
$\bar{x} = \frac{\sum fx}{\sum f} = \frac{240}{20} = 12$ years old

b. Complete the relevant columns of the table (see below) and apply the formula: $MD = \frac{\sum f|x-\bar{x}|}{\sum f} = \frac{54}{20} = 2.7$

c. Probability $= \frac{\text{children 11 or older}}{\text{all children}} = \frac{4+9}{20} = \frac{13}{20} = 0.65$

Interval	Mid-point (x)	f	fx	$x-\bar{x}$	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
5-7	6	2	12	$6-12=-6$	6	12
8-10	9	5	45	$9-12=-3$	3	15
11-13	12	4	48	$12-12=0$	0	0
14-16	15	9	135	$15-12=+3$	3	27
	Totals:	$\sum f = 20$	$\sum fx = 240$			$\sum f\lvert x-\bar{x}\rvert = 54$

3. The frequency distribution shows the marks scored by 30 pupils on a Maths exam.

Marks (%)	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	2	4	5	8	7	3	1

a. Draw a cumulative frequency curve for the distribution.
b. Use the graph to find the 45th percentile.

c. If students must score more than 68% to pass, use the graph to find the probability that a student chosen at random passed the test.

Solutions:

a. Before drawing the cumulative frequency curve, complete a cumulative frequency table:

Pupils' Marks			
Marks	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
30 – 39	2	39.5	2
40 – 49	4	49.5	2+4=6
50 – 59	5	59.5	6+5=11
60 – 69	8	69.5	11+8=19
70 – 79	7	79.5	19+7=26
80 – 89	3	89.5	26+3=29
90 – 99	1	99.5	29+1=30
Total	30		

See curve below.

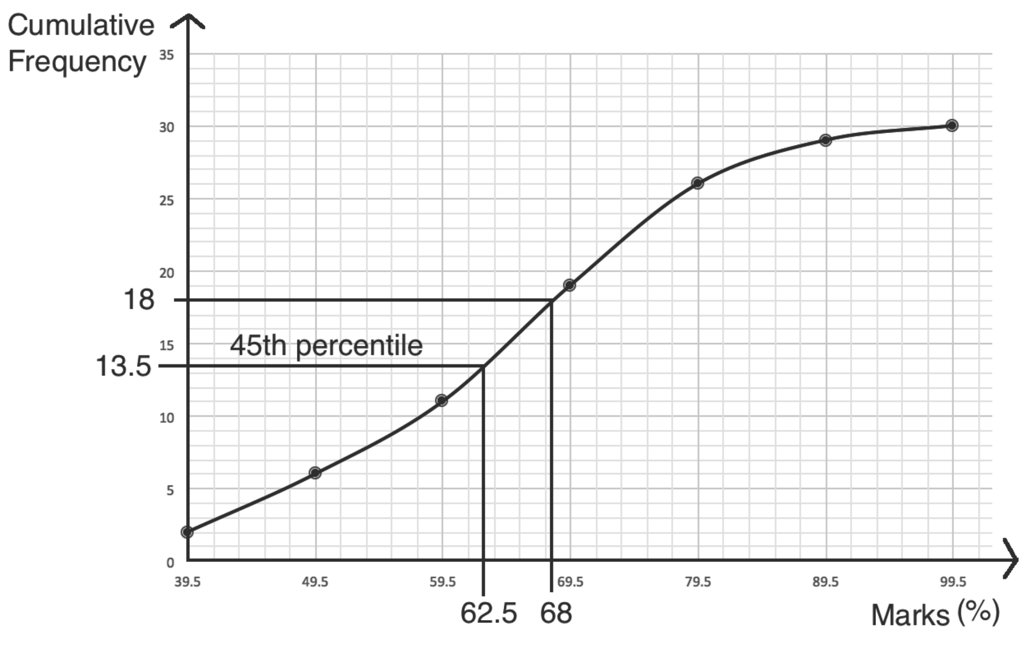
b. Find the position of the 45th percentile: $\frac{n}{100}\sum f = \frac{45}{100}(30) = \frac{1350}{100} = 13.5$

Identify the 45th percentile on the curve as 62.5 marks. (see curve below).

c. To identify the number of pupils scoring above 68%, first identify 68 marks on the c.f. curve. 68 marks corresponds to a cumulative frequency of 18. If 18 pupils scored 68 or lower, then the number that passed is $30 - 18 = 12$.

Probability that a student passed $= \frac{\text{passing students}}{\text{all students}} = \frac{12}{30} = 0.4$.

Curve:



4. The weights of 8 babies in kilogrammes are: 3.5, 5.0, 5.5, 3.0, 4.0, 3.5, 4.5, and 3.0. Calculate:
   a. The mean.
   b. The standard deviation.
   c. The probability that a baby chosen at random will weigh at least 4 kilogrammes.

   Solutions:

   a. Mean: $\bar{x} = \frac{3.5+5+5.5+3.0+4.0+3.5+4.5+3.0}{8} = \frac{32}{8} = 4$ kg
   b. Standard deviation: $s = \sqrt{\frac{1}{n}\sum(x-\bar{x})^2} = \sqrt{\frac{6}{8}} = \sqrt{0.75} \approx 0.87$

x	$x - \bar{x}$	$(x - \bar{x})^2$
3.5	$3.5 - 4 = -0.5$	0.25
5.0	$5 - 4 = +1$	1
5.5	$5.5 - 4 = +1.5$	2.25
3.0	$3 - 4 = -1$	1
4.0	$4 - 4 = 0$	0
3.5	$3.5 - 4 = -0.5$	0.25
4.5	$4.5 - 4 = +0.5$	0.25
3.0	$3 - 4 = -1$	1
Total	$= \sum(x-\bar{x})^2 =$	6

   c. Probability $= \frac{\text{babies 4.0 kg and heavier}}{\text{all babies}} = \frac{4}{8} = 0.5$

Practice

1. The following are the ages of 20 football players: 25, 22, 23, 26, 27, 23, 24, 24, 25, 26, 27, 26, 22, 24, 25, 21, 22, 23, 25, 27.
   a. Construct a frequency table for the grouped data, without class intervals.
   b. Calculate, correct to 2 decimal places: i. The mean; ii. The standard deviation.
   c. Find the probability that a football player selected at random is under 25 years old.
2. The table below gives the scores of 20 pupils on an exam.

Score	0-4	5-9	10-14	15-19
Frequency (f)	2	6	7	5

   Calculate:

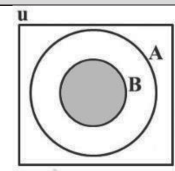
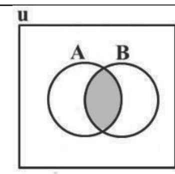
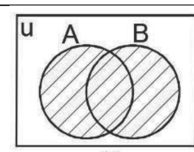
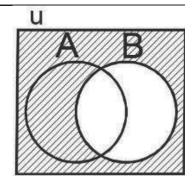
   a. The mean score.
   b. The mean deviation.
   c. The probability that a pupil selected at random scored 10 or more points.

Lesson Title: Sets	**Theme:** Review
Practice Activity: PHM3-L121	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems on sets.

Overview

Problems on sets may involve 2 or 3 sets. It is important to know common notation for sets, and how they are displayed using Venn diagrams. These are shown below.

Characteristic	Explanation	Notation	Reading	Diagram
Subset	If every element in set B is present in set A, then B is a subset of A.	$B \subset A$	"set B is a subset of set A"	
Intersection	The intersection of 2 sets A and B is the element(s) that is/are common to both sets A and B.	A ∩ B	"A intersection B"	
Union	The union of A and B is the set formed by combining the elements in both sets.	$\mathrm{A} \cup \mathrm{B}$	"A union B"	
Complement	If B is a subset of the universal set U, then the complement of B is the set of members which belong to U but not to B.	B'	"B prime"	

The "cardinality of a set" or the "cardinal number of a set" is the number of elements in the set. It is denoted with the letter n and brackets. For example, consider $n(A) = 4$. This statement says that the set A has 4 elements.

When we have two sets A and B then:

- $n(\mathrm{A} \cup \mathrm{B})$ is the number of elements present in either of the sets A or B.
- $n(\mathrm{A} \cap \mathrm{B})$ is the number of elements present in both the sets A and B.
- For two sets A and B: $n(\mathrm{A} \cup \mathrm{B}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cap \mathrm{B})$

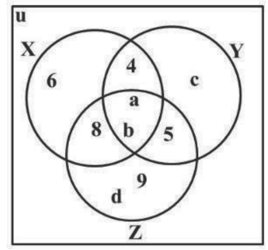
This formula may be needed for questions that appear on the WASSCE exam.

Solved Examples

1. For the sets X = {4, 6, a, b, 8}, Y = {4, a, b, c, 5} and Z = {5, a, b, d, 8, 9},
   a. Find X∩Y, Y∩Z, X∩Z, and X∩Y∩Z.
   b. Illustrate your answer with a Venn diagram.

   Solutions:

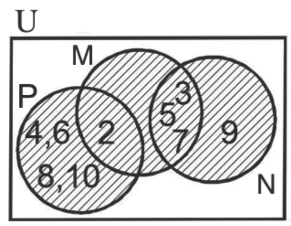
   a. $X \cap$ Y = {4, a, b}; Y∩ Z = {5, a, b}; X∩Z = {8, a, b} and X∩Y∩Z = {a, b}
   b. See Venn diagram at right.



2. Given the sets M = $\{x: x$ is a prime number up to 10}, N = $\{x: x$ is an odd positive integer up to 10} and P = $\{x: x$ is an even positive integer up to 10}:
   a. List the elements of sets M, N and P
   b. Find the union of sets M, N and P.
   c. Show the union of the sets on a Venn diagram

   Solutions:

   a. M = {2, 3, 5, 7}, N = {3, 5, 7, 9} and P = {2, 4, 6, 8, 10}
   b. M∪N∪P = {2, 3, 4, 5, 6, 7, 8, 9, 10}
   b. See the Venn diagram at right.



3. If U = $\{-3 \leq x < 3\}$ and A = $\{-1 \leq x \leq 2\}$, list the elements of U, A and A'.

   Solution:

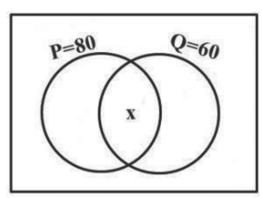
   From the problem, U = $\{-3, -2, -1, 0, 1, 2\}$ and A = $\{-1, 0, 1, 2\}$. A' is the set of elements in U but not A, so A'= $\{-3, -2\}$.

4. If A and B are two finite sets such that n(A) = 32, n(B) = 18 and n(A∪B) = 42, find n(A∩B).

   Solution:

   n(A∪B) = n(A) + n(B) – n(A∩B)
   n(A∩B) = n(A) + n(B) – n(A∪B)
   = 32+18 – 42
   = 50 – 42 = 8

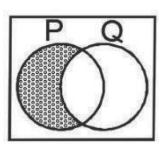
5. In the Venn diagram, set P has 80 members, set Q has 60 members while there are members in P∩Q and 120 members in P∪Q. Use the Venn diagram to answer the following questions:



   a. Shade the region which shows P only.
   b. Find the number of members in P∩Q.
   c. Find the number of members in Q only.

   Solutions:

   a. See Venn diagram at right.



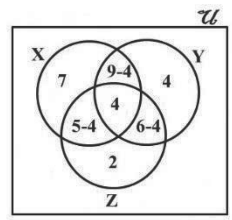
   b. P∩Q = x
      Region for P only = $80 - x$
      Region for Q only = $60 - x$

Add the three regions and equate to n(P∪Q)= 120

$$\begin{aligned} 80 - x + 60 - x + x &= 0 \\ 80 + 60 - x &= 0 \\ 140 - x &= 0 \\ x &= 0 - 120 = 20 \end{aligned}$$

c. Members in Q only $= 60 - x = 60 - 20 = 40$

6. Given three sets X, Y and Z is such that n(X∩Y∩Z) = 4, n(X∩Y) = 9, n(X∩Z) = 5, n(Y∩Z) = 6, n(X∩Y'∩Z') = 7, n(X'∩Y'∩Z) = 2 and n(X'∩Y∩Z') = 4,
    a. Draw the Venn diagram
    b. Find: i. n(X)   ii. n(Y)   iii. n(Z)

    Solutions:
    a. See Venn diagram on the right.
    b. i. Find n(X) by adding all elements in set X:
    n(X) = 7 + (9 – 4) + 4 + (5 – 4) = 7 + 5 + 4 + 1 = 17
    ii. Find n(Y) by adding all elements in set Y:
    n(Y) = 4 + (9 – 4) + (6 – 4) + 4 = 4 + 5 + 2 + 4 = 15
    iii. Find n(Z) by adding all elements in set Z:
    n(Z) = 4 + (5 – 4) + 2 + (6 – 4) = 4 + 1 + 2 + 2 = 9

Practice

1. Represent the relationship between the following set B and its subsets V and C on a Venn diagram: B = {x: x is an alphabet between a and f}, V = {x: x is a vowel} and C = {x: x is a consonant}.
2. Consider the following sets, where U is the universal set: U = {2, 4, 6, 8...20}, A = {multiples of 4 less than 20}, B = {even numbers less than 14} and C = {8, 12, 18, 16, 20}.
    a. Find (i) A∩B, (ii) B∩C, (iii) A∩C, (iv) A∩B∩C
    b. Draw a Venn diagram to illustrate A∩ B∩ C
3. In a school of 120 students, 70 are in the Lawn tennis team and 80 are in the Volleyball team. Each student is on at least one team.
    a. Illustrate this information on a Venn diagram.
    b. Find how many students are in both teams.
    c. Find how many students are in Lawn tennis team only.
4. In a class of 49 pupils, 27 offer French and 32 offer Krio. Nine pupils do not offer any of the languages.
    a. Draw a Venn diagram to illustrate this information.
    b. How many stunts offer: i. Both languages; ii. Only French; iii. Only Krio

Lesson Title: Indices and logarithms	**Theme:** Review
Practice Activity: PHM3-L122	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems on indices and logarithms.

Overview

Numbers written with a base and a power are written in **index form**. For example, 7^3 is in index form, and can be written as $7^3 = 7 \times 7 \times 7$. Seven is the **base** and three is the **power** or **index**. $\text{base} \rightarrow 7^{3 \leftarrow \text{power/index}}$

The **laws of indices** are applied when simplifying or solving problems. These are:

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $a^0 = 1$
4. $(a^x)^y = a^{xy}$

When numbers are raised to negative powers, the expression can be written as a fraction. A number (a) raised to a negative index ($-n$) is written as: $a^{-n} = \frac{1}{a^n}$.

The square root of a number can be rewritten as a number raised to a fractional index. For example, $\sqrt{4} = 4^{\frac{1}{2}}$. The general form is $\sqrt[n]{x} = x^{\frac{1}{n}}$.

A number can be raised to a fractional power with an integer numerator other than 1, such as $5^{\frac{2}{3}}$. The numerator is treated as a power, and the denominator is the root. This can be written as $5^{\frac{2}{3}} = \sqrt[3]{5^2}$.

When fractions are raised to powers, remember to distribute the power both to the numerator and denominator. The general rule is $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$. If there is a negative power on a fraction, take the reciprocal of the fraction and raise it to the same **positive** index. The general rule is $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ provided $a \neq 0$ and $b \neq 0$.

Logarithms and indices are related. Logarithms have the form $\log_b y = x$, where b is the base. $\log_b y = x$ can be read "log to base b of y equals x". Each equation written as a logarithm has an equivalent equation written as an index.

This relationship is shown below:

$y = b^{x}$ ← power; ↑ base

$\log_{b} y = x$ ← power; ↑ base

The following are **laws of logarithms:**

1. $\log_{10} pq = \log_{10} p + \log_{10} q$
2. $\log_{10} \left(\frac{p}{q}\right) = \log_{10} p - \log_{10} q$
3. $\log_{10}(P)^{n} = n \log_{10} P$

There are some other basic rules to keep in mind when simplifying logarithms. The logarithm of any number to the base of the same number is 1, that is $\log_{a} a = 1$. The logarithm of 1 to any base is 0, that is $\log_{a} 1 = 0$.

Solved Examples

1. Simplify the following expressions:

   a. $10x^{4}y \times 2x^{3}y^{2}$

   b. $\frac{75a^{2}b^{-2}}{5a^{3}b^{-3}}$

   c. $8b^{5}a^{3} \div 4b^{3}a$

   d. $(n^{-3})^{4} \times n^{7}$

   e. $\frac{24xy^{5}}{6xy^{4}}$

   f. $125^{\frac{2}{3}}$

Solutions:

a. $$\begin{aligned}10x^{4}y \times 2x^{3}y^{2} &= 10 \times 2 \times x^{4} \times x^{3} \times y \times y^{2}\\ &= 20 \times x^{4+3} \times y^{1+2}\\ &= 20x^{7}y^{3}\end{aligned}$$

b. $$\begin{aligned}\frac{75a^{2}b^{-2}}{5a^{3}b^{-3}} &= 75a^{2}b^{-2} \div 5a^{3}b^{-3}\\ &= (75 \div 5)(a^{2} \div a^{3})(b^{-2} \div b^{-3})\\ &= 15a^{2-3}b^{-2-(-3)}\\ &= 15a^{-1}b^{-2+3}\\ &= 15a^{-1}b^{1}\\ &= \frac{15b}{a}\end{aligned}$$

c. $$\begin{aligned}8b^{5}a^{3} \div 4b^{3}a &= (8 \div 4)b^{5-3}a^{3-1}\\ &= 2b^{2}a^{2}\end{aligned}$$

d. $$\begin{aligned}(n^{-3})^{4} \times n^{7} &= n^{-3\times4} \times n^{7}\\ &= n^{-12} \times n^{7}\\ &= n^{-12+7}\\ &= n^{-5}\\ &= \frac{1}{n^{5}}\end{aligned}$$

e. $$\begin{aligned}\frac{24xy^{5}}{6xy^{4}} &= 24xy^{5} \div 6xy^{4}\\ &= (24 \div 6)x^{1-1}y^{5-4}\\ &= 4x^{0}y^{1}\\ &= 4y\end{aligned}$$

f. $$\begin{aligned}125^{\frac{2}{3}} &= \sqrt[3]{125^{2}} = (\sqrt[3]{125})^{2} = 5^{2}\\ &= 25\end{aligned}$$

2. Solve the following expressions for x: a. $3^{x+1} = 81$ b. $10^{2x-2} = \frac{1}{10,000}$

Solutions:

$$\begin{aligned}3^{x+1} &= 81\\ 3^{x+1} &= 3^{4}\\ x+1 &= 4\end{aligned}$$

$$\begin{aligned}10^{2x-2} &= \frac{1}{10,000}\\ 10^{2x-2} &= (10,000)^{-1}\\ 10^{2x-2} &= (10^{4})^{-1}\end{aligned}$$

$$x = 4 - 1$$
$$x = 3$$

$$10^{2x-2} = 10^{-4}$$
$$2x - 2 = -4$$
$$2x = -4 + 2$$
$$2x = -2$$
$$x = -1$$

3. Solve for the variable in each of the following: a. $x = \log_3 9$
   b. $\log_{10} 0.0001 = p$ c. $y = \log_3 \sqrt{27}$

   Solutions:

   a.
   $$x = \log_3 9$$
   $$9 = 3^x$$
   $$3^2 = 3^x$$
   $$2 = x$$

   b.
   $$\log_{10} 0.0001 = p$$
   $$0.0001 = 10^p$$
   $$\frac{1}{10{,}000} = 10^p$$
   $$\frac{1}{10^4} = 10^p$$
   $$10^{-4} = 10^p$$
   $$-4 = p$$

   c. Let $\log_3 \sqrt{27} = y$
   $$\sqrt{27} = 3^y$$
   $$(3^3)^{\frac{1}{2}} = 3^y$$
   $$3^{\frac{3}{2}} = 3^y$$
   $$y = \frac{3}{2} = 1\frac{1}{2}$$

4. Simplify: a. $\frac{\log_a 8 + \log_a 16 - \log_a 2}{\log_a 32}$ b. $\frac{\log_2 8^3}{\log_2 8}$ c. $\log_3 15 + \log_3 9 - \log_3 5$

   Solutions:

   a. $\frac{\log_a 8 + \log_a 16 - \log_a 2}{\log_a 32} = \frac{\log_a\left(\frac{8 \times 16}{2}\right)}{\log_a 32} = \frac{\log_a 64}{\log_a 32} = \frac{\log_a 2^6}{\log_a 2^5} = \frac{6(\log_a 2)}{5(\log_a 2)} = \frac{6}{5} = 1\frac{1}{5}$

   b. $\frac{\log_2 8^3}{\log_2 8} = \frac{3(\log_2 8)}{(\log_2 8)} = 3$

   c. $\log_3 15 + \log_3 9 - \log_3 5 = \log_3\left(\frac{15 \times 9}{5}\right) = \log_3 27 = \log_3 3^3 = 3(\log_3 3) = 3 \times 1 = 3$

Practice

1. Simplify the following: a. $9a^3b^4 \times 3ab^2$ b. $\frac{45mn^3}{9mn^2}$
2. Simplify: a. $(2m^3n^4)^2$ b. $(4x^3y)^2 \div (2xy^2)^2$ c. $v^{-7} \div v^{-7}$
3. Simplify: a. $125^{\frac{1}{3}}$ b. $729^{\frac{2}{3}}$
4. Simplify: a. $25^{\frac{1}{2}} \times 5^{\frac{1}{2}} \times 625^{\frac{1}{4}}$ b. $\left(\frac{125}{27}\right)^{\frac{1}{3}} \times \left(\frac{4}{9}\right)^{-\frac{1}{2}}$
5. Solve for x in each expression: a. $2^{x+1} = 2^5$ b. $10^{2x-1} = 10^3$ c. $5^x = \frac{1}{125}$
6. Solve for the variable: a. $y = \log_3 27$ b. $\log_3 \frac{1}{243} = x$.
7. Simplify: a. $\log_4 8 + \log_4 8 + \log_4 2$ b. $\log_3 9 + \log_3 2 + \log_3 2$
8. Expand: a. $\log_7 5\frac{2}{3}$ b. $\log_9 7\frac{4}{5}$
9. Simplify: a. $\log_4 32 - \log_4 8$ b. $\log_3 48 - \log_3 27$
10. Simplify: a. $\frac{\log_{10} 625}{\log_{10} 25}$ b. $\frac{\log_{10} a^4 - \log_{10} a^2}{\log_{10} a^3}$

Lesson Title: Sequences and series	Theme: Review
Practice Activity: PHM3-L123	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems on sequences and series.

Overview

Arithmetic Sequences

A sequence in which the terms either increase or decrease by a common difference is an arithmetic sequence, or arithmetic progression (AP).

Consider the sequence: 5, 7, 9, 11, 13, …. The first term is 5 and the **common difference** is 2. The common difference is added to each term to find the next term.

The letter a is commonly used to describe the first term, and the letter d is used for common difference. This formula is used to find the nth term of an AP: $U_n = a + (n-1)d$, where U_n is the nth term of the AP.

Geometric Sequences

A sequence in which the terms either increase or decrease by a common ratio is a geometric sequence, or geometric progression (GP).

Consider the sequence: 1, 2, 4, 8, 16, 32, …. The first term is 1 and the **common ratio** is 2. The common ratio is multiplied by each term to get the next term.

The letter a is commonly used to describe the first term, and the letter r is used for a common ratio. This formula is used to find the nth term of a GP: $U_n = ar^{n-1}$, where U_n is the nth term of the GP.

Note these special cases for GPs:

- If the numbers decrease as the GP progresses, the common ratio must be a fraction. (Example: The sequence$32, 16, 8, 4, ...$, where $r = \frac{1}{2}$.)
- If the numbers alternate between positive and negative digits, the common ratio must be a negative value. (Example: The sequence $1, -2, 4, -8, 16, ...$, where $r = -2$)

Series

When the terms of a sequence are added together, the result is a series. Some sequences are infinite, and carry on forever. It is impossible to find the sum of an infinite sequence, but we can find the sum of the first n terms. Finite sequences have a certain number of terms, and end at a certain point. It is always possible to find the sum of a finite sequence.

The following formula is used to find the sum of the first n terms of an AP: $S_n = \frac{1}{2}n[2a + (n-1)d$, where n is the number of terms, a gives the first term, and d is the common difference.

There are 2 formulae for finding the sum of a GP. The following formula is used to find the sum of the first n terms of a GP when r is a fraction $|r| < 1$: $S_n = \frac{a(1-r^n)}{1-r}$.

The following formula is used to find the sum when $|r| > 1$: $S_n = \frac{a(r^n-1)}{r-1}$.

In both formulae, the variable n is the number of terms, a is the first term, and r is the common ratio.

Solved Examples

1. Find the 7th term of the sequence. $24, 19, 14, 9, \ldots$
   Solution:
   The first term is $a = 24$, the common difference is $d = -5$. Apply the formula for AP:

U_n	$=$	$a + (n-1)d$	
U_7	$=$	$24 + (7-1)(-5)$	Substitute for a, d and n
	$=$	$24 + 6 \times -5$	Clear the brackets
	$=$	-6	Simplify

2. The fifth term of a sequence is 27, and the common difference is 5. Find the eighth term.
   Solution:
   This is an AP, because it has a common difference. Find the value of a, and use it to find the eighth term.
   Step 1. Find a:
   Given $U_5 = 27$ and $d = 5$,

U_n	$=$	$a + (n-1)d$	
27	$=$	$a + (5-1)(5)$	Substitute $U_5 = 27, n = 5, d = 5$
27	$=$	$a + 20$	Clear the brackets
a	$=$	7	Solve for a

   Step 2. Find the eighth term:

U_8	$=$	$7 + (8-1)5$	Substitute $a = 7, d = 5, n = 8$
U_8	$=$	42	Clear the brackets and simplify.

   The eighth term is 42.

3. Find the eighth term of the sequence $6, -3, \frac{3}{2}, -\frac{3}{4}, \ldots$
   Solution:
   This is a geometric progression with common ratio $r = -\frac{1}{2}$. Apply the formula for GP:

U_n	$=$	ar^{n-1}	
U_8	$=$	$6 \times \left(-\frac{1}{2}\right)^{8-1}$	Substitute $n = 8, r = -\frac{1}{2}, a = 6$

$= \quad 6 \times \left(-\frac{1}{2^7}\right)$ Simplify

$= \quad -\frac{6}{128}$

$= \quad -\frac{3}{64}$

The eighth term is $-\frac{3}{64}$.

4. Find the sum of the first nine terms of the AP: $12 + 11\frac{1}{2} + 11 + 10\frac{1}{2} + \cdots$

   Solution:

   Given: $a = 12, d = -\frac{1}{2}, n = 9$

   $S_n \quad = \frac{1}{2}n[2a + (n-1)d]$

   $S_9 \quad = \frac{1}{2}(9)[2(12) + (9-1)(-\frac{1}{2})]$ Substitute n, a, and d

   $= \left(\frac{9}{2}\right)\left[24 - \frac{8}{2}\right]$ Simplify

   $= \left(\frac{9}{2}\right)[24 - 4]$

   $= \left(\frac{9}{2}\right)(20)$

   $= 90$

   The sum of the first nine terms is 90.

5. Find the sum of the first eight terms of the series $4 + 12 + 36 + 108 + \cdots$

   Solution:

   The series is a GP with common ratio of 3.

   Given: $a = 4, r = 3, n = 8$

   $S_n \quad = \frac{a(r^n - 1)}{r - 1}$

   $S_8 \quad = \frac{4(3^8 - 1)}{3 - 1}$ Substitute n, a, and r

   $= \frac{4}{2}(3^8 - 1)$

   $= (2)(6561 - 1)$ Simplify

   $= 13{,}120$

   The sum of the first eight terms is $13{,}120$.

Practice

1. Find the sixth term of the sequence 15, 21, 27, 33, …
2. Find the seventh term of the arithmetic sequence 3, -2, -7, -12, …
3. Find the number of terms in the arithmetic sequence 7,11,15, … , 79
4. Find the 8^{th} term of the geometric sequence, $3, 9, 27, 81, \ldots$
5. Write down the 9^{th} term of the GP $3, -2, \frac{4}{3}, \ldots$
6. Find the number of terms in the geometric sequence 2,4,8, … , 512
7. Find the sum of the first eight terms of the series $25 + 17 + 9 + \ldots$
8. Find the sum of the first five terms of the geometric series $1 + 3 + 9 + \ldots$
9. Find the sum of the first ten terms of the series $6 - 12 + 24 - 48 + \ldots$

Lesson Title: Ratio/Proportion/Rate/ Percentages	**Theme:** Review
Practice Activity: PHM3-L124	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems on ratio, proportion, rate, and percentages.

Overview

Recall that ratio, rate, and percentage were covered in lessons M3-T2-W13-L049 through M3-T2-W18-L072 of this course. This lesson provides a brief review.

- **Ratio:** Used to compare quantities of the same type. Example: 4 girls : 5 boys
- **Rate:** Used to compare quantities of different types. Example: 70 km/hour
- **Proportional division:** Refers to a quantity shared in a given ratio.
- There are many applications of **percentage** in everyday life, including in calculating profit and loss, discount, commission, and so forth.

Recall the following useful information:

- To **increase or decrease a quantity** Q by a ratio $m : n$, use the formula $\frac{m}{n} \times Q$.
- When asked to **compare 2 or more ratios**, write them as unit ratios or with the LCM as the denominator, then compare them.
- To **share something in a given ratio**, find the total number of parts in that ratio by adding all of the numbers in the ratio. Then, find one part of the whole by multiplying by a fraction with the total number of parts in the denominator.
- To find the **percentage of a quantity**, multiply the quantity by the percentage (as a fraction over 100).
- To calculate **percentage change**, use the formula: $\text{percentage change} = \frac{\text{change}}{\text{original quantity}} \times 100$.
- When asked **to increase or decrease a quantity by x%,** take the original quantity as 100%, and increase or decrease it by x. The new quantity is $(100 + x)\%$ of the original, or $(100 - x)\%$ of the original.

Solved Examples

1. Increase Le 40,000.00.00 in the ratio 6 **:** 5.
   Solution:
   To increase in the ratio 6 **:** 5 means that every Le 5.00 is increased to Le 6.00. We know it is an increase because the first part of the ratio is larger than the second part of the ratio.

   New amount $= \frac{6}{5} \times 40{,}000$ Change the ratios to their fraction forms.
   $= \text{Le } 48{,}000.00$

2. Find which of the pair of ratios is greater: 3 : 7 or 2 : 5.

Solution:

Two methods may be used: either convert to unitary ratios, or use the LCM of the denominators to write like fractions.

The LCM of 5 and 7 is 35, so write each ratio as a like fraction:

$$3 : 7 = \frac{3}{7} = \frac{15}{35}$$

$$2 : 5 = \frac{2}{5} = \frac{14}{35}$$

Compare the fractions. Since $\frac{15}{35} > \frac{14}{35}$, the answer is $3 : 7 > 2 : 5$

3. A car travels a distance of 270 km in 3 hours.
   a. What is the average speed in kilometres per hour (km/hr)?
   b. How far will it travel in 10 hours?

Solutions:

a. Convert to a fraction and simplify to a unit rate.

270 km	:	3 hrs	Write as a ratio
rate	=	$\frac{270 \text{ km}}{3 \text{ hrs}}$	Write as a fraction
	=	$\frac{90 \text{ km}}{1 \text{ hr}}$	Write in the form $m : 1$ by dividing numerator and denominator by 3
	=	90 km/hr	Write as a rate in km/hr

b. Multiply the unit rate by the number of hours.

speed	=	$\frac{90 \text{ km}}{1 \text{ hr}}$	90 km in 1 hr
Distance in 10 hours	=	$\frac{90 \text{ km}}{1 \text{ hr}} \times 10$ hrs	
	=	900 km	The hours cancel each other

4. Share Le 150,000.00 between 2 children in the ratio 5 : 7. How much will each child receive?

Solution:

Step 1. Find the total number of parts to the ratio: $5 + 7 = 12$

Note that the ratio 5 : 7 means that for every Le 12.00 of the amount to be shared, Le5.00 will go to Child 1 and Le 7.00 will go to Child 2.

Step 2. Find what proportion (fraction) of the total is given to each part.

Child 1 receives: $\frac{5}{12} \times 150{,}000 = \text{Le } 62{,}500.00$

Child 2 receives: $\frac{7}{12} \times 150{,}000 = \text{Le } 87{,}500.00$

5. If x% of 360 equals 12, find x.

Solution:

Set up an equation for finding percentage of a quantity, and solve for x:

$360 \times \frac{x}{100}$	=	12	Equation
$3.6x$	=	12	Simplify

$$x = \frac{12}{3.6}$$
$$x = 3.3$$

6. M naira invested for 5 years at $x\%$ simple interest per annum yields 0.50M naira interest. Find the value of x.
   Solution:
   Apply the simple interest formula $I = \frac{PRT}{100}$, where I is the interest earned (0.50M), P is the principal (M), R is the rate (x), and T is the time (5).
   $$I = \frac{PRT}{100}$$
   $$0.50M = \frac{Mx5}{100}$$
   $$0.50 = \frac{5x}{100} \qquad M \text{ cancels}$$
   $$0.50 = \frac{x}{20}$$
   $$20 \times 0.50 = x$$
   $$10\% = x$$

7. A trader bought 100 cassava at 5 sets for Le 2,500.00. She sold them in sets of 4 for Le 3,000.00. Find her percentage profit.
   Solution:
   Recall the formulae: percentage profit = $\frac{SP-CP}{CP} \times 100$, where SP is the selling price and CP is cost price.
   Calculate her cost price: $\frac{100}{5} \times 2{,}500 = \text{Le } 50{,}000.00$
   Calculate her selling price: $\frac{100}{4} \times 3{,}000 = \text{Le } 75{,}000.00$
   Percentage profit = $\frac{SP-CP}{CP} \times 100 = \frac{75{,}000-50{,}000}{50{,}000} \times 100 = \frac{1}{2} \times 100 = 50\%$.

Practice

1. Increase a mass of 80 kg in the ratio 5 : 2.
2. A certain shirt costs Le 12,000.00. If the cost decreased at a ratio of 5 : 8, what is the new cost of the shirt?
3. A man invests £4,000.00 for 2 years at compound interest. After one year, his money amounts to £4,160.00. Find the:
   a. Rate of interest
   b. Interest for the second year
4. A vehicle consumes 100 litres of fuel for a distance of 450 km. How many litres of fuel will be consumed for a distance of 1,620 km?
5. A piece of work takes 20 labourers 10 days to complete. How many labourers will complete the same job in 8 days if they work at the same rate?
6. The ratio of the pocket money received by three friends, Momodu, Musa and Amadu is 5 : 3 : 4. If Amadu received Le 1,200.00 in pocket money, find out how much Momodu and Musa received.

Lesson Title: Linear equations	**Theme:** Review
Practice Activity: PHM3-L125	**Class:** SSS 3

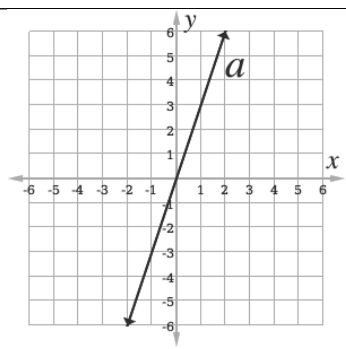
Learning Outcome
By the end of the lesson, you will be able to graph and solve problems on linear equations

Overview

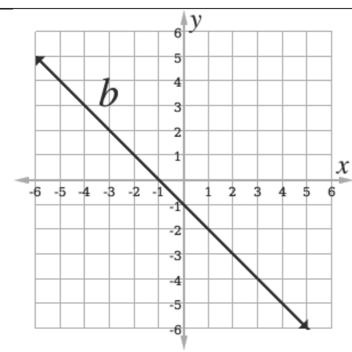
Slope-intercept form of a line is given by $y = mx + c$, where m is the gradient and c is the y-intercept of the line. The y-intercept is the point where the line crosses the y-axis. A line should be written in slope-intercept form before graphing it.

A line can be graphed in 2 common ways. The first way is to find solutions to the linear equation, complete a table of values, and plot each point (see Solved Example 1). The second way is to identify the gradient and y-intercept, and use them to graph the line (see Solved Example 3).

Gradient is a number that tells us in which direction a line increases, and how steep it is. The greater the absolute value of a gradient, the steeper the line is. See the examples below:



- Line a increases as it goes to the **right**, or the positive x-direction. Line a has a **positive** gradient.
- Line a is steeper than line b. It has a gradient of +3.



- Line b increases as it goes to the **left**, or the negative x-direction. Line b has a **negative** gradient.
- Line b is not as steep as line a. It has a gradient of -1.

The gradient of a line can be calculated using any 2 points on the line. It is calculated by dividing the change in y by the change in x between those 2 points.

$$gradient = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x}$$

Gradient m is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ for any 2 points (x_1, y_1) and (x_2, y_2) on a line.

Linear equations can be given by the formula $y - y_1 = m(x - x_1)$ where m is the gradient and (x_1, y_1) is a specific point on the line. (x, y) is a general point on the line.

Using this formula, we can find the equation of a line if we are given **either** of the following:

- The gradient m and a given point (x_1, y_1)
- Two given points (x_1, y_1) and (x_2, y_2)

To find the equation of a line given the gradient and a point, substitute the values of m, x_1, and y_1 into the formula $y - y_1 = m(x - x_1)$. Simplify and write the equation in slope-intercept form.

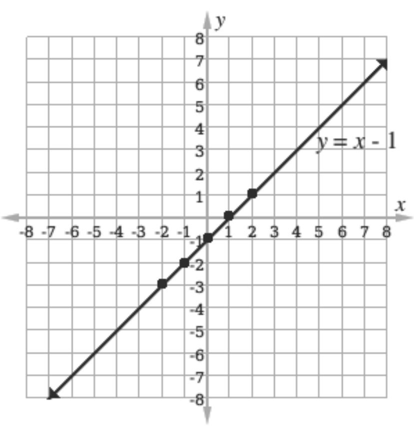
To find the equation of a line given two points, use the formula for gradient ($m = \frac{y_2 - y_1}{x_2 - x_1}$) to find the gradient. Then, follow the process above, using the gradient and **one** of the points on the line.

Solved Examples

1. Complete a table of values for relation $y = x - 1$ for the values of x from -2 to 2. Graph the relation on the Cartesian plane.

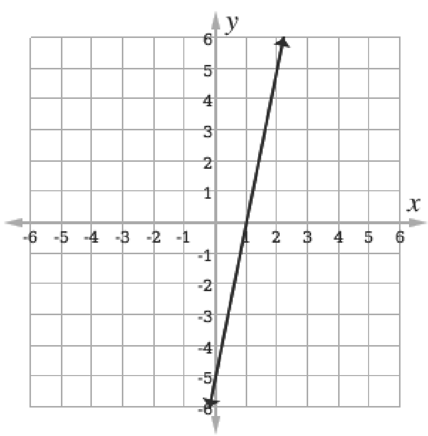
$$\begin{aligned} y &= (-2) - 1 \\ &= -3 \\ y &= (-1) - 1 \\ &= -2 \\ y &= (0) - 1 \\ &= -1 \\ y &= (1) - 1 \\ &= 0 \\ y &= (2) - 1 \\ &= 1 \end{aligned}$$

x	-2	-1	0	1	2
y	-3	-2	-1	0	1



2. Calculate the gradient of the line shown at right.
Solution:

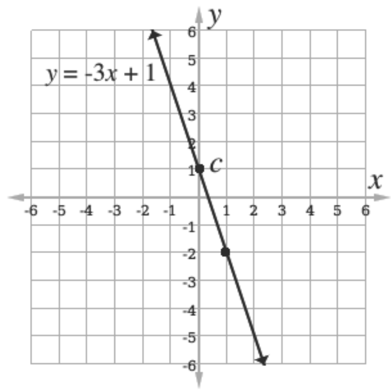
Choose any 2 points on the line. For example: $(1, 0)$ and $(2, 5)$. Use these points to calculate m: $m = \frac{5-0}{2-1} = \frac{5}{1} = 5$



3. Graph $y = -3x + 1$ without creating a table of values.

   Solution:

   - Identify and plot the y-intercept, $c = 1$.
   - Using $m = -3 = \frac{-3}{1}$, find another point. From c, count **down** 3 units in the y-direction. Count to the right 1 unit in the x-direction. Plot this point $(1, -2)$.
   - Draw a line that passes through $(1, -2)$ and c.
   - Label the line $y = -3x + 1$.



4. Determine the equation of the straight line passing through points $(3, 4)$ and $(1, -2)$. Graph it on the Cartesian plane.

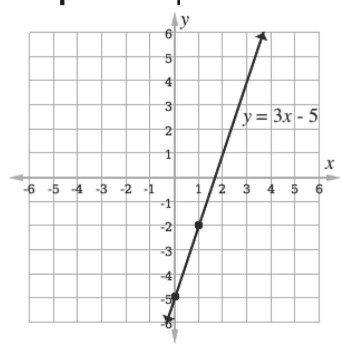
   Solution:

   Step 1. Find the gradient: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-4}{1-3} = \frac{-6}{-2} = 3$

   Step 2. Find the equation:

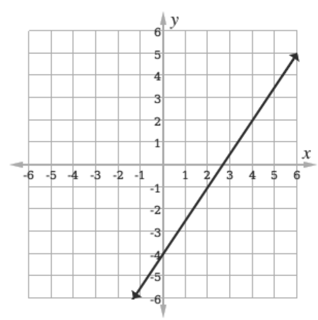
$y - y_1$	$=$	$m(x - x_1)$	Equation of a straight line
$y - 4$	$=$	$3(x - 3)$	Substitute the values
$y - 4$	$=$	$3x - 9$	Simplify
y	$=$	$3x - 9 + 4$	Transpose -4
y	$=$	$3x - 5$	

   Step 3. Graph:



Practice

1. Graph the equation $y + 3x = 4$ using the values of $-2 \leq x \leq 2$.
2. Find the gradient of the line which passes through the points:
   a. $S(2, 3)$ and $T(6, -5)$
   b. $L(0, -4)$ and $M(-3, 0)$
3. Calculate the slope of the line shown:



4. Draw the graph of $y = -2x + 1$.
5. Draw the graph of $y = \frac{1}{2}x + 3$.
6. Write the equation (in slope-intercept form) of the lines with:
   a. Gradient -2 and passing through the point $(-1, -3)$
   b. Gradient 2 and passing through the point $(2, 3)$
7. Find the equation of the line passing through the points $(1, 2)$ and $(-1, -4)$. Graph the line on the Cartesian plane.
8. Find the equation of the line with gradient 2 and passing through the point $(2, 3)$. Then, graph the line on the Cartesian plane.

Lesson Title: Quadratic equations	**Theme:** Review
Practice Activity: PHM3-L126	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to graph and solve problems on quadratic equations

Overview

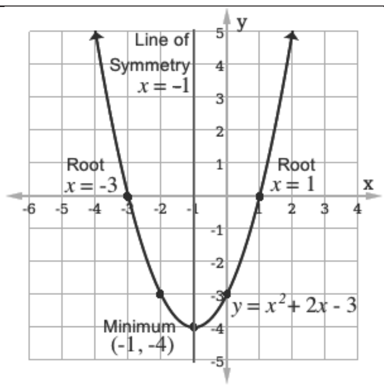
A "quadratic equation" is given in a way that allows you to solve for a variable and does not have a y-value (Example: $x^2 + 4x + 3 = 0$). A "quadratic function" has a y-value and can be graphed (Example: $y = x^2 + 4x + 3$). Quadratic equations and functions have a term with a variable raised to the power 2 (x^2).

Quadratic equations can be solved by graphing the related quadratic function. The graph of a quadratic function is a parabola. The **solutions** (or **roots**) of a quadratic equation are the values of x where the parabola crosses the x-axis.

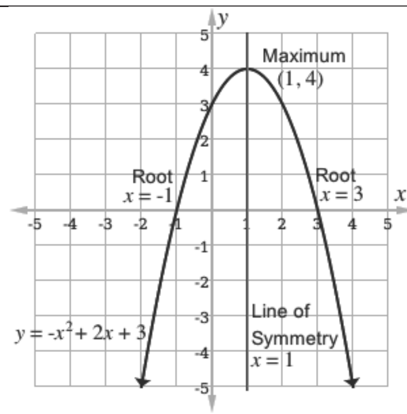
The **maximum** or **minimum** of a quadratic function is the turning point of the graphed parabola. The minimum is the lowest point of a parabola that opens up. The maximum is the highest point of a parabola that opens down. The minimum or maximum can be given as an ordered pair, (x, y).

The **line of symmetry** is a vertical line that passes through the maximum or minimum. It is parallel to the y-axis. It can be written as a linear equation with one variable, x (Example: $x = 2$).

Examples:



- The graph of $y = x^2 + 2x - 3$ opens up. It has a line of symmetry at $x = -1$, and a minimum at $(-1, -4)$.
- The solutions of $x^2 + 2x - 3 = 0$ are the roots, $x = -3$ and $x = 1$.



- The graph of $y = -x^2 + 2x + 3$ opens down. It has a line of symmetry at $x = 1$, and a maximum at $(1, 4)$.
- The solutions of $-x^2 + 2x + 3 = 0$ are the roots, $x = -1$ and $x = 3$.

We can also find the solutions to quadratic equations using several algebraic methods: using factorisation, completing the square, or using the quadratic formula. Factorisation is the algebraic method shown here.

Solved Examples

1. Graph the quadratic function $y = -x^2 + x + 2$ for the interval $-2 \leq x \leq 3$. Use it to answer the following questions:
    a. What is the maximum value of $y = -x^2 + x + 2$?
    b. What is the solution set of the equation $-x^2 + x + 2 = 0$?
    c. What is the equation of the line of symmetry?

Solutions:

Step 1. Graph the function using a table of values with x-values from -2 to 3:

x	-2	-1	0	1	2	3
y	-4	0	2	2	0	-4

$$\begin{aligned} y &= -(-2)^2 + (-2) + 2 \\ &= -4 - 2 + 2 \\ &= -4 \end{aligned}$$

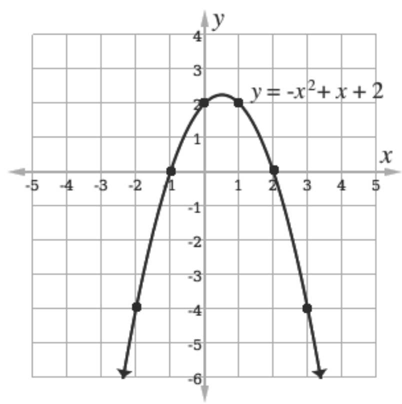
$$\begin{aligned} y &= -(-1)^2 + (-1) + 2 \\ &= -1 - 1 + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y &= -(0)^2 + (0) + 2 \\ &= 0 + 0 + 2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} y &= -(1)^2 + (1) + 2 \\ &= -1 + 1 + 2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} y &= -(2)^2 + (2) + 2 \\ &= -4 + 2 + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y &= -(3)^2 + (3) + 2 \\ &= -9 + 3 + 2 \\ &= -4 \end{aligned}$$



Step 2. Use the graph to answer the questions:

a. The maximum is at $x = 0.5$. To write this as an ordered pair, substitute $x = 0.5$ into the quadratic equation and solve for y:

$$\begin{aligned} y &= -x^2 + x + 2 \\ &= -0.5^2 + 0.5 + 2 \\ &= -0.25 + 0.5 + 2 \\ &= 2.25 \end{aligned}$$

The maximum point is $(0.5, 2.25)$.

b. The solution set consists of the x-values where the parabola crosses the x-axis, $x = -1, 2$.

c. The line of symmetry is the vertical line that passes through the maximum, $x = 0.5$.

2. Solve the quadratic equation $x^2 + 2x - 3 = 0$ using factorisation.

Factorisation:

Factorisation involves rewriting the quadratic equation in the form $x^2 + 2x - 3 = (x + a)(x + b)$, where a and b are numbers. a and b should sum to the coefficient of the second term of the equation, and multiply to get the third term. After finding a and b, set each binomial equal to 0 and solve for x to find the solutions, or roots.

$x^2 + 2x - 3 \;=\; + a)(x + b)$		Set up the equation
$a + b = 2$	$a \times b = -3$	Note that the values of a and b must be 3 and -1 to satisfy these equations
$x^2 + 2x - 3 \;=\; (x + 3)(x - 1)$		Substitute values of a and b
$x + 3 = 0$ $x = -3$	$x - 1 = 0$ $x = 1$	Set each binomial equal to 0 and solve for x. These are the roots.

Practice

Solve the following quadratic equations graphically, using factorisation, and using the quadratic formula. Check that you arrive at the same answer using each method.

1. Graph the function $y = x^2 + 5x - 6$ using the interval $-5 \leq x \leq 1$. From your graph:
   a. Find the solution set of the equation $x^2 + 5x - 6 = 0$.
   b. Write down the minimum of the function.
   c. What is the equation of the line of symmetry?
2. Solve using an algebraic method: $x^2 - 4x - 5 = 0$
3. Solve the following equation by graphing: $x^2 + 2x + 1 = 0$
4. Copy and complete the following table of values for the relation: $y = 10 + 6x - 3x^2$ for $-3 \leq x \leq 5$

x	-3	-2	-1	0	1	2	3	4	5
y		-14		10			1		

   a. Use a scale of 2 cm to 1 unit on the x-axis and 2 cm to 5 units on the y-axis, draw the graph of the relation.
   b. From the graph, solve the equation $10 + 6x - 3x^2 = 0$.
   c. What is the maximum of the function?
   d. Find the equation of the axis of symmetry.
   e. Find the approximate value y when $x = 1.5$.

Lesson Title: Simultaneous equations	**Theme:** Review
Practice Activity: PHM3-L127	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve simultaneous equations using graphical and algebraic methods.

Overview

This lesson covers simultaneous equations where they are both linear, and where one of them is a quadratic equation. Simultaneous equations are 2 equations that are solved at the same time (or simultaneously), and which have the same answer. We must solve for 2 unknown variables (usually x and y), and the two results should satisfy both equations. This lesson covers 2 methods of solving simultaneous equations: substitution, and graphing.

For example, this is a set of simultaneous linear equations: $2x + 5y = -1$ and $8x + 5y = 11$. This is the answer to the simultaneous equations: $x = 2, y = -1$, or $(2, -1)$

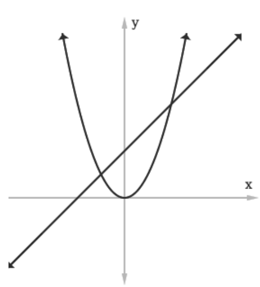
To solve by **substitution**, first change the subject. Choose one of the given equations and make one of the variables the subject of the other one. After changing the subject, we substitute the expression into the other linear equation.

To solve by **graphing,** graph both lines on a plane. The solution is the point where the lines intersect. At this point, the x-value and y-value satisfy both of the equations.

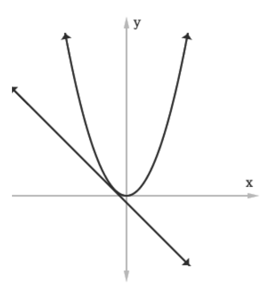
Simultaneous equations may have **a linear and a quadratic equation**. These are simultaneous linear and quadratic equations: $y = x + 2$ and $y = x^2$.

Simultaneous linear and quadratic equations can be solved using substitution or graphing. They can have 0, 1 or 2 solutions. The solutions are ordered pairs, (x, y).

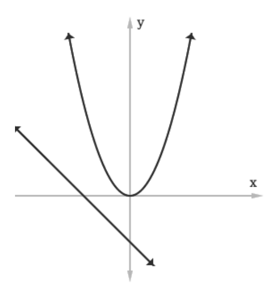
When we graph simultaneous linear and quadratic equations, the intersection points of the curve and line are the solutions to the simultaneous equations.



2 solutions



1 solution



0 solutions

To solve simultaneous linear and quadratic equations using **substitution**, solve one equation for x or y, then substitute it into the other equation. Simplify until it has the form of a standard quadratic equation ($ax^2 + bx + c = 0$).Then, solve the quadratic equation using any method. Factorisation is shown here. Substitute each result into one of the original equations to find the corresponding x- or y-values.

To solve simultaneous linear and quadratic equations by **graphing**, simply graph both relations on the same axes and observe any points of intersection. The solution is written as an ordered pair (x, y).

Solved Examples

1. Five cups and 3 plates cost Le 19,000.00. Four cups and 6 plates cost Le 26,000.00. What is the cost of a cup and a plate?

   Solution:

   Step 1. Identify the unknown variables: c: cost of a cup, p: cost of a plate

   Step 2. Create the linear equations:

   5 cups and 3 plates cost Le 19,000.00: $5c + 3p = 19{,}000$ (1)
   4 cups and 6 plates cost Le 26,000.00: $4c + 6p = 26{,}000$ (2)

   Step 3. Solve by substitution:

$$
\begin{aligned}
c &= 3{,}800 - \tfrac{3}{5}p && (1)\\
4\left(3{,}800 - \tfrac{3}{5}p\right) + 6p &= 26{,}000 && (2)\\
4(19{,}000 - 3p) + 30p &= 130{,}000\\
38{,}000 - 6p + 15p &= 65{,}000\\
9p &= 27{,}000\\
p &= 3{,}000\\
c &= \tfrac{19{,}000}{5} - \tfrac{3}{5}p && (1)\\
c &= \tfrac{19{,}000}{5} - \tfrac{3(3{,}000)}{5}\\
c &= 2{,}000
\end{aligned}
$$

   Answer: The cost is Le 2,000.00 for a cup and Le 3,000.00 for a plate.

2. Solve **the simultaneous equations:**

Equation 1: $y = x^2 - 5x + 7$ and Equation 2: $y - 2x = 1$

Solution:

$(x^2 - 5x + 7) - 2x$	$=$	1	(2)	Substitute equation (1) into equation (2)
$x^2 - 7x + 7$	$=$	1		Simplify
$x^2 - 7x + 7 - 1$	$=$	0		Transpose 1
$x^2 - 7x + 6$	$=$	0		
$(x - 6)(x - 1)$	$=$	0		Factorise the quadratic equation
$x - 6 = 0$	or	$x - 1 = 0$		Set each binomial equal to 0
$x = 6$	or	$x = 1$		Transpose -6 and -1

$y - 2(6) = 1$ Substitute $x = 6$ into equation (2)
$y - 12 = 1$
$y = 1 + 12$ Transpose -12
$y = 13$

$y - 2(1) = 1$ Substitute $x = 1$ into equation (2)
$y - 2 = 1$
$y = 1 + 2$ Transpose -2
$y = 3$

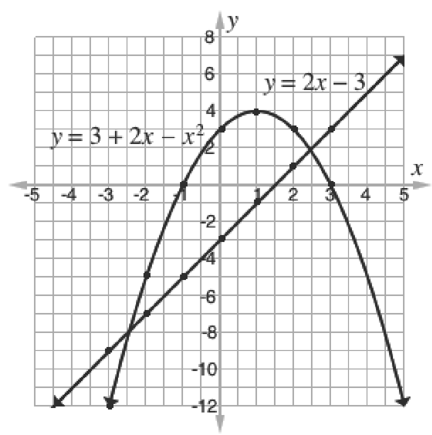
Solutions: $(6, 13)$ and $(1, 3)$

3. Using the scale of 2 cm to 1 unit on the x-axis and 1 cm to 1 unit on the y-axis, draw on the same axes the graphs of $y = 3 + 2x - x^2$ and $y = 2x - 3$ on the interval $-3 \leq x \leq 3$. Using your graph, find the solutions to the simultaneous equations $y = 3 + 2x - x^2$ and $y = 2x - 3$.

   Solution:
   Create a table of values for the graphs on the interval $-3 \leq x \leq 3$.

x	-3	-2	-1	0	1	2	3
$y = 3 + 2x - x^2$	-12	-5	0	3	4	3	0
$y = 2x - 3$	-9	-7	-5	-3	-1	1	3

   Graph the lines on the same axes (graph not to scale):



   The parabola and line intersect at $(-2.5, -8)$ and $(2.5, 2)$. These are the solutions.

Practice

1. Solve the following simultaneous equations by using the substitution method:
   $2x + 3y = 5$ and $x + 2y = 4$
2. Solve the simultaneous equations by graphing: $y = 4x + 3$ and $y + x - 3 = 0$.
3. Solve the equations using substitution: $y = x^2 + 3x + 1$, $y = -2x - 5$
4. Solve the following simultaneous equations by graphing:
   $y = x + 3$ and $y = x^2 - 2x + 3$
5. Four pineapples and 2 mangoes cost Le 30,000.00, 2 pineapples and 6 mangoes cost Le 22,000.00. What is the cost of each pineapple and mango?

Lesson Title: Variation	Theme: Review
Practice Activity: PHM3-L128	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems on variation.

Overview

This lesson covers 4 types of variation: direct, indirect, joint, and partial. Each is described below.

Direct Variation:

Direct variation means that two quantities, x and y, are related such that an increase in one results in an increase in the other in the same ratio. At the same time, a decrease in one results in a decrease in the other in the same ratio. We can also say that x and y are "directly proportional".

Direct variation can be shown with the symbol $\propto$. The statement $x \propto y$ means that x and y are directly proportional. Direct variation can also be shown with the equation $x = ky$, where k is a constant.

Indirect Variation:

Indirect variation means that two quantities x and y are related such that an increase in one results in a decrease in the other. At the same time, a decrease in one results in an increase in the other. We can also say that x and y are "indirectly proportional".

Indirect variation uses the same symbol as direct variation: $\propto$. The statement $x \propto \frac{1}{y}$ means that x and y are indirectly proportional. Indirect variation can also be shown with the equation $x = k\frac{1}{y}$ or $x = \frac{k}{y}$, where k is a constant.

Joint Variation:

Joint variation occurs when a variable varies directly or inversely with multiple variables. For example:

- If x varies directly with both y and z, we have $x \propto yz$ or $x = kyz$.
- If x varies directly with y and inversely with z, we have $x \propto \frac{y}{z}$ or $x = \frac{ky}{z}$.

Joint variation problems can be solved using the same process as other variation problems: Set up the equation and substitute the given set of 3 values to find the constant k. Write the equation with constant k, and substitute the other 2 given values to find the answer.

Partial Variation:

Partial variation occurs when a variable is related to two or more other variables added together. This lesson focuses on the case where a variable (such as x) is partly a constant, and partly varies directly with another variable (such as y).

For example, $x = k_1 + ky$ states that x is partly related to the constant k_1, and varies partly as y. In many cases, you will be asked to determine the relationship between two variables (such as x and y) that are related by partial variation. This involves finding two constants, k_1 and k, using two simultaneous linear equations.

Solved Examples

1. If y varies directly as x and $y = 9$ when $x = 3$, find the value of y when $x = 4$.
   Solution:

y	$\propto$	x	Identify the relationship between y and x
y	$=$	kx	
9	$=$	$k(3)$	Substitute known values for y and x
k	$=$	$\frac{9}{3}$	Solve for the constant, k
k	$=$	3	
y	$=$	$3x$	Write the formula
y	$=$	$3(4)$	Find y when $x = 4$
y	$=$	12	

2. Michael is traveling to Freetown. If he drives at the rate of 60 kph it will take him 4 hours. How long will it take him to reach Freetown if he drives at the rate of 80 kph?
   Solution:
   Speed is inversely proportional to time. If Michael drives faster, it will take less time to reach Freetown. If he drives slower, it will take more time.

s	$\propto$	$\frac{1}{t}$	The relationship between speed and time
s	$=$	$\frac{k}{t}$	
60	$=$	$\frac{k}{4}$	Substitute known values for s and t
k	$=$	60×4	Solve for the constant, k
k	$=$	240	
s	$=$	$\frac{240}{t}$	Write the formula
80	$=$	$\frac{240}{t}$	Substitute $s = 80$
t	$=$	$\frac{240}{80}$	Solve for t
t	$=$	3 hr	

3. z varies directly as the product of x and y. When $x = 3$, $y = 8$, and $z = 6$. Find z when $x = 6$ and $y = 4$.
   Solution:

$$z \propto xy \quad \text{Identify the relationship between } z, x \text{ and } y$$
$$z = kxy$$
$$6 = k(3)(8) \quad \text{Substitute known values for } z, x \text{ and } y$$
$$6 = 24k \quad \text{Solve for the constant, } k$$
$$k = \frac{6}{24} = \frac{1}{4}$$
$$z = \frac{1}{4}xy \quad \text{Write the formula}$$
$$z = \frac{1}{4}(6)(4) \quad \text{Substitute } x = 6 \text{ and } y = 4 \text{ into the formula}$$
$$z = \frac{1}{4}(24) \quad \text{Simplify}$$
$$z = 6$$

4. Fatu hired Mohamed to take her to the market with her goods. His base rate is Le 15,000.00, and he charges an additional Le 1,000.00 per kilometre. If she travelled 7 kilometres, how much did she pay him?

   Solution:

   Set up the equation: $C = k_1 + kd$, which says that cost is partly a constant k_1, and partially varies with distance, d.

   From the problem, we have $k_1 = 15{,}000$, $k = 1{,}000$, and $d = 7$.

$$C = k_1 + kd$$
$$= 15{,}000 + (1{,}000)(7) \quad \text{Substitute } k_1, k, \text{ and } d$$
$$= 15{,}000 + 7{,}000 \quad \text{Simplify}$$
$$= 22{,}000$$

   Answer: Fatu paid him Le 22,000.00.

Practice

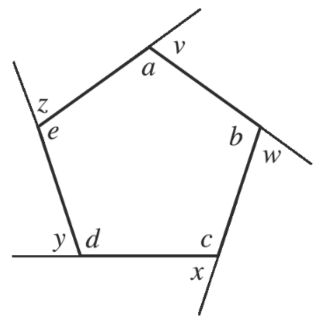
1. If y varies directly as x and $y = 24$ when $x = 8$, find the value of y when $x = 12$.
2. The amount of rice consumed by customers of a hotel varies directly as their number. If 900 kg of rice are needed in a month by 50 customers, how many customers are there if the hotel needs 2,610 kg of rice?
3. If y varies inversely as x, and $y = 4$ when $x = 24$, find:
   a. The formula that connects x and y.
   b. The value of y when $x = 32$.
4. The variable x, y, and z are related by joint variation. When $x = 3$ and $y = 4, z = 36$. Find z when $x = 4$ and $y = 6$ if: a. $z \propto xy$ b. $z \propto \frac{x}{y}$
5. The cost of using mobile internet is partially constant, and varies partially as the number of gigabytes (GB) used. The company charges Le20,000.00 for the first 2 GB, and Le12,000.00 for each GB after that.
   a. Write a formula for the relationship between internet usage and cost.
   b. If Michael uses 12 GB internet, how much will he pay?

Lesson Title: Angles of polygons	**Theme:** Review
Practice Activity: PHM3-L129	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems on the angles of polygons.

Overview

This lesson is on solving problems on angles of a polygon. These can be interior or exterior angles. In the diagram, the following angles are interior angles: a, b, c, d, e. The following angles are exterior angles: v, w, x, y, z.



Exterior angles are formed by extending the sides of the polygon. Every polygon has the same number of interior and exterior angles as it has sides.

This is the formula for finding the sum of the interior angles in a polygon:

$(n - 2) \times 180°$ where n is the number of sides.

The sum of interior angles up to a decagon are given in the table.

Sides	Name	Sum of Interior Angles
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	900°
8	Octagon	1,080°
9	Nonagon	1,260°
10	Decagon	1,440°

We can use the sums to calculate the interior angles of polygons. For regular polygons, all interior angles are equal. There is a formula for finding the measure of each interior angle of a regular polygon: $\frac{(n-2)\times 180°}{n}$ where n is the number of sides.

For polygons that are not regular, missing angles can be found by subtracting known angles from the sum of the angles for that type of polygon.

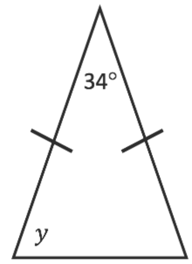
Exterior angles can be found if you know the corresponding interior angle. Note that each exterior angle forms a straight line with an interior angle. This means that the exterior and interior angles sum to 180°.

Unknown exterior angles can also be found if from given exterior angles. All of the exterior angles of **any** polygon sum to 360°.

The exterior angles of a regular polygon are all equal. We can use the following formula to find the measure of each exterior angle: $\frac{360°}{n}$ where n is the number of sides.

Solved Examples

1. Find the missing angle y in the triangle.



Solution:

This is an isosceles triangle. The bottom 2 angles are equal to each other. The measure of either can be found by subtracting 34° from 180°, and dividing the result by 2.

$$180° - 34° = 146°$$
$$y = 146° \div 2 = 73°$$

2. Calculate the sum of the interior angles of a polygon with 15 sides.

Solution:

Substitute $n = 15$ in the formula and solve:

$$\begin{aligned} \text{Sum of angles} &= (n-2) \times 180° \\ &= (15-2) \times 180° \\ &= 13 \times 180° \\ &= 2{,}340° \end{aligned}$$

3. The sum of the interior angles of a polygon is 1,440°. Calculate the number of sides.

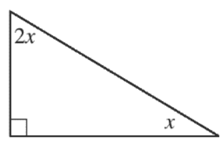
Solution:

This can be solved by substituting 1,440° into the interior angle formula, then solving for n.

$$\begin{aligned} \text{Sum of angles} &= (n-2) \times 180° & \\ 1{,}440° &= (n-2) \times 180° & \text{Substitute the sum} \\ 8 &= n - 2 & \text{Divide throughout by 180°} \\ 8 + 2 &= n & \text{Transpose } -2 \\ n &= 10 & \end{aligned}$$

The polygon has 10 sides. It is a decagon.

4. In the triangle below, find the value of x.



Solution:

Use the fact that the angles of a triangle sum to 180°. Add the given angles, and solve for x.

$$\begin{aligned} 180° &= 90° + 2x + x & \text{Set up equation} \\ 180° &= 90° + 3x & \text{Solve for } x \\ 180° - 90° &= 3x & \\ 90° &= 3x & \\ 30° &= x & \end{aligned}$$

5. Find the interior angle of a regular hexagon.
   Solution:
   Apply the formula and substitute $n = 6$, because a hexagon has 6 sides.

$$\begin{aligned} \text{Interior angle} &= \frac{(n-2)\times180°}{n} \\ &= \frac{(6-2)\times180°}{6} \\ &= \frac{4\times180°}{6} \\ &= \frac{720°}{6} \\ &= 120° \end{aligned}$$

6. The interior angle of a regular polygon is $108°$. Find the number of sides of the polygon.
   Solution:
   Apply the formula for an interior angle, and solve for n:

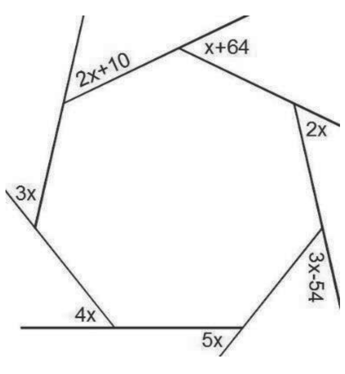
$$\begin{aligned} 108° &= \frac{(n-2)\times180°}{n} \\ 108°n &= (n-2)\times180° \\ 108°n &= 180°n - 360° \\ 108°n - 180°n &= -360° \\ -72°n &= -360° \\ \frac{-72°n}{-72°} &= \frac{-360°}{-72°} \\ n &= 5 \end{aligned}$$

   The polygon has 5 sides. It is a pentagon.

7. Find the exterior angle of a regular hexagon.
   Solution:
   Apply the formula for finding the exterior angle of a regular polygon.
   Exterior angle $= \frac{360°}{n} = \frac{360°}{6} = 60°$

Practice

1. A regular polygon has 20 sides. Find:
   a. The measure of each interior angle.
   b. The measure of each exterior angle.
2. The sum of the interior angles of a regular polygon is 3,600°. Calculate:
   a. The number of sides.
   b. The measure of one exterior angle.
3. Each interior angle of a regular polygon is $140°$. Find the number of sides of the polygon.
4. A regular polygon has exterior angles measuring 30°. How many sides does the polygon have?
5. Find the value of x in the diagram on the right.



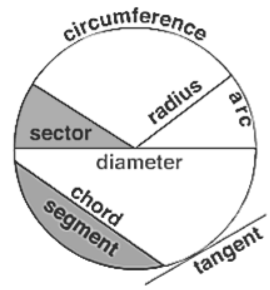
Lesson Title: Circles	**Theme:** Review
Practice Activity: PHM3-L130	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems on circumference and area of circles.

Overview

This lesson is a review lesson on calculating the circumference and area of a circle. From the circumference and area, we are also able to calculate the length of an arc and area of a sector.

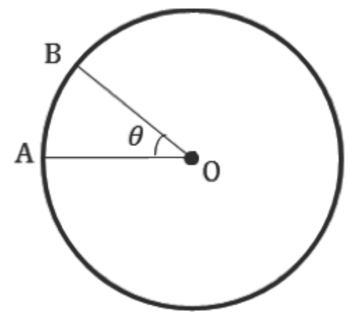
In order to perform calculations on circles, you must have an understanding of its parts. The parts of a circle are shown in the diagram on the right.



The **circumference** of a circle is given by the formula $C = 2\pi r$ where r is the radius of the circle. The **area** of a circle is given by the formula $A = \pi r^2$ where r is the radius of the circle. π is a constant, and we use estimated values of 3.14 or $\frac{22}{7}$ for π.

An **arc** is a part of the circumference of a circle.

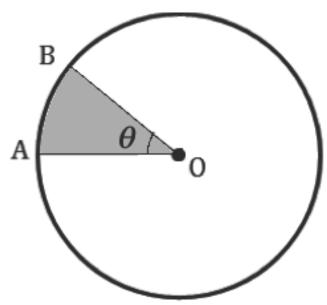
The circle to the right shows an arc AB subtended by an angle θ at the centre, O, of a circle. We know that the angle subtended by the circumference of a circle is $360°$ (a full revolution). The lengths of any arcs of the circle are in proportion to the angles they subtend. In the diagram, the length of arc AB is proportional to θ. To find the length of the arc, multiply the circumference by θ as a fraction of $360°$.



Length of arc $= \frac{\theta}{360°} \times C = \frac{\theta}{360°} \times 2\pi r$

Similarly, a **sector** is part of the area of a circle.

The area of a sector of a circle is proportional to the angle of the sector. The circle to the right shows sector OAB subtended by the angle θ at the centre, O, of a circle. An entire circle is a sector with an angle of $360°$ and area πr^2. All other sectors have areas in proportion to the angle of the sector.



Area of sector $= \frac{\theta}{360°} \times A = \frac{\theta}{360°} \times \pi r^2$

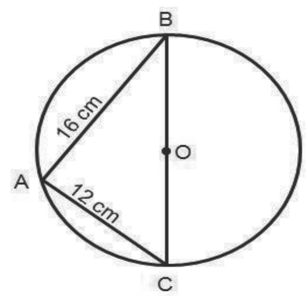
Solved Examples

1. The radius of a circle is 7 cm. Using $\pi = \frac{22}{7}$, find:
   a. The length of its diameter.
   b. The length of its circumference.
   c. The area of the circle.

   Solutions:

   a. Diameter is twice radius: $d = 2r = 2(7) = 14$ cm
   b. Circumference: $C = 2\pi r = 2 \times \frac{22}{7} \times 7 = \frac{308}{7} = 44$ cm
   c. Area: $A = \pi r^2 = \frac{22}{7} \times 7^2 = 22 \times 7 = 154$ cm^2

2. In the diagram at right, triangle ABC is cut out from the circle with centre O. If $|AB| = 16 \text{ cm}$ and $|AC| = 12 \text{ cm}$, find the area of the remaining part of the circle. (Use $\pi = 3.14$)



   Solution:

   To find the area of the remaining part, find the area of the triangle and subtract from the area of the circle. ΔABC is a right angled triangle since it is incribed in a semi-circle.

   Triangle Area $= \frac{1}{2}|\text{AB}| \times |\text{AC}|$

   $= \frac{1}{2} \times 16 \times 12$ Substitute the given sides

   $= 96 \text{ cm}^2$

   To find the area of the circle, you need the radius, which is $\frac{1}{2}$|BC|. Find |BC| Using Pythagoras' theorem:

   $|BC|^2 = |\text{AB}|^2 + |\text{AC}|^2$

   $= 16^2 + 12^2$ Substitute the given sides

   $= 400$

   |BC| $= 20$

   Circle Area $= \pi r^2$

   $= 3.14 \times (10)^2$ Substitute $r = \frac{1}{2}|\text{BC}| = \frac{20}{2} = 10$

   $= 314 \text{ cm}^2$

   Area of the remaining part of the circle = Area of circle – Area of triangle:

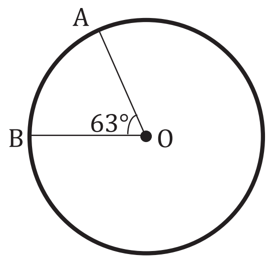
   Area of the remaining part $= 314 - 96$

   $= 218 \text{ cm}^2$

3. An arc subtends an angle of 63° at the centre of a circle with a radius of 12 cm. Using $\pi = \frac{22}{7}$, find the following, correct to 1 decimal place:
   a. The length of the subtended arc.
   b. The area of the sector.

   Solutions:

   a. First draw a diagram of the problem (shown below). Use the formula to find the length of the arc:



$|AB| = \frac{\theta}{360} \times 2\pi r$

$= \frac{63}{360} \times 2 \times \frac{22}{7} \times 12$ Substitute $\theta = 63, r = 12$

$= \frac{63 \times 2 \times 22 \times 12}{360 \times 7}$ Simplify

$|AB| = 13.2$ cm

The length of the arc is 13.2 cm.

b. Use the formula to find the area of the sector:

$A = \frac{\theta}{360} \times \pi r^2$

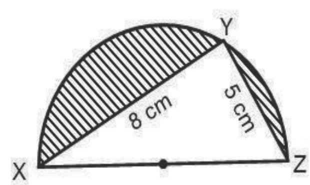
$= \frac{63}{360} \times \frac{22}{7} \times 12^2$ Substitute $\theta = 63°, r = 12$

$= 79.2$ cm^2 Simplify

The area of the sector is 79.2 cm^2

Practice

1. The area of a circular track is $1{,}386 \text{ m}^2$. Find the radius of the track. Use $\pi = \frac{22}{7}$.
2. Find the perimeter of a semi-circle with radius 7 metres. Use $\pi = \frac{22}{7}$.
3. Find the length of an arc with a radius of 12 cm if it subtends an angle of 100° at its centre. Give your answer to 2 significant figures. [Use $\pi = \frac{22}{7}$]
4. The angle of a sector of a circle with a diameter of 30 cm is $50°$. Find the area of the sector, to the nearest whole number. Take $\pi = 3.14$.
5. A sector of a circle with a radius of 13 cm has an area 64.6 cm^2. Calculate the angle of the sector, correct to the nearest degree. Take $\pi = 3.14$.
6. The area of the sector of a circle is 114 cm^2. If the angle of the sector is $108°$, find the radius of the circle to the nearest whole number. [Use $\pi = \frac{22}{7}$]
7. In the diagram below at right, XYZ is a semi-circle. If |XY|= 8 cm and |YZ|= 5 cm, calculate correct to 3 significant figures:
   a. The radius of the circle.
   b. The area of the shaded part. $(\pi = 3.142)$

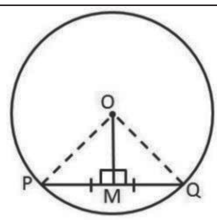
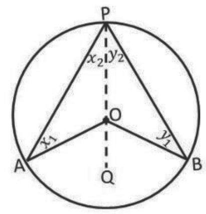
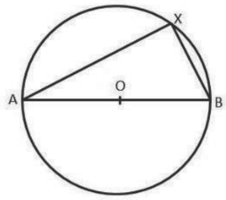
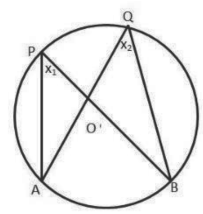
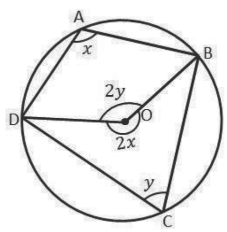
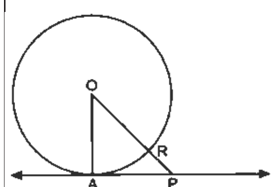
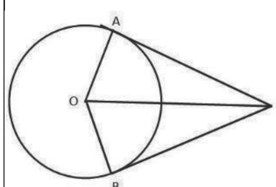
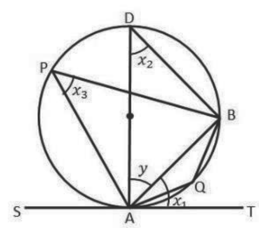


Lesson Title: Circle theorems	**Theme:** Review
Practice Activity: PHM3-L131	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems using circle theorems.

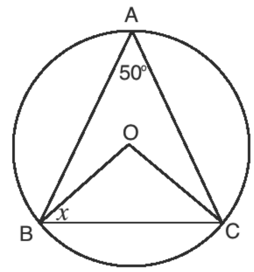
Overview

This lesson is a review lesson on applying the circle theorems to solve problems.

Circle Theorem 1: A straight line from the centre of a circle that bisects a chord is at right angles to the chord. $\lvert PM\rvert = \lvert QM\rvert$ and OM ⊥ PQ.		**Circle Theorem 2:** The angle subtended at the centre of a circle is twice that subtended at the circumference. ∠AOB=2×∠APB.	
Circle Theorem 3: The angle in a semi-circle is a right angle. $\angle AXB$=90°		**Circle Theorem 4:** Angles subtended at the circumference by a chord or arc in the same segment of a circle are equal. $\angle APB$=$\angle AQB$.	
Circle Theorem 5: The opposite angles of a cyclic quadrilateral are supplementary. $\angle BAD + \angle BCD = 180°$ and $\angle ABC + \angle ADC = 180°$.		**Circle Theorem 6:** The angle between a tangent and a radius is equal to $90°$. OA ⊥ l.	
Circle Theorem 7: The lengths of the two tangents from a point to a circle are equal. |TA| = |TB|. Also, ∠AOT=∠BOT and ∠ATO=∠BTO.		**Circle Theorem 8:** The angle between a chord and a tangent at the end of the chord equals the angle in the alternate segment. ∠TAB = ∠APB and ∠SAB = ∠AQB. This is the alternate segment theorem.	

Solved Examples

1. O is the centre of the circle in the diagram. Find the value of x.



Solution:

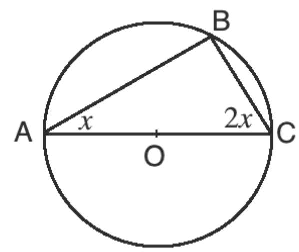
According to circle theorem 2, ∠BOC=2∠BAC. Apply this to find ∠BOC:

$$\angle BOC = 2 \times 50°$$
$$= 100°$$

Note that triangle BOC is isosceles, because 2 sides are the radius, r=|BO|=|OC|. Therefore, $x = \angle OBC = \angle OCB$. Subtract the known angle in the triangle ($100°$) from $180°$, and divide by 2 to find x.

$$x = \frac{180° - 100°}{2}$$
$$= \frac{80°}{2} = 40°$$

2. In the diagram, AC is the diameter of the circle. Find:
a. ∠ABC b. The value of x.



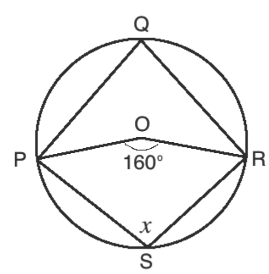
Solutions:

a. According to circle theorem 3, the angle in a semi-circle is a right angle. Therefore, ∠ABC=90°.

b. Because they are angles in a triangle, ∠ABC$+x + 2x = 180°$. Solve this formula for x:

$$90° + x + 2x = 180°$$
$$3x = 180° - 90°$$
$$3x = 90°$$
$$x = \frac{90°}{3}$$
$$x = 30°$$

3. O is the centre of the circle PQRS. Find the measure of x.



Solution:

According to circle theorem 2, ∠POR=2∠PQR. Apply this to find ∠PQR:

$$\angle PQR = \frac{1}{2} \times 160°$$
$$= 80°$$

According to circle theorem 5, opposite angles of a cyclic quadrilateral are supplementary. Thus, ∠PQR$+x = 180°$. Apply this to find x:

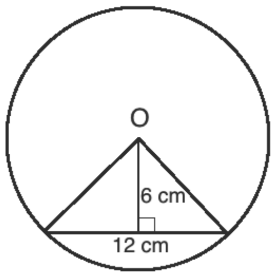
$$x = 180° - \angle PQR$$
$$= 180° - 80°$$
$$= 100°$$

4. A chord of length 12 cm is 6 cm away from the centre of the circle. What is the radius of the circle? Give your answer as a surd.

Solution:

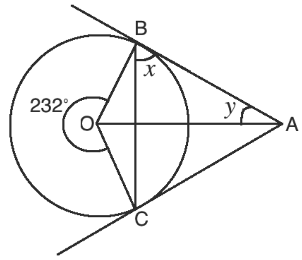
First, draw a diagram. Note that this is an application of circle theorem 1.

Use Pythagoras' theorem to find the radius, which is the hypotenuse of the triangle. Note that the side of the triangle is half the chord, 6 cm.



$$r^2 = 6^2 + 6^2$$
$$r^2 = 72$$
$$r = \sqrt{72} = \sqrt{36 \times 2}$$
$$r = 6\sqrt{2} \text{ cm}$$

5. In the diagram, AB and AC are tangents to the circle, and O is the centre. Find the measures of angles x and y.



Solution:

Note that $\angle BOC = 360° - 232° = 128°$.

Because $\triangle BOC$ is isosceles, $\angle OBC = \angle OCB$. Therefore:

$\angle OBC = \frac{180° - 128°}{2} = 26°$

According to circle theorem 6, tangent AB forms a right angle with radius OB.

Therefore, $x = 90° - 26° = 64°$.

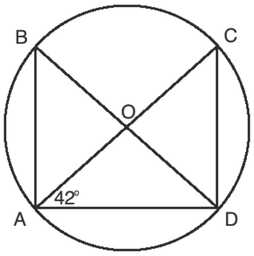
Find angle y using $\triangle ABO$. Note that OA bisects $\angle BOC$. Therefore, $\angle BOA = \frac{1}{2}\angle BOC = \frac{1}{2}128° = 64°$. Subtract the known angles of $\triangle ABO$ from 180° to find the measure of y: $y = 180° - \angle BOA - \angle OBA = 180° - 64° - 90° = 26°$

Answer: $x = 64°$, $y = 26°$

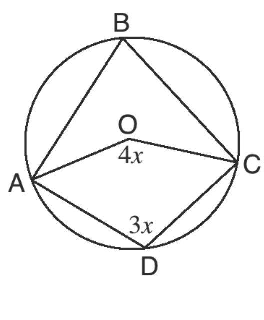
Practice

1. O is the centre of circle $ABCD$. Find:

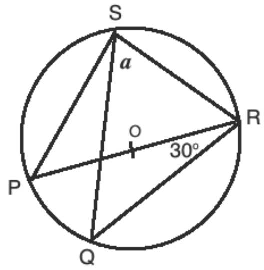
a. $\angle DOC$ b. $\angle ACD$



2. The diagram is a circle with centre O. ABCD are points on the circle. Find the value of $\angle ABC$.



3. Find the measure of a in the diagram:



Lesson Title: Transformations on the Cartesian plane	**Theme:** Review
Practice Activity: PHM3-L132	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to transform figures on the Cartesian plane.

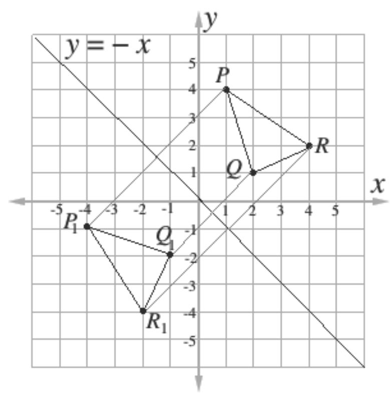
Overview

This lesson is a review of transformations on the Cartesian plane. This topic was covered in lessons 87 through 96 of this course. Recall that 4 types of transformations were covered: translation, reflection, rotation, and enlargement. Refer to the relevant sections of the Pupil Handbook for an overview of each of these.

Solved Examples

1. Triangle PQR has coordinates $P(1,4), Q(2,1)$ and $R(4,2)$. Find the coordinates, P_1, Q_1 and R_1 of the image of the triangle formed under reflection in the line $y = -x$.

   Solution:

   Step 1. Assess and extract the given information from the problem. Given: points $P(1,4), Q(2,1)$ and $R(4,2)$, line $y = -x$

   Step 2. Draw the x- and y- axes. Locate the points P, Q and R on the graph. Draw the lines joining the points.

   Step 3. Draw the line $y = -x$

   Step 4. Draw a line at right angles from P to the mirror line ($y = -x$). Measure this distance.

   Step 5. Measure the same distance on the opposite side of the mirror line ($y = x$) to locate the point P_1 on the graph.

   Step 6. Write the new co-ordinates: $P_1(-4,-1)$, $Q_1(-1,-2)$ and $R_1(-2,-4)$

2. Use the appropriate formula to find the coordinates of the image point when point $X(-3,-2)$ is rotated $90°$ clockwise about the point $(0,-4)$.

   Solution:

   Apply the following formula for rotation:

   $$\begin{pmatrix} x-a \\ y-b \end{pmatrix} \longrightarrow \begin{pmatrix} (y-b)+a \\ -(x-a)+b \end{pmatrix} = \begin{pmatrix} (y-(-4))+0 \\ -(x-0)+(-4) \end{pmatrix} = \begin{pmatrix} y+4 \\ -x-4 \end{pmatrix}$$

   $$\begin{pmatrix} -3 \\ -2 \end{pmatrix} \longrightarrow \begin{matrix} -2+4 \\ (-3)-4 \end{matrix}\Bigg) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$ ply the formula

   $X(-3,-2)$ rotated $90°$ clockwise about the point $(0,-4)$ gives $(2,-1)$

3. Points $A(2,4), B(6,3)$ and $C(3,1)$ are points on the Cartesian plane. Find the coordinates, A_1, B_1 and C_1 of the image of the triangle formed under an anti-clockwise rotation of $90°$ about the origin, O.

Solution:

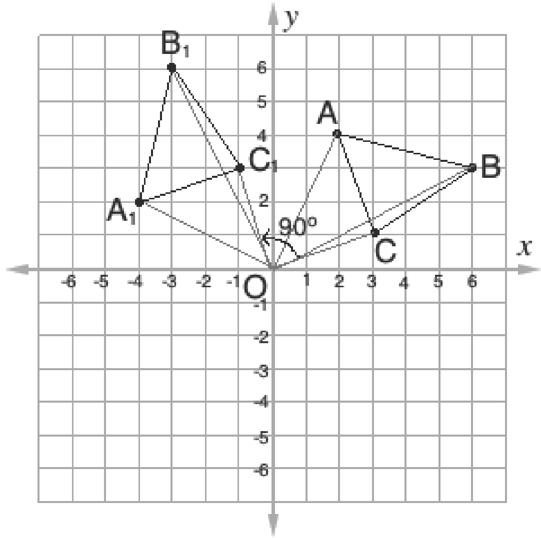
Step 1. Draw the Cartesian plane and locate points A, B and C. Draw lines connecting the points.

Step 2. Mark the centre of rotation $O(0,0)$. Draw a straight line from O to P. Measure the distance OP.

Step 3. Measure an angle of $90°$ in an anti-clockwise direction.

Step 4. Identify A_1 on the plane. Write the co-ordinates of $A_1 = (-4,2)$.

Step 5. Follow the same procedure for points B and C giving $B_1(-3,6)$ and $C_1(-1, 3)$.



4. Find the image of $(1, -3)$ under the enlargement with scale factor of 3 from:

a. The origin   b. The point $(2,4)$

Solutions:

a. Multiply by the scale factor:

$$\begin{pmatrix}1\\-3\end{pmatrix} \longrightarrow \begin{matrix}1\\-3\end{matrix}\Big) \quad = \begin{pmatrix}3\times(1)\\3\times(-3)\end{pmatrix} = \begin{pmatrix}3\\-9\end{pmatrix}$$

b. Apply the formula for enlargement:

$$\begin{pmatrix}x-a\\y-b\end{pmatrix} \longrightarrow \begin{matrix}-2\\3-4\end{matrix}\Big) \quad = \begin{pmatrix}-1\\-7\end{pmatrix}$$ subtract components (2,4) from (1, -3)

$$\begin{pmatrix}-1\\-7\end{pmatrix} \longrightarrow \begin{matrix}-1\\-7\end{matrix}\Big) \quad = \begin{pmatrix}-3\\-21\end{pmatrix}$$ enlarge using given scale factor

$$\begin{pmatrix}-3\\-21\end{pmatrix} \longrightarrow \begin{matrix}\cdot 3+2\\21+4\end{matrix}\Big) \quad = \begin{pmatrix}-1\\-17\end{pmatrix}$$ add back components of (2,4)

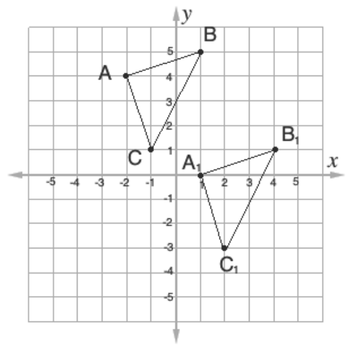
5. Triangle ABC has coordinates $A(-2,4)$, $B(1,5)$ and $C(-1,1)$.
   a. Draw triangle ABC on the Cartesian plane.
   b. Draw triangle $A_1B_1C_1$, which is ABC translated by the vector $\begin{pmatrix}3\\-4\end{pmatrix}$.

   Solutions:

   a. Identify on the Cartesian plane, and connect them in a triangle as shown.
   b. Identify point A_1 by translating $A(-2,4)$ by the vector $\begin{pmatrix}3\\-4\end{pmatrix}$:

$$\begin{pmatrix}-2\\4\end{pmatrix} \longrightarrow \begin{pmatrix}-2\\4\end{pmatrix} + \begin{pmatrix}3\\-4\end{pmatrix}$$
$$= \begin{pmatrix}-2+3\\4-4\end{pmatrix}$$
$$= \begin{pmatrix}1\\0\end{pmatrix}$$

Draw the image of $A_1B_1C_1$ using A_1 as a reference point. Note that it has the same shape and size as ABC.



Practice

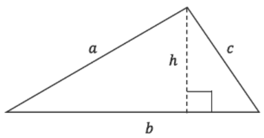
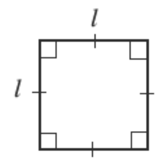
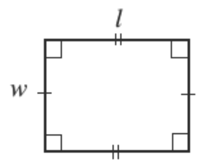
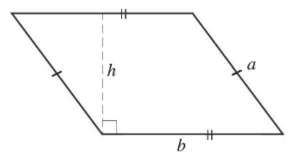
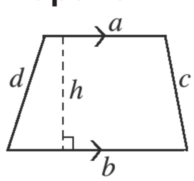
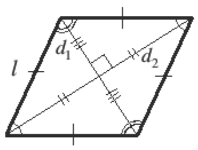
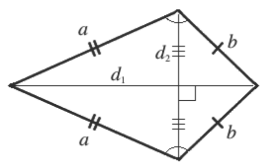
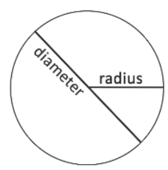
1. Triangle DEF has coordinates $D(-4,1), E(-2,3)$ and $F(-5,4)$. Find the co-ordinates, D_1, E_1 and F_1 of the image of the triangle formed under reflection in the line $y = x$.
2. Find the image of $(-5,7)$ under the enlargement with a scale factor of 5 from the point $(-1,-1)$.
3. Triangle ABC has co-ordinates $A(-2,-4)$, $B(-2,-1)$ and $C(-5,-1)$. Draw triangle $A_1B_1C_1$, which is ABC translated by the vector $\binom{7}{5}$.
4. Points $A(2,4), B(6,3)$ and $C(3,1)$ are points on the Cartesian plane. Draw a diagram and find the coordinates, A_1, $B_1$and $C_1$of the image of the triangle formed under a **clockwise** rotation of $90°$ about the origin, O.

Lesson Title: Area and surface area	**Theme:** Review
Practice Activity: PHM3-L133	**Class:** SSS 3

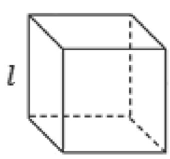
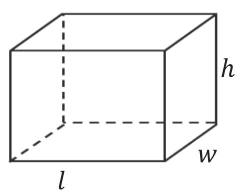
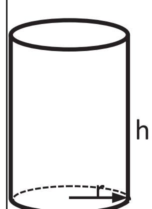
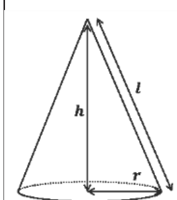
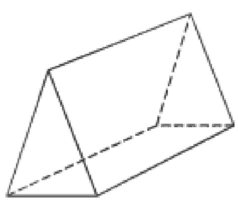
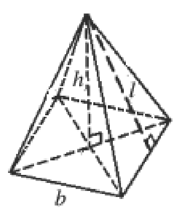
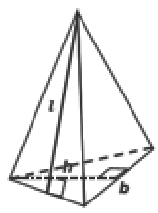
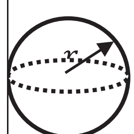
Learning Outcome
By the end of the lesson, you will be able to calculate the area of 2-dimensional figures and the surface area of 3-dimensional figures.

Overview

This lesson reviews area and surface area.

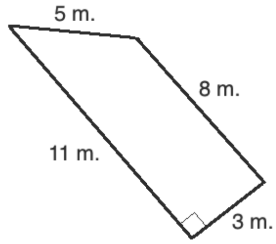
Triangle	**Square**	**Rectangle**	**Parallelogram**
 $A = \frac{1}{2} b \times h$	 $A = l \times l = l^2$	 $A = l \times w$	 $A = b \times h$
Trapezium	**Rhombus**	**Kite**	**Circle**
 $A = \frac{1}{2}(a + b)h$	 $A = \frac{1}{2} d_1 \times d_2$	 $A = \frac{1}{2} d_1 \times d_2$	 $A = \pi r^2$

The surface area of a solid is the total area of its outside surface. The surface area is the sum of the areas of the faces of the three-dimensional shape. The formulae for surface area of some common solids are given below.

Cube	**Cuboid**	**Cylinder**	**Cone**
 $SA = 6 \times l^2$	 $SA = 2(lh + hw + lw)$	 $SA = 2\pi r(r + h)$	 $SA = \pi r(l + r)$
Triangular Prism	**Pyramid with a rectangular base**	**Pyramid with a triangular base**	**Sphere**
 No formula	 $SA = b^2 + 2bl$	 $SA = \frac{1}{2} b(h + 3l)$	 $SA = 4\pi r^2$

Solved Examples

1. Find the area of the trapezium.



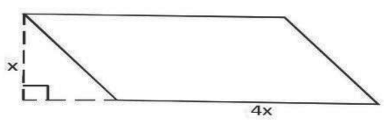
Solution:

A	$=$	$\frac{1}{2}(a+b)h$	Apply the formula
	$=$	$\frac{1}{2}(8+11)3$	Substitute known values
	$=$	$\frac{1}{2}(19)3$	Subtract $14\ \text{cm}$ from both sides
	$=$	$\frac{1}{2}(57)$	
	$=$	$28.5\ \text{m}^2$	

2. The base of a parallelogram is four times its height. If its area is $676\ \text{cm}^2$, find the base and the height.

Solution:

Step 1. Draw a diagram:



Step 2. Substitute the area, x, and 4x into the area formula, and solve for x:

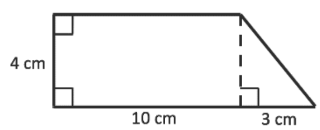
A	$=$	$b \times h$	
$676\ \text{cm}^2$	$=$	$4x \times x$	
$676\ \text{cm}^2$	$=$	$4x^2$	
$\frac{676\ \text{cm}^2}{4}$	$=$	x^2	Divide throughout by 4
169	$=$	x^2	
x^2	$=$	169	
$\sqrt{x^2}$	$=$	$\sqrt{169}$	Take the square root of both sides
x	$=$	$13\ \text{cm}$	

Step 3. Find the lengths of the base and height:

$b = 4x = 4 \times 13 = 52\ \text{cm}$

$h = x = 13\ \text{cm}$

3. Find the area of the shape at right:



Solution:

Find area by finding the areas of the rectangle and triangle, then adding them together:

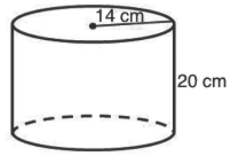
Area of rectangle: $A = l \times w = 10 \times 4 = 40\ \text{cm}^2$

Area of triangle: $A = \frac{1}{2} b \times h = \frac{1}{2}(3)(4) = 3 \times 2 = 6\ \text{cm}^2$

Total area: $40\ \text{cm}^2 + 6\ \text{cm}^2 = 46\ \text{cm}^2$

4. Find the surface area of each shape [Use $\pi = \frac{22}{7}$]:

a. 

b. 

Solutions:

a. Apply the formula for surface area of a cuboid:

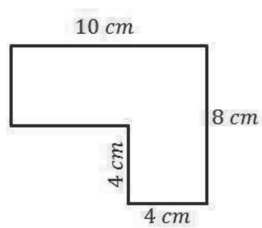
$$\begin{aligned} SA &= 2(lh + hw + lw) && \\ &= 2(13 \times 5 + 5 \times 4 + 13 \times 4) && \text{Substitute} \\ &= 2(65 + 20 + 52) && \text{Simplify} \\ &= 2(137) = 274 \text{ m}^2 && \end{aligned}$$

b. Apply the formula for surface area of a cylinder:

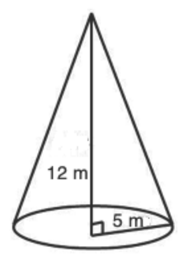
$$\begin{aligned} SA &= 2\pi r(r + h) && \\ &= 2\left(\frac{22}{7}\right)(14)(14 + 20) && \text{Substitute} \\ &= 2(22)(2)(34) && \text{Simplify} \\ &= 2{,}992 \text{ cm}^2 && \end{aligned}$$

Practice

1. A trapezium has parallel sides that are 12 m and 17 m. If its area is 290 m^2, what is its height?
2. The area of the triangle is 108 cm^2. If its height is 18 cm, find the length of its base.
3. The base of a parallelogram is five times its height. If its area is 180 cm^2, find the base and the height.
4. Calculate the area of the rhombus with sides of length 8.3 m, and diagonals 9 m and 14 m.
5. A kite has one diagonal that is 30 mm in length. If its area is $1{,}500 \text{ mm}^2$, what is the length of its other diagonal? Use the formula $A = \frac{1}{2}d_1 \times d_2$.
6. The area of a circle is 154 cm^2. Find its radius to the nearest whole number. (Take $\pi = 3.14$)
7. Find the area of the shape:



8. The surface area of a cube is 150 m^2. What is the length of each side?
9. The height of a right circular cone is 12 m. The radius of its base is 5 m. Find its surface area to 1 decimal place. Use $\pi = \frac{22}{7}$.

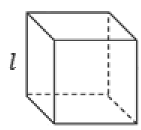
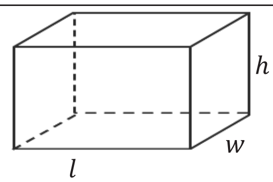
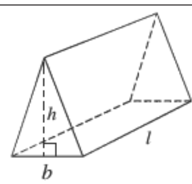
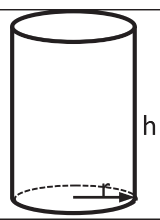
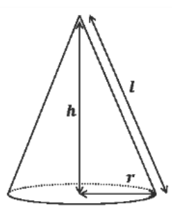
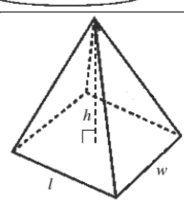


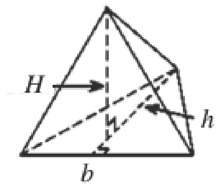
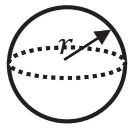
Lesson Title: Volume	Theme: Review
Practice Activity: PHM3-L134	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to calculate the volume of 3-dimensional figures.

Overview

The volume of a three-dimensional solid is a measurement of the space occupied by the shape. This involves multiplication. The volume of common shapes can be found with formulae, which are given below.

Solid	Diagram	Formula
Cube		$V = l^3$ where l is the length of a side of the cube.
Cuboid		$V = lwh$ where l and w are the length and width of the base, and h is the height of the cuboid.
Triangular Prism		$V = \frac{1}{2}bhl$ where b and h are the base and height of the triangular face, and l is the length of the prism.
Cylinder		$V = \pi r^2 \times h$ where r is the radius of the circular face and h is the height of the cylinder.
Cone		$V = \frac{1}{3}\pi r^2 h$ where r is the radius and h the height of the cone.
Pyramid with a rectangular base		$V = \frac{1}{3}lwh$ where l and w are the length and width of the base rectangle, and h is the height of the pyramid.

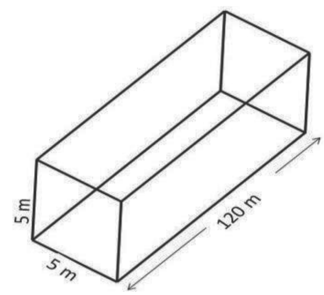
Pyramid with a triangular base		$V = \frac{1}{3}AH$ where A is the area of the triangular base, and H is the height of the pyramid.
Sphere		$V = \frac{4}{3}\pi r^3$ where r is the radius.

To find the volume of a composite solid, first identify the solids it is made up of. Find the individual volume using the appropriate formula. Finally, add the volumes together.

For some WASSCE problems, you may need to use the fact that $1{,}000\ \text{cm}^3$ contains 1 litre of liquid ($1\ l = 1{,}000\ \text{cm}^3$).

Solved Examples

1. A tunnel with a length 120 m is to be bored with a square cross-section 5 m wide and 5 m in height.
   a. What volume of spoil has to be evacuated?
   b. If the spoil is to be taken away using trucks with a capacity of $80\ \text{m}^3$, how many truck loads will be moved?



Solutions:

The tunnel is in the form of a cuboid of cross-sectional area 5 m $\times$ 5 m and length 120 m. Draw a diagram. Note that the diagram shown is not proportional, but it is sufficient to visualise the problem.

a. Calculate the volume:

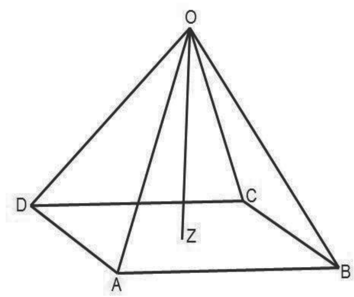
$$\begin{aligned} \text{Volume of spoil} &= \text{Area of cross-section} \times \text{length of tunnel} \\ \text{Area of cross-section} &= 5\ \text{m} \times 5\ \text{m} \\ &= 25\ \text{m}^2 \\ \text{Volume of spoil} &= 25 \times 120\ \text{m}^3 \\ &= 3{,}000\ \text{m}^3 \end{aligned}$$

b. Calculate the truck loads: If the capacity of one truck is $80\ \text{m}^3$ and n trucks are used to clear the $3{,}000\ \text{m}^3$, then:

$$\begin{aligned} 80n &= 3{,}000 \\ n &= 37\tfrac{1}{2} \end{aligned}$$

$37\frac{1}{2}$ truck loads will be moved.

2. The diagram shows a right pyramid with a rectangular base ABCD and vertex O. If |AB|= 10 cm, |DA|= 8 cm and |OD|= 12 cm, find:



   a. The height of the pyramid.
   b. The volume of the pyramid.

   Solutions:

   a. The height of the pyramid OZ is obtained from ΔOZD, which is right angled at Z. We must find the length of DZ, which is half of the diagonal of the base. DZ = $\frac{1}{2}$(diagonal of the base).

   Calculate DB, the length of the full diagonal, using triangle DAB:

$$
\begin{aligned}
|DB| &= \sqrt{|AB|^2 + |DA|^2} \\
&= \sqrt{10^2 + 8^2} \\
&= \sqrt{164} = 2\sqrt{41}
\end{aligned}
$$

   Divide the result by 2 to find the length of DZ: $|DZ| = 2\sqrt{41} \div 2 = \sqrt{41}$

   Calculate the height using triangle OZD:

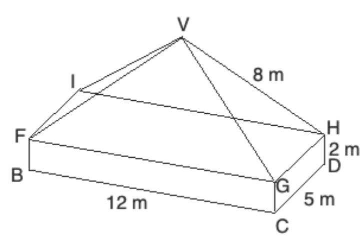
$$
\begin{aligned}
|OZ|^2 + |DZ|^2 &= |OD|^2 && \text{Using Pythagoras' theorem on } \Delta OZD \\
|OZ| &= \sqrt{|OD|^2 - |DZ|^2} && \text{Height of pyramid (from } \Delta ODZ) \\
&= \sqrt{12^2 - \left(\sqrt{41}\right)^2} \\
&= \sqrt{103} \\
\text{Height} &= 10.15 \text{ cm}
\end{aligned}
$$

   b. Calculate the volume:

$$
\begin{aligned}
V &= \frac{1}{3} \times |\mathrm{DA}| \times |\mathrm{AB}| \times |\mathrm{OZ}| \\
V &= \frac{1}{3} \times 8 \times 10 \times 10.15 \\
&= 270.67 \text{ cm}^3
\end{aligned}
$$

Practice

1. Oil fills an inverted metal cone to a depth of 24 cm.
   a. If the radius of the surface of the oil is 20 cm, find the volume of oil. Let $\pi = 3.14$.
   b. The oil is then poured into a rectangular can of base 25 cm by 18 cm. Find, correct to two decimal places, the depth of oil in the can.
2. A steel cuboid measuring 94.2 mm by 36 mm by 16 mm is melted down and cast into ball bearings of radius 6 mm. How many ball bearings are cast? Ball bearings are in the form of a sphere. $(\pi = 3.14)$
3. The diagram at right shows a pyramid standing on a cuboid. The dimensions of the cuboid are 2 m x 5 m x 12 m, and the slant edge of the pyramid is 8 m. Calculate the volume of the shape, correct to 1 decimal place.



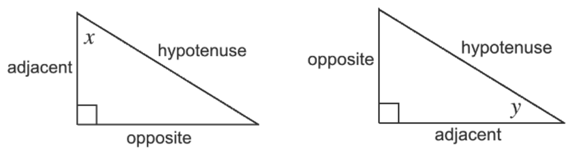
Lesson Title: Trigonometry	**Theme:** Review
Practice Activity: PHM3-L135	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to apply trigonometric ratios to solve triangles.

Overview

To "solve" a triangle means to find any missing side or angle measures. Trigonometric functions are used to find missing sides in a right-angled triangle, and inverse trigonometric functions are used to find missing angles. You may use other methods for solving triangles, including Pythagoras' theorem, and finding angle measures by subtracting from 180°.

We use 3 types of sides (adjacent, opposite and hypotenuse) in trigonometric ratios. "Adjacent" and "opposite" are determined by their relationship to the angle in question. For example, consider angles x and y, and the side labels of each triangle:



The 3 trigonometric ratios are:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{O}}{\text{H}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\text{A}}{\text{H}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Sine, cosine, and tangent are functions of angles. The theta symbol (θ) is shown here, and it is often used to represent angles.

We use the term SOHCAHTOA as a way of remembering the ratios. SOH stands for "sine equals opposite over hypotenuse". CAH stands for "cosine equals adjacent over hypotenuse". TOA stands for "tangent equals opposite over adjacent".

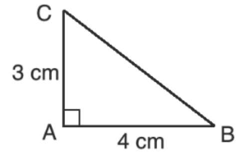
Trigonometric tables can be used to find the trigonometric function of a given angle to 4 decimal places. If you do not have access to trigonometric tables, a calculator may be used.

The inverse of a function is its opposite. It's another function that can undo the given function. Inverse functions are shown with a power of -1. For example, inverse sine is $\sin^{-1} x$. Inverse sine "undoes" sine: $\sin^{-1}(\sin\theta) = \theta$. The other inverse trigonometric functions are $\cos^{-1} x$ and $\tan^{-1} x$. The inverse functions are also sometimes called "arcsine", "arccosine", and "arctangent".

You can use inverse trigonometric functions to find the degree measure of an angle. You can use the trigonometric tables ("log books") or calculators. Using trigonometric tables, you will work backwards. Find the decimal number in the chart, and identify the angle that it corresponds to.

Solved Examples

1. Find the measure of angle B and the length $|BC|$:



Solution:
Step 1. Identify which function to use. The opposite and adjacent sides are known, so we will use $\tan^{-1}$.
Step 2. Find the tangent ratio. This is the ratio that you will "undo" with $\tan^{-1}$ to find the angle:
$\tan B = \frac{3}{4} = 0.75$
Step 3. Find $\tan^{-1}$ of both sides to find the angle measure:

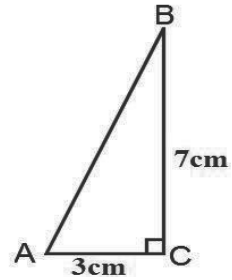
$$\begin{aligned} \tan B &= 0.75 \\ \tan^{-1}(\tan B) &= \tan^{-1}(0.75) \\ B &= \tan^{-1}(0.75) \end{aligned}$$

Calculate $\tan^{-1}(0.75)$ using either a calculator or trigonometry table.
To calculate $\tan^{-1}(0.75)$ using the tangent table: Look for 0.75 in the table. It is not there, but 0.7481 is there. If we add 0.0018 to 0.7481, it will give us 0.75. Find 18 in the "add differences" table, and it corresponds to 7. Therefore, $B = 36.87°$.

Use Pythagoras' theorem to find $|BC|$:
$BC^2 = AC^2 + AB^2 = 3^2 + 4^2 = 25$
$|BC| = \sqrt{25} = 5 \text{ cm}$

2. Solve the triangle ABC in which ∠C= $90°$, $a = 7$ cm and $b = 3$ cm.
Solution:
Note that to solve a triangle means finding all the unknown side(s) and angle(s).
Use Pythagoras' theorem to find the side c, since the other two sides are known;

$$\begin{aligned} c^2 &= a^2 + b^2 && \text{Pythagoras' theorem} \\ c^2 &= 7^2 + 3^2 && \text{Substitute values for } a \text{ and } b \\ c^2 &= 58 \\ c &= \sqrt{58} \\ c &= 7.62 \text{ cm} \end{aligned}$$



To find any of the missing angles, use any of the trigonometric ratios to find 1 angle. Then, subtract the known angles from $180°$ to find the other.

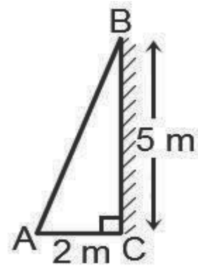
$\tan A$	$=$	$\frac{7}{3}$	Apply the tangent ratio to A
$\tan A$	$=$	2.3333	Substitute values for a and b
A	$=$	$\tan^{-1} 2.3333$	Take the inverse tangent of both sides
A	$=$	$66.8°$	

Use the known angles to find B: $B = 180° - 66.8° - 90° = 23.2°$

3. A stick leans on a vertical wall of height 5 m. If the foot of the stick is 2 m from the base of the wall, find:
   a. The length of the stick.
   b. The angle between the stick and the wall.

   Solutions:

   Step 1. Draw a diagram (at right).



   a. From the diagram, apply Pythagoras' theorem to find the length of the stick, which is AB.

$\lvert AB\rvert^2$	$=$	$2^2 + 5^2$	Apply Pythagoras' Theorem
	$=$	29	Simplify
$\lvert AB\rvert$	$=$	$\sqrt{29}$	
	$=$	5.39 m	Length of stick

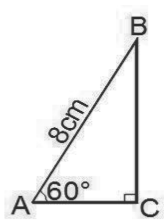
   b. To find the angle between the wall and the stick, find ∠B using any of the trigonometric ratios.

$\tan B$	$=$	$\frac{2}{5}$	Apply the tangent ratio
	$=$	0.4	
B	$=$	$\tan^{-1} 0.4$	Take the inverse tangent
	$=$	$21.8°$	Angle between the wall and stick

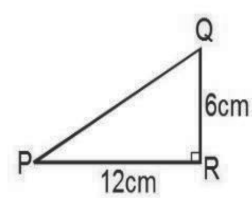
Practice

1. In a right-angle triangle ABC, $\angle C = 90°, \angle A = 70°$ and |AB| = 5 cm. Find the other sides and angle of the triangle.
2. The two legs of a right-angle triangle are 8 cm and 6 cm. Find the hypotenuse side and the angles of the triangle.
3. A ladder 6 metres long leans against a vertical wall. If the foot of the ladder is 2 metres from the base of the wall, find: a. The angle between the ladder and the wall; b. The height of the wall.
4. Find the missing sides and angles in the figures:

a.



b.



Lesson Title: Angles of elevation and depression	Theme: Review
Practice Activity: PHM3-L136	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems on angles of elevation and depression.

Overview

"Elevation" is related to height. Problems on angles of elevation handle the angle that is associated with the height of an object. An angle of elevation is measured a certain distance away from an object.

Angle of elevation problems generally deal with 3 measures: the angle, the distance from the object, and the height of the object. You may be asked to solve for any of these measures. These can be solved by using trigonometry to find distances, and inverse trigonometry to find angles.

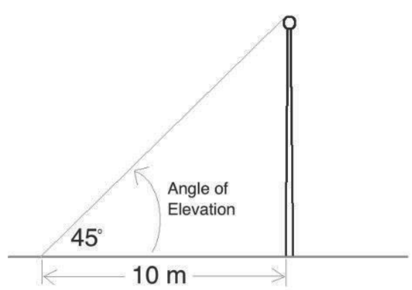
"Depression" is the opposite of elevation. "Depressed" means downward. So, if there is an angle of depression, it is an angle in the downward direction.

The angle of depression is the angle made with the **horizontal** line. See the diagram in Solved Example 2. The horizontal line is at the height of the cliff.

Angle of depression problems generally deal with 3 measures: the angle, the horizontal distance, and the depth of the object. Depth is the opposite of height. It is the distance downward. You may be asked to solve for any of these 3 measures.

Solved Examples

1. At a point 10 metres away from a flag pole, the angle of elevation of the top of the pole is $45°$. What is the height of the pole?
   Solution:
   First, draw a diagram:



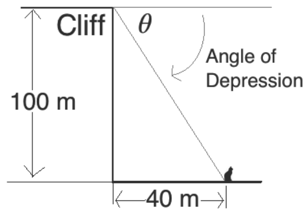
Solve using the tangent ratio, because we are concerned with the sides opposite and adjacent to the angle.

$$\tan 45° = \frac{h}{10} \quad \text{Set up the equation}$$

$$1 = \frac{h}{10} \quad \text{Substitute } \tan 45° = 1$$

$$10 \text{ m} = h$$

2. A cliff is 100 metres tall. At a distance of 40 metres from the base of the cliff, there is a cat sitting on the ground. What is the angle of depression of the cat from the cliff?
   Solution:
   First, draw a diagram:



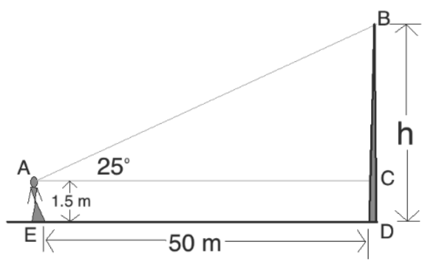
Solve using the tangent ratio, because we are concerned with the sides opposite and adjacent to the angle.

$$\tan\theta = \frac{100}{40} = 2.5 \quad \text{Set up the equation}$$

$$\tan^{-1}(\tan\theta) = \tan^{-1}(2.5) \quad \text{Take the inverse tangent}$$

$$\theta = 68.2 \quad \text{Use the tangent tables}$$

3. A woman standing 50 metres from a flag pole observes that the angle of elevation of the top of the pole is 25°. Assuming her eye is 1.5 metres above the ground, calculate the height of the pole to the nearest metre.
   Solution:
   First, draw a diagram:



To find the height of the flag pole, we must find the length of BC, then add it to the height of the woman's eye (CD or AE), which is 1.5 metres.

Step 1. Find $\overline{BC}$:

$$\tan 25° = \frac{\overline{BC}}{50} \quad \text{Set up the equation}$$

$$0.4663 = \frac{\overline{BC}}{50} \quad \text{Substitute } \tan 25° = 0.4663 \text{ (from table)}$$

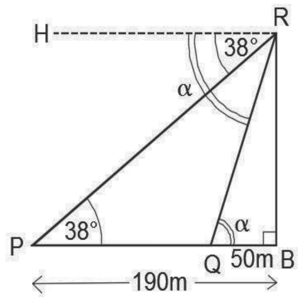
$$50 \times 0.4663 = \overline{BC} \quad \text{Multiply throughout by 50}$$

$$\overline{BC} = 23.315 \quad \text{Metres}$$

Step 2. Add: $h = BC + CD = 23.315 + 1.5 = 24.815$

Rounded to the nearest metre, the height of the pole is 25 m.

4. From the top of a cliff, the angle of depression of a boat P on the sea, which is 190 m from the foot of the cliff B, is $38°$. Q is another boat on the same horizontal line PB such that |QB| is 50 m. Find, correct to one decimal place:
   a. The height of the cliff;
   b. The angle of depression of the boat Q from the top of the cliff.



Solutions:

Draw a diagram (see below). From the diagram, the height of the cliff is |RB|, let that be h. Let ∠RQB, which is the same as $\angle HRQ$, the angle of depression, be α.

a. To find the height of the cliff, use the angle at P and the distance of P from the cliff.

$$\frac{h}{190} = \tan 38°$$
$$h = 190 \tan 38°$$
$$= 190 \times 0.7813$$
$$= 148.4 \text{ m}$$ Height of the cliff

b. To find α, use the height of the cliff and the distance of Q from the base of the cliff.

$$\tan \alpha = \frac{h}{50}$$
$$\tan \alpha = \frac{148.4}{50}$$
$$\tan \alpha = 2.968$$
$$\angle \alpha = \tan^{-1} 2.968$$
$$= 71.38°$$ Angle of depression of boat Q from the top of the cliff.

Practice

1. There is a dog sitting 60 metres from the base of a cliff. If the angle of depression of the dog from the top of the cliff is 45°, how tall is the cliff?
2. Point A is on the ground 10 metres away from a tree. If the tree is 12 metres tall, what is the angle of elevation at point A? Give your answer to 3 significant figures.
3. A man 1.6 m tall is standing 15 m from a tree 25 m tall. Find, correct to 1 decimal place, the angle of elevation of the top of the tree observed by the man.
4. Boat A is on the sea, 150 m from the foot of the cliff, F. The angle of depression from the top of the cliff to boat A is $41°$. Boat B is on the same horizontal line AF such that |BF| is 40 m. Find correct to one decimal place:
   a. The height of the cliff;
   b. The angle of depression of the boat B from the top of the cliff.

Lesson Title: Bearings and distances	**Theme:** Review
Practice Activity: PHM3-L137	**Class:** SSS 3

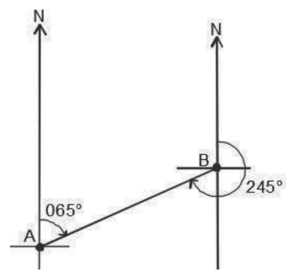
Learning Outcome
By the end of the lesson, you will be able to solve problems on bearings and distance.

Overview

Three-figure bearings are bearings given in 3 digits. These 3 digits give the angle of the bearing in the clockwise direction from the geographic north. The angles range from 000° to 360°.

When we talk about "reverse" bearings, we must have 2 points. Consider 2 points A and B. We have the bearing from A to B, and we have the bearing from B to A. These are different, because bearings are about direction. A to B is a different direction than B to A. They are reverse. Reverse bearings are sometimes called "back bearings".

Consider the example:

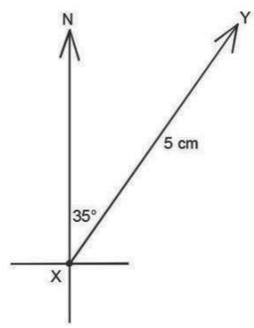


The bearing from A to B is 065°, and the bearing from B to A is 245°. For both bearings, we use the line that joins them and the north direction. We find the bearing of the line joining them from north.

Depending on the size of the first bearing, you will add or subtract 180° to find the reverse bearing:

- Reverse bearing = $\theta + 180°$ if θ is less than $180°$
- Reverse bearing = $\theta - 180°$ if θ is more than $180°$

Distance-bearing form gives the bearing, as well as the distance between two points.

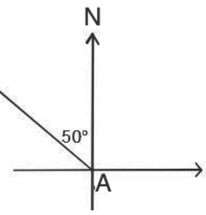


Consider the diagram on the right. The bearing from X to Y can be written as $\overrightarrow{XY} = (5 \text{ cm}, 035°)$. The distance and three-point bearing are given in brackets.
In general, the position of point Q from another point P can be represented by $\overrightarrow{PQ} = (r, \theta)$, where r is the distance between the 2 points, and θ is the three-point bearing from P to Q.

When you encounter a bearings problem, the first step is to draw a diagram. At times, this diagram forms a right-angled triangle. You can apply Pythagoras' theorem and trigonometry to find missing sides and angles in such triangles.

Solved Examples

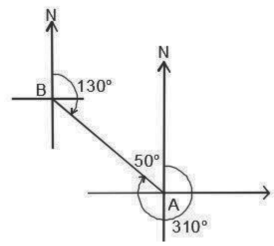
1. In the diagram, find:
    a. The bearing of B from A
    b. The bearing of A from B



Solutions:

First, draw a diagram showing the bearings.

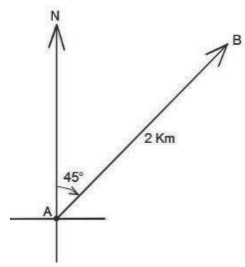
a. Find the bearing from north. Subtract the given angle (50°) from 360°: $360° - 50° = 310°$.
b. Find the reverse bearing using the result from part a.: $310° - 180° = 130°$.



2. A hunter starts at point A and travels through the bush 2 km in the direction 045° to point B. Give the bearing and draw a diagram.

Solution:

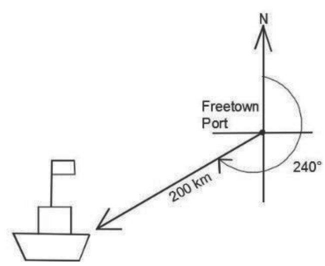
Bearing: $\overrightarrow{AB} = (2 \text{ km}, 045°)$ Diagram:



3. A boat sailed from Freetown port at a bearing of 240°. It is now 200 km from Freetown. Write the ship's bearing and draw a diagram.

Solution:

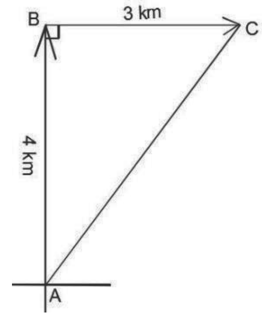
Bearing: $(200 \text{ km}, 240°)$ Diagram:



4. Hawa walked 4 km from point A to B in the north direction, then 3 km from point B to C in the east direction.
    a. How far is she from her original position?
    b. What is the bearing from A to C?

Solutions:

First, draw a diagram. →



Draw her movement in the north direction and the east direction. These two lengths can be connected to form a triangle, as shown above.

a. Use Pythagoras' theorem to find the distance from C to A:

$\lvert AB\rvert^2 + \lvert BC\rvert^2$	$=$	$\lvert AC\rvert^2$	Apply Pythagoras' theorem
$4^2 + 3^2$	$=$	$\lvert AC\rvert^2$	Substitute known lengths
$16 + 9$	$=$	$\lvert AC\rvert^2$	Simplify
25	$=$	$\lvert AC\rvert^2$	
$\sqrt{25}$	$=$	$\sqrt{\lvert AC\rvert^2}$	Take the square root of both sides
5 km	$=$	$\lvert AC\rvert$	

She is 5 km from her original position.

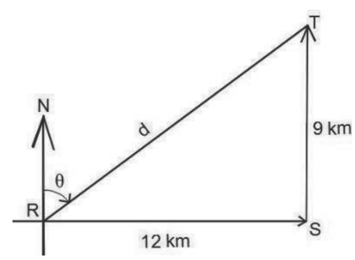
b. Use trigonometry to find the angle of the bearing from A to C. We can choose any trigonometric function, because we know the lengths of all 3 sides. Let's use tangent:

$\tan A$	$=$	$\frac{3}{4} = 0.75$	Apply the tangent ratio
$\tan^{-1}(\tan\ A)$	$=$	$\tan^{-1}(0.75)$	Take the inverse tangent of both sides
A	$=$	$\tan^{-1}(0.75)$	
A	$=$	$36.87°$	From the tangent table

The bearing from A to C is $\overrightarrow{AC} = (5\text{ km}, 037°)$.

Practice

1. The bearing of X from Y is 072°. Draw a diagram and find the bearing of Y from X.
2. Sia walked 250 metres due north, then 150 metres due west. She then walked 300 metres on a bearing of 155°.
   a. Write the bearings for each of her 3 walks.
   b. Draw a diagram of her movement.
3. A ship travelled 5 km due east from point X to point Y, then 12 km due south from point Y to point Z.
   a. Draw a diagram for the problem.
   b. Find the distance from point X to point Z.
   c. Find the bearing from point X to point Z.
4. Find the bearing from R to T in the diagram at right.



5. A farmer travels 10 km due north to reach his land. He then travels 24 km due east to bring his harvest to a market.
   a. Draw a diagram for the problem.
   b. Find the distance from his starting point to the market.
   c. Find the bearing from his starting point to the market.

Lesson Title: Probability	**Theme:** Review
Practice Activity: PHM3-L138	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems on probability.

Overview

This lesson is on solving probability problems. Probability is covered in the first 12 lessons of this book, lessons 97 through 108. The Solved Examples below illustrate the types of problems that may appear on the WASSCE exam.

Solved Examples

1. A letter is selected at random from the letters of the English alphabet. What is the probability that the letter selected is from the words SIERRA LEONE?
   Solution:
   Identify the number of letters that appear in "Sierra Leone". There are 8 letters: s, i, e, r, a, l, o, n. Some letters appear more than once, but are only counted once. The probability is: $\frac{8}{26}=\frac{4}{13}$.

2. The probabilities that Sia, Hawa, and Foday will score a goal are $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{1}{4}$, respectively. Find the probability that only Sia will score a goal.
   Solution:
   If only Sia scores a goal, then Hawa and Foday will not score a goal. Subtract from 1 to find the probability that each will not score a goal:
   Pr(Hawa will not score) $= 1-\frac{2}{3}=\frac{1}{3}$
   Pr(Foday will not score) $= 1-\frac{1}{4}=\frac{3}{4}$
   Multiply the probability that Sia will score by the probabilities that Hawa and Foday will not score:
   Pr(only Sia will score) $= \frac{3}{5}\times\frac{1}{3}\times\frac{3}{4}=\frac{9}{60}=\frac{3}{20}$

3. Mohamed applied to enroll at 2 universities, A and B. The probability that he will be accepted to university A is 0.6. The probability that he will **not** be accepted to university B is 0.3. What is the probability that he will:
   a. Be accepted to both universities
   b. Be accepted to exactly 1 university
   c. Be accepted to neither university
   Solutions:
   a. First, find the probability that he will be accepted to university B:
      Pr(Accepted to B) $= 1-0.3=0.7$
      Multiply the probability that he will be accepted to university A by the probability that he will be accepted to university B.

Pr(Accepted to A and B) $= 0.6 \times 0.7 = 0.42$

b. Find the probability that he will be accepted to only A and not B by subtracting the probability that he will be accepted to both by the probability that he will be accepted to A:

Pr(Accepted to only A) $= 0.6 - 0.42 = 0.18$

Follow the same process for B:

Pr(Accepted to only B) $= 0.7 - 0.42 = 0.28$

Add to find the probability that he will be accepted to only A or only B:

Pr(Accepted to only A or B) $= 0.18 + 0.28 = 0.46$

c. Multiply the probability that he will not be accepted to university A by the probability that he will not be accepted to university B:

Pr(Accepted to neither A nor B) $= 0.4 \times 0.3 = 0.12$

4. Fatu has 30 oranges in a box. Some are ripe while others are not. If the probability of selecting a ripe orange is $\frac{1}{5}$, calculate the number of:
   a. Unripe oranges
   b. Ripe oranges which should be added to the box such that the probability of picking a ripe orange will be $\frac{1}{2}$.

Solutions:

a. Note that the total number of oranges is 30, and the probability of selecting an unripe orange is $1 - \frac{1}{5} = \frac{4}{5}$. We also know that:

Pr(choosing an unripe orange) $= \frac{\text{No of unripe oranges}}{\text{Total no of oranges}}$

Therefore, we have $\frac{\text{No of unripe oranges}}{\text{Total no of oranges}} = \frac{\text{U}}{30} = \frac{4}{5}$

Cross multiply and solve for the number of unripe oranges (U):

$$\begin{aligned} \frac{\text{U}}{30} &= \frac{4}{5} \\ 5U &= 30 \times 4 \\ U &= \frac{120}{5} = 24 \end{aligned}$$

Answer: There are 24 unripe oranges.

b. The probability of selecting a ripe orange is $\frac{1}{5}$, so the number of ripe oranges currently is $\frac{1}{5} \times 30 = 6$. A certain number of ripe oranges should be added to the total to create a probability of $\frac{1}{2}$. Let's call that number r. Then we have: $\frac{1}{2} = \frac{6+r}{30+r}$. This is based on the new probability, and adding r to both the total number, and the number of ripe oranges.

$$\begin{aligned} \frac{1}{2} &= \frac{6+r}{30+r} \\ 30 + r &= 2(6 + r) \qquad \text{Cross-multiply} \\ 30 + r &= 12 + 2r \\ 30 - 12 &= 2r - r \\ 18 &= r \end{aligned}$$

Answer: 18 ripe oranges should be added.

Practice

1. The table below gives the ages of 50 students in a university course.

Age (years)	18	19	20	21
Frequency	5	11	21	13

   Find the probability that a pupil chosen at random from the class is:
   a. 18 years old
   b. Not 19 years old
   c. 19 or 20 years old
   d. More than 19 years old
2. A bag contains papers with the names of 60 pupils in a class. If a paper is chosen at random, the probability of selecting a female pupil is $\frac{3}{5}$. How many males are in the class?
3. A letter is chosen at random from the alphabet. Find the probability that it is in either the word MATHS or SCIENCE.
4. A fair six-sided die is thrown. Find the probability of getting:
   a. Three
   b. An odd number
   c. A number greater than 4
5. A number has 3 digits formed by arranging 3, 4, and 5 randomly. Find the probability that the number is divisible by:
   a. Two
   b. Five
   c. Ten

Lesson Title: Statistics – ungrouped data	**Theme:** Review
Practice Activity: PHM3-L139	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve statistics problems with ungrouped data.

Overview

This lesson is on statistics problems that handle ungrouped data. Such problems may ask you to draw and interpret a pie chart or bar chart. They may ask you to calculate mean, median, or mode. The solved examples below illustrate the types of problems that may appear on the WASSCE exam.

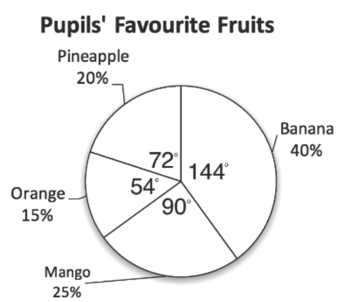
Solved Examples

1. A group of pupils was surveyed to find their favourite fruits. The data is in the table below. Create a pie chart for this data.

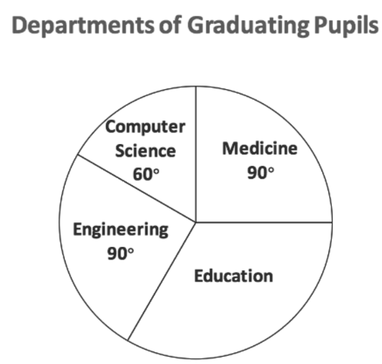
Favourite Fruit	Frequency	Percentage
Banana	16	40%
Mango	10	25%
Orange	6	15%
Pineapple	8	20%
TOTAL	40	100%

Solution:
First, calculate the degree measure for each fruit. Give each frequency as a fraction of the whole (40), and multiply that fraction by 360°. Draw the pie chart using the degrees you find.

$$\text{Banana} = \frac{16}{40} \times 360° = 144°$$
$$\text{Mango} = \frac{10}{40} \times 360° = 90°$$
$$\text{Orange} = \frac{6}{40} \times 360° = 54°$$
$$\text{Pineapple} = \frac{8}{40} \times 360° = 72°$$



2. This year, 1,000 pupils graduated from a certain university. The pie chart shows the departments they graduated from. Use it to answer the questions:
   a. How many pupils graduated from the education department?
   b. What percentage of the total graduated from the medicine department?



Solutions:

a. This solution involves multiply steps:

Step 1. Find the degree measure of Education:

Education measure $= 360° - (90° + 60° + 90°) = 120°$

Step 2. Multiply the proportion by the total number of pupils to calculate those studying Education:

Number in Education $= \frac{120}{360} \times 1{,}000$

$= \frac{1}{3} \times 1{,}000$

$= 333.3$

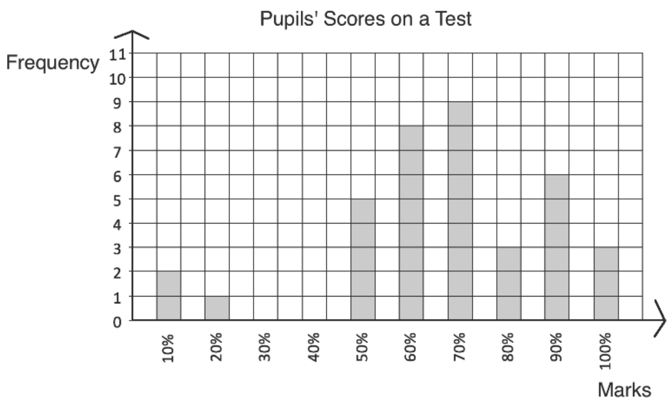
$= 333$ Round to a whole number

Answer: 333 pupils are graduating from education.

b. Write medicine as a percentage of the whole using its degree measure:

Pupils graduating from medicine $= \frac{90}{360} \times 100\% = 25\%$

3. The bar chart below shows marks that pupils achieved on a test, as percentages. Find the mean, median, and mode.



Solution:

Mean. Use the formula:

$$\frac{\sum fx}{\sum f} = \frac{2(10)+20+5(50)+8(60)+9(70)+3(80)+6(90)+3(100)}{2+1+5+8+9+3+6+3}$$

$$= \frac{20+20+250+480+630+240+540+300}{37} = \frac{2{,}480}{37} = 67.03\%$$

Median.

The median is at the position $\frac{n+1}{2}$. Since there are 37 pupils in the class, the 19th score is the median ($\frac{37+1}{2} = \frac{38}{2} = 19$). Locate the 19th pupil in the bar chart by counting the bars, from least to greatest. The 19th pupil is within 70%. Therefore, median = 70%

Mode. The highest bar is at 70%; therefore, the mode is 70%.

4. On her exams, Fatu scored x% in Mathematics, 90% in English, 95% in Biology, and 80% in Chemistry. If her mean score for all subjects was 88%, what is the value of x?

Solution:

$$\begin{aligned} 88 &= \frac{x+90+95+80}{4} && \text{Set up the equation} \\ 88 &= \frac{x+265}{4} && \text{Simplify} \\ 4 \times 88 &= x + 265 && \text{Multiply throughout by 4} \\ 352 &= x + 265 \\ 352 - 265 &= x && \text{Subtract 265 from both sides} \\ 87 &= x \end{aligned}$$

Practice

1. The table below shows how a family spends their money in one day.

Items	**Amount Spent**
Food	Le 15,000.00
House rent	Le 9,000.00
Electricity	Le 10,000.00
Transportation	Le 24,000.00
Other	Le 2,000.00

   a. Represent the information on a pie chart.
   b. What percentage does the family spend on transportation?

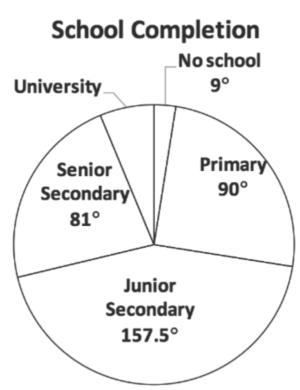
2. A teacher surveyed a class to learn the favourite fruits of the class members. The result of the survey is show in the frequency table:

Favourite Fruits	
Mango	10
Pawpaw	12
Orange	3
Banana	16
Pineapple	8

   a. Draw a bar chart to represent the data.
   b. Which fruit is most popular?
   c. Which fruit is least popular?
   d. What percentage of pupils prefer banana?
   e. How many pupils prefer either mango or pawpaw?

3. Fatu has 4 children. Their mean weight at birth was 2.8 kg. Three of their weights are 2.1 kg, 2.3 kg, and 3.5 kg. How much does the fourth baby weigh?

4. The pie chart below shows the highest level of education achieved by 800 people in a village. Use it to answer the questions.
   a. How many people have received a university education?
   b. What percentage of people have a junior secondary education or higher?



Lesson Title: Statistics – grouped data	**Theme:** Review
Practice Activity: PHM3-L140	**Class:** SSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems with grouped data.

Overview

This lesson is on statistics problems that handle grouped data. Such problems may ask you to draw and interpret a histogram or frequency polygon. They may ask you to calculate mean, median, or mode. The Solved Examples below illustrate the types of problems that may appear on the WASSCE exam. Additional statistics problems also appear, such as those handling standard deviation and mean deviation. Those topics are covered earlier in this book.

Solved Examples

1. A chimpanzee reserve houses 39 chimpanzees that have been rescued from hunters and people who kept them as pets. The weights of the chimpanzees are displayed in the frequency table:

Weight (kg)	10-19	20-29	30-39	40-49	50-59	60-69
Frequency	6	7	12	5	7	2

   a. Draw a histogram for the data.
   b. What is the median class?
   c. What is the modal class?
   d. Estimate the mode.

Solutions:

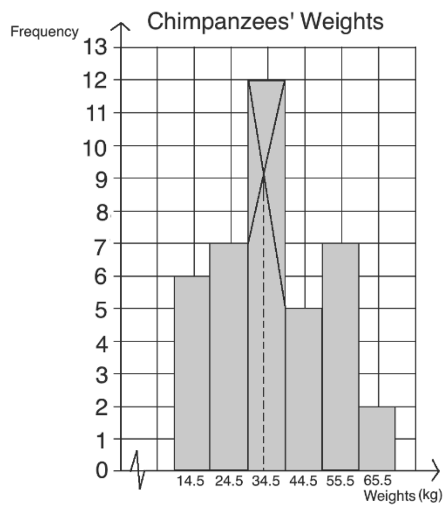
a. Find the class mid-points, which are used to plot the histogram. These can be added to the table (see below). Draw the histogram as shown.

Weight (kg)	10-19	20-29	30-39	40-49	50-59	60-69
Frequency	6	7	12	5	7	2
Class mid-point	14.5	24.5	34.5	44.5	54.5	64.5

b. The median class is the class where the chimpanzee with the median weight falls. The 20th chimpanzee has the median weight. This is in class interval 30-39, which is the median class.

   c. The modal class is the tallest bar, which is 30-39.

   d. To estimate the mode, draw lines as shown on the histogram. The mode is approximately 34 kg.



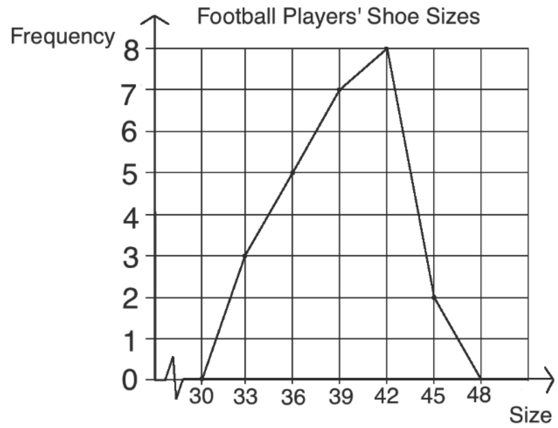
2. A school wants to buy shoes for its football team. They measured the shoe sizes of 25 football players and displayed them in the table below.

Shoe Size	32-34	35-37	38-40	41-43	44-46
Frequency	3	5	7	8	2

a. Draw a frequency polygon to display the data.
b. What is the modal class?
c. What is the median class?
d. The market only has football shoes in size 41 and larger this week. How many football players have to wait to receive their shoes?

Solutions:

a. Frequency polygon (using class mid-points 33, 36, 39, 42, 45) →



b. The modal class is 41-43.
c. The median class is where the 13th pupil falls, which is 38-40.
d. Add the frequencies of classes less than 41: $3 + 5 + 7 = 15$ pupils.

3. In one village, 17 farmers have just harvested cassava. The table below shows the amount of cassava they harvested in kilogrammes. Estimate the mean and median, correct to 2 decimal places.

Farmers' Harvests	
Cassava (kg)	**Frequency**
10 – 14	1
15 – 19	3
20 – 24	6
25 – 29	5
30 – 34	2
Total	17

Solution:

Mean. To apply the formula for mean ($\bar{x} = \frac{\sum fx}{\sum f}$), first find value to use for x. This can be done by finding the class mid-points (see the table).

Farmers' Harvests		
Cassava (kg)	**Frequency**	**Mid-point**
10 – 14	1	12
15 – 19	3	17
20 – 24	6	22
25 – 29	5	27
30 – 34	2	32
Total	17	

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{12+3(17)+6(22)+5(27)+2(32)}{1+3+6+5+2} = \frac{12+51+132+135+64}{17} = \frac{394}{17} = 23.18 \text{ kg}$$

Median. First, identify that the median falls into class interval 20-24. Since there are 17 farmers, the 9th farmer has the median harvest. Eight farmers harvested more, and 8 farmers harvested less. The 9th farmer is in interval 20-24.

Identify the value for each variable in the formula, substitute them, and evaluate.

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\Sigma f)_L}{f_m}\right] \times c = 20 + \left[\frac{\frac{17}{2} - (1+3)}{6}\right] \times 5 \quad \text{Substitute values}$$

$$= 20 + \left[\frac{8.5-4}{6}\right] \times 5 \quad \text{Simplify}$$

$$= 20 + \left[\frac{4.5}{6}\right] \times 5$$

$$= 20 + 0.75 \times 5$$

$$= 20 + 3.75$$

$$= 23.75 \text{ kg}$$

Practice

1. The table below shows the distribution of marks on an assignment that a class completed. No one scored below 6 marks. Find the mean, median, and mode of the scores.

Marks	6	7	8	9	10
Frequency	3	9	4	3	1

2. A certain women's group has 25 members. Their ages are 21, 42, 35, 26, 32, 19, 23, 27, 29, 38, 41, 42, 27, 35, 18, 30, 31, 26, 24, 41, 22, 35, 37, 23, 20.
   a. Draw a frequency table using class intervals 16-20, 21-25, 26-30, 31-35, 36-40, 41-45.
   b. Draw a frequency polygon.
   c. Identify the modal class and the median class.
3. The ages of 25 community members are given: 25, 31, 45, 42, 36, 28, 43, 49, 52, 28, 24, 40, 44, 36, 48, 52, 41, 54, 32, 38, 39, 41, 54, 50, 28.
   a. Create a frequency distribution table using intervals 21-25, 26-30, 31-35, and so forth.
   b. Draw the histogram of the distribution.
   c. What is the class interval with the greatest number of pupils?
   d. How many people are more than 35 years old?
4. The children's ward of a hospital has 25 patients. Their ages in years are: 1, 13, 17, 5, 2, 6, 4, 2, 6, 12, 15, 3, 2, 16, 4, 14, 12, 10, 9, 5, 10, 8, 7, 5, 11.
   a. Draw a frequency table using class intervals 1-3, 4-6, 7-9, 10-12, 13-15, 16-18.
   b. Draw a histogram to display the data.
   c. One doctor is assigned to all patients under the age of 10. How many patients does she have?
   d. Which class interval contains the median?
   e. Which is the modal class?
   f. Use the histogram to estimate the mode.

SSS 3 Maths Term 3 Answer Key

Lesson Title: Introduction to probability – Part 1	
Lesson Number: PHM3-L097	

1. a. Likely if you have a mobile phone, unlikely if you do not
   b. Certain
   c. Impossible
   d. Unlikely as it is Harmattan season
   e. Unlikely – very few people born on February 29 as it happens only once every 4 years

2. Pupils' own answers.

3. a. $S = \{12, 14, 16, 18, 20\}$
   b. $S = \{1, 2, 3, 4, 5, 6\}$
   c. $S = \{\text{head, tail}\}$
   d. $S = \{3, 5, 7, 9, 11, 13\}$
   e. $S = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40\}$
   f. $S = \{1, 2, 3, 4, 5, 6, \ldots 50, 51, 52\}$
   g. $S = \{5, 6, 7, 8, \ldots 18, 19, 20, \}$

Lesson Title: Introduction to probability – Part 2	
Lesson Number: PHM3-L098	

1. a. $\frac{7}{14} = \frac{1}{2}$ b. $\frac{3}{14}$ c. $\frac{4}{14} = \frac{2}{7}$ d. 0
2. $\frac{3}{10}, 30\%$ 3. a. $\frac{11}{20}$ b. $\frac{9}{20}$
4. $\frac{2}{5}$. It is more likely to rain as it has a higher probability.
5. a. $\frac{10}{20} = \frac{1}{2}$ b. $\frac{10}{20} = \frac{1}{2}$ c. $\frac{4}{20} = \frac{1}{5}$ d. $\frac{5}{20} = \frac{1}{4}$

Lesson Title: Addition law for mutually exclusive events – Part 1	
Lesson Number: TGM3-L099	

1. b. and d.   2. $\frac{13}{16}$
3. a. $\frac{9}{21}$ b. $\frac{17}{21}$ c. $\frac{14}{21} = \frac{2}{3}$
4. a. $\frac{8}{10} = \frac{4}{5}$ b. $\frac{4}{10} = \frac{2}{5}$ c. $\frac{8}{10} = \frac{4}{5}$ d. $\frac{2}{10} = \frac{1}{5}$
5. a. 0.2   b. 0.6

Lesson Title: Addition law for mutually exclusive events – Part 2
Lesson Number: PHM3-L100

1. 0.35 2. a. $\frac{6}{8} = \frac{3}{4}$ b. $\frac{7}{8}$ c. $\frac{1}{8}$
3. a. $\frac{11}{20}$ b. $\frac{9}{20}$ c. $\frac{5}{20} = \frac{1}{4}$
4. a. $\frac{3}{7}$ b. 12 c. 14
5. Yellow: $\frac{2}{7}$ Black: $\frac{11}{35}$ Pink: $\frac{14}{35} = \frac{2}{5}$

Lesson Title: Multiplication law for independent events – Part 1
Lesson Number: PHM3-L101

1. $\frac{6}{12} = \frac{1}{2}$ 2. a. $\frac{5}{11}$ b. $\frac{25}{121}$ c. $\frac{61}{121}$
3. a. 0.28 b. 0.42 c. 0.12 d. 0.18
4. a. $\frac{9}{36} = \frac{1}{4}$ b. $\frac{6}{36} = \frac{1}{6}$ 5. a. 0.52 b. 0.07

Lesson Title: Multiplication law for independent events – Part 2
Lesson Number: PHM3-L102

1. a. 0.00625 b. 0.04875 c. 120
2. a. $\frac{1}{12}$ b. $\frac{21}{144} = \frac{7}{48}$
3. a. 0.42 b. 0.144 c. 0.336
4. $\frac{1}{64}$ 5. $\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{46,656}$

Lesson Title: Application of the addition and multiplication laws
Lesson Number: PHM3-L103

1. $\frac{62}{121}$ 2. a. 0.28 b. 0.54 c. 0.82
3. a. $\frac{1}{6}$ b. $\frac{1}{36}$ c. $\frac{10}{36} = \frac{5}{18}$ d. $\frac{75}{216} = \frac{25}{72}$
4. a. $\frac{1}{216}$ b. $\frac{5}{216}$ c. $\frac{15}{216} = \frac{5}{72}$ d. $\frac{6}{216} = \frac{1}{36}$

Lesson Title: Outcome Tables
Lesson Number: PHM3-L104

1. a. b. $\frac{4}{16} = \frac{1}{4}$ c. $\frac{2}{16} = \frac{1}{8}$ d. 0

	Spinner A			
Spinner B	1	2	3	4
1	0	1	2	3
2	1	0	1	2
3	2	1	0	1
4	3	2	1	0

e. $\frac{10}{16} = \frac{5}{8}$

2. a.

	Packet A			
Packet B	P	B	Y	W
P	PP	BP	YP	WP
B	PB	BB	YB	WB
Y	PY	BY	YY	WY

b. $\frac{1}{12}$ c. $\frac{3}{12} = \frac{1}{4}$ d. $\frac{8}{12} = \frac{3}{4}$

3. a.

	First day		
Second day	C	P	G
C	CC	PC	GC
P	CP	PP	GP
G	CG	PG	GG

b. $\frac{1}{9}$ c. $\frac{3}{9} = \frac{1}{3}$ d. $\frac{8}{9}$ e. $\frac{1}{9}$

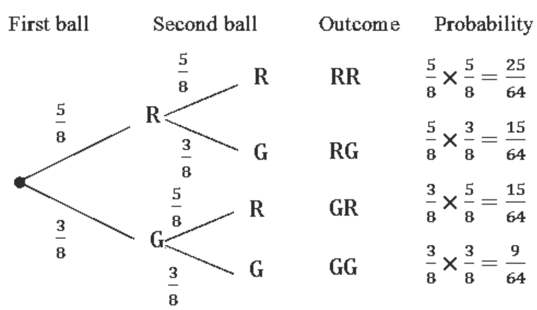
4. a.

	Second game				
First game	A	B	C	D	E
A	AA	AB	AC	AD	AE
B	BA	BB	BC	BD	BE
C	CA	CB	CC	CD	CE
D	DA	DB	DC	DD	DE
E	EA	EB	EC	ED	EE

b. $\frac{1}{25}$ c. $\frac{5}{25} = \frac{1}{5}$ d. $\frac{9}{25}$

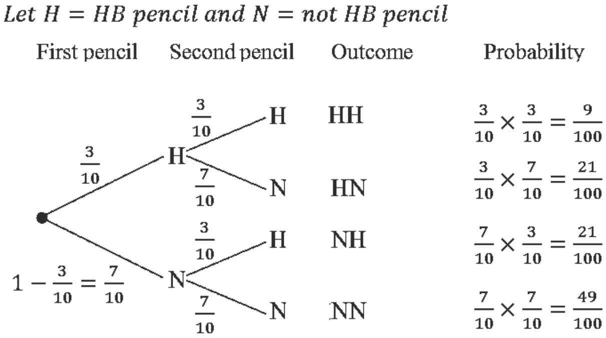
Lesson Title: Tree diagrams – Part 1
Lesson Number: PHM3-L105

1. a.



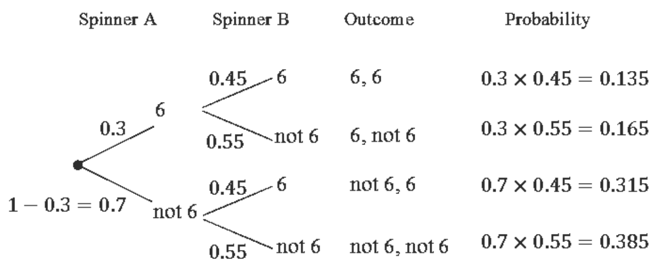
b. $\frac{25}{64}$ c. $\frac{15}{32}$ d. $\frac{55}{64}$ e. $\frac{39}{64}$

2. a. *Let H = HB pencil and N = not HB pencil*



b. $\frac{21}{50}$

3. a.

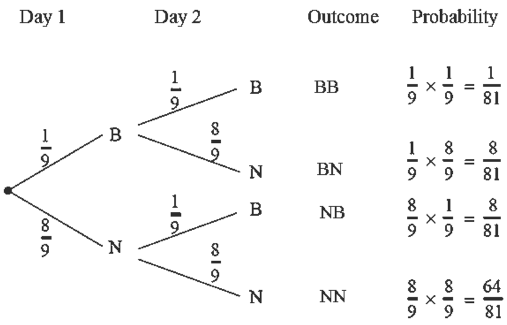


b. 0.385

c. 0.48

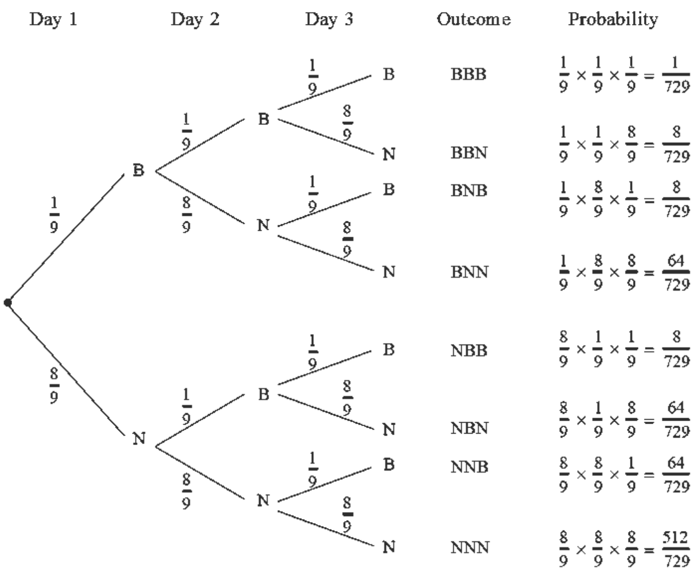
d. 0.315

4. a. *Let B = Burnt and N = Not burnt*



$\frac{17}{81}$

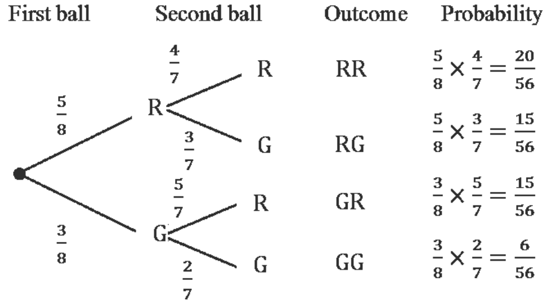
b. *Let B = Burnt and N = Not burnt*



$\frac{217}{729}$

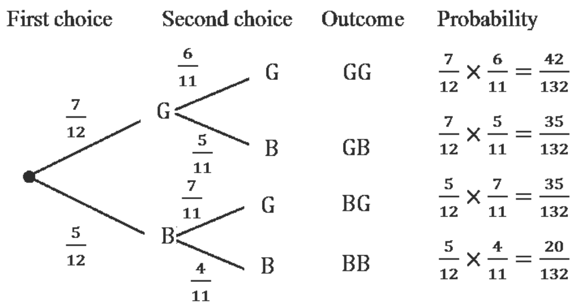
Lesson Title: Tree diagrams – Part 2
Lesson Number: PHM3-L106

1. a.



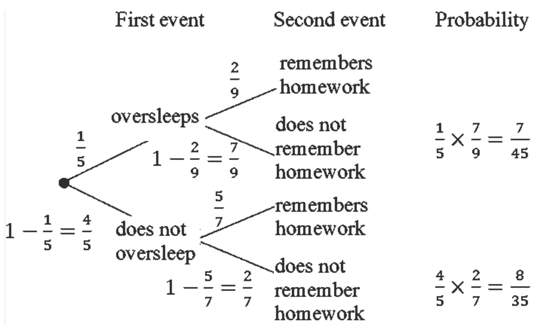
b. $\frac{20}{56}=\frac{5}{14}$ c. $\frac{50}{56}=\frac{25}{28}$ d. $\frac{30}{56}=\frac{15}{28}$

2. a.



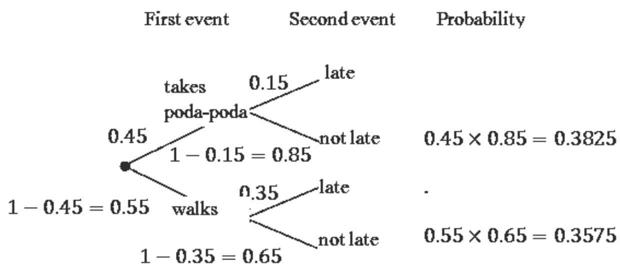
b. $\frac{42}{132}=\frac{7}{22}$ c. $\frac{70}{132}=\frac{35}{66}$ d. $\frac{112}{132}=\frac{28}{33}$

3. a.



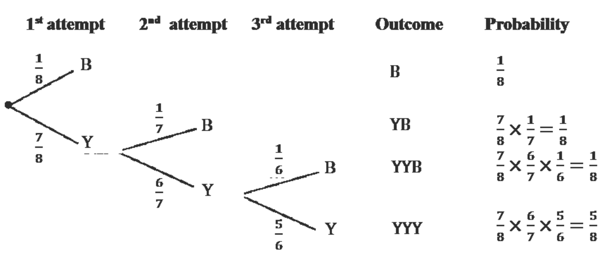
b. $\frac{605}{1575}=\frac{121}{315}$

4. a.



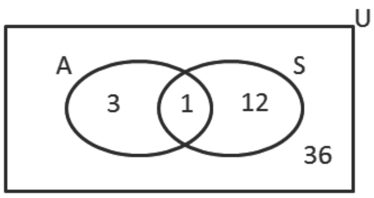
b. 0.74

5. a.

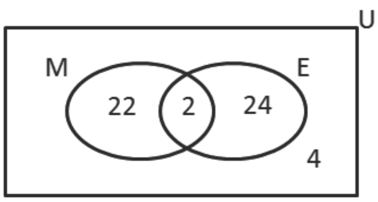


b. $\frac{3}{8}$

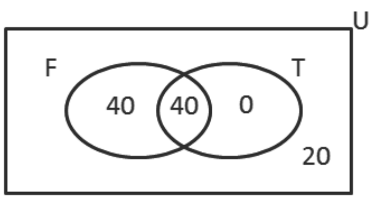
Lesson Title: Venn diagrams
Lesson Number: PHM3-L107

1. a. 

b. $\frac{16}{52} = \frac{4}{13}$ c. $\frac{1}{52}$

2. a. 

b. $\frac{22}{52} = \frac{11}{26}$ c. $\frac{26}{52} = \frac{1}{2}$ d. $\frac{24}{52} = \frac{6}{13}$
e. $\frac{4}{52} = \frac{1}{13}$ f. $\frac{48}{52} = \frac{12}{13}$

3. a. 

b. $\frac{40}{100} = \frac{2}{5}$ c. 0 d. $\frac{40}{100} = \frac{2}{5}$

4. a. $\frac{36}{80}$ b. $\frac{57}{80}$

Lesson Title: Solve probability problems
Lesson Number: PHM3-L108

1. a. 40 pupils b. $\frac{23}{40}$ c. $\frac{11}{40}$ d. $\frac{11}{40}$
2. a. $\frac{1}{100}$ b. $\frac{5}{100} = \frac{1}{20}$
3. a. $\frac{82}{231}$ b. $\frac{149}{231}$
4. a. $\frac{4}{36} = \frac{1}{9}$ b. $\frac{6}{36} = \frac{1}{6}$ c. $\frac{14}{36} = \frac{7}{18}$
5. $\frac{40}{81}$
6. a. $\frac{11}{357}$ b. $\frac{44}{1,785}$ c. $\frac{209}{1,785}$

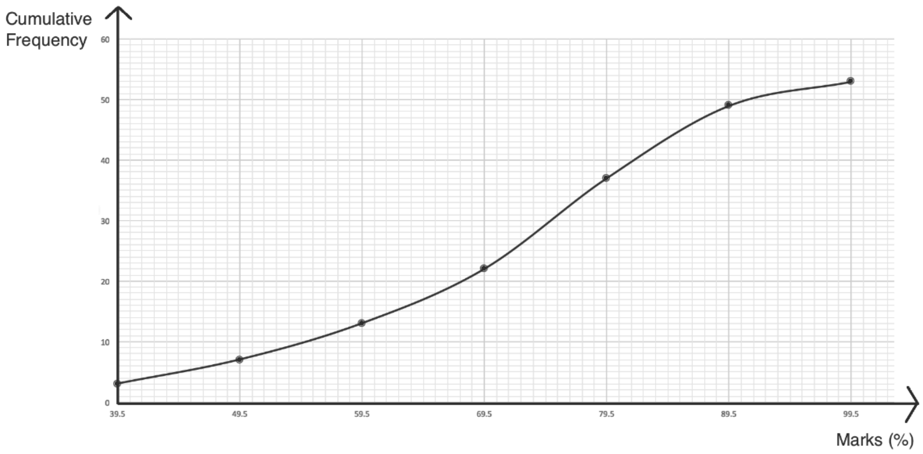
Lesson Title: Review of cumulative frequency curve
Practice Activity: PHM3-L109

1. a. Cumulative frequency table:

Pupils' Scores on a Test			
Marks	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
30-39	3	39.5	3
40-49	4	49.5	3+4=7
50-59	6	59.5	7+6=13

60-69	9	69.5	13+9=22
70-79	15	79.5	22+15=37
80-89	12	89.5	37+12=49
90-99	4	99.5	49+4=53
Total	53		

b. Cumulative frequency curve:



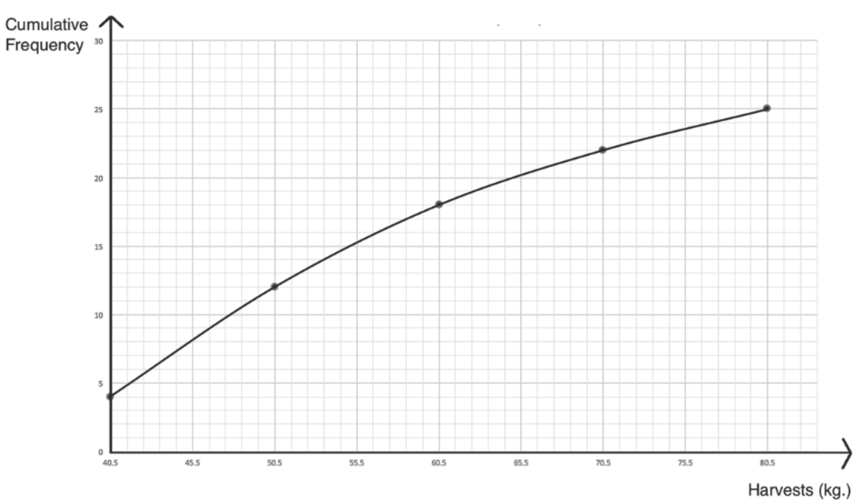
c. Median: 72.5 marks (estimated)

d. Interquartile range: $82.5 - 60.5 = 22.5$ marks (estimated)

2. a. Cumulative frequency table:

Cassava harvests			
Marks	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
31-40	4	40.5	4
41-50	8	50.5	4+8=12
51-60	6	60.5	12+6=18
61-70	4	70.5	18+4=22
71-80	3	80.5	22+3=25
Total	25		

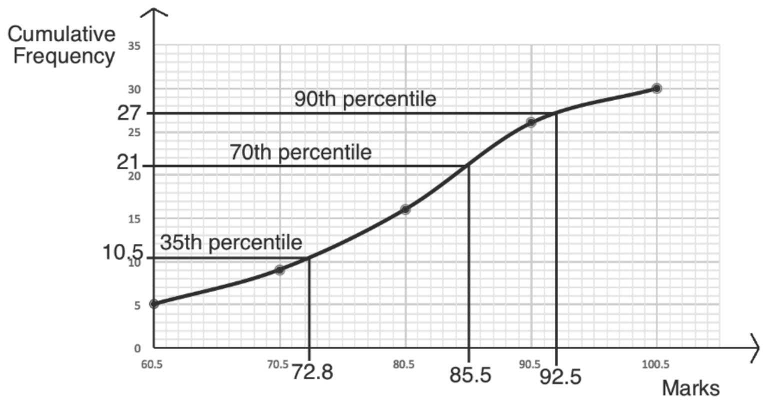
b. Cumulative frequency curve:



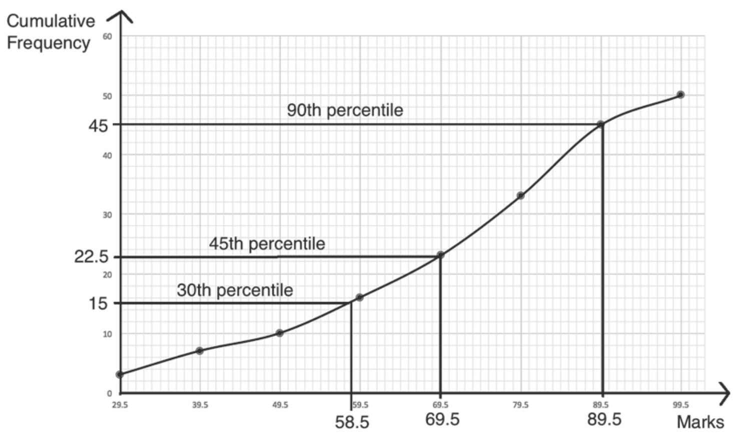
c. Semi-interquartile range: $\frac{63.5-43.5}{2} = \frac{20}{2} = 10$ kg (estimate)

Lesson Title: Percentiles
Practice Activity: PHM3-L110

1. Estimated values: a. 72.8; b. 85.5; c. 92.5. See graph:



2. Estimated values: a. 58.5; b. 69.5; c. 89.5. See graph:



Lesson Title: Applications of percentiles
Practice Activity: PHM3-L111

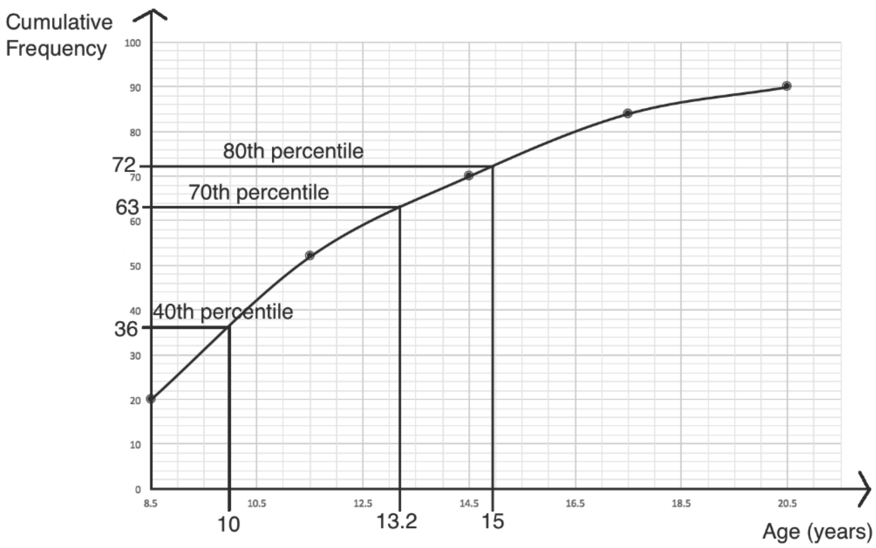
1.
   a. Cumulative frequency table:

Pupils' ages			
Age (years)	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
5 – 8	20	8.5	20
9 – 11	32	11.5	20+32=52
12 – 14	18	14.5	52+18=70
15 – 17	14	17.5	70+14=84
18 – 20	6	20.5	84+6=90
Total	90		

b. Cumulative frequency curve: see below.
c. 80th percentile (see curve below): 15 years old
d. 36 pupils
e. 10 years old
f. 27 pupils
g. 13 years old

Lesson Title: Measures of dispersion	
Practice Activity: PHM3-L112	

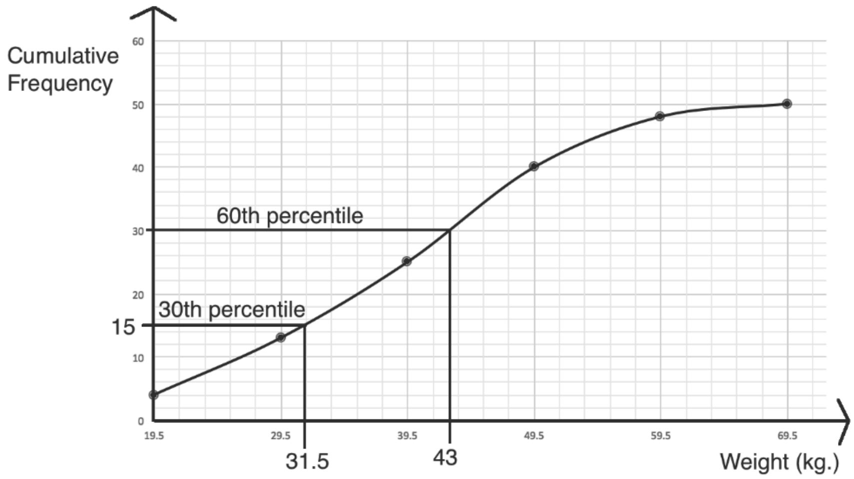
1. a. 10 cm; b. 125 cm; c. 11.33
2. Range: 20 marks; variance: 46.6
3. a. 15 kg; b. 27 kg; c. 21.5



4.
a. Cumulative frequency table:

Chimpanzees' weights			
Age (years)	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
10 – 19	4	19.5	4
20 – 29	9	29.5	4+9=13
30 – 39	12	39.5	13+12=25
40 – 49	15	49.5	25+15=40
50 – 59	8	59.5	40+8=48
60 – 69	2	69.5	48+2=50
Total	50		

b. Cumulative frequency curve: see below.
c. 15 chimpanzees
d. 20 chimpanzees
e. 43 kg



Lesson Title:	Standard deviation of ungrouped data
Practice Activity:	PHM3-L113

1. $s \approx 5.92$ 2. $s \approx 7.36$ 3. $s \approx 8.22$ 4. $s \approx 3.54$

Lesson Title:	Standard deviation of grouped data – Part 1
Practice Activity:	PHM3-L114

1. $\bar{x} = 37.1; s = 1.3$ 2. $s \approx 1.11$ 3. $\bar{x} \approx 1.77; s \approx 0.1$

Lesson Title:	Standard deviation of grouped data – Part 2
Practice Activity:	PHM3-L115

1. $\bar{x} = 31.25; s \approx 5.54$ 2. $s \approx 3.26$ 3. $\bar{x} \approx 77.17; s \approx 10.65$

Lesson Title:	Standard deviation practice
Practice Activity:	PHM3-L116

1. a. 7 years; b. 6 years old; c. 2.35
2. $\bar{x} = 7.9$ marks; $s = 3.2$
3. $\bar{x} = 3.38$ kg.; $s = 0.51$

Lesson Title:	Mean deviation of ungrouped data
Practice Activity:	PHM3-L117

1. $MD = 2$
2. $\bar{x} = 40; MD = 1.2$
3. a. 22 kg; b. $\bar{x} = 77$ kg; c. $MD = 5.8$
4. $\bar{x} = 1.1$ kg; $MD = 0.2$

Lesson Title:	Mean deviation of grouped data – Part 1
Practice Activity:	PHM3-L118

1. $MD = 102$
2. a. $\bar{x} = 156$ cm; b. $MD = 1.4$
3. $\bar{x} = 3.5$ kg; c. $MD = 0.47$
4. $\bar{x} = 40$ kg; $MD = 4.5$

Lesson Title:	Mean deviation of grouped data – Part 2
Practice Activity:	PHM3-L119

1. $\bar{x} = 130$ cm.; $\mathrm{MD} = 4.2$
2. $\mathrm{MD} = 5.2$
3. $\bar{x} = 77$ kg; $\mathrm{MD} = 4.17$

Lesson Title:	Statistics and probability
Practice Activity:	PHM3-L120

1. a. Frequency table:

Age (years)	21	22	23	24	25	26	27
Frequency (f)	1	3	3	3	4	3	3

b. $\bar{x} = 24.35$; $s = 1.79$

c. 0.5

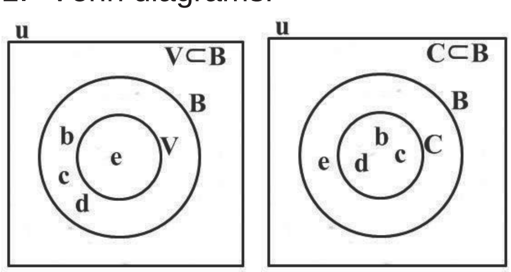
2. a. $\bar{x} = 10.75$

b. $MD = 4$

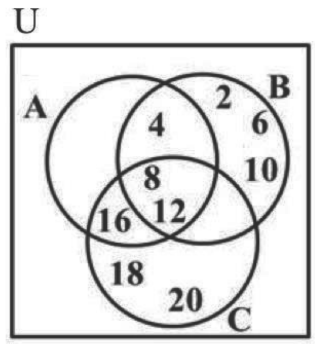
c. 0.6

Lesson Title:	Sets
Practice Activity:	PHM3-L121

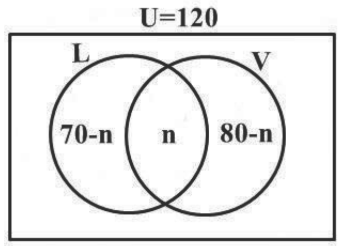
1. Venn diagrams:



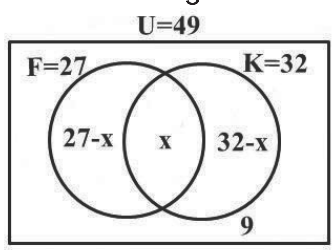
2. a. i{4, 8, 12}; ii. {8, 12}; iii. {8, 12, 16}; iv. {8, 12}; b. Venn diagram:



3. See Venn diagram below; b. 30 students; c. 40 played Lawn tennis only



4. a. See Venn diagram below; b. i. 19 ii. 8 iii. 13



Lesson Title:	Indices and logarithms
Practice Activity:	PHM3-L122

1. a. $27a^4b^6$; b. $5n$
2. a. $4m^6n^8$; b. $\frac{4x^4}{y^2}$; c. 1
3. a. 5; b. 81
4. a. $25\sqrt{5}$; b. $2\frac{1}{2}$
5. a. $x = 4$; b. $x = 2$; c. $x = -3$
6. a. $y = 3$; b. $x = -5$
7. a. $\log_4 128$; b. $\log_3 36$
8. a. $\log_7 17 - \log_7 3$; b. $\log_9 39 - \log_9 5$
9. a. 1; b. $\log_3 \frac{16}{9}$
10. a. 2; b. $\frac{2}{3}$

Lesson Title:	Sequences and series
Practice Activity:	PHM3-L123

1. 45 2. -27 3. 19 4. 6,561 5. $\frac{256}{2,187}$

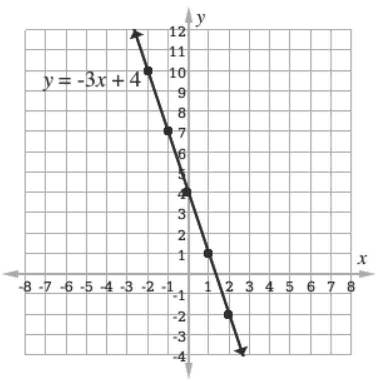
6. 9 7. -24 8. 121 9. $-2,046$

Lesson Title:	Ratio/Proportion/Rate/Percentages
Practice Activity:	PHM3-L124

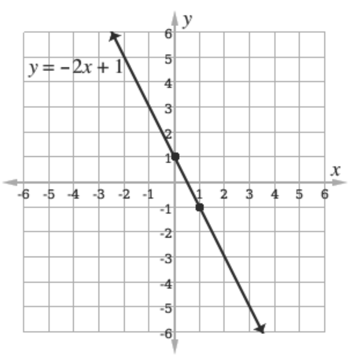
1. 200 kg 2. Le 7500.00 3. a. 4%; b. £166.40 4. 360 litres
5. 25 labourers
6. Momodu received Le 1,500.00 and Musa received Le 900.00.

Lesson Title:	Linear equations
Practice Activity:	PHM3-L125

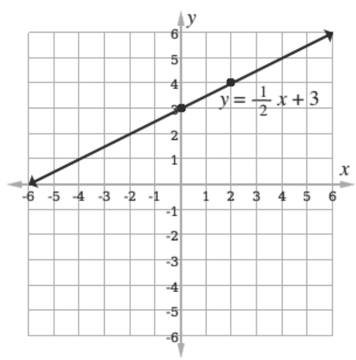
1. Graph:



2. a. $m = -2$; b. $m = -1\frac{1}{3}$
3. $m = 1\frac{1}{2}$
4.

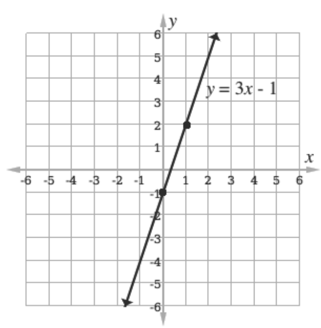


5.

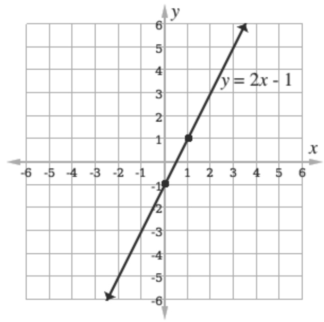


6. a. $y = -2x - 5$; b. $y = 2x - 1$

7. Equation: $y = 3x - 1$
Graph:

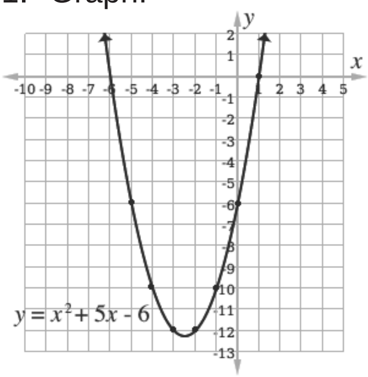


8. Equation: $y = 2x - 1$
Graph:



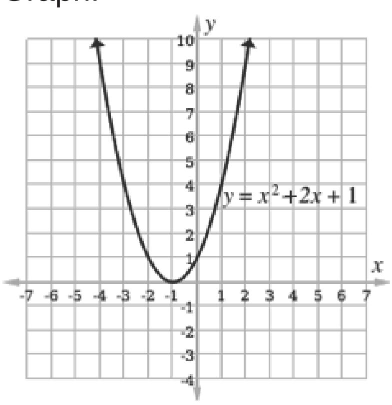
Lesson Title:	Quadratic equations
Practice Activity:	PHM3-L126

1. Graph:



   a. $x = -6, 1$; b. $(-2.5, -12.25)$; c. $x = -\frac{5}{2}$ or $x = -2.5$
2. $x = -1$ and $x = 5$

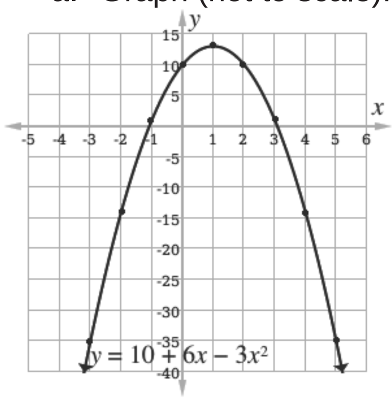
3. Solution: $x = -1$.
Graph:



4. Table of values:

x	-3	-2	-1	0	1	2	3	4	5
y	-35	-14	1	10	13	10	1	-14	-35

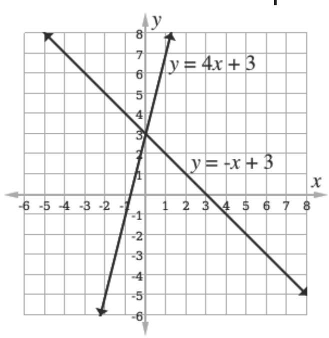
a. Graph (not to scale):



b. $x = -1.08,\ 3.08$; c. $(1, 13)$; d. $x = 1$; e. $y = 11.25$

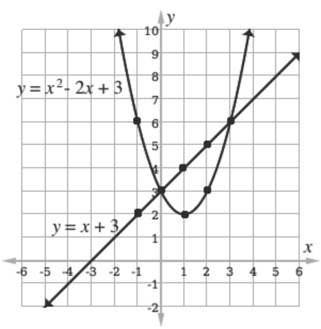
Lesson Title:	Simultaneous equations
Practice Activity:	PHM3-L127

1. $(-2, 3)$
2. The solution is the point of intersection, $(0, 3)$.



3. $(-3,1)(-2,-1)$

4. Graph:



Solutions: $(0, 3)$ and $(3, 6)$

5. One pineapple costs Le 6,800.00, one mango costs Le 1,400.00.

Lesson Title:	Variation
Practice Activity:	PHM3-L128

1. $y = 36$
2. 145 customers
3. a. $y = \frac{96}{x}$; b. $y = 3$
4. a. 72; b. 32
5. a. $c = 20{,}000 + 12{,}000u$, where c is cost and u is internet usage.
   b. He will pay Le 164,000.00.

Lesson Title:	Angles of polygons
Practice Activity:	PHM3-L129

1. a. $162°$; b. $18°$
2. a. 22; b. $16\frac{4}{11}°$
3. 9
4. 12
5. $x = 17°$

Lesson Title:	Circles
Practice Activity:	PHM3-L130

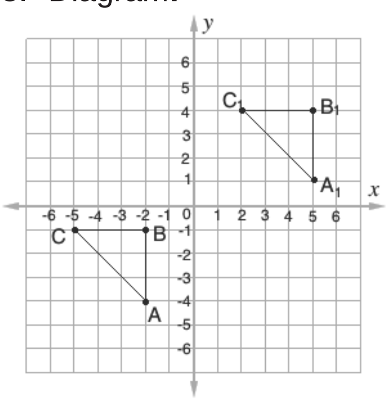
1. 21 m
2. 36 m
3. 21 cm
4. 98 cm^2
5. 44°
6. 11 cm
7. a. 4.72 cm; b. 15.0 cm

Lesson Title:	Circle theorems
Practice Activity:	PHM3-L131

1. a. $\angle DOC = 84°$; b. $\angle ACD = 48°$
2. 72°
3. 60°

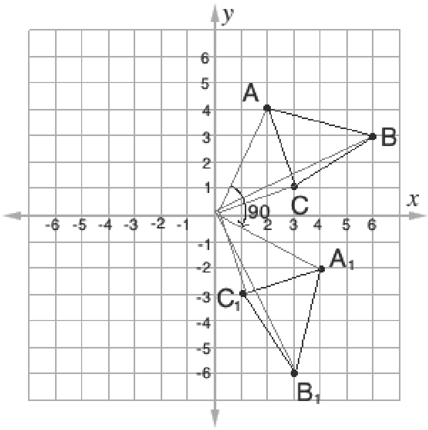
Lesson Title:	Transformations on the Cartesian Plan
Practice Activity:	PHM3-L132

1. $D_1(1, -4)$, $E_1(3, -2)$, $F_1(4, -5)$
2. $\binom{-21}{39}$
3. Diagram:



4. $A_1(4, -2)$, $B_1(3, -6)$, $C_1(1, -3)$

Diagram:



Lesson Title:	Area and surface area
Practice Activity:	PHM3-L133

1. 20 m
2. 12 cm
3. $b = 30$ cm; $h = 6$ cm
4. 63 m^2
5. 100 mm
6. 7 cm
7. 56 cm^2
8. 5 m
9. 282.9 m^2

Lesson Title:	Volume
Practice Activity:	PHM3-L134

1. Volume of oil= $10{,}048\ \text{cm}^3$, height= 22.33 cm
2. 60 ball bearings
3. $213.2\ \text{m}^3$

Lesson Title:	Trigonometry
Practice Activity:	PHM3-L135

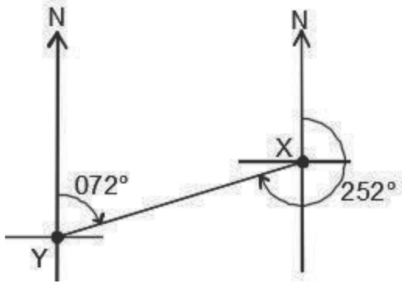
1. |BC|= 4.70 cm, |AC|= 1.71 cm, ∠B = 20°
2. hypotenuse: 10 cm; angles: 53.13°, 36.87°
3. a. 19.47°; b. $4\sqrt{2}$ m or 5.66 m
4. a. |AB| = 4 cm, |BC| = 6.93 cm, $\angle B = 30°$; b. |PQ| = 13.42 cm, $\angle P = 26.57°$, $\angle Q = 63.43°$

Lesson Title:	Angles of elevation and depression
Practice Activity:	PHM3-L136

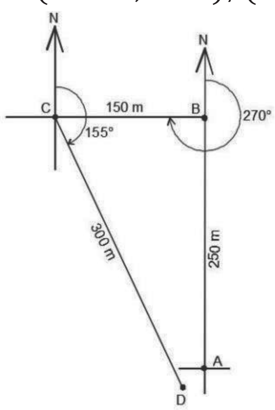
1. 60 m
2. 50.2°
3. 57.3°
4. a. 130.4 m; b. 72.9°

Lesson Title:	Bearings and distances
Practice Activity:	PHM3-L137

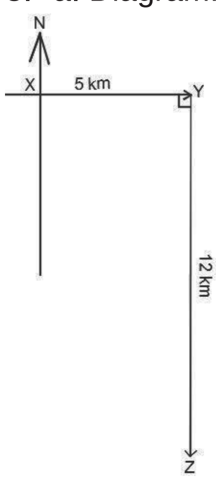
1. $252°$; Diagram:



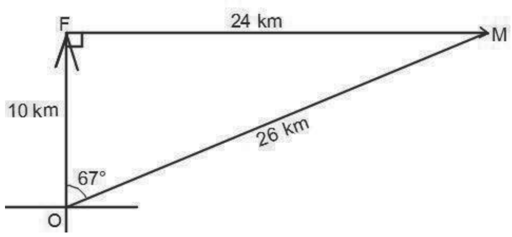
2. a. $(250\text{ m}, 000°)$, $(150\text{ m}, 270°)$, $(300\text{ m}, 155°)$; b. diagram:



3. a. Diagram: see below; b. $|XZ| = 13$ km; c. $\overrightarrow{XZ} = (13\text{ km}, 157°)$



4. $\overrightarrow{RT} = (15\text{ km}, 053°)$
5. a. Diagram: see below; b. 26 km; c. $\overrightarrow{OM} = (26\text{ km}, 067°)$

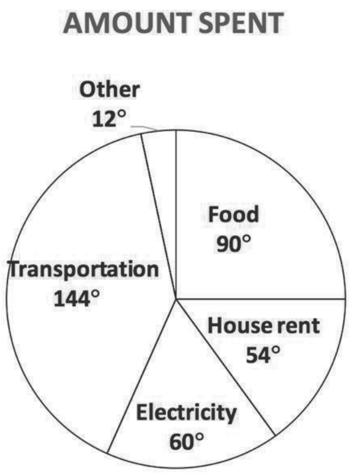


Lesson Title:	Probability
Practice Activity:	PHM3-L138

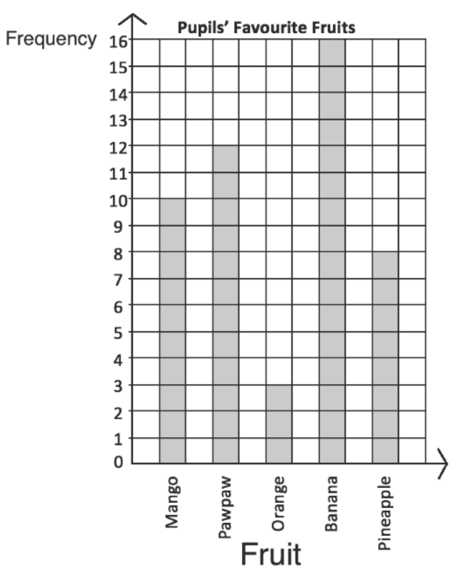
1. a. $\frac{1}{10}$; b. $\frac{39}{50}$; c. $\frac{16}{25}$; d. $\frac{17}{25}$ 2. 24 males 3. $\frac{9}{26}$
4. a. $\frac{1}{6}$; b. $\frac{1}{2}$; c. $\frac{1}{3}$ 5. a. $\frac{1}{3}$; b. $\frac{1}{3}$; c. 0

Lesson Title:	Statistics – ungrouped data
Practice Activity:	PHM3-L139

1. a. See pie chart below; b. 40%



2. a. See bar chart below; b. Banana is the most popular; c. Orange is the least popular; d. 32.7%; e. 22 pupils.

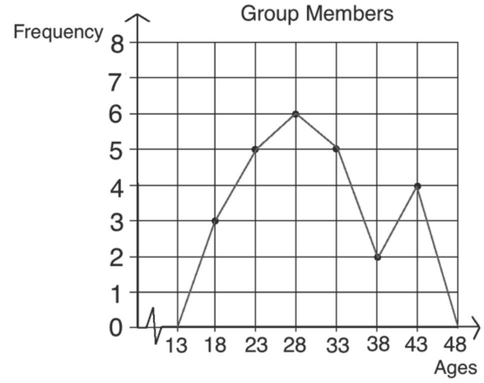


3. 3.3 kg
4. a. 50 people; b. 72.5%

Lesson Title:	Statistics – grouped data
Practice Activity:	PHM3-L140

1. mean = 7.5 marks; median = 7 marks; mode = 7 marks
2. a. Frequency table:

Group Members	
Age	**Frequency**
16-20	3
21-25	5
26-30	6
31-35	5
36-40	2
41-45	4



b. Frequency polygon:

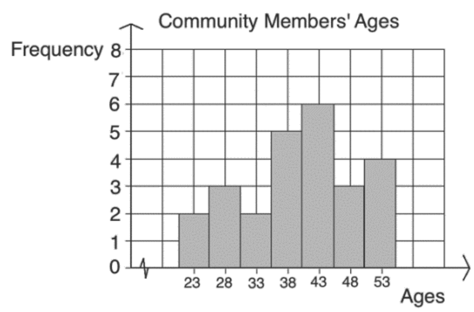
c. Modal class: 26-30; Median class: 26-30

3. a. and b.: see below; c. 41-45; d. 18 people.

Frequency Table:

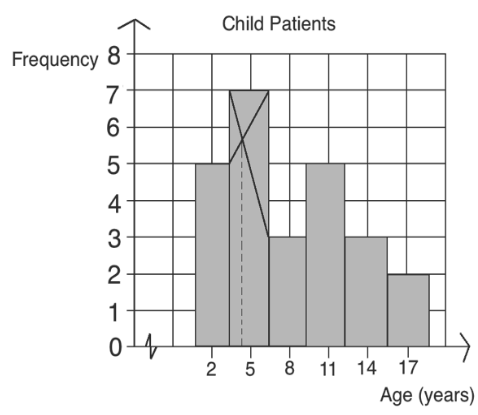
Ages	**Frequency (f)**	**Class mid-points**
21-25	2	23
26-30	3	28
31-35	2	33
36-40	5	38
41-45	6	43
46-50	3	48
51-55	4	53

Histogram:



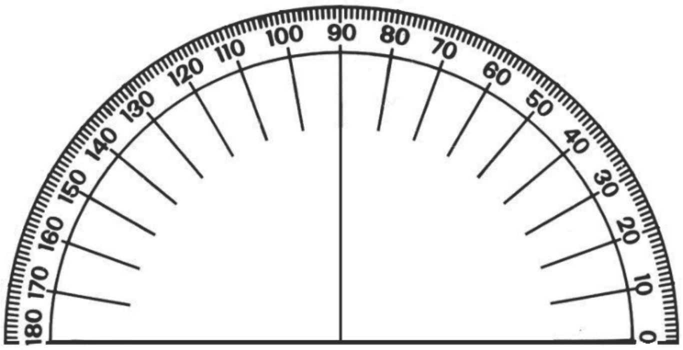
4. a. and b:. see below; c. 15 patients; d. 7-9 years; e. 4-6 years;
   f. reasonable estimates are around 4-4.5 years.

Child Patients	
Ages	**Frequency**
1-3	5
4-6	7
7-9	3
10-12	5
13-15	3
16-18	2



Appendix I: Protractor

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.



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