Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' handbook for

JSS Mathematics

JSS 3

Term 1

STRICTLY NOT FOR SALE

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Please see final page for further information.

FOREWORD

The production of Teachers' Guides and Pupils' handbooks in respect of English and Mathematics for Junior Secondary Schools (JSSs) in Sierra Leone is an innovation. This would undoubtedly lead to improvement in the performance of pupils in the Basic Education Certificate Examination in these subjects. As Minister of Basic and Senior Secondary Education, I am pleased with this development in the educational sector.

The Teachers' Guides give teachers the support they need to utilize appropriate pedagogical skills to teach; and the Pupils' Handbooks are designed to support self-study by the pupils, and to give them additional opportunities to learn independently.

These Teachers' Guides and Pupils' Handbooks had been written by experienced Sierra Leonean and international educators. They have been reviewed by officials of my Ministry to ensure that they meet specific needs of the Sierra Leonean population.

I call on the teachers and pupils across the country to make the best use of these educational resources.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd. Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank the Department for International Development (DFID) for their continued support. Finally, I also thank the teachers of our country - for their hard work in securing our future.

Timbo

Mr. Alpha Osman Timbo
Minister of Basic and Senior Secondary Education

The Ministry of Basic and Senior Secondary Education, Sierra Leone, policy stipulates that every printed book should have a lifespan of 3 years.

To achieve this DO NOT WRITE IN THE BOOKS.

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Introduction

to the Pupils’ Handbook

These practice activities are aligned to the lesson plans in the Teachers’ Guide, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Education, Science and Technology.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.

   Learning Outcomes
5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

Lesson Title: Sorting Objects	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-001	**Class:** JSS 3

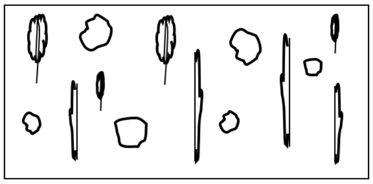
Learning Outcomes
By the end of the lesson, you will be able to:
1. Collect and sort objects into groups.
2. Describe groups of objects.

Overview

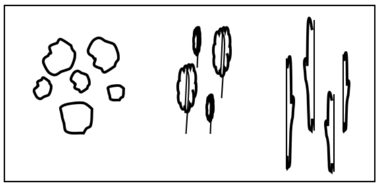
In this lesson, you will practise sorting objects according to properties they have in common. To sort means to separate things according to the type of object.

When we have a collection of objects, we can use a common property to sort them into groups. This property can be used to describe the sort of group we make with the objects.

For example, here is a group of stones, sticks, and leaves:



We can sort them according to the type of object. We can group all the stones together. We can also group all the leaves together, and all the sticks together.



We can sort the objects by any other common property. You could sort the stones into 2 groups: large stones and small stones. You could also sort them by colour. For example, black stones and white stones.

Solved Examples

1. Use the words in the box below. Write the words down in 2 lists according their common properties. Give each list a name.

Doctor	Teacher	Shop	Tailor	Market	School
Mechanic	Hospital	Engineer	Radio Station		

Solution

Look at the words and see if you notice any common properties. You will notice that some of the words are jobs that people have. Some of the words are places in a community.

Write the lists:

Jobs: Doctor, Teacher, Tailor, Mechanic, Engineer
Places: Shop, Market, School, Hospital, Radio Station

2. This is a list of pupils in the front row of a JSS 3 class:

   John, Hawa, Sara, Mustapha, Aminata, Charles, Ahmed, Sia, Fatu

   The teacher wants to put the pupils into groups. Help her sort them into the following groups. Write a list for each.

   a. Boys and girls
   b. Names starting with letters A-M, names starting with letters N-Z.
   c. Names with more than 4 letters, names with 4 letters or fewer.

   Solutions

   a. Boys: John, Mustapha, Charles, Ahmed
      Girls: Hawa, Sara, Aminata, Sia, Fatu

   b. Names starting with letters A-M: John, Hawa, Mustapha, Aminata, Charles, Ahmed, Fatu
      Names starting with letters N-Z: Sara, Sia

   c. Names with more than 4 letters: Mustapha, Aminata, Charles, Ahmed
      Names with 4 letters or fewer: John, Hawa, Sara, Sia, Fatu

Practice

1. Use the words in the box below. Write the words in 2 lists based on their common properties. Give each list a name.

Math	Exercise book	Pen	Chemistry	Ruler	Desk
English	Pencil	Social Studies	Protractor	Biology	

2. Use the letters in the box below. Write the letters in 2 lists based on their common properties. Give each list a name.

B	T	A	S	O	P	R	L	E	N	Q	U

3. This is a list of teachers in a school:

   Hawa Bangura, Issa Koroma, John Kamara, Mohamed Bah, Aminata Kamara, Juliet Nyalloma

   They will work in 2 groups to plan an activity for the school. Help her sort them into the following groups. Write a list for each.

   a. Males and females
   b. Surnames starting with letters A-M, surnames starting with N-Z.
   c. Surnames starting with K, surnames starting with any other letters.

Lesson Title: Introduction to Sets	Theme: Numbers and Numeration
Practice Activity: PHM-09-002	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify a set as a well-defined collection of objects or ideas.

Overview

Maths has its own language. In maths, a **set** is a collection of objects. We study sets so that we can collect and examine objects and ideas. These objects and ideas share a common property. This helps us to classify and count them**.**

When we want to talk about sets, we use special language and notation. We will use a set of stationery items to learn some of the basic language and set notation. These are stationery items: book, pencil, paper, ruler, pen.

We can describe this set with S = {stationery items}. The curly brackets {} tell us that this is a set of objects. S = {stationery items} is read as "S is the set of stationery items".

We can also write S = {book, pencil, paper, ruler, pen}. We have listed all of the objects in the set S. It does not matter in what order the objects are written. We just need to list all of them. We call the objects in the set **members** or **elements**. We separate them using commas.

We give the members or elements of the set by this symbol: $\in$

We can write that the pen belongs to set S as: pen $\in$ S. We read it as: "Pen is an element of set S."

There are 5 elements in the set S. The number of elements in a set is given by a lowercase n and brackets. $n(S) = 5$ says that there are 5 elements in S.

A set is a well-defined collection of objects or ideas. If the set is not well-defined, we will not be able to identify all its elements. For example, the set of greatest football players is not well-defined because everybody has their own opinion of what makes a football player great. We cannot say with certainty who should belong to the set.

Solved Examples

1. Decide whether each set is well-defined. If it is a well-defined set, list its elements.
    a. Vowels
    b. Tall people in your class
    c. Positive numbers less than 5
    d. The best foods in Sierra Leone

Solutions

a. This set is well-defined because we can identify all of the members.
   Vowels = {a, e, i, o, u}
b. 'Tall people in your class' is not a well-defined set. People might disagree on what 'tall' means. Some of your classmates might think they are tall, while others disagree.
c. This set is well-defined because we can identify all of the members.
   Positive numbers less than 5 = {1, 2, 3, 4}
d. 'The best foods in Sierra Leone' is not a well-defined set. People might disagree on what the best foods are.

2. Consider P = {Provinces in Sierra Leone}. For this set:
   a. Describe the set in words.
   b. Is the set well-defined? Explain.

   If the set is well-defined:

   c. List the elements of the set;
   d. Select the third element of the set and use set notation to show that it belongs to the set;
   e. Write the number of elements in the set using set notation.

Solutions

a. P is the set of provinces in Sierra Leone.
b. Yes, the set is well-defined because we can identify all of its members.
c. The members are P = {Eastern Province, Northern Province, Southern Province, Western Area}
d. The third element in this list is Southern Province. Note that you could write the provinces in any order in part c. In set notation, Southern Province $\in$ P.
e. n(P) = 4

3. Consider the set Fruit = {Banana, Orange, Pineapple, Watermelon, Plum}.
   a. Write the number of elements in the set using set notation.
   b. Use set notation to show that Plum belongs to the set.
   c. Use set notation to show that Orange belongs to the set.

Solutions

a. There are 5 elements in the set. In set notation, n(Fruit) = 5.
b. Plum $\in$ Fruit
c. Orange $\in$ Fruit

Practice

1. Consider the set of Colours = {White, Red, Blue, Yellow, Orange, Purple}.
    a. Write the number of elements in the set using set notation.
    b. Use set notation to show that Blue belongs to the set.
    c. Use set notation to show that Orange belongs to the set.

2. Consider the set Continents = {Africa, Antarctica, Australia, Asia, Europe, North America, South America}
    a. Write the number of elements in the set using set notation
    b. Use set notation to show that Africa belongs to the set.
    c. Use set notation to show that Asia belongs to the set.

3. Consider the set M = {Months having 31 days}
    a. Describe the set in words.
    b. Is the set well-defined? Explain.

    If the set is well-defined:

    c. List the elements of the set;
    d. Select the fifth element of the set and use set notation to show that it belongs to the set;
    e. Write the number of elements in the set using set notation.

Lesson Title: Sets in Real Life	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-003	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify sets of objects or ideas from everyday life.
2. Sort objects or ideas from everyday life into sets.

Overview

Sets can be made from things we can see and touch which are called **objects**, and things we cannot see and touch which are called **ideas**. We do not need to worry whether they are objects or ideas as long as we can group them using a common property. We use the general term **element** or **member** to identify them.

Remember that it does not matter what order the elements of a set are listed in. For example, the set of Colours can be written in the following ways and more. These are all the same:

Colours = {red, blue, yellow}

Colours = {blue, red, yellow}

Colours = {yellow, red, blue}

Consider the set D of daily activities that people do. Here are some elements of D: D = {eat, sleep}. D can have many more elements.

D is well-defined. However, there are a lot of activities people do daily, and we cannot list them all. D is a special type of set called an **infinite set**. We will talk about this type of set in another lesson. For now, we will specify the number of elements we want to list in our sets.

Solved Examples

1. List 5 elements from the set of fruits, F.

   Solution
   You may list any 5 fruits that you can think of. For example:

   F = {banana, orange, plum, pineapple, avocado}

2. Fatu made a list of items from her everyday life. They are given in the box below. Sort the items into sets, and write each set using set notation.

mother	exercise book	school	brother	
market	house	friend	father	pen
maths book	uncle	bag	bank	teacher

Solution

Sort the items into reasonable sets. Looking at the words in the box, there are people, locations, and things. List the items in each of these sets. Make sure each word from the box appears in a set:

The set of people, P = {mother, brother, friend, father, uncle, teacher}

The set of locations, L = {school, market, house, bank}

The set of things, T = {exercise book, pen, maths book, bag}

3. List 3 items from the set C, cities in Sierra Leone.

Solution

You may list any 3 cities in Sierra Leone. For example:

C = {Freetown, Bo, Kenema}

Practice

1. List 5 items from the set D, districts in Sierra Leone.
2. List 4 items from the set A, countries in Africa.
3. List 3 items from the set F, football teams.
4. Foday made a list of items from his everyday life. They are given in the box below. Sort the items into sets, and write each set using set notation.

Abu	desk	trousers	Fatu	hat	bed	
Hawa	shirt	David	jacket	table	chair	
shoes	Mustapha	socks	Alice			

Lesson Title: Describe Sets of Objects	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-004	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Describe sets using words.
2. Define the properties of a set of objects or ideas.

Overview

In the previous lessons, you practised sorting into sets and listing the elements of sets. In this lesson, you will practise describing sets and you will define the properties of a set.

Consider the set P: P = {Eastern Province, Northern Province, Southern Province, Western Area}.

Recall that we can describe P by saying, "P is the set of provinces in Sierra Leone."

Another way we can identify a set is by defining the properties of the set. We use a special notation to do this.

We can write the set P in this way: P = {x : x is a province in Sierra Leone}

This says that P is the set of all x, such that x is a province in Sierra Leone. We use a small letter to represent the elements. For this set, we have used 'x'. We use the colon symbol : and we read it as, "such that".

Consider another example: J = {p : p is a pupil of JSS3 }

This says, "J is the set of all p such that p is a pupil of JSS3." Because you are a pupil in JSS3, you are an element of this set.

Solved Examples

1. D is the set of days of the week. Write the set D using all of the ways you know.

   Solution
   The set D can be written in the following ways. These all describe the same set.

   D = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
   D = {days of the week}
   D = {x : x is a day of the week}

2. F is the set of letters in the word 'French'. Write the set F by:
   a. Defining its properties
   b. Listing its elements

Solutions

a. F = {x : x is a letter in the word French}
b. F = {f, r, e, n, c, h}

3. Write each of the following sets by defining its properties:
   a. V is the set of vegetables
   b. D is the set of districts in Sierra Leone
   c. C is the set of countries in Europe
   d. M is the set of letters in the word 'monkey'

Solutions

a. V = {w : w is a vegetable}
b. D = {x : x is a district in Sierra Leone}
c. C = {y : y is a country in Europe}
d. M = {z : z is a letter in the word 'monkey'}

4. Write each of the following out in words:
   a. F = {x : x is a fruit}
   b. B = {y : y is a boy}
   c. G = {z : z is a letter in the word girl}

Solutions

a. F is the set of all x such that x is a fruit.
b. B is the set of all y such that y is a boy.
c. G is the set of all z such that z is letter in the word girl

Practice

1. M is the set of months of the year. Write the set M using all the ways you know.
2. C is the set of letters in the word 'chemistry'. Write the set C using all the ways you know.
3. P is the set of letters in the word 'phone'. Write the set P by:
   a. Defining its elements
   b. Listing its elements
4. F is the set of colours on the flag of Sierra Leone. Write the set F by:
   a. Defining its elements
   b. Listing its elements
5. Write each of the following out in words:
   a. S = {x : x is a sauce}
   b. F = {y : y is a female}
   c. M = {z : z is a letter in the word male}

Lesson Title: Write Sets of Numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-005	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. List the numbers in a set using brackets.
2. Identify and interpret set notation.

Overview

In the previous lesson, you worked on sets of real objects or ideas. In this lesson, you will work on sets of numbers.

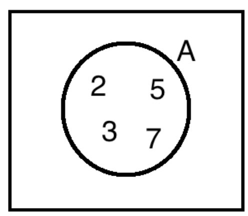
For example, consider the set A. A = $\{x : x$ is a prime number less than 10$\}$. This is read as, "A is the set of all x such that x is a prime number less than 10." Every element in the set A is a number.

We can identify set A another way using set notation: A = $\{x : x$ is prime, $x < 10\}$

We can also write set A by listing its elements. The elements of A are all prime numbers less than 10. Remember that a prime number can only be divided by 1 and itself. The elements of A are: A = {2, 3, 5, 7}.

We use the symbol $\in$ to show that a number belongs to A. For example, $3 \in A$ states that "3 is an element of A".

We can also use a Venn diagram, at right, to show the elements of A. The circle represents the set A. We list all of the elements of set A inside the circle. We have drawn set A inside a rectangle. This rectangle is the set from which we have taken the elements of set A.

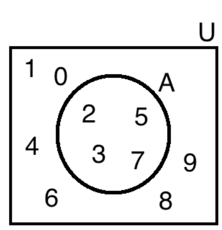


We have taken the elements of A from the set of numbers less than 10. This set is called the **universal set**. We define it as the set of all the elements under consideration. We represent it with the letter U. We will assume that the set has no fractions or decimals less than 10, just whole numbers.

We use the symbol $\notin$ to show that a number does not belong to set A. For example, $6 \notin A$ states that "6 is not an element of A". We know that 6 is not an element of A because it is not a prime number.

We refer to the set of elements in set U which are not in set A as the **complement** of A. We denote this by the symbol A^C or A'. In the Venn diagram at right, A^C can be shown by drawing the numbers in A^C outside of the circle.

This Venn diagram shows A, U, and A^C:



Solved Examples

1. Consider the set S = {x : x is a counting number, x < 12}
    a. Describe the set S in words.
    b. List the elements of S.
    c. Write using set notation: "7 is an element of S"
    d. Write using set notation: "15 is not an element of S"

    Solutions
    a. S is the set of all x such that x is a counting number less than 12.
    b. S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}
    c. $7 \in S$
    d. $15 \notin S$

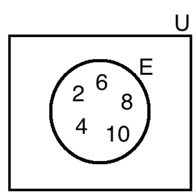
2. List the elements of the set F, where F = {x : x is a multiple of 5 less than 40}.

    Solution
    F is the set of x such that x is a multiple of 5 less than 40. List the elements of x by writing the multiples of 5. Stop at 35, which is the last multiple of 5 before 40.

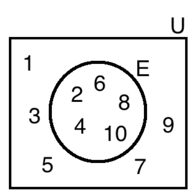
    F = {5, 10, 15, 20, 25, 30, 35}

3. Let the universal set be the counting numbers 1 to 10.
    a. List the elements of the universal set, U.
    b. Define the set of even numbers in the universal set. Represent the set by E, and the elements by s.
    c. Draw the Venn diagram for the set E.
    d. Write the elements of E^c in the Venn diagram.

    Solutions
    a. U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
    b. E = {s : s is an even number, $s \leq 10$}
    c. The Venn diagram showing E has a circle with all of the elements of E inside:



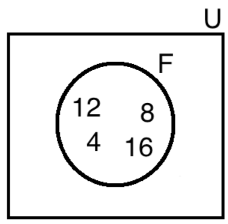
    d. Add the elements of E^c to your Venn diagram. Remember that the complement of E is all of the elements in the universal set U which are not in E.



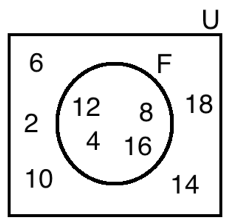
4. Let the universal set be U = $\{x : x \text{ is even}, x < 20\}$
    a. List the elements of the universal set, U.
    b. Define the set of multiples of 4 that are less than 20. Represent the set by F, and the elements by z.
    c. Draw the Venn diagram for the set F.
    d. List the elements of F^c.
    e. Write the elements of F^c in the Venn diagram.

    Solutions

    a. U = {2, 4, 6, 8, 10, 12, 14, 16, 18}
    b. F = $\{z : z \text{ is a multiple of } 4, z < 20\}$
    c. The Venn diagram showing F has a circle with all of the elements of F inside:



    d. F^c is the numbers in the universal set U that are not in F. F^c = {2, 6, 10, 14, 18}
    e. Add the elements of E^c to your Venn diagram:



Practice

1. List the elements of the set G, where G = $\{ x : x \text{ is a multiple of 4 less than } 45\}$.
2. List the elements of the set Z, where Z = $\{ x : 5 \leq x \leq 15\}$.
3. Consider the set S = $\{ x : x \text{ is a multiple of } 10, x < 100\}$.
    a. Describe the set S in words.
    b. List the elements of S.
    c. Write using set notation: "30 is an element of S".
    d. Write using set notation: "72 is not an element of S".
4. Let the universal set be U = $\{x : x \text{ is a multiple of } 3, x < 30\}$.
    a. List the elements of the universal set, U.
    b. Define the set of multiples of 6 that are less than 30. Represent the set by T, and the elements by y.
    c. Draw the Venn diagram for the set T.
    d. List the elements of T^c.
    e. Write the elements of T^c in the Venn diagram.

Lesson Title: Finite Sets	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-006	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify a unit set as one with one element; an empty set as one with no element.

Overview

A finite set is a set that comes to an end. We can list and count the elements of a finite set. For example, consider E = {positive even numbers less than 10}. The elements of this set are E = {2, 4, 6, 8}. We can write the number of elements in E: n(E) = 4. E is a finite set because we can list and count its elements.

Solved Examples

1. Decide whether each set is finite. If it is a finite set, list its elements. Use ellipses (…) if there are more than 8 elements in the set.
    a. Days of the week
    b. The alphabet
    c. Odd numbers
    d. Whole numbers greater than 100
    e. Prime numbers less than 35

    Solutions
    a. Finite; {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}.
    b. Finite; {a, b, c, …, z}
    c. Not finite. The list of odd numbers continues forever. We cannot list or count them.
    d. Not finite. The list of whole numbers greater than 100 continues forever. We cannot list or count them.
    e. Finite; {2, 3, 5, 7, …, 31}

2. Decide whether each set is finite. If it is a finite set, list its elements and give the number of elements in each set.
    a. $A = \{x : x$ is a factor of 12$\}$
    b. $B = \{x : x$ is a multiple of 5$\}$
    c. $C = \{x : x$ is a multiple of 6 less than 50$\}$
    d. $E = \{x : x$ is an even number less than 30$\}$

Solutions

a. Finite; A = {1, 2, 3, 4, 6, 12}; n(A) = 6
b. B is not finite. The multiples of 5 continue forever. We cannot list or count them.
c. Finite; C = {6, 12, 18, 24, 30, 36, 42, 48}; n(C) = 8
d. Finite; E = {2, 4, 6, 8, 10, …, 28}; n(E) = 14

3. Let the universal set be U = {whole numbers from 1 to 30}. List the elements in the following sets and give the number of elements in each set. Use ellipses (…) if there are more than 8 elements in the set.
   a. A = {even numbers}
   b. B = {square numbers}
   c. C = {numbers divisible by 6}
   d. D = {multiples of 7}

   Solutions

   a. A = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30}
   n(A) = 15
   b. B = {1, 4, 9, 16, 25}
   n(B) = 5
   c. C = {6, 12, 18, 24, 30}
   n(C) = 5
   d. D = {7, 14, 21, 28}
   n(D) = 4

Practice

1. List the elements of F = {Factors of 18}, and give the number of elements in the set.
2. Determine whether the following sets are finite. For each finite set, list its elements and give the number of elements. Use ellipses where needed.
   a. M = {Months of the year}
   b. W = {Multiples of 8 less than 50}
   c. X = {Even numbers}
   d. Y = {Even numbers from 10 to 20}
3. Let the universal set be U = {whole numbers from 20 to 30}. List the elements in the following sets and give the number of elements in each set. Use ellipses (…) if there are more than 8 elements in the set.
   a. A = {even numbers}
   b. B = {square numbers}
   c. C = {numbers divisible by 5}
   d. D = {multiples of 10}

Lesson Title: Infinite Sets	Theme: Numbers and Numeration
Practice Activity: PHM-09-007	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify sets of objects, things, ideas and numbers that are infinite.

Overview

Something is infinite if it goes on forever without coming to an end. Consider the counting of numbers: 1, 2, 3, 4, 5, …. You can continue counting forever. Counting numbers is infinite.

An infinite set is a set where we **cannot** list or count all the elements or members. No matter how many elements we list, there will always be more.

In an infinite set, we add ellipses (…) at the end to show that the elements go on forever. For example, consider the set of all multiples of 3. There are an infinite number of multiples of 3, and they are {3, 6, 9, 12, 15, 18, 21, …}.

Note that this is different from when we added ellipses to finite sets. In a finite set, we know the number of elements. We add ellipsis to show some elements have been left out. In an infinite set, we do not know the number of elements. We add ellipsis to show that there are infinitely more elements to come.

Solved Examples

1. Determine whether each set is finite or infinite. Explain.
   a. A = {1, 2, 3, …, 10}
   b. B = {2, 4, 6, 8, …}
   c. C = {3, 6, 9, …}
   d. D = {4, 8, 12, …, 40}

   Solutions
   a. **Finite**. The ellipse is within the list, and the list ends with 10. There are a finite number of elements.
   b. **Infinite**. The ellipse is at the end of the list. The list has no end. There are an infinite number of elements.
   c. **Infinite**. The ellipse is at the end of the list. The list has no end. There are an infinite number of elements.
   d. **Finite**. The ellipse is within the list, and the list ends with 40. There are a finite number of elements.

2. For each set below, determine whether the set is finite or infinite. List the elements of the set using set notation.
    a. X = {whole numbers}
    b. Y = {whole numbers between 45 and 50}
    c. Z = {whole numbers greater than 51}

    Solutions
    a. Infinite. X = {0, 1, 2, 3, … }
    b. Finite. Y = {45, 46, 47, 48, 49, 50}
    c. Infinite. Z = {52, 53, 54, …}

3. For each set below, determine whether the set is finite or infinite. List the elements of the set using set notation.
    a. A = {odd numbers less than 20}
    b. B = {odd numbers between 0 and 100}
    c. C = {odd numbers}

    Solutions
    a. Finite. A = {1, 3, 5, …, 19}
    b. Finite. B = {1, 3, 5, …, 99}
    c. Infinite. C = {1, 3, 5, … }

4. For each set below, determine whether the set is finite or infinite. List the elements of the set using set notation.
    a. R = {multiples of 5}
    b. S = {multiples of 5 between 0 and 100}
    c. T = {multiples of 5 greater than 100}

    Solutions
    a. Infinite. R = {5, 10, 15, … }
    b. Finite. S = {5, 10, 15, …, 100}
    c. Infinite. T = {105, 110, 115, … }

Practice

1. Determine whether each set is finite or infinite. Explain.
    a. A = {1, 2, 3, …, 100}
    b. B = {1, 3, 5, …, 101}
    c. C = {1, 3, 5, …}
    d. D = {2, 4, 6, 8, 10, …}
2. For each set below, determine whether the set is finite or infinite. List the elements of the set using set notation.
    a. A = {even numbers greater than 5}
    b. B = {even numbers between 5 and 10}
    c. C = {even numbers between 5 and 100}
    d. D = {even numbers}
3. For each set below, determine whether the set is finite or infinite. List the elements of the set using set notation.
    a. X = {multiples of 3}
    b. Y = {multiples of 3 between 1 and 10}
    c. Z = {multiples of 3 greater than 10}

Lesson Title: Unit and Empty Sets	Theme: Numbers and Numeration
Practice Activity: PHM-09-008	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify a unit set as one with one element, and an empty set as one with no elements.

Overview

This lesson is on how to define and identify a unit set and an empty set.

A **unit set** is a set that has only one element, no more or less. For example, consider the set F $= \{x: x$ is a month with less than 30 days}. We know that F = {February}, which is a unit set. We can write n(F) = 1.

A set that does not contain any element is called an **empty set**. Such a set is represented by curly bracket with nothing inside { } or the null symbol ∅. For example, the set A of cats with horns is a null set. We can write A = { } or A = ∅. We can also write n(A) = 0.

Solved Examples

1. Identify the sets below as an empty set or not:
   a. A = {Goats with five legs}
   b. B = {Months of the year with 45 days}
   c. C = {Families with 2 children}

   Solutions
   a. A is an empty set
   b. B is an empty set
   c. C is not an empty set

2. Decide whether the following sets are unit sets or not:
   a. W = {0}
   b. X = $\{x: x = 2\}$
   c. Y = {4}
   d. Z = {0, 1}

   Solutions
   a. W only has 1 element, the number 0.
   n(W) = 1, so W is a unit set
   b. X only has 1 element, 2. X = {2}.
   n(W) = 1, so W is a unit set
   c. Y only has 1 element, 4.
   N(Y) = 1, so Y is a unit set.

d. Z has 2 elements, 0 and 1.
   n(Z) = 2, so Z is not a unit set

3. Determine whether each of the following sets is a unit set, empty set, or neither:
   a. Q = {countries in West Africa}
   b. R = {Countries with the capital city as London}
   c. S = {countries in West Africa with the capital city as Freetown}
   d. T = {countries in West Africa with the capital city as London}

   Solutions
   a. Q has more than 1 element: Q = {Sierra Leone, Guinea, Liberia, …}
      Q is **neither** a unit set nor an empty set.
   b. R has exactly 1 element, the United Kingdom: R = {United Kingdom}.
      n(R) = 1, so R is a **unit set**.
   c. S has exactly 1 element, Sierra Leone: S = {Sierra Leone}.
      n(S) = 1, so S is a **unit set**.
   d. T has 0 elements. The only country with the capital city of London is the United Kingdom, and it is not in West Africa.
      n(T) = 0, so T is an **empty set**.

Practice

1. Decide whether each of the following sets are empty sets or not.
   a. G = {Dogs that can swim}
   b. R = {Reptiles that can breast feed}
   c. M = {Countries that have landed men on the moon}
   d. P = {Female Presidents of Sierra Leone}
2. Decide whether each of the following sets are unit sets or not.
   a. D = {Months of the year with three letters}
   b. E = {Months of the year with four letters}
   c. W = {The first day of the week}
   d. Y = {The days of the week beginning with the letter 'y'}
3. Determine whether each of the following sets is a unit set, empty set, or neither:
   a. A = {odd numbers}
   b. B = {odd numbers divisible by 2}
   c. C = {odd numbers greater than 2 but less than 4}
   d. D = {odd numbers greater than 100}

Lesson Title: Equal Sets	Theme: Numbers and Numeration
Practice Activity: PHM-09-009	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify sets with the same elements.

Overview

Two sets are equal if and only if they have the same elements.

Note that the order in which the members of a set are written does not matter in determining whether they are equal. For example, if P = {1, 2, 3} and Q = {2, 3, 1} then P = Q. P and Q are equal sets.

It also does not matter if elements of the set are written more than once. When counting the number of elements in a set, we only count the unique elements. This means we do not count any element more than once. For example, if P = {1, 2, 3} and R = {1, 1, 2, 3, 3}, then P = R. P and R are equal sets.

Solved Examples

1. Consider the following sets and identify whether they are equal or not. Explain your reasons.
    a. W = $\{x: 0 < x < 4\}$ and V = $\{x: 0 \leq x \leq 4\}$
    b. C = {spoon, saucer, plate, pot}, and S = {pot, spoon, plate, saucer}
    c. X = $\{x : x$ < 10, x is even} and Y = {2, 4, 8}
    d. P = {even numbers greater than 10}, R = {12, 14, 16, …}

    Solutions
    a. W = {1, 2, 3} and V ={0, 1, 2, 3, 4}. Since the elements in W and V are not all the same, then W ≠ V. These are not equal sets.
    b. The elements in C and S are the same, so C = S. These are equal sets.
    c. Note that X = {2, 4, 6, 8}. The element 6 is in X but not Y, so X ≠ Y. These are not equal sets.
    d. Note that P = {12, 14, 16, …}. This is the same as R, so P = R. These are equal sets.

2. Write 5 different sets that are equal to X = {x : x < 10, x is even}

Solution

Start by listing the elements of the set X: X = {2, 4, 6, 8}

Remember that order does not matter, and we can give each element more than once. There are 5 examples of sets equal to X below. There are many other examples you could write.

A = {2, 2, 4, 6, 8}
B = {2, 2, 4, 4, 6, 6, 8, 8}
C = {8, 6, 4, 2}
D = {2, 4, 8, 6}
E = {2, 4, 6, 8, 8, 8, 8, 8}

Practice

1. Consider the following pairs of sets and identify whether they are equal or not.
    a. V = $\{x : x > 0\}$ and W = $\{x : x < 0\}$
    b. C = {January, February, March, April} and D = {December, November, October, September}
    c. A = {x : x is an odd number between 0 and 10} and $B = \{1, 3, 5, 7, 9\}$
    d. X = $\{x : x = 5\}$ and Y = $\{5\}$
    e. R = {Monday, Tuesday, Wednesday} and S = {Tuesday, Monday, Thursday}
    f. P = {apple, plum, avocado} and Q = {avocado, apple, apple, plum}

2. Write 5 different sets that are equal to X = {5, 10, 15, 20, 25}

Lesson Title: Equivalent Sets	Theme: Numbers and Numeration
Practice Activity: PHM-09-010	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that an equivalent set has the same number of elements.
2. Distinguish between equal and equivalent sets.

Overview

To be equivalent, any pair of sets should have the same number of elements. This implies that there is a one-to-one correspondence between the elements of any two sets for them to be equivalent. For example, consider 2 sets: $A = \{1, 2, 3, 4, 5\}$ and $B = \{a, e, i, o, u\}$. They both have 5 elements: $n(A) = n(B) = 5$. Thus, A and B are equivalent. We can write this using a double-headed arrow as: $A \leftrightarrow \mathrm{B}$.

Note that all empty sets are equivalent. Similarly, all unit sets are equivalent.

All equal sets are equivalent but not all equivalent sets are equal.

Solved Examples

1. Consider the following sets. Establish whether they are equivalent or not.
   a. P = {5, 4, 3, 2, 1} and Q = {J, U, L, Y, S}
   b. A = {a, b, c, d, e} and R = {m, n, o, p}
   c. F = {Hen, goats, cow, pig, cock} and G = $\{x : x > 5\}$

   Solutions
   a. Since n(P) = n(Q) = 5, P is equivalent to Q
   b. n(A) = 5 and n(R) = 4. Since n(A) ≠ n(R) then set A is not equivalent to R.
   c. n(F) = 5 and n(G) is infinity so set F is not equivalent to G

2. Consider the sets:
   A = {a, b, c, d}   B = {a, a, b, c, d}   C = {a, b, c}   D = {1, 2, 3, 4}
   a. Which sets are equivalent? Give your reasons.
   b. Which sets are equal? Give your reasons.

   Solutions
   a. Equivalent sets are those with the same number of elements. Note that we only count unique elements. For set B, we have B = {a, a, b, c, d} = {a, b, c, d}. It actually has 4 elements. Sets A, B, and D all have 4 elements, so they are all equivalent. There is no set equivalent to C, which has 3 elements.
   Answer: A ↔ B, B ↔ D, A ↔ D

b. Equal sets are those with exactly the same elements. Notice that A and B have the same elements, {a, b, c, d}. There are no other sets with the same elements.

   Answer: A = B

3. The 4 sets Q, R, S and T are described as shown:

   Q = {circle, oval, triangle, square}
   R = {odd numbers between 0 and 8}
   S = {square numbers between 0 and 20}
   T = {1, 3, 5, 7}

   a. List the elements for the sets R and S.
   b. Use ↔ (is equivalent to) or = (is equal to) between the 2 sets to make the statements true:

   Q ☐ R   R ☐ S   S ☐ T   R ☐ T

   Solutions

   a. R = {1, 3, 5, 7} and S = {1, 4, 9, 16}
   b. Write ↔ between any 2 sets with the same number of elements, and = between any 2 sets with exactly the same elements:

   Q ↔ R   R ↔ S   S ↔ T   R ↔ T or R = T

   Note that all of the sets are equivalent because they all have 4 elements. Sets R and T are also equal, because they have exactly the same elements.

Practice

1. Consider the following pairs of sets. Identify whether they are equivalent or not.
   a. K = {0, −1, 1, 5, 6} and L = $\{\pi, \Phi, \rho, \ \gamma, \ \theta\}$
   b. A = {u, v, w, x, y} and B = {9, 10, 11}
   c. C = {Toyota, Benz, Lexus, Nissan} and T = {Chrysler, Dodge, Pontiac}
   d. N = {Lucy, Brima, Joe, Mary} and M = $\{x : 4 < x < 9\}$

2. Consider the sets:

   W = {s, r, t}   X = {r, s, t}   Y = {r, s, t, u}   Z = {u, r, s, t}

   a. Which sets are equivalent? Give your reasons.
   b. Which sets are equal? Give your reasons.

3. The 4 sets A, B, C and D are described as shown:

   A = {maths, English, biology, French, chemistry}
   B = {even numbers from 10 to 18}
   C = {1, 2, 3, 4, 5}
   D = $\{x : 1 \leq x \leq 5\}$

   a. List the elements for the sets B and D.
   b. Use ↔ (is equivalent to) or = (is equal to) between the 2 sets to make the statements true:

   A ☐ B   B ☐ C   C ☐ D   A ☐ D

Lesson Title: Introduction to Subsets	Theme: Numbers and Numeration
Practice Activity: PHM-09-011	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify subsets as a collection of objects within a set.

Overview

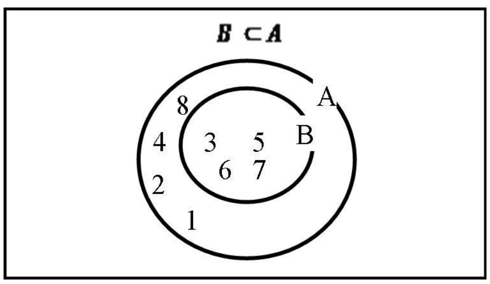
For any two sets A and B, if every element in set B is present in set A, then B is a subset of A.

A special notation is used to represent the relationships between subsets. In the case of A and B we say $B \subset A$. This is read as "set B is a subset of set A" or "B contained A". The symbol ⊂ means "is a subset of" or "is found in". The notation ⊄ means "not a subset of".

If two sets are equal then, then they are also subsets of each other. For example, consider sets A = {1, 2, 3, 4} and B = {4, 3, 2, 1}. They have the same elements, so $B \subset A$ and $A \subset B$.

A Venn diagram is a diagram that shows the relationship between sets. It is most commonly drawn with circles and squares. A circle inside of another circle commonly shows subsets.

For example, the Venn diagram below shows $B \subset A$, where $A = \{1, 2, 3, 4, 5, 6 ,7 ,8\}$ and $B = \{3, 5, 6, 7\}$.



Note that the empty set is a subset of every set, and every set is a subset of itself.

Solved Examples

1. Given the set X = {m, n, o, p, q}, list any ten subsets of X.

   Solution

   Given X = {m, n, o, p, q}, ten subsets are:

   X_1 = {m, n, o, p}, X_2 = {m, n, o, q}, X_3 = {m, n, p, q}, X_4 = {m, o, p, q}
   X_5 = {n, o, p}, X_6 = {m, n}, X_7 = {m, o}, X_8 = {m, p}, X_9 = {m, q}, X_{10} = {m}

   Your answers may be different because there are more than 10 possible subsets.

2. If A = {a, b, c, d, e, f, g, h} identify whether the following sets are subsets of A:

   a. B = {b, d, f, h}
   b. C = {a, c, e, g}
   c. D = {a, b, c, d, e}
   d. E = {a, c, l, g}

   Solutions

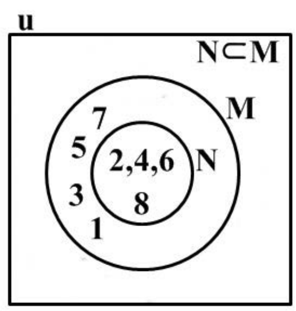
   a. B is a subset of A; All elements of B are found in the set A
   b. C is a subset of A; All the elements in set C are found in set A
   c. D is a subset of A; All the elements in the set D are found in set A
   d. E is **not** a subset of A; l is an element of E but it is not found in A

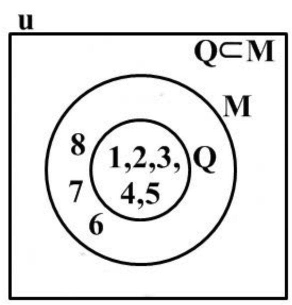
3. Consider the set M = {1, 2, 3, 4, 5, 6, 7, 8}.
   a. Identify which of the following sets are subsets of M:
      i. N = {2, 4, 6, 8}
      ii. P = {1, 2, 5, 7, 9}
      iii. Q = {1, 2, 3, 4, 5}
   b. Draw Venn diagrams to illustrate the relationships between set M and each of its subsets in part a.

   Solutions

   a. i. N is a subset of M; All the elements in N are in M
      ii. P is **not** a subset of M; 9 is an element of P but not in M
      iii. Q is a subset of M; All the elements in Q are in M

   b. Diagrams showing that N and Q are subsets of M:





4. List all the subsets of the set Z = {U, V, W}.

   Solution

   The subsets of Z = {U, V, W} are:

   Z_1 = {U, V, W}, Z_2 = {U, V}, Z_3 = {U, W}, Z_4 = {V, W}, Z_5 = {U}, Z_6 = {V}, Z_7 = {W} and Z_8= { }.

Practice

1. Given the set S = {a, b, c, d} list all its subsets.

2. Given the set X = $\{x : 0 \leq x \leq 20$, where x is an integer}, identify which of the following sets are subsets of X:

   a. W = $\{x : 3 < x < 9$, where x is an integer}
   b. V = $\{x : 0 \leq x \leq 5$, where x is an integer}
   c. T = $\{x : x > 0\}$

3. Represent the relationship between the following set B and its subset V on a Venn diagram:

   B = $\{x : x$ is an alphabet between a and f}
   V = $\{x : x$ is a vowel}

4. List all the subsets of the set M = $\{x : x$ is a multiple of 2 up to 6}

Lesson Title: Identifying Subsets of the Set of Real Numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-012	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to identify subsets of real numbers: natural numbers, whole numbers, rational numbers (integers, fractions and decimals).

Overview

All the numbers we will look at today are subsets of real numbers. The set of real numbers is the universal set for the sets we are considering. It is denoted by R.

The first subset is the set of natural numbers, N. These are the counting numbers we learned in primary school: N = {1, 2, 3, 4, …}

The next subset is the set of whole numbers, W. These are the natural numbers plus 0: W = {0, 1, 2, 3, …}

The next subset is the set of integers, Z. It is made up of positive natural numbers, 0, and negative natural numbers: Z = {…, -3, -2, -1, 0, 1, 2, 3, …}

The next subset is the set of rational numbers, Q. These are all real numbers that can be written as an integer, fraction, or decimal. However, remember that the denominator of a real number cannot be 0. Examples of rational numbers are: Q = {…, -2, $-\frac{11}{2}$, -0.5, 0, 1, $\frac{33}{4}$, . . . }

Natural numbers, whole numbers, integers, and rational numbers are all types of real numbers. Therefore, they are subsets of real numbers. We can write the following:

$N \subset R$ $W \subset R$ $Z \subset R$ $Q \subset R$

Solved Examples

1. Are there any subsets of R given in the overview that are also subsets of Z, integers? If so, list them.

   Solution
   The integers are negative and positive numbers, and 0. Thus, the integers include the natural numbers and the whole numbers.

   We have $N \subset Z$ because {1, 2, 3, 4, …} $\subset$ {…, -3, -2, -1, 0, 1, 2, 3, …}

   We have $W \subset Z$ because {0, 1, 2, 3, …} $\subset$ {…, -3, -2, -1, 0, 1, 2, 3, …}

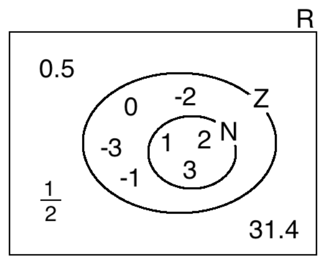
   Remember that any set is a subset of itself. Thus, we have $Z \subset Z$.

   Answer: Yes. N, W and Z are subsets of Z.

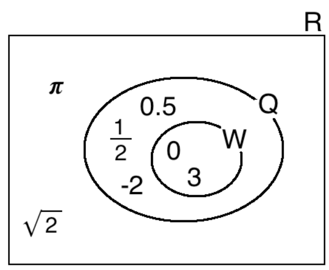
2. Draw the following relationships with a Venn diagram. In your diagram, give a few examples of numbers that belong in each set. Remember that the universal set is R.
   a. $N \subset Z$
   b. $W \subset Q$
   c. $Z \subset R$

   Solutions

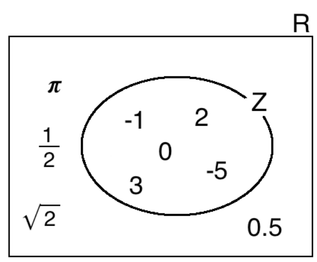
   a. First, draw a rectangle for the universal set R. Draw a circle inside R for Z, and a circle inside Z for N. Write examples in the diagram in this order:
      - Give examples of natural numbers in the circle for N. In the example below, 1, 2 and 3 are shown.
      - Give examples of integers in the circle for Z. Do not include examples of natural numbers, which are already given in the circle for N.
      - Finally, give some examples of real numbers in the rectangle for R. Do not include examples of natural numbers or integers, which are already given in the circles.



   b. Follow the same process as above, drawing circles for whole numbers and rational numbers in the rectangle for R:



   c. Follow the same process as above. This states that integers are a subset of the universal set real numbers, so we only need 1 circle in the rectangle for real numbers:



Practice

1. Are there any subsets of R given in the overview that are also subsets of Q, rational numbers? If so, list them.

2. Draw the following relationships with a Venn diagram. In your diagram, give a few examples of numbers that belong in each set. Remember that the universal set is R.

   a. $N \subset Q$ b. $W \subset Z$ c. $W \subset R$

Lesson Title: Comparing Sets of Real Numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-013	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Compare sets of real numbers.
2. Use a Venn diagram to compare sets of real numbers.

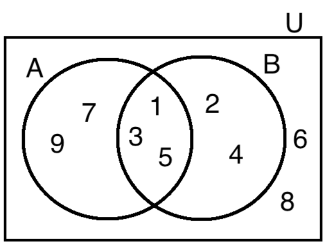
Overview

Today we are going to compare sets of real numbers. This will include how to use Venn diagrams to compare sets of real numbers.

We compare sets of numbers to determine the similarities and differences between them. This is useful in fields such as science, banking, and accounting where large amounts of data are processed. Data are 'collections' of objects and ideas that have been sorted into groups.

One way to compare sets of data is to examine the lists of elements from the sets. There are special symbols and vocabulary we use in set notation to show the similarities and differences between sets.

Consider 2 sets, A= {1, 3, 5, 7, 9} and B = {1, 2, 3, 4, 5} in the universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9}. They are shown with the Venn diagram on the right.



Elements that are in **both** sets A and B are called the **intersection** of the sets. The symbol ∩ is used to show an intersection. A ∩ B says "the intersection of sets A and B". In this example, A ∩ B = {1, 3, 5}.

Elements that are in **either** sets A or B are called the **union** of the sets. The union includes all of the elements in A and B. The symbol ∪ is used to show a union. A ∪ B says "the union of sets A and B". In this example, A ∪ B = {1, 2, 3, 4, 5, 7, 9}.

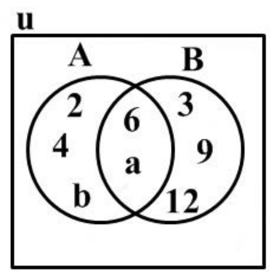
Recall that A^C means the **complement** of A. That is, the elements of the universal set that are not in A. If the universal set is U = {1, 2, 3, 4, 5, 7, 9}, then the complement of A is A^C = {2, 4, 6, 8} and the complement of B is B^C = {6, 7, 8, 9 }.

Solved Examples

1. Consider the two sets A and B:

$$A = \{2, 4, 6, a, b\}$$
$$B = \{3, 6, 9, a, 12\}$$

   a. Draw a Venn diagram to illustrate both sets
   b. Find A ∩ B.
   c. Find A ∪ B.
   d. Find B^C.



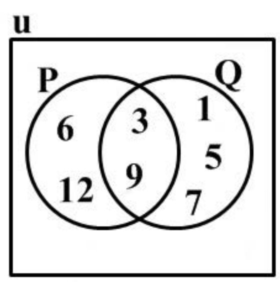
Solutions

   a. See the Venn diagram at right.
   b. A ∩ B = {6, a}, because 6 and a are common to both sets A and B.
   c. List the elements in either A or B: A ∪ B = {2, 3, 4, 6, 9, 12, a, b}
   d. B^C is the set of elements in the universal set that are not in B. In this case, the universal set includes only the elements in A and B. Thus, B^C is the set of elements in A but not B.
   B^C = {2, 4, b}

2. Consider the two sets P and Q:

$$P = \{\text{multiples of 3 less than 13}\}$$
$$Q = \{\text{odd numbers less than 10}\}$$

   a. List the members of both sets P and Q.
   b. Draw a Venn diagram to illustrate P ∩ Q.
   c. Find P ∩ Q.
   d. Find P ∪ Q.
   e. Find P^C.



Solutions

   a. P = {3, 6, 9, 12} and Q = {1, 3, 5, 7, 9}
   b. See the Venn diagram.
   c. P ∩ Q = {3, 9}, because 3 and 9 are common to both sets.
   d. List the elements in either P or Q: P ∪ Q = {1, 3, 5, 6, 7, 9, 12}
   e. List the elements that are in Q but not P.
   P^C = {1, 5, 7}.

3. Set A is prime numbers less than 15, and set B is even numbers less than 15. The universal set U is all natural numbers less than 15.
    a. List the elements of A, B and U.
    b. Draw a Venn diagram to illustrate the sets.
    c. Find A ∩ B.
    d. Find A ∪ B.
    e. Find A^C.

Solutions

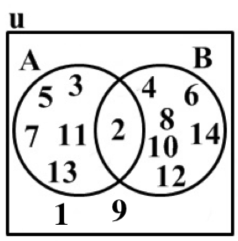
a. List the elements of each set:

A = {prime numbers less than 15} = {2, 3, 5, 7, 11, 13}

B = {even numbers less than 15} = {2, 4, 6, 8, 10, 12, 14}

U = {natural numbers less than 15} = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}

b. Venn diagram:



c. A ∩ B = {2}, because 2 is the only common element.

d. List the elements in either A or B: A ∪ B = {2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14}

e. List the elements of the universal set that are not in A.

A^C = {1, 4, 6, 8, 9, 10, 12, 14}.

Practice

1. Consider two sets P and Q:

P = {6, 12, 18, 24}

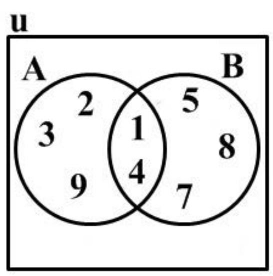
Q = {12, 24, 36, 48}

    a. Draw a Venn diagram to illustrate set P and set Q.
    b. Find P ∩ Q.
    c. Find P ∪ Q.
    d. Find P^C and Q^C.

2. Consider two sets R and S, where R = {factors of 12} and S = {even numbers less than 13}. The universal set is U = {natural numbers less than 13}
    a. List the elements R, S and U.
    b. Illustrate the sets in a Venn diagram.
    c. Find R ∩ S.
    d. Find R ∪ S
    e. Find R^C.

3. Consider two sets, A = {multiples of 4 less than 19} and B = {even numbers greater than 7 but less than 19}.
    a. List the elements of set A and set B.
    b. Illustrate set A and set B on a Venn diagram.
    c. Find A ∩ B.
    d. Find A ∪ B.
    e. Find B^C.

4. From the Venn diagram at right:
    a. List the elements of set A and set B.
    b. Identify A ∩ B.
    c. Identify A ∪ B.
    d. Identify A^C and B^C.



Lesson Title: Ordering Sets of Real Numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-014	**Class:** JSS 3

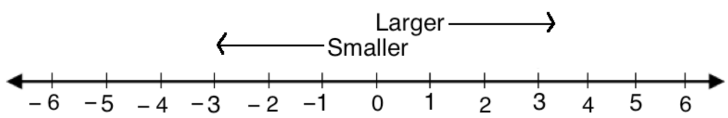
Learning Outcome
By the end of the lesson, you will be able to order sets of real numbers.

Overview

This lesson is on comparing and ordering real numbers. Recall that integers, fractions and decimals are all types of real numbers. You will need to recall how to compare and order fractions, decimals, and negative numbers.

The following symbols are used to compare 2 numbers: < (less than), > (greater than), ≤ (less than or equal to), ≥ (greater than or equal to). Note that the symbol opens to the larger number. For example, $\frac{3}{5} > \frac{4}{7}$ says that $\frac{3}{5}$ is greater than $\frac{4}{7}$.

Negative numbers are real numbers. Remember that negative integers get smaller the further away from 0 they are. Positive integers get larger the further away from 0 they are.



The greater the absolute value of a negative number, the less its value is. For example, we have $-5 < -2$.

To compare **fractions**, make the denominators equal using the LCM. For example, if you are asked to compare $\frac{3}{5}$ and $\frac{4}{7}$, make the denominators equal to 35, the LCM of 5 and 7: $\frac{3}{5} = \frac{21}{35}$ and $\frac{4}{7} = \frac{20}{35}$. We can see that $\frac{21}{35} > \frac{20}{35}$, so we also have $\frac{3}{5} > \frac{4}{7}$.

To compare **decimals**, look at the whole number part first. A decimal number with a greater whole number part is a greater number. For example, $7.5 > 6.9$ and $-5.2 > -8.5$. If the whole number part is the same, look at the digits after the decimal point. A greater digit in tenths place gives a greater number. For example, $7.5 > 7.3$. If the tenths digits are equal, look at the hundredths place, and so on.

It is best to compare only fractions, or only decimals. If you have a mixture of decimals and fractions, you may convert them all to 1 type before comparing or ordering them.

When you are asked to order numbers, **ascending** order means from least to greatest. **Descending** order is the opposite, from greatest to least.

Solved Examples

1. Write the set of real numbers in ascending order: $\{6, -3, 4, -9, 0\}$

   Solution
   Recall that negative numbers are always less than positive numbers, so they will come first. The greater the absolute value of a negative number, the less it actually is.

   The answer is $\{-9, -3, 0, 4, 6\}$

2. Write the set of real numbers in descending order: $\{4, -3, -1, 8, -10, 7\}$.

   Solution
   Note that you are asked to write the numbers in descending order this time, from greatest to least.

   The answer is $\{8, 7, 4, -1, -3, -10\}$.

3. Write the following numbers in ascending order: $\{0.3, 2.4, -3.6, 2.7, 2.04, -3.8\}$.

   Solution
   Start with negative numbers. Note that $-3.6 > -3.8$, because it has a lower absolute value and is closer to 0.

   Next, order the positive decimal numbers. 0.3 is less than the decimal numbers with a whole number 2. There are 3 decimal numbers with a whole number 2: 2.4, 2.7, 2.04. Write these in order of the digit in the tenths place. This is the ascending order: 2.04, 2.4, 2.7.

   The answer is $\{-3.8, -3.6, 0.3, 2.04, 2.4, 2.7\}$.

4. Write the set of numbers in descending order: $\{\frac{5}{6}, \frac{1}{3}, \frac{1}{2}, \frac{1}{6}, \frac{2}{3}\}$

   Solution
   Change the denominators of the fractions to the LCM so they can be compared. The LCM of 6, 3 and 2 is 6.

   The new fractions are: $\frac{5}{6}$, $\frac{1}{3} = \frac{2}{6}$, $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{6}$, $\frac{2}{3} = \frac{4}{6}$,

   To write fractions with the same denominator in descending order, write them with the numerators in descending order: $\{\frac{5}{6}, \frac{4}{6}, \frac{3}{6}, \frac{2}{6}, \frac{1}{6}\}$.

   Replace these fractions with the equivalent fractions given in the problem: $\{\frac{5}{6}, \frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}\}$. This is the answer.

5. Write the numbers in ascending order: $\{0.5, \frac{3}{4}, \frac{1}{4}, -0.25, -\frac{1}{2}\}$

Solution

Write all of the numbers as either decimals or fractions. You may recognize that the decimals are common decimals that can easily be written as fractions:

$0.5 = \frac{1}{2}$ and $-0.25 = -\frac{1}{4}$

Otherwise, convert the decimals to fractions by writing them over a power of 10 and simplifying. Decimal numbers with tenths should be written over 10, and decimal numbers with hundredths should be written over 100:

$0.5 = \frac{5}{10} = \frac{1}{2}$ and $-0.25 = -\frac{25}{100} = -\frac{1}{4}$

Write all of the numbers in the set as fractions: $\{\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, -\frac{1}{4}, -\frac{1}{2}\}$

Write all of the numbers with the LCM, 4, in the denominator: $\{\frac{2}{4}, \frac{3}{4}, \frac{1}{4}, -\frac{1}{4}, -\frac{2}{4}\}$

Write the fractions in ascending order, using the numerators: $\{-\frac{2}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}\}$

Replace these fractions with the numbers given in the problem: $\{-\frac{1}{2}, -0.25, \frac{1}{4},\ 0.5, \frac{3}{4}\}$

This is the answer.

Practice

1. Write the following sets of real numbers in ascending order:
   a. $\{2, 4, -5, 0, -8, -12, 7, 13\}$
   b. $\{\frac{1}{5}, \frac{3}{10}, \frac{4}{5}, \frac{1}{10}, \frac{2}{5}\}$
   c. $\{0.5, -5.05, 5.5, 5.05, -0.5, -5.0\}$
   d. $\{0.2, \frac{3}{5}, -\frac{1}{5}, -0.4, 0.8\}$

2. Write the following sets of real numbers in descending order:
   a. $\{15, -12, 27, -4, -18, 6, 14\}$
   b. $\{\frac{1}{6}, \frac{1}{5}, \frac{4}{5}, \frac{5}{6}, \frac{2}{5}\}$
   c. $\{2.5, 3.5, -2.5, -2.05, -2.2, 3.1, -3, 3.05\}$
   d. $\{0.25, \frac{1}{2}, -\frac{1}{4}, -0.5, 1, \frac{3}{4}\}$

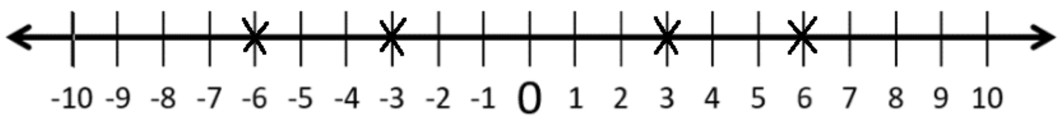
Lesson Title: Real Numbers on a Number Line	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-015	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to locate real numbers on a number line.

Overview

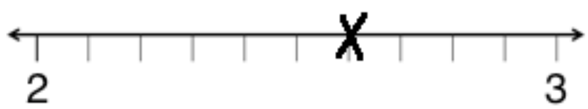
Today you are going to learn how to locate real numbers on a number line. Recall that real numbers can be positive, negative or zero. They can also be integers, fractions or decimals.

Any point that can be identified on the number line is a real number. Each real number is represented with an X or a dot at the appropriate point on the number line. For example, the set of numbers $\{-6, 3, 6, -3\}$ is shown on the number line below:



From this number line, notice that -3 and 3 are the same distance from 0. Similarly, -6 and 6 are the same distance from 0. Every real number has positive and negative counterparts. They are the same distance away from zero. They will always equal to 0 when we add them together.

We can also show fractions and decimals on the number line. We can use any scale to draw number line. For example, the number line below shows tenths between 2 and 3. That is, the marks are 2.0, 2.1, 2.2, etc. The number 2.6 is marked with an X.

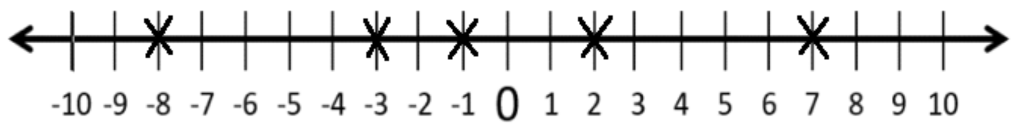


Solved Examples

1. Represent the set of numbers on a number line: $\{-3, -8, 2, -1, 7\}$

   Solution
   First, choose an appropriate size and scale for the number line. The numbers given are positive and negative whole numbers between -10 and 10. Therefore, a number line showing whole numbers between -10 and 10 is best.

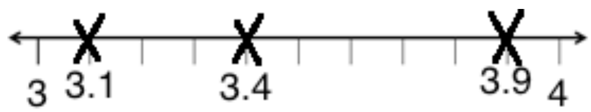
Draw the number line, and draw an X at each number listed:



2. Represent the following on a number line: $\{3.1, 3.4, 3.9\}$

Solution
First, choose an appropriate size and scale for the number line. These numbers are between 3 and 4, and have decimal places with tenths. Therefore, a number line showing tenths from 3 to 4 is best.

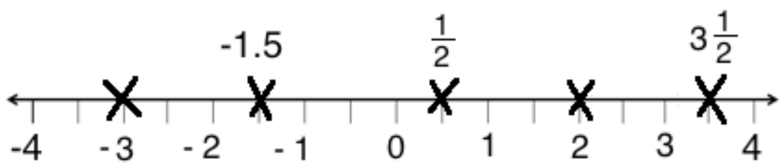
Draw the number line, and draw an X at each number listed:



3. Represent the following on a number line: $\{-1.5, \frac{1}{2}, 2, 3\frac{1}{2}, -3\}$

Solution
First, choose an appropriate size and scale for the number line. These numbers are between -4 and 4, and have halves. Therefore, a number line showing halves from -4 to 4 is best.

Draw the number line, and draw an X at each number listed:



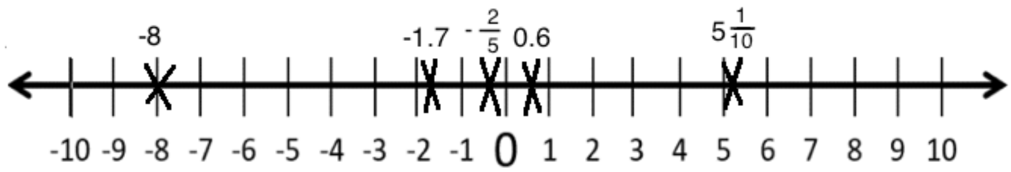
4. Represent the following on the number line: $\{-\frac{2}{5}, 5\frac{1}{10}, 0.6, -8, -1.7\}$

Solution
We can show all of these on a number line from -10 to 10. For the decimals and fractions, we will show approximately where they are.

First, convert fractions to decimals so they are easy to plot:

$-\frac{2}{5} = -0.4$ $\qquad$ $5\frac{1}{10} = 5.1$

Draw the number line, and draw an X at each number listed:



Practice

1. Draw a number line marked from -10 to 10. Locate the following set of real numbers: $\{-5, 5, -10, 7, -9, 6, 1\}$

2. Represent the following set of numbers on a number line: $\{-4, 4, 0, 6, -8, -1\}$

3. Draw a number line with tenths marked from -4 to -3. Locate the following set of numbers: $\{-3.5, -4.0, -3.2, -3.9\}$

4. Represent the following numbers on a number line: $\{-\frac{3}{2}, \frac{5}{2}, -\frac{1}{2}, -2\frac{1}{2}\}$

5. Draw a number line with tenths marked from 0 to 1. Represent the following set of numbers on the number line: $\{\frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{7}{10}\}$

Lesson Title: The Roman Numeral System	Theme: Numbers and Numeration
Practice Activity: PHM-09-016	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify the symbols used to show a Roman numeral.
2. Read, write and count numbers up to 20.

Overview

Today we are going to identify the symbols used to show a Roman numeral. You will be able to read, write and count numbers up to 20. Romans are people who lived in Rome in the country of Italy. The Romans wrote their numbers in a different way from what we are used to. Their numbers are used on clocks, watches and similar objects. Roman numerals were invented by the ancient Romans between the years 900 and 800 B.C.

The Romans used 7 basic symbols to represent their numbers. We can match these numbers to our base 10 numbers. Our base 10 numbers are derived from the Hindu-Arabic numbers.

Symbols used in Roman Numerals	
Hindu-Arabic	Roman
1	I
5	V
10	X
50	L
100	C
500	D
1000	M

These symbols can be written in capital or small letters. They mean the same thing. For example, maths problems are sometimes numbered I, ii, iii. These are the Roman numerals for 1, 2 and 3.

The Romans tried to show numbers using as few symbols as possible. They wrote a smaller digit **after** a larger digit to show that it is **added**. For example, VI means 5 + 1, which is 6. Thus, we know that VI is 6. They wrote a smaller digit **before** a larger digit to show that it is **subtracted**. For example, IV means 5 – 1, which is 4. Thus, we know that IV is 4.

There is no symbol for 0 in Roman numerals.

Solved Examples

1. Complete the table below. You will write the numbers 1 through 10 as Roman numerals, and show the calculation used to find each Roman numeral.

Numerals from 1-20		
Hindu-Arabic	**Calculation**	**Roman**
1	1	I
2	1 + 1	II
3		
4	5 – 1	IV
5		
6		VI
7	5 + 2	
8		VIII
9	10 – 1	
10	10	X

Solution

Determine what calculation will give you each Roman numeral. You may add or subtract from 5 or 10 to find the Roman numeral.

Numerals from 1-20		
Hindu-Arabic	**Calculation**	**Roman**
1	1	I
2	1 + 1	II
3	1 + 1 + 1	III
4	5 – 1	IV
5	5	V
6	5 + 1	VI
7	5 + 2	VII
8	5 + 3	VIII
9	10 – 1	IX
10	10	X

2. Hawa's telephone number is +432-26584193. Help her write each digit of her phone number as a Roman numeral. The first digit is completed for you.

4	3	2	2	6	5	8	4	1	9	3
IV										

Solution

Write each digit as a Roman numeral. You may use the table in Solved Example 1 as a guide.

4	3	2	2	6	5	8	4	1	9	3
IV	III	II	II	VI	V	VIII	IV	I	IX	III

3. Write the following numbers as Roman numerals:
    a. 20
    b. 18
    c. 14
    d. 17

    Solutions

    Follow the same pattern for numbers 11 – 20 as you did for numbers 1 – 10. Use addition from 10. In other words, write the Roman numeral for 10, which is X. Then, write the digits that must be added to give the number between 11 and 20.

    a. 20 can be shown as the addition of 2 tens: 10 + 10 = 20. In Roman numerals, this is XX.
    b. 18 can be shown using addition: 10 + 8. In Roman numerals, this is XVIII.
    c. 14 can be shown using addition: 10 + 4. In roman numerals, this is XIV.
    d. 17 can be shown using addition: 10 + 7. In Roman numerals, this is XVII.

Practice

1. Mark's telephone number is +432-38857695. Help him write each digit of his phone number as a Roman numeral. The first digit is completed for you.

4	3	2	9	8	8	5	7	6	9	5
IV										

2. Write the following numbers as Roman numerals:
    a. 13 b. 15 c. 19 d. 16

Lesson Title: Converting between Base 10 and Roman Numerals	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-017	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to convert numbers up to 100 from base 10 to Roman numerals and vice versa.

Overview

In the previous lesson, you learned to write the Roman numerals up to 20. In this lesson, you will learn to identify and write the Roman numerals up to 100. In this lesson, you will use two new Roman numerals to construct the others. L is 50, and C is 100.

The tens are written in Roman numerals using a similar system to the numbers 1 – 10. Remember that a smaller digit **after** a larger digit shows that it is **added**. A smaller digit **before** a larger digit shows that it is **subtracted**.

Recall that VII can be represented as 5 + 2 = 7. Similarly, LXX can be represented by 50 + 20 = 70. Recall that IV can be represented as 5 – 1 = 4. Similarly, XL can be represented as 50 – 10 = 40.

The following table shows the tens in Roman numerals:

Tens from 10 – 100		
Hindu-Arabic	**Calculation**	**Roman**
10	10	X
20	10 + 10	XX
30	10 + 10 + 10	XXX
40	50 – 10	XL
50	50	L
60	50 + 10	LX
70	50 + 20	LXX
80	50 + 30	LXXX
90	100 – 10	XC
100	100	C

Other numbers up to 100 can be made using the tens in the table above, and adding digits 1-9 from the previous lesson. For example:

- 28 = 20 + 8 = XX + VIII = XXVIII
- 62 = 60 + 2 = LX + II = LXII
- 89 = 80 + 9 = LXXX + IX = LXXXIX

Solved Examples

1. Convert 37 to Roman numerals.

   Solution
   Break the number into tens and ones: 37 = 30 + 7

   Write the Roman numerals for 30 and 7: XXX + VII

   Write them together: 37 = XXXVII

2. Convert LXXIV to a base 10 number.

   Solution
   Look for the tens and ones. If needed, use the tables of Roman numerals in the previous lesson and this lesson.

   LXX is a multiple of 10, and IV is the ones digit. Break each one down to its parts:

   LXX = 50 + 20 = 70
   IV = 5 – 1 = 4

   The tens is 70 and the ones is 4. This gives LXXIV = 74.

3. Convert the following numbers to Roman numerals:
   a. 61 b. 92 c. 97 d. 23 e. 46

   Solutions
   Break each number down to tens and ones, then write each part as a Roman numeral. Finally, write the Roman numerals all together:

   a. 61 = 60 + 1 = LX + I = LXI
   b. 92 = 90 + 2 = XC + II = XCII
   c. 97 = 90 + 7 = XC + VII = XCVII
   d. 23 = 20 + 3 = XX + III = XXIII
   e. 46 = 40 + 6 = XL + VI = XLVI

4. Convert the following Roman numerals to base 10 numbers:
   a. XXXVI b. XLIV c. XLVIII d. LXXVII e. XCIII

Solutions

Break each Roman numeral down to tens and ones, then write each part as a base 10 numeral. Finally, add the base 10 numbers together:

a. XXXVI = XXX + VI = 30 + 6 = 36
b. XLIV = XL + IV = 40 + 4 = 44
c. XLVIII = XL + VIII = 40 + 8 = 48
d. LXXVII = LXX + VII = 70 + 7 = 77
e. XCIII = XC + III = 90 + 3 = 93

Practice

1. Convert the following base 10 numbers to Roman numerals:
   a. 19 b. 29 c. 55 d. 78 e. 83

2. Convert the following Roman numerals to base 10 numbers:
   a. XCIV b. XIV c. LXIII d. XLIX e. XXXVIII

Lesson Title: Introduction to Base 2	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-018	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify the numerals used to read and write in base 2.
2. Count up to 20 in base 2 and work out the pattern of numbers.

Overview

In this lesson, you will learn how to identify the numerals used to read and write in base 2.

Base 10 is the set of numbers that you are used to working with. There are 10 digits in base 10. These are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. All numbers in base 10 can be written with these digits.

Base 2 is another system of writing numbers. There are 2 digits in base 2. These are 0 and 1.

Base 2 numbers are also called 'binary' numbers. They are called binary numbers because 2 different digits are used. All base 2 numbers are made using the 2 numerals, 0 and 1. We use base 2 or binary numbers in all our mobile phones, laptops, computers, DVDs, and so on. All the information in these devices is stored and sent in 0s and 1s.

When we learned how to count in base 10, we used units, tens, hundreds, thousands and so on. These are all powers of 10.

To count with base 2, we will use the powers of 2. Recall the first 5 powers of 2: $1 = 2^0$; $2 = 2^1$; $4 = 2^2$; $8 = 2^3$; $16 = 2^4$.

Any base 10 number can be written as a base 2 number. For example, the number 5 in base 10 is 101 in base 2. To show the base of the numbers, use a subscript. 5_{ten} shows that 5 is a base 10 number. 101_{two} shows that 101 is a base 2 number.

To write base 10 numbers as base 2 numbers, we need to decide how many of each power of 2 we need to make the base 10 number. In the table below, base 10 numbers are listed on the right. At the top are the first 5 powers of 2. We write binary numbers by writing '1' under the numbers that we sum to get the base 10 number.

These are the first 5 base 2 numbers:

16	***8***	***4***	***2***	***1***		
				1	=	***1***
			1	*0*	=	***2***
			1	*1*	=	***3***
		1	*0*	*0*	=	***4***
		1	*0*	*1*	=	***5***

The first 5 binary numbers are 1, 10, 11, 100, 101. These are read differently than base 10 numbers. For example, 101_{ten} is read "one hundred and one". However, 101_{two} is read "one zero one".

To understand how the table works, consider the base 5_{ten}. We know that 4 + 1 = 5. In the table, write '1' under the digits 4 and 1. Write '0' in the other columns. We have 5_{ten} = 101_{two}.

We can also write this out as multiplication of the powers of 2 by 1 or 0:

$$\begin{aligned} 101_{\text{two}} &= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 4 + 0 + 1 \\ &= 5_{\text{ten}} \end{aligned}$$

Solved Examples

1. Complete the table below to find the next 5 binary numbers:

16	***8***	***4***	***2***	***1***		
					=	***6***
					=	***7***
					=	***8***
					=	***9***
					=	***10***

Solution

For each row, determine how many of each base 2 numbers are needed to make the given number. For example, 6 = 4 + 2. Write 1 in the columns for 4 and 2. Write 0 in the column for 1.

Continue with the other numbers until the table is filled:

16	***8***	***4***	***2***	***1***		
		1	*1*	*0*	=	***6***
		1	*1*	*1*	=	***7***
	1	*0*	*0*	*0*	=	***8***
	1	*0*	*0*	*1*	=	***9***
	1	*0*	*1*	*0*	=	***10***

2. Write each binary number as a base 10 number:

   a. 110 b. 1011 c. 1010 d. 1111

Solutions

For each number, identify each of the powers of 2 that correspond to the 1 digits. Add them together. You may draw a new table and use it to find each number. A table is shown below with a row for each of these numbers.

a. $110_{two} = 4 + 2 = 6_{ten}$

b. $1011_{two} = 8 + 2 + 1 = 11_{ten}$

c. $1010_{two} = 8 + 2 = 10_{ten}$

d. $1111_{two} = 8 + 4 + 2 + 1 = 15_{ten}$

16	***8***	***4***	***2***	***1***		
		1	*1*	*0*	=	***6***
	1	*0*	*1*	*1*	=	***11***
	1	*0*	*1*	*0*	=	***10***
	1	*1*	*1*	*1*	=	***15***

Practice

1. Complete the table below to find the binary numbers for base 10 numbers 16 to 20.

16	***8***	***4***	***2***	***1***		
					=	***16***
					=	***17***
					=	***18***
					=	***19***
					=	***20***

2. Write each binary number as a base 10 number:

   a. 1110 b. 1100 c. 10011 d. 101 e. 111

Lesson Title: Ordering and Comparing Numbers in Base 2	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-019	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. List sets of base 2 numbers up to 20 in ascending order.
2. Compare base 2 numbers up to 20.

Overview

Comparing and ordering numbers in base 2 is similar to comparing and ordering numbers in base 10.

Numbers with more digits are larger. For example, 10 > 1. Recall that if you convert 10 to a base 10 number, it is 2. When you convert 1 to a base 10 number, it is still 1. Therefore, 10 > 1 in base 2 is the same as 2 > 1 in base 10.

If numbers have the same digits, look at the digits starting from the left to determine which number is greater. For example, 111 > 101. The first digit is the same: 1. Look at the second digit. 111 is greater than 101 because its second digit is greater. It does not matter what the next digit is.

If you can compare 2 numbers, then you can also write a list of numbers in order according to size. Remember that **ascending** means from least to greatest, and **descending** means from greatest to least.

Solved Examples

1. Put < or > to make the following statements true:

   a. 1111 ☐ 111 b. 10111 ☐ 10110

   c. 1000 ☐ 1011 d. 101 ☐ 1000

   Solutions

   Consider each set of numbers.

   a. 1111 > 111 because 1111 has more digits.
   b. 10111 > 10110. The first 4 digits are the same, so look at the fifth digit. 1 > 0, so we have 10111 > 10110.
   c. 1000 < 1011. The first 2 digits are the same, so look at the third digit. 0 < 1, so we have 1000 < 1011.
   d. 101 < 1000 because 1000 has more digits.

2. Write the sets of base 2 numbers in ascending order:
    a. {101, 110, 111, 100}
    b. {1100, 1010, 1000, 1001, 1111}
    c. {11, 1, 10}
    d. {11011, 1011, 101, 10111, 100}

    Solutions

    First, look at the number of digits in each number. List numbers with fewer digits first, because they are smaller. After grouping the numbers according to their digits, look at the digits from left to right to determine their size.

    a. All of the numbers are 3 digits, so list them according to their size. For example, numbers with 0 in the second place are less than numbers with 1 in the second place. The answer is {100, 101, 110, 111}.
    b. All of the numbers are 4 digits, so list them according to their size. The answer is {1000, 1001, 1010, 1100, 1111}
    c. The numbers have a different number of digits. Numbers with 1 digit (1) come before numbers with 2 digits (10 and 11). The answer is {1, 10, 11}.
    d. The numbers have 3, 4 and 5 digits. First, write them in order from fewer digits to more digits: {101, 100, 1011, 11011, 10111}.
    Now consider each set of numbers with the same number of digits. For example, 100 < 101, so we want to write them in this order. The answer is {100, 101, 1011, 10111, 11011}.

3. Write the sets of base 2 numbers in descending order:
    a. {11, 111, 10, 101, 100}
    b. {1011, 1101, 110, 101, 1000, 10001}
    c. {1, 10, 100, 11, 101, 110}
    d. {1010, 1100, 1011, 1000, 1001}

    Solutions

    Follow the same process as in Solved Example 2, but list the numbers from largest to smallest.

    a. {111, 101, 100, 11, 10}
    b. {10001, 1101, 1011, 1000, 110, 101}
    c. {110, 101, 100, 11, 10, 1}
    d. {1100, 1011, 1010, 1001, 1000}

Practice

1. Put $<$ or $>$ to make the following statements true:

   a. 101 $\square$ 1000   b. 1110 $\square$ 11011

   c. 1011 $\square$ 1100   d. 11101 $\square$ 11001

2. Write the sets of base 2 numbers in ascending order:
   a. {111, 11, 1, 110, 10, 101, 100}
   b. {1011, 1100, 1101}
   c. {11011, 10011, 11111, 10101, 10111}
   d. {101, 1011, 1010, 111, 110}

3. Write the sets of base 2 numbers in descending order:
   a. {101, 100, 111}
   b. {10111, 1011, 1101, 11101}
   c. {11, 101, 110, 10, 111}
   d. {11011, 1011, 1000, 10000, 10101, 1110}

Lesson Title: Converting between Base 10 and Base 2	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-020	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to convert numbers up to 50 from base 10 to base 2 and vice versa.

Overview

In this lesson, you will convert numbers up to 50 from base 10 to base 2. You will also convert numbers from base 2 to base 10. Remember that the powers of 2 are used to convert to base 2. We have worked with powers up to $2^4 = 16$ to convert from base 10 numbers up to 20. In this lesson, you will work with larger numbers, so you will use the next power of 2, $2^5 = 32$. Another column is added to the base 2 table (see below).

To **convert a base 10 number to a base 2 number**, find which powers of 2 add up to the given number. Write a 1 in the column for each base 2 number you need.

For example, consider 34_{ten}. You can add 32 and 2 to make 34_{ten}: 32 + 2 = 34. Write 1 in the columns for 32 and 2. Write 0 in all the other columns.

32	**16**	**8**	**4**	**2**	**1**		
1	0	0	0	1	0	=	34_{ten}

The base 2 number is 100010_{two}. We have $34_{ten} = 100010_{two}$.

To **convert a base 2 number to a base 10 number**, write the 1 and 0 digits in the conversion table. Remember to line the numbers up on the right side of the table. Add the powers of 2 wherever you have a 1.

For example, consider 11001_{two}. Write the digits in the table:

32	**16**	**8**	**4**	**2**	**1**		
	1	1	0	0	1	=	25_{ten}

Add the numbers at the top of each column with a 1: 16 + 8 + 1 = 25_{ten}. We have 11001_{two} = 25_{ten}.

Solved Examples

1. Convert the following base 10 numbers to base 2 numbers:
   a. 50_{ten} b. 48_{ten} c. 35_{ten} d. 29_{ten} e. 22_{ten}

Solutions

Draw a table with powers of 2, and use a row to convert each number to base 2:

32	16	8	4	2	1		
1	1	0	0	1	0	=	50_{ten}
1	1	0	0	0	0	=	48_{ten}
1	0	0	0	1	1	=	35_{ten}
	1	1	1	0	1	=	29_{ten}
	1	0	1	1	0	=	22_{ten}

The answers are:

a. 110010_{two}
b. 110000_{two}
c. 100011_{two}
d. 11101_{two}
e. 10110_{two}

2. Convert the following base 2 numbers to base 10 numbers:
   a. 11100 b. 101000 c. 101010 d. 101100 e. 10111

Solutions

Draw a table and write each binary number in it. Remember to line the digits up from the right. Add the powers of 2 to find the base 10 number. The working and answers are shown below the table.

32	16	8	4	2	1		
	1	1	1	0	0	=	28_{ten}
1	0	1	0	0	0	=	40_{ten}
1	0	1	0	1	0	=	42_{ten}
1	0	1	1	0	0	=	44_{ten}
	1	0	1	1	1	=	23_{ten}

The answers are:

a. $16 + 8 + 4 = 28_{ten}$
b. $32 + 8 = 40_{ten}$
c. $32 + 8 + 2 = 42_{ten}$
d. $32 + 8 + 4 = 44_{ten}$
e. $16 + 4 + 2 + 1 = 23_{ten}$

Practice

1. Convert the following base 10 numbers to base 2 numbers:
   a. 49_{ten} b. 32_{ten} c. 21_{ten} d. 43_{ten} e. 36_{ten}

2. Convert the following base 2 numbers to base 10 numbers:
   a. 101001 b. 11111 c. 100001 d. 11011 e. 101101

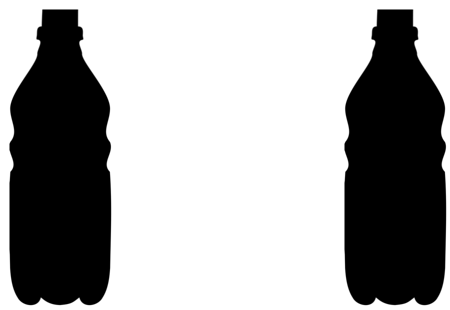
Lesson Title: Capacity and Mass	**Theme:** Everyday Arithmetic
Practice Activity: PHM-09-021	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Differentiate between mass and capacity.
2. Solve problems with masses and problems with capacities.

Overview

In this lesson, you will learn how to differentiate between mass and capacity.

Imagine that you have 2 bottles of the same size. Each holds 1 litre of liquid. One is full of water, the other one is full of palm oil. One litre of water has a mass of 1000 g. One litre of palm oil has a mass of 890 g.

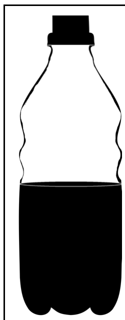


1 litre water
1000 g water

1 litre palm oil
890 g palm oil

The **mass** of an object is the amount of matter it contains. The **volume** is the amount of space the object takes up. Two objects can have the same volume but a different mass. On the other hand, 2 objects can have the same mass but different volume.

The words 'volume' and 'capacity' have a very similar meaning, and you will hear them used in the same way. **Capacity** is the amount of space inside of something, and **volume** is the amount of space that is taken up. For example, you may have a bottle with a capacity of 1 litre. If the bottle is half filled with water, the volume of the water is 0.5 litre, or 500 millilitres.



Bottle capacity: 1 litre

Water volume: 0.5 litre or 500 ml

We use the metric system of measurement for mass and capacity.

- Common units used to measure mass are: gramme, milligramme, kilogramme, tonne
- Common units used to measure volume are: litre, millilitre

The relationships between units can be used to convert from one unit to another. The relationships are:

Mass		
1000 mg	=	1 g
1000 g	=	1 kg
1000 kg	=	1 tonne (t)

Capacity		
1000 ml	=	1 l
10 decilitres (dl)	=	1 l
1 kilolitre (kl)	=	1000 l

To convert from a larger unit to a smaller one, multiply. To convert from a smaller unit to a larger one, divide. Here are some examples:

- Convert 4 litres to ml: $4 \times 1{,}000 = 4{,}000$ ml
- Convert 3,000 ml to litres: $3{,}000 \div 1{,}000 = 3$ l

In this lesson, you will solve story problems involving mass and capacity. Note that quantities should be in the same units before performing operations on them.

Solved Examples

1. Hawa bought a 20 kg bag of rice. In one week, her family ate 8.5 kg of rice. How much rice is left? Give your answer in: a. kg b. g

Solutions

The words "how much is left" tell us to subtract: 20 kg – 8.5 kg.

a. Work the subtraction problem:

$$\begin{array}{rr} {}^{19} & \\ \cancel{20}. & {}^{1}0 \\ -\ 8. & 5 \\ \hline 11. & 5 \end{array}$$

11.5 kg of rice.

b. Convert 11.5 kg to grammes. Multiply by 1000, because you are converting from a larger unit to a smaller unit. Remember that to multiply by 1000, you move the decimal point 3 spaces to the right.

$11.5 \times 1{,}000 = 11{,}500$ g of rice.

2. A bottle contains 1 litre of water. Mohamed drinks 500 ml, and Abu drinks 300 ml. How much water is left? Give your answer in millilitres.

Solution

The words "how much is left" tell us to subtract. Quantities must be in the same units before we can subtract. Convert 1 litre to millimetres, then subtract the amounts they drank.

Step 1. Convert 1 litre to ml: $1 \times 1{,}000 = 1{,}000$ ml

Step 2. Subtract: $1{,}000 - (500 + 300) = 1{,}000 - 800 = 200$ ml

There are 200 ml of water left.

3. Juliet owns a restaurant. She uses 500 ml of oil each day. How much oil does she use in 7 days? Give your answer in litres.

   Solution
   In this example, the problem is worked in millilitres, then converted to litres. You could also convert 500 ml to litres first, then work the problem.

   Step 1. Multiply to find how much is used in 7 days:

   $500 \times 7 = 3{,}500$ ml

   Step 2. Convert to litres. Remember that when dividing by 1,000, the decimal place moves 3 places to the left.

   $3{,}500$ ml $= 3{,}500 \div 1{,}000 = 3.5$ litres

Practice

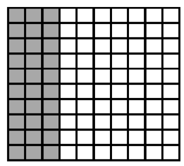
1. A baby weighed 4.5 kg. She gained 2.5 kg. How much does she weigh now? Give your answer in: a. kg b. g
2. There is 1 litre of cola in a bottle. It is divided evenly between 5 people. How much does each person drink? Give your answer in millilitres.
3. Fatu drinks 500 ml of orange juice each day. How much does she drink in 3 days? Give your answer in: a. millilitres b. litres
4. David has a bottle with 2 litres of water. If he lets his brother drink 700 millilitres, how much water is left? Give your answer in litres.
5. Ahmed eats 250 grammes of rice each day. How much does he eat in 8 days? Give your answer in kilogrammes.

Lesson Title: Percentages of Quantities	**Theme:** Everyday Arithmetic
Practice Activity: PHM-09-022	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to find percentages of quantities.

Overview

Percent means part of 100, or out of 100. For example, 30 percent means 30 out of one hundred. The square below is divided into 100 small pieces. Thirty of the squares are shaded. This shows 30%. This can also be written as a fraction: $30\% = \frac{30}{100}$.



Any percentage can be written as a fraction over 100. For example: $25\% = \frac{25}{100}$ and $7\% = \frac{7}{100}$.

To find the percentage of a given quantity, express the percentage as a fraction and then multiply the fraction by the given quantity.

Solved Examples

1. Calculate 15% of 500.

   Solution

   Step 1. Express 15% as a fraction: $\frac{15}{100}$

   Step 2. Find 15% of 500: $\frac{15}{100} \times \frac{500}{1} = \frac{7500}{100} = 75$

2. Calculate 65% of Le 50,000.00.

   Solution

   When there are units in a problem (for example, Leones) make sure you include the units with your answer. Note that money sometimes includes 2 extra zeros. These are for showing cents. If the two zeros are given in the problem, give them in the answer too. In this problem, Le 50,000.00 = 50,000. These amounts are the same.

   Step 1. Express 65% as a fraction: $\frac{65}{100}$

   Step 2. Find 65% of Le 50,000.00: $\frac{65}{100} \times \frac{50,000}{1} = \frac{65 \times 500}{1} =$ Le 32,500.00

3. Find 20% of 90 mangoes.

Solution

Step 1. Express 20% as a fraction: $20\% = \frac{20}{100}$

Step 2. Multiply the fraction by 90: $\frac{20}{100} \times 90 = \frac{1}{5} \times 90 = 18$ mangoes

4. Musu has 50 oranges and gave 30% of her oranges to her sister. How many oranges did she give away?

Solution

Step 1. Express the percent as a fraction: $30\% = \frac{30}{100}$

Step 2. Multiply the fraction by 50: $\frac{30}{100} \times 50 = \frac{3}{10} \times 50 = 3 \times 5 = 15$ oranges

5. Joe is given $\text{Le } 15{,}000.00$ as lunch and transport to and from school every day. If he spends 40% of this amount as transport to and from school, how much is left for lunch?

Solution

There are 2 ways to solve this problem. Both methods are shown below.

Method 1. Find how much he spends on transportation. Then, subtract that amount from 15,000 to find the amount left for lunch.

Money spent on transportation $= \frac{40}{100} \times 15{,}000 = \text{Le } 6{,}000.00$

Money left for lunch $= \text{Le}15{,}000 - \text{Le } 6{,}000 = \text{Le } 9{,}000.00$

Method 2. Find the percentage he has left for lunch, then use the percentage to calculate the amount of money left.

Percentage spent on lunch: $100\% - 40\% = 60\%$

Money left for lunch $= \frac{60}{100} \times 15{,}000 = \text{Le } 9{,}000.00$

6. 60% of the pupils in a school are girls. If 540 pupils are girls, find:
   a. How many pupils there are in the school in total.
   b. How many boys there are in the school.

Solutions

a. This problem is different because you want to solve for the total number of pupils. You are given the percentage and the quantity of girls. Set up the equation and solve for the total:

Number of girls	=	Percentage × Total population	Formula
540	=	$\frac{60}{100}$ × Total population	Substitute
540×100	=	$60 \times$ Total population	Multiply by 100
$54{,}000$	=	$60 \times$ Total population	Divide by 60
$\frac{54{,}000}{60}$	=	Total population	
900	=	Total population	

There are 900 pupils in the school.

b. There are 900 pupils and 540 of them are girls. Subtract the girls from the total to find how many boys there are.

Number of boys $= 900 - 540 = 360$ boys

Practice

1. Find 60% of 800.
2. Find 5% of 1,000.
3. Find 2% of Le 48,000.00.
4. Find 35% of 120 mangoes.
5. Fatu bought a bag containing 150 oranges, but 10% were rotten. How many were rotten?
6. A village has a population of 1,500 people. 28% of the population are children.
   a. How many children are there?
   b. How many adults are there?
7. A newspaper vendor has 500 newspapers to sell. He sold 25% of them.
   a. How many did he sell?
   b. How many newspapers remain?
8. 25% of a certain number is 16. What is the number?
9. What is $5\frac{1}{2}\%$ of Le $600{,}000.00$?
10. A father left some amount of money for his children, Abu and Fatu. Fatu received Le $32{,}000.00$, which is 40% of the amount. Abu received the rest.
    a. Find how much money the father left them in total.
    b. Find how much money Abu received.

Lesson Title: Percentage Increase and Decrease	**Theme:** Everyday Arithmetic
Practice Activity: PHM-09-023	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Calculate the percentage increase or decrease, given 2 numbers.
2. Increase and decrease quantities by a percentage.

Overview

Percentage change is all about comparing old to new values. A change can either be an increase or a decrease. When the new value is greater than the old value, it is a percentage increase. When the new value is less than the old value, it is a decrease. This lesson handles two types of problems. You will be able to calculate the percentage change. You will also be able to increase or decrease a quantity by a given percentage.

To **find percentage change**, express the change in quantity as a fraction of the original quantity and then multiply by 100. The formula for calculating percentage change is:

$$\text{Percentage change} = \frac{\text{change in quantity}}{\text{original quantity}} \times 100$$

To calculate percentage change for an **increase**, subtract the original quantity from the new quantity (new quantity – original quantity). To calculate change in quantity for a **decrease**, subtract the new quantity from the original quantity (original quantity – new quantity).

You can also **find the new quantity** after an increase or decrease. If there is a percentage increase, it means we add to the original amount. If there is a percentage decrease, it means we subtract from the original amount.

To calculate a new quantity given the percentage increase or decrease upon the original, given number, follow these steps:

- State the increase or decrease in percent.
- For percent increase, **add** the percentage to 100%.
- For percent decrease, **subtract** the percentage from 100%.
- Since it is percent, divide the answer by 100 to cancel the percentage.
- Multiply the answer by the given number to find the new number.

Use the following formulae:

Percentage increase: New number $= \frac{100+\text{percentage increase}}{100} \times \frac{\text{the given number}}{1}$

Percentage decrease: New number $= \frac{100-\text{percentage decreae}}{100} \times \frac{\text{given number}}{1}$

Solved Examples

1. The cost of petrol increased from Le $4{,}500.00$ to Le $6{,}300.00$ per litre. Calculate the percentage increase.

   Solution
   Step 1. Calculate the change in quantity: $6{,}300 - 4{,}500 = \text{Le } 1{,}800.00$
   Step 2. Calculate percentage increase using the formula:
   $$\text{Percentage increase} = \frac{1800}{4500} \times 100 = \frac{1800}{45} = 40\%$$

2. Martin is a farmer. One week, he harvested 12 kg of cassava. The next week, he harvested 21 kg of cassava. What was the percentage increase?

   Solution
   Step 1. Calculate the change in quantity: $21 - 12 = 9$ kg
   Step 2. Calculate percentage increase using the formula:
   $$\text{Percentage increase} = \frac{9}{12} \times 100 = \frac{3}{4} \times 100 = \frac{300}{4} = 75\%$$

3. Mustapha is a farmer. Last week, he sold his cassava for Le 4,000.00 per kilogramme. This week, there is more cassava on the market. He could only sell his cassava for Le 3,800.00 per kilogramme. What is the percentage decrease in the price?

   Solution
   Step 1. Calculate the change in quantity: $4{,}000 - 3{,}800 = \text{Le } 200.00$
   Step 2. Calculate percentage decrease using the formula:
   $$\text{Percentage decrease} = \frac{200}{4000} \times 100 = \frac{200}{40} = \frac{20}{4} = 5\%$$

4. The number 600 is increased by 35%. Find the new number.

   Solution
   Use the formula for percentage increase:
   $$\begin{aligned}\text{New number} &= \frac{100+\text{percentage increase}}{100} \times \frac{\text{the given number}}{1} \\ &= \frac{100+35}{100} \times \frac{600}{1} \\ &= \frac{135}{1} \times \frac{6}{1} \\ &= 135 \times 6 \\ &= 810\end{aligned}$$

5. The number 600 is decreased by 35%. Find the new number.

   Solution

   Use the formula for percentage decrease:

$$\begin{aligned}\text{New number} &= \frac{100-\text{percentage decrease}}{100} \times \frac{\text{the given number}}{1} \\ &= \frac{100-35}{100} \times \frac{600}{1} \\ &= \frac{65}{1} \times \frac{6}{1} \\ &= 65 \times 6 \\ &= 390\end{aligned}$$

6. The population of a village was 5,600 people. If the population increased by 12%, what is the new population?

   Solution

   Use the formula for percentage increase to find the new population:

$$\begin{aligned}\text{New number} &= \frac{100+\text{percentage increase}}{100} \times \frac{\text{the given number}}{1} \\ &= \frac{100+12}{100} \times \frac{5{,}600}{1} \\ &= \frac{112}{1} \times \frac{56}{1} \\ &= 112 \times 56 \\ &= 6{,}272 \text{ people}\end{aligned}$$

Practice

1. A factory increases its annual production of shoes from 4,000 to 4,600. Calculate the percentage increase in the number of shoes.
2. The population of a village increased from 500 to 525. Calculate the percentage increase.
3. Fatu scored 60 marks on her maths exam. She was not happy with her score, and decided to study a lot more. On her next maths exam, she scored 90 marks. What was the percentage increase of her score?
4. A litre of petrol cost Le 8,000.00, and the price decreased to Le 7,400.00. What was the percentage decrease?
5. Last year, there were 8,000 patients treated at a certain hospital. This year, 7,600 patients were treated at the same hospital. What is the percentage decrease?
6. A seamstress gives a discount of 5% for customers who pay in advance. Calculate the reduced price of a dress that originally cost Le 70,000.00.
7. An athlete took 10 seconds to sprint 100 m during practice. If in the actual race he reduced his time by 8%, how long did it take him to run the actual race?
8. Increase a length of 80 cm by 30%.
9. The number of pupils enrolled in a certain school was 520 last year. This year, enrolment increased by 15%. How many pupils are enrolled in the school this year?
10. Isatu's salary is Le 480,000.00. If it is increased by 20%, what is her new salary?

Lesson Title: Ratios	**Theme:** Everyday Arithmetic
Practice Activity: PHM-09-024	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Review the forms of ratio: m : n and $\frac{m}{n}$.
2. Divide a number into a given ratio.
3. Solve ratio problems and simplify answers to the lowest terms.

Overview

Ratio is a way of comparing two or more quantities. For example, if you compared the number of boys and girls in your class, that would be a ratio.

Say that you are in a JSS 3 class where there are 22 boys and 24 girls. This can be written as a ratio in two ways: 22 : 24 or $\frac{22}{24}$. The ratio is read as "22 is to 24". It can also be said "22 boys is to 24 girls".

A ratio can be **simplified to its lowest terms**, like any fraction. It is easiest to write the ratio in the fractional form before simplifying: $\frac{22}{24} = \frac{22 \div 2}{24 \div 2} = \frac{11}{12}$. The simplified form can be written $\frac{11}{12}$ or 11 : 12. This means that for every 11 boys in the class, there are 12 girls.

The order in which ratios are written is very important and must be maintained when solving a problem. A ratio written as 2 : 3 means $\frac{2}{3}$, while a ratio written as 3 : 2 means $\frac{3}{2}$. The fractions are different and give different answers.

We can only simplify ratios when the quantities are in the same units. If the quantities are not in the same unit, we must convert one to the other before we simplify.

We can also **share a number in a given ratio**. For example, say you have 75 oranges. You want to share them between Foday and Fatu in the ratio 2 : 3. This means that for every 2 oranges Foday gets, Fatu will get 3 oranges. Follow these steps to divide a number into a given ratio:

Steps	Example Problem
1. Treat the parts of the ratio as shares. Add them to find how many shares there are all together.	In the example, there are 2 + 3 = 5 shares.
2. Divide the number you are sharing by the number of shares from step 1. This gives the size of 1 share.	The number of oranges in 1 share is $75 \div 5 = 15$ oranges.
3. Multiply each part of the ratio by the size of 1 share to find how much that person gets.	Foday gets $2 \times 15 = 30$ oranges Fatu gets $3 \times 15 = 45$ oranges

You can check your answer by adding the results. They should equal the original amount you shared. In this example problem, $30 + 45 = 75$ oranges.

Solved Examples

1. What is 20 centimetres as a ratio of 1 metre? Give your answer as a fraction in its simplified form.

   Solution
   Quantities must be in the same unit before writing them as a ratio. Convert 1 metre to centimetres. We know that 1 m = 100 cm.

   The ratio is 20 : 100. Write this as a fraction and simplify it: $20 : 100 = \frac{20}{100} = \frac{1}{5}$

2. Mohamed received an 85% mark on an exam. What ratio of correct answers did Mohamed get? Write your answer as a fraction in its simplified form.

   Solution
   To solve this, we must remember that when we talk about percent, we compare a number to 100. 85% is the same as $\frac{85}{100}$. We also know that $\frac{85}{100} = 85 : 100$. This is the ratio that we want to simplify: $\frac{85}{100} = \frac{17}{20}$

3. The length and width of a rectangle are 45 cm and 30 cm respectively. What is the ratio of its length to its width?

   Solution
   The ratio of its length to its width is 45 : 30. Write this as a fraction and simplify it to its lowest term: $\frac{45}{30} = \frac{3}{2}$. The answer is $\frac{3}{2}$ or 3 : 2.

4. Divide 90 in the ratio 2 : 7.

   Solution
   Follow the steps given in the overview to divide 90 in the given ratio.

   Step 1. Find the total number of shares: $2 + 7 = 9$ shares

   Step 2. Find the size of 1 share. Divide the number you are sharing by the number of shares: $90 \div 9 = 10$.

   Step 3. Multiply each part of the ratio by the size of 1 share:

   $$2 \times 10 = 20 \qquad\qquad 7 \times 10 = 70$$

   Answer: 20, 70.

5. Mustapha picks 48 mangoes from a tree. He wants to share them between his 2 friends in the ratio 3 : 1. How much does each friend get?

Solution

Step 1. Find the total number of shares: $3 + 1 = 4$ shares

Step 2. Find the size of 1 share. Divide the number Mustapha is sharing by the number of shares: $48 \div 4 = 12$.

Step 3. Multiply each part of the ratio by the size of 1 share:

$$3 \times 12 = 36 \qquad 1 \times 12 = 12$$

Answer: His friends get 36 mangoes and 12 mangoes.

6. Share 96 ml in the ratio 5 : 3.

Solution

Step 1. Find the total number of shares: $5 + 3 = 8$ shares

Step 2. Find the size of 1 share. Divide the number you are sharing by the number of shares: $96 \div 8 = 12$.

Step 3. Multiply each part of the ratio by the size of 1 share:

$$5 \times 12 = 60 \qquad 3 \times 12 = 36$$

Answer: 60 ml and 36 ml

Practice

1. Express $20 : 35$ as a fraction in its lowest terms.
2. Hawa received a 95% mark on an exam. What ratio of correct answers did Hawa get? Write your answer as a fraction in its simplified form.
3. Mohamed has a shop near a school. He sold 150 exercise books this year, and 100 exercise books remain in his shop. What is the ratio of sold exercise books to unsold exercise books? Give your answer as a fraction in its lowest terms.
4. Sia has 108 bananas. She wants to share them between 2 of her friends in the ratio 4 : 5. How many bananas does each friend get?
5. Share 200 grammes in the ratio 1 : 3.
6. Share 150 exercise books in the ratio 2 : 3.

Lesson Title: Rates	Theme: Everyday Arithmetic
Practice Activity: PHM-09-025	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that rate is a special ratio that compares 2 units of measurement.
2. Solve problems involving rate.

Overview

We use ratios to compare two or more 'like' quantities. This means that they are of the same kind. For example, they are both people (boys and girls), or they are both measurements of weight (kg and g). Remember that measurements must be expressed in the same unit for them to be compared as a ratio.

Rate is a special ratio that compares two **different** type of units of measurement. For example, how much money someone is paid per month at their job. The different quantities are time and money.

The quantities in a ratio are measured with the same unit. When we write the ratio as a fraction, the units in the ratio cancel each other out because they are the same. For example, $\frac{1\text{ km}}{2\text{ km}} = \frac{1\cancel{\text{ km}}}{2\cancel{\text{ km}}} = \frac{1}{2}$.

The quantities in a rate are measured with 2 different units. The units in a rate take on the unit from the numerator and the unit from the denominator. For example, if a car travels 3 kilometres in 1 hour, we have the rate $\frac{\text{distance}}{\text{time}} = \frac{3\text{ km}}{1\text{ hr}} = 3\frac{\text{km}}{\text{hr}} = 3\text{ km/hr}$. This can be read as '3 kilometres per hour'.

Rates use words and symbols such as **per** (/), **each** (ea) and **at** (@).

Solved Examples

1. A man buys 100 grammes of rice for Le 3,600.00.
   a. Write this as a rate of Leones per grammes.
   b. Write the rate in its simplest form.

   Solutions
   a. The rate is $\frac{\text{Le }3{,}600.00}{100\text{ g.}}$. This is read as "Le 3,600 per 100 g of rice". This means that for Le 3,600 you can purchase 100 g of rice.
   b. Simplify the fraction: $\frac{\text{Le }3{,}600.00}{100\text{ g.}} = \frac{\text{Le }36.00}{1\text{ g.}}$. This ratio is commonly written as $\text{Le }36.00/\text{g}$. It is read as "Le 36 per gramme of rice".

2. If you walk 12 miles in 3 hours, at what rate are you traveling? Give your answer in miles per hour.

   Solution

   You are walking at a rate of 12 miles per 3 hours, which can be written as $\frac{12\text{ mi.}}{3\text{ hr}}$. This can be simplified: $\frac{12\text{ mi.}}{3\text{ hr}} = \frac{4\text{ mi.}}{1\text{ hr}} = 4\text{ mi./hr.}$

   Note that miles per hour is often written mph. We can write the answer as 4 mph.

3. A car covered a distance of 210 km in $3\frac{1}{2}$ hours. Calculate the average speed of the car.

   Solution

   When you are asked to calculate average speed, you simply use the numbers in the problem to find the rate at which the car travelled. The car may have been going faster or slower at some points, but this calculation will give you the average speed.

   Divide and simplify: $\frac{210\text{ km}}{3.5\text{ hr}} = \frac{2{,}100\text{ km}}{35\text{ hr}} = \frac{420\text{ km}}{7\text{ hr}} = 60\text{ km/hr}$

   The average speed of the care was 60 kph.

   - Recall the rules for dividing decimals. The divisor can be changed to a whole number. This will make the rate easier to divide or simplify: $\frac{210\text{ km}}{3.5\text{ hr}} = \frac{2{,}100\text{ km}}{35\text{ hr}}$

4. Abigail is a tailor. She can sew 10 dresses in 5 days. What is her rate for sewing dresses?

   Solution

   Her rate is $\frac{10\text{ dresses}}{5\text{ days}} = \frac{2\text{ dresses}}{1\text{ day}} = 2\text{ dresses/day.}$

5. Mustapha sells 4 kilogrammes of chicken for Le 100,000.00. What is the rate of the price for chicken? Give your answer in Leones per kilogramme.

   Solution

   The rate of the price for chicken is $\frac{\text{Le }100{,}000}{4\text{ kg}} = \frac{\text{Le }25{,}000}{1\text{ kg}} = \text{Le }25{,}000.00\text{ Le/kg}$

6. John is working on his maths assignment. There are 10 problems on the assignment. If it takes him 45 minutes to solve the problems, what is his rate in minutes per problem?

   Solution

   His rate in problems per minute is $\frac{45\text{ minutes}}{10\text{ problem}} = \frac{9}{2}\text{ minutes/problem}$

   You may also write your answer as a decimal: $4.5\text{ minutes/problem}$. Rates are often given as decimal numbers.

Practice

1. A car travels 60 kilometres in 2 hours. Find the car's rate in km/hr.
2. A man walks 15 kilometres in 3 hours. Find the man's rate in km/hr.
3. Mohamed is an auto mechanic. He can fix 20 cars in 8 days. What is his rate of fixing cars?
4. Fatu sat for a maths exam. She solved 20 problems in 40 minutes. What is her rate in minutes per problem?
5. Foday harvested his pepper. He worked for 4 hours and harvested 12 kg of pepper. At what rate does he harvest pepper? Give your answer in kg/hr.
6. A factory produces 6,800 cartons of soap in 4 hours. Find the company's production rate of cartons per hour.
7. If USD $1 equals Le 7,200.00, how much will USD $350.00 give in Leones?
8. A customer used 2,468 Kwh of electricity in January. If the cost of 1 Kwh is Le 860.00, how much will the customer have to pay?

Lesson Title: Direct Proportions	**Theme:** Everyday Arithmetic
Practice Activity: PHM-09-026	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify the symbol for proportionality ($\propto$), the means and extremes.
2. Solve direct proportion problems.

Overview

A proportion is a pair of equivalent ratios in which the units must be the same. Direct proportions mean that as one ratio increases, the other does too. On the other hand, as one ratio goes down the other does too.

For example, this is a proportion: $\frac{2}{4}=\frac{5}{10}$. If one ratio changes, the other must also change so that they stay equal. For example, if 2 increases to 4, then 5 increases to 10.

Direct proportions are shown by the relationship one value (y) is equal to another value (x) multiplied by a constant (k): $y = kx$. Remember that a constant is a number that does not change. We call k the **constant of proportionality**, because it does not change. It shows the relationship between the two ratios in the proportion.

When two values are directly proportional to each other, we can use the symbol $\propto$. This symbol means 'is proportional to'. For $y = kx$, we can also write $y \propto x$, meaning that y is directly proportional to x.

When we have a proportion with two equivalent ratios, we can use **cross-multiplication**. Cross-multiplication is multiplying the diagonals of equal fractions. If we multiply the numbers diagonally across from each other, the two products will be equal.

When you cross-multiply you multiply the **extremes** first. The extremes are the outside terms. In our example $\frac{2}{4}=\frac{5}{10}$, 2 and 10 are the extremes. Then you multiply the **means**. The means are the inside terms. In our example, 4 and 5 are the means.

$$\frac{2}{4}=\frac{5}{10}$$

The product of the extremes should be equal to the product of the means: $2 \times 10 = 5 \times 4 = 20$. Multiplying the means and extremes like this shows that the proportions are equivalent. Since these ratios are equal, the cross products are equal too.

In this lesson, you will solve real-life problems involving proportion. For example**: If Joe can read 4 books in 2 days, how many books can he read in 8 days?** There are 2 ways to solve the problems you will see in this lesson: the unitary method and the ratio method.

For the **unitary method**, you first find the rate for 1 unit. In this example, Joe reads 4 books in 2 days. The first step would be to find out how many books Joe can read in **1 day**. Then, you would multiply the unitary rate by the number given in the problem. In this example, multiply the rate for 1 day by 8 days.

For the **ratio method**, write the proportions as fractions and solve using cross-multiplication.

Examples of both methods are shown in the Solved Examples. You may choose one method to solve the Practice problems.

Solved Examples

1. y and x are directly proportional. When $x = 20$, $y = 4$. Find the constant of proportionality.

   Solution
   Substitute the values $x = 20$ and $y = 4$ into the equation and solve for k.

$$
\begin{aligned}
y &= kx \\
4 &= k(20) && \text{Substitute } x = 20 \text{ and } y = 4 \\
4 &= 20k \\
\frac{4}{20} &= \frac{20k}{20} && \text{Divide both sides by 20} \\
\frac{1}{5} &= k && \text{Simplify the fraction}
\end{aligned}
$$

   We have found the constant of proportionality, $k = \frac{1}{5}$. The equation is $y = \frac{1}{5}x$.

2. If Joe can read 4 books in 2 days, how many books can he read in 8 days?

   Solution
   Using the **unitary method:** Find how many books Joe reads in 1 day:

$$4 \text{ books} \div 2 \text{ days} = 2 \text{ books in } 1 \text{ day}$$

   Multiply the rate for 1 day by the number of days:

$$\text{Number of books in } 8 \text{ days} = 2 \text{ books } \times \ 8 = 16 \text{ books}$$

   Using the **ratio method:** Write a ratio for the problem: $\frac{4 \text{ books}}{2 \text{ days}} = \frac{y}{8 \text{ days}}$. The unknown value y is the number of books he can read in 8 days.

   Solve using cross-multiplication:

$$
\begin{aligned}
4 \times 8 &= 2 \times y && \text{Cross-multiply} \\
32 &= 2y && \text{Simplify} \\
\frac{32}{2} &= y && \text{Divide both sides by 2} \\
16 &= y
\end{aligned}
$$

   Answer: Joe can read 16 books in 8 days.

3. Mustapha can paint 80 m^2 with 2 cans of paint. What area can he paint with 7 cans of paint?

Solution

Using the **unitary method:** Find how many m^2 Mustapha can paint with 1 can:

$$80 \text{ m}^2 \div 2 \text{ days} = 40 \text{ m}^2 \text{ with 1 can}$$

Multiply the rate for 1 can by the number of cans:

$$\text{Number of m}^2 \text{with 7 cans} = 40 \text{ m}^2 \times 7 = 280 \text{ m}^2$$

Answer: Mustapha can paint 280 m^2 with 7 cans of paint.

4. Bendu drove her car 120 km in 4 hours. What distance could she drive in 6 hours, if she kept the same rate?

Solution

Using the **ratio method:** Write a ratio for the problem: $\frac{120 \text{ km}}{4 \text{ hr}} = \frac{z}{6 \text{ hr}}$. The unknown value z is the number of kilometres she can drive in 6 hours.

Solve using cross-multiplication:

$120 \times 6 = 4 \times z$		Cross-multiply
$720 = 4z$		Simplify
$\frac{720}{4} = z$		Divide both sides by 2
$180 = z$		

Answer: Bendu can drive 180 km in 6 hours.

Practice

1. y and x are directly proportional. When $x = 10$, $y = 4$. Find the constant of proportionality.
2. y is directly proportional to x; when $x = 5, y = 20$. Find y when $x = 2$.
3. Abu walked 3 kilometres in 18 minutes. How long would it take him to walk 10 kilometres?
4. Mr. Bangura uses 3 pieces of chalk every week to teach 18 lessons. How many pieces of chalk does he need to teach 60 lessons?
5. Abass is a farmer. He uses 2 bottles of fertiliser for 6 hectares of land. How many bottles of fertiliser does he need for 42 hectares?
6. Alice solved 5 maths problems in 30 minutes. How long would it take her to solve 16 maths problems?
7. There are 8 members of Hawa's family. Together, they eat 12 cups of rice each day. One day, there are only 6 members of her family present. How many cups of rice should she prepare?

Lesson Title: Indirect Proportions	**Theme:** Everyday Arithmetic
Practice Activity: PHM-09-027	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify the form of an indirectly proportional relationship ($t \propto \frac{1}{s}$).
2. Solve indirect proportion problems.

Overview

Recall that a proportion is a pair of equivalent ratios in which the units must be the same. The previous lesson was on direct proportion. Direct proportions mean that as one ratio increases, the other does too. On the other hand, as one ratio goes down the other does too.

Indirect proportions mean that as one ratio goes up, the other goes down. On the other hand, as one ratio goes down the other goes up. The ratios move in opposite directions instead of in the same direction like a directly proportional relationship.

For example, consider a group of people doing a piece of work. If it takes 10 people 1 hour to brush the piece of land, how long will the same job take for 5 people? It will take more than 1 hour. When there are fewer people doing a job, the job takes more time.

Inverse proportions are shown with the equation $y = k\frac{1}{x}$ or $y = \frac{k}{x}$. These equations are the same. Indirect proportions are shown by the relationship one value (y) is equal to another value ($\frac{1}{x}$) multiplied by a constant (k).

When two values are indirectly proportional to each other, we can still use the special symbol ∝, because it just means it's proportional and then the numbers around it tell what kind of relationship exists. However, this time we would write: $y \propto \frac{1}{x}$, meaning that y is **indirectly** proportional to x.

When we have an inverse proportion with two equivalent ratios, we can still use **cross-multiplication,** however it needs to be set up differently, because the ratios have a different relationship to each other.

To use cross-multiplication to solve an inverse proportion, follow these steps:

1. Make sure each ratio is one unit, do not mix units.
2. Flip one of the ratios upside down.
3. Solve as normal.

Since the relationship is indirect ($y \propto \frac{1}{x}$), not direct ($y \propto x$) we cannot say that the 2 ratios are equal until we flip one of them.

You will solve real-life indirect proportion problems. For example: **If 5 people dig a hole for construction, it takes 8 hours. If 10 people are digging, how many hours will it take?**

There are 2 ways to solve the problems you will see in this lesson: the unitary method and the ratio method.

For the **unitary method**, you first find out how many hours it would take 1 person to do all the work. You would use this to find how many hours it takes 10 people to do the same work.

For the **ratio method**, write the proportions as fractions. You will **keep the units together** so that the same units are in each fraction. Remember to flip one of the fractions upside down, then solve using cross multiplication.

Solved Examples

1. Find the value of b that completes the indirect proportion $3 : b \propto 8 : 4$.

   Solution

   Write the ratios as fractions: $\frac{3}{b}$ and $\frac{8}{4}$. Flip one fraction upside down and set them equal: $\frac{3}{b} = \frac{4}{8}$. Now you can use cross-multiplication to find b:

$3 \times 8 = b \times 4$		Cross-multiply
$24 = 4b$		Simplify
$\frac{24}{4} = b$		Divide both sides by 4
$6 = b$		

   The answer is $b = 6$. The complete proportion is $3 : 6 \propto 8 : 4$.

2. If 5 people dig a hole for construction, it takes 8 hours. If 10 people are digging, how many hours will it take?

   Solution

   Unitary Method: Find how long it would take for 1 person to do the work:

   $$5 \times 8 = 40 \text{ hours}$$

   Divide by 10 to find how long it takes 10 people to do the work:

   $$40 \text{ hr} \div 10 = 4 \text{ hours}$$

   Ratio Method: Let h represent the number of hours. We have the ratios $5 \text{ people} : 10 \text{ people}$ and $8 \text{ hours} : h \text{ hours}$. Remember to keep the same units together.

Write the ratios as fractions: $\frac{5}{10}$ and $\frac{8}{h}$. Flip one fraction upside down and set them equal: $\frac{5}{10} = \frac{h}{8}$. Now you can use cross-multiplication to find h:

$$\begin{aligned} 5 \times 8 &= 10 \times h && \text{Cross-multiply} \\ 40 &= 10h && \text{Simplify} \\ \frac{40}{10} &= h && \text{Divide both sides by 10} \\ 4 &= h \end{aligned}$$

Answer: It will take 10 people 4 hours to dig the hole.

3. 5 kg rice can last 12 people for 4 days. How many days would the rice last if there were 8 people?

Solution

Ratio Method: Let d represent the number of days. We have the ratios $12 \text{ people} : 8 \text{ people}$ and $4 \text{ days} : d$ days. Remember to keep the same units together.

Write the ratios as fractions: $\frac{12}{8}$ and $\frac{4}{d}$. Flip one fraction upside down and set them equal: $\frac{12}{8} = \frac{d}{4}$. Now you can use cross-multiplication to find d:

$$\begin{aligned} 12 \times 4 &= 8 \times d && \text{Cross-multiply} \\ 48 &= 8d && \text{Simplify} \\ \frac{48}{8} &= d && \text{Divide both sides by 8} \\ 6 &= d \end{aligned}$$

Answer: $d = 6$. The rice would last 8 people 6 days.

Practice

1. Find the missing values that complete each **indirect** proportion:
    a. $a : 20 \propto 48 : 12$
    b. $30 : b \propto 1 : 5$
    c. $10 : 4 \propto c : 5$
2. Thirty workers can complete a piece of work in 80 days. How many workers will you need to complete the same piece of work in 20 days?
3. Fifty litres of drinking water can last 6 people for 5 days. How many days would the drinking water last if there were 10 people?
4. In a village, there is enough food for 500 people for 30 days. If 100 more people move to the village, how many days will the food last?
5. Mr. Bangura has a plum tree. He picked enough plums to give 4 plums to each of 15 of his friends. If he decided to share the same plums with 20 friends, how many would each friend get?

Lesson Title: Proportion Problem Solving	**Theme:** Everyday Arithmetic
Practice Activity: PHM-09-028	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to solve direct and indirect proportion problems.

Overview

In this lesson, you will decide if a problem describes direct or indirect proportion. You will use the methods you learned in lessons 26-27 to solve the problems. Remember:

- In **direct proportion**, both ratios move in the same direction. They either both increase or both decrease.
- In **indirect proportion**, the ratios move in opposite directions. As one decreases, the other increases. As one increases, the other decreases.

Solved Examples

1. There are 3 cooks at the school who prepare 250 school meals in 50 minutes every day. The principal wants to shorten the time it takes to prepare the meals by hiring more cooks. How many cooks would the school need to prepare the meals in 30 minutes, assuming they all work at the same rate?

   Solution
   First, determine the type of proportion. This is an indirect proportion, because the time for the meal will go down, while the number of cooks will increase.

   Write the 2 fractions. Let c represent the unknown number of cooks: $\frac{50\text{ min.}}{30\text{ min.}}$ and $\frac{3\text{ cooks}}{c}$.
   Flip one of them upside down: $\frac{50\text{ min.}}{30\text{ min.}} = \frac{c}{3\text{ cooks}}$

   Cross-multiply and solve:

   $$\begin{aligned} 50 \times 3 &= 30 \times c && \text{Cross-multiply} \\ 150 &= 30c && \text{Simplify} \\ \tfrac{150}{30} &= c && \text{Divide both sides by 30} \\ 5 &= c \end{aligned}$$

   Answer: The school would need 5 cooks to prepare the meals in 30 minutes.

2. Foday is drawing a scale map of his community. He measured the distance he walks from his home to the school every day at 4 km. The scale on his map is 1 cm = 50 m. How long should the distance be on his map?

Solution

First, determine the type of proportion. This is a direct proportion, because as the distance in Foday's community increases, the distance on the map increases. Remember that each ratio must have the same units. Convert 4 km to metres: 4 km = 4,000 m

The ratios are $\frac{50\text{ m}}{4{,}000\text{ m}} = \frac{1\text{ cm}}{x}$, where x is the distance from his home to school on the map. Cross-multiply and solve:

$$\begin{aligned} 50 \times x &= 4{,}000 \times 1 && \text{Cross-multiply} \\ 50x &= 4{,}000 && \text{Simplify} \\ x &= \tfrac{4{,}000}{50} && \text{Divide both sides by 50} \\ x &= 80 && \text{Simplify} \end{aligned}$$

The distance on the map is 80 cm.

3. Alpha and Hawa are both driving from Bo to Freetown. Hawa travels at 60 kph and reaches Freetown in 4 hours. Alpha wants to reach Freetown in 3 hours for an appointment. How fast does he need to drive?

Solution

First, determine the type of proportion. This is an indirect proportion, because as the speed increases, the time it takes decreases. The ratios are $\frac{4\text{ h}}{3\ h}$ and $\frac{60\text{ kph}}{s}$, where s is the speed at which Alpha needs to drive. Flip one ratio upside down: $\frac{4\text{ h}}{3\ h} = \frac{s}{60\text{ kp}h}$.

Cross-multiply and solve:

$$\begin{aligned} 4 \times 60 &= 3 \times s && \text{Cross-multiply} \\ 240 &= 3s && \text{Simplify} \\ \tfrac{240}{3} &= s && \text{Divide both sides by 50} \\ 80 &= s && \text{Simplify} \end{aligned}$$

Alpha needs to drive at 80 kph to reach Freetown in 3 hours.

4. Hawa is traveling to Bo. If she drives at the rate of 40 kph it will take her 2 hours. How much faster would she get to Bo if she drove at the rate of 50 kph?

Solution

Notice that this question asks how much faster she would get there. Once we find out how long it would take Hawa to get to Bo traveling at 50 kph, we need to find the difference between the two lengths of time.

Step 1. Find how long it will take her at 50 kph:

Let h represent the number of hours. Write the ratios as fractions: $\frac{40 \text{ kph}}{50 \text{ kph}}$ and $\frac{2 \text{ hr}}{h}$. Flip one fraction upside down and set them equal: $\frac{40 \text{ kph}}{50 \text{ kph}} = \frac{h}{2 \text{ hr}}$. Cross-multiply to find h:

40×2	$=$	$50 \times h$	Cross-multiply
80	$=$	$50h$	Simplify
$\frac{80}{50}$	$=$	h	Divide both sides by 50
$\frac{8}{5}$	$=$	h	
$1\frac{3}{5}$	$=$	h	

It would take her $1\frac{3}{5}$ hours.

Step 2. Find the difference between the 2 lengths of time:

$2 - 1\frac{3}{5}$	$=$	$2 - \frac{8}{5}$	Cross-multiply
	$=$	$\frac{10}{5} - \frac{8}{5}$	Simplify
	$=$	$\frac{10-8}{5}$	Divide both sides by 50
	$=$	$\frac{2}{5}$ hours	

She will reach Bo $\frac{2}{5}$ hours faster. We can write this in minutes: $\frac{2}{5} \times 60 = \frac{120}{5} = 24$ minutes.

Practice

1. Mustapha is a tailor. He wants to estimate how much money he makes in 1 year. He knows that he earned Le 700,000.00 in the previous 4 weeks. If he earns money at the same rate all year, how much does he make in a year?
2. Mohamed and David are farmers. They work together on the farm, and share all the costs and profits. This week, Mohamed contributed Le 20,000.00 and David contributed Le 24,000.00. They agreed to split this week's income from the farm according to how much they put in. Mohamed received Le 60,000.00. What is David's portion?
3. At a school event, there are 4 big pots of rice, enough to feed 500 people. If 125 more people come to the event than planned, how much more rice is needed?
4. Francis and Juliet are tailors. They were hired to make school uniforms for 30 pupils and expect it will take them 6 days. The academic year begins in 4 days, so they will ask other tailors to help them complete the job on time. How many more tailors do they need?
5. Twelve girls can do a job in 45 minutes.
   a. How many girls can do the same job in 9 minutes?
   b. How long will it take 20 girls to do the same job?

Lesson Title: Financial Literacy I	Theme: Everyday Arithmetic
Practice Activity: PHM-09-029	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to solve problems with wages, salaries, and income tax.

Overview

In this lesson you will solve problems about the money people earn.

The money that people make can be called wages or salary. **Wages** are the money people earn who get paid by the hour or by the day. They get paid at the end of the month only for the hours or days they work. **Salaries** do not depend on the number of hours or days someone works. It is a fixed amount paid at the end of the month regardless of the number of hours or days someone works.

The money that people earn is taxed by the government. It is used to provide services to the country such as education, health and social welfare. The tax that people pay on their wages or salary is called **income tax**. No matter whether you earn a wage or a salary, you still have to pay income tax on it.

Employee taxes are deducted from their salaries by their employers using a method called PAYE. PAYE stands for Pay As You Earn and the 2017 rates are shown on the table on the board.

Sierra Leone PAYE Tax Rate	
Not over Le 500,000 per month	Nil
Next Le 500,000 per month	15%
Next Le 500,000 per month	20%
Next Le 500,000 per month	30%
Above Le 2 million per month	35%

Every employee has a tax-free income. This is the income below which you do not have to pay any income tax. The current tax-free income is Le 500,000.00.

The **gross income** is the amount of money a person earns before taxes. The **net income** is the amount of money left after tax has been deducted. Calculate net income by subtracting taxes from the gross salary.

Solved Examples

1. Adama earns Le 950,000.00 per month. She is given a 5% rise in salary.
    a. How much is her new monthly salary?
    b. How much tax does Adama pay each month?
    c. What is her **net** monthly salary?

Solutions

a. Use the formula for percentage increase to find her new salary:

$$\begin{aligned}\text{New salary} &= \frac{100+\text{percentage increase}}{100} \times \frac{\text{the given salary}}{1}\\ &= \frac{100+5}{100} \times \frac{950{,}000}{1}\\ &= \frac{105}{1} \times \frac{9500}{1}\\ &= 105 \times 9500\\ &= \text{Le } 997{,}500.00\end{aligned}$$

b. The first thing we need to do when calculating income tax is to find the taxable income. Remember that the first Le 500,000.00 that people earn is not taxed. We need to subtract tax-free income from her total monthly income.

Taxable income: $997{,}500 - 500{,}000 = \text{Le } 497{,}500.00$

Adama's taxable income is Le 497,500, which is less than Le 500,000. Look at the table in the overview, and notice that the next Le 500,000 after the first Le 500,000 is taxed at 15% per month. This is Adama's tax rate.

Find 15% of Le $497{,}500.00$:

$$\begin{aligned}\text{Income tax} &= \frac{15}{100} \times 497{,}500\\ &= 15 \times 4975\\ &= \text{Le } 74{,}625.00\end{aligned}$$

Adama pays Le 74,625.00 in income tax each month.

c. Subtract the amount of tax she pays from her total (gross) salary:

$$\begin{aligned}\text{Net monthly salary} &= \text{Gross monthly salary} - \text{Income tax}\\ &= \text{Le}997{,}500 - \text{Le}74{,}625\\ &= \text{Le } 922{,}875.00\end{aligned}$$

2. A labourer earns wages of Le 270,000.00 for 9 days of work.
   a. If he works 24 days in a month, how much does he earn in the month?
   b. How much income tax does he pay each month?

Solutions

a. This is a direct proportion problem. Write a ratio for the problem: $\frac{270{,}000}{9 \text{ days}} = \frac{x}{24 \text{ days}}$.

   The unknown value x is the amount he earns in 24 days.

   Solve using cross-multiplication:

$$\begin{aligned} 270{,}000 \times 24 &= 9 \times x && \text{Cross-multiply} \\ 6{,}480{,}000 &= 9x && \text{Simplify} \\ \tfrac{6{,}480{,}000}{9} &= x && \text{Divide both sides by 2} \\ 720{,}000 &= x \end{aligned}$$

   He earns Le 720,000.00 in 24 days.

b. Subtract to find his taxable income: $720{,}000 - 500{,}000 = \text{Le } 220{,}000.00$.
   His taxable income is Le 220,000, which is less than Le 500,000. His tax rate is 15%. Find 15% of $\text{Le } 220{,}000.00$:

$$\begin{aligned} \text{Income tax} &= \tfrac{15}{100} \times 220{,}000 \\ &= 15 \times 2200 \\ &= \text{Le } 33{,}000.00 \end{aligned}$$

   The labourer pays Le 33,000.00 in income tax each month.

3. Hawa earns a monthly salary of Le 1,800,000.00. Calculate the amount of income tax she pays each month.

Solution

First, find her taxable income: $1{,}800{,}000 - 500{,}000 = \text{Le } 1{,}300{,}000.00$

Her taxable income is 1,300,000.00. Use the table to find the amount of tax she pays. She pays a different amount on each Le 500,000 after the first tax-free Le 500,000.00.

Note that her taxable income can be divided into parts:

$$1{,}300{,}000 = 500{,}000 + 500{,}000 + 300{,}000$$

The tax rate on the first 500,000 was 0. On the second 500,000, the tax is 15%. On the third 500,000 the tax is 20%. On the last 300,000, the tax is 30%.

Calculate her total tax:

$$\begin{aligned}
\text{Income tax} &= \tfrac{15}{100} \times 500{,}000 + \tfrac{20}{100} \times 500{,}000 + \tfrac{30}{100} \times 300{,}000 \\
&= 15 \times 5000 + 20 \times 5000 + 30 \times 3000 \\
&= 75{,}000 + 100{,}000 + 90{,}000 \\
&= 265{,}000
\end{aligned}$$

Hawa pays Le 265,000.00 in income tax each month.

Practice

1. Fatu's monthly salary was Le 750,000.00. If it was increased by 15%, what is her new monthly salary?
2. Alpha's monthly salary was Le 900,000.00. His company is not doing well, and it decided to decrease each employee's salary by 10%. What is Alpha's salary now?
3. Emmanuel earns a monthly salary of Le 2,000,000.00. Calculate the amount of income tax he pays each month.
4. Bendu earned Le 1,500,000.00 each month.
    a. If her salary increased by 20%, what is her new salary?
    b. How much tax will she pay each month on her new salary?
5. Gibril is a driver. His wages are Le 240,000.00 for 6 days.
    a. If he works 26 days in a month, how much does he earn in a month?
    b. How much income tax does he pay each month?
    c. What is his net monthly salary?

Lesson Title: Financial Literacy II	Theme: Everyday Arithmetic
Practice Activity: PHM-09-030	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to solve simple interest problems.

Overview

When someone puts money in a bank, the bank pays them an additional amount of money. This money depends on the amount put in by the person and how long they keep the money in the bank. Simple interest is calculated as a percentage of the money put in the bank on an annual, or yearly, rate.

You do not need to memorise a formula. You can remember how to calculate simple interest with percentages.

Consider a problem: Find the simple interest on Le 400,000.00 for 3 years at an interest rate of 5% per annum.

Remember how to calculate percentage of a number: write the percentage as a fraction and multiply it by the number. We will do the same here: $\frac{5}{100} \times 400{,}000.00$. This calculation will give us the interest for 1 year. We want to find the interest on the money over 3 years. Multiply the calculation by 3: $\frac{5}{100} \times 400{,}000.00 \times 3$. This will give us the answer to the problem, the simple interest on the money over 3 years.

We can write a simple equation to calculate simple interest: $I = PRT$, where P is the principal, R is the rate, and T is time in years. Principal is the original amount of money borrowed, lent, or invested at the given rate.

Note that in the above formula, R is a fraction or decimal. So if the rate is 5%, you should convert it to a fraction ($\frac{5}{100}$) or decimal (0.05) before substituting it into the formula.

You may be asked to find the interest, principal, rate, or time. Use the formula $I = PRT$ and substitute the values you are given.

Solved Examples

1. Find the simple interest on Le 400,000.00 for 3 years at an interest rate of 5% per annum.

Solution

Substitute the given numbers into the formula $I = PRT$ and find the simple interest, I.

$$\begin{aligned} I &= PRT \\ &= 400{,}000 \times \tfrac{5}{100} \times 3 \\ &= 4000 \times 5 \times 3 \\ &= \text{Le } 60{,}000.00 \end{aligned}$$

The simple interest is Le 60,000.00

2. Find the amount of interest on Le 350,000.00 invested in a bank for a period of 4 years at a rate of 12%.

Solution

Substitute the given numbers into the formula $I = PRT$ and find the simple interest, I.

$$\begin{aligned} I &= PRT \\ &= 350{,}000 \times \tfrac{12}{100} \times 4 \\ &= 3500 \times 12 \times 4 \\ &= \text{Le } 168{,}000.00 \end{aligned}$$

The simple interest is Le 168,000.00

3. If Le 10,000,000.00 yielded an interest of Le 500,000.00 in 5 years, calculate the rate per annum.

Solution

Substitute the given numbers into the formula $I = PRT$ and find the rate, R.

$$\begin{aligned} I &= PRT \\ 500{,}000 &= 10{,}000{,}000 \times R \times 5 \\ 500{,}000 &= 50{,}000{,}000 \times R \\ \tfrac{500{,}000}{50{,}000{,}000} &= R \\ \tfrac{5}{500} &= R \\ \tfrac{1}{100} &= R \end{aligned}$$

Recall that $\frac{1}{100}$ is the same as 0.01 or 1%.

The interest rate is 1%.

4. In how many years will Le 60,000.00 earn an interest of Le 12,000.00 at 2.5% interest per annum?

Solution

Substitute the given numbers into the formula $I = PRT$ and find the time, T.

$$\begin{aligned} I &= PRT \\ 12{,}000 &= 60{,}000 \times \tfrac{2.5}{100} \times T \\ 12{,}000 &= 600 \times 2.5 \times T \\ \tfrac{12{,}000}{600} &= 2.5 \times T \\ 20 &= 2.5 \times T \\ \tfrac{20}{2.5} &= T \\ 8 &= T \end{aligned}$$

It will take 8 years.

Practice

1. Find the interest on Le 500,000.00 for 9 years at 6% interest per annum.
2. Find the simple interest on Le 700,000.00 for 5 years at 4% per annum.
3. Find the simple interest on Le 120,000.00 at 2% for 5 years.
4. At which rate will Le 30,000.00 earn an interest of Le 6,000.00 in 5 years?
5. If Le 600,000.00 yielded an interest of Le 60,000.00 in 2 years, calculate the rate per annum.
6. In how many years will Le 100,000.00 earn an interest of Le 20,000.00 at 8% interest per annum?

Lesson Title: Index Notation and the Laws of Indices	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-031	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Interpret numbers in index notation.
2. State the 6 laws of indices and solve simple examples for each.

Overview

Numbers are written in **index notation** when there is a power. A number and its power are called an **index**. For example, 2^5 is an index. The plural form of index (more than 1) is **indices**. For example, 2^5 and 3^2 are indices.

The bottom number is the 'base' and it is written bigger than the other number. The smaller, raised number is called the index. Index is another way to say **power**.

base → 3^2 ← power or index

The power of a number says how many times to multiply that number by itself. A power can be any number. For example, $2^5 = 2 \times 2 \times 2 \times 2 \times 2$. 2^5 can be read as "2 to the power of 5".

Note the following helpful rules about indices:

Rule	Example
1 raised to any power is 1	$1^4 = 1 \times 1 \times 1 \times 1 = 1$
0 raised to any power is 0	$0^5 = 0 \times 0 \times 0 \times 0 \times 0 = 0$
A number to the power 1 = itself	$2^1 = 2$

The general rule for the last rule in the table is $a^1 = a$. This means that any number to the power 1 is itself.

Recall the 6 laws of indices:

First Law: multiplication of indices	$a^m \times a^n = a^{m+n}$
Second Law: division of indices	$a^m \div a^n = a^{m-n}$
Third Law: power of zero	$(a^0 = 1)$
Fourth Law: powers of indices	$(a^m)^n = a^{mn}$
Fifth Law: power of a product	$(a \times b)^n = a^n \times b^n$
Sixth Law: power of a quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = b \neq 0$

Solved Examples

1. Simplify: $7^3 \times 7^4$

   Solution
   Add the powers: $7^3 \times 7^4 = 7^{3+4} = 7^7$

2. Simplify: $3^7 \times 3$

   Solution
   Remember that a number without a power on it is the same as that number raised to the power of 1. In this problem, $3 = 3^1$.

   $3^7 \times 3 = 3^7 \times 3^1 = 3^{7+1} = 3^8$

3. Simplify: $\frac{p^7}{p^5}$

   Solution
   Fractions are the same as division. Follow the second law of indices and subtract the indices: $\frac{p^7}{p^5} = p^7 \div p^5 = p^{7-5} = p^2$

4. Divide: $7^{12} \div 7$

   Solution
   Remember that $7 = 7^1$.

   Subtract the powers: $7^{12} \div 7 = 7^{12} \div 7^1 = 7^{12-1} = 7^{11}$

5. Simplify: $3^9 \div 3^0$

   Solution
   Subtract the powers, applying the second law of indices: $3^9 \div 3^0 = 3^{9-0} = 3^9$

6. Simplify: $2^7 \div 2^7$

   Solution
   Apply the second law of indices, then the third law: $2^7 \div 2^7 = 2^{7-7} = 2^0 = 1$

7. Simplify: $(2^2)^3$

   Solution
   Apply the fourth law of indices: $(2^2)^3 = 2^{2\times3} = 2^6$

8. Simplify: $(5^5)^0$

Solution

Apply the fourth law of indices, then simplify using the third law:

$(5^5)^0 = 5^{5\times 0} = 5^0 = 1$

9. Use the fifth law of indices to rewrite the following: a. $(2 \times 9)^4$ b. $(3a)^2$

Solutions

Distribute the power to each factor inside brackets:

a. $(2 \times 9)^4 = 2^4 \times 9^4$
b. Note that $3a = 3 \times a$. Therefore, $(3a)^2 = 3^2a^2 = 9a^2$

10. Use the sixth law of indices to rewrite the following: a. $\left(\frac{5}{9}\right)^4$ b. $(c \div d)^2$

Solutions

a. $\left(\frac{5}{9}\right)^4 = \frac{5^4}{9^4}$
b. $(c \div d)^2 = c^2 \div d^2$

Practice

Simplify the following expressions using the laws of indices:

1. $11^4 \times 11^9$
2. $10^4 \times 10$
3. $9^6 \times 9^7 \times 9^{10}$
4. 123^0
5. $\frac{b^9}{b}$
6. $12^7 \div 12^7$
7. $5^9 \div 5^3$
8. $a^{10} \div a$
9. $3^5 \times 3^4 \times 3^0$
10. $(1^4)^{12}$
11. $(7^4)^0$
12. $(9^7)^3$
13. $(5x)^2$
14. $\left(\frac{1}{8}\right)^{12}$

Lesson Title: Application of the Laws of Indices	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-032	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to apply the 6 laws of indices to simplify problems.

Overview

You have now learned all of the laws of indices. In this lesson, you will use all 6 of them to simplify expressions with indices. For example, consider the expression $(2^3)^4 \times 2^5$. It involves a power of an index, and multiplication of indices. We will use 2 different laws to simplify this expression.

Remember the order of operations. The order is very important when applying the laws of indices. It is BODMAS: Brackets Of Division Multiplication Addition Subtraction. Remember that indices fall under 'Of'. In the example above, you would first work $(2^3)^4$, then you would multiply by 2^5.

You will solve problems with indices in fractions. Remember to simplify the numerator and denominator first, if possible. You can cancel any like terms from the numerator and denominator. Consider the fraction $\frac{2^{10}}{2^{10}\times 5^{10}}$. 2^{10} can be canceled from the numerator and denominator.

Solved Examples

1. Simplify: $(2^3)^4 \times 2^5$

   Solution

$$\begin{aligned}(2^3)^4 \times 2^5 &= 2^{3\times 4} \times 2^5 && \text{Index law 4: Power of an index}\\ &= 2^{12} \times 2^5 \\ &= 2^{12+5} && \text{Index law 1: Multiplication}\\ &= 2^{17}\end{aligned}$$

2. Simplify: $\frac{(2^5)^2}{(2\times5)^{10}}$

Solution

First, simplify the numerator and denominator separately. Then apply the division rule.

$$\begin{aligned} \frac{(2^5)^2}{(2\times5)^{10}} &= \frac{2^{5\times2}}{2^{10}\times5^{10}} && \text{Index law 4 and Index law 1} \\ &= \frac{2^{10}}{2^{10}\times5^{10}} \\ &= \frac{1}{5^{10}} && \text{Cancel } 2^{10} \end{aligned}$$

3. Simplify: $\frac{a^3}{(3a)^4}$

Solution

$$\begin{aligned} \frac{a^3}{(3a)^4} &= \frac{a^3}{3^4\times a^4} && \text{Index law 5: Power of a product} \\ &= \frac{1}{3^4\times a} && \text{Cancel } a^3 \end{aligned}$$

Note about cancelling indices:

- Remember that cancelling is the same as dividing the numerator and denominator by the same amount.
- To cancel a^3, divide both the numerator and denominator by a^3.
- Dividing the numerator gives 1: $a^3 \div a^3 = a^{3-3} = a^0 = 1$
- Dividing the denominator gives a: $a^4 \div a^3 = a^{4-3} = a^1 = a$

4. Simplify: $(a^4)^5 \times (a^3)^2$

Solution

$$\begin{aligned} (a^4)^5 \times (a^3)^2 &= a^{4\times5} \times a^{3\times2} && \text{Index law 4: Power of an index} \\ &= a^{20} \times a^6 \\ &= a^{20+6} && \text{Index law 1: Multiplication} \\ &= a^{26} \end{aligned}$$

5. Simplify: $\frac{(5\times3)^7}{3^7}$

Solution

$$\begin{aligned} \frac{(5\times3)^7}{3^7} &= \frac{5^7\times3^7}{3^7} && \text{Index law 5: Power of a product} \\ &= \frac{5^7}{1} && \text{Cancel } 3^7 \\ &= 5^7 \end{aligned}$$

6. Simplify: $2^4 \div 3^{10} \div 3^2$

Solution

Two of the indices have the same base. We can divide $3^{10} \div 3^2$, but 2^4 cannot be divided with the others.

$$2^4 \div 3^{10} \div 3^2 = 2^4 \div 3^{10-2} = 2^4 \div 3^8$$

$2^4 \div 3^8$ is the simplified form of the expression.

7. Simplify: $\left(\frac{ab^2}{a^3b}\right)^3$

Solution

$\left(\frac{ab^2}{a^3b}\right)^3$	$=$	$\left(\frac{b^{2-1}}{a^{3-1}}\right)^3$	Cancel a and b from the numerator and denominator
	$=$	$\left(\frac{b^1}{a^2}\right)^3$	Simplify
	$=$	$\left(\frac{b}{a^2}\right)^3$	
	$=$	$\frac{b^3}{(a^2)^3}$	Index law 6: Power of a quotient
	$=$	$\frac{b^3}{a^{2\times3}}$	Index law 4: Power of an index
	$=$	$\frac{b^3}{a^6}$	

Practice

Simplify the following expressions:

1. $\frac{(5\times7)^2}{7^2}$
2. $(3^2 \times 3^4)^8$
3. $2^{20} \times (2^8)^5$
4. $\frac{5^3}{5^4\times3^4}$
5. $\frac{b^4}{(2b)^3}$
6. $(2^8)^2 \times (2^{15})^3$
7. $\left(\frac{c^2b}{cb^2}\right)^4$
8. $4a^7 \times a^3 \times 2b^2 \times b^5$
9. $2b^5c^3 \times 7b^6c$

Lesson Title: Indices with Negative Powers	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-033	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that a number with a negative index can be rewritten as a fraction ($a^{-n} = \frac{1}{a^n}$).
2. Apply the laws for multiplying and dividing indices to those with positive and negative powers.

Overview

Negative powers are the opposite of positive powers. Positive powers told us to multiply. The opposite of multiplication is division, so negative powers tell us to divide. A negative power tells us how many times to divide by that number.

For example, consider 2^{-3}. This 2 has a power of **negative** 3. That means we **divide** by 2 three times:

$$2^{-3} = 1 \div (2 \times 2 \times 2) = \frac{1}{2\times2\times2} = \frac{1}{2^3}$$

When any number has a negative power, it can be rewritten in the denominator of a fraction with a positive power. The general rule is $a^{-n} = \frac{1}{a^n}$.

The same 6 laws of indices are applied to indices with negative powers. The laws of indices are in lesson PHM-09-031.

You will need to apply the rules for basic operations on negative numbers:

- When **adding** two negative numbers, we add their values and carry the negative sign to the answer. For example, $(-3) + (-5) = -8$
- **Subtracting** a negative is the same as adding. For example, $(-3) - (-5) = (-3) + 5$. When we add a negative and positive, we do a subtraction problem and write the sign of the larger number on the answer. For example: $(-3) - (-5) = (-3) + 5 = 5 - 3 = 2$.
- When **multiplying,** a negative times a negative is a positive, and a negative times a positive is a negative.

Remember to use the correct order of operations (BODMAS) to evaluate problems.

Solved Examples

1. Simplify the following: a. 12^{-5} b. 100^{-3} c. 1^{-8} d. 8^{-40}

   Solutions

   Write each index as a fraction with a positive power:

   a. $12^{-5} = \frac{1}{12^5}$
   b. $100^{-3} = \frac{1}{100^3}$
   c. $1^{-8} = \frac{1}{1^8} = \frac{1}{1} = 1$
   d. $8^{-40} = \frac{1}{8^{40}}$

2. Simplify: $2^4 \times 2^{-3}$

   Solution

   Change 2^{-3} to a fraction with a positive power in the denominator. 2^4 will not change. It will stay in the numerator of the fraction. You will then be able to cancel:

$$\begin{aligned} 2^4 \times 2^{-3} &= \frac{2^4}{2^3} && \text{Index law 5: Power of a product} \\ &= \frac{2}{1} && \text{Cancel } 2^3 \\ &= 2 \end{aligned}$$

3. Simplify: $18^{-12} \times 18^{-3}$

   Solution

$$\begin{aligned} 18^{-12} \times 18^{-3} &= 18^{(-12)+(-3)} && \text{Add the powers} \\ &= 18^{-15} \\ &= \frac{1}{18^{15}} && \text{Simplify} \end{aligned}$$

4. Evaluate: $5^{-7} \div 5^{-10}$

   Solution

$$\begin{aligned} 5^{-7} \div 5^{-10} &= 5^{(-7)-(-10)} && \text{Subtract the powers} \\ &= 5^{-7+10} \\ &= 5^{10-7} \\ &= 5^3 \end{aligned}$$

5. Evaluate $2^{-6} \div 2^{-6}$

Solution

$$\begin{aligned} 2^{-6} \div 2^{-6} &= 2^{(-6)-(-6)} && \text{Subtract the powers} \\ &= 2^{-6+6} \\ &= 5^{0} \\ &= 1 && \text{Recall that } a^0 = 1 \end{aligned}$$

10. Evaluate: $(2^3)^{-2} \div 2^{-4}$

Solution

$$\begin{aligned} (2^3)^{-2} \div 2^{-4} &= 2^{3\times(-2)} \div 2^{-4} && \text{Power of an index} \\ &= 2^{-6} \div 2^{-4} \\ &= 2^{-6-(-4)} && \text{Divide} \\ &= 2^{-6+4} \\ &= 2^{-2} \\ &= \frac{1}{2^2} \end{aligned}$$

11. Evaluate: $a^8 \times (a \times b)^{-3}$

$$\begin{aligned} a^8 \times (a \times b)^{-3} &= a^8 \times a^{-3} \times b^{-3} && \text{Power of a product} \\ &= a^{8+(-3)} \times b^{-3} && \text{Divide} \\ &= a^{8-3} \times b^{-3} \\ &= a^5 \times b^{-3} \\ &= \frac{a^5}{b^3} \end{aligned}$$

Practice

Simplify the following:

1. 10^{-9}
2. 7×3^{-3}
3. $2^5 \times 2^{-3}$
4. $5^{-12} \div 5^{-4}$
5. $10^{-12} \div 10^{-17}$
6. $21^{-10} \times 21^{-8}$
7. $3^{-5} \times 3^7 \div 3^{-4}$
8. $2^{-4} \times 2^2 \times 5^4 \times 5^{-3}$
9. $(7^9)^{-3}$
10. $(2^2)^{-5} \times 2^{-3}$
11. $\left(\frac{1}{3}\right)^{-8}$

Lesson Title: Indices with Fractional Powers	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-034	**Class:** JSS 3

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that a number with a fractional power can be rewritten as a root $\left(a^{\frac{1}{n}} = \sqrt[n]{a}\right)$.
2. Simplify simple indices with fractional powers.

Overview

Consider how we take the square root of a number, for example $\sqrt{4} = 2$. Recall that taking the square root is the opposite of squaring a number. We have $\sqrt{4} = 2$ because $2^2 = 4$.

The square root of a number can be rewritten as a number raised to a fractional index. In this case, the power is $\frac{1}{2}$. So we have $\sqrt{4} = 4^{\frac{1}{2}}$.

A number raised to the power $\frac{1}{3}$ is known as the cube root. For example, $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$. This is true because $2^3 = 2 \times 2 \times 2 = 8$.

Generally, if we need the nth root of any number, say x, we can write it as $\sqrt[n]{x}$. This, when translated into index notation, becomes $\sqrt[n]{x} = x^{\frac{1}{n}}$. This can be interpreted as, "what number can you multiply by itself n times to get x?" For example: $\sqrt{9} = 9^{\frac{1}{2}} = 3$ means if you multiply 3 by itself two times you get 9 ($3 \times 3 = 9$).

A number can be raised to a fractional power with an integer numerator other than 1, such as $5^{\frac{2}{3}}$. The numerator is treated as a power, and the denominator is the root. This can be written as $5^{\frac{2}{3}} = \sqrt[3]{5^2}$.

Generally, for any number x raised to a fractional power $\frac{m}{n}$ such that m is not 1, we can write the number in the index notation as $\sqrt[n]{x^m} = x^{\frac{m}{n}}$.

Apply the laws of indices and BODMAS as you normally would to simplify expressions containing indices with fractional powers.

Solved Examples

1. Write the following as fractional indices:
   a. $\sqrt{13}$
   b. $\sqrt[7]{x^4}$
   c. $\sqrt[b]{2^a}$

   Solutions

   a. $13^{\frac{1}{2}}$
   b. $x^{\frac{4}{7}}$
   c. $2^{\frac{a}{b}}$

2. Calculate the following roots: a. $25^{\frac{1}{2}}$ b. $25^{-\frac{1}{2}}$ c. $27^{\frac{1}{3}}$ d. $16^{-\frac{1}{4}}$

   Solutions

   Write each fractional index as a root and solve. Remember that if the power is negative, the index is written in the denominator with a positive power.

   a. $25^{\frac{1}{2}} = \sqrt{25} = 5$
   b. $25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$
   c. $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$
   d. $16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}} = \frac{1}{\sqrt[4]{16}} = \frac{1}{2}$

3. Simplify: $7^{\frac{1}{2}} \times 7^{\frac{1}{2}}$

   Solution

   Apply the first law of indices. Add the powers:

   $7^{\frac{1}{2}} \times 7^{\frac{1}{2}} = 7^{\frac{1}{2}+\frac{1}{2}} = 7^1 = 7$

4. Simplify: $12^{\frac{2}{3}} \div 12^{\frac{1}{3}}$

   Solution

   Apply the second law of indices. Subtract the powers:

   $12^{\frac{2}{3}} \div 12^{\frac{1}{3}} = 12^{\frac{2}{3}-\frac{1}{3}} = 12^{\frac{1}{3}} = \sqrt[3]{12}$

5. Simplify: $(4^2 \times 4^4)^{\frac{1}{2}}$

Solution

$(4^2 \times 4^4)^{\frac{1}{2}}$	=	$(4^{2+4})^{\frac{1}{2}}$	Index law 1: Multiplication
	=	$(4^6)^{\frac{1}{2}}$	
	=	$4^{6\times\frac{1}{2}}$	Index law 4: Power of an index
	=	$4^{\frac{6}{2}}$	
	=	4^3	Simplify the fraction

Practice

1. Write as fractional indices:
   a. $\sqrt{43}$ c. $\sqrt[3]{17}$ b. $\sqrt[8]{11^2}$
2. Simplify:
   a. $4^{\frac{1}{2}}$ b. $64^{\frac{1}{2}}$ c. $1{,}000^{\frac{1}{3}}$
3. Simplify:
   a. $2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}}$
   b. $3^{\frac{1}{4}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{4}}$
   c. $5^{\frac{3}{4}} \div 5^{\frac{1}{2}}$
   d. $2 \div 2^{\frac{1}{2}}$
4. Simplify:
   a. $(2^6 \times 2^4)^{\frac{1}{2}}$
   b. $(3^8 \times 5^{10})^{\frac{1}{2}}$

Lesson Title: Multiplying and Dividing Indices with Fractional Powers	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-035	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to apply the laws for multiplying and dividing indices to those with positive and negative fractional powers.

Overview

In this lesson, you will continue practising with indices with fractional powers. Apply the laws of indices and BODMAS as you normally would to simplify expressions containing indices with fractional powers.

You will work with fractional powers that are negative. Remember the rule for negative indices: $a^{-n} = \frac{1}{a^n}$.

From the rule for negative indices, we can derive another rule. If there is a negative power on a fraction, take the reciprocal of the fraction and raise it to the same **positive** index. The general rule is $\left(\frac{\mathrm{a}}{\mathrm{b}}\right)^{-\mathrm{n}} = \left(\frac{\mathrm{b}}{\mathrm{a}}\right)^{\mathrm{n}}$ provided $a \neq 0$ and $b \neq 0$.

For example, you would rewrite $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$ as $\left(\frac{125}{8}\right)^{\frac{1}{3}}$ before solving.

Solved Examples

1. Simplify: $\left(\frac{p^8}{p^4}\right)^{\frac{1}{2}}$

   Solution
   Divide first because it is inside brackets. Then, apply the fractional power.

$$\begin{aligned}\left(\frac{\mathrm{p}^8}{\mathrm{p}^4}\right)^{\frac{1}{2}} &= (\mathrm{p}^{8-4})^{\frac{1}{2}} \\ &= (\mathrm{p}^4)^{\frac{1}{2}} \\ &= \mathrm{p}^{4\times\frac{1}{2}} \\ &= \mathrm{p}^2\end{aligned}$$

2. Simplify: $(\frac{16}{25})^{\frac{1}{2}}$

Solution

$$(\frac{16}{25})^{\frac{1}{2}} = \frac{16^{\frac{1}{2}}}{25^{\frac{1}{2}}}$$ Apply the power to the numerator and denominator

$$= \frac{\sqrt{16}}{\sqrt{25}}$$ Apply the rule $\sqrt[n]{x} = x^{\frac{1}{n}}$

$$= \frac{4}{5}$$ Find the square roots

3. Simplify: $(\frac{8}{125})^{-\frac{1}{3}}$

Solution

$$\left(\frac{8}{125}\right)^{-\frac{1}{3}} = \left(\frac{125}{8}\right)^{\frac{1}{3}}$$ Apply the rule $\left(\frac{\mathrm{a}}{\mathrm{b}}\right)^{-\mathrm{n}} = \left(\frac{\mathrm{b}}{\mathrm{a}}\right)^{\mathrm{n}}$

$$= \frac{(125)^{\frac{1}{3}}}{(8)^{\frac{1}{3}}}$$ Apply the power to the numerator and denominator.

$$= \frac{\sqrt[3]{125}}{\sqrt[3]{8}}$$ Apply the rule $\sqrt[n]{x} = x^{\frac{1}{n}}$

$$= \frac{5}{2}$$ Find the cube roots

$$= 2\frac{1}{2}$$ Convert to mixed fraction

4. Simplify: $(\mathrm{u}^{10})^{-\frac{1}{2}}$

Solution

$$(\mathrm{u}^{10})^{-\frac{1}{2}} = \mathrm{u}^{10\times(-\frac{1}{2})}$$ Apply Index law 4, power of an index

$$= \mathrm{u}^{-\frac{10}{2})}$$

$$= \mathrm{u}^{-5}$$ Simplify

$$= \frac{1}{u^5}$$ Apply the rule $a^{-n} = \frac{1}{a^n}$

5. Simplify: $\left(\frac{x^2y^6}{z^4}\right)^{-\frac{1}{2}}$

Solution

$\left(\frac{x^2y^6}{z^4}\right)^{-\frac{1}{2}}$	$=\left(\frac{z^4}{x^2y^6}\right)^{\frac{1}{2}}$	Apply the rule $\left(\frac{\mathrm{a}}{\mathrm{b}}\right)^{-\mathrm{n}}=\left(\frac{\mathrm{b}}{\mathrm{a}}\right)^{\mathrm{n}}$
	$=\frac{(z^4)^{\frac{1}{2}}}{(x^2y^6)^{\frac{1}{2}}}$	Apply the power to the numerator and denominator.
	$=\frac{(z^4)^{\frac{1}{2}}}{(x^2)^{\frac{1}{2}}(y^6)^{\frac{1}{2}}}$	Apply Index law 1 to the denominator
	$=\frac{z^{\frac{4}{2}}}{x^{\frac{2}{2}}y^{\frac{6}{2}}}$	Apply Index law 4; multiply each set of powers
	$=\frac{z^2}{xy^3}$	Simplify powers

Practice

Simplify the following expressions. You will need to apply the laws of indices, as well as the rules for negative and fractional powers.

1. $(x^9 \times y^6)^{\frac{1}{3}}$
2. $(x^4 \times x^{12})^{\frac{1}{4}}$
3. $\left(\frac{a^4}{a^{10}}\right)^{\frac{1}{2}}$
4. $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$
5. $(a^{36})^{-\frac{1}{3}}$
6. $\left(\frac{a^{12}b^8}{c^6}\right)^{-\frac{1}{2}}$
7. $\frac{x^{\frac{1}{2}}}{x}$
8. $(2^{15})^{-\frac{1}{3}}$

Lesson Title: Multiplying and Dividing by Powers of 10	Theme: Numbers and Numeration
Practice Activity: PHM-09-036	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to multiply and divide whole numbers and decimals by powers of 10.

Overview

This lesson is on multiplying and dividing by the powers of 10. These are the first 5 powers of 10. The pattern continues with higher powers. Notice that the number of zeros in the number and the index are the same.

Index	Whole Number
10^0	1
10^1	10
10^2	100
10^3	1,000
10^4	10,000

Remember the rules for multiplying and dividing by 10, 100 or 1,000. To multiply, move the decimal point to the right by the number of zeros. To divide, move the decimal point to the left by the number of zeros. For example:

$$4.25 \times 10 = 42.5 \qquad 102.5 \div 100 = 1.025 \qquad 51 \times 1{,}000 = 51{,}000$$

For whole numbers, remember that the decimal point is at the end of the number. When multiplying a whole number by a power of 10, add zeros if needed to hold the places. For example, 51 is the same as 51.000. We can write the whole number with zeros before multiplying by 1,000. This helps us to keep count: $51 \times 1{,}000 = 51.000 \times 1{,}000 = 51{,}000$

Follow the same rules for multiplying by an index with a base of 10. To multiply, move the decimal point to the right by the power. To divide, move the decimal point to the left by the power. The same examples from above are given below with indices:

$$4.25 \times 10^1 = 42.5 \qquad 102.5 \div 10^2 = 1.025 \qquad 51 \times 10^3 = 51{,}000$$

Solved Examples

1. Multiply the following whole numbers by powers of 10:
   a. 31×10 b. 4×100 c. 17×10^2 d. 82×10^3

Solutions

Move each decimal point to the right by the number of zeros in the power of 10, or by the power on the index.

a. $31 \times 10 = 31.0 \times 10 = 310$
b. $4 \times 100 = 4.00 \times 100 = 400$
c. $17 \times 10^2 = 17.00 \times 10^2 = 1700$
d. $82 \times 10^3 = 82.000 \times 10^3 = 82{,}000$

2. Multiply the following decimal numbers by powers of 10:

a. 2.865×100 b. 0.45×10^3 c. 1.034×10^2 d. $0.304 \times 1{,}000$

Solutions

Move each decimal point to the right by the number of zeros in the power of 10, or by the power on the index.

a. $2.865 \times 100 = 286.5$
b. $0.45 \times 10^3 = 0.450 \times 10^3 = 450$
c. $1.034 \times 10^2 = 103.4$
d. $0.304 \times 1000 = 304$

3. Divide the following whole numbers by powers of 10:

a. $12 \div 10$ b. $1{,}349 \div 100$ c. $12{,}000 \div 10^2$ d. $34 \div 10^3$

Solutions

Move each decimal point to the left by the number of zeros in the power of 10, or by the power on the index.

a. $12 \div 10 = 1.2$
b. $1{,}349 \div 100 = 13.49$
c. $12{,}000 \div 10^2 = 120.00 = 120$
d. $34 \div 10^3 = 0.034$

4. Divide the following decimal numbers by powers of 10:

a. $1.4 \div 100$ b. $235.67 \div 10$ c. $0.34 \div 10^2$ d. $90.5 \div 10^3$

Solutions

a. $1.4 \div 100 = 0.014$
b. $235.67 \div 10 = 23.567$
c. $0.34 \div 10^2 = 0.0034$
d. $90.5 \div 10^3 = 0.0905$

Practice

Multiply or divide the following numbers:

1. 0.567×10^3
2. $25.46 \div 100$
3. $487 \div 10$
4. 231×10^2
5. 2.4×100
6. $0.9 \div 10^2$
7. $2{,}309 \div 10^3$
8. 1.871×10^2
9. $245.98 \div 1{,}000$
10. $2.6 \times 1{,}000$

Lesson Title: Standard Form of Large Numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-037	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to interpret and write large numbers in standard form (scientific notation): notation $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.

Overview

Every number can be expressed and interpreted as a product of the number and a power of ten. Numbers written in this form are said to be in **standard form,** which is the focus of this lesson.

Standard form is a system of working with very large or very small numbers. For example, 0.00000024 is a very small number, and 5,400,000 is a very large number. Working with very small or very large numbers can be difficult because of the number of zeroes you have to write, as in the examples above. Expressing numbers in standard form gives us a shorter way out of the difficulty.

Standard form is also called scientific notation. Science deals with very large numbers such as the distance between the earth and the sun. Science also deals with very small numbers such as the distance between atoms in a molecule.

Every number can be written in standard form using the notation $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.

Note that the number a **must** be between 1 and 10. It can be a whole or decimal number.

When **very small numbers** are changed to standard form, they have a **negative power.** When **very large numbers** are changed to standard form, they have a **positive power.**

In this lesson, you will handle large numbers with positive powers. In the next lesson, you will handle small numbers with negative powers.

To change a number to standard form, first write it as a number between 1 and 10. Then, count the number of spaces you need to move the decimal point to get the new decimal number. The number of spaces you move the decimal point tells you what power to use on the 10.

Solved Examples

1. Which of the following numbers are in standard form? Circle the numbers in standard form.

   81×10^3 $\quad$ 6.781×10^2 $\quad$ 7×10^{12} $\quad$ 0.5×10^4

   Solution
   Remember that standard form numbers have the form $a \times 10^n$, where $1 \leq a < 10$.
   You should have circled 6.781×10^2 and 7×10^{12}, because a is between 1 and 10.

   81×10^3 is **not** in standard form, because $81 > 10$. Also, 0.5×10^4 is **not** in standard from because $0.5 < 1$.

2. Write 5,400,000 in standard form.

   Solution
   To get a number between 1 and 10, the decimal point needs to be moved 6 places to the left, to come after the first integer, 5.

   $$5\,.\,4\;\;0\;\;0\;\;0\;\;0\;\;0$$

   Thus, we have $5{,}400{,}000 = 5.4 \times 10^6$. The power on the 10 is 6 because that's how many places we moved the decimal point. It is positive because we are dealing with a very big number.

3. Write the following numbers in standard form:
   a. 7,102.3 $\quad$ b. 31,000 $\quad$ c. 432 $\quad$ d. 425,000

   Solutions
   a. Shift the decimal point 3 spaces to the left: $7{,}102.3 = 7.1023 \times 10^3$
   b. Shift the decimal point 4 spaces to the left: $31{,}000 = 3.1 \times 10^4$
   c. Shift the decimal point 2 spaces to the left: $432 = 4.32 \times 10^2$
   d. Shift the decimal point 5 spaces to the left: $425{,}000 = 4.25 \times 10^5$

4. Write 87.2×10^6 in standard form.

Solution

The whole number part (87) is not between 1 and 10, so the number is not in standard form. To write it in standard form, shift the decimal point to the left to come after 8 (the first non-zero digit) and multiply by 10^{+1}. This adds one more to the power on the 10, making it 7.

$$\begin{aligned} 87.2 \times 10^6 &= 8.72 \times 10^1 \times 10^6 \\ &= 8.72 \times 10^{1+6} \\ &= 8.72 \times 10^7 \end{aligned}$$

5. Convert these numbers from standard form to ordinary form:
   a. 2.69×10^7 b. 1.7×10^3 c. 9.8×10 d. 7.136×10^2

   Solutions
   a. Shift the decimal point 7 places to the right, and fill the empty spaces with zeros: $2.69 \times 10^7 = 26{,}900{,}000$
   b. Shift the decimal point 3 places to the right, and fill the empty spaces with zeros: $1.7 \times 10^3 = 1{,}700$
   c. Shift the decimal point 1 place to the right: $9.8 \times 10 = 98$
   d. Shift the decimal point 2 places to the right: $7.136 \times 10^2 = 713.6$

Practice

1. Write the following numbers in standard form:
   a. 3,201.7
   b. 90,000
   c. 2,134
   d. 54
   e. 99,895,600

2. Write the following standard form numbers in ordinary form:
   a. 6.75×10^4
   b. 1.90×10^6
   c. 9.99×10^2
   d. 1.715×10^2
   e. 8×10^3

3. Write 30.2×10^3 in standard form.

4. Write 112.5×10^2 in standard form.

Lesson Title: Standard Form of Small Numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM-09-038	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to interpret and write small numbers in standard form (scientific notation): notation $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.

Overview

This is the second lesson on standard form. In the previous lesson, you handled the standard form of large numbers, which have a positive power on 10. In this lesson, you will handle standard form of small numbers. These numbers will have a negative power on 10.

Follow the same process to change a number to standard form. First write it as a number between 1 and 10. Then, count the number of spaces you need to move the decimal point to get the new decimal number. The number of spaces you move the decimal point tells you what power to use on the 10.

Solved Examples

1. Which of the following numbers are in standard form? Circle the numbers in standard form.

   1.6×10^{-3} $\quad$ 13.4×10^{-12} $\quad$ 0.91×10^{-9} $\quad$ 8×10^{-4}

   Solution
   Remember that standard form numbers have the form $a \times 10^n$, where $1 \leq a < 10$.
   You should have circled 1.6×10^{-3} and 8×10^{-4}, because a is between 1 and 10.

   13.4×10^{-12} is **not** in standard form, because $13.4 > 10$. Also, 0.91×10^{-9} is **not** in standard from because $0.91 < 1$.

2. Write 0.00000024 in standard form.

Solution

To get a number between 1 and 10, the decimal point needs to move 7 places to the right, to come after the first non-zero integer.

$$0\ 0\ 0\ 0\ 0\ 0\ 0\ 2\ .\ 4$$

Thus, we have $0.00000024 = 2.4 \times 10^{-7}$. The power on the 10 is 7 because that's how many places we moved the decimal point. It is negative because we are dealing with a very small number.

3. Write the following numbers in standard form:

a. 0.00012 b. 0.0291 c. 0.999 d. 0.00008

Solutions

a. Shift the decimal point 4 spaces to the right: $0.00012 = 1.2 \times 10^{-4}$
b. Shift the decimal point 2 spaces to the right: $0.0291 = 2.91 \times 10^{-2}$
c. Shift the decimal point 1 space to the right: $0.999 = 9.99 \times 10^{-1}$
d. Shift the decimal point 5 spaces to the right: $0.00008 = 8 \times 10^{-5}$

4. Write 0.257×10^{-4} in standard form.

Solution

The whole number part is 0, which is not between 1 and 10. To change the number to standard form, shift the decimal point to the right to come after 2 (the first non-zero digit) and multiply by 10^{-1}.

$$\begin{aligned} 0.257 \times 10^{-4} &= 2.57 \times 10^{-1} \times 10^{-4} \\ &= 2.57 \times 10^{-1+(-4)} \\ &= 2.57 \times 10^{-5} \end{aligned}$$

5. Convert these numbers from standard form to ordinary form:

a. 3.4×10^{-5} b. 5×10^{-3} c. 7.4×10^{-1} d. 1.872×10^{-8}

Solutions

a. Shift the decimal point 5 places to the left, and fill the empty spaces with zeros: $3.4 \times 10^{-5} = 0.000034$
b. Shift the decimal point 3 places to the left, and fill the empty spaces with zeros: $5 \times 10^{-3} = 0.005$
c. Shift the decimal point 1 place to the left: $7.4 \times 10^{-1} = 0.74$
d. Shift the decimal point 8 places to the left, and fill the empty spaces with zeros: $1.872 \times 10^{-8} = 0.00000001872$

Practice

1. Write the following numbers in standard form:
    a. 0.00000513
    b. 0.0000002
    c. 0.718
    d. 0.0051
    e. 0.0000109

2. Write the following standard form numbers in ordinary form:
    a. 2.752×10^{-2}
    b. 4.51×10^{-3}
    c. 7×10^{-6}
    d. 2.541×10^{-1}
    e. 1.05×10^{-4}

3. Write 0.56×10^{-2} in standard form.

4. Write 13.9×10^{-4} in standard form.

Lesson Title: Conversion to and from Standard Form	Theme: Numbers and Numeration
Practice Activity: PHM-09-039	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to convert from whole numbers and decimals to standard form and vice versa.

Overview

In this lesson you will practice converting to and from standard form. Remember that standard form is used to write very large and very small numbers in maths, science and everyday life.

People often use standard form if the number is smaller than 0.001 or if it is equal to or greater than 10,000. Otherwise, we normally write them as ordinary whole numbers or decimal numbers.

Remember when converting from standard form to an ordinary number, the power of 10 gives the number of places to move the decimal point. Move the decimal point to the right for a positive power, and to the left for a negative power.

Solved Examples

1. The distance of the Earth from the Sun is approximately 149,600,000,000 metres. Write this number in standard form.

   Solution
   The number in standard form is 1.496×10^{11}.

   Remember that the sign on the power of 10 is positive because this is a very large number. The number is 11 because the decimal point must be moved 11 places to the left to get a number between 1 and 10.

2. The mass of a particle of dust is 7.53×10^{-10} kg. Write this as an ordinary number.

   Solution
   Move the decimal point 10 places to the left, because the power on 10 is -10. The ordinary number is 0.000000000753.

3. Scientists have estimated that there are 5.5 million species of insects in the world. Write this number in:
   a. Ordinary form
   b. Standard form

Solutions

In everyday life, very large numbers are sometimes written as whole or decimal numbers with the word 'millions' or 'billions' written after them. Remember that 1 million has 6 zeros (1,000,000) and 1 billion has 9 zeros (1,000,000,000).

a. 5.5 million in ordinary form is 5,500,000.
b. The standard form is 5.5×10^6. The decimal must be moved 6 places to the left to get a number between 1 and 10.

4. The populations of 5 countries in West Africa are given in the table. Write each country's population in standard form.

Country	Population	Standard Form
Ghana	28,210,000	
Guinea	12,400,000	
Liberia	4,614,000	
Nigeria	186,000,000	
Sierra Leone	7,396,000	

Solution

Convert each population to standard form. Remember that the first number must be between 1 and 10.

Country	Population	Standard Form
Ghana	28,210,000	2.821×10^7
Guinea	12,400,000	1.24×10^7
Liberia	4,614,000	4.614×10^6
Nigeria	186,000,000	1.86×10^8
Sierra Leone	7,396,000	7.396×10^6

5. Express the following ordinary numbers in standard form:

a. 0.000617 b. 4,570,000 c. 0.004 d. 50,000,000,000

Solutions

a. Shift the decimal point 4 spaces to the right: $0.000617 = 6.17 \times 10^{-4}$
b. Shift the decimal point 6 spaces to the left: $4{,}570{,}000 = 4.57 \times 10^6$
c. Shift the decimal point 3 spaces to the right: $0.004 = 4 \times 10^{-3}$
d. Shift the decimal point 10 spaces to the right: $50{,}000{,}000{,}000 = 5 \times 10^{10}$

6. Convert these standard form numbers to ordinary numbers:
   a. 4.15×10^{-6} b. 3.9×10^{3} c. 9.89×10^{8} d. 5×10^{-7}

   Solutions
   a. Shift the decimal point 6 places to the left, and fill the empty spaces with zeros: $4.15 \times 10^{-6} = 0.00000415$
   b. Shift the decimal point 3 places to the right, and fill the empty spaces with zeros: $3.9 \times 10^{3} = 3{,}900$
   c. Shift the decimal point 8 places to the right, and fill the empty spaces with zeros: $9.89 \times 10^{8} = 989{,}000{,}000$
   d. Shift the decimal point 7 places to the left, and fill the empty spaces with zeros: $5 \times 10^{-7} = 0.0000005$

Practice

1. A single grain of rice has mass of 0.029 grammes. Write this in standard form.
2. One sesame seed weighs approximately 3.64×10^{-6} kg. Write this as an ordinary number.
3. The table below gives the populations of the 5 most populous countries. Write each country's population in ordinary form and standard form. Part of the table has been completed for you.

Country	Population	Ordinary Form	Standard Form
China	1.38 billion	1,380,000,000	
India	1.297 billion		
United States	329 million	329,000,000	
Indonesia	263 million		
Brazil	208.8 million		

4. Express the following ordinary numbers in standard form:
   a. 678,000 b. 0.0008193 c. 945,100,000 d. 0.00275

5. Express the following standard form numbers in ordinary form:
   a. 6×10^{-9} b. 5.55×10^{-3} c. 8.005×10^{7} d. 6.9×10^{10}

Lesson Title: Multiplying and Dividing Small and Large Numbers	Theme: Numbers and Numeration
Practice Activity: PHM-09-040	Class: JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. Do simple multiplication and division problems with whole numbers, decimals and fractions.
2. Give answers to problems in standard form.

Overview

In this lesson, you will multiply and divide numbers in standard form.

To **multiply** numbers in standard form, multiply the number parts and multiply the powers of 10. Apply the laws of indices in multiplying the powers of 10. That is, add the powers.

To **divide** in standard form, divide the number parts and divide the powers of 10. Apply the laws of indices in dividing the powers of 10. That is, subtract the powers.

After applying the operation, check that your answer is in standard form. If it is not in standard form, you must change it. For example, in Solved Example 1 the answer to the multiplication problem is 18.2×10^8. This is not in standard form, because 18.2 is greater than 10. It must be changed to standard form: 1.82×10^9.

Solved Examples

1. Simplify $(9.1 \times 10^5) \times (2 \times 10^3)$. Give your answer in standard form.

Solution

$(9.1 \times 10^5) \times (2 \times 10^3)$	$= (9.1 \times 2) \times (10^5 \times 10^3)$	Collect like terms
	$= 18.2 \times 10^{5+3}$	Multiply
	$= 18.2 \times 10^8$	Apply law of indices
	$= (1.82 \times 10^1) \times 10^8$	Change to standard form
	$= 1.82 \times 10^9$	

2. Simplify $(6 \times 10^{-3}) \div (2 \times 10^3)$ and leave your answer in standard form.

Solution

$(6 \times 10^{-3}) \div (2 \times 10^3)$	$= \frac{6\times10^{-3}}{2\times10^3}$	Write as a fraction
	$= \frac{6}{2} \times \frac{10^{-3}}{10^3}$	Collect like terms
	$= 3 \times 10^{-3-3}$	Divide
	$= 3 \times 10^{-6}$	Apply law of indices

3. Simplify $(6.4 \times 10^{-5}) \times (3.2 \times 10^{-3})$. Give your answer in standard form.

Solution

$$
\begin{aligned}
(6.4 \times 10^{-5}) \times (3.2 \times 10^{-3}) &= (6.4 \times 3.2) \times (10^{-5} \times 10^{-3}) && \text{Group like terms} \\
&= 20.48 \times 10^{-5+(-3)} && \text{Multiply} \\
&= 20.48 \times 10^{-5-3} && \text{Apply law of indices} \\
&= 20.48 \times 10^{-8} \\
&= (2.048 \times 10^{1}) \times 10^{-8} && \text{Change to standard form} \\
&= 2.048 \times 10^{1-8} \\
&= 2.048 \times 10^{-7}
\end{aligned}
$$

4. Simplify $\frac{7.5\times10^{-8}}{2.5\times10^{-7}}$. Give your answer in standard form.

Solution

$$
\begin{aligned}
\frac{7.5\times10^{-8}}{2.5\times10^{-7}} &= \frac{7.5}{2.5} \times \frac{10^{-8}}{10^{-7}} && \text{Collect like terms} \\
&= 3 \times 10^{-8-(-7)} && \text{Divide} \\
&= 3 \times 10^{-8+7} && \text{Apply law of indices} \\
&= 3 \times 10^{-1}
\end{aligned}
$$

5. Simplify $\frac{8.6\times10^{17}}{2.5\times10^{7}}$, and leave your answer in standard form.

Solution

$$
\begin{aligned}
\frac{8.6\times10^{17}}{2.5\times10^{7}} &= \frac{8.6}{2.5} \times \frac{10^{17}}{10^{7}} \\
&= 3.44 \times 10^{17-7} \\
&= 3.44 \times 10^{10}
\end{aligned}
$$

Practice

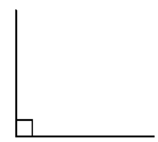
1. Simplify $(5.7 \times 10^{8}) \times (4 \times 10^{3})$ and give your answer in standard form.
2. Simplify $(7.46 \times 10^{-3}) \times (1.2 \times 10^{-1})$. Give your answer in standard form.
3. Simplify $(6.6 \times 10^{-5}) \div (2.2 \times 10^{-4})$ and give the answer in standard form.
4. Calculate $(8.632 \times 10^{5}) \div (2.6 \times 10^{4})$ and give the answer in standard form.
5. Simplify $\frac{9.43\times10^{15}}{2.3\times10^{4}}$ and give the answer in standard form.

Lesson Title: Right-angled Triangles (Revision)	**Theme:** Geometry
Practice Activity: PHM-09-041	**Class:** JSS 3

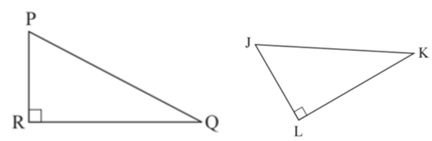
Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify the parts of a right-angled triangle.
2. Identify the properties of a right-angled triangle.

Overview

A right angle is formed by 2 perpendicular lines. There are many right angles in everyday life. For example, the corner of a room or the corner of a book are right angles. Recall that we measure angles with degrees, which have the symbol °. A right angle is always 90°.



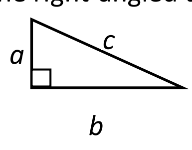
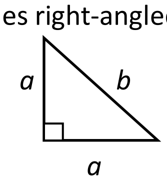
A right-angled triangle is a triangle which has a 90° angle. The following triangles are examples of right-angled triangles:



We can name triangles using the letters that their angles are labeled with. The above triangles are ΔPRQ and ΔJKL.

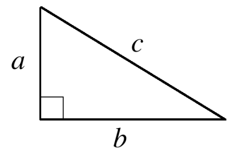
Each right-angled triangle has a hypotenuse. The hypotenuse is the side opposite the right angle, and it is the longest side. In the triangles above, PQ is the hypotenuse of ΔPRQ, and JK is the hypotenuse of ΔJKL. Additional properties of right-angled triangles are in the table below.

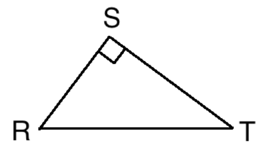
Properties of Right-Angled Triangles	

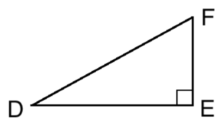
Scalene right-angled triangle  Isosceles right-angled triangle 	• All 3 angles add up to 180°
	• The 2 acute angles are complementary angles, and sum to 90°.
	• The 2 short sides, a and b form the 90° angle
	• The hypotenuse, c, is the side opposite the 90° angle
	• There are 2 types of right-angled triangles: scalene and isosceles
	• Scalene right-angled triangle
	  o All 3 sides are different from each other
	  o One 90° angle, 2 other angles different from each other
	• Isosceles right-angled triangle
	  o Have 2 equal sides which form the 90° angle
	  o The 2 angles are 45° each

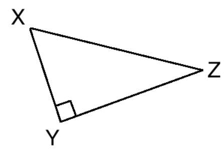
Solved Examples

1. Identify the hypotenuse of each triangle below:

a.


b.


c.


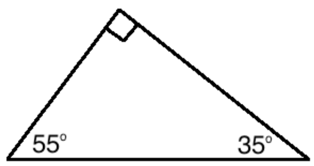
d.


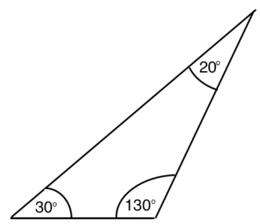
Solutions

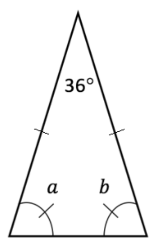
Remember that the hypotenuse is the side opposite the right angle. The answers are:

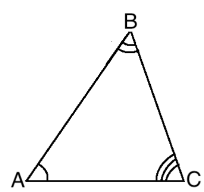
a. c
b. RT
c. DF
d. XZ

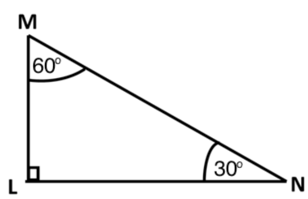
2. Identify which of the triangles below are right-angled triangles.

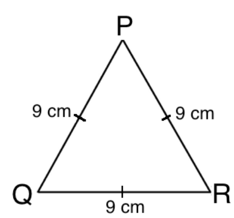
a.


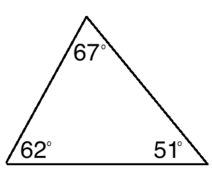
b.


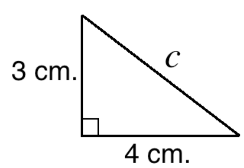
c.


d.


e.


f.


g.


h.


Solutions

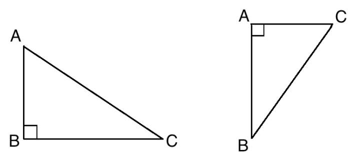
Any triangle with a right angle marked by a small square is a right-angled triangle. Among the triangles above, the following are right-angled: a, e and h.

The other triangles are **not** right-angled triangles. You may recognize them as various other types. For example, c is an isosceles triangle and f is an equilateral triangle.

3. Draw a right-angled triangle and label it ABC.

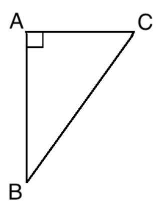
Solution

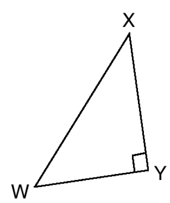
Your right-angled triangle may be any size and face any direction. Its 3 angles should be labeled with the labeled with the letters A, B and C. These are examples:

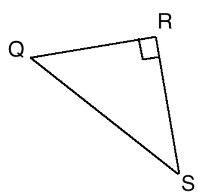


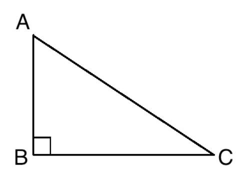
Practice

1. Identify the hypotenuse of each triangle below:

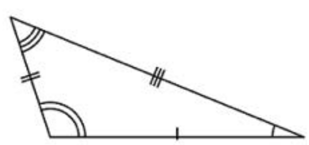
a.


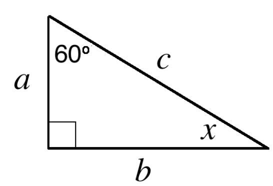
b.


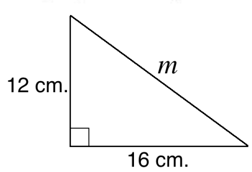
c.


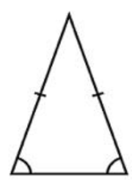
d.


2. Identify which of the triangles below are right-angled triangles.

a.


b.


c.


d.


3. Draw a right-angled triangle and label it XYZ.

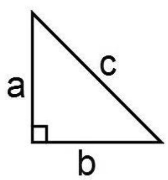
Lesson Title: Introduction to Pythagoras' Theorem	**Theme:** Geometry
Practice Activity: PHM-09-042	**Class:** JSS 3

Learning Outcomes
By the end of the lesson, you will be able to:
1. State Pythagoras' theorem.
2. Identify that the formula $a^2 + b^2 = c^2$ can be used to find the sides of a right-angled triangle.

Overview

In mathematics, Pythagoras' theorem, also known as the Pythagorean Theorem, is a relation among the three sides of a right triangle.

The general formula for Pythagoras' theorem is:



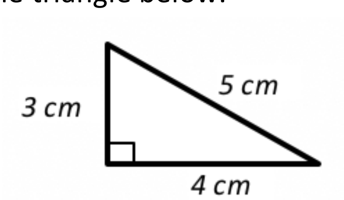
$$a^2 + b^2 = c^2$$

Pythagoras' theorem states that for any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. It is true for all right-angled triangles.

On the last page of this activity you will find a table of common squares and square roots. You may use this to help you quickly do the calculations in this lesson and the next lessons.

Solved Examples

1. Verify Pythagoras' theorem for the triangle below:



Solution

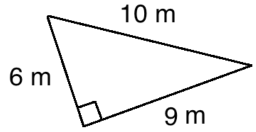
First, determine which side is the hypotenuse. This side will be c. The other 2 sides can be a or b. In this triangle, the hypotenuse is $c = 5$ cm. Let the other 2 sides be $a = 3$ cm and $b = 4$ cm.

Substitute $a = 3$, $b = 4$ and $c = 5$ into the formula for Pythagoras' theorem, and simplify:

$$
\begin{aligned}
a^2 + b^2 &= c^2 && \text{Pythagoras' theorem} \\
3^2 + 4^2 &= 5^2 && \text{Substitute} \\
9 + 16 &= 25 && \text{Simplify} \\
25 &= 25 && \\
\text{LHS} &= \text{RHS} &&
\end{aligned}
$$

The 2 sides of the equation are equal, so Pythagoras' theorem holds true.

2. Use Pythagoras' theorem to determine if the triangle below is actually a right-angled triangle:



Solution

First, determine which side is the hypotenuse. This side will be c. The other 2 sides can be a or b. In this triangle, the hypotenuse is $c = 10$ m. Let the other 2 sides be $a = 6$ m and $b = 9$ m.

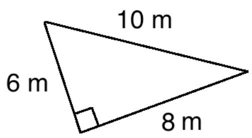
Substitute $a = 6$, $b = 9$ and $c = 10$ into the formula for Pythagoras' theorem, and simplify:

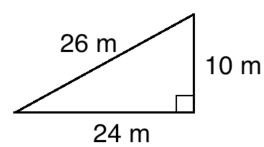
$$
\begin{aligned}
a^2 + b^2 &= c^2 && \text{Pythagoras' theorem} \\
6^2 + 9^2 &= 10^2 && \text{Substitute} \\
36 + 81 &= 100 && \text{Simplify} \\
117 &= 100 && \\
\text{LHS} &\neq \text{RHS} &&
\end{aligned}
$$

This is **not** a right-angled triangle, because the left-hand side and right-hand side of the equation are not equal. The triangle is incorrectly labelled with a right angle.

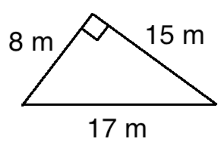
Practice

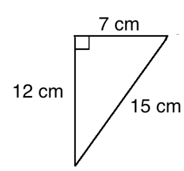
1. Verify Pythagoras' theorem for the triangles below:

a. 

b. 

2. Use Pythagoras' theorem to determine if the triangles below are actually right-angled triangles:

a. 

b. 

Squares - Cubes - Square Root (

number n	square n^2	cube n^3	square root $\sqrt{n}$	number n	square n^2	cube n^3	square root $\sqrt{n}$
1	1	1	1.0000	41	1681	68921	6.4031
2	4	8	1.4142	42	1764	74088	6.4807
3	9	27	1.7321	43	1849	79507	6.5574
4	16	64	2.0000	44	1936	85184	6.6332
5	25	125	2.2361	45	2025	91125	6.7082
6	36	216	2.4495	46	2116	97336	6.7823
7	49	343	2.6458	47	2209	103823	6.8557
8	64	512	2.8284	48	2304	110592	6.9282
9	81	729	3.0000	49	2401	117649	7.0000
10	100	1000	3.1623	50	2500	125000	7.0711
11	121	1331	3.3166	51	2601	132651	7.1414
12	144	1728	3.4641	52	2704	140608	7.2111
13	169	2197	3.6056	53	2809	148877	7.2801
14	196	2744	3.7417	54	2916	157464	7.3485
15	225	3375	3.8730	55	3025	166375	7.4162
16	256	4096	4.0000	56	3136	175616	7.4833
17	289	4913	4.1231	57	3249	185193	7.5498
18	324	5832	4.2426	58	3364	195112	7.6158
19	361	6859	4.3589	59	3481	205379	7.6811
20	400	8000	4.4721	60	3600	216000	7.7460
21	441	9261	4.5826	61	3721	226981	7.8102
22	484	10648	4.6904	62	3844	238328	7.8740
23	529	12167	4.7958	63	3969	250047	7.9373
24	576	13824	4.8990	64	4096	262144	8.0000
25	625	15625	5.0000	65	4225	274625	8.0623
26	676	17576	5.0990	66	4356	287496	8.1240
27	729	19683	5.1962	67	4489	300763	8.1854
28	784	21952	5.2915	68	4624	314432	8.2462
29	841	24389	5.3852	69	4761	328509	8.3066
30	900	27000	5.4772	70	4900	343000	8.3666
31	961	29791	5.5678	71	5041	357911	8.4261
32	1024	32768	5.6569	72	5184	373248	8.4853
33	1089	35937	5.7446	73	5329	389017	8.5440
34	1156	39304	5.8310	74	5476	405224	8.6023
35	1225	42875	5.9161	75	5625	421875	8.6603
36	1296	46656	6.0000	76	5776	438976	8.7178
37	1369	50653	6.0828	77	5929	456533	8.7750
38	1444	54872	6.1644	78	6084	474552	8.8318
39	1521	59319	6.2450	79	6241	493039	8.8882
40	1600	64000	6.3246	80	6400	512000	8.9443

Lesson Title: Finding the Hypotenuse of a Right-Angled Triangle	Theme: Geometry
Practice Activity: PHM-09-043	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to find the hypotenuse of a right-angled triangle using Pythagoras' theorem.

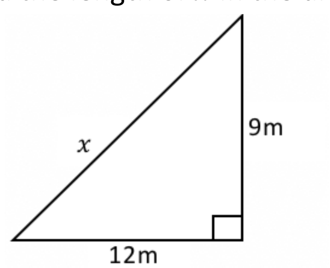
Overview

In the last lesson, we used Pythagoras' theorem to verify if a triangle is a right-angled triangle. The main use of Pythagoras' theorem is to find the lengths of unknown sides of a triangle. In this lesson, you will use Pythagoras' theorem to find the length of the hypotenuse of a triangle. This can only be done if the lengths of the other 2 sides are known.

Recall that Pythagoras' theorem is $a^2 + b^2 = c^2$, where c is the hypotenuse.

Solved Examples

1. Find the length of x in the diagram below.



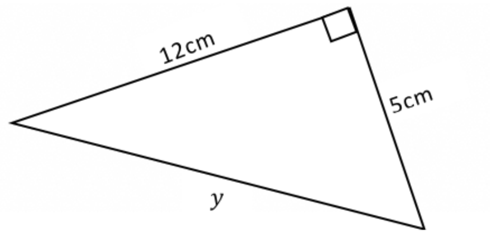
Solution

Apply Pythagoras theorem $c^2 = a^2 + b^2$. Substitute the known sides and solve for x.

$$\begin{aligned} c^2 &= a^2 + b^2 && \text{Pythagoras' theorem} \\ x^2 &= 12^2 + 9^2 && \text{Substitute the sides of the triangle} \\ &= 144 + 81 && \text{Simplify} \\ &= 225 && \\ x &= \sqrt{225} && \text{Take the square root of both sides} \\ &= 15 \text{ m} && \end{aligned}$$

Note that $\sqrt{225} = 15$ or $\sqrt{225} = -15$. However, the length of the hypotenuse cannot be a negative number. It must be the positive result. The answer is 15 m.

2. Find the values of y in the diagram below:

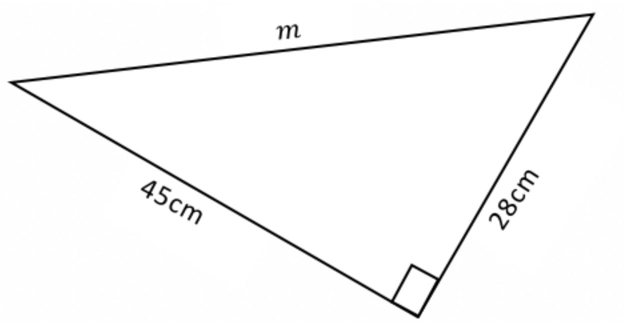


Solution

Apply Pythagoras theorem $c^2 = a^2 + b^2$. Substitute the known sides and solve for y.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ y^2 &= 5^2 + 12^2 \\ &= 25 + 144 \\ &= 169 \\ y &= \sqrt{169} \\ &= 13 \text{ cm} \end{aligned}$$

3. Find the value of m in the diagram below:

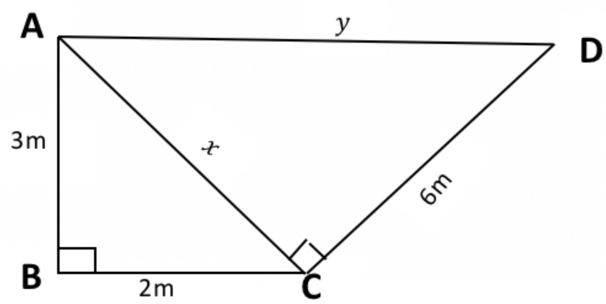


Solution

Apply Pythagoras theorem $c^2 = a^2 + b^2$. Substitute the known sides and solve for m.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ m^2 &= 28^2 + 45^2 \\ &= 784 + 2025 \\ &= 2{,}809 \\ m &= \sqrt{2{,}809} \\ &= 53 \text{ cm} \end{aligned}$$

4. Calculate the length of line AD in the diagram below:



Solution

In triangle ABC, x is the hypotenuse. In triangle ACD, y is the hypotenuse. Find the length of x using triangle ABC, then use it to find y in triangle ACD.

In triangle ABC:

$$
\begin{aligned}
x^2 &= 3^2 + 2^2 \\
&= 9 + 4 \\
&= \sqrt{13}
\end{aligned}
$$

In triangle ACD:

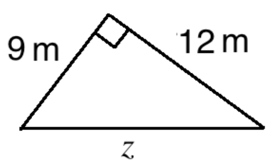
$$
\begin{aligned}
y^2 &= x^2 + 6^2 \\
&= \left(\sqrt{13}\right)^2 + 36 \\
&= 49 \\
y &= \sqrt{49} \\
&= 7 \text{ cm}
\end{aligned}
$$

The answer is $y = |AD| = 7$ cm

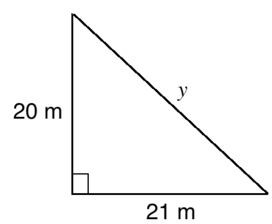
Practice

1. Calculate the length of the sides marked with letters in the following triangles:

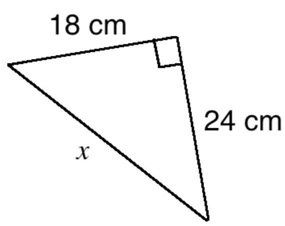
a.



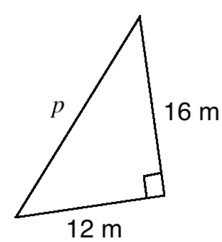
b.



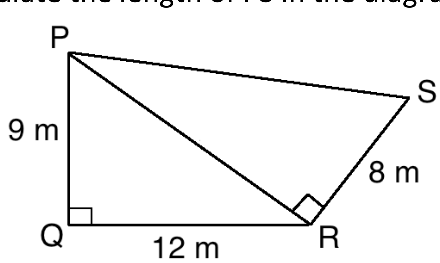
c.



d.



2. Calculate the length of PS in the diagram below:



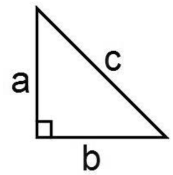
Lesson Title: Finding the Other Sides of a Right-Angled Triangle	Theme: Geometry
Practice Activity: PHM-09-044	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to apply Pythagoras' theorem to find the length of the other two sides of a right-angled triangle.

Overview

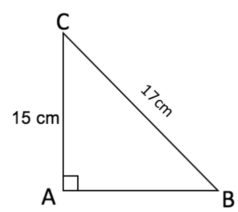
Recall that the hypotenuse is always opposite the right angle and it is always the longest side of the triangle. Pythagoras' theorem can be used to find the third side of a right-angled triangle if the other two sides are known.

To find the sides a and b, change the subject of the formula for Pythagoras' theorem. Recall that the theorem is $c^2 = a^2 + b^2$. Substitute the known sides and change the subject of the equation to solve for the unknown side, a or b.



Solved Examples

1. Find AB in the triangle below.

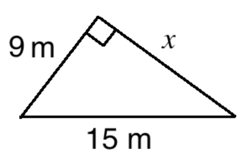


Solution

Identify the known sides: $|AC| = 15$ cm and $|BC| = 17$ cm. Substitute these into Pythagoras' theorem:

$$\begin{aligned}
|AC|^2 + |AB|^2 &= |BC|^2 && \text{Pythagoras' theorem} \\
15^2 + |AB|^2 &= 17^2 && \text{Substitute known values} \\
225 + |AB|^2 &= 289 && \text{Solve for } |AB| \\
|AB|^2 &= 289 - 225 \\
|AB|^2 &= 64 \\
AB &= \sqrt{64} \\
AB &= 8 \text{ cm}
\end{aligned}$$

2. In the triangle below, calculate the length of x.

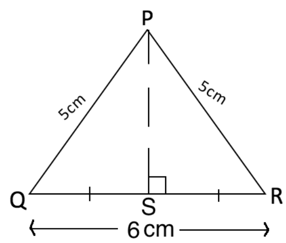


Solution

Substitute the known sides into Pythagoras' theorem:

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagoras' theorem} \\ 9^2 + x^2 &= 15^2 && \text{Substitute known values} \\ 81 + x^2 &= 225 && \text{Solve for } x \\ x^2 &= 225 - 81 \\ x^2 &= 144 \\ x &= \sqrt{144} \\ x &= 12 \text{ m} \end{aligned}$$

3. In triangle PQR, $|PQ| = |PR| = 5$ cm, $|QR| = 6$ cm, and $|QS| = |SR|$. Find $|PS|$.



Solution

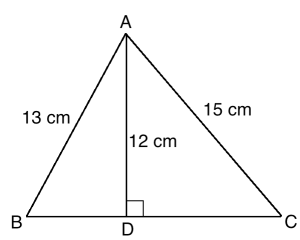
Find the length of QS and SR, which can be used to find PS. The length of each segment is half the length of QR.

$$QS = SR = \frac{1}{2}QR = \frac{1}{2} \times 6 \text{ cm} = 3 \text{ cm}$$

In triangle PQS:

$$\begin{aligned} |PS|^2 + |QS|^2 &= |PQ|^2 && \text{Pythagoras' theorem} \\ |PS|^2 + 3^2 &= 5^2 && \text{Substitute known values} \\ |PS|^2 + 9 &= 25 && \text{Solve for } |PS| \\ |PS|^2 &= 25 - 9 \\ |PS|^2 &= 16 \\ PS &= \sqrt{16} \\ PS &= 4 \end{aligned}$$

4. In the triangle below, calculate the length of BC.



Solution

Divide the shape into 2 triangles, ABD and ADC. Find the missing side of each triangle: BD and DC. Then, add the 2 lengths together to find BC: $|BC| = |BD| + |DC|$

In triangle ABD:

$$\begin{aligned} |AD|^2 + |BD|^2 &= |AB|^2 \\ 12^2 + |BD|^2 &= 13^2 \\ 144 + |BD|^2 &= 169 \\ |BD|^2 &= 169 - 144 \\ |BD|^2 &= 25 \\ |BD| &= \sqrt{25} \\ |BD| &= 5 \text{ cm} \end{aligned}$$

In triangle ADC:

$$\begin{aligned} |AD|^2 + |DC|^2 &= |AC|^2 \\ 12^2 + |DC|^2 &= 15^2 \\ 144 + |DC|^2 &= 225 \\ |DC|^2 &= 225 - 144 \\ |DC|^2 &= 81 \\ |DC| &= \sqrt{81} \\ |DC| &= 9 \text{ cm} \end{aligned}$$

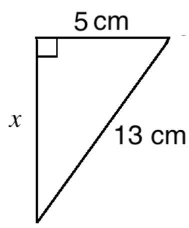
Now add the 2 sides: $BC = BD + DC = 5 \text{ cm} + 9 \text{ cm} = 14 \text{ cm}$

The answer is $BC = 14$ cm

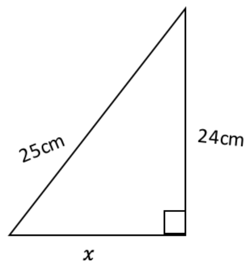
Practice

1. In the diagrams below, calculate the length of each side marked x.

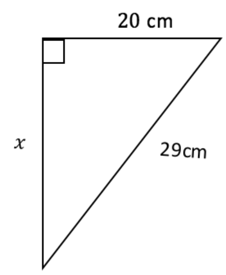
a.



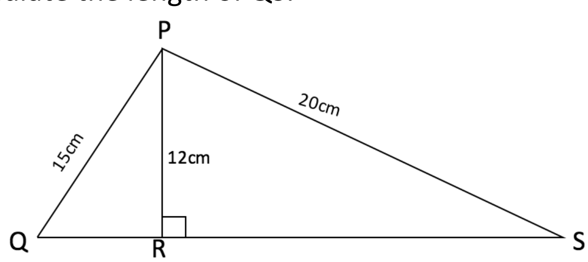
b.



c.



2. In the figure below, calculate the length of QS.



Lesson Title: Applying Pythagoras' Theorem	Theme: Geometry
Practice Activity: PHM-09-045	**Class:** JSS 3

Learning Outcome
By the end of the lesson, you will be able to solve diagram and word problems involving Pythagoras' theorem.

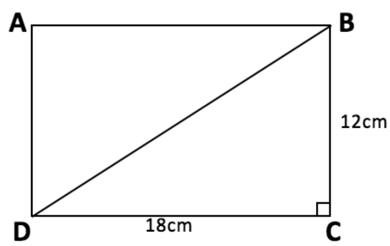
Overview

This lesson is on solving problems using Pythagoras' theorem. You will solve word problems that require Pythagoras' theorem. It is very important to draw a diagram first to help with solving such problems.

Solved Examples

1. Find the length of the diagonal of a rectangle with sides of length 12 cm and 18 cm. Give your answer to 1 decimal place.

   Solution
   First, draw the rectangle showing its diagonal:



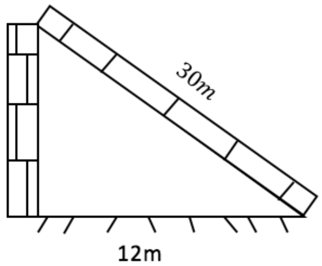
In rectangle ABCD, the diagonal is the hypotenuse of triangle ABD and BDC.

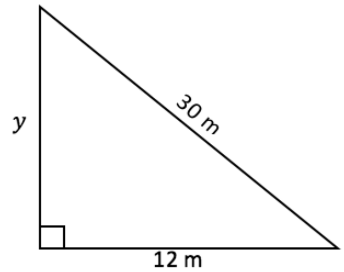
$$\begin{aligned} |BD|^2 &= |DC|^2 + |BC|^2 \\ |BD|^2 &= 18^2 + 12^2 \\ |BD|^2 &= 324 + 144 \\ |BD|^2 &= 468 \\ BD &= \sqrt{468} \\ BD &= 21.6 \text{ cm} \end{aligned}$$

2. A ladder 30 m long rests against a vertical wall. The distance between the foot of the ladder and the wall is 12 m. How far up the wall is the top of the ladder?

Solution

First, draw a diagram:





Use Pythagoras' theorem to find the height, y:

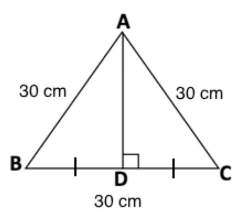
$$\begin{aligned} y^2 + 12^2 &= 30^2 \\ y^2 + 144 &= 900 \\ y^2 &= 900 - 144 \\ y^2 &= 756 \\ y &= \sqrt{756} \\ y &= 27.5 \text{ m} \end{aligned}$$

The top of the ladder is 27.5 metres from the ground.

3. The lengths of the sides of an equilateral triangle are 30 cm. Find the height of the triangle. Give your answer to the nearest whole number.

Solution

First, draw a diagram:



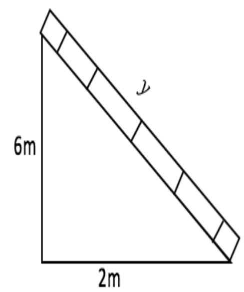
From the diagram, note that $BD = \frac{1}{2}\, BC = \frac{30}{2} = 15$ cm. Use this to find $|AD|$:

$$\begin{aligned} |AD|^2 + |BD|^2 &= |AB|^2 \\ |AD|^2 + 15^2 &= 30^2 \\ |AD|^2 + 225 &= 900 \\ |AD|^2 &= 900 - 225 \\ |AD|^2 &= 675 \\ |AD| &= \sqrt{675} \\ |AD| &= 25.98 = 26 \text{ cm} \end{aligned}$$

4. The foot of a ladder is 2 m from a wall and the top of the ladder is 6 m above the ground. Calculate the length of the ladder, correct to one decimal place.

Solution

First, draw a diagram:



Apply Pythagoras' Theorem to find y:

$$
\begin{aligned}
y^2 &= 6^2 + 2^2 \\
&= 36 + 4 \\
&= 40 \\
y &= \sqrt{40} \\
&= 6.3 \text{ m}
\end{aligned}
$$

Practice

1. The length of a rectangle is 6 cm and the diagonal is 8 cm. How long is the width of the rectangle?
2. A ladder 9 m long leans against a wall. If the foot of the ladder is 4.5 m away from the wall, how far up the wall does the ladder reach?
3. The sides of an equilateral triangle are 8 cm long. Calculate the vertical height of the triangle.
4. A rectangular piece of cardboard is 40 cm long. The length of the diagonal is 50 cm. How wide is the cardboard?
5. A ladder leans against a vertical wall of height 15 m, if the foot of the ladder is 8 m away from the wall. Calculate the length of the ladder.

Answer Key – JSS 3 Term 1

Lesson Title: Sorting Objects	
Practice Activity: PHM-09-001	

1. School subjects: Math, Chemistry, English, Social Studies, Biology
   Stationery/objects: Exercise book, Pen, Ruler, Desk, Pencil, Protractor
2. Vowels: A, O, E, U; Consonants: B, T, S, P, R, L, N, Q
3. a. Males: Issa Koroma, John Kamara, Mohamed Bah; females: Hawa Bangura, Aminata Kamara, Juliet Nyalloma
   b. Surnames starting with letters A-M: Hawa Bangura, Issa Koroma, John Kamara, Mohamed Bah, Aminata Kamara; Surnames starting with letters N-Z: Juliet Nyalloma
   c. Surnames starting with K: Issa Koroma, John Kamara, Aminata Kamara; Surnames starting with any other letters: Hawa Bangura, Mohamed Bah, Juliet Nyalloma

Lesson Title: Introduction to Sets	
Practice Activity: PHM-09-002	

1. a. n(Colours) = 6; b. Blue ∈ Colours; c. Orange ∈ Colours
2. a. n(Continents) = 7; b. Africa ∈ Continents; c. Asia ∈ Continents
3. a. M is the set of months having 31 days; b. Yes, the set is well-defined because we can identify all of its members; c. M = {January, March, May, July, August, October, December}; d. August ∈ M; e. n(M) = 7

Lesson Title: Sets in Real Life	
Practice Activity: PHM-09-003	

1. D = {Port Loko, Bombali, Kailahun, Koinadugu, Moyamba}
2. A = {Sierra Leone, Kenya, Mozambique, Liberia}
3. F = {Manchester United, Chelsea, Arsenal}
4. The set of people, P = {Abu, Fatu, Hawa, David, Mustapha, Alice}
   The set of furniture, F = {desk, bed, table, chair}
   The set of clothes, C = {trousers, hat, shirt, jacket, shoes, socks}

Lesson Title: Describe Sets of Objects	
Practice Activity: PHM-09-004	

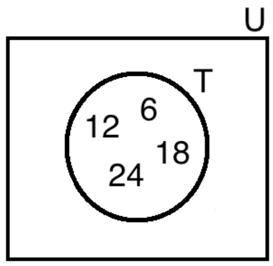
1. M = {January, February, March, April, May, June, July, August, September, October, November, December}
   M = {months of the year}
   M = {x : x is a month of the year}
2. C = {c, h, e, m, i, s, t, r, y}
   C = {letters in the word chemistry}
   C = {x : x is a letter in chemistry}

3. a. P = {y : y is a letter in phone}; P = {p, h, o, n, e}
4. a. F = {z : z is a colour on the flag of Sierra Leone}; b. F = {green, white, blue}
5. a. S is the set of all x such that x is a sauce; b. F is the set of all y such that y is a female; c. M is the set of all z such that z is a letter in the word male.

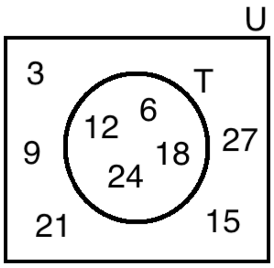
Lesson Title: Write Sets of Numbers
Practice Activity: PHM-09-005

1. G = {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44}
2. Z = {5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
3. a. S is the set of all x such that x is a multiple of 10 and x is less than 100; b. S = {10, 20, 30, 40, 50, 60, 70, 80, 90}; c. 30 ∈ S; d. 72 ∉ S
4. a. U = {3, 6, 9, 12, 15, 18, 21, 24, 27}; b. T = {y : y is a multiple of 6, y < 30}; c. See below; d. T^c = {3, 9, 15, 21, 27}; e. See below.

c.



e.



Lesson Title: Finite Sets
Practice Activity: PHM-09-006

1. F = {1, 2, 3, 6, 9, 18}; n(F) = 6
2. a. Finite, M = {January, February, March, … , December}, n(M) = 12; b. Finite, W = {8, 16, 24, 32, 40, 48}, n(W) = 6; c. Not finite, the list of all even numbers cannot be listed or counted; d. Finite, Y = {10, 12, 14, 16, 18, 20}, n(Y) = 6
3. a. A = {20, 22, 24, 26, 28, 30}, n(A) = 6; b. B = {25}, n{B} = 1; c. C = {20, 25, 30}, n(C) = 3; d. D = {20, 30}, n(D) = 2

Lesson Title: Infinite Sets
Practice Activity: PHM-09-007

1. a. Finite. The list ends with 100; b. Finite. The list ends with 101; c. Infinite. The ellipses show that it continues forever. d. Infinite. The ellipses show that it continues forever.
2. a. Infinite, A = {6, 8, 10, … }; b. Finite, B = {6, 8, 10}; c. Finite, C = {6, 8, 10, … , 100}; d. Infinite, D = {2, 4, 6, 8, … }
3. a. Infinite, X = {3, 6, 9, 12, … }; b. Finite, Y = {3, 6, 9}; c. Infinite, Z = {12, 15, 18, …}

Lesson Title:	Unit and Empty Sets
Practice Activity:	PHM-09-008

1. a. Not an empty set; b. Empty set; c. Not an empty set; d. Empty set
2. a. Unit set; b. Not a unit set; c. Unit set; d. Not a unit set
3. a. Neither; b. Empty set; c. Unit set; d. Neither

Lesson Title:	Equal Sets
Practice Activity:	PHM-09-009

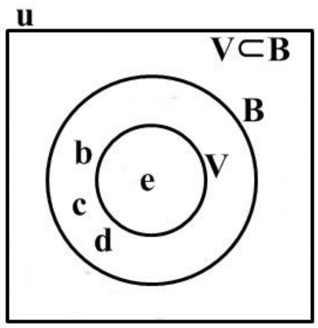
1. a. Not equal; b. Not equal; c. Equal; d. Equal; e. Not equal; f. Equal
2. The following are 5 examples. Your own may be different:
   A = {5, 5, 10, 10, 15, 15, 20, 20, 25}
   B = {25, 20, 15, 10, 5}
   C = {5, 15, 10, 25, 20}
   D = {5, 5, 15, 20, 20, 25, 10}
   E = {10, 5, 25, 20, 15}

Lesson Title:	Equivalent Sets
Practice Activity:	PHM-09-010

1. a. Equivalent; b. Not equivalent; c. Not equivalent; d. Equivalent
2. a. W and X are equivalent because n(W) = n(X) = 3. Y and Z are equivalent because n(Y) = n(Z) = 4.
   b. W and X are equal because they have exactly the same elements, W = X = {r, s, t}. Y and Z are equal because they have exactly the same elements, Y = Z = {r, s, t, u}
3. a. B = {10, 12, 14, 16, 18}, D = {1, 2, 3, 4, 5}; b. $A \leftrightarrow B$, $B \leftrightarrow C$, $C = D$, $A \leftrightarrow D$.

Lesson Title:	Introduction to Subsets
Practice Activity:	PHM-09-011

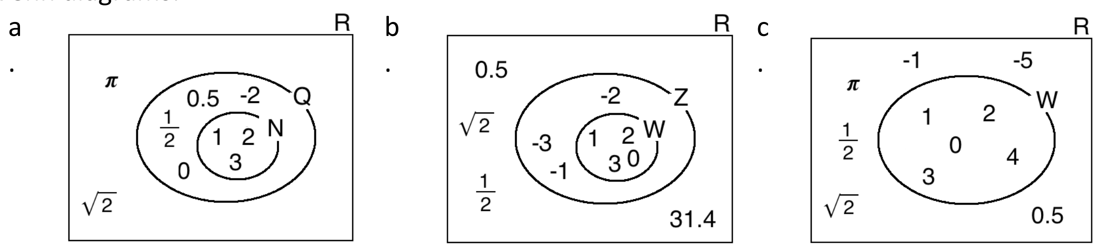
1. S_0= {a, b, c, d}, S_1 = {a, b, c}, S_2 = {b, c, d}, S_3 = {c, d, a}, S_4 = {d, a, b}, S_5 = {a, b}, S_6 = {a, c}, S_7 = {a, d}, S_8 = {b, c }, S_9 = {b, d}, S10 = {c, d}, S_{11} = {a}, S_{12} = {b}, S_{13} = {c}, S_{14} = {d}, S_{15} = Φ = {}
2. a. W is a subset of X; b. V is a subset of X; c. T is not a subset of X.
3. Venn diagram:



4. $M_1 = \{2, 4, 6\}$, $M_2 = \{2, 4\}$, $M_3 = \{2, 6\}$, $M_4 = \{4, 6\}$, $M_5 = \{2\}$, $M_6 = \{4\}$, $M_7 = \{6\}$, $M_8 = \{\}$

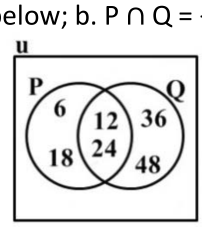
Lesson Title: Identifying Subsets of the set of Real Numbers
Practice Activity: PHM-09-012

1. Yes. N, W and Z are all subsets of Q.
2. Venn diagrams:

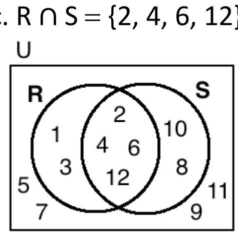


Lesson Title: Comparing Sets of Real Numbers
Practice Activity: PHM-09-013

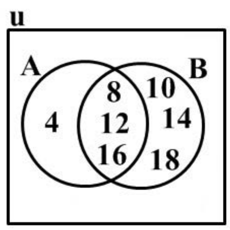
1. a. See below; b. $P \cap Q = \{12, 24\}$; c. $P \cup Q = \{6, 12, 18, 24, 36, 48\}$; d. $P^C = \{36, 48\}$, $Q^C = \{6, 18\}$



2. a. $R = \{1, 2, 3, 4, 6, 12\}$, $S = \{2, 4, 6, 8, 10, 12\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$; b. See below; c. $R \cap S = \{2, 4, 6, 12\}$; d. $R \cup S = \{1, 2, 3, 4, 6, 8, 10, 12\}$; e. $R^C = \{5, 7, 8, 9, 10, 11\}$



3. a. $A = \{4, 8, 12, 16\}$, $B = \{8, 10, 12, 14, 16, 18\}$; b. See below; c. $A \cap B = \{8, 12, 16\}$; d. $A \cup B = \{4, 8, 10, 12, 14, 16, 18\}$; e. $B^C = \{4\}$



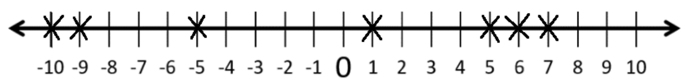
4. a. A = {1, 2, 3, 4, 9}, B = {1, 4, 5, 7, 8}; b. A ∩ B = {1, 4}; c. A ∪ B = {1, 2, 3, 4, 5, 7, 8, 9}; d. A^C = {5, 7, 8}, B^C = {2, 3, 9}

Lesson Title:	Ordering Sets of Real Numbers
Practice Activity:	PHM-09-014

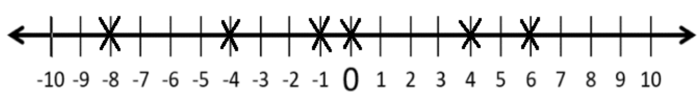
1. a. $\{-12, -8, -5, 0, 2, 4, 7, 13\}$; b. $\{\frac{1}{10}, \frac{1}{5}, \frac{3}{10}, \frac{2}{5}, \frac{4}{5}\}$; c. $\{-5.05, -5.0, -0.5, 0.5, 5.05, 5.5\}$; d. $\{-0.4, -\frac{1}{5}, 0.2, \frac{3}{5}, 0.8\}$
2. a. $\{27, 15, 14, 6, -4, -12, -18\}$; b. $\{\frac{5}{6}, \frac{4}{5}, \frac{2}{5}, \frac{1}{5}, \frac{1}{6}\}$; c. $\{3.5, 3.1, 3.05, 2.5, -2.05, -2.2, -2.5, -3\}$; d. $\{1, \frac{3}{4}, \frac{1}{2}, 0.25, -\frac{1}{4}, -0.5\}$

Lesson Title:	Real Numbers on a Number Line
Practice Activity:	PHM-09-015

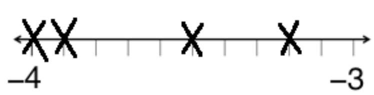
1. Number line:



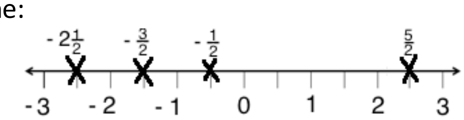
2. Number line:



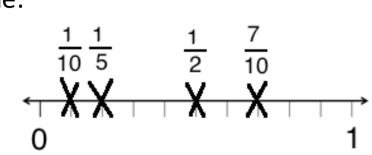
3. Number line:



4. Number line:



5. Number line:



Lesson Title: The Roman Numeral System	
Practice Activity: PHM-09-016	

1. Mark's phone number:

4	3	2	9	8	8	5	7	6	9	5
IV	III	II	IX	VIII	VIII	V	VII	VI	IX	V

2. a. XIII; b. XV; c. XIX; d. XVI

Lesson Title: Converting between Base 10 and Roman Numerals	
Practice Activity: PHM-09-017	

1. a. XIX; b. XXIX; c. LV; d. LXXVIII; e. LXXXIII
2. a. 94; b. 14; c. 63; d. 49; e. 38

Lesson Title: Introduction to Base 2	
Practice Activity: PHM-09-018	

1. Completed table:

16	***8***	***4***	***2***	***1***		
1	*0*	*0*	*0*	*0*	=	***16***
1	*0*	*0*	*0*	*1*	=	***17***
1	*0*	*0*	*1*	*0*	=	***18***
1	*0*	*0*	*1*	*1*	=	***19***
1	*0*	*1*	*0*	*0*	=	***20***

2. a. 14; b. 12; c. 19; d. 5; e. 7

Lesson Title: Ordering and Comparing Numbers in Base 2	
Practice Activity: PHM-09-019	

1. a. 101 < 1000; b. 1110 < 11011; c. 1011 < 1100; d. 11101 > 11001
2. a. {1, 10, 11, 100, 101, 110, 111}; b. {1011, 1100, 1101}; c. {10011, 10101, 10111, 11011, 11111}; d. {101, 110, 111, 1010, 1011}

3. a. { 111; 101; 100}, b. { 11101; 10111; 1101; 1011}; c. { 111; 110; 101; 11; 10}; d. { 11011; 10101; 10000; 1110; 1011; 1000}

Lesson Title: Converting between Base 10 and Base 2
Practice Activity: PHM-09-020

1. a. 110001_{two}; b. 100000_{two}; c. 10101_{two}; d. 101011_{two}; e. 100100_{two}
2. a. 41_{ten}; b. 31_{ten}; c. 33_{ten}; d. 27_{ten}; e. 45_{ten}

Lesson Title: Capacity and Mass
Practice Activity: PHM-09-021

1. a. 7 kg; b. 7,000 g
2. 200 ml
3. a. 1,500 ml; b. 1.5 l
4. 1.3 l
5. 2 kg

Lesson Title: Percentages of Quantities
Practice Activity: PHM-09-022

1. 480
2. 50
3. Le 960.00
4. 42 mangoes
5. 15 oranges
6. a. 420 children; b. 1,080 adults
7. a. He sold 125 newspapers; b. 375 newspapers remain
8. 64
9. Le 33,000.00
10. a. Le 80,000.00 b. Le 48,000.00

Lesson Title: Percentage Increase and Decrease
Practice Activity: PHM-09-023

1. 15%
2. 5%
3. 50%
4. 7.5%
5. 5%
6. Le 66,500.00

7. 9.2 seconds
8. 104 cm
9. 598 pupils
10. Le 576,000.00

Lesson Title: Ratios
Practice Activity: PHM-09-024

1. $\frac{4}{7}$
2. $\frac{19}{20}$
3. $\frac{3}{2}$
4. 48 bananas and 60 bananas
5. 50 grammes and 150 grammes
6. 60 exercise books and 90 exercise books

Lesson Title: Rates
Practice Activity: PHM-09-025

1. 30 km/hr
2. 5 km/hr
3. 2.5 cars/day
4. 2 minutes/problem
5. 3 kg/hr
6. 1,700 cartons/hour
7. Le 2,520,000.00
8. Le 2,122,480.00

Lesson Title: Direct Proportions
Practice Activity: PHM-09-026

1. $\frac{2}{5}$
2. $y = 8$
3. 60 minutes, or 1 hour
4. 10 pieces of chalk
5. 14 bottles of fertiliser
6. 96 minutes, or 1 hour and 36 minutes
7. 9 cups of rice

Lesson Title: Indirect Proportions
Practice Activity: PHM-09-027

1. a. $a = 5$; b. $b = 6$; c. $c = 2$

2. 120 workers
3. 3 days
4. 25 days
5. 3 plums each

Lesson Title: Proportion Problem Solving
Practice Activity: PHM-09-028

1. Le 9,100,000.00
2. Le 72,000.00
3. 1 more pot of rice
4. They need 1 more tailor
5. a. 60 girls; b. 27 minutes

Lesson Title: Financial Literacy I
Practice Activity: PHM-09-029

1. Le 862,500.00
2. Le 810,000.00
3. Le 325,000.00
4. a. Le 1,800,000.00; b. Le 265,000.00
5. a. Le 1,040,000.00; b. Le 83,000.00; c. Le 957,000.00

Lesson Title: Financial Literacy II
Practice Activity: PHM-09-030

1. Le 270,000.00
2. Le 140,000.00
3. Le 12,000.00
4. 4%
5. 5%
6. 2.5 years

Lesson Title: Index Notation and the Laws of Indices
Practice Activity: PHM-09-031

15. 11^{13}
16. 10^5
17. 9^{23}
18. 1
19. b^8
20. 1
21. 5^6
22. a^9

23. 3^9
24. 1
25. 1
26. 9^{21}
27. $25x^2$
28. $\frac{1}{8^{12}}$

Lesson Title: Application of the Laws of Indices
Practice Activity: PHM-09-032

12. 5^2
13. 3^{48}
14. 2^{60}
15. $\frac{1}{5\times 3^4}$
16. $\frac{b}{2^3}$
17. 2^{61}
18. $\frac{c^4}{b^4}$
19. $8a^{10}b^7$
20. $14b^{11}c^4$

Lesson Title: Indices with Negative Powers
Practice Activity: PHM-09-033

12. $\frac{1}{10^9}$
13. $\frac{7}{3^3}$
14. 2^2
15. $\frac{1}{5^8}$
16. 10^5
17. $\frac{1}{21^{18}}$
18. $\frac{1}{3^2}$
19. $\frac{5}{2^2}$
20. $\frac{1}{7^{27}}$
21. $\frac{1}{2^{13}}$
22. 3^8

Lesson Title: Indices with Fractional Powers
Practice Activity: PHM-09-034

5. a. $43^{\frac{1}{2}}$; b. $17^{\frac{1}{3}}$; c. $11^{\frac{1}{4}}$
6. a. 2; b. 8; c. 10

7. a. 2; b. 3; c. $5^{\frac{1}{4}}$ or $\sqrt[4]{5}$; d. $2^{\frac{1}{2}}$ or $\sqrt{2}$
8. a. 2^5; b. $3^4 \times 5^5$

Lesson Title: Multiplying and Dividing Indices with Fractional Powers
Practice Activity: PHM-09-035

4. x^3y^2
5. x^4
6. $\frac{1}{a^3}$
7. $1\frac{1}{5}$
8. $\frac{1}{a^{12}}$
9. $\frac{c^3}{a^6b^4}$
10. $\frac{1}{\sqrt{x}}$
11. $\frac{1}{2^5}$

Lesson Title: Multiplying and Dividing by Powers of 10
Practice Activity: PHM-09-036

1. 567
2. 0.2546
3. 48.7
4. 23,100
5. 240
6. 0.009
7. 2.309
8. 187.1
9. 0.24598
10. 2,600

Lesson Title: Standard Form of Large Numbers
Practice Activity: PHM-09-037

1. a. 3.2017×10^3; b. 9×10^4; c. 2.134×10^3; d. 5.4×10; e. 9.98956×10^7
2. a. 67,500; b. 1,900,000; c. 999; d. 171.5; e. 8,000
3. 3.02×10^4
4. 1.125×10^4

Lesson Title: Standard Form of Small Numbers
Practice Activity: PHM-09-038

1. a. 5.13×10^{-6}; b. 2×10^{-7}; c. 7.18×10^{-1}; d. 5.1×10^{-3}; e. 1.09×10^{-5}

2. a. 0.02752; b. 0.00451; c. 0.000007; d. 0.2541; e. 0.000105
3. 5.6×10^{-3}
4. 1.39×10^{-3}

Lesson Title: Conversion to and from Standard Form
Practice Activity: PHM-09-039

1. 2.9×10^{-2}
2. 0.00000364
3. Completed table:

Country	Population	Ordinary Form	Standard Form
China	1.38 billion	1,380,000,000	1.38×10^{9}
India	1.297 billion	1,297,000,000	1.297×10^{9}
United States	329 million	329,000,000	3.29×10^{8}
Indonesia	263 million	263,000,000	2.63×10^{8}
Brazil	208.8 million	208,800,000	2.088×10^{8}

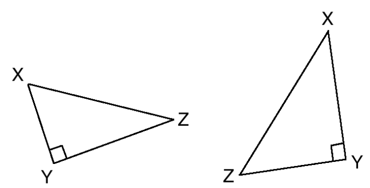
4. a. 6.78×10^{5}; b. 8.193×10^{-4}; c. 9.451×10^{8}; d. 2.75×10^{-3}
5. a. 0.000000006; b. 0.00555; c. $80{,}050{,}000$; d. $69{,}000{,}000{,}000$

Lesson Title: Multiplying and Dividing Small and Large Numbers
Practice Activity: PHM-09-040

1. 2.28×10^{12}
2. 8.952×10^{-4}
3. 3×10^{-1}
4. 3.32×10^{1} or 3.32×10
5. 4.1×10^{11}

Lesson Title: Right-angled Triangles (Revision)
Practice Activity: PHM-09-041

1. a. BC; b. WX; c. QS; d. AC
2. b, c
3. Example answers:



Lesson Title: Introduction to Pythagoras' Theorem
Practice Activity: PHM-09-042

1. Substitute the given values for each triangle into Pythagoras' theorem and verify that LHS = RHS. This is true for both triangles a and b, which are actually right-angled triangles.
2. a. Right-angled triangle; b. Not a right-angled triangle

Lesson Title: Finding the Hypotenuse of a Right-Angled Triangle
Practice Activity: PHM-09-043

1. a. $z = 15$ m; b. $y = 29$ m; c. $x = 30$ cm; d. $p = 20$ m
2. PS = 17 m

Lesson Title: Finding the Other Sides of a Right-Angled Triangle
Practice Activity: PHM-09-044

1. a. $x = 12$ cm; b. $x = 7$ cm; c. $x = 21$ cm
2. $QS = 25$

Lesson Title: Applying Pythagoras' Theorem
Practice Activity: PHM-09-045

1. 5.3 cm
2. 7.8 m
3. 6.9 cm
4. 30 cm
5. 17 m

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