Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' handbook for

JSS Mathematics

JSS 2

Term 1

STRICTLY NOT FOR SALE

DOI: 10.5281/zenodo.3745218
Please see final page for further information.

FOREWORD

The production of Teachers' Guides and Pupils' handbooks in respect of English and Mathematics for Junior Secondary Schools (JSSs) in Sierra Leone is an innovation. This would undoubtedly lead to improvement in the performance of pupils in the Basic Education Certificate Examination in these subjects. As Minister of Basic and Senior Secondary Education, I am pleased with this development in the educational sector.

The Teachers' Guides give teachers the support they need to utilize appropriate pedagogical skills to teach; and the Pupils' Handbooks are designed to support self-study by the pupils, and to give them additional opportunities to learn independently.

These Teachers' Guides and Pupils' Handbooks had been written by experienced Sierra Leonean and international educators. They have been reviewed by officials of my Ministry to ensure that they meet specific needs of the Sierra Leonean population.

I call on the teachers and pupils across the country to make the best use of these educational resources.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd. Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank the Department for International Development (DFID) for their continued support. Finally, I also thank the teachers of our country - for their hard work in securing our future.

Timbo

Mr. Alpha Osman Timbo
Minister of Basic and Senior Secondary Education

The Ministry of Basic and Senior Secondary Education, Sierra Leone, policy stipulates that every printed book should have a lifespan of 3 years.

To achieve this DO NOT WRITE IN THE BOOKS.

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Introduction
to the Pupils' Handbook

These practice activities are aligned to the lesson plans in the Teachers' Guide, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Education, Science and Technology.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.

   Learning Outcomes
5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

Lesson Title: Converting Between Mixed and Improper Fractions	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-001	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Express mixed numbers as improper fractions.
2. Express improper fractions as mixed numbers.

Overview

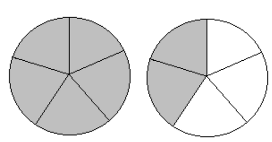
A **mixed number** is a whole number and a fractional part in which the numerator is less than the denominator. For example, $1\frac{2}{5}$ is a mixed number. An **improper fraction** is a fraction in which the numerator is bigger than the denominator. For example, $\frac{7}{5}$ is an improper fraction.

Mixed numbers and improper fractions are greater than 1. A **proper fraction** is one with the numerator less than the denominator. Proper fractions are always less than 1.
We can convert mixed numbers to improper fractions, and improper fractions to mixed numbers.

An improper fraction can be converted to a mixed number. To convert $\frac{7}{5}$ to a mixed number, write it as 2 fractions. In the first fraction, the numerator must be evenly divisible by the denominator. The second fraction should be a proper fraction. Simplify the first fraction so that it is a whole number. For this example, we have $\frac{7}{5}=\frac{5+2}{5}=\frac{5}{5}+\frac{2}{5}=1\frac{2}{5}$.

Another way to convert $\frac{7}{5}$ to a mixed number is by dividing. Remember that $\frac{7}{5}$ is the same as $7 \div 5$. Divide: $7 \div 5 = 1$ remainder 2. The number 1 gives the whole number, and the remainder is the numerator of the fraction part. The mixed fraction is $1\frac{2}{5}$.

To change a mixed fraction to an improper fraction, multiply the whole number part by the denominator, and add the numerator. Write this in the numerator of the result. Use the same denominator as in the original fraction. For example: $1\frac{2}{5}=\frac{1\times5+2}{5}=\frac{7}{5}$.
We can show $\frac{7}{5}=1\frac{2}{5}$ with a picture:



Simplify your answers if possible.

Solved Examples

1. Convert $\frac{3}{2}$ to a mixed number.

   Solution

   Method 1. $\frac{3}{2}$ is the same as $3 \div 2$. Divide, and find $3 \div 2 = 1$ remainder 1.

   This gives $\frac{3}{2} = 1\frac{1}{2}$.

   Method 2. Write $\frac{3}{2}$ as 2 fractions: $\frac{3}{2} = \frac{2+1}{2} = \frac{2}{2} + \frac{1}{2} = 1\frac{1}{2}$

2. Convert the following to mixed numbers:

   a. $\frac{5}{4}$ b. $\frac{21}{5}$ c. $\frac{32}{10}$

   Solutions

   The first method is shown for solving each problem.

   a. $\frac{5}{4} = \frac{4+1}{4} = \frac{4}{4} + \frac{1}{4} = 1\frac{1}{4}$

   b. $\frac{21}{5} = \frac{20}{5} + \frac{1}{5} = 4\frac{1}{5}$

   c. $\frac{32}{10} = \frac{30}{10} + \frac{2}{10} = 3\frac{2}{10} = 3\frac{1}{5}$ Remember to simplify the fraction part: $\frac{2}{10} = \frac{1}{5}$

3. Convert the following to mixed numbers:

   a. $\frac{27}{4}$ b. $\frac{7}{6}$ c. $\frac{16}{3}$

   Solutions

   The second method is shown for solving each problem.

   a. $\frac{27}{4} = 27 \div 4 = 6$ remainder 3, which gives $6\frac{3}{4}$

   b. $\frac{7}{6} = 7 \div 6 = 1$ remainder 1, which gives $1\frac{1}{6}$

   c. $\frac{16}{3} = 16 \div 3 = 5$ remainder 1, which gives $5\frac{1}{3}$

4. Convert $2\frac{1}{8}$ to an improper fraction.

   Solution

   Multiply the whole number part by the denominator, and add the numerator. Write this in the numerator of the result: $2\frac{1}{8} = \frac{2\times8+1}{8} = \frac{16+1}{8} = \frac{17}{8}$

5. Convert the following mixed numbers to improper fractions:

   a. $6\frac{3}{4}$ b. $1\frac{3}{20}$ c. $12\frac{2}{3}$

   Solutions

   a. $6\frac{3}{4} = \frac{6\times4+3}{4} = \frac{24+3}{4} = \frac{27}{4}$

   b. $1\frac{3}{20} = \frac{1\times20+3}{20} = \frac{20+3}{20} = \frac{23}{20}$

   c. $12\frac{2}{3} = \frac{12\times3+2}{3} = \frac{36+2}{3} = \frac{38}{3}$

Practice

1. Convert the following improper fractions to mixed numbers:

   a. $\frac{23}{3}$ b. $\frac{20}{6}$ c. $\frac{9}{8}$ d. $\frac{8}{3}$

2. Convert the following mixed numbers to improper fractions:

   a. $2\frac{1}{8}$ b. $8\frac{3}{4}$ c. $11\frac{2}{5}$ d. $1\frac{7}{40}$

Lesson Title: Converting Decimals to Fractions	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-002	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to express decimals as fractions.

Overview

Decimal numbers are sometimes called fractional numbers because they can be easily expressed as fractions. To change decimals to fractions, we look at the number of decimal places or the numbers after the point. We write the decimal digits over a power of 10, and simplify. Follow these rules to choose the power of 10 for the denominator:

- If there is 1 decimal place, then the number is expressed over 10.
- If there are 2 decimal places, then the number is expressed over 100.
- If there are more decimal places, the number is expressed over 1,000 and so on.

If the decimal has a whole number part, keep the whole number and change the decimal numbers to a fraction. For example, $2.3 = 2\frac{3}{10}$.
Simplify your answers if possible.

Solved Examples

1. Convert 0.25 to a fraction.

   Solution
   There are 2 decimal places, so express the digits (25) over 100. Then, simplify.

$$0.25 = \frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

2. Convert 0.025 to a fraction.

   Solution
   There are 3 decimal places, so express the digits (025) over 1000. Then, simplify.

$$0.025 = \frac{025}{1,000} = \frac{25}{1,000} = \frac{1}{40}$$

3. Convert 2.8 to a fraction.

   Solution
   Keep the whole number (2) and change the decimal numbers (0.8) to a fraction.

$$2.8 = 2\frac{8}{10} = 2\frac{4}{5}$$

4. Convert the following numbers to fractions:
    a. 0.6
    b. 1.35
    c. 2.302

    Solutions

    a. $0.6 = \frac{6}{10} = \frac{3}{5}$
    b. $1.35 = 1\frac{35}{100} = 1\frac{7}{20}$
    c. $2.302 = 2\frac{302}{1{,}000} = 2\frac{151}{500}$

Practice

Convert the decimals to fractions:

1. 0.45
2. 5.26
3. 0.005
4. 10.05
5. 25.25
6. 9.7
7. 0.08
8. 3.30

Lesson Title: Converting Fractions to Decimals	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-003	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to express fractions as decimals.

Overview

If the denominator of a fraction is a power of 10 such as 10, 100, or 1000, we can easily change the fraction to a decimal. We follow rules that are the opposite of the rules in the previous lesson:

- If the fraction is over 10, the decimal number has 1 decimal place. Example: $\frac{3}{10} = 0.3$
- If the fraction is over 100, the decimal number has 2 decimal places. Example: $\frac{17}{100} = 0.17$
- If the fraction is over 1000, the decimal number has 3 decimal places. Example: $\frac{4}{1,000} = 0.004$

To express any other fraction as a decimal, divide the numerator by the denominator. You will do long division. Make sure you write a decimal number after the numerator in the long division problem. You will place a decimal number directly above it in the answer.

For example, this long division shows $\frac{1}{5} = 0.2$. Look at the where the decimal is placed. Be very careful to write the decimal in the correct place in your problems and answers.

$$\begin{array}{r|l} & 0.\ 2 \\ \hline 5 & 1.\ 0 \\ & -1\ 0 \\ \hline & 0 \end{array}$$

Solved Examples

1. Convert the following fractions to decimals:
   a. $\frac{7}{10}$
   b. $\frac{3}{100}$
   c. $8\frac{17}{100}$
   d. $\frac{312}{1000}$
   e. $3\frac{13}{1,000}$

Solutions

Follow the rules from the Overview to decide how many decimal places to give each answer:

a. $\frac{7}{10} = 0.7$

b. $\frac{3}{100} = 0.03$

c. $8\frac{17}{100} = \frac{817}{100} = 8.17$

d. $\frac{312}{1,000} = 0.312$

e. $3\frac{13}{1,000} = \frac{3013}{1,000} = 3.013$

2. Convert $\frac{1}{2}$ to a decimal.

Solution

Divide the numerator (1) by the denominator (2).

$$\begin{array}{r|ll} & 0. & 5 \\ \hline 2 & 1. & 0 \\ - & 1 & 0 \\ \hline & & 0 \end{array}$$

Answer: $\frac{1}{2} = 0.5$

3. Convert $\frac{7}{8}$ to a decimal.

Solution

Divide the numerator (7) by the denominator (8).

$$\begin{array}{r|llll} & 0. & 8 & 7 & 5 \\ \hline 8 & 7. & 0 & 0 & 0 \\ - & 6 & 4 & & \\ \hline & & 6 & 0 & \\ & - & 5 & 6 & \\ \hline & & & 4 & 0 \\ & & - & 4 & 0 \\ \hline & & & & 0 \end{array}$$

Answer: $\frac{7}{8} = 0.875$

4. Convert $\frac{7}{20}$ to a decimal.

Solution

Divide the numerator (7) by the denominator (20).

$$
\begin{array}{r|lll}
 & 0. & 3 & 5 \\ \cline{2-4}
20 & 7. & 0 & 0 \\
- & 6 & 0 & \\ \hline
 & 1 & 0 & 0 \\
- & 1 & 0 & 0 \\ \cline{2-4}
 & & & 0
\end{array}
$$

5. Convert $1\frac{1}{2}$ to a decimal.

Solution

Step 1. Convert the mixed numbers to improper fraction: $1\frac{1}{2} = \frac{1\times2+1}{2} = \frac{3}{2}$

Step 2. Divide:

$$
\begin{array}{r|ll}
 & 1. & 5 \\ \cline{2-3}
2 & 3. & 0 \\
- & 2 & 0 \\ \hline
 & 1 & 0 \\
- & 1 & 0 \\ \cline{2-3}
 & & 0
\end{array}
$$

Answer: $1\frac{1}{2} = 1.5$

Practice

Convert the fractions to decimals:

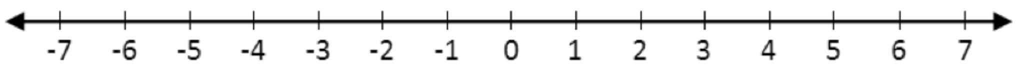
1. $\frac{7}{10}$
2. $\frac{31}{1{,}000}$
3. $1\frac{99}{100}$
4. $\frac{3}{5}$
5. $\frac{3}{8}$
6. $\frac{9}{20}$
7. $4\frac{1}{2}$
8. $\frac{1}{16}$

Lesson Title: Comparing and Ordering a Mixture of Numbers	Theme: Numbers and Numeration
Practice Activity: PHM-08-004	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to order and compare integers, decimals and fractions using a number line.

Overview

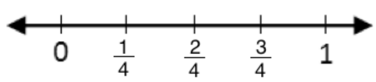
This lesson is on comparing and ordering different types of numbers on a number line. A number line is a way to see the relationship between numbers. Here is what a number line looks like:



This number line shows negative and positive numbers between -7 and 7. The arrows on the ends of the number line show that it goes forever in both directions. Number lines do not end. Number lines always increase, or go up, from left to right. This means that a number to the right is greater than any number to its left.

Number lines include marks to show where the numbers are on the line. These marks are all the same distance apart.

Number lines can show different ranges of numbers by using a different **scale**. For example, we could create a number line that shows certain fractions from 0 to 1 like the one below. This number line has a scale of $\frac{1}{4}$.



Number lines can show any real number, including integers, fractions or decimals.

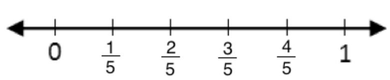
Solved Examples

1. Create a number line that shows the fractions in fifths from 0 to 1.

   Solution

   First, recall all of the fractions in fifths from 0 to 1. 0 is equal to $\frac{0}{5}$, and 1 is equal to $\frac{5}{5}$. The fractions in between $\frac{0}{5}$ and $\frac{5}{5}$ are $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}$, and $\frac{4}{5}$.

   Your number line should show the fractions for fifths an equal distance apart, like this:

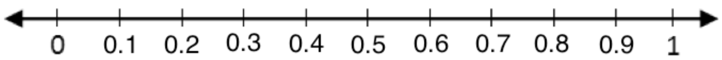


2. Create a number line that illustrates these numbers:

0.1, 0.2, 0.4, 0.8, 0.9, 0.3, 0.5, 0.6, 0.7, 0, 1

Solution
First, write the numbers in order from least to greatest: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1

Note that the decimal numbers in this list are all tenths between 0 and 1. Your number line should show the decimal numbers an equal distance apart, like this:

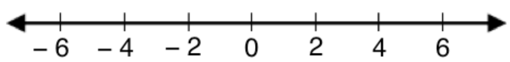


3. Create a number line that shows even integers from -6 to 6.

Solution
Remember that even numbers can be divided by 2. Positive even numbers are 2, 4, 6,…. Negative numbers can also be even. Negative even numbers are -2, -4, -6,….

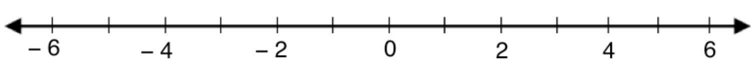
When writing these on a number line, remember that negative numbers are less than positive numbers. Also, the greater the absolute value of the negative number, the less it actually is. For example, $-6 < -4$. Remember that numbers increase to the right.

The number line will look like this:



All of the even numbers are the same distance apart. Although the odd numbers are actually in between each of the even numbers, it is not necessary to show them on the number line.

You may draw your number line like this, and it would also be correct. The odd numbers are shown with marks:



Practice

1. Create a number line that shows even integers from -12 to 2.
2. Create a number line that shows the decimals in tenths from 1 to 2.
3. Create a number line that shows the fractions in thirds from 0 to 1.
4. Create a number line that shows the fractions in thirds from -1 to 0.

Lesson Title: Locating a Mixture of Numbers on the Number Line	Theme: Numbers and Numeration
Practice Activity: PHM-08-005	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to locate integers, decimals and fractions on the number line.

Overview

In this lesson, you will identify numbers on the number line that are not labelled. Finding the value of a number shown on a number line is easy if you follow these steps:

Step 1. Find 2 numbers so that your mystery number is between them.
Step 2. Find out what scale is being used in the number line.
Step 3. Write the value of the mystery number on the number line.

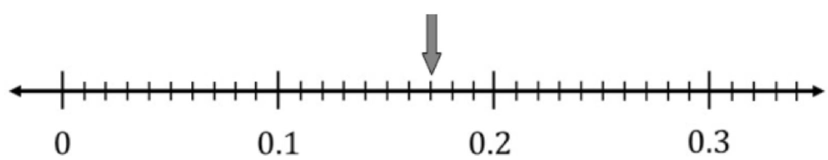
Solved Examples

1. Identify the number shown with the arrow:

Solution

Step 1. The mystery number is between 2 and 3.
Step 2. The scale being used is fifths. Each unit is divided into 5 equal parts.
Step 3. The value of the mystery number is $2\frac{1}{5}$.

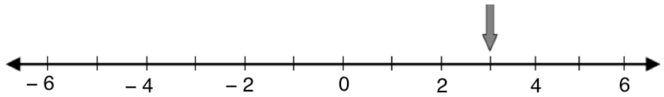
2. Identify the number shown with the arrow:



Solution

Step 1. The mystery number is between 0.1 and 0.2.
Step 2. The scale being used is hundredths. The labelled numbers are tenths: 0.1, 0.2, 0.3. Each tenth is divided into 10 equal parts, which are hundredths.
Step 3. The value of the mystery number is 0.17.

3. Identify the number shown with the arrow:



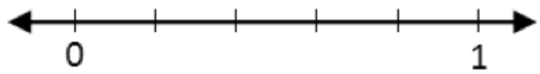
Solution

Step 1. The mystery number is between 2 and 4.

Step 2. The scale being used is ones, but only the even numbers are labelled. Each marked number is an integer.

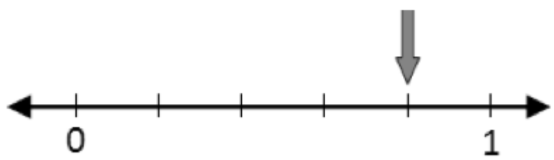
Step 3. The value of the mystery number is 3.

4. On the number line below, draw an arrow at $\frac{4}{5}$:

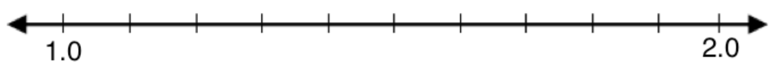


Solution

The number line is between 0 and 1. The scale being used is fifths. Count up to 4 fifths, and label the number line with an arrow:

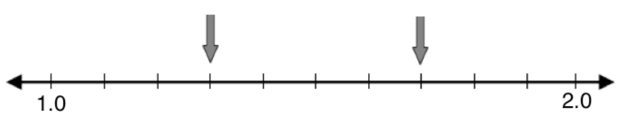


5. On the number line below, draw arrows at $1\frac{3}{10}$ and 1.7:



Solution

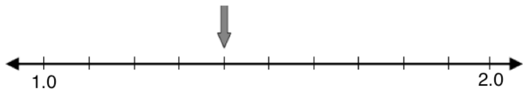
The number line is between 1 and 2. The scale being used is tenths. Remember that the fraction $\frac{3}{10}$ shows tenths, and the decimal 0.7 also shows tenths. Count up starting from 1.0 to $1\frac{3}{10}$ and 1.7, and draw the arrows:

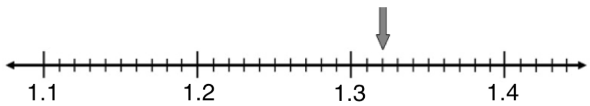


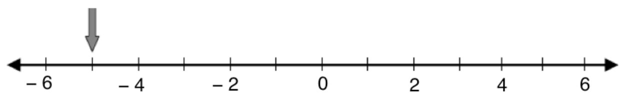
Practice

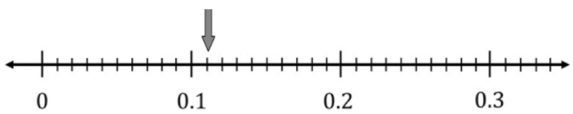
1. Identify each number shown on the number lines:

a. 

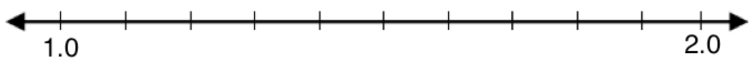
b. 

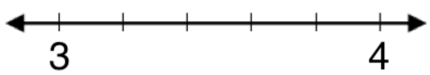
c. 

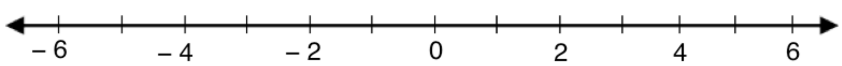
d. 

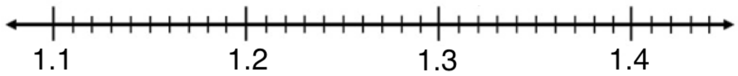
e. 

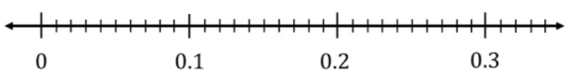
2. Draw an arrow to show each number on the given number line:

a. 1.2 

b. $3\frac{2}{5}$ 

c. −3 

d. 1.23 

e. 0.27 

Lesson Title: Classification of Decimal Numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-006	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify terminating decimals.
2. Identify recurring decimals.

Overview

This lesson covers two types of decimal numbers: terminating and recurring. **Terminate** means to stop or end, and **recur** means to repeat.

A **terminating** decimal is a decimal that has an end. In other words, terminating decimals have a finite number of digits. For example, consider $\frac{3}{4}$. We can convert $\frac{3}{4}$ to a decimal by dividing:

	0.	7	5
4	3.	0	
-	2	8	
		2	0
	-	2	0
			0

Note that in this example the division works out exactly and the decimal comes to an end (terminates).

In some cases, the division does not stop, and the decimal does not terminate. The same digit or group of digits keeps repeating infinitely. We call such decimals **recurring decimals**. For example, consider $\frac{2}{9}$:

	0.	2	2	2
9	2.	0	0	0
-	1	8		
		2	0	
	-	1	8	
			2	0
		-	1	8
				2

In this example, you will notice that the 2 will never stop repeating. Repeating numbers are shown with a dot or line over the repeating number, for example: $0.\dot{2}$ or $0.\overline{2}$. This notation is used to show that the digit 2 repeats forever. Repeating decimals can also be shown with 3 dots: $0.\overline{2} = 0.2222\ldots$

Solved Examples

1. Write the following decimal numbers in their shortened notation. For example, $0.3333\ldots = 0.\overline{3}$.
   a. $0.666666\ldots$
   b. $3.25252525\ldots$
   c. $0.123123123123\ldots$
   d. $1.5454545454\ldots$

   Solutions

   All of the repeating digits should be under the bar. For example, if 2 digits repeat, they are both under the bar.
   a. $0.666666\ldots = 0.\overline{6}$
   b. $3.25252525\ldots = 3.\overline{25}$
   c. $0.123123123123\ldots = 0.\overline{123}$
   d. $1.5454545454\ldots = 1.\overline{54}$

2. Determine whether the following decimal numbers are recurring or terminating:
   a. 0.125 b. $6.\overline{4}$ c. 99.99 d. $8.888\ldots$

   Solutions

   Remember that a bar or 3 dots (…) show a recurring decimal. Other decimal numbers are terminating.
   a. Terminating
   b. Recurring
   c. Terminating
   d. Recurring

3. Determine whether the following fractions are equal to, recurring, or terminating decimals:
   a. $\frac{7}{100}$ b. $\frac{2}{3}$ c. $\frac{1}{6}$ d. $\frac{1}{5}$

Solutions

Convert each fraction to a decimal number, then decide if it is recurring or terminating.

a. Fractions over 100 can easily be written as decimals: $\frac{7}{100} = 0.07$

   This is a **terminating** decimal.

b. Divide to convert $\frac{2}{3}$ to a decimal:

$$\begin{array}{cllll} & 0. & 6 & 6 \\ 3 & \big| \; 2. & 0 & 0 \\ - & 1 & 8 & \\ \hline & & 2 & 0 \\ & - & 1 & 8 \\ \hline & & & 2 \end{array}$$

   This pattern will continue forever, giving $\frac{2}{3} = 0.666 \ldots$. This is a **recurring** decimal.

c. Divide to convert $\frac{1}{6}$ to a decimal:

$$\begin{array}{cllll} & 0. & 1 & 6 & 6 \\ 6 & \big| \; 1. & 0 & 0 & 0 \\ - & & 6 & & \\ \hline & & 4 & 0 & \\ & - & 3 & 6 & \\ \hline & & & 4 & 0 \end{array}$$

   This pattern will continue forever, giving $\frac{1}{6} = 0.1666 \ldots$. This is a **recurring** decimal. It can be written as $0.1\overline{6}$

d. Divide to convert $\frac{1}{5}$ to a decimal:

$$\begin{array}{clll} & 0. & 2 \\ 5 & \big| \; 1 & 0 \\ - & 1 & 0 \\ \hline & & 0 \end{array}$$

   The decimal stops at 2. This gives $\frac{1}{5} = 0.2$, a **terminating** decimal.

Practice

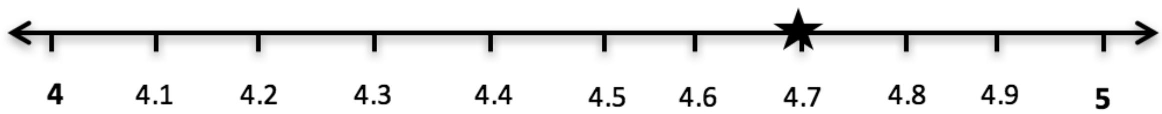
1. Write the following decimal numbers in their shortened notation:
   a. $9.888888\ldots$
   b. $3.12121212\ldots$
   c. $123.456456456456\ldots$
   d. $8.68686868\ldots$
2. Determine whether the following decimal numbers are recurring or terminating:
   a. $9.\overline{9}$ b. 3.8261 c. $2.999\ldots$ d. $0.\overline{05}$
3. Determine whether the following fractions are equal to recurring or terminating decimals:
   a. $\frac{9}{10}$ b. $\frac{2}{11}$ c. $\frac{4}{9}$ d. $\frac{1}{8}$

Lesson Title: Rounding off Decimal Numbers to the Nearest Whole	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-007	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to round decimal numbers to the nearest whole number.

Overview

To round a decimal number to a whole number, we must find the whole number that it is nearest to. For example, consider 4.7, which is between whole numbers 4 and 5. We can show this on a number line:



Since 4.7 is closer to the whole number 5**,** 4.7 rounds up to 5. The decimal 4.7 is rounded up to 5 by adding 1 onto the ones digit (4) and removing the decimal point.

We do not need to draw a number line to round numbers. Use these 3 rules for rounding decimals to whole numbers:

a. If the digit in the tenths' place is greater than 5, then round **up** to the next consecutive whole number. For example, 56.7 would round to 57.
b. If the digit in the tenths' place is less than 5, then leave the whole number the same. For example, 673.2 would round to 673.
c. If the digit in the tenths' place is equal to 5, then round **up** to the next consecutive whole number. For example, 1,326.5 would round to 1,327.

Solved Examples

1. Round 13.29 to the nearest whole number.

   Solution
   We only consider the digit after the decimal point, 2. This is less than 5, so we round down.
   Answer: 13

2. Round 412.5 to the nearest whole number.

   Solution
   Numbers 5 and greater tell us to round up. Add 1 to 412 to round up.
   Answer: 413

3. Round the following numbers to the nearest whole number:
    a. 20.3
    b. 59.9
    c. 1,000.82
    d. 0.28

    Solutions
    a. Round down: 20
    b. Round up: 60
    c. Round up: 1,001
    d. Round down: 0

4. Six farmers harvested their peppers and brought them to the market. They weighed their peppers and recorded the weight in the table below. Round each weight to the nearest kilogramme.

FARMER	WEIGHT (KG)	WEIGHT TO THE NEAREST KG.
Hawa	50.68	
Juliet	37.09	
Martin	18.389	
Abass	48.218	
Alice	30.9	
Mohamed	45.2	

Solutions

Round each decimal number up or down based on the digit after the decimal place. The answers are given below.

FARMER	WEIGHT (KG)	WEIGHT TO THE NEAREST KG.
Hawa	50.68	51
Juliet	37.09	37
Martin	18.389	18
Abass	48.218	48
Alice	30.9	31
Mohamed	45.2	45

Practice

1. Round the following numbers to the nearest whole number:
    a. 317.95
    b. 0.399
    c. 1.500
    d. 70.8
    e. 200.999

2. Six pupils measured the distance of their houses from school in kilometres. They recorded the distances in the table below. Round each distance to the nearest whole number.

PUPIL	DISTANCE (KM)	DISTANCE TO THE NEAREST KM.
Sia	3.8	
David	1.75	
Annette	0.3	
Yusuf	2.5	
Mary	2.189	
Foday	1.09	

Lesson Title: Rounding off Decimal Numbers to Stated Decimal Places	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-008	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to round decimal numbers to a given number of decimal places.

Overview

Use the same approach as in the previous lesson to round a decimal to a given number of decimal places. Remember that if a digit is less than 5, we round **down**. If a digit is 5 or more, we round **up**.

To round to a given decimal place, look at the digit after the decimal place you are rounding to. For example, consider 6.47. To round to 1 decimal place, look at the digit in the second decimal place (7). Seven is greater than 5, so we round up. Add 1 to the digit in the first decimal place. The number 6.47 rounded to 1 decimal place is 6.5.

Rounding to a given place value is the same as rounding to a given number of decimal places.

- A number rounded to the nearest **tenth** is the same as a number rounded to **1 decimal place.**
- A number rounded to the nearest **hundredth** is the same as a number rounded to **two decimal places**.
- A number rounded to the nearest **thousandth** is the same as a number rounded to **three decimal places.**

We can round to any number of decimal places!

Solved Examples

1. Round 21.2391 to:
    a. 1 decimal place
    b. 2 decimal places
    c. 3 decimal places

    Solutions
    a. The digit in the 2^{nd} decimal place is 3, so we round down: 21.2
    b. The digit in the 3^{rd} decimal place is 9, so we round up: 21.24
    c. The digit in the 4^{th} decimal place is 1, so we round down: 21.239

2. Round 4.97 to 1 decimal place.

   Solution
   The digit in the 2nd decimal place is 7, so we round up. When we add 1 to the 9 in the first decimal place, we get 10. This carries over. We add 1 to the whole number 4, and leave 0 in the first decimal place.

   Answer: 5.0

3. Round the following numbers to the nearest hundredth:
   a. 312.201
   b. 54.058
   c. 0.58291
   d. 1.2763

   Solutions
   Recall that the hundredths place is the second place after the decimal. We want to round to 2 decimal places.
   a. Round down: 312.20
   b. Round up: 54.06
   c. Round down: 0.58
   d. Round up: 1.28

4. Dr. Bangura delivered 5 babies today. She recorded their weights in the table below. Round each weight to 1 decimal place.

BABY	WEIGHT (KG)	WEIGHT (KG) TO 1 DECIMAL PLACE
1	3.125	
2	2.987	
3	2.45	
4	3.001	
5	2.78	

Solutions

Round each decimal number up or down based on the digit in the second decimal place. Write all of your answers in the table as shown.

BABY	WEIGHT (KG)	WEIGHT (KG) TO 1 DECIMAL PLACE
1	3.125	3.1
2	2.987	3.0
3	2.45	2.5
4	3.001	3.0
5	2.78	2.8

Practice

1. Round 2.1982 to 1 decimal place.
2. Express 1,787.421 to the nearest tenth.
3. Round 0.981 to the nearest tenth.
4. Round the following numbers to the nearest thousandth:
   a. 2.10481
   b. 0.59198
   c. 21.021021
   d. 9.090909
   e. 310.3579
5. Mustapha has a small shop. He runs his generator every evening. He recorded the amount of fuel that he used each day. Round each number to 1 decimal place.

DAY	FUEL USED (L)	FUEL (L) TO 1 DECIMAL PLACE
Monday	4.578	
Tuesday	3.45	
Wednesday	5.093	
Thursday	0.995	
Friday	3.72	

Lesson Title: Introduction to Significant Figures	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-009	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to identify significant figures in whole numbers and decimals.

Overview

A **significant figure** (sometimes also called a **significant digit**) is each of the digits of a number that are used to express it to the required degree of accuracy, starting with the first non-zero digit.

Rules for identifying significant figures (s.f.):

Rules	Examples	Number of s.f.
1. All non-zero digits are significant.	123	(3 s.f.)
2. All zeros between significant digits are significant.	12.507 304	(5 s.f.) (3 s.f.)
3. Zeros to the left of the first non-zero digit are not significant.	1.02 0.12 0.012	(3 s.f.) (2 s.f.) (2 s.f.)
4. If a number ends in zeros to the right of the decimal point, those zeros are significant.	2.0 2.00	(2 s.f.) (3 s.f.)
5. Zeros at the end of a whole number are not significant, unless there is a decimal point.	4300 4300.0	(2 s.f.) (5 s.f.)

Solved Examples

1. Identify how many significant figures are in each number:
   a. 2,100 b. 900 c. 201 d. 30.0

   Solutions

   Reasons are given for each answer below.

   a. 2 significant figures – Zeros at the end of a whole number are not significant.
   b. 1 significant figure – Zeros at the end of a whole number are not significant.
   c. 3 significant figures – Zeros between significant figures are significant.
   d. 3 significant figures – Zeros to the right of a decimal point are significant. They also make the zero in the whole number 30 significant.

2. How many significant figures are there in each number?
    a. 0.897 b. 89.001 c. 400.0 d. 0.7

    Solutions

    a. 3 significant figures – zeros to the left of the first non-zero digit are not significant.
    b. 5 significant figures – zeros to the right of a decimal point are significant.
    c. 4 significant figures – Zeros to the right of a decimal point are significant. They also make the zeros in the whole number 400 significant.
    d. 1 significant figure – zeros to the left of a decimal point are not significant.

Practice

Identify how many significant figures there are in each number:

1. 21.050
2. 6.0
3. 0.9800
4. 0.18
5. 10.01
6. 4,000
7. 69.010
8. 580
9. 0.04
10. 12.120

Lesson Title: Rounding off Decimal Numbers to Significant Figures	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-010	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to round decimal numbers to a given number of significant figures.

Overview

This lesson is on rounding to a given number of significant figures. Review the previous lesson and make sure you can identify how many significant figures are in a number.
Follow these steps to round to a stated number of significant figures:

1. Find the last significant figure you want to round to.
2. Look at the next significant figure (to the right).
3. If the next significant figure is less than 5, leave the last significant figure you want as it is. If the next significant figure is 5 or more, add 1 to the last significant figure you want.

Solved Examples

1. Round 287,540 to: a. 4 s.f. b. 3 s.f. c. 2 s.f.

   Solutions
   Note that the number 287,540 has 5 significant figures. All of the non-zero digits are significant.
   a. To round to 4 s.f., look at the first 4 s.f. in the number: 2875. The next digit is 4, so we round down: 287,500
   b. To round to 3 s.f., look at the first 3 s.f. in the number: 287. The next digit is 5, so we round up: 288,000
   c. To round to 2 s.f., look at the first 2 s.f. in the number: 28. The next digit is 7, so we round up: 290,000

2. Round 0.0397 to 2 s.f.

   Solution
   Note that the number 0.0397 has 3 significant figures, 397. Look at the first 2 s.f. in the number, 39. The next digit is 7, so we round up: 0.040

3. Approximate the following numbers to 3 s.f.:
   a. 0.032847 b. 4.29984 c. 31.2511

Solutions

Find the first 3 s.f. in each digit. Then, look at the 4th s.f. to determine whether to round up or down.

a. 0.0328
b. 4.30
c. 31.3

4. Correct each of these numbers to 2 s.f.:

a. 12,304 b. 23.104 c. 0.29467

Solutions

Find the first 2 s.f. in each digit. Then, look at the 3rd s.f. to determine whether to round up or down.

a. 12,000
b. 23
c. 0.29

5. Of all of the Eastern Provinces, Kenema has the highest population. The population was 609,873 people according to the 2015 census. Find the population of Kenema correct to:

a. 2 s.f.
b. 3 s.f.
c. 4 s.f.
d. 5 s.f.

Solutions

For each problem, find the digit you are being asked to round to. Look at the next digit and decide to round up or round down.

a. 610,000
b. 610,000 (the 3rd s.f. is 9, which rounds up to 10)
c. 609,900
d. 609,870

Practice

1. Round 587,257 to: a. 4 s.f. b. 3 s.f. c. 2 s.f.
2. Round 0.04662578 to 5 s.f.
3. Approximate the following numbers to 2 s.f.:
    a. 0.032847 b. 4.29984 c. 31.2511
4. Correct each of these numbers to 1 s.f.:
    a. 23,000 b. 539 c. 0.583
5. Mrs. Bangura owns a shop where she sells goods. In one day, her profit was 315,700 Leones. Round her profit to 2 significant figures.
6. According to the national census, the population of Freetown was 1,050,301 in 2015. Correct this number to 3 s.f.
7. Express 0.0058432 correct to three significant figures.

Lesson Title: Adding and Subtracting Integers and Decimals	Theme: Numbers and Numeration
Practice Activity: PHM-08-011	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to add and subtract a mixture of integers and decimals.

Overview

Addition and subtraction of integers and decimal numbers can be done vertically. Make sure the digits are lined up correctly (ones under ones, tens under tens, and so on). Also make sure the decimal points are lined up vertically in the problem and answer.
Apply the same rules for carry over and borrowing whether you are working with integers or decimal numbers.

In addition, when you get a number greater than 9, you will **carry over**. Consider the example $345+72$. In this example the tens column gives $4+7=11$. Carry the 1 to the hundreds column, and write 1 in the tens place of the answer.

$$\begin{array}{r} {}^{1} \\ 3\ 4\ 5 \\ +\ \ \ 7\ 2 \\ \hline 4\ 1\ 7 \end{array}$$

In subtraction, if the number on top is less than the number on bottom, you need to **borrow**. Consider the example $562-28$. In this example, 2 is less than 8. We borrow one 10 from the tens column. In the tens column, 6 becomes 5. In the ones column, 2 becomes 12.

$$\begin{array}{r} 5\ \overset{5}{\cancel{6}}\ {}^{1}2 \\ -\ \ \ 2\ 8 \\ \hline 5\ 3\ 4 \end{array}$$

Solved Examples

1. Add: $34.5+12$

   Solution
   Remember that $12=12.0$. When working, write a decimal and zero after the whole number to hold the tenths' place. Write the numbers with the decimal points lined up vertically, then add.

$$\begin{array}{r} 3\ 4.\ 5 \\ +1\ 2.\ 0 \\ \hline 4\ 6.\ 5 \end{array}$$

   Answer: 46.5

2. Add: $29.52 + 12.81$

Solution
This problem requires carry over. Note that 1 is carried over from the tenths place (to the right of the decimal point) to the ones place (to the left of the decimal point).

$$\begin{array}{r} {}^{1}\ \ {}^{1}\quad\ \ \\ 29.52 \\ +12.81 \\ \hline 42.33 \end{array}$$

Answer: 42.33

3. Subtract: $215.93 - 42$

Solution
Remember that $42 = 42.00$. Write the numbers vertically, with the larger number on top.

$$\begin{array}{r} \overset{1}{\cancel{2}}\ {}^{1}1\,5.93 \\ -\quad 42.00 \\ \hline 173.93 \end{array}$$

Answer: 173.39

4. Subtract: $19.05 - 5.32$

Solution
This problem requires borrowing. Note that 1 is borrowed from the ones place (to the left of the decimal point) to the tenths place (to the right of the decimal point).

$$\begin{array}{r} 1\overset{8}{\cancel{9.}}\ {}^{1}0\,5 \\ -\quad 5.\ 3\,2 \\ \hline 13.\ 7\,3 \end{array}$$

5. Mr. Bangura sells rice in his shop. He sold 20.5 kg of rice in the morning, and 15 kg of rice in the afternoon. How much did he sell in total?

   Solution

   "In total" tells us to add the numbers:

$$\begin{array}{r} 20.5 \\ +15.0 \\ \hline 35.5 \end{array}$$

   Answer: Mr. Bangura sold 35.5 kg of rice in total.

Practice

Add or subtract the numbers:

1. $215.98 + 125.2$
2. $1.5 - 0.9$
3. $2.25 - 1.81$
4. $18.9 + 21$
5. $4.8 + 20.345$
6. $247 - 21.8$
7. $314.98 - 42.7$
8. $2.98 + 3.762$
9. Mary had 6.25 yards of fabric. She used 2.5 yards to make a dress. How much fabric does she have left?
10. Foday sells petrol. This morning, he had 25.8 litres of petrol. If he sold 16 litres, how much petrol does he have left?

Lesson Title: Adding and Subtracting Fractions with Integers and Decimals	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-012	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to add and subtract a mixture of fractions, integers and decimals.

Overview

This lesson handles problems with more than 1 operation (addition or subtraction), and more than 1 type of number (fractions, integers and decimals). Remember that operations are applied according to BODMAS, the order of operations. The letters BODMAS stand for: Bracket, Of, Division Multiplication, Addition and Subtraction. However, addition and subtraction are at the same level in BODMAS. We can do them in the order we come to them. For example, consider the problem $5 - \frac{7}{5} + 2.9$. We can solve this by subtracting first.

When you have a fraction and decimal in the same problem, you should convert one of them. Numbers should be of the same type before adding or subtracting them. In our example problem above, we can convert the improper fraction to a decimal and add decimals, OR convert the decimal to an improper fraction and add fractions.

Solved Examples

1. Evaluate $5 - \frac{7}{5} + 2.9$.

   Solution
   Apply addition and subtraction in the order that you come to them.

   Step 1. Subtract $5 - \frac{7}{5}$:
   $$5 - \frac{7}{5} = \frac{25}{5} - \frac{7}{5} = \frac{18}{5}$$
   This gives the problem $5 - \frac{7}{5} + 2.9 = \frac{18}{5} + 2.9$

   Step 2. Add $\frac{18}{5} + 2.9$

   Convert $\frac{18}{5}$ to a decimal by dividing. We find that $\frac{18}{5} = 3.6$.

		3	.6
5	1	8	.0
-	1	5	
		3	0
	-	3	0
			0

Add the decimal numbers: $3.6 + 2.9 = 6.5$

$$\begin{array}{r} \overset{1}{3}.6 \\ +\ 2.9 \\ \hline 6.5 \end{array}$$

Answer: $5 - \frac{7}{5} + 2.9 = 6.5$

2. Evaluate $2.5 + 10 - \frac{1}{4}$.

Solution

Apply addition and subtraction in the order that you come to them.

Step 1. Add $2.5 + 10$:

$$\begin{array}{r} 10.0 \\ +\ 2.5 \\ \hline 12.5 \end{array}$$

This gives the problem $2.5 + 10 - \frac{1}{4} = 12.5 - \frac{1}{4}$.

Step 2. Subtract $12.5 - \frac{1}{4}$.

Remember that we can either convert 12.5 to a fraction, or $\frac{1}{4}$ to a decimal. This time, let's convert 12.5 to a fraction.

Convert 12.5 to an improper fraction. $12.5 = 12\frac{5}{10} = 12\frac{1}{2} = \frac{12\times2+1}{2} = \frac{25}{2}$

Subtract:

$12.5 - \frac{1}{4}$	$= \frac{25}{2} - \frac{1}{4}$	Write both numbers as fractions
	$= \frac{25\times2}{2\times2} - \frac{1}{4}$	Change the denominators to the LCM
	$= \frac{50}{4} - \frac{1}{4}$	Subtract
	$= \frac{50-1}{4}$	
	$= \frac{49}{4}$	
	$= 12\frac{1}{4}$	Convert to a mixed fraction

Answer: $2.5 + 10 - \frac{1}{4} = 12\frac{1}{4}$

Practice

Evaluate the following:

1. $20 + 0.4 - \frac{3}{2}$
2. $2.8 - \frac{3}{5} + 4.1$
3. $\frac{21}{4} + 0.25 - 3$
4. $31 - 10\frac{1}{2} + 0.5$

Lesson Title: Multiplying and Dividing by Integers and Decimals	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-013	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to multiply and divide a mixture of integers and decimals.

Overview

This lesson handles problems with more than one operation (multiplication or division), and more than one type of number (integers and decimals). Remember that operations are applied according to BODMAS, the order of operations. The letters BODMAS stand for: Bracket, Of, Division Multiplication, Addition and Subtraction. However, multiplication and division are at the same level in BODMAS. We can do them in the order we come to them. For example, consider the problem $5 \div 2 \times 1.35$. We can solve this by dividing first. This is in Solved Example 3.

There are certain rules for multiplying and dividing decimal numbers:

For **multiplication of decimals**, multiply the numbers vertically as you would with whole numbers. The number of decimal places in the answer should be the same as the total number of decimal places in the problem. For example, $0.25 \times 0.3 = 0.075$. There are 3 decimal places in total in the problem, and 3 decimal places in the answer. See Solved Example 1 for the full solution to this problem.

For **division of decimals**, we must make the divisor a whole number by multiplying by 10 (if it has 1 decimal place), by 100 (if it has 2 decimal places) or by 1,000 (if it has 3 decimal places). Multiply the dividend by the same number you multiplied by the divisor. Then, divide the numbers as we do in whole numbers. For example, consider the problem $1.68 \div 0.2$. The divisor is 0.2. We need to multiply it by 10 to get a whole number: $0.2 \times 10 = 2$. We also multiply 1.68 by 10, $1.68 \times 10 = 16.8$. Rewrite the problem: $1.68 \div 0.2 = 16.8 \div 2$. The problem can now be solved using long division. See Solved Example 2 for the full solution to this problem.

Solved Examples

1. Multiply: 0.25×0.3

   Solution
   You may work the problem with or without the decimal places.
   Method 1. Multiply the decimal numbers. The answer will have 3 decimal places, the same as the problem:

		0 .	2	5
	×		0 .	3
		0	7	5
	0	0	0	0
	0 .	0	7	5

Answer: 0.075

Method 2. Omit the decimal point and multiply the numbers as whole numbers. Make sure you have the same number of decimal places in your answer as it is in the question.

	2	5
×		3
	7	5

Since we have 3 decimal places in the question, move the decimal point 3 places from right to left.

Answer: 0.075

2. Divide: $1.68 \div 0.2$

Solution

Multiply both the dividend and the divisor by 10, to make the divisor a whole number:
$1.68 \times 10 = 16.8$ and $0.2 \times 10 = 2$

Thus, $1.68 \div 0.2 = 16.8 \div 2$.

Carry out the long division →

```
       8 . 4
    ----------
   2| 1 6 . 8
   - 1 6    ↓
   --------
       0    8
     -      8
     ---------
            0
```

Answer: $1.68 \div 0.2 = 8.4$

3. Evaluate: $5 \div 2 \times 1.35$

Solution

Apply multiplication and division in the order that you come to them.

Step 1. Divide: $5 \div 2 = \frac{5}{2}$.

We can leave this as a fraction, but then we have the problem $\frac{5}{2} \times 1.35$. It will be easier to change $\frac{5}{2}$ to a decimal number, and multiply 2 decimals. We have $\frac{5}{2} = 2.5$.

This gives the problem $5 \div 2 \times 1.35 = 2.5 \times 1.35$

Step 2. Multiply 2.5×1.35:

$$
\begin{array}{r}
1.35 \\
\times \quad 2.5 \\
\hline
675 \\
2700 \\
\hline
3.375
\end{array}
$$

Answer: $5 \div 2 \times 1.35 = 3.375$

4. Evaluate: $4.5 \times 4 \div 0.25$

   Solution

   Step 1. Multiply 4.5×4:

$$
\begin{array}{r}
{}^{2} \\
4.5 \\
\times \quad 4 \\
\hline
18.0
\end{array}
$$

   This gives the problem $4.5 \times 4 \div 0.25 = 18 \div 0.25$.

   Step 2. Divide $18 \div 0.25$.
   Multiply both the dividend and the divisor by 100, to make the divisor a whole number: $18 \times 100 = 1{,}800$ and $0.25 \times 100 = 25$

   Thus, $18 \div 0.25 = 1{,}800 \div 25$.

   Carry out the long division:

$$
\begin{array}{r|l}
 & 72 \\
\hline
25 & 1800 \\
 & -175\downarrow \\
\hline
 & 50 \\
 & -50 \\
\hline
 & 0
\end{array}
$$

   Answer: $4.5 \times 4 \div 0.25 = 72$

Practice

1. $12.6 \div 0.3$
2. 1.4×2.3
3. $20 \div 8 \times 3.5$
4. $2.8 \times 1.3 \div 0.4$
5. $6 \times 0.32 \div 3$

Lesson Title: Multiplying and Dividing Fractions by Integers and Decimals	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-014	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to multiply and divide a mixture of fractions, integers and decimals.

Overview

This lesson handles problems with more than one operation (multiplication or division), and more than one type of number (fractions, integers and decimals). Remember that operations are applied according to BODMAS, the order of operations.

When you have a fraction and decimal in the same problem, you should convert one of them. Numbers should be of the same type before multiplying or dividing them. For example, consider $5.2 \div 8 \times \frac{2}{5}$. We can convert the fraction to a decimal, OR convert the decimal to a fraction.

Use the rules for multiplying and dividing decimal numbers from the previous lesson. You will also use the rules for multiplying and dividing **fractions:**

For **multiplication of fractions**, multiply the numerators and multiply the denominators. Simplify the answer if possible. For example, $\frac{1}{3} \times \frac{3}{4} = \frac{1\times3}{3\times4} = \frac{3}{12} = \frac{1}{4}$.

For **division of fractions**, change the division sign to a multiplication sign. The fraction **after** the division sign is changed to its reciprocal. For example, $\frac{2}{5} \div \frac{3}{5} = \frac{2}{5} \times \frac{5}{3} = \frac{2}{1} \times \frac{1}{3} = \frac{2\times1}{1\times3} = \frac{2}{3}$.

Solved Examples

1. Evaluate $5.2 \div 8 \times \frac{2}{5}$.

   Solution
   Remember that we can work division and multiplication in the order that we come to them.

   Step 1. Divide: $5.2 \div 8$.

   Solve this using long division. The answer is $5.2 \div 8 = 0.65$

	0.	6	5
8	5.	2	0
-	4	8	
		4	0
	-	4	0
			0

   Step 2. Multiply $0.65 \times \frac{2}{5}$.

We can either convert 0.65 to a fraction, or convert $\frac{2}{5}$ to a decimal. Both methods are shown below.

Solve with decimals:

Convert $\frac{2}{5}$ to a decimal: $\frac{2}{5} = 0.4$.

Multiply the decimals: $0.65 \times 0.4 = 0.26$

$$\begin{array}{cccc} & ^{2} & ^{2} & \\ & 0. & 6 & 5 \\ \times & & 0. & 4 \\ \hline 0 & .2 & 6 & 0 \end{array}$$

Solve with fractions:

Convert 0.65 to a fraction: $0.65 = \frac{65}{100} = \frac{13}{20}$

Multiply the fractions: $\frac{13}{20} \times \frac{2}{5} = \frac{13\times2}{20\times5} = \frac{26}{100} = \frac{13}{50}$

Answer: 0.26 or $\frac{13}{50}$

2. Evaluate $3 \times \frac{2}{5} \div 0.2$.

Solution

Step 1. Multiply: $3 \times \frac{2}{5} = \frac{3\times2}{5} = \frac{6}{5}$

Step 2. Divide: $\frac{6}{5} \div 0.2$:

We can easily convert 0.2 to a fraction: $0.2 = \frac{2}{10} = \frac{1}{5}$.

Divide the fractions: $\frac{6}{5} \div 0.2 = \frac{6}{5} \div \frac{1}{5} = \frac{6}{5} \times \frac{5}{1} = \frac{6}{1} = 6$

Answer: 6

Practice

Evaluate the following:

1. $12 \div \frac{6}{15}$
2. $1.4 \times 12 \div \frac{3}{5}$
3. $1.2 \div 0.4 \times \frac{1}{6}$
4. $3.5 \times \frac{1}{2} \div \frac{1}{4}$
5. $\frac{1}{6} \div \frac{1}{2} \times 2.5$

Lesson Title: Story Problems with Operations on Different Number Types	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-015	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Apply operations to different number types in story problems.
2. Give answers to required degree of accuracy.

Overview

In this lesson, you will solve story problems with operations on fractions, decimals, and integers. Story problems are maths problems that are expressed as stories. Take your time to read story problems. Find the numbers, and decide which operation to use.

Here are some key words to look for in story problems. These will help you decide which operation to use:

- **Addition** words: sum, total, more than, all together
- **Subtraction** words: difference, less than, left
- **Multiplication** words: times, each, total, all together
- **Division** words: share, each

Make sure to write your answer with units. For example, in Solved Example 1, the unit is cups. This must be included with the answer.

Solved Examples

1. Sia had $\frac{1}{2}$ cup of rice, and her sister gave her $\frac{1}{4}$ cup more. How much rice did she have in total?

   Solution

   Words like **more** and **total** tell us to add. Identify the 2 numbers to be added: $\frac{1}{2}$ and $\frac{1}{4}$.

   Write the problem and carry out the addition:

   $\frac{1}{2}+\frac{1}{4} = \frac{2}{4}+\frac{1}{4}$ Change the denominator

   $= \frac{2+1}{4}$ Add

   $= \frac{3}{4}$ cups

   Sia had $\frac{3}{4}$ cups of rice in total.

2. Martin uses $\frac{1}{4}$ of his money to buy rice, and $\frac{1}{8}$ to buy bananas. What fraction of his money is left?

Solution

Words like **left** tell us to subtract. We want to subtract the money that Martin spent from 1 whole. Subtract $1 - \frac{1}{4} - \frac{1}{8}$ or $1 - \left(\frac{1}{4} + \frac{1}{8}\right)$.

$1 - \left(\frac{1}{4} + \frac{1}{8}\right) = 1 - \left(\frac{2}{8} + \frac{1}{8}\right)$ Change the denominator

$= 1 - \left(\frac{2+1}{8}\right)$ Add

$= 1 - \frac{3}{8}$

$= \frac{8}{8} - \frac{3}{8}$ Substitute $\frac{8}{8} = 1$

$= \frac{8-3}{8}$ Subtract

$= \frac{5}{8}$ of his money

3. Three farmers harvested crops together. They harvested $\frac{3}{4}$ kilogrammes pepper, which they will share equally between themselves. How much does each farmer get?

Solution

Divide the amount they harvested ($\frac{3}{4}$) by the number of farmers (3) to find how much each person gets:

$\frac{3}{4} \div 3 = \frac{3}{4} \div \frac{3}{1}$ Write 4 as a fraction

$= \frac{3}{4} \times \frac{1}{3}$ Change to multiplication

$= \frac{1}{4} \times \frac{1}{1}$ Cancel 3

$= \frac{1 \times 1}{4 \times 1}$

$= \frac{1}{4}$ kg

Each farmer gets $\frac{1}{4}$ kg of pepper.

4. Abu's height is 1.5 m. and Foday's height is $1\frac{3}{10}$ m. How much taller is Abu than Foday?

Solution

We have a decimal and a fraction, so we need to convert one of them. It is simple to convert $1\frac{3}{10}$, because we know $\frac{3}{10} = 0.3$. Therefore, use $1\frac{3}{10} = 1.3$.

None of the key words are here. However, the words 'how much taller' tell us that we want to find the **difference** in their heights. We will subtract: $1.5 - 1.3$

Abu is 0.2 metres taller than Foday.

$$\begin{array}{r} 1.5 \\ -\ 1.3 \\ \hline 0.2 \end{array}$$

5. This morning, Mr. Bangura harvested 25.92 kg pepper from his farm. If he sold $12\frac{1}{2}$ kg of the pepper, how much does he have left?

Solution

We have a decimal and a fraction, so we need to convert one of them. It is simple to convert $12\frac{1}{2}$, because we know $\frac{1}{2} = 0.5$. Therefore, use $12\frac{1}{2} = 12.5$.

The words 'how much does he have left' tell us to subtract: $25.92 - 12.5$

Mr. Bangura has 13.42 kg of pepper left.

$$\begin{array}{r} 25.92 \\ -\ 12.50 \\ \hline 13.42 \end{array}$$

Practice

1. There are 20 books in a stack. The weight of each book is $1\frac{3}{4}$ kg. Find the total weight of the books.
2. Abu has a shop. He sells 3.8 kg of sugar each day. If his store is open for 6 days in a week, how much sugar does he sell in 1 week?
3. You go out for a long walk. You walk $\frac{3}{4}$ mile and then sit down to take rest. Then you walk another $\frac{3}{8}$ mile. How far did you walk in total?
4. Juliet is 167 cm tall, and Alice is 159.5 cm tall. How much taller is Juliet than Alice?
5. Juliet is a baker. It takes her $\frac{1}{8}$ of a working day to make one cake. How many cakes can she bake in $2\frac{1}{2}$ working days?
6. Joseph walks $\frac{7}{8}$ mile to school. Paul walks $\frac{1}{2}$ mile to school. How much farther does Joseph walk than Paul?
7. Mustapha harvested the cassava from his farm. It took him 3 days to sell all of it. He recorded the weight of the cassava he sold each day: 21.5 kg, 30.6 kg, 9.3 kg. How much cassava did he have all together?
8. David weighs 87.4 kg. If he reduced his weight by $4\frac{1}{2}$ kg, how much would he weigh?

Lesson Title: Review the Concept and Vocabulary of Factors and Multiples	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-016	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to identify factors and multiples of given numbers.

Overview

Factors are numbers that can divide another number exactly. For example, 6 can divide 12 two times, therefore 6 is a factor of 12. Factors of 12 include 1, 2, 3, 6, and 12.

A **multiple** of a number is formed by multiplying it with another number. 10, 20, 30, and 40 are the multiples of 10. A list of multiples can go on forever. The multiples of 10 are unlimited.

Here are some facts about multiples:

a. Each number is a multiple of itself.
b. Every number is a multiple of 1.
c. A multiple of a number cannot be less than the number.
d. The list of multiples of any number is infinite, meaning it can continue on and on.

Solved Examples

1. Write down all multiples of 3 greater than 10 but less than 20.

   Solutions
   The multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, …

   Choose the multiples of 3 from the list that are greater than 10 but less than 20. These are 12, 15, 18.

2. Find the factors of 15.

   Solutions
   The factors of 15 are all of the numbers that divide it evenly. These are 1, 3, 5, 15. Check this answer by dividing 15 by each factor:

   $15 \div 1 = 15 \quad 15 \div 3 = 5 \quad 15 \div 5 = 3 \quad 15 \div 15 = 1$

   Note that each of these factors can be multiplied to get 15:

$1 \times 15 = 15 \quad 3 \times 5 = 15$

3. List the first 10 multiples of 4.
    a. Are any of the multiples of 4 also multiples of 5? If so, what are they?
    b. Are any of the multiples of 4 factors of 100? If so, what are they?

Solutions

First 10 multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40

a. Yes. 20 and 40 are also multiples of 5.
b. Yes.
   4 is a factor of 100, because $100 \div 4 = 25$
   20 is a factor of 100, because $100 \div 20 = 5$.
   None of the other multiples of 4 in this list can divide 100 evenly.

4. List the factors of the following numbers:
    a. 6
    b. 60
    c. 81

Solutions

a. 1, 2, 3, 6
b. 1, 2, 3, 4, 5, 6, 10, 15, 20, 30, 60
c. 1, 3, 9, 27, 81

Practice

1. List the first 10 multiples of 3.
2. List the factors of the following numbers: a. 20 b. 36 c. 80
3. List the multiples of 3 greater than 40 but less than 60.
4. Write down all of the multiples of 9 greater than 20 but less than 80.
5. List the first 10 multiples of 6.
    a. Are any of the multiples of 6 also multiples of 4? If so, what are they?
    b. Are any of the multiples of 6 factors of 100? If so, what are they?

Lesson Title: Review Prime and Composite Numbers	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-017	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to identify prime and composite numbers.

Overview

A **prime number** is a number that is greater than 1 and cannot be divided evenly by any other number except 1 and itself. Therefore, a prime number has exactly two factors. Zero and 1 are not considered prime numbers. Examples of prime numbers between 1 and 15 are 2, 3, 5, 7, 11, and 13.

A **composite number** is any whole number other than 1 that is not a prime number, meaning it has factors other than 1 and the number itself. Therefore, a composite number has more than two factors. Examples of composite numbers between 0 and 15 are 4, 6, 8, 9, 10, 12, 14, and 15.

Solved Examples

1. For each of the numbers in the table below, list all of its factors. Then, decide if it is prime or composite.

Number	Factors	Prime or Composite
18		
31		
36		

Solutions
The factors are listed in the table below. Note that if a number has more than 2 factors listed, it is composite.

Number	Factors	Prime or Composite
18	1, 2, 3, 6, 9, 18	Composite
31	1, 31	Prime
36	1, 2, 4, 6, 9, 18, 36	Composite

2. Write down all of the prime numbers greater than 30 but less than 40.

Solution
Consider all of the numbers between 30 and 40: 31, 32, 33, 34, 35, 36, 37, 38, 39.
Note that even numbers are never prime (except the number 2). We can eliminate the even numbers (32, 34, 36, 38) because they are composite.

Consider the odd numbers: 31, 33, 35, 37, 39

Eliminate the numbers that can be divided by any number besides 1 and itself. For example, 33 and 39 can be divided by 3. Thirty-five can be divided by 5.

- Remember that you can use a multiplication table to find the factors of many numbers. If you have a difficult time determining if a number is prime or composite, look for it in the multiplication table. For example, 35 is in the multiplication table because $5 \times 7 = 35$. Therefore, it is composite.

This leaves 31 and 37.
Answer: 31, 37

Practice

1. For each of the numbers in the table below, list all of its factors. Then, decide if it is prime or composite.

Numbers	Factors	Prime or Composite
16		
24		
29		
41		
49		
54		
59		

2. Write down all of the composite numbers greater than 20 but less than 35.
3. Write down all of the prime numbers greater than 50 but less than 70.

Lesson Title: Prime Factors of Whole Numbers	Theme: Numbers and Numeration
Practice Activity: PHM-08-018	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to find the prime factors of given numbers.

Overview
For this lesson, recall the following definitions from previous lessons:

- **Factors** are numbers that divide other numbers exactly.
- A **prime number** can only be divided by 1 and itself.

The factors of a number that are also prime numbers are referred to as **prime factors**. For example, 2 and 3 are factors of 12 and they are also prime numbers. Therefore, 2 and 3 are prime factors of 12.

Learning to find prime factors of numbers will help you to find the highest common factors of numbers (HCF) and the least common multiple (LCM) of numbers.

Solved Examples

1. Identify the prime factors of 50.

   Solution
   First, list all of the factors of 50: 1, 2, 5, 10, 25, 50

   Identify whether each factor is also a prime number. In this list, only 2 and 5 are prime numbers.

   The prime factors of 50 are 2 and 5.

2. For each number in the table below, list its factors and prime factors:

Numbers	Factors	Prime factors
20		
34		
40		
81		
100		

Solutions

For each number in the table, first list its factors. Then, look at each factor and determine if it is prime. List the prime factors in the last column.

Numbers	Factors	Prime factors
20	1, 2, 4, 5, 10, 20	2, 5
34	1, 2, 17, 34	2, 17
40	1, 2, 4, 5, 8, 10, 20, 40	2, 5
81	1, 3, 9, 27, 81	3
100	1, 2, 4, 5, 10, 20, 25, 50, 100	2, 5

Practice

1. Identify the prime factors of 30.
2. Identify the prime factors of 22.
3. For each number in the table below, list its factors and prime factors:

Numbers	Factors	Prime factors
21		
35		
42		
56		
90		

Lesson Title: Calculating the Least Common Multiple (LCM)	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-019	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to find the least common multiple (LCM) of given numbers using prime factorisation.

Overview

This lesson is on finding the least common multiple (LCM), the smallest multiple that two numbers share. If two numbers have a multiple that is the same, that number is a **common multiple** of the numbers. Two numbers have many common multiples, and the LCM is the smallest one.

For example, consider 3 and 5. These are the first 10 multiples of 3 and 5:

3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50

The numbers 15 and 30 are in both lists. Therefore, 15 and 30 are common multiples of 3 and 5. The LCM is 15, because it is the smallest common multiple.

We will use the prime factorisation method to find the LCM of 2 numbers.

These are the steps to **find the prime factors** of numbers with a factor tree:

a. Write down the numbers.
b. Underneath, write two numbers you multiply to get the number at the top.
c. Continue until you don't have any composite numbers. When you come to a prime number, the branch stops there.

After you find the prime factors of each number using a factor tree, you will use the prime factors to **find the LCM**. Follow these steps:

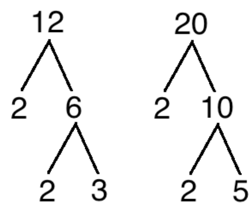
a. List all of the prime factors from both factor trees. List each prime factor the maximum number of times it appears in one tree.
b. Multiply the numbers in your list.

Solved Examples

1. Find the lowest common multiple (LCM) of 12 and 20.

   Solution

   Step 1. Factor each number with a factor tree:



   Step 2. Find the LCM:

   Identify the prime factors for both numbers.

   12: 2, 2, 3
   20: 2, 2, 5

   The maximum number of times the prime factor 2 occurs is two. This is for 12 and 20. The prime factors 3 and 5 only occur once. We will list these together: 2, 2, 3, 5

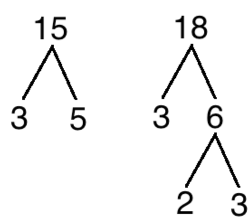
   Multiply the prime factors to find the LCM: $2 \times 2 \times 3 \times 5 = 60$

   The LCM of 12 and 20 is 60.

2. Find the LCM of 15 and 18.

   Solution

   Step 1. Factor each number:



   Step 2. Find the LCM:

   Identify the prime factors for both numbers.

   15: 3, 5
   18: 2, 3, 3

   The maximum number of times the prime factor 3 occurs is two. It should be listed twice. The prime factors 2 and 5 only occur once. We will list these together: 2, 3, 3, 5

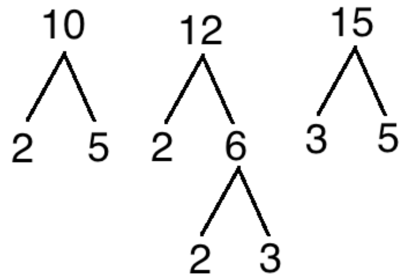
   Multiply the prime factors to find the LCM: $2 \times 3 \times 3 \times 5 = 90$

The LCM of 15 and 18 is 90.

3. Find the LCM of 10, 12, and 15

Solution

Step 1. Factor each number:



Step 2. Identify the prime factors of each number

10: 2, 5
12: 2, 2, 3
15: 3, 5

The maximum number of times the prime factor 2 occurs is two. It should be listed twice. The prime factors 3 and 5 only occur once. We will list these together: 2, 2, 3, 5

Step 3. Multiply to find the LCM of 10, 12 and 15: $2 \times 2 \times 3 \times 5 = 60$
The LCM of 10, 12 and 15 is 60.

Practice

Find the LCM of the following numbers:

1. 2 and 4
2. 3 and 6
3. 6 and 24
4. 15 and 20
5. 18 and 12
6. 9, 15 and 8
7. 4, 5, 6 and 9

Lesson Title: Calculating the Highest Common Factor (HCF)	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-020	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to find the highest common factor (HCF) of given numbers using prime factorisation.

Overview

Recall that **factors** are numbers that divide other numbers exactly. If two numbers have a factor that is the same, that factor is a **common factor** of the numbers. For example, consider the factors of 9 and 12:

9: 1, 3, 9
12: 1, 2, 3, 4, 6, 12

The numbers 1 and 3 are in both lists. Therefore, 1 and 3 are common factors of 9 and 12. All numbers have a common factor of 1.

The HCF of two (or more) numbers is the largest number that divides evenly into both numbers. HCF is the largest of all the common factors. The HCF of 9 and 12 is 3.

It is very easy to find the HCF of small numbers like 9 and 12. To find the HCF of big numbers, we use the factor tree.

Follow these steps to use the factor tree method to find the HCF of numbers:

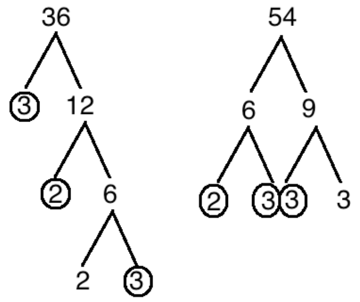
a. Draw the factor tree for each number. Remember to continue until each branch ends in a prime number.
b. Circle the prime factors that the two original numbers have in common.
c. Multiply the common prime factors of each original number to get the HCF.

Solved Examples

1. Find the HCF of 36 and 54.

   Solution

   Draw the factor trees and circle the common prime factors:



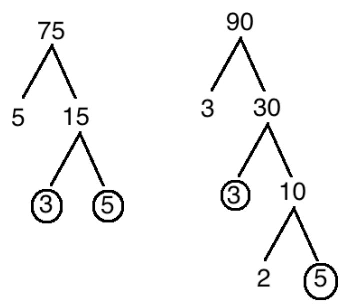
   Multiply the common prime factors: $2 \times 3 \times 3 = 18$

   The HCF of 36 and 54 is 18.

2. Find the HCF of 75 and 90.

   Solution

   Draw the factor trees and circle the common prime factors:



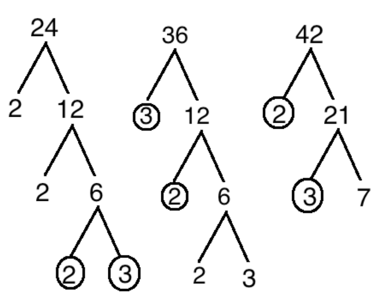
   Multiply the common prime factors: $3 \times 5 = 15$

   The HCF of 75 and 90 is 15.

3. Find the HCF of 24,36 and 42.

   Solution

   Use the factor tree as explained in problem 2 above for all the 3 numbers.



Multiply the common prime factors: $2 \times 3 = 6$
The HCF of 24, 36 and 42 is 6.

Practice

Find the HCF of the following numbers:

1. 8 and 12
2. 5 and 20
3. 48 and 60
4. 56 and 84
5. 36 and 54
6. 24, 40 and 60

Lesson Title: Index Notation	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-021	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify the index and base in index notation.
2. Identify that the index indicates the number of times the base is multiplied by itself.
3. Identify that any integer raised to the power of one gives itself ($a^1 = a$).

Overview

Numbers are written in **index notation** when there is a power. A number and its power are called an **index**. For example, 2^5 is an index. The plural form of index (more than 1) is **indices**. For example, 2^5 and 3^2 are indices.

The bottom number is the **base** and it is written bigger than the other number. The smaller, raised number is called the **index**. Index is another way to say **power**.

base → 3^2 ← power or index

The power of a number says how many times to multiply that number by itself. A power can be any number. For example, $2^5 = 2 \times 2 \times 2 \times 2 \times 2$. 2^5 can be read as "2 to the power of 5".

To **square** a number means to multiply the number by itself. For example, $3^2 = 3 \times 3 = 9$. 3^2 can be read as "3 squared" or "3 to the power of 2".

To **cube** a number means to multiply the number by itself 3 times. For example, $2^3 = 2 \times 2 \times 2 = 8$. 2^3 can be read as "2 cubed" or "2 to the power of 3".

Note the following helpful rules about indices:

Rule	Example
1 raised to any power is 1	$1^4 = 1 \times 1 \times 1 \times 1 = 1$
0 raised to any power is 0	$0^5 = 0 \times 0 \times 0 \times 0 \times 0 = 0$
A number to the power 1 = itself	$2^1 = 2$

The general rule for the last rule in the table is $a^1 = a$. This means that any number to the power 1 is itself.

Solved Examples

1. Evaluate the following: a. 7^2 b. 5^1 c. 8 squared

   Solutions

   Multiply each number by itself:

   a. $7^2 = 7 \times 7 = 49$

   b. $5^1 = 5$

   c. $8 \text{ squared} = 8^2 = 8 \times 8 = 64$

2. Evaluate: 3^3

   Solution

   Multiply 3 by itself 3 times: $3 \times 3 \times 3$. The multiplication can be done in 2 steps:

   Step 1. $3 \times 3 = 9$

   Step 2. $9 \times 3 = 27$

   Answer: $3^3 = 27$

3. Find the value of: 8^3

   Solution

   Multiply 8 by itself 3 times: $8 \times 8 \times 8$. The multiplication can be done in 2 steps. Use vertical multiplication to solve step 2.

   Step 1. $8 \times 8 = 64$

   Step 2. $64 \times 8 = 512$

   Answer: $8^3 = 512$

$$\begin{array}{r} {}^{3} \\ 6\,4 \\ \times \quad 8 \\ \hline 5\,1\,2 \end{array}$$

4. Expand: 8^5

   Solution

   Write 8^5 in its expanded form: $8^5 = 8 \times 8 \times 8 \times 8 \times 8$

5. Simplify and leave your answer in index form: $7 \times 7 \times 7 \times 7 \times 7 \times 7$.

   Solution

   7 is multiplied by itself 6 times, so 6 is the power of 7: 7^6

6. Evaluate: 0^6

   Solution

   0 raised to any power is 0: $0^6 = 0 \times 0 \times 0 \times 0 \times 0 \times 0 = 0$

7. Find the value of: 3^4

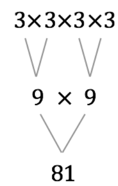
Solution

Multiply 3 by itself 4 times: $3 \times 3 \times 3 \times 3$. The multiplication can be done in 3 steps:

Step 1. $3 \times 3 = 9$
Step 2. $9 \times 3 = 27$
Step 3. $27 \times 3 = 81$

Answer: $3^4 = 81$

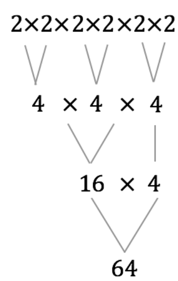
We can also find 3^4 by multiplying 2 of the sets of 3 ($3 \times 3 = 9$ and $3 \times 3 = 9$), then multiplying the results, $9 \times 9 = 81$.



8. Evaluate: 2^6

Solution

Multiply 2 by itself 6 times: $2 \times 2 \times 2 \times 2 \times 2 \times 2$.



Answer: $2^6 = 64$

Practice

1. Find the value of each square: a. 1^2 b. 4 squared c. 30^2
2. Evaluate the following: a. 10^3 b. 4 cubed c. 0^3
3. Expand the following: a. 3^8 b. 8^4 c. 9^7
4. Write the following in index form:
    a. $7 \times 7 \times 7 \times 7$
    b. $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
    c. $3 \times 3 \times 3 \times 3 \times 3$
5. Find the values of the following: a. 5^4 b. 1^9 c. 12^2

Lesson Title: Index Law 1: Multiplication of Indices	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-022	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that $a^m \times a^n = a^{m+n}$.
2. Multiply two or more indices.

Overview

To multiply two or more indices with the same base, add the powers. For example, $3^2 \times 3^5 = 3^{2+5} = 3^7$.

To see why this is true, write each index in its expanded form:

$$\begin{aligned} 3^2 \times 3^5 &= 3 \times 3 \quad \times \quad 3 \times 3 \times 3 \times 3 \times 3 \\ &= 3^7 \end{aligned}$$

This is what we refer to as the first law of indices, which has the general formula $a^m \times a^n = a^{m+n}$. Here m and n are the powers of a, and a is the base. This rule only works if the bases are the same.

Solved Examples

1. Simplify: $7^3 \times 7^4$

   Solution
   Add the powers: $7^3 \times 7^4 = 7^{3+4} = 7^7$

2. Simplify: $r^3 \times r^4$

   Solution
   $r^3 \times r^4 = r^{3+4} = r^7$

3. Simplify: $3^7 \times 3$

   Solution
   Remember that a number without a power on it is the same as that number raised to the power of 1. In this problem, $3 = 3^1$.

   $3^7 \times 3 = 3^7 \times 3^1 = 3^{7+1} = 3^8$

4. Simplify: $2^8 \times 2^5$

   Solution

   $2^8 \times 2^5 = 2^{8+5} = 2^{13}$

5. Simplify: $3^8 \times 3^7 \times 3$

   Solution

   When there are more than 2 indices, we simply add all of the powers together. Remember that $3 = 3^1$.

   $3^8 \times 3^7 \times 3 = 3^8 \times 3^7 \times 3^1 = 3^{8+7+1} = 3^{16}$

6. Simplify: $2^3 \times 3^4$

   Solution

   These indices cannot be combined because the bases are different. This expression is already in its simplest form.

7. Simplify: $2^3 \times 3^9 \times 3^4$

   Solution

   Two of the indices have the same base. We can multiply $3^9 \times 3^4$, but 2^3 cannot be multiplied together with the others.

   $2^3 \times 3^9 \times 3^4 = 2^3 \times 3^{9+4} = 2^3 \times 3^{13}$

   $2^3 \times 3^{13}$ is the simplified form of the expression.

Practice

Simplify the following:

1. $a^{12} \times a^3$
2. $u^4 \times u^3$
3. $9^8 \times 9^3$
4. $11^4 \times 11^9$
5. $10^4 \times 10$
6. $9^6 \times 9^7 \times 9^{10}$
7. $7 \times 7^9 \times 7^4$
8. $5 \times 2^5 \times 2^8$
9. $7^6 \times 5^2 \times 5^{10}$
10. $3^5 \times 11^8$

Lesson Title: Index Law 2: Division of Indices	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-023	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that $a^m \div a^n = a^{m-n}$.
2. Divide two or more indices.

Overview

To divide two or more indices with the same base, subtract the powers. For example, $2^5 \div 2^2 = 2^{5-2} = 2^3$. This is referred to as the second law of indices, which has the general formula $a^m \div a^n = a^{m-n}$. Here m and n are the powers of a, and a is the base. This rule only works if the bases are the same.

To see why this rule for indices is true, write the division as a fraction. Then, write each index in its expanded form and cancel:

$$
\begin{aligned}
3^5 \div 3^2 &= \frac{3^5}{3^2} && \text{Write as a fraction} \\
&= \frac{3\times3\times3\times3\times3}{3\times3} && \text{Expand the indices} \\
&= \frac{\cancel{3}\times\cancel{3}\times3\times3\times3}{\cancel{3}\times\cancel{3}} && \text{Cancel two 3's} \\
&= 3\times3\times3 && \\
&= 3^3 && \text{Simplify}
\end{aligned}
$$

We don't need to write the expanded form each time. We can simply subtract the powers: $3^5 \div 3^2 = 3^{5-2} = 3^3$.

Solved Examples

1. Simplify: $3^6 \div 3^2$

   Solution

   Subtract the powers: $3^6 \div 3^2 = 3^{6-2} = 3^4$

2. Simplify: $\frac{p^7}{p^5}$

   Solution

   Fractions are the same as division. Follow the second law of indices and subtract the indices.

   $\frac{p^7}{p^5} = p^7 \div p^5 = p^{7-5} = p^2$

3. Divide: $7^{12} \div 7$

Solution

Remember that $7 = 7^1$.

Subtract the powers: $7^{12} \div 7 = 7^{12} \div 7^1 = 7^{12-1} = 7^{11}$

4. Simplify: $9^{10} \div 9^2 \div 9^3$

Solution

The second law of indices also applies to dividing more than 2 indices. We simply subtract each of the powers of the indices we are dividing by.

$9^{10} \div 9^2 \div 9^3 = 9^{10-2-3} = 9^5$

5. Simplify: $7^{10} \div 3^2$

Solution

This expression cannot be simplified because the bases are different. It is already in its simplest form.

6. Simplify: $2^4 \div 3^{10} \div 3^2$

Solution

Two of the indices have the same base. We can divide $3^{10} \div 3^2$, but 2^4 cannot be divided with the others.

$2^4 \div 3^{10} \div 3^2 = 2^4 \div 3^{10-2} = 2^4 \div 3^8$

$2^4 \div 3^8$ is the simplified form of the expression.

Practice

Simplify the following:

1. $5^9 \div 5^3$
2. $a^{10} \div a$
3. $2^{30} \div 2^5$
4. $\frac{4^{13}}{4^3}$
5. $7^3 \div 7^2$
6. $\frac{b^9}{b}$
7. $8^{10} \div 8$

8. $2^{12} \div 2^{8} \div 2$
9. $7^{10} \div 7^{5} \div 7^{3}$
10. $5^{10} \div 3^{5}$

Lesson Title: Index Law 3: Power of Zero	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-024	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to identify that any integer raised to the power of zero equals one ($a^0 = 1$).

Overview

The third law of indices says that any number raised to the power of zero equals one. Consider an example, 3^0. This is 3 raised to the power 0, which means we multiply 3 by itself zero times. This gives us 1. We have $3^0 = 1$.

To understand why this is true, consider $3^2 \div 3^2$. We can use the second law of indices to simplify this expression: $3^2 \div 3^2 = 3^{2-2} = 3^0$.

Another way to write 3^2 is 9, because $3^2 = 3 \times 3 = 9$. We know that $9 \div 9 = 1$, so we also must have $3^2 \div 3^2 = 1$.

In summary, $3^0 = 3^2 \div 3^2 = 9 \div 9 = 1$.

The general form for this rule is $a^0 = 1$.

Solved Examples

1. Simplify the following: a. 7^0 b. 13^0 c. 207^0

   Solution

   Any number raised to the power 0 is equal to 1. Therefore, each answer is 1:

   a. 1 b. 1 c. 1

2. Simplify: $3^9 \div 3^0$

   Solution

   Subtract the powers, applying the second law of indices: $3^9 \div 3^0 = 3^{9-0} = 3^9$

   Also note that any number divided by 1 equals itself (for example, $3 \div 1 = 3$). This is true for powers as well: $3^9 \div 1 = 3^9$. Since we know $3^0 = 1$, we can quickly determine that the answer to this problem is 3^9.

3. Simplify: $2^7 \div 2^7$

   Solution
   Apply the second law of indices, then the third law: $2^7 \div 2^7 = 2^{7-7} = 2^0 = 1$

   Also note that any number divided by itself equals 1 (for example, $2 \div 2 = 1$). This is true for powers as well: $2^7 \div 2^7 = 1$. We can use this to quickly determine that the answer is 1.

4. Simplify: $a^{15} \div a^{15}$

   Solution
   Apply the second law of indices, then the third law: $a^{15} \div a^{15} = a^{15-15} = a^0 = 1$

5. Simplify: $5^{12} \times 5^0 \times 5^2$

   Solution
   Apply the first law of indices: $5^{12} \times 5^0 \times 5^2 = 5^{12+0+2} = 5^{14}$

Practice

Simplify the following:

1. 8^0
2. 17^0
3. 123^0
4. $12^7 \div 12^7$
5. $15^8 \div 15^8$
6. $7^{12} \div 7^0$
7. $5^9 \times 5^0$
8. $m^{10} \div m^{10}$
9. $3^5 \times 3^4 \times 3^0$

Lesson Title: Index Law 4: Powers of Indices	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-025	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that $(a^m)^n = a^{mn}$.
2. Apply an additional power to an index.

Overview

The fourth law is about raising an index to another power. For example, $(2^3)^4$. This says that the index 'two cubed' is raised to the power 4. In other words, 2 cubed is multiplied by itself 4 times: $(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$.

The fourth law of indices says that if an index is raised to another power, we can multiply the two powers together to get the power in the answer. The base stays the same. In our example, we have $(2^3)^4 = 2^{3\times 4} = 2^{12}$.

The general form for this rule is $(a^m)^n = a^{mn}$.

Solved Examples

1. Simplify: $(2^2)^3$

   Solution
   Apply the fourth law of indices: $(2^2)^3 = 2^{2\times 3} = 2^6$

2. Simplify: $(b^3)^3$

   Solution
   Apply the fourth law of indices: $(b^3)^3 = b^{3\times 3} = b^9$

3. Simplify: $(n^{23})^4$

   Solution
   Apply the fourth law of indices: $(n^{23})^4 = n^{23\times 4} = n^{92}$

   You may multiply vertically to find 23×4:

$$\begin{array}{r} {}^{1} \\ 2\ 3 \\ \times\ \ \ 4 \\ \hline 9\ 2 \end{array}$$

4. Simplify: $(5^5)^0$

   Solution
   Apply the fourth law of indices, then simplify using the third law:
   $$(5^5)^0 = 5^{5\times0} = 5^0 = 1$$

5. Simplify: $(1^7)^9$

   Solution
   Apply the fourth law of indices: $(1^7)^9 = 1^{7\times9} = 1^{63} = 1$
   Recall that 1 to any power is equal to 1, so $1^{63} = 1$.

6. Simplify: $(0^5)^2$

   Solution
   Apply the fourth law of indices: $(0^5)^2 = 0^{5\times2} = 0^{10} = 0$
   Recall that 0 to any power is equal to 0, so $0^{10} = 0$.

Practice

Simplify the following:

1. $(9^7)^3$
2. $(2^{20})^5$
3. $(0^4)^6$
4. $(a^4)^2$
5. $(1^4)^{12}$
6. $(u^4)^{15}$
7. $(7^4)^0$

Lesson Title: Index Laws 5 and 6: Power of a Product and Quotient	Theme: Numbers and Numeration
Practice Activity: PHM-08-026	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that $(a \times b)^n = a^n \times b^n$ and $(\frac{a}{b})^n = \frac{a^n}{b^n}$, where $b \neq 0$.
2. Apply index laws 5 and 6 to simplifying problems.

Overview

The fifth and sixth laws are about finding the power of a product and a quotient.
The **fifth law** is about finding the **power of a product**. Consider $(2 \times 3)^2$. This is the power of the product of 2 and 3. We can rewrite this: $(2 \times 3)^2 = (2 \times 3) \times (2 \times 3) = 2 \times 3 \times 2 \times 3 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$.

This gives us $(2 \times 3)^2 = 2^2 \times 3^2$. The power is distributed to each factor in the brackets.
The general form for the fifth law is $(a \times b)^n = a^n \times b^n$.

The **sixth law** is about finding the **power of a quotient**. Consider $\left(\frac{2}{3}\right)^2$.This is the power of the quotient of 2 divided by 3. We can rewrite this: $\left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) = \frac{2 \times 2}{3 \times 3} = \frac{2^2}{3^2}$.

This gives us $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$. The power is distributed to both the divided (numerator) and divisor (denominator).

The general form for the sixth law is $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, where $b \neq 0$.

Solved Examples

1. Use the fifth law of indices to rewrite the following:
   a. $(2 \times 9)^4$
   b. $(x \times y)^5$
   c. $(3a)^2$

   Solutions
   Distribute the power to each factor inside brackets:
   a. $(2 \times 9)^4 = 2^4 \times 9^4$
   b. $(x \times y)^5 = x^5 \times y^5$
   c. Note that $3a = 3 \times a$.
   Therefore, $(3a)^2 = 3^2a^2$

If you were asked to evaluate this completely, you would have
$(3a)^2 = 3^2a^2 = 9a^2$.

2. Use the sixth law of indices to rewrite the following:
   a. $\left(\frac{5}{9}\right)^4$
   b. $\left(\frac{1}{7}\right)^3$
   c. $(c \div d)^2$
   d. $(9 \div 2)^{13}$

   Solutions
   a. $\left(\frac{5}{9}\right)^4 = \frac{5^4}{9^4}$
   b. $\left(\frac{1}{7}\right)^3 = \frac{1^3}{7^3} = \frac{1}{7^3}$
   c. $(c \div d)^2 = c^2 \div d^2$
   d. $(9 \div 2)^{13} = 9^{13} \div 2^{13}$

Practice

Use the fifth and sixth laws of indices to rewrite the following:

1. $(s \times t)^{100}$
2. $(5x)^6$
3. $(2 \times 4)^{15}$
4. $\left(\frac{1}{8}\right)^{12}$
5. $\left(\frac{x}{y}\right)^3$
6. $(12 \div 11)^{31}$
7. $(51 \div 17)^2$

Lesson Title: Application of the Laws of Indices	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-027	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to use the six laws of indices to simplify problems.

Overview

You have now learned all of the laws of indices. In this lesson, you will use all 6 of them to simplify expressions with indices. For example, consider the expression $(2^3)^4 \times 2^5$. It involves a power of an index, and multiplication of indices. We will use multiple laws to simplify this expression.

Recall the 6 laws of indices:

1. Multiplication: $a^m \times a^n = a^{m+n}$
2. Division: $a^m \div a^n = a^{m-n}$
3. Power of zero: $a^0 = 1$
4. Power of an index: $(a^m)^n = a^{mn}$
5. Power of a product: $(a \times b)^n = a^n \times b^n$
6. Power of a quotient: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, where $b \neq 0$

Remember the order of operations. The order is very important when applying the laws of indices. It is BODMAS: Brackets Of Division Multiplication Addition Subtraction. Remember that indices fall under 'Of'.

You will solve problems with indices in fractions. You can cancel any like terms from the numerator and denominator. Consider the fraction $\frac{2^{10}}{2^{10} \times 5^{10}}$. 2^{10} can be canceled from the numerator and denominator.

Solved Examples

1. Simplify: $(2^3)^4 \times 2^5$

   Solution

$$
\begin{aligned}
(2^3)^4 \times 2^5 &= 2^{3\times 4} \times 2^5 && \text{Index law 4: Power of an index} \\
&= 2^{12} \times 2^5 \\
&= 2^{12+5} && \text{Index law 1: Multiplication} \\
&= 2^{17}
\end{aligned}
$$

2. Simplify: $\frac{(2^5)^2}{(2\times5)^{10}}$

Solution

First, simplify the numerator and denominator separately. Then apply the division rule.

$$\begin{aligned} \frac{(2^5)^2}{(2\times5)^{10}} &= \frac{2^{5\times2}}{2^{10}\times5^{10}} && \text{Index law 4 and Index law 1} \\ &= \frac{2^{10}}{2^{10}\times5^{10}} \\ &= \frac{1}{5^{10}} && \text{Cancel } 2^{10} \end{aligned}$$

3. Simplify: $\frac{a^3}{(3a)^4}$.

Solution

$$\begin{aligned} \frac{a^3}{(3a)^4} &= \frac{a^3}{3^4\times a^4} && \text{Index law 5: Power of a product} \\ &= \frac{1}{3^4\times a} && \text{Cancel } a^3 \end{aligned}$$

Note about cancelling indices:

- Remember that cancelling is the same as dividing the numerator and denominator by the same amount.
- To cancel a^3, divide both the numerator and denominator by a^3.
- Dividing the numerator gives 1: $a^3 \div a^3 = a^{3-3} = a^0 = 1$
- Dividing the denominator gives a: $a^4 \div a^3 = a^{4-3} = a^1 = a$

4. Simplify $(a^4)^5 \times (a^3)^2$

Solution

$$\begin{aligned} (a^4)^5 \times (a^3)^2 &= a^{4\times5} \times a^{3\times2} && \text{Index law 4: Power of an index} \\ &= a^{20} \times a^6 \\ &= a^{20+6} && \text{Index law 1: Multiplication} \\ &= a^{26} \end{aligned}$$

5. Simplify: $\frac{(5\times3)^7}{3^7}$

Solution

$$\begin{aligned} \frac{(5\times3)^7}{3^7} &= \frac{5^7\times3^7}{3^7} && \text{Index law 5: Power of a product} \\ &= \frac{5^7}{1} && \text{Cancel } 3^7 \\ &= 5^7 \end{aligned}$$

Practice

Simplify the following expressions:

1. $\frac{(5\times7)^2}{7^2}$
2. $(3^2 \times 3^4)^8$
3. $2^{20} \times (2^8)^5$
4. $\frac{5^3}{5^4\times3^4}$
5. $\frac{b^4}{(2b)^3}$
6. $(2^8)^2 \times (2^{15})^3$
7. Simplify: $\frac{a^3\times a^4}{a}$

Lesson Title: Indices with Negative Powers	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-028	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that a number with a negative index can be rewritten as a fraction ($a^{-n} = \frac{1}{a^n}$).
2. Simplify simple indices with negative powers.

Overview

Negative powers are the opposite of positive powers. Positive powers tell us to multiply. The opposite of multiplication is division, so negative powers tell us to divide. A negative power tells us how many times to divide by that number.

For example, consider 2^{-3}. This 2 has a power of **negative** 3. That means we **divide** by 2 three times:

$$2^{-3} = 1 \div (2 \times 2 \times 2) = \frac{1}{2\times2\times2} = \frac{1}{2^3}$$

When any number has a negative power, it can be rewritten in the denominator of a fraction with a positive power. The general rule is $a^{-n} = \frac{1}{a^n}$.

Solved Examples

1. Simplify: 19^{-4}

   Solution
   In this case, simplify means to write the index as a fraction with a positive power. The answer is $19^{-4} = \frac{1}{19^4}$.

2. Simplify the following: a. 12^{-5} b. 100^{-3} c. 1^{-8} d. 8^{-40}

   Solution
   Write each index as a fraction with a positive power:
   a. $12^{-5} = \frac{1}{12^5}$
   b. $100^{-3} = \frac{1}{100^3}$
   c. $1^{-8} = \frac{1}{1^8} = \frac{1}{1} = 1$
   d. $8^{-40} = \frac{1}{8^{40}}$

3. Simplify: 2×5^{-3}

   Solution

   Change 5^{-3} to a fraction with a positive power in the denominator. Two will not change. It will stay in the numerator of the fraction.

   Answer: $2 \times 5^{-3} = \frac{2}{5^3}$

4. Simplify: $2^4 \times 2^{-3}$

   Solution

   Change 2^{-3} to a fraction with a positive power in the denominator. 2^4 will not change. It will stay in the numerator of the fraction. You will then be able to cancel:

$2^4 \times 2^{-3}$	$=$	$\frac{2^4}{2^3}$	Index law 5: Power of a product
	$=$	$\frac{2}{1}$	Cancel 2^3
	$=$	2	

Practice

Simplify the following:

1. 21^{-3}
2. 10^{-9}
3. 130^{-4}
4. a^{-12}
5. b^{-100}
6. 7×3^{-3}
7. 10×2^{-14}
8. $2^5 \times 2^{-3}$
9. $3^7 \times 3^{-6}$

Lesson Title: Multiplying and Dividing Indices with Negative Powers	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-029	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to apply the laws for multiplying and dividing indices to those with negative powers.

Overview

When multiplying and dividing indices with positive powers, we add the powers for multiplication and subtract the powers for division. Follow the same rules to multiply and divide indices with negative powers. Remember that the bases should be the same in order to apply the multiplication and division laws to indices.

Remember the rules for adding and subtracting negative numbers:

When **adding** two negative numbers, we add their values and carry the negative sign to the answer. For example, $(-3) + (-5) = -8$

Subtracting a negative is the same as adding. For example, $(-3) - (-5) = (-3) + 5$. When we add a negative and positive, we do a subtraction problem and write the sign of the larger number on the answer. For example: $(-3) - (-5) = (-3) + 5 = 5 - 3 = 2$.

Solved Examples

1. Evaluate: $2^{-3} \times 2^{-5}$

   Solution

   $2^{-3} \times 2^{-5} = 2^{(-3)+(-5)}$ Add the powers
   $= 2^{-8}$
   $= \frac{1}{2^8}$ Simplify

2. Evaluate: $4^{-6} \div 4^{-4}$

   Solution

   $4^{-6} \div 4^{-4} = 4^{(-6)-(-4)}$ Subtract the powers
   $= 4^{-6+4}$
   $= 4^{-2}$
   $= \frac{1}{4^2}$ Simplify

3. Simplify: $18^{-12} \times 18^{-3}$

Solution

$$
\begin{aligned}
18^{-12} \times 18^{-3} &= 18^{(-12)+(-3)} && \text{Add the powers} \\
&= 18^{-15} \\
&= \frac{1}{18^{15}} && \text{Simplify}
\end{aligned}
$$

4. Evaluate: $5^{-7} \div 5^{-10}$

Solution

$$
\begin{aligned}
5^{-7} \div 5^{-10} &= 5^{(-7)-(-10)} && \text{Subtract the powers} \\
&= 5^{-7+10} \\
&= 5^{10-7} \\
&= 5^{3}
\end{aligned}
$$

5. Evaluate: $2^{-6} \div 2^{-6}$

Solution

$$
\begin{aligned}
2^{-6} \div 2^{-6} &= 2^{(-6)-(-6)} && \text{Subtract the powers} \\
&= 2^{-6+6} \\
&= 2^{0} \\
&= 1 && \text{Recall that } a^{0} = 1
\end{aligned}
$$

Practice

Evaluate the following expressions:

1. $3^{-2} \times 3^{-3}$
2. $5^{-12} \div 5^{-4}$
3. $10^{-12} \div 10^{-17}$
4. $21^{-10} \times 21^{-8}$
5. $3^{-50} \times 3^{-3}$
6. $6^{-22} \div 6^{-22}$
7. $14^{-5} \div 14^{-6}$
8. $7^{-4} \times 7^{0}$

Lesson Title: Negative Powers and the Index Laws	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-030	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to apply the index laws to simplifying expressions containing positive and negative powers.

Overview

In this lesson, you will use all 6 index laws to evaluate expressions with positive and negative powers. The index laws are:

1. Multiplication: $a^m \times a^n = a^{m+n}$
2. Division: $a^m \div a^n = a^{m-n}$
3. Power of zero: $a^0 = 1$
4. Power of an index: $(a^m)^n = a^{mn}$
5. Power of a product: $(a \times b)^n = a^n \times b^n$
6. Power of a quotient: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, where $b \neq 0$

You will also use the rule for simplifying negative powers, $a^{-n} = \frac{1}{a^n}$. Remember to use the correct order of operations (BODMAS) to evaluate problems.

Solved Examples

1. Simplify: $a^4 \times a^{-2} \div a^3$

Solution

$$\begin{aligned} a^4 \times a^{-2} \div a^3 &= a^{4+(-2)} \div a^3 && \text{Multiply} \\ &= a^{4-2} \div a^3 \\ &= a^2 \div a^3 \\ &= a^{2-3} && \text{Divide} \\ &= a^{-1} \\ &= \frac{1}{a^1} && \text{Recall that } a^{-1} = \frac{1}{a^1} \\ &= \frac{1}{a} && \text{Recall that } a^1 = a \end{aligned}$$

2. Simplify: $a^5 \times a^{-2} \times b \times b^4$

Solution

First, recall that the first index law can only be applied to indices with the same base. We can multiply the indices with base a separately from those with base b.

$$
\begin{aligned}
a^5 \times a^{-2} \times b \times b^4 &= a^{5+(-2)} \times b^{1+4} && \text{Multiply} \\
&= a^{5-2} \times b^5 && \text{Simplify} \\
&= a^3 \times b^5
\end{aligned}
$$

3. Evaluate: $5^5 \div 5^4 \times 5^{-7}$

Solution

Multiplication and division on indices with the same base can be applied at the same time. We could also work them in separate steps.

$$
\begin{aligned}
5^5 \div 5^4 \times 5^{-7} &= 5^{5-4+(-7)} && \text{Multiply and Divide} \\
&= 5^{5-4-7} && \text{Simplify} \\
&= 5^{-6} \\
&= \frac{1}{5^6}
\end{aligned}
$$

4. Simplify: $(5^{-4})^3$

Solution

Apply the 4th law of indices:

$$
\begin{aligned}
(5^{-4})^3 &= 5^{-4\times 3} && \text{Power of an index} \\
&= 5^{-12} \\
&= \frac{1}{5^{12}}
\end{aligned}
$$

5. Simplify: $\left(\frac{1}{4}\right)^{-3}$

Solution

Apply the 6th law of indices. Note that if $a^{-n} = \frac{1}{a^n}$, then we can also say $a^n = \frac{1}{a^{-n}}$. We should not have a negative power in the denominator of a fraction. It can be moved to the numerator, and the power made positive.

$$
\begin{aligned}
\left(\frac{1}{4}\right)^{-3} &= \frac{1^{-3}}{4^{-3}} && \text{Power of a quotient} \\
&= \frac{4^3}{1^3} && \text{Simplify using } a^{-n} = \frac{1}{a^n} \\
&= \frac{4^3}{1} \\
&= 4^3
\end{aligned}
$$

6. Evaluate: $(2^3)^{-2} \div 2^{-4}$

   Solution

$$
\begin{aligned}
(2^3)^{-2} \div 2^{-4} &= 2^{3\times(-2)} \div 2^{-4} && \text{Power of an index} \\
&= 2^{-6} \div 2^{-4} \\
&= 2^{-6-(-4)} && \text{Divide} \\
&= 2^{-6+4} \\
&= 2^{-2} \\
&= \frac{1}{2^2}
\end{aligned}
$$

7. Evaluate: $a^8 \times (a \times b)^{-3}$

$$
\begin{aligned}
a^8 \times (a \times b)^{-3} &= a^8 \times a^{-3} \times b^{-3} && \text{Power of a product} \\
&= a^{8+(-3)} \times b^{-3} && \text{Divide} \\
&= a^{8-3} \times b^{-3} \\
&= a^5 \times b^{-3} \\
&= \frac{a^5}{b^3}
\end{aligned}
$$

Practice

Evaluate the following expressions:

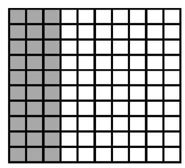
1. $3^{-5} \times 3^7 \div 3^{-4}$
2. $2^{-4} \times 2^2 \times 5^4 \times 5^{-3}$
3. $(7^9)^{-3}$
4. $(2^2)^{-5} \times 2^{-3}$
5. $c^{-4} \times (c \times d)^{-1}$
6. $\left(\frac{1}{3}\right)^{-8}$

Lesson Title: Identifying the Percentage of a Given Quantity	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-031	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the given percentage of a given quantity.

Overview

Percent means part of 100, or out of 100. For example, 30 percent means 30 out of one hundred. The square below is divided into 100 small pieces. Thirty of the squares are shaded. This shows 30%. This can also be written as a fraction: $30\% = \frac{30}{100}$.



Any percentage can be written as a fraction over 100. For example: $25\% = \frac{25}{100}$ and $7\% = \frac{7}{100}$.

To find the percentage of a given quantity, express the percentage as a fraction and then multiply the fraction by the given quantity.

Solved Examples

1. Calculate 15% of 500.

   Solution

   Step 1. Express 15% as a fraction: $\frac{15}{100}$

   Step 2. Find 15% of 500: $\frac{15}{100} \times \frac{500}{1} = \frac{7,500}{100} = 75$

2. Calculate 65% of Le 50,000.00.

   Solution

   When there are units in a problem (for example, Leones) make sure you include the units with your answer. Note that money sometimes includes 2 extra zeros. These are for showing cents. If the two zeros are given in the problem, give them in the answer too. In this problem, Le 50,000.00 = 50,000. These amounts are the same.

   Step 1. Express 65% as a fraction: $\frac{65}{100}$

Step 2. Find 65% of Le 50,000.00: $\frac{65}{100} \times \frac{50,000}{1} = \frac{65 \times 500}{1} =$ Le 32,500.00

3. Find 20% of 90 mangoes.

Solution

Step 1. Express 20% as a fraction: $20\% = \frac{20}{100}$

Step 2. Multiply the fraction by 90: $\frac{20}{100} \times 90 = \frac{1}{5} \times 90 = 18$ mangoes

4. Musu had a total of 50 oranges and she gave 30% of her oranges to her sister. How many oranges did she give away?

Solution

Step 1. Express the percent as a fraction: $30\% = \frac{30}{100}$

Step 2. Multiply the fraction by 50: $\frac{30}{100} \times 50 = \frac{3}{10} \times 50 = 3 \times 5 = 15$ oranges

5. Joe is given $\text{Le } 15{,}000.00$ as lunch and transport to and from school every day. If he spends 40% of this amount as transport to and from school, how much is left for lunch?

Solution

We must first find how much he spends on transportation. Then, subtract that amount from 15,000 to find the amount left for lunch.

Money spent on transportation $= \frac{40}{100} \times 15{,}000 = \text{Le } 6{,}000.00$

Money left for lunch $= \text{Le } 15{,}000 - \text{Le } 6{,}000 = \text{Le } 9{,}000.00$

6. In a school with a pupil population of 900, 55% are girls. How many boys are there in the school?

Solution

We must first find the number of girls. Then, we subtract our answer from the total population.

Step 1. Number of girls $= \frac{55}{100} \times 900 = 55 \times 9 = 495$ girls

Step 2. Number of boys: $= 900 - 495 = 405$ boys

Practice

1. Find 60% of 800.
2. Find 5% of 1,000.
3. Find 2% of Le 48,000.00.
4. Find 35% of 120 mangoes.
5. Fatu bought a bag containing 150 oranges, but 10% were rotten. How many were rotten?
6. A village has a population of 1,500 people. If 28% of the population are children, then how many children are there?
7. A newspaper vendor has 500 newspapers to sell. He sold 25% of them. How many did he sell?

Lesson Title: Expressing One Quantity as a Percentage of Another	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-032	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate one quantity as a percentage of another.

Overview

To express one quantity as a percentage of another, make sure both are in the same unit. You may see problems with different units, for example kilometres and metres. Always convert the bigger units to the smaller one to avoid complications. If you have a problem with both kilometres and metres, convert the kilometres to metres before solving.

To express a quantity as a percentage of another (both in the same unit), write the given quantity as a fraction of the total. Multiply by 100% and simplify.

Solved Examples

1. In a bag containing 250 mangoes, 30 became rotten. What percentage of the mangoes are rotten?

   Solution

$$\begin{aligned}\text{Percentage of rotten mangoes} &= \frac{\text{number of rotten mangoes}}{\text{total number of mangoes}} \times 100\% \\ &= \frac{30}{250} \times \frac{100}{1}\% \\ &= \frac{3}{25} \times \frac{100}{1}\% \\ &= \frac{3}{1} \times \frac{4}{1}\% \\ &= 12\%\end{aligned}$$

2. What percentage of Le 72,000.00 is Le 1,800.00?

   Solution
   Calculate Le 1,800 as a percentage of Le 72,000:

$$\frac{1{,}800}{72{,}000} \times 100\% = \frac{18}{720} \times 100\% = \frac{180}{72} = \frac{5}{2} = 2.5\% \text{ or } 2\frac{1}{2}\%$$

3. On a maths exam, Fatu scored 38 marks out of a total of 40 marks. What percentage did she score?

Solution

Calculate 38 as a percentage of 40:

$$\frac{38}{40} \times 100\% = \frac{19}{20} \times 100\% = \frac{1{,}900}{20}\% = 95\%$$

4. Express 60 g as a percentage of 2 kg.

Solution

Step 1. Use the fact that 1 kg = 1,000 g. Convert kg to g, because grammes are the smaller unit:

$$2\text{ kg} = 2\text{ kg} \times \frac{1{,}000\text{ g}}{1\text{ kg}} = 2{,}000\text{ g}$$

Step 2. Write the given quantity (60 g) as a fraction of the total (2,000 g): $\frac{60}{2{,}000}$

Step 3. Multiply the fraction by 100%:

$$\frac{60}{2{,}000} \times 100\% = \frac{60}{20}\% = 3\%$$

5. In a class of 50 pupils, 35 are girls. Find the percentage of:
   a. Girls in the class
   b. Boys in the class

Solutions

a.

$$\text{Percentage of girls} = \frac{number\ of\ girls\ in\ class}{number\ of\ pupils\ in\ class} \times 100$$
$$= \frac{35}{50} \times 100$$
$$= 70\%$$

b.

$$\text{Number of boys} = number\ of\ pupils - number\ of\ girls$$
$$= 50 - 35$$
$$= 15\ boys$$
$$\text{Percentage of boys} = \frac{number\ of\ boys\ in\ class}{number\ of\ pupils\ in\ class} \times 100$$
$$= \frac{15}{50} \times 100$$
$$= 30\%$$

Practice

1. Express Le 100.00 as a percentage of Le $1,000.00$.
2. Express 400 g as a percentage of 2 kg.
3. Express 36 minutes as a percentage of 75 minutes.
4. During a Mathematics test lasting 1 hour, a student took 9 minutes to answer one question. What percentage of the test time was used to answer the question?
5. Koroma had 300 mangoes and sold 240 of them.
    a. What percentage of the mangoes did he sell?
    b. What is the percentage of mangoes left?
6. In a farm, there are 100 chickens, 700 goats, and 200 sheep. What percentage of the total number of animals on the farm are:
    a. Chickens
    b. Goats
    c. Sheep

Lesson Title: Percentage Increase	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-033	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to calculate the percentage increase given two numbers.

Overview

Percentage change is all about comparing old to new values. A change can be either an increase or a decrease. When the new value is greater than the old value, it is a percentage increase. When the new value is less than the old value, it is a decrease.

This lesson is on percentage increase. To find percentage increase, express the change in quantity as a fraction of the original quantity and then multiply by 100.

The formula for calculating percentage change is:

$$\text{Percentage change} = \frac{\text{change in quantity}}{\text{original quantity}} \times 100$$

To find the percentage increase, we need the change in quantity and the original quantity. Then we substitute the numbers into this formula. We divide them and multiply by 100%. To calculate change in quantity for an increase, subtract the original quantity from the new quantity (New quantity – Original quantity).

Solved Examples

1. The cost of petrol increased from $\text{Le } 4{,}500.00$ to $\text{Le } 6{,}300.00$ per litre. Calculate the percentage increase.

   Solution

   Step 1. Calculate the change in quantity: $6{,}300 - 4{,}500 = \text{Le } 1{,}800.00$

   Step 2. Calculate percentage increase using the formula:

   Percentage increase = $\frac{1{,}800}{4{,}500} \times 100 = \frac{1{,}800}{45} = 40\%$

2. Martin is a farmer. One week, he harvested 12 kg of cassava. The next week, he harvested 21 kg of cassava. What was the percentage increase?

   Solution

   Step 1. Calculate the change in quantity: $21 - 12 = 9$ kg

   Step 2. Calculate percentage increase using the formula:

   Percentage increase = $\frac{9}{12} \times 100 = \frac{3}{4} \times 100 = \frac{300}{4} = 75\%$

3. There were 44 pupils in a class. If 11 more pupils enrol in the class, what is the percentage increase?

   Solution

   Step 1. The change in quantity is given in the problem. It is 11.

   Step 2. Calculate the percentage increase using the formula:

$$\text{Percentage increase} = \frac{11}{44} \times 100 = 25\%$$

Practice

1. A factory increases its annual production of shoes from 4,000 to 4,600. Calculate the percentage increase in the number of shoes.
2. A litre of petrol costs Le 8,000.00, but the price increased to Le 8,500.00. What was the percentage increase?
3. The population of a village increased from 500 to 525. Calculate the percentage increase.
4. Last year, there were 50 babies born at a certain hospital. This year, there were 65 babies born at the same hospital. What is the percentage increase?
5. Fatu scored 60 marks on her maths exam. She was not happy with her score, and decided to study a lot. On her next maths exam, she scored 90 marks. What was the percentage increase of her score?

Lesson Title: Percentage Decrease	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-034	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the percentage decrease given 2 numbers.

Overview

Finding percentage decrease is similar to finding percentage increase, which was the topic of the previous lesson. We use the same formula. To find a percentage decrease, express the change in quantity as a fraction of the original quantity and then multiply by 100.

The formula for calculating percentage change is:

$$\text{Percentage change} = \frac{\text{change in quantity}}{\text{original quantity}} \times 100$$

The difference is how we find the change in quantity. To calculate change in quantity for a **decrease**, subtract the new quantity from the original quantity (original quantity – new quantity). This is the opposite of what we did to find the change in quantity for an increase. In summary, this is how to find the change in quantity for each:

- Increase: new quantity – original quantity
- Decrease: original quantity – new quantity

Note that we always subtract the smaller number from the larger number.

Solved Examples

1. Mustapha is a farmer. Last week, he sold his cassava for Le 4,000.00 per kilogramme. This week, there is more cassava on the market. He could only sell his cassava for Le 3,800.00 per kilogramme. What is the percentage decrease in the price?

   Solution

   Step 1. Calculate the change in quantity: $4{,}000 - 3{,}800 = \text{Le } 200.00$

   Step 2. Calculate percentage decrease using the formula:

   $$\text{Percentage decrease} = \frac{200}{4{,}000} \times 100 = \frac{200}{40} = \frac{20}{4} = 5\%$$

2. A new health centre was built in a particular town and the number of babies dying per month decreased from 20 to 8. Calculate the percentage decrease.

Solution

Step 1. Calculate the change in quantity: $20 - 8 = 12$ babies

Step 2. Calculate percentage increase using the formula:

$$\text{Percentage decrease} = \frac{12}{20} \times 100 = \frac{120}{2} = 60\%$$

3. The population of a village was 800 people. Twenty young people moved away to enrol in university. What is the percentage decrease?

Solution

Step 1. The change in quantity is given in the problem. It is 20.

Step 2. Calculate the percentage decrease using the formula:

$$\text{Percentage decrease} = \frac{20}{800} \times 100 = \frac{20}{8} = 2.5\%$$

Practice

1. Fatu scored 90 marks on her previous maths exam. She was very happy with her score, and she did not study for the next exam. She scored 45 marks on her next exam. What was the percentage decrease of her score?
2. A litre of petrol cost Le 8,000.00, but the price decreased to Le 7,400.00. What is the percentage decrease?
3. The population of a village decreased from 640 to 560. Calculate the percentage decrease.
4. Last year, there were 8,000 patients treated at a certain hospital. This year, 7,600 patients were treated at the same hospital. What is the percentage decrease?
5. Hawa sells oranges at the market. She had 100 oranges, and she sold 51 of them. Calculate the percentage decrease.
6. A factory produced 3,456 bicycles in 1968 and 2,880 in 1969. Calculate the percentage decrease in production from 1968 to 1969.

Lesson Title: Applying Percentage Increase and Decrease	Theme: Numbers and Numeration
Practice Activity: PHM-08-035	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to calculate a number given the percentage increase or decrease upon a given number.

Overview

The previous lessons focused on finding the percentage increase or decrease given 2 numbers. In this lesson, you will find the new quantity after an increase or decrease. If there is a percentage increase, it means we add to the original amount. If there is a percentage decrease, it means we subtract from the original amount.

To calculate a new quantity given the percentage increase or decrease upon the original, given number, follow these steps:

- State the increase or decrease in percent.
- For percent increase, **add** the percentage to 100%.
- For percent decrease, **subtract** the percentage from 100%.
- Since it is percent, divide the answer by 100 to cancel the percentage.
- Multiply the answer by the given number to give the new number.

Use the following formulae:

Percentage increase: $\text{New number} = \frac{100+\text{percentage increase}}{100} \times \frac{\text{the given number}}{1}$

Percentage decrease: $\text{New number} = \frac{100-\text{percentage decreae}}{100} \times \frac{\text{the given number}}{1}$

Solved Examples

1. The number 600 is increased by 35%. Find the new number.

Solution

Use the formula for percentage increase:

$$\begin{aligned} \text{New number} &= \frac{100+\text{percentage increase}}{100} \times \frac{\text{the given number}}{1} \\ &= \frac{100+35}{100} \times \frac{600}{1} \\ &= \frac{135}{1} \times \frac{6}{1} \\ &= 135 \times 6 \end{aligned}$$

$= \quad 810$

2. The number 600 is decreased by 35%. Find the new number.

   Solution

   Use the formula for percentage decrease:

$$\text{New number} = \frac{100-\text{percentage decrease}}{100} \times \frac{\text{the given number}}{1}$$

$$= \frac{100-35}{100} \times \frac{600}{1}$$

$$= \frac{65}{1} \times \frac{6}{1}$$

$$= 65 \times 6$$

$$= 390$$

3. The population of a certain village was 5,600 people. If the population increased by 12%, what is the new population?

   Solution

   Use the formula for percentage increase to find the new population:

$$\text{New number} = \frac{100+\text{percentage increase}}{100} \times \frac{\text{the given number}}{1}$$

$$= \frac{100+12}{100} \times \frac{5{,}600}{1}$$

$$= \frac{112}{1} \times \frac{56}{1}$$

$$= 112 \times 56$$

$$= 6{,}272 \text{ people}$$

4. A man brought a piece of land for Le 500,000.00. Ten years later, the value of the land had increase by 60%. Calculate the new value of the land.

   Solution

$$\text{The new value} = \frac{100+60}{100} \times \frac{\text{Le } 500{,}000}{1}$$

$$= \frac{160}{100} \times \frac{\text{Le } 500{,}000}{1}$$

$$= 160 \times \text{Le } 5{,}000$$

$$= \text{Le } 800{,}000.00$$

5. A track which was 60 m long was decreased by 15%. Calculate the new length of the track.

Solution

The new length $= \frac{100-15}{100} \times \frac{60}{1}$

$= \frac{85}{100} \times \frac{60}{1}$

$= \frac{85}{10} \times 6$

$= 8.5 \times 6$

$= 51$ m

6. The price of a phone increased 10%. The price is now Le 550,000.00. Calculate the price of the phone before the increase.

Solution

In this case, we are given the new number, and we want to find the given number. Remember that a 'given number' is the original amount before a percentage increase or decrease. Substitute the numbers from the problem into the formula and solve:

$$\text{New number} = \frac{100+\text{percentage increase}}{100} \times \frac{\text{the given number}}{1}$$

$$550{,}000 = \frac{100+10}{100} \times \frac{\text{original price}}{1}$$

$$550{,}000 = \frac{110}{100} \times \frac{\text{original price}}{1}$$

$$550{,}000 \times 100 = 110 \times \text{original price}$$

$$\frac{55{,}000{,}000}{110} = \text{original price}$$

$$\text{Le } 500{,}000.00 = \text{original price}$$

The original price of the phone was Le 500,000.00.

Practice

1. A seamstress gives a discount of 5% for customers who pay beforehand. Calculate the reduced price of a dress that originally cost Le 70,000.00.
2. An athlete took 10 seconds to sprint 100 m during practice. If in the actual race, he reduced his time by 8%, how long did it take him to run the actual race?
3. Increase a length of 80 cm by 30%.
4. A messenger received a salary of Le 68,500.00. She is promoted to a higher grade and his salary increases by 14%. Calculate her new salary.
5. The number of pupils enrolled in a certain school was 520 last year. This year, enrolment increased by 15%. How many pupils are enrolled in the school this year?
6. During a clearance sale, a shop reduced the prices of all its goods by 20%. A customer bought a pair of shoes for Le 35,000.00 during the sale. Calculate the price of the shoes before the sale.

Lesson Title: Introduction to Profit and Loss	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-036	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Compare profit to loss.
2. Identify that profit is a percentage increase and loss is a percentage decrease.

Overview

In this lesson, you will identify what profit and loss are and compare them.

We have some special terms for businesses. When you have a business that is making money, you have made a profit. When your business is losing money, it is called a loss.

You can use percentage increase or decrease to calculate how much money you are making or losing in business. We can calculate the percent of profit and percent of loss using formulae that are similar to percent increase and percent decrease. Remember that the formula for finding percent increase and percent decrease is $\frac{\text{change in quantity}}{\text{original quantity}} \times 100\%$.

Here are the formulae for calculating percentage profit and loss:

$$\text{Percent profit} = \frac{\text{profit}}{\text{capital}} \times 100\%$$

$$\text{Percent loss} = \frac{\text{loss}}{\text{capital}} \times 100\%$$

Capital is the money you have to start and run a business. In these formulae, you can see that profit or loss is the change in quantity and capital is the original quantity.

With percent increase and decrease remember the change in quantity was the difference between the new quantity and the original quantity. For profit and loss, we find the difference between the money spent (cost/capital) and the money you earn (sales).

Profit - Money made after costs have been subtracted from sales: $\text{Profit} = \text{sales} - \text{cost}$

Loss - Money lost after costs have been subtracted from sales: $\text{Loss} = \text{cost} - \text{sales}$

If sales are greater than costs, then the business made a profit. If costs are greater than sales, then the business has a loss. Remember that profit is an increase and loss is a decrease.

Solved Examples

1. Zinab wants to start a rice farm. She spends Le 100,000.00 to buy new land, Le 25,000.00 on new tools, Le 15,000.00 on seeds, and Le 30,000.00 for Musa to help her harvest the rice. She sells all of her rice for Le 300,000.00.
    a. What are the total costs and sales?
    b. Did Zinab's farm have a profit or a loss? Explain.
    c. Calculate her profit or loss.

    Solutions

    a. Calculate Zinab's total costs. To find this we must add up her costs:

    $\text{Cost} = \text{Le } 100{,}000 + \text{Le } 25{,}000 + \text{Le } 15{,}000 + \text{Le } 30{,}000 = \text{Le } 170{,}000.00$

    Her total sales is how much she sells the rice for: $\text{Sales} = \text{Le } 300{,}000.00$

    b. Zinab's farm had a profit, because her sales are more than her cost: $300{,}000 > 170{,}000$

    c. Use the formula for profit:

    $$\text{Profit} = \text{sales} - \text{cost} = \text{Le } 300{,}000 - \text{Le } 170{,}000 = \text{Le } 130{,}000.00$$

2. Mohamed is a taxi driver. He owns his car. In January, he spent Le 400,000.00 on petrol. His car had problems, and he spent Le 300,000.00 to repair it. He also had to pay Le 50,000.00 in taxi union fees. He earned Le 700,000.00 in taxi fares from his passengers.
    a. What are Mohamed's total costs and sales?
    b. Did Mohamed have a profit or loss? Explain.
    c. Calculate his profit or loss.

    Solutions

    a. Calculate Mohamed's total cost:

    $$\text{Cost} = \text{Le } 400{,}000 + \text{Le } 300{,}000 + \text{Le } 50{,}000 = \text{Le } 750{,}000.00$$

    His total sales are $\text{Le } 700{,}000.00$.

    b. Mohamed had a loss, because his sales are less than his costs ($700{,}000 < 750{,}000$).

    c. Use the formula for loss:

    $$\text{Loss} = \text{cost} - \text{sales} = 750{,}000 - 700{,}000 = \text{Le } 50{,}000.00$$

3. In February, Mohamed spent Le 420,000.00 on petrol and Le 50,000.00 on union fees. His car didn't have any problems. He earned Le 600,000.00 in taxi fares from his passengers.
    a. What are Mohamed's total costs and sales?
    b. Did Mohamed have a profit or loss? Explain.
    c. Calculate his profit or loss.

Solutions

a. Calculate Mohamed's total cost:

$$\text{Cost} = \text{Le } 420{,}000 + \text{Le } 50{,}000 = \text{Le } 470{,}000.00$$

His total sales are Le $600{,}000.00$.

b. Mohamed had a profit, because his sales are greater than his costs ($600{,}000 < 470{,}000$).

c. Use the formula for profit:

$$\text{Profit} = \text{sales} - \text{cost} = 600{,}000 - 470{,}000 = \text{Le } 130{,}000.00$$

Practice

1. Fatu is an artist. This month, she spent Le 200,000.00 on new paint, and Le 300,000.00 on a new canvas. She sold 2 paintings. One painting was sold for 350,000.00 and the other painting was sold for Le 180,000.00.
    a. What are the total costs and sales?
    b. Did Fatu have a profit or a loss? Explain.
    c. Calculate her profit or loss.

2. Mustapha is a baker. This month, he spent Le 500,000.00 on flour and Le 100,000.00 on oil. He replaced the roof on his shop, which cost him Le 1,000,000.00. Mustapha made Le 1,400,000.00 in profit from selling his bread this month.
    a. What are the total costs and sales?
    b. Did Mustapha have a profit or a loss? Explain.
    c. Calculate his profit or loss.

Lesson Title: Calculating Profit	Theme: Everyday Arithmetic
Practice Activity: PHM-08-037	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to apply percentages to calculate profit on a transaction.

Overview

Recall that profit is the amount of money gained after the cost is subtracted from sales:

$$\text{Profit} = \text{sales} - \text{cost}$$

We can calculate percentage profit with a formula: $\text{Percentage profit} = \frac{\text{profit}}{\text{capital}} \times 100\%$.

Percentage profit is useful to understand how much your profit is compared to the amount of capital that you spent. For example, consider 2 business investments:

a. Fatu spent Le 100,000.00 on her business.
b. Martin spent Le 200,000.00 on his business.

Let's say both Fatu and Martin made a profit of Le 50,000.00. To understand how successful their businesses are, we should calculate their percentage profits:

a. Fatu's percentage profit $= \frac{50{,}000}{100{,}000} \times 100\% = 50\%$
b. Martin's percentage profit $= \frac{50{,}000}{200{,}000} \times 100\% = 25\%$

Although they both made the same profit, Fatu's percentage profit is greater. She made more money as a percentage of her capital. It looks like her business is more successful. To understand the success of your business, it is important to calculate profit and percentage profit.

Solved Examples

1. Jeneba owns a car. She pays Le 22,500.00 for gasoline. She drives 5 passengers for Le 8,100.00 each. Calculate Jeneba's percent profit.

Solution

Step 1. Calculate her profit using the formula: $\text{Profit} = \text{sales} - \text{cost}$

Sales: $5 \times \text{Le } 8{,}100 = \text{Le } 40{,}500$

Costs (gasoline): $\text{Le } 22{,}500$

Profit: $\text{sales} - \text{cost} = 40{,}500 - 22{,}500 = 18{,}000$

Step 2. Calculate her percentage profit:

$$\text{Percentage profit} = \frac{\text{profit}}{\text{capital}} \times 100\%$$
$$= \frac{18{,}000}{22{,}500} \times 100\%$$
$$= 80\%$$

2. A watermelon which was bought for GH₵1.00 was sold at GH₵1.70. Calculate the profit percent.

Solution

On the BECE exam, you may see currencies that are not Leones. Treat the problem the same as you would treat Leones. Write the correct unit on the answer. This problem handles the currency of Ghana, the Ghanaian cedi, which has the symbol GH₵. You may also see dollars ($).

Step 1. Calculate profit using the formula: $\text{Profit} = \text{sales} - \text{cost}$

Sales: GH₵1.70

Costs: GH₵1.00

Profit: $\text{sales} - \text{cost} = 1.70 - 1.00 = 0.70$

Step 2. Calculate the percentage profit:

$$\text{Percentage profit} = \frac{\text{profit}}{\text{capital}} \times 100\%$$
$$= \frac{0.70}{1.00} \times 100\%$$
$$= \frac{70\%}{1.00}$$
$$= 70\%$$

3. Esther sells goods in the market in her village. She pays Le 20,000.00 for a carton of 30 eggs, which she buys in Freetown. The cost of her travel from Freetown to her village is Le 40,000.00. She sells the 30 eggs for Le 3,000.00 each. Calculate Esther's percent profit.

Solution

Step 1. Calculate her profit using the formula: $\text{Profit} = \text{sales} - \text{cost}$

Sales: $30 \times \text{Le } 3{,}000 = \text{Le } 90{,}000.00$

Costs: $\text{Carton of eggs} + \text{transport} = 20{,}000 + 40{,}000 = \text{Le } 60{,}000.00$

Profit: $\text{sales} - \text{cost} = 90{,}000 - 60{,}000 = \text{Le } 30{,}000.00$

Step 2. Calculate her percentage profit:

$$\text{Percentage profit} = \frac{\text{profit}}{\text{capital}} \times 100\%$$
$$= \frac{30{,}000}{60{,}000} \times 100\%$$
$$= 50\%$$

Practice

1. A book which was bought for Le 8,000.00 was sold at Le 10,000.00. Calculate the profit percent.
2. A bag of oranges which was bought for GH₵5.00 was sold at GH₵7.50. Calculate the percentage profit.
3. Francis is a tailor. He spent Le 100,000.00 on cloth to make a dress. He spent Le 25,000.00 on other materials. If he sold the dress for Le 200,000.00, what was his percentage profit?
4. Hawa sells fruit in the market. One day, she paid Le 220,000.00 for pineapples. She also paid Le 5,000.00 to rent a market table. She sold all of the pineapples for Le 315,000.00. What was her percentage profit?

Lesson Title: Calculating Loss	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-038	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to apply percentages to calculate loss on a transaction.

Overview

Recall that loss is the amount of money lost after the sales are subtracted from the cost:

$$\text{Loss} = \text{cost} - \text{sales}$$

We can calculate percentage loss with a formula: $\text{Percentage loss} = \frac{\text{loss}}{\text{capital}} \times 100\%$

Percentage loss is useful to understand in terms of how much your loss is compared to the amount of capital that you spent. For example, consider 2 business investments:

a. Mustapha spent Le 100,000.00 on his business.
b. Hawa spent Le 200,000.00 on her business.

Let's say both Mustapha and Hawa had a loss of Le 50,000.00. To understand how successful their businesses are, we should calculate their percentage loss:

a. Mustapha's percentage loss $= \frac{50{,}000}{100{,}000} \times 100\% = 50\%$
b. Hawa's percentage loss $= \frac{50{,}000}{200{,}000} \times 100\% = 25\%$

Although they both had the same loss, Mustapha's loss is a greater percentage of his cost. He lost more money as a percentage of his capital. His business is less successful because he has lost more.

To understand the success of your business, it is important to calculate loss and percentage loss. A greater percentage loss means that your business has been less successful. You would want to make different decisions to try to make a profit.

Solved Examples

1. Amad spent Le 120,000.00 on supplies to start a carpentry business, and Le 130,000.00 on building a shop. During the first month, he sold 4 pieces of furniture each costing Le 50,000.00. Calculate Amad's percentage loss.

   Solution

   Step 1. Calculate his loss using the formula: $\text{Loss} = \text{cost} - \text{sales}$

   Cost: $\text{Le } 120{,}000 + \text{Le } 130{,}000 = \text{Le } 250{,}000.00$

   Sales: $4 \times \text{Le } 50{,}000.00 = \text{Le } 200{,}000.00$

Loss: $\text{cost} - \text{sales} \;=\; 250{,}000 - 200{,}000 \;=\; 50{,}000$

Step 2. Calculate his percentage loss:

$$\begin{aligned}\text{Percentage loss} &= \frac{\text{loss}}{\text{capital}} \times 100\% \\ &= \frac{\text{Le } 50{,}000}{\text{Le } 250{,}000} \times 100\% \\ &= 20\%\end{aligned}$$

2. Fatima wants to start a tailoring business. She pays Le 780,000.00 for a sewing machine and Le 20,000.00 for other supplies. After one month, she has tailored two suits for Le 290,000.00 each. Calculate Fatima's percentage loss.

Solution

Step 1. Calculate her loss using the formula: $\text{Loss} \;=\; \text{cost} - \text{sales}$

Cost: $\text{Le } 780{,}000 + \text{Le } 20{,}000 \;=\; \text{Le } 800{,}000.00$

Sales: $2 \times \text{Le } 290{,}000.00 = \text{Le } 580{,}000.00$

Loss: $\text{cost} - \text{sales} \;=\; \text{Le } 800{,}000 - \text{Le } 580{,}000 = \text{Le } 220{,}000.00$

Step 2. Calculate her percentage loss:

$$\begin{aligned}\text{Percentage loss} &= \frac{\text{loss}}{\text{capital}} \times 100\% \\ &= \frac{\text{Le } 220{,}000}{\text{Le } 800{,}000} \times 100\% \\ &= 27.5\%\end{aligned}$$

3. A pineapple which was bought for GH₵1.50 was sold at GH₵1.20. Calculate the percentage loss.

Solution

Step 1. Calculate the loss using the formula: $\text{Loss} \;=\; \text{cost} - \text{sales}$

Cost: GH₵1.50

Sales: GH₵1.20

Loss: $\text{cost} - \text{sales} \;=\; 1.50 - 1.20 \;= \text{GH₵}0.30$

Step 2. Calculate the percentage loss:

$$\begin{aligned}\text{Percentage loss} &= \frac{\text{loss}}{\text{capital}} \times 100\% \\ &= \frac{0.30}{1.50} \times 100\% \\ &= 20\%\end{aligned}$$

Practice

1. A book which was bought for Le 10,000.00 was sold at Le 8,000.00. Calculate the percentage loss.
2. A bag of cassava which was bought for GH₵12.00 was sold at GH₵9.00. Calculate the percentage loss.
3. Francis is a tailor. He spent Le 120,000.00 on the cloth to make a dress. He spent Le 20,000.00 on other materials. His machine broke, and he paid Le 60,000.00 for repairs. If he sold the dress for Le 190,000.00, what was his percentage loss?
4. Hawa sells fruit in the market. One day, she paid Le 235,000.00 for watermelons. She also paid Le 5,000.00 to rent a market table. Some of the watermelons were spoiled. She sold the good watermelons for a total of Le 180,000.00. What was her percentage loss?

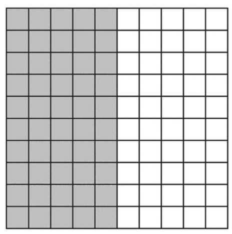
Lesson Title: Introduction to Percentages Greater than 100	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-039	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to identify percentages greater than 100 as more than one whole.

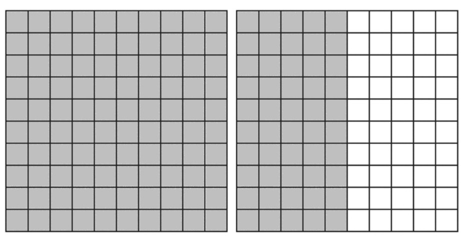
Overview

This lesson is on percentages greater than 100. We know that a percentage is any part or share of a whole. Percentages can also be more that 100 if you have more than the whole.

Consider 50%. We can show this as a part of 1 whole. In the diagram below, 50 out of 100 small squares are shaded:



Now consider 150%. This is 100% + 50%. This is 1 whole, and another 50%. It can be shown in a diagram like this:



Percentages over 100 follow the same rules as other percentages. For example, to express them as a fraction you still put them over 100. 150% as a fraction is $\frac{150}{100}$.

Solved Examples

1. Calculate 100 as a percentage of 80.

   Solution
   Recall that to calculate one number as a percentage of another, we write the numbers as a fraction and multiply by 100%:

$$\frac{100}{80} \times 100\% = \frac{10}{8} \times 100\% = \frac{1000}{8}\% = 125\%$$

2. Write the following percentages as fractions over 100:
    a. 117% b. 140% c. 187% d. 219%

    Solution

    In the next lesson, you will use fractions like these to perform calculations. The answers are:

    a. $\frac{117}{100}$ b. $\frac{140}{100}$ c. $\frac{187}{100}$ d. $\frac{219}{100}$

3. A pupil studied for 20 minutes one night. The next night she studied for 50 minutes. What is the percentage increase?

    Solution

    Recall the formula for percentage increase:

    $$\text{Percentage increase} = \frac{\text{change in quantity}}{\text{original quantity}} \times 100\%$$

    Step 1. Calculate the change in quantity: 50 – 20 = 30 minutes

    Step 2. Calculate the percentage increase:

    $$\begin{aligned} \text{Percentage increase} &= \frac{\text{change in quantity}}{\text{original quantity}} \times 100\% \\ &= \frac{30}{20} \times 100\% \\ &= \frac{3}{2} \times 100\% \\ &= \frac{300\%}{2} \\ &= 150\% \end{aligned}$$

    Answer: The pupil studied 150% more on the second night.

Practice

1. Calculate 90 as a percentage of 60.
2. Calculate 100 as a percentage of 40.
3. Calculate 180 as a percentage of 160.
4. Write the following percentages as fractions over 100:
    a. 102%
    b. 199%
    c. 200%
5. Mohamed sold 20 pineapples one day, and 25 pineapples the next day. What was the percentage increase?

Lesson Title: Calculations with Percentages Greater than 100	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-040	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to calculate the percentage of a number where the percentage is greater than 100.

Overview

In this lesson, you will calculate the percentage of a number where the percentage is greater than 100. If needed, review lesson PHM-08-031. That lesson was on calculating the percentage of a number when the percentage was less than 100%. We will use the same process.

To find the percentage of a given quantity, express the percentage as a fraction and then multiply the fraction by the given quantity.

Note that if the percentage is less than 100%, the new quantity will be less than the original quantity. If the percentage is greater than 100% (as in this lesson), the new quantity will be more than the original quantity.

Solved Examples

1. Calculate 120% of 20.

   Solution

   Write 120% as a fraction over 100, and multiply it by 20:

   120% of 20 $= \frac{120}{100} \times \frac{20}{1} = \frac{2,400}{100} = 24$

2. Calculate 200% of 50.

   Solution

   Write 200% as a fraction over 100, and multiply it by 50:

   200% of 50 $= \frac{200}{100} \times \frac{50}{1} = 100$

   - Note that 200% of 50 is 100, which is twice as much as 50. Finding 200% of a number is the same as doubling that number.

3. Calculate 140% of Le 20,000.00.

Solution

Treat money the same as any other number. Write 140% as a fraction over 100, and multiply it by Le 20,000.00:

140% of 20,000.00 $= \frac{140}{100} \times \frac{20,000}{1} = 140 \times 200 = \text{Le } 28,000.00$

4. Last year, there were 85 pupils enrolled in JSS 2. The enrolment this year is 120% of last year's enrolment. What is the enrolment in JSS 2 this year?

Solution

We want to find 120% of 85 pupils. Write 120% as a fraction over 100, and multiply by 85:

120% of 85 pupils $= \frac{120}{100} \times \frac{85}{1} = 102$ pupils

There are 102 pupils enrolled in JSS 2 this year.

Practice

1. Find 120% of 80.
2. Find 300% of 4.
3. Find 175% of 60.
4. Calculate 250% of Le 8,000.00.
5. Calculate 140% of Le 10,000.00.
6. Last year, there were 140 people living in a village. The population this year is 105% of the population last year. How many people live in the village now?
7. Last week, Mustapha sold 92 kilogrammes of cassava. The amount he sold this week is 125% of what he sold last week. How many kilogrammes of cassava did he sell this week?

Lesson Title: Ratio	**Theme:** Numbers and Numeration
Practice Activity: PHM-08-041	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to identify the forms of ratio: m:n and m/n and simplify ratios to their lowest terms.

Overview

Ratio is a way of comparing two or more quantities. For example, if you compare the number of boys and girls in your class, that would be a ratio.

Say that you are in a JSS 2 class where there are 22 boys and 24 girls. This can be written as a ratio in two ways: 22 : 24 or $\frac{22}{24}$. The ratio is read as "22 is to 24". It can also be said as "22 boys is to 24 girls".

A ratio can be simplified to its lowest terms, like any fraction. It is easiest to write the ratio in the fractional form before simplifying: $\frac{22}{24}=\frac{22\div2}{24\div2}=\frac{11}{12}$. The simplified form can be written $\frac{11}{12}$ or 11 : 12. This means that for every 11 boys in the class, there are 12 girls.

The order in which ratios are written is very important and must be maintained when solving a problem. A ratio written as 2 : 3 means $\frac{2}{3}$, while a ratio written as 3 : 2 means $\frac{3}{2}$. The fractions are different and give different answers.

We can only simplify ratios when the quantities are in the same units. If the quantities are not in the same unit, we must convert one to the other before we simplify.

Solved Examples

1. Express $24 : 14$ as a fraction in its lowest terms.

   Solution
   Write $24 : 14$ as a fraction and simplify it: $24 : 14 = \frac{24}{14} = \frac{12}{7}$

2. What is 20 centimetres as a ratio of 1 metre? Give your answer as a fraction in its simplified form.

   Solution
   Quantities must be in the same unit before writing them as a ratio. Convert 1 metre to centimetres. We know that 1 m = 100 cm.
   The ratio is 20 : 100. Write this as a fraction and simplify it: $20 : 100 = \frac{20}{100} = \frac{1}{5}$

3. Mohamed received an 85% mark on an exam. What ratio of correct answers did Mohamed get? Write your answer as a fraction in its simplified form.

   Solution
   To solve this, we must remember that when we talk about percent, we compare a number to 100. 85% is the same as $\frac{85}{100}$. We also know that $\frac{85}{100} = 85 : 100$. This is the ratio that we want to simplify: $\frac{85}{100} = \frac{17}{20}$.

4. Two sellers travelled different distances to the market. Samuel went 6 km and Alice went 8 km. What is the ratio of Samuel's travel to Alice's in the lowest term?

   Solution
   The ratio of Samuel's travel to Alice's travel is 6 : 8. We can write these as a ratio because they are in the same units.
   Write the ratio as a fraction and simplify it: $\frac{6}{8} = \frac{3}{4}$

5. Marie sold 250 kg of cassava. She has 750 kg remaining. What is the ratio of sold cassava to unsold cassava expressed as a fraction in the lowest term?

   Solution
   The ratio of sold cassava to unsold cassava is 250 : 750. We can write these as a ratio because they are in the same units.
   Write the ratio as a fraction and simplify it: $\frac{250}{750} = \frac{1}{3}$

Practice

1. Express $20 : 35$ as a fraction in its lowest terms.
2. Express $12 : 48$ as a fraction in its lowest terms.
3. Express $200 : 800$ as a fraction in its lowest terms.
4. Express $150 : 450$ as a fraction in its lowest terms.
5. Hawa received a 95% mark on an exam. What ratio of correct answers did Hawa get? Write your answer as a fraction in its simplified form.
6. Mohamed has a shop near a school. He sold 150 exercise books this year, and 50 exercise books remain in his shop. What is the ratio of sold exercise books to unsold exercise books? Give your answer as a fraction in its lowest terms.

Lesson Title: Rate	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-042	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that rate is a special ratio that compares two units of measurement.
2. Identify notation for rates.

Overview

We use ratios to compare two or more **like** quantities. This means that they are of the same kind. For example, they are both people (boys and girls), or they are both measurements of weight (kg and g). Remember that measurements must be expressed in the same unit for them to be compared as a ratio.

Rate is a special ratio that compares two **different** type of units of measurement. For example, how much money someone is paid per month at their job. The different quantities are time and money.

The quantities in a ratio are measured with the same unit. When we write the ratio as a fraction, the units in the ratio cancel each other out because they are the same. For example, $\frac{1\text{ km}}{2\text{ km}} = \frac{1\,\cancel{\text{km}}}{2\,\cancel{\text{km}}} = \frac{1}{2}$.

The quantities in a rate are measured with 2 different units. The units in a rate take on the unit from the numerator and the unit from the denominator. For example, if a car travels 3 kilometres in 1 hour, we have the rate $\frac{\text{distance}}{\text{time}} = \frac{3\text{ km}}{1\text{ hr}} = 3\frac{\text{km}}{\text{hr}} = 3\text{ km/hr}$. This can be read as "3 kilometres per hour".

Solved Examples

1. A man buys 100 grammes of rice for Le 3,600.00.
   a. Write this as a rate of Leones per grammes.
   b. Write the rate in its simplest form.

   Solutions

   a. The rate is $\frac{\text{Le }3{,}600.00}{100\text{ g.}}$. This is read as "Le 3,600 per 100 g of rice". This means that for Le 3,600 you can purchase 100 g of rice.
   b. Simplify the fraction: $\frac{\text{Le }3{,}600.00}{100\text{ g.}} = \frac{\text{Le }36.00}{1\text{ g.}}$. This ratio is commonly written as $\text{Le }36.00/\text{g}$. It is read as "Le 36 per gramme of rice".

2. If you walk 12 miles in 3 hours, at what rate are you traveling? Give your answer in miles per hour.

   Solution

   You are walking at a rate of 12 miles per 3 hours, which can be written as $\frac{12\text{ mi}}{3\text{ hr}}$. This can be simplified: $\frac{12\text{ mi}}{3\text{ hr}} = \frac{4\text{ mi}}{1\text{ hr}} = 4\text{ mi/hr}$.
   Note that miles per hour is often written mph. We can write the answer as 4 mph.

3. Calculate the average speed of a vehicle that travels 100 miles in 4 hours.

   Solution

   To calculate the average speed, simply find the rate at which the vehicle travelled. The process is the same as in problem 2.

   The average speed is $\frac{100\text{ mi}}{4\text{ hr}} = \frac{25\text{ mi}}{1\text{ hr}} = 25\text{ mph}$

4. Abigail is a tailor. She can sew 10 dresses in 5 days. What is her rate for sewing dresses?

   Solution

   Her rate is $\frac{10\text{ dresses}}{5\text{ days}} = \frac{2\text{ dresses}}{1\text{ day}} = 2\text{ dresses/day}$.

5. Mustapha sells chicken 4 kilogrammes for Le 100,000.00. What is the rate of the price for chicken? Give your answer in Leones per kilogramme.

   Solution

   The rate of the price for chicken is $\frac{\text{Le }100{,}000}{4\text{ kg}} = \frac{\text{Le }25{,}000}{1\text{ kg}} = \text{Le }25{,}000.00/\text{kg}$

6. John is working on his maths assignment. There are 10 problems on the assignment. If it takes him 45 minutes to solve the problems, what is his rate in minutes per problem?

   Solution

   His rate in problems per minute is $\frac{45\text{ minutes}}{10\text{ problem}} = \frac{9}{2}\text{ minutes/problem}$.
   You may also write your answer as a decimal: $4.5\text{ minutes/problem}$. Rates are often given as decimal numbers.

Practice

1. A car travels 60 kilometres in 2 hours. Find the car's rate in km/hr.
2. A man walks 15 kilometres in 3 hours. Find the man's rate in km/hr.
3. Mohamed is an auto mechanic. He can fix 20 cars in 8 days. What is his rate of fixing cars?
4. Fatu sat a maths exam. She solved 20 problems in 40 minutes. What is her rate in minutes per problem?
5. Foday harvested his peppers. He worked for 4 hours and harvested 12 kg of peppers. At what rate does he harvest pepper? Give your answer in kg/hr.
6. Calculate the average speed of a vehicle travelling from Moyamba junction to Masiaka, a distance of 85 km in 5 hours.
7. A car needs 4 litres of petrol to travel 45 km. What is its rate of petrol consumption?

Lesson Title: Unit Rate	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-043	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Perform basic calculations to find unit rate.
2. Convert different rates to their unit rates.

Overview

When a rate has the number 1 in the denominator, we call it a **unit rate**, because it is the rate per 1 unit. For example, Foday is a writer. He writes 500 words per hour. This rate is $\frac{500 \text{ words}}{1 \text{ hour}}$. The denominator is 1. This rate can be written as 500 words/hr.

We had problems that have unit rate answers in the previous lesson. For example, in Solved Example 2, you found that you travelled at $\frac{4 \text{ mi}}{1 \text{ hr}}$, which can be written as 4 mi/hr or 4 mph. Unit rates can be written as one number, with the rate given after.

Solved Examples

1. I can carry 45 coconuts in 3 bags. How many coconuts fit per bag?

   Solution

   Write the ratio of coconuts per bag, and simplify: $\frac{45 \text{ coconuts}}{3 \text{ bag}} = \frac{15 \text{ coconuts}}{1 \text{ bag}} =$ 15 coconuts/ bag

2. Sando is traveling home to the village. It takes her 3 hours to travel 210 km. What is her rate of travel in kph?

   Solution

   This problem asks us to find the rate in kph, which is kilometres/hour. We need to find out the number of kilometres that Sando travelled in one hour.

   Sando's rate of travel is $\frac{210 \text{ km}}{3 \text{ hours}}$. We can simplify this to get 1 in the denominator, and make this a unit ratio:

   $\frac{210 \text{ km}}{3 \text{ hours}} = \frac{70 \text{ km}}{1 \text{ hr}} = 70 \text{ km/hour or } 70 \text{ kph}$

3. There are 350 pupils divided among 7 classrooms. What is the unit rate for 1 classroom?

Solution

Write the ratio of pupils per classroom: $\frac{350 \text{ pupils}}{7 \text{ classrooms}}$

Simplify the fraction to find the number of pupils per 1 classroom:

$\frac{350 \text{ pupils}}{7 \text{ classrooms}} = \frac{50 \text{ pupils}}{1 \text{ classroom}} = 50 \text{ pupils/classroom}$

Practice

1. A farmer brings peppers from his farm to the market. He brings 150 kg of peppers in 10 bags. How many kilogrammes of pepper fit per bag?
2. There are 90 football players divided among 6 teams. What is the unit rate for 1 team?
3. Sia is an artist. She can paint 36 paintings in 4 weeks. What is her unit rate of paintings per week?
4. Eighteen teachers from one school travel to Freetown for a meeting. If they take 3 cars, what is the unit rate of teachers per car?
5. It takes a car 3 hours to travel from Freetown to Bo, a distance of 240 km. At what rate did the car travel in kph?
6. It takes the same car 4 hours to return from Bo to Freetown, the same distance of 240 km. At what rate did the car travel in kph?

Lesson Title: Calculation of Unit Price	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-044	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to calculate the unit price of goods sold by various units (litres, kilogrammes, and so on).

Overview

This lesson is on calculating the **unit price** of goods. You will use the same math concepts from the previous lesson on calculating the unit rate. For unit price, the first unit in the rate is money (Leones, dollars, and so on). The second unit is for the good you are buying. It could be a measurement such as kilogrammes or litres. It could also be real items such as exercise books or chickens.

These are some examples of unit price:

- Le 6,500.00/litre "6,500 Leones per litre"
- \$5.00/kg. "5 dollars per kilogramme"
- GH₵6.00/chicken "6 Ghanaian cedi per chicken"
- ₦8.00/book "8 Nigerian naira per book"

Solved Examples

1. Five kilogrammes of cassava cost Le 6,000.00. What does 1 kg of cassava cost?

   Solution

   This question is asking us to find the unit price for 1 kg of cassava.

   The rate for cassava is $\frac{\text{Le } 6{,}000}{5 \text{ kg.}}$.

   Simplify this to find the unit price: $\frac{\text{Le } 6{,}000}{5 \text{ kg.}} = \frac{\text{Le } 1{,}200}{1 \text{ kg.}} = \text{Le } 1{,}200.00/\text{ kg}$

   - Remember to give your answer with cents (.00) when cents are given in the problem. You do not need to show the cents in your working.

2. Bendu paid Le 80,000.00 for 20 litres of petrol. What is the unit price for each litre of petrol?

   Solution

   Find the rate that Bendu paid: $\frac{\text{Le } 80{,}000}{20 \text{ litres}}$

   Simplify the fraction to find the unit price, or the cost of each litre of petrol:

   $\frac{\text{Le } 80{,}000}{20 \text{ litres}} = \frac{\text{Le } 4{,}000}{1 \text{ litre}} = \text{Le } 4{,}000.00/\text{litre}$

3. Three kg of peanuts costs Le 4,500. What does 1 kg cost?

   Solution

   Find the rate for peanuts: $\frac{\text{Le } 4{,}500}{3 \text{ kg}}$

   Simplify the fraction to find the unit price, or the cost of each kg of peanuts:

   $\frac{\text{Le } 4{,}500}{3 \text{ kg}} = \frac{\text{Le } 1{,}500}{1 \text{ kg}} = \text{ Le } 1{,}500.00/\text{kg}$

4. Juliet sells palm oil in large bottles that are 5 litres. She sells each bottle for Le 65,000.00. What is the unit cost for each litre of palm oil?

   Solution

   Find the rate of Leones per litre: $\frac{\text{Le } 65{,}000}{5 \text{ l}}$

   Simplify the fraction to find the unit price, or the cost of each litre of oil:

   $\frac{\text{Le } 65{,}000}{5 \text{ l}} = \frac{\text{Le } 13{,}000}{1 \text{ l}} = \text{Le}13{,}000.00/\text{l}$

Practice

1. David paid Le 30,000.00 for 6 exercise books. What is the unit price for 1 exercise book?
2. Fatu paid Le 100,000.00 for 5 kg of fish. What is the unit price for 1 kg of fish?
3. Mohamed is a fisherman. This morning he sold 6 large fish. If he earned Le 72,000.00, what was the unit price per fish?
4. Sia owns a bookstore. This morning, she sold 5 maths books for GH₵30.00. What is the unit price for each maths book?
5. Mustapha has a carton of 30 boiled eggs in his shop. If he sold them for Le 120,000.00 in total, what was the unit price per egg?

Lesson Title: Making Comparisons with Unit Price	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-045	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to compare goods to find which one has a better unit price.

Overview

Today we will learn how to compare the prices of goods using unit price. Unit price can be very helpful to allow us to compare prices of different quantities. Sellers sometimes sell the same good in different amounts. Unit price will help you find the best price and decide which goods to buy. A lower unit price for the same product is a better deal.

Solved Examples

1. Seller A has 50 kilogrammes of rice for Le 200,000.00. Seller B has 60 kg of rice for Le 222,000.00. Which rice has a lower unit price?

   Solution
   Find the unit rate of each seller's rice. That is, the price of 1 kg.

   - The rate for Seller A's rice is: $\frac{\text{Le } 200{,}000}{50 \text{ kg}} = \text{Le } 4{,}000/\text{ kg}$
   - The rate for Seller B's rice is: $\frac{\text{Le } 222{,}000}{60 \text{ kg}} = \text{Le } 3{,}700/\text{ kg}$

   The unit price of Seller B's rice is lower.

   Although Seller B is selling the rice for more money, the unit price is better. Each kilogramme of rice will cost less money if you buy Seller B's rice.

2. In the market, you can buy 2 litres of cooking oil for Le 80,000.00 or 3.5 litres of cooking oil for Le 122,500.00. Which option has the lower unit price?

   Solution
   Find the unit rate for each option. That is, the price of 1 litre.

   - The rate for the first option is: $\frac{\text{Le } 80{,}000}{2 \text{ litres}} = \text{Le } 40{,}000 \text{ /litre}$
   - The rate for the second option is: $\frac{\text{Le } 122{,}500}{3.5 \text{ litres}} = \text{Le } 35{,}000\text{/litre}$

   The unit rate for the second option is lower. It is better to buy 3.5 litres of oil for Le 122,500.00.

3. Bendu sells pieces of lappa that are 3 yards for Le 120,000.00. She sells pieces of lappa that are 5 yards for Le 210,000.00. Which option has the lower unit price?

Solution
Find the unit rate of for each option. That is, the price of 1 yard.

- The rate for the first option is: $\frac{\text{Le } 120{,}000}{3 \text{ yards}} = \text{ Le } 40{,}000 \text{ /yard}$
- The rate for the second option is: $\frac{\text{Le } 210{,}000}{5 \text{ yards}} = \text{Le } 42{,}000\text{/yard}$

The unit rate for the first option is lower. It is better to buy 3 yards of lappa for Le 120,000.00.

4. John sells exercise books in his shop. He sells 1 exercise book for Le 8,000.00. He sells 3 exercise books for Le 21,000.00 and 10 exercise books for Le 75,000.00. Which option has the best unit price?

Solution
Find the unit rate of for each option. That is, the price of 1 exercise book.

- The rate for the first option is: $\frac{\text{Le } 8{,}000}{1 \text{ exercise book}} = \text{ Le } 8{,}000\text{/ exercise book}$
- The rate for the second option is: $\frac{\text{Le } 21{,}000}{3 \text{ exercise books}} = \text{Le } 7{,}000 \text{ /exercise book}$
- The rate for the third option is: $\frac{\text{Le } 75{,}000}{10 \text{ exercise books}} = \text{Le } 7{,}500 \text{ /exercise book}$

The unit rate for the second option is lowest. If you buy 3 exercise books for Le 21,000.00 you will get the best price.

Practice

1. Michael sells beans. He sells 3 kg of beans for 42,000.00, and 5 kg of beans for Le 65,000.00. Which option has the better unit price?
2. Bendu bought 10 litres of petrol for Le 70,000.00. Foday bought 14 litres of petrol for Le 112,000.00. Who paid a better price for petrol?
3. Ama sells pens in her shop. She sells 1 pen for Le 5,000.00. She sells 3 pens for Le 12,000.00 and 8 pens for Le 36,000.00. Which option has the best unit price?
4. In the market, you can buy 30 kg of rice for Le 240,000.00. You can buy 50 kg of rice for Le 375,000.00. Which option has the lower unit price?
5. The Rokel Commercial Bank consumed 317 units of electricity in one month. The first 100 units were charged at Le 35.00 per unit and the rest at Le 17.00 per unit. How much will the bank pay for the electricity?

Lesson Title: Direct Proportion	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-046	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that a proportion is two ratios set equal to each other.
2. Identify the symbol for proportionality (∝), the means and extremes.

Overview

A proportion is a pair of equivalent ratios in which the units must be the same. Direct proportions mean that as one ratio increases, the other does too. On the other hand, as one ratio goes down the other does as well.

For example, this is a proportion: $\frac{2}{4} = \frac{5}{10}$

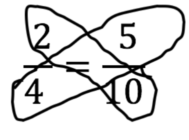
If one ratio changes, the other must also change so that they stay equal. For example, if 2 increases to 4, then 5 increases to 10.

Direct proportions are shown by the relationship where one value (y) is equal to another value (x) multiplied by a constant (k): $y = kx$. Remember that a constant is a number that does not change.

When two values are directly proportional to each other, we can use the symbol ∝. This symbol means 'is proportional to'. For $y = kx$, we can also write $y \propto x$, meaning that y is directly proportional to x.

When we have a proportion with two equivalent ratios, we can use **cross-multiplication**. Cross multiplication is multiplying the diagonals of equal fractions. If we multiply the numbers diagonally across from each other, the two products will be equal.

When you cross multiply, you multiply the **extremes** first. The extremes are the outside terms. In our example $\frac{2}{4} = \frac{5}{10}$, 2 and 10 are the extremes. Then you multiply the **means**. The means are the inside terms. In our example, 4 and 5 are the means.



The product of the extremes should be equal to the product of the means: $2 \times 10 = 5 \times 4 = 20$. Multiplying the means and extremes like this shows that the proportions are equivalent. Since these ratios are equal, the cross products are equal too.

Solved Examples

1. Express the following direct proportion as fractions, $3 : 6 = 9 : 18$. What are the means and extremes of the proportion? Use the means and extremes to demonstrate the ratios are equal.

   Solution

   Write the direct proportion as fractions: $\frac{3}{6} = \frac{9}{18}$

   The extremes are 3 and 18, and the means are 6 and 9.

   Multiply the extremes and means: $3 \times 18 = 54$ and $6 \times 9 = 54$

   We have shown that the ratios are equal because the cross products are equal.

2. Write each of the following ratios as fractions. Determine whether each represents a direct proportion:
   a. $2 : 5 = 8 : 20$
   b. $5 : 8 = 2 : 3$
   c. $2 : 8 = 25 : 100$

   Solutions

   a. Fractions: $\frac{2}{5} = \frac{8}{20}$

      Multiply the extremes and means: $2 \times 20 = 40$ and $5 \times 8 = 40$

      This is a direct proportion, because the cross products are equal.

   b. Fractions: $\frac{5}{8} = \frac{2}{3}$

      Multiply the extremes and means: $5 \times 3 = 15$ and $8 \times 2 = 16$

      This is **not** a direct proportion, because the cross products are different.

   c. Fractions: $\frac{2}{8} = \frac{25}{100}$

      Multiply the extremes and means: $2 \times 100 = 200$ and $8 \times 25 = 200$

      This is a direct proportion, because the cross products are equal.

Practice

1. Consider the ratios $3 : 12 = 5 : 20$.
   a. Write the ratios as fractions.
   b. What are the extremes and means?
   c. Is this a direct proportion?

2. Consider the ratios $5 : 15 = 7 : 20$.
   a. Write the ratios as fractions.
   b. What are the extremes and means?
   c. Is this a direct proportion?

3. Write each of the following ratios as fractions. Determine whether each represents a direct proportion:
    a. $1 : 9 = 3 : 27$
    b. $6 : 10 = 21 : 35$
    c. $5 : 8 = 20 : 30$

Lesson Title: Identifying Direct Proportions	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-047	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify true proportions.
2. Find the constant of proportionality.

Overview

In this lesson, you will use the equation for direct proportionality: $y = kx$. This equation was introduced in the previous lesson.

Remember that k is a constant, a number that does not change. We call k the **constant of proportionality**, because it does not change. It shows the relationship between the two ratios in the proportion.

For example, consider the ratios $\frac{3}{6} = \frac{9}{18}$. In the previous lesson, we showed that this is a direct proportion.

The value of k for this proportion is 2, and we have the equation $y = 2x$. We can substitute the numbers from the fractions into this equation, and show that this is true.

- For $\frac{3}{6}$, let $x = 3$ and $y = 6$. Substitute these values into the equation: $6 = 2(3)$. This is true.
- For $\frac{9}{18}$, let $x = 9$ and $y = 18$. Substitute these values into the equation: $18 = 2(9)$. This is true.

Every relationship that is directly proportional has a value of k. In this lesson, you will be given values for x and y, and you will be asked to find the value of k. In other words, you are asked to find the relationship between x and y.

From the previous lesson, two proportions can be called **true proportions** when the product of the means equals the product of the extremes. If you have the constant of proportionality (k), you can find proportions that are true proportions by substituting any value for x and finding the corresponding value of y. See Solved Example 3.

Solved Examples

1. $y \propto x$. When $x = 2$, $y = 10$. Find the constant of proportionality.

   Solution

   Recall that $y \propto x$ means 'y is directly proportional to x'. When y and x are directly proportional, we can use the equation $y = kx$.

   Substitute the values $x = 2$ and $y = 10$ into the equation and solve for k.

   $$\begin{aligned} y &= kx \\ 10 &= k(2) && \text{Substitute } x = 2 \text{ and } y = 10 \\ 10 &= 2k \\ \frac{10}{2} &= \frac{2k}{2} && \text{Divide both sides by 2} \\ 5 &= k \end{aligned}$$

   We have found the constant of proportionality, $k = 5$. The equation is $y = 5x$.

2. y and x are directly proportional. When $x = 20$, $y = 4$. Find the constant of proportionality.

   Solution

   Substitute the values $x = 20$ and $y = 4$ into the equation and solve for k.

   $$\begin{aligned} y &= kx \\ 4 &= k(20) && \text{Substitute } x = 20 \text{ and } y = 4 \\ 4 &= 20k \\ \frac{4}{20} &= \frac{20k}{20} && \text{Divide both sides by 20} \\ \frac{1}{5} &= k && \text{Simplify the fraction} \end{aligned}$$

   We have found the constant of proportionality, $k = \frac{1}{5}$. The equation is $y = \frac{1}{5}x$.

3. Two values x and y are directly proportional, such that $y = 3x$.
   a. Use the equation to find 2 proportions that are true proportions.
   b. Check that the proportions are true proportions using cross multiplication.

   Solutions

   a. You may use any value for x. Substitute your chosen values into the equation $y = 3x$ and find the corresponding value of y. This will give you your 2 proportions. Use the x- and y-values for your proportions: $x : y$

      Let's use $x = 2$ and $x = 3$.

Substitute $x = 2$:

$y = 3x$	Equation
$y = 3(2)$	Substitute $x = 2$
$y = 6$	

Substitute $x = 3$:

$y = 3x$	Equation
$y = 3(3)$	Substitute $x = 3$
$y = 9$	

This gives us 2 proportions: $2 : 6$ and $3 : 9$.

b. Write the proportions as fractions: $\frac{2}{6} = \frac{3}{9}$
Multiply the extremes and means: $2 \times 9 = 18$ and $6 \times 3 = 18$
The proportions are true proportions because the cross products are equal.

Practice

1. $y \propto x$. When $x = 5$, $y = 30$. Find the constant of proportionality.
2. y and x are directly proportional. When $x = 10$, $y = 4$. Find the constant of proportionality.
3. y and x can be represented with the equation $y = kx$. When $x = 25$, $y = 75$. Find the value of k.
4. Two values x and y are directly proportional, such that $y = 4x$.
   a. Use the equation to find 2 proportions that are true proportions.
   b. Check that the proportions are true proportions using cross multiplication.
5. Two values x and y are directly proportional, such that $y = 10x$.
   a. Use the equation to find 2 proportions that are true proportions.
   b. Check that the proportions are true proportions using cross multiplication.

Lesson Title: Solving Direct Proportions	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-048	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to find the value of an unknown term in a direct proportion.

Overview

In this lesson, you will find the value of an unknown term in a direct proportion. For example, consider the proportion $\frac{3}{5} = \frac{c}{10}$. c is a variable that represents an unknown number. You can use cross multiplication to find the value of c. See Solved Example 1 for the solution.

Solved Examples

1. Find the value of c that completes the direct proportion $\frac{3}{5} = \frac{c}{10}$.

   Solution
   Remember that for a direct proportion, the product of the means equals the product of the extremes. In this case, $3 \times 10 = 5 \times c$. Use this to solve for c.

   $$\begin{aligned} 3 \times 10 &= 5 \times c && \text{Cross Multiply} \\ 30 &= 5c && \text{Simplify} \\ \tfrac{30}{5} &= c && \text{Divide both sides by 5} \\ 6 &= c \end{aligned}$$

   The answer is $c = 6$. The complete proportion is $\frac{3}{5} = \frac{6}{10}$.

   You can always check your answer by cross-multiplying the complete proportion: $3 \times 10 = 5 \times 6 = 30$.

2. Find the value for a that completes the direct proportion $\frac{a}{16} = \frac{7}{28}$.

   Solution

   $$\begin{aligned} a \times 28 &= 16 \times 7 && \text{Cross-multiply} \\ 28a &= 112 && \text{Simplify} \\ a &= \tfrac{112}{28} && \text{Divide both sides by 28} \\ a &= 4 \end{aligned}$$

   The answer is $a = 4$, and the complete proportion is $\frac{4}{16} = \frac{7}{28}$.

3. Find the missing value that completes the direct proportion $\frac{4}{60} = \frac{5}{x}$.

Solution

$$
\begin{aligned}
4 \times x &= 60 \times 5 && \text{Cross-multiply} \\
4x &= 300 && \text{Simplify} \\
x &= \tfrac{300}{4} && \text{Divide both sides by 4} \\
x &= 75
\end{aligned}
$$

The answer is $x = 75$, and the complete proportion is $\frac{4}{60} = \frac{5}{75}$.

Practice

Find the missing values that complete each proportion:

1. $\frac{a}{2} = \frac{6}{4}$
2. $\frac{1}{b} = \frac{7}{21}$
3. $\frac{5}{35} = \frac{7}{x}$
4. $\frac{4}{24} = \frac{y}{18}$
5. $\frac{z}{150} = \frac{3}{75}$
6. $\frac{3}{45} = \frac{8}{z}$

Lesson Title: Applications of Direct Proportions	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-049	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Solve problems with direct proportions.
2. Solve proportions that include units.

Overview

In this lesson, you will apply your understanding of direct proportions to solve real-life problems with units. For example, consider the story problem:

If Joe can read 4 books in 2 days, how many books can he read in 8 days?

There are 2 ways to solve the problems you will see in this lesson: the unitary method and the ratio method.

For the **unitary method**, you must first find the rate for 1 unit. In this example, Joe reads 4 books in 2 days. The first step would be to find out how many books Joe can read in **1 day**. Then, you would multiply the unitary rate by the number given in the problem. In this example, multiply the rate for 1 day by 8 days.

For the **ratio method**, write the proportions as fractions and solve using cross multiplication. This method was used to solve problems in the previous lesson.

Both methods are shown in the Solved Examples. You may choose one method to solve the Practice problems. See Solved Example 1 for the solution to the story problem above.

Solved Examples

1. If Joe can read 4 books in 2 days, how many books can he read in 8 days?

   Solution

   Using the **unitary method:**

   Find how many books Joe reads in 1 day:

   $$4 \text{ books} \div 2 \text{ days} = 2 \text{ books in } 1 \text{ day}$$

   Multiply the rate for 1 day by the number of days:

   $$\text{Number of books in } 8 \text{ days} = 2 \text{ books } \times 8 = 16 \text{ books}$$

   Using the **ratio method:**

Write a ratio for the problem: $\frac{4 \text{ books}}{2 \text{ days}} = \frac{y}{8 \text{ days}}$. The unknown value y is the number of books he can read in 8 days.

Solve using cross-multiplication:

4×8	$=$	$2 \times y$	Cross-multiply
32	$=$	$2y$	Simplify
$\frac{32}{2}$	$=$	y	Divide both sides by 2
16	$=$	y	

Answer: Joe can read 16 books in 8 days.

2. Mustapha can paint 80 m^2 with 2 cans of paint. What area can he paint with 7 cans of paint?

Solution

Using the **unitary method:**

Find how many square metres Mustapha can paint with 1 can:

$$80 \text{ m}^2 \div 2 \text{ days} = 40 \text{ m}^2 \text{ with 1 can}$$

Multiply the rate for 1 can by the number of cans:

$$\text{Number of m}^2 \text{with 7 cans} = 40 \text{ m}^2 \times 7 = 280 \text{ m}^2$$

Using the **ratio method:**

Write a ratio for the problem: $\frac{80 \text{ m.}^2}{2 \text{ cans}} = \frac{x}{7 \text{ cans}}$. The unknown value x is the number of square metres he can paint with 7 cans.

Solve using cross-multiplication:

80×7	$=$	$2 \times x$	Cross-multiply
560	$=$	$2x$	Simplify
$\frac{560}{2}$	$=$	x	Divide both sides by 2
280	$=$	x	

Answer: Mustapha can paint 280 m^2 with 7 cans of paint.

3. Bendu drove her car 120 km in 4 hours. What distance could she drive in 6 hours, if she kept the same rate?

Solution

Using the **unitary method:**

Find how many km Bendu can drive in 1 hour:

$120 \text{ km} \div 4 \text{ hours} = 30 \text{ km}$ in 1 hour

Multiply the rate for 1 hour by the number of hours:

$\text{Kilometres in 6 hours} = 30 \text{ km} \times 6 = 180 \text{ km}$

Using the **ratio method:**

Write a ratio for the problem: $\frac{120 \text{ km}}{4 \text{ hr}} = \frac{z}{6 \text{ hr}}$. The unknown value z is the number of kilometres she can drive in 6 hours.

Solve using cross-multiplication:

120×6	$=$	$4 \times z$	Cross-multiply
720	$=$	$4z$	Simplify
$\frac{720}{4}$	$=$	z	Divide both sides by 2
180	$=$	z	

Answer: Bendu can drive 180 km in 6 hours.

Practice

1. Abu walked 3 kilometres in 18 minutes. How long would it take him to walk 10 kilometres?
2. Mr. Bangura uses 3 pieces of chalk every week to teach 18 lessons. How many pieces of chalk does he need to teach 60 lessons?
3. Abass is a farmer. He uses 2 bottles of fertiliser for 6 hectares of land. How many bottles of fertiliser does he need for 42 hectares?
4. Alice solved 5 maths problems in 30 minutes. How long would it take her to solve 16 maths problems?
5. There are 8 members of Hawa’s family. Together, they eat 12 cups of rice each day. One day, there are only 6 members of her family present. How many cups of rice should she prepare?
6. Five men took four days to plough a field. How long would it take 4 men working at the same rate to plough the same field?

Lesson Title: Direct Proportion Story Problems	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-050	**Class:** JSS 2

Learning Outcome

By the end of the lesson, you will be able to solve story problems involving direct proportion.

Overview

In this lesson, you will practise solving story problems on direct proportion. These are a little more challenging than the problems in the previous lesson. You should now be able to identify situations that involve direct proportion. You may use either the unitary method or ratio method to solve these problems. Only one method is given for each problem below.

Solved Examples

1. A farmer can harvest 600 cassava in 3 hours. How many cassava can he harvest in 5 hours?

   Solution

   Using the **ratio method**, write a ratio for the problem: $\frac{600 \text{ cassava}}{3 \text{ hr}} = \frac{x}{5 \text{ hr}}$. The unknown value x is the number of cassava he can harvest in 5 hours.

   Solve using cross-multiplication:

$$\begin{aligned} 600 \times 5 &= 3 \times x && \text{Cross-multiply} \\ 3{,}000 &= 3x && \text{Simplify} \\ \tfrac{3{,}000}{3} &= x && \text{Divide both sides by 3} \\ 1{,}000 &= x \end{aligned}$$

   Answer: The farmer can harvest 1,000 cassava in 5 hours.

2. A teacher can mark 50 exams in 2 hours. How long would it take the teacher to mark 120 exams?

   Solution

   Using the **ratio method**, write a ratio for the problem: $\frac{50 \text{ exams}}{2 \text{ hr}} = \frac{120 \text{ exams}}{a}$. The unknown value a is the number of hours it would take the teacher to mark 120 exams.

Solve using cross-multiplication:

$$\begin{aligned} 50 \times a &= 2 \times 120 && \text{Cross-multiply} \\ 50a &= 240 && \text{Simplify} \\ a &= \tfrac{240}{50} && \text{Divide both sides by 50} \\ a &= 4\tfrac{4}{5} \text{ or } 4.8 \end{aligned}$$

Answer: It would take the teacher $4\frac{4}{5}$ or 4.8 hours to mark 120 exams.

3. Abu is a writer. He can write 3 pages of a book in 51 minutes. How long would it take him to write 10 pages?

   Solution

   Using the unitary method, Find how long it takes him to write 1 page:

   $$51 \text{ min} \div 3 \text{ pages} = 17 \text{ minutes to write 1 page}$$

   Multiply the rate for 1 page by the number of pages:

   $$\text{Time for 10 pages} = 17 \text{ min } \times \ 10 = 170 \text{ minutes}$$

   Answer: It would take him 170 minutes to write 10 pages.

   - The answer can also be given in hours by dividing by 60: $170 \div 60 = 2$ remainder 50. It would take him 2 hours and 50 minutes to write 10 pages.

4. Bendu and Hawa started a business together. Bendu contributed Le 400,000.00 and Hawa contributed Le 360,000.00. They agreed to split income from the business according to how much they put in. Bendu received Le 50,000.00. What is Hawa's portion?

   Solution

   The income from the business is in the same proportion as the money contributed. This means that we can write the ratio: $\frac{\text{Le } 400{,}000}{\text{Le } 360{,}000} = \frac{\text{Le } 50{,}000}{H}$, where H is the portion of the income that Hawa receives.

   We will solve for H using cross-multiplication. We can cancel some zeros in the first ratio to make the multiplication easier: $\frac{\text{Le } 400{,}000}{\text{Le } 360{,}000} = \frac{\text{Le } 40}{\text{Le } 36}$

Now we have $\frac{\text{Le } 40}{\text{Le } 36} = \frac{\text{Le } 50{,}000}{H}$. Cross-multiply and solve:

$$\begin{aligned} 40 \times H &= 36 \times 50{,}000 && \text{Cross-multiply} \\ 40H &= 1{,}800{,}000 && \text{Simplify} \\ H &= \frac{1{,}800{,}000}{40} && \text{Divide both sides by 40} \\ H &= \frac{180{,}000}{4} && \text{Simplify} \\ H &= 45{,}000 \end{aligned}$$

Answer: Hawa's portion is Le 45,000.00.

Practice

1. A woman sold 50 oranges in 4 hours. If she continues selling them at the same rate, how many can she sell in 6 hours?
2. Foday is a farmer. He has 32 rows of eggplant on his farm. It took Foday 3 hours to weed 8 rows of the eggplant. How long will it take him in total to weed the 32 rows?
3. Fatu used 10 litres of fuel to travel 80 miles. How much fuel will she need to travel 200 miles?
4. Martin and Issa started a business together. Martin contributed Le 300,000.00 and Issa contributed Le 450,000.00. They agreed to split income from the business according to how much they put in. Martin received Le 100,000. What is Issa's portion?
5. A bottle contains 150 ml of medicine. The label on the bottle reads: **Take two teaspoonful three times a day**. If one teaspoonful can hold 5 ml of medicine, how many days will the medicine last?

Lesson Title: Indirect Proportion	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-051	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify the form of an indirectly proportional relationship ($t \propto \frac{1}{d}$).
2. Compare indirect proportion to direct proportion.

Overview

Recall that a proportion is a pair of equivalent ratios in which the units must be the same. The previous lessons have been on direct proportion. Direct proportions mean that as one ratio increases, the other does too. On the other hand, as one ratio goes down the other does too.

This is the first lesson on indirect proportion. Indirect proportions mean that as one ratio goes up, the other goes down. On the other hand, as one ratio goes down the other goes up. The ratios move in opposite directions instead of in the same direction like a directly proportional relationship.

For example, consider a group of people doing a piece of work. If it takes 10 people 1 hour to brush the piece of land, how long will the same job take for 5 people? It will take more than 1 hour. When there are fewer people doing a job, the job takes more time.

Inverse proportions are shown with the equation $y = k\frac{1}{x}$ or $y = \frac{k}{x}$. These equations are the same. Indirect proportions are shown by the relationship one value (y) is equal to another value ($\frac{1}{x}$) multiplied by a constant (k).

When two values are indirectly proportional to each other, we can still use the special symbol $\propto$, because it just means proportional and then the numbers around it tell what kind of relationship exists. However, this time we would write: $y \propto \frac{1}{x}$, meaning that y is **indirectly** proportional to x.

When we have an inverse proportion with two equivalent ratios, we can still use **cross-multiplication**, however, it needs to be set up differently, because the ratios have a different relationship to each other.

To use cross-multiplication to solve an inverse proportion, follow these steps:

1. Make sure each ratio is one unit, do not mix units.

2. Flip one of the ratios upside down.
3. Solve as normal.

Since the relationship is indirect ($y \propto \frac{1}{x}$), not direct ($y \propto x$), we cannot say that the 2 ratios are equal until we flip one of them.

Solved Examples

1. Consider the indirectly proportional ratios 2 : 4 ∝ 10 : 5. Use them to complete the following:
    a. Write the ratios as fractions.
    b. Rearrange the fractions so you can accurately cross-multiply.
    c. What are the means and extremes of the proportion?
    d. Show that it is a true proportion by cross-multiplying.

    Solutions

    a. Write the ratios as fractions: $\frac{2}{4}$ and $\frac{10}{5}$.
    Note that these cannot be set equal. The first fraction is proper (less than 1), and the second fraction is improper (greater than 1).
    b. Rearrange the fractions: $\frac{2}{4} = \frac{5}{10}$.
    They can now be set equal and cross-multiplied.
    c. Means: 4 and 5; Extremes: 2 and 10
    d. Cross multiply: $2 \times 10 = 4 \times 5 = 20$. This is a true proportion.

2. Consider the indirectly proportional ratios 1 : 4 ∝ 12 : 3. Use them to complete the following:
    a. Write the ratios as fractions.
    b. Rearrange the fractions so you could accurately cross-multiply.
    c. What are the means and extremes of the proportion?
    d. Show that it is a true proportion by cross-multiplying.

    Solutions

    a. Write the ratios as fractions: $\frac{1}{4}$ and $\frac{12}{3}$.
    b. Rearrange the fractions: $\frac{1}{4} = \frac{3}{12}$.
    c. Means: 4 and 3; Extremes: 1 and 12
    d. Cross-multiply: $1 \times 12 = 4 \times 3 = 12$. This is a true proportion.

3. Write each of the following ratios as fractions. Determine whether each represents an indirect proportion:

    a. $2 : 5 \propto 20 : 8$
    b. $3 : 4 \propto 20 : 15$
    c. $3 : 8 \propto 2 : 1$

    Solutions

    a. Fractions: $\frac{2}{5}$ and $\frac{20}{8}$
    Rearrange the fractions: $\frac{2}{5} = \frac{8}{20}$
    Multiply the extremes and means: $2 \times 20 = 40$ and $5 \times 8 = 40$
    This is an indirect proportion, because the cross products are equal.

    b. Fractions: $\frac{3}{4}$ and $\frac{20}{15}$
    Rearrange the fractions: $\frac{3}{4} = \frac{15}{20}$
    Multiply the extremes and means: $3 \times 20 = 60$ and $4 \times 15 = 60$
    This is an indirect proportion, because the cross products are the same.

    c. Fractions: $\frac{3}{8}$ and $\frac{2}{1}$
    Rearrange the fractions: $\frac{3}{8} = \frac{1}{2}$
    Multiply the extremes and means: $3 \times 2 = 6$ and $8 \times 1 = 8$
    This is **not** an indirect proportion, because the cross products are different.

Practice

1. Consider the indirectly proportional ratios 3 : 9 ∝ 12 : 4. Use them to complete the following:

    a. Write the ratios as fractions.
    b. Rearrange the fractions so you can accurately cross-multiply.
    c. What are the means and extremes of the proportion?
    d. Show that it is a true proportion by cross-multiplying.

2. Determine whether each of the following represents an indirect proportion:

    a. $1 : 5 \propto 20 : 5$
    b. $4 : 8 \propto 20 : 10$
    c. $1 : 6 \propto 30 : 5$
    d. $3 : 10 \propto 12 : 4$

Lesson Title: Solving Indirect Proportions	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-052	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to find the value of an unknown term in an indirect proportion.

Overview

In this lesson, you will find the value of an unknown term in an indirect proportion. For example, consider the indirect proportion $3 : b \propto 8 : 4$. b is a variable that represents an unknown number. You can write the ratios as fractions, and use cross-multiplication to find the value of b. Remember that for indirect proportions you should flip 1 fraction upside down. See Solved Example 1 for the solution.

Solved Examples

1. Find the value of b that completes the indirect proportion $3 : b \propto 8 : 4$.

   Solution
   Write the ratios as fractions: $\frac{3}{b}$ and $\frac{8}{4}$. Flip one fraction upside down and set them as equal: $\frac{3}{b} = \frac{4}{8}$. Now you can use cross-multiplication to find b:

$$\begin{aligned} 3 \times 8 &= b \times 4 && \text{Cross-multiply} \\ 24 &= 4b && \text{Simplify} \\ \tfrac{24}{4} &= b && \text{Divide both sides by 4} \\ 6 &= b \end{aligned}$$

   The answer is $b = 6$. The complete proportion is $3 : 6 \propto 8 : 4$.

2. $a : 16 \propto 10 : 5$ are two indirectly proportional ratios. Find the value for a that completes the proportion.

   Solution
   Write the ratios as fractions: $\frac{a}{16}$ and $\frac{10}{5}$. Flip one fraction upside down and set them equal: $\frac{a}{16} = \frac{5}{10}$. Now you can use cross-multiplication to find a:

$$\begin{aligned} a \times 10 &= 16 \times 5 && \text{Cross-multiply} \\ 10a &= 80 && \text{Simplify} \\ a &= \tfrac{80}{10} && \text{Divide both sides by 10} \\ a &= 8 \end{aligned}$$

   The answer is $a = 8$. The complete proportion is $8 : 16 \propto 10 : 5$.

3. $150 : 20 \propto c : 15$ are two indirectly proportional ratios. Find the value for c that completes the proportion.

Solution

Write the ratios as fractions: $\frac{150}{20}$ and $\frac{c}{15}$. Flip one fraction upside down and set them equal: $\frac{150}{20} = \frac{15}{c}$. Now you can use cross-multiplication to find c:

$150 \times c$	$=$	20×15	Cross-multiply
$150c$	$=$	300	Simplify
c	$=$	$\frac{300}{150}$	Divide both sides by 150
c	$=$	2	

The answer is $c = 2$. The complete proportion is $150 : 20 \propto 2 : 15$.

Practice

Find the missing values that complete each indirect proportion:

1. $a : 20 \propto 48 : 12$
2. $30 : b \propto 1 : 5$
3. $10 : 4 \propto c : 5$
4. $100 : 25 \propto 5 : d$
5. $250 : x \propto 5 : 25$
6. $38 : 12 \propto 36 : z$

Lesson Title: Applications of Indirect Proportions	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-053	**Class:** JSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Solve problems with indirect proportions.
2. Solve indirect proportions that include units.

Overview

In this lesson, you will apply your understanding of indirect proportions to solve real-life problems with units. For example, consider the story problem:

> If 5 people dig a hole for construction, it takes 8 hours. If 10 people are digging, how many hours will it take?

There are 2 ways to solve the problems you will see in this lesson: the unitary method and the ratio method.

For the **unitary method**, you must first find out how many hours it would take 1 person to do all the work. You would use this to find how many hours it takes 10 people to do the same work.

For the **ratio method**, you write the proportions as fractions. You will **keep the units together** so that the same units are in each fraction. Remember to flip one of the fractions upside down, then solve using cross-multiplication.

Both methods are shown in the Solved Examples. You may choose one method to solve the Practice problems. See Solved Example 1 for the solution to the story problem above.

Solved Examples

1. If 5 people dig a hole for construction, it takes 8 hours. If 10 people are digging, how many hours will it take?

   Solution

   Unitary Method:

   Find how long it would take for 1 person to do the work:

   $$5 \times 8 = 40 \text{ hours}$$

   Divide by 10 to find how long it takes 10 people to do the work:

   $$40 \text{ hr } \div 10 = 4 \text{ hours}$$

Ratio Method: Let h represent the number of hours. We have the ratios 5 people : 10 people and 8 hours : h hours. Remember to keep the same units together. Write the ratios as fractions: $\frac{5}{10}$ and $\frac{8}{h}$. Flip one fraction upside down and set them equal: $\frac{5}{10} = \frac{h}{8}$. Now you can use cross-multiplication to find h:

$$\begin{aligned} 5 \times 8 &= 10 \times h && \text{Cross-multiply} \\ 40 &= 10h && \text{Simplify} \\ \frac{40}{10} &= h && \text{Divide both sides by 10} \\ 4 &= h \end{aligned}$$

Answer: It will take 10 people 4 hours to dig the hole.

2. If 5 boys took 14 hours to brush a piece of land, how long will it take 7 boys working at the same rate to brush the land?

Solution

Unitary Method:

Find how long it would take for 1 person to do the work:

$$5 \times 14 = 70 \text{ hours}$$

Divide by 7 to find how long it takes 7 people to do the work:

$$70 \text{ hr} \div 7 = 10 \text{ hours}$$

Ratio Method: Let h represent the number of hours. We have the ratios 5 boys : 7 boys and 14 hours : h hours. Remember to keep the same units together. Write the ratios as fractions: $\frac{5}{7}$ and $\frac{14}{h}$. Flip one fraction upside down and set them equal: $\frac{5}{7} = \frac{h}{14}$. Now you can use cross-multiplication to find h:

$$\begin{aligned} 5 \times 14 &= 7 \times h && \text{Cross-multiply} \\ 70 &= 7h && \text{Simplify} \\ \frac{70}{7} &= h && \text{Divide both sides by 7} \\ 10 &= h \end{aligned}$$

Answer: $h = 10$. It will take 7 boys people 10 hours to brush the land.

3. Five kg rice can last 12 people for 4 days. How many days would the rice last if there were 8 people?

Solution

Unitary Method:

Find how the rice would last for 1 person:

$$12 \times 4 = 48 \text{ days}$$

Divide by 8 to find how long the rice would last for 8 people:

$$48 \text{ days } \div 8 = 6 \text{ days}$$

Ratio Method: Let d represent the number of days. We have the ratios 12 people $:$ 8 people and 4 days $:$ d days. Remember to keep the same units together. Write the ratios as fractions: $\frac{12}{8}$ and $\frac{4}{d}$. Flip one fraction upside down and set them equal: $\frac{12}{8} = \frac{d}{4}$. Now you can use cross-multiplication to find d:

12×4	$=$	$8 \times d$	Cross-multiply
48	$=$	$8d$	Simplify
$\frac{48}{8}$	$=$	d	Divide both sides by 8
6	$=$	d	

Answer: $d = 6$. The rice would last 8 people 6 days.

Practice

1. Thirty workers can complete a piece of work in 80 days. How many workers will you need to complete the same piece of work in 20 days?
2. Fifty litres of drinking water can last 6 people for 5 days. How many days would the drinking water last if there were 10 people?
3. In a village, there is enough food for 500 people for 30 days. If 100 more people moved to the village, how many days would the food last?
4. Mr. Bangura has a plum tree. He picked enough plums to give 4 plums to each of 15 of his friends. If he decided to share the same plums with 20 friends, how many would each friend get?

Lesson Title: Indirect Proportion Story Problems	Theme: Everyday Arithmetic
Practice Activity: PHM-08-054	**Class:** JSS 2

Learning Outcomes
By the end of the lesson, you will be able to solve story problems involving indirect proportions.

Overview

In this lesson, you will practise solving story problems on indirect proportion. These are a little more challenging than the problems in the previous lesson. You should now be able to identify situations that involve indirect proportion. You may use either the unitary method or ratio method to solve these problems. Only one method is given for each problem below.

Solved Examples

1. Kumba has land to allow his animals to graze. There is enough grass for 50 animals for 6 days. How many days would the food last if there were:
   a. 75 animals?
   b. 100 animals?

   Solutions
   This is an indirect proportion because as the numbers of animals goes up, the food goes down. The solution is shown below using the ratio method. You may use either the ratio or unitary method, but use the same method for both parts a. and b.

   a. Let x represent the number of days. Write the ratios as fractions: $\frac{\text{50 animals}}{\text{75 animals}}$ and $\frac{\text{6 days}}{x}$. Flip one fraction upside down and set them equal: $\frac{\text{50 animals}}{\text{75 animals}} = \frac{x}{\text{6 days}}$. Now you can use cross-multiplication to find x:

   $$\begin{aligned} 50 \times 6 &= 75 \times x && \text{Cross-multiply} \\ 300 &= 75x && \text{Simplify} \\ \tfrac{300}{75} &= x && \text{Divide both sides by 75} \\ 4 &= x \end{aligned}$$

   $x = 4$. The rice would last 75 animals 4 days.

b. Apply the same method for 100 animals: $\frac{50 \text{ animals}}{100 \text{ animals}} = \frac{x}{6 \text{ days}}$

$$\begin{aligned} 50 \times 6 &= 100 \times x && \text{Cross-multiply} \\ 300 &= 100x && \text{Simplify} \\ \frac{300}{100} &= x && \text{Divide both sides by 100} \\ 3 &= x \end{aligned}$$

$x = 3$. The rice would last 100 animals 3 days.

2. Fatu, Martin and Issa were hired to harvest a farm and expect it will take them 6 days. If they complete the harvest in 4 days, they will be paid a bonus of Le 500,000.00. How many friends do they need to ask to join them to get the bonus?

Solution

With 3 people, the job takes 6 days. We need to find how many people can finish the job in 4 days.

Let p represent the number of people. Write the ratios as fractions: $\frac{6 \text{ days}}{4 \text{ days}}$ and $\frac{3 \text{ people}}{p}$. Flip one fraction upside down and set them equal: $\frac{6 \text{ days}}{4 \text{ days}} = \frac{p}{3 \text{ people}}$. Now you can use cross-multiplication to find p:

$$\begin{aligned} 6 \times 3 &= 4 \times p && \text{Cross-multiply} \\ 18 &= 4p && \text{Simplify} \\ \frac{18}{4} &= p && \text{Divide both sides by 4} \\ 4\frac{1}{2} &= p \end{aligned}$$

To finish the job in 4 days, they need $4\frac{1}{2}$ people. We can't have $\frac{1}{2}$ of a person! They will need 5 people to finish the job on time. They already have 3 people, so they will need to invite 2 more friends to finish the job on time.

Answer: 2 friends

3. Alpha is traveling to Conakry to buy goods. If he drives at the rate of 50 kph it will take him 14 hours. How long would it take him to get to Conakry if he drove at the rate of 70 kph?

Solution

Let h represent the number of hours. Write the ratios as fractions: $\frac{50 \text{ kph}}{70 \text{ kph}}$ and $\frac{14 \text{ hr}}{h}$. Flip one fraction upside down and set them equal: $\frac{50 \text{ kph}}{70 \text{ kph}} = \frac{h}{14 \text{ hr}}$. Now you can use cross-multiplication to find h:

$$\begin{aligned} 50 \times 14 &= 70 \times h && \text{Cross-multiply} \\ 700 &= 70h && \text{Simplify} \\ \frac{700}{70} &= h && \text{Divide both sides by 70} \\ 10 &= h \end{aligned}$$

It would take him 10 hours.

4. Hawa is traveling to Bo. If she drives at the rate of 40 kph it will take her 2 hours. How much faster would she get to Bo if she drove at the rate of 50 kph?

Solution

Notice that this question asks how much faster she would get there. Once we find out how long it would take Hawa to get to Bo traveling at 50 kph, we need to find the difference between the two lengths of time.

Step 1. Find how long it will take her at 50 kph:

Let h represent the number of hours. Write the ratios as fractions: $\frac{40 \text{ kph}}{50 \text{ kph}}$ and $\frac{2 \text{ hr}}{h}$. Flip one fraction upside down and set them equal: $\frac{40 \text{ kph}}{50 \text{ kph}} = \frac{h}{2 \text{ hr}}$. Now you can use cross-multiplication to find h:

$$\begin{aligned} 40 \times 2 &= 50 \times h && \text{Cross-multiply} \\ 80 &= 50h && \text{Simplify} \\ \frac{80}{50} &= h && \text{Divide both sides by 50} \\ \frac{8}{5} &= h \\ 1\frac{3}{5} &= h \end{aligned}$$

It would take her $1\frac{3}{5}$ hours.

Step 2. Find the difference between the 2 lengths of time:

$$\begin{aligned} 2 - 1\frac{3}{5} &= 2 - \frac{8}{5} && \text{Cross-multiply} \\ &= \frac{10}{5} - \frac{8}{5} && \text{Simplify} \\ &= \frac{10-8}{5} && \text{Divide both sides by 50} \\ &= \frac{2}{5} \text{ hours} \end{aligned}$$

She will reach Bo $\frac{2}{5}$ hours faster. We can write this in minutes: $\frac{2}{5} \times 60 = \frac{120}{5} = 24$ minutes.

Practice

1. Fatu saved her money and bought a bicycle. It took her 4 hours to travel to her village, a distance of 30 km. She wants to go to her friend's village 50 km away. How long will it take her?
2. Mustapha is traveling to Accra. If he drives at the rate of 50 kph it will take him 3 days. How long would it take him to get to Accra if he drove at the rate of 60 kph?
3. Francis and Juliet are tailors. They were hired to make school uniforms for 30 pupils and expect it will take them 6 days. The academic year begins in 4 days, so they will ask other tailors to help them complete the job on time. How many more tailors do they need?

Lesson Title: Practice with Proportion	**Theme:** Everyday Arithmetic
Practice Activity: PHM-08-055	**Class:** JSS 2

Learning Outcome
By the end of the lesson, you will be able to solve number and story problems with direct and indirect proportion.

Overview

In this lesson, you will decide if a problem describes direct or indirect proportion. You will use the methods you learned in lessons 46 to 54 to solve the problems. Remember:

- In **direct proportion**, both ratios move in the same direction. They either both increase or both decrease.
- In **indirect proportion**, the ratios move in opposite directions. As one decreases, the other increases. As one increases, the other decreases.

Solved Examples

1. There are 3 cooks at a school who prepare 250 school meals in 50 minutes every day. The principal wants to shorten the time it takes to prepare the meals by hiring more cooks. How many cooks would the school need to prepare the meals in 30 minutes, assuming they all work at the same rate?

   Solution
   First, determine the type of proportion. This is an indirect proportion, because the time for the meal will go down, while the number of cooks will increase.

   Write the 2 fractions. Let c represent the unknown number of cooks: $\frac{50\text{ min.}}{30\text{ min.}}$ and $\frac{3\text{ cooks}}{c}$.
   Flip one of them upside down: $\frac{50\text{ min.}}{30\text{ min.}} = \frac{c}{3\text{ cooks}}$

   Cross-multiply and solve:

$$\begin{aligned} 50 \times 3 &= 30 \times c && \text{Cross-multiply} \\ 150 &= 30c && \text{Simplify} \\ \frac{150}{30} &= c && \text{Divide both sides by 30} \\ 5 &= c && \end{aligned}$$

   Answer: The school would need 5 cooks to prepare the meals in 30 minutes.

2. Foday is drawing a scale map of his community. He measured the distance he walks from his home to the school every day at 4 km. The scale on his map is 1 cm = 50 m. How long should the distance be on his map?

   Solution

   First, determine the type of proportion. This is a direct proportion, because as the distance in Foday's community increases, the distance on the map increases. Remember that each ratio must have the same units. Convert 4 km to metres: 4 km = 4,000 m

   The ratios are $\frac{50\text{ m}}{4{,}000\text{ m}} = \frac{1\text{ cm}}{x}$, where x is the distance from his home to school on the map. Cross-multiply and solve:

$50 \times x$	$=$	$4{,}000 \times 1$	Cross-multiply
$50x$	$=$	$4{,}000$	Simplify
x	$=$	$\frac{4{,}000}{50}$	Divide both sides by 50
x	$=$	80	Simplify

   The distance on the map is 80 cm.

3. Alpha and Hawa are both driving from Bo to Freetown. Hawa travels at 60 kph and reaches Freetown in 4 hours. Alpha wants to reach Freetown in 3 hours for an appointment. How fast does he need to drive?

   Solution

   First, determine the type of proportion. This is an indirect proportion, because as the speed increases, the time it takes decreases.

   The ratios are $\frac{4\text{ h}}{3\ h}$ and $\frac{60\text{ kph}}{s}$, where s is the speed at which Alpha needs to drive. Flip one ratio upside down: $\frac{4\text{ h}}{3\ h} = \frac{s}{60\text{ kp}h}$.

   Cross-multiply and solve:

4×60	$=$	$3 \times s$	Cross-multiply
240	$=$	$3s$	Simplify
$\frac{240}{3}$	$=$	s	Divide both sides by 50
80	$=$	s	Simplify

   Alpha needs to drive at 80 kph to reach Freetown in 3 hours.

Practice

1. Mustapha is a tailor. He wants to estimate how much money he makes in 1 year. He knows that he earned Le 700,000.00 in the previous 4 weeks. If he earns money at the same rate all year, how much does he make in a year?

2. Mohamed and David are farmers. They work together on the farm, and share all of the costs and profits. This week, Mohamed contributed Le 20,000.00 and David contributed Le 24,000.00. They agreed to split this week's income from the farm according to how much they put in. Mohamed received Le 60,000.00. What is David's portion?

3. At a school event, there are 4 big pots of rice, enough to feed 500 people. If 125 more people come to the event than planned, how much more rice is needed?

JSS2 Answer Key – Term 1

Lesson Title: Converting Between Mixed and Improper Fractions
Practice Activity: PHM-08-001

1. a. $7\frac{2}{3}$; b. $3\frac{1}{3}$; c. $1\frac{1}{8}$; d. $2\frac{2}{3}$
2. a. $\frac{17}{8}$; b. $\frac{35}{4}$; c. $\frac{57}{5}$; d. $\frac{47}{40}$

Lesson Title: Converting Decimals to Fractions
Practice Activity: PHM-08-002

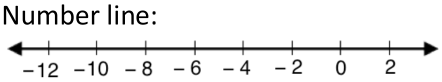
1. $\frac{9}{20}$
2. $5\frac{13}{50}$
3. $\frac{1}{200}$
4. $10\frac{1}{20}$
5. $25\frac{1}{4}$
6. $9\frac{7}{10}$
7. $\frac{2}{25}$
8. $3\frac{3}{10}$

Lesson Title: Converting Fractions to Decimals
Practice Activity: PHM-08-003

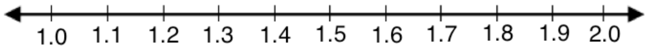
1. 0.7
2. 0.031
3. 1.99
4. 0.6
5. 0.375
6. 0.45
7. 4.5
8. 0.0625

Lesson Title: Comparing and Ordering a Mixture of Numbers
Practice Activity: PHM-08-004

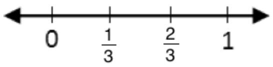
1. Number line:



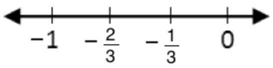
2. Number line:



3. Number line:



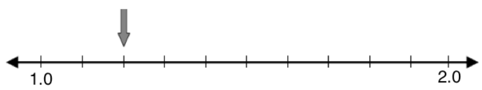
4. Number line:



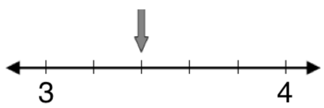
Lesson Title: Locating a Mixture of Numbers on the Number Line
Practice Activity: PHM-08-005

1. a. $\frac{4}{5}$; b. 1.4 or $1\frac{4}{10}$; c. 1.32; d. -5; e. 0.11
2. Number lines with arrows:

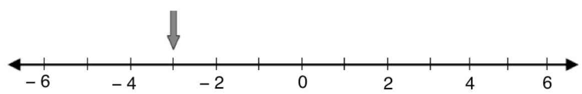
f. 1.2



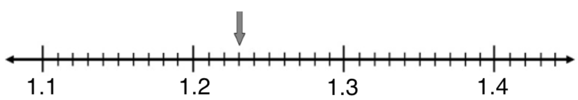
g. $3\frac{2}{5}$



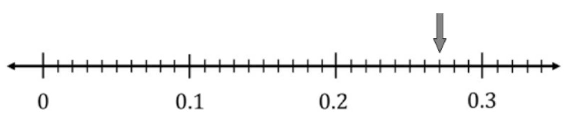
h. -3



i. 1.23



j. 0.27



Lesson Title: Classification of Decimal Numbers
Practice Activity: PHM-08-006

1. a. $9.\overline{8}$; b. $3.\overline{12}$; c. $123.\overline{456}$; d. $8.\overline{68}$
2. a. recurring; b. terminating; c. recurring; d. recurring
3. a. terminating; b. recurring; c. recurring; d. terminating

Lesson Title: Rounding off Decimal Numbers to the Nearest Whole
Practice Activity: PHM-08-007

1. a. 318; b. 0; c. 2; d. 71; e. 201
2. Completed table:

PUPIL	DISTANCE (KM)	DISTANCE TO THE NEAREST KM
Sia	3.8	4
David	1.75	2
Annette	0.3	0
Yusuf	2.5	3
Mary	2.189	2
Foday	1.09	1

Lesson Title: Rounding off Decimal Numbers to Stated Decimal Places
Practice Activity: PHM-08-008

1. 2.2
2. 1787.4
3. 1.0
4. a. 2.105; b. 0.592; c. 21.021; d. 9.091; e. 310.358
5. Completed table:

DAY	FUEL USED (L.)	FUEL (L) TO 1 DECIMAL PLACE
Monday	4.578	4.6
Tuesday	3.45	3.5
Wednesday	5.093	5.1
Thursday	0.995	1.0
Friday	3.72	3.7

Lesson Title: Introduction to Significant Figures
Practice Activity: PHM-08-009

1. 5 s.f.
2. 2 s.f.
3. 4 s.f.
4. 2 s.f.
5. 4 s.f.
6. 1 s.f.

7. 5 s.f.
8. 2 s.f.
9. 1 s.f.
10. 5 s.f.

Lesson Title: Rounding off Decimal Numbers to Significant Figures
Practice Activity: PHM-08-010

1. a. 587,300, b. 587,000, c. 590,000
2. 0.046626
3. a. 0.033, b. 4.3, c. 31
4. a. 20,000, b. 500, c. 0.6
5. Le 320,000
6. 1,050,000
7. 0.00584

Lesson Title: Adding and Subtracting Integers and Decimals
Practice Activity: PHM-08-011

1. 341.18
2. 0.6
3. 0.44
4. 39.9
5. 25.145
6. 225.2
7. 272.28
8. 6.742
9. 3.75 yards
10. 9.8 litres

Lesson Title: Adding and Subtracting Fractions with Integers and Decimals
Practice Activity: PHM-08-012

1. 18.9 or $18\frac{9}{10}$
2. 6.3 or $6\frac{3}{10}$
3. 2.5 or $2\frac{1}{2}$
4. 21

Lesson Title: Multiplying and Dividing by Integers and Decimals
Practice Activity: PHM-08-013

1. 42
2. 3.22

3. 8.75
4. 9.1
5. 0.64

Lesson Title: Multiplying and Dividing Fractions by Integers and Decimals
Practice Activity: PHM-08-014

1. 30
2. 28
3. $\frac{1}{2}$
4. 7
5. $\frac{5}{6}$

Lesson Title: Story Problems with Operations on Different Number Types
Practice Activity: PHM-08-015

1. 35 kg
2. 22.8 kg
3. $1\frac{1}{8}$ mile
4. 7.5 cm
5. 20 cakes
6. $\frac{3}{8}$ mile
7. 61.4 kg
8. 82.9 kg

Lesson Title: Review the Concept and Vocabulary of Factors and Multiples
Practice Activity: PHM-08-016

1. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
2. a. 1, 2, 4, 5, 10, 20; b. 1, 2, 3, 4, 6, 9, 12, 18, 36; c. 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
3. 42, 45, 48, 51, 54, 57
4. 27, 36, 45, 54, 63, 72
5. 6, 12, 18, 24, 30, 36, 42, 48, 54, 60; a. Yes; 12, 24, 36, 48, 60; b. No

Lesson Title: Review Prime and Composite Numbers
Practice Activity: PHM-08-017

1. Completed table:

Numbers	**Factors**	**Prime or Composite**
16	1, 2, 4, 8, 16	Composite
24	1, 2, 3, 4, 6, 8, 12, 24	Composite
29	1, 29	Prime
41	1, 41	Prime
49	1, 7, 49	Composite
54	1, 2, 3, 6, 9, 18, 27, 54	Composite
59	1, 59	Prime

2. 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34
3. 53, 57, 59, 61, 67

Lesson Title: Prime Factors of Whole Numbers
Practice Activity: PHM-08-018

1. 2, 3, 5
2. 2, 11
3. Completed table:

Numbers	**Factors**	**Prime factors**
21	1, 3, 7, 21	3, 7
35	1, 5, 7, 35	5, 7
42	1, 2, 3, 6, 7, 14, 21, 42	2, 3, 7
56	1, 2, 4, 7, 8, 14, 28, 56	2, 7
90	1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90	2, 3, 5

Lesson Title: Calculating the Least Common Multiple (LCM)	
Practice Activity: PHM-08-019	

1. 4
2. 6
3. 24
4. 60
5. 36
6. 360
7. 180

Lesson Title: Calculating the Highest Common Factor (HCF)	
Practice Activity: PHM-08-020	

1. 4
2. 5
3. 12
4. 28
5. 18
6. 4

Lesson Title: Index Notation	
Practice Activity: PHM-08-021	

1. a. 1; b. 16; c. 900
2. a. 1000; b. 64; c. 0
3. a. $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$; b. $8 \times 8 \times 8 \times 8$; c. $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
4. a. 7^4; b. 4^7; c. 3^5
5. a. 625; b. 1; c. 144

Lesson Title: Index Law 1: Multiplication of Indices	
Practice Activity: PHM-08-022	

1. a^{15}
2. u^7
3. 9^{11}
4. 11^{13}
5. 10^5
6. 9^{23}
7. 7^{14}
8. 5×2^{13}
9. $7^6 \times 5^{12}$
10. $3^5 \times 11^8$; the expression is already in its simplest form

Lesson Title: Index Law 2: Division of Indices	
Practice Activity: PHM-08-023	

1. 5^6
2. a^9
3. 2^{25}
4. 4^{10}
5. 7
6. b^8
7. 8^9
8. 2^3
9. 7^2
10. $5^{10} \div 3^5$; the expression is already in its simplest form

Lesson Title: Index Law 3: Power of Zero	
Practice Activity: PHM-08-024	

1. 1
2. 1
3. 1
4. 1
5. 1
6. 7^{12}
7. 5^9
8. 1
9. 3^9

Lesson Title: Index Law 4: Powers of Indices	
Practice Activity: PHM-08-025	

1. 9^{21}
2. 2^{100}
3. 0
4. a^8
5. 1
6. u^{60}
7. 1

Lesson Title: Index Laws 5 and 6: Power of a Product and Quotient
Practice Activity: PHM-08-026

1. $s^{100} \times t^{100}$
2. $5^6 x^6$
3. $2^{15} \times 4^{15}$
4. $\frac{1^{12}}{8^{12}}$ or $\frac{1}{8^{12}}$
5. $\frac{x^3}{y^3}$
6. $12^{31} \div 11^{31}$
7. $51^2 \div 17^2$

Lesson Title: Application of the Laws of Indices
Practice Activity: PHM-08-027

1. 5^2
2. 3^{48}
3. 2^{60}
4. $\frac{1}{5\times3^4}$
5. $\frac{b}{2^3}$
6. 2^{61}
7. a^6

Lesson Title: Indices with Negative Powers
Practice Activity: PHM-08-028

1. $\frac{1}{21^3}$
2. $\frac{1}{10^9}$
3. $\frac{1}{130^4}$
4. $\frac{1}{a^{12}}$
5. $\frac{1}{b^{100}}$
6. $\frac{7}{3^3}$
7. $\frac{10}{2^{14}}$
8. 2^2
9. 3

Lesson Title: Multiplying and Dividing Indices with Negative Powers	
Practice Activity: PHM-08-029	

1. $\frac{1}{3^5}$
2. $\frac{1}{5^8}$
3. 10^5
4. $\frac{1}{21^{18}}$
5. $\frac{1}{3^{53}}$
6. 1
7. 14
8. $\frac{1}{7^4}$

Lesson Title: Negative Powers and the Index Laws	
Practice Activity: PHM-08-030	

1. 3^6
2. $\frac{5}{2^2}$
3. $\frac{1}{7^{27}}$
4. $\frac{1}{2^{13}}$
5. $\frac{1}{c^5 d}$
6. 3^8

Lesson Title: Identifying the Percentage of a Given Quantity	
Practice Activity: PHM-08-031	

1. 480
2. 50
3. Le 960.00
4. 42 mangoes
5. 15 oranges
6. 420 children
7. 125 newspapers

Lesson Title: Expressing One Quantity as a Percentage of Another	
Practice Activity: PHM-08-032	

1. 10%
2. 20%

3. 48%
4. 15%
5. a. 80%; b. 20%
6. a. 10%; b. 70%; 20%

Lesson Title: Percentage Increase	
Practice Activity: PHM-08-033	

1. 15%
2. 6.25%
3. 5%
4. 30%
5. 50%

Lesson Title: Percentage Decrease	
Practice Activity: PHM-08-034	

1. 50%
2. 7.5%
3. 12.5%
4. 5%
5. 51%
6. $16\frac{2}{3}\%$

Lesson Title: Applying Percentage Increase and Decrease	
Practice Activity: PHM-08-035	

1. Le 66,500.00
2. 9.2 seconds
3. 104 cm.
4. Le 78,090.00
5. 598 pupils
6. Le 43,750.00

Lesson Title: Introduction to Profit and Loss	
Practice Activity: PHM-08-036	

1. a. Total costs: Le 500,000.00; Total sales: 530,000.00; b. Profit, her sales are greater than her costs; c. Her profit is Le 30,000.00
2. a. Total costs: Le 1,600,000.00; Total sales: Le 1,400,000.00; b. Loss, his sales are less than his costs; c. His loss is Le 200,000.00

Lesson Title: Calculating Profit	
Practice Activity: PHM-08-037	

1. 25%
2. 50%
3. 60%
4. 40%

Lesson Title: Calculating Loss	
Practice Activity: PHM-08-038	

1. 20%
2. 25%
3. 5%
4. 25%

Lesson Title: Introduction to Percentages Greater than 100	
Practice Activity: PHM-08-039	

1. 150%
2. 250%
3. 112.5%
4. a. $\frac{102}{100}$; b. $\frac{199}{100}$; c. $\frac{200}{100}$
5. 125%

Lesson Title: Calculations with Percentages Greater than 100	
Practice Activity: PHM-08-040	

1. 96
2. 12
3. 105
4. Le 20,000.00
5. Le 14,000.00
6. 147 people
7. 115 kg of cassava

Lesson Title: Ratio	
Practice Activity: PHM-08-041	

1. $\frac{4}{7}$
2. $\frac{1}{4}$
3. $\frac{1}{4}$
4. $\frac{1}{3}$
5. $\frac{19}{20}$
6. $\frac{3}{1}$

Lesson Title: Rate	
Practice Activity: PHM-08-042	

1. 30 km/hr
2. 5 km/hr
3. $\frac{5}{2}$ cars/day or 2.5 cars/day
4. 2 minutes/problem
5. 3 kg/hr
6. 17 km/hr
7. 11.25 km/l

Lesson Title: Unit Rate	
Practice Activity: PHM-08-043	

1. 15 kg/bag
2. 15 players/team
3. 9 paintings/week
4. 6 teachers/car
5. 80 kph or 80 km/hr
6. 60 kph or 60 km/hr

Lesson Title: Calculation of Unit Price	
Practice Activity: PHM-08-044	

1. Le 5,000.00/exercise book
2. Le 20,000.00/kg.
3. Le 12,000.00/fish
4. GH₵6.00/book
5. Le 4,000.00/egg

Lesson Title: Making Comparisons with Unit Price	
Practice Activity: PHM-08-045	

1. Option 2: 5 kg of beans for Le 65,000.00
2. Bendu
3. Option 2: 3 pens for Le 12,000.00
4. Option 2: 50 kg. of rice for Le 375,000.00
5. Le 7,189.00

Lesson Title: Direct Proportion	
Practice Activity: PHM-08-046	

1. a. $\frac{3}{12} = \frac{5}{20}$; b. Extremes: 3 and 20, Means: 12 and 5; c. Yes
2. a. $\frac{5}{15} = \frac{7}{20}$; b. Extremes: 5 and 20, Means: 15 and 7; c. No
3. a. $\frac{1}{9} = \frac{3}{27}$, Yes; b. $\frac{6}{10} = \frac{21}{35}$, Yes; c. $\frac{5}{8} = \frac{20}{30}$; No

Lesson Title: Identifying Direct Proportions	
Practice Activity: PHM-08-047	

1. $k = 6$
2. $k = \frac{2}{5}$
3. $k = 3$
4. a. There are many different answers, depending on the values you choose for x. Example answers: $1 : 4, 2 : 8, 3 : 12$.
   b. Write your proportions as fractions and cross multiply. If the cross products are equal, they are true proportions.
5. a. There are many different answers, depending on the values you choose for x. Example answers: $1 : 10, 2 : 20, 3 : 30$.
   c. Write your proportions as fractions and cross multiply. If the cross products are equal, they are true proportions.
   d.

Lesson Title: Solving Direct Proportions
Practice Activity: PHM-08-048

1. $a = 3$
2. $b = 3$
3. $x = 49$
4. $y = 3$
5. $z = 6$
6. $z = 120$

Lesson Title: Applications of Direct Proportions
Practice Activity: PHM-08-049

1. 60 minutes, or 1 hour
2. 10 pieces of chalk
3. 14 bottles of fertilizer
4. 96 minutes, or 1 hour and 36 minutes
5. 9 cups of rice
6. 5 days

Lesson Title: Direct Proportion Story Problems
Practice Activity: PHM-08-050

1. 75 oranges
2. 12 hours
3. 25 litres of fuel
4. Le 150,000.00
5. 5 days

Lesson Title: Indirect Proportion
Practice Activity: PHM-08-051

1. a. $\frac{3}{9}$ and $\frac{12}{4}$; b. $\frac{3}{9} = \frac{4}{12}$; c. Means: 9 and 4, Extremes: 3 and 12; d. $9 \times 4 = 3 \times 12 = 36$
2. a. No; b. Yes; c. Yes; d. No

Lesson Title: Solving Indirect Proportions	
Practice Activity: PHM-08-052	

1. $a = 5$
2. $b = 6$
3. $c = 2$
4. $d = 20$
5. $x = 50$
6. $z = 114$

Lesson Title: Applications of Indirect Proportions	
Practice Activity: PHM-08-053	

1. 120 workers
2. 3 days
3. 25 days
4. 3 plums each

Lesson Title: Indirect Proportion Story Problems	
Practice Activity: PHM-08-054	

1. $2\frac{2}{5}$ hours, or 2 hours and 24 minutes
2. 2.5 days
3. They need 1 more tailor

Lesson Title: Practice with Proportion	
Practice Activity: PHM-08-055	

1. Le 9,100,000.00
2. Le 72,000.00
3. 1 more pot of rice

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Document information:

Leh Wi Learn (2019). *"Math, Class 08, Term 01 Full, pupil handbook."* A resource produced by the Sierra Leone Secondary Education Improvement Programme (SSEIP). DOI: 10.5281/zenodo.3745218.

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Archived on Zenodo: April 2020.
DOI: 10.5281/zenodo.3745218