THE PRESIDENT'S RECOVERY PRIORITIES

Education

Ministry of Education, Science and Technology

Lesson plans for

JSS Mathematics

JSS 3

TERM 3

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Please see final page for further information.

Foreword

Our country's future lies in the education of our children. The Government of Sierra Leone is committed to doing whatever it takes to secure this future.

As Minister of Education, Science and Technology since 2007, I have worked every day to improve our country's education. We have faced challenges, not least the Ebola epidemic which as we all know hit our sector hard. The Government's response to this crisis – led by our President – showed first-hand how we acted decisively in the face of those challenges, to make things better than they were in the first place.

One great success in our response was the publication of the Accelerated Teaching Syllabi in August 2015. This gave teachers the tools they needed to make up for lost time whilst ensuring pupils received an adequate level of knowledge across each part of the curriculum. The Accelerated Teaching syllabi also provided the pedagogical resource and impetus for the successful national radio and TV teaching programs during the Ebola epidemic.

It is now time to build on this success. I am pleased to issue new lesson plans across all primary and JSS school grades in Language Arts and Mathematics. These plans give teachers the support they need to cover each element of the national curriculum. In total, we are producing 2,700 lesson plans – one for each lesson, in each term, in each year for each class. This is a remarkable achievement in a matter of months.

These plans have been written by experienced Sierra Leonean educators together with international experts. They have been reviewed by officials of my Ministry to ensure they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognised techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new plans. It is really important that these Lesson Plans are used, together with any other materials you may have.

This is just the start of education transformation in Sierra Leone. I am committed to continue to strive for the changes that will make our country stronger.

I want to thank our partners for their continued support. Finally, I also want to thank you – the teachers of our country – for your hard work in securing our future.

Dr. Minkailu Bah

Minister of Education, Science and Technology

Table of Contents

JSS
Mathematics

Introduction

to the Lesson Plan Manual

These lesson plans are based on the National Curriculum
and meet the requirements established
by the Ministry of Education, Science and Technology.

1. The lesson plans will not take the whole term, so use spare time to review material or prepare for exams
2. Teachers can use other textbooks alongside or instead of these lesson plans.
3. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
4. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
5. Quickly review what you taught last time before starting each lesson.
6. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
7. Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
8. Use the board and other visual aids as you teach.
9. Interact with all pupils in the class – including the quiet ones.
10. Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Learning outcomes

Teaching aids

Preparation

Lesson Title: Review of the Number Line	**Theme:** Algebra	
Lesson Number: M-09-106	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Locate positive and negative numbers on the number line. 2. Compare and order numbers on the number line.	None	1. 1. Draw 6 number lines from $-7\ to\ +7$, spaced apart in the centre of the board (see end of Lesson Plan). 2. Write the questions from Independent Practice on the board.

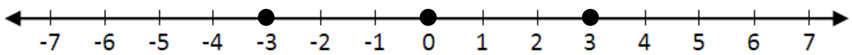
Opening *(3 minutes)*

1. **Ask**: Who can remind the class of what numbers make up the rational numbers?
2. Guide a pupil to answer: Positive and negative whole numbers, fractions, decimals and zero.
3. Point to the number line drawn on the board.
4. **Ask**: Who can identify what this is?
5. Have a pupil volunteer to answer. Guide the pupil to answer: A number line.
6. **Say**: Today we are going to locate positive and negative numbers on the number line. We will also compare and order numbers on the number line.

Introduction to the New Material *(10 minutes)*

Wait a few moments after each question, before having pupils volunteer to answer.

1. **Say**: We looked at rational numbers at the beginning of the academic year.
   Let us see how much of it we remember.
2. Write this set of numbers on the board: $\{\,3, -3,\ 0\,\}$
3. **Say**: Can someone show the class where the numbers *0, 3* and *-3* fit on the first number line?
4. Have a volunteer pupil show these on the number line on the board.



5. **Ask**: What do you notice about the positions of the *3* and *-3* in relation to *0?*
   (Answer: They are the same distance away from 0.)
6. **Say**: Every rational number has positive and negative counterparts. They are the same distance away from zero. They always equal to *0* when we add them together.

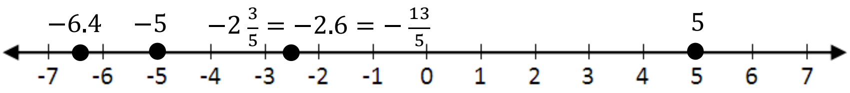
$$3 + (-3) \quad = \quad 0 \qquad \text{3 plus negative 3 equals zero}$$
$$\frac{1}{2} + \left(-\frac{1}{2}\right) \quad = \quad 0 \qquad \text{half plus negative half equals zero}$$

   We used this fact before to balance our equations when we solved for the variables in linear equations. We will use this fact again later when we solve for variables in inequalities.
7. **Ask**: Imagine we have a giant number line marked on the floor. We are standing at *0.* Will the numbers get bigger or smaller if we start to walk right?
8. Allow pupils a few moments to think about this. Have a pupil volunteer to answer.
   (Answer: Numbers will get bigger)

9. **Ask:** Now imagine we are back at *0* on our giant number line. We start to walk left. Will our numbers get bigger or smaller? (Answer: Smaller)
10. **Say**: Draw and mark a number line in your exercise books from -7 to $+7$.
11. While the pupils are doing so, **write** this set of numbers on the board:

$$\{-2\tfrac{3}{5}, -5, -2.6, -6.4, 5, -\tfrac{13}{5}\}$$

12. **Say**: Work in pairs to locate these numbers on your number lines.
    Mark the number line with a ● and write the number above the mark.
13. Allow time for pupils to mark the numbers on their number lines.
14. Have pupils from around the classroom volunteer to mark 2 numbers each on one of the



    number lines on the board.
15. **Ask**: What do you notice about -$2\frac{3}{5}$, -2.6 and $-\frac{13}{5}$? (Answer: They represent the same number.)
16. **Ask**: What do you notice about -5 and 5? (Answer: They are an equal distance away from 0).
17. **Ask**: Let us compare 2 of the numbers on our number line. Look at -6.4 and 5. How do we know which one is bigger? (Answer: Since the bigger number is on the right of the other number, 5 is bigger than -6.4.)
18. **Write** this set of numbers on the board: $\{5.9, -3\frac{1}{3}, -1.5, 5\frac{4}{5}, -1\frac{1}{5}\}$
19. **Say**: Use the number line on the board to help you put these numbers in ascending order. You can change the fractions to decimal numbers. Check with your neighbour when you finish.
20. Allow pupils time to do this. Have a pupil volunteer to answer.
    (Answer: $\{-3\frac{1}{3}, -1.5, -1\frac{1}{5}, 5\frac{4}{5}, 5.9\}$)

Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs.
2. **Write** on the board:

    Put a circle around the smaller of the 2 numbers shown below.

    i. $-12.7, -5.8$ *ii.* $6\frac{2}{5}, 6.25$ *iii.* $-8.7, -7.8$

    Put these numbers in ascending order

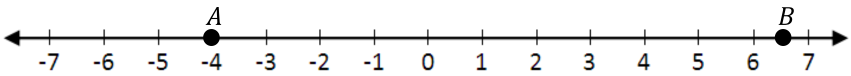
    iv. $\{3, -5, 1, -3\}$ *v.* $\{\frac{3}{5}, \frac{2}{5}, \frac{1}{2}, \frac{4}{7}\}$ *vi.* $\{3.4, -3.4, 3.04, 0.34\}$

3. Walk around, if possible, to check answers and clear misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Correct any errors in the answers on the board. Ask all pupils to check their work. (Answers: *i.* -12.7*; ii.* 6.25*, iii.* -8.7;
    iv. $\{-5, -3, 1, 3\}$; v*i.* $\{\frac{2}{5}, \frac{4}{7}, \frac{1}{2}, \frac{3}{5}\}$; *vi.* $\{-3.4, 0.34, 3.04, 3.4\}$)

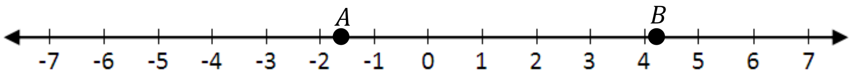
Independent Practice *(10 minutes)*

1. Ask the pupils to work independently.
2. Point to these questions on the board:

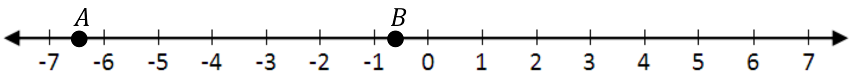
i. Which letter best shows 6.5? Write the number for the other letter.



ii. Which letter best shows 4.2? Write the number for the other letter.



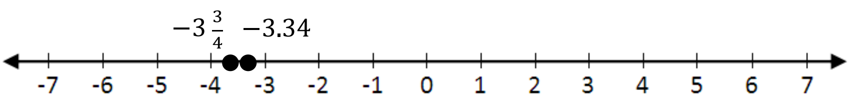
iii. Which letter best shows -6.5? Write the number for the other letter.



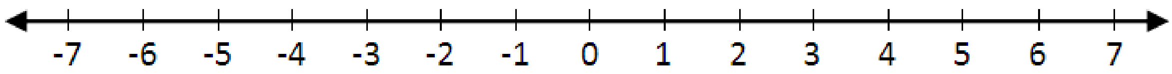
3. Walk around, if possible, to check answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Correct any errors and write the answers on the board. Ask pupils to check their work. (Answers: i. B, -4 ; ii. B, accept -1.7 or -1.8; iii. A, accept -0.7 or -0.8)

Closing *(2 minutes)*

1. **Ask**: Look for the number 4.5 on the number line. What number is an equal distance away from 0 as 4.5? (Answer: -4.5)
2. **Ask**: Which of these 2 numbers is bigger: -3.34 or $-3\frac{3}{4}$? Give a reason for your answer.
3. Have a pupil volunteer to give their reason for their choice. Ask the class if they agree. (Example answers:
   - -3.34 is bigger; it is closer to 0 than $-3\frac{3}{4}$,
   - $-3\frac{3}{4} = -3.75$ so it is smaller than -3.34;
   - -3.34 is on the right of $-3\frac{3}{4}$ so it is bigger.
   - Accept all reasonable answers.)
4. Ask pupils to confirm this by locating the 2 numbers on a number line in their exercise books.
5. Have a pupil volunteer to show the numbers on a number line on the board. (Answer: Shown below)



[NUMBER LINE -7 TO 7]



Lesson Title: Introduction to Inequality	**Theme**: Algebra	
Lesson Number: M-09-107	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify the 'greater than' and 'less than' symbols and use them to compare positive and negative numbers. 2. Show 'greater than' and 'less than' on the number line.	None	1. Write on the top centre of the board: $=, \neq, <, >$ 2. Draw 4 number lines from $-7\ to\ +7$ underneath in the centre of the board. 3. Write the question for the different sections of this lesson on the right (or left) hand side of the board (see the end of the lesson plan).

Opening *(3 minutes)*

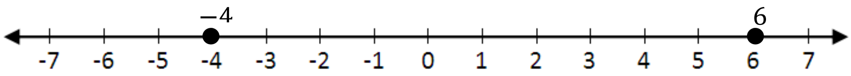
1. **Ask:** Who can remind the class of what we did last lesson?
2. Have a pupil volunteer to answer. (Answer: Located numbers on a number line; ordered and compared numbers. Accept all reasonable answers.)
3. Write this set of numbers on the board: $\{ -\frac{14}{3}, \frac{19}{5}, -0.5, 1.62 \}$
4. **Say**: Put these numbers in descending order. Raise your hand when you finish.
5. Allow pupils time to do this. Have a pupil volunteer to answer. (Answer: $\frac{19}{5}, 1.62, -0.5, -\frac{14}{3}$)
6. **Say**: Today, we are going to identify the 'greater than' and 'less than' symbols and use them to compare positive and negative numbers. We will also show 'greater than' and 'less than' on the number line.

Introduction to the New Material *(10 minutes)*

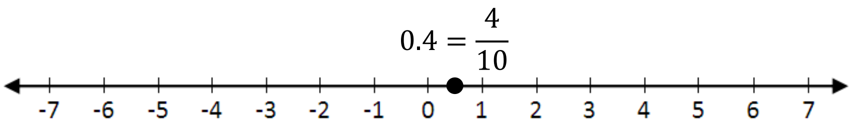
1. Point to the symbols on the board.
2. **Ask**: Who can remind the class of what the symbols on the board mean?
3. Allow a few moments for pupils to discuss and share ideas.
   Point to each symbol and guide different pupils to say:
   - $=$ is equal to
   - $\neq$ is not equal to
   - $<$ is less than
   - $>$ is greater than
4. **Say:** The first symbol shows equality between 2 expressions or numbers.
   The other 3 symbols are called inequality symbols.
5. **Ask:** What do you think these 3 symbols show?
6. Guide a volunteer pupil to give the answer to what the symbols show. (Answer: They show that 2 expressions or numbers are not equal; they show inequality. Accept all reasonable answers.)
7. **Write**: $4 \neq 3$
8. **Say**: It is quite easy to see that 4 is not equal to 3, and we write it as shown. Let us see how the other 2 symbols are used. We will also show the result on a number line. Look at Question a. on the board.

9. **Ask**: Which of these symbols can we put between the 2 numbers to make the statement true? Show this on a number line.
10. Allow time for pupils to answer. They can work in pairs to discuss and share ideas.
11. Have pupils volunteer to show their answers on a number line.
    (Answer: shown below)

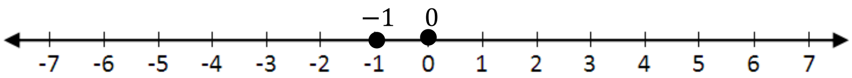
    iv. $6 > -4$



    v. $0.4 = \frac{4}{10}$



    vi. $-1 < 0$



12. **Ask**: How do we know which is the smaller of 2 numbers? (Answer: The smaller number is on the left of the other number.)
13. **Say**: It is sometimes difficult to know which symbol to use for less than and which one for greater than. The point on the symbol always points to the smaller number. Check the numbers on the board and verify it for yourself.
14. Allow a few moments for pupils to check and discuss this amongst themselves.

Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs to answer the questions for Guided Practice.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Select volunteer pupils from around the classroom to give their answers to the questions.
4. Correct any errors and write the answers on the board. Have pupils check their work. (Answers: Shown below the questions at the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs.
2. **Say**: Write 3 questions similar to the ones you have just answered. You must first solve the questions yourselves. Then ask your partner to solve the questions as well.
3. Allow time for pupils to write and solve questions, and to swap for their partners to solve.
4. Walk around, if possible, to check answers and clear up any misconceptions.
5. **Say**: Check each other's answers and make any corrections.
6. Have pairs from around the classroom volunteer to present their questions and answers to the class.

7. Correct any errors and write the answers on the board. Ask pupils to check their work. (Answers: Various, depending on pupils' questions)

Closing *(2 minutes)*

1. **Say:** Write down in your pairs 2 different things you learnt today.
2. Allow pupils time to discuss and share ideas.
3. Have 2 pupils volunteer to share their answers. (Example answers: Learnt to show 'greater than' and 'less than' on a number line; Wrote questions for self and others to solve. Accept all reasonable answers.)

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIALS]

1. Use one of the symbols $<$, $>$ or $=$ to make the statement true:

a. $6 \;\square\; -4$ b. $0.1 \;\square\; \frac{1}{10}$ c. $-1 \;\square\; 0$

[QUESTIONS FOR GUIDED PRACTICE]

2. Use one of the symbols $<$, $>$ or $=$ to make the statement true:

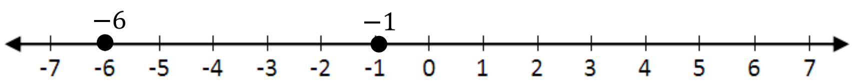
a. $7 + 13 \;\square\; -4 + 8$ b. $5 - 2.5 \;\square\; -2\frac{1}{2} + 5$ c. $7 - 9 \;\square\; -6 + 6$

d. $3.3 + 2.7 \;\square\; 5.5$ e. $8 \;\square\; -1\frac{1}{4} + 9\frac{1}{4}$ f. $-6 \;\square\; -7 + 6$

g. Show your answer for f. on a number line.

Answers: a. $20 > 4$; b. $2.5 = 2.5$; c. $-2 < 0$; d. $6 > 5.5$; e. $8 = 8$; f. $-6 < -1$

g.

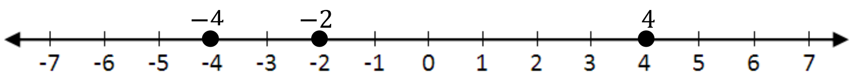


Lesson Title: Linear Inequality	Theme: Algebra	
Lesson Number: M-09-108	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Interpret simple linear inequalities. 2. Represent simple linear inequalities on the number line.	None	1. Draw 4 number lines from $-7\ to\ +7$ in the centre of the board. 2. Write the question for the different sections of this lesson on the right (or left) hand side of the board (see the end of the lesson plan).

Opening *(3 minutes)*

1. Write on the board: Show the numbers $4, -4$ and -2 on a number line.
2. Allow pupils time to do this. Have a pupil volunteer to show the answer on the board. (Answer: shown below).



3. **Say**: Today we are going interpret simple linear inequalities. We will also represent simple linear inequalities on the number line.

Introduction to the New Material *(10 minutes)*

1. **Ask**: Last lesson, we used 3 symbols to show the relationship between 2 numbers. Who can remind the class of what they were?
2. Have a pupil volunteer to answer. Write the symbols on the board as they answer (Answer: $<$, *less than;* $>$, *greater than;* $=$, *equal to*)
3. **Ask**: What are the symbols called? (Answer: *Inequality* symbols)
4. The next 2 questions can be answered in any order by the pupils.
5. **Ask:** How can we show the relationship between 4 and –2 using one of the symbols on the board? (Answer: $4 > -2$)
6. **Ask:** Is there any other way we can show this relationship? (Answer: $-2 < 4$)
7. Point to the number line which shows the relationship.
8. **Say:** We have shown here how we represent this relationship on a number line.
9. **Write** on the board:

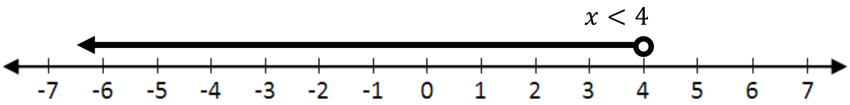
$$x = 4 \qquad (1)$$

$$x < 4 \qquad (2)$$

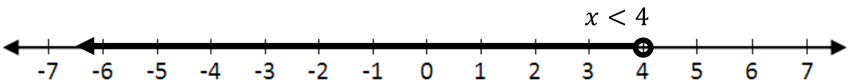
$$x > 4 \qquad (3)$$

10. **Ask**: Who can tell the class what the equal sign denote in line (1)?
11. Guide a volunteer pupil to say: equality (or a linear equation).
12. **Ask**: Who can tell the class what the symbols denote in lines (2) and (3)?
13. Guide a volunteer pupil to say: inequality.

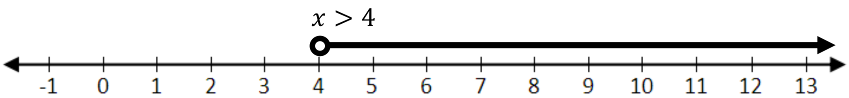
14. **Say**: $x < 4$ and $x > 4$ are called linear inequalities denoted by the 'less than' and 'greater than' symbols. We read them as 'x is less than 4' and 'x is greater than 4'. There is only one answer for $x = 4$ as shown on the number line.
15. **Ask**: What answers do we have for $x < 4$?
16. Have pupils from around the classroom volunteer to give answers.
17. Keep asking for answers until hopefully they realise that there is an infinite number of answers. (Example answers: $3, 2, 1, 0, 1.5, -1, -2.5, -3\frac{3}{4}$ . . .).
18. If they do not realise that there is an infinite number of answers, **say**: We can keep giving answers for $x < 4$ for ever and we will not reach a final answer. We can show this relationship on a number line.



19. (An alternative way of showing the linear inequality is shown below. Choose the method with which you are familiar.)
20. **Say:** Note that the open circle means that $x = 4$ is not a possible solution to the linear inequality $x < 4$. The arrow shows that all numbers to the left of 4 going to negative infinity are answers or solutions to the linear inequality $x < 4$. $x = 4$ is called a boundary point for the inequality.



21. **Ask**: Work with the pupil beside you to show the relationship for $x > 4$ on a number line.
22. **Say**: Start by drawing a number line with a suitable scale.
23. Allow time for pupils to discuss and share ideas.
24. Have a pupil volunteer to show their answer on the board. (Answer: Shown below, allow flexibility in choice of scale)



25. **Say:** Note again that the open circle means that $x = 4$ is not a possible solution to the linear inequality $x > 4$. The arrow shows that all numbers to the right of 4 going to infinity are answers or solutions to the linear inequality $x > 4$. $x = 4$ is the boundary point for the inequality.

Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs to answer the questions for Guided Practice.
2. Walk around, if possible, to check answers and correct any misconceptions. For example, make sure pupils are not shading or filling in the circles for either $<$ or $>$.
3. Have pupils from around the classroom volunteer to draw their linear inequality on the board.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs to answer the questions for Independent Practice.
2. Walk around, if possible, to check answers and clear misconceptions.
3. **Say**: Check each other's answers and make any corrections.

4. Have pupils from around the classroom volunteer to draw their linear inequality on the board. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Closing *(2 minutes)*

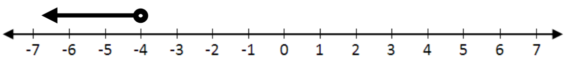
1. **Say:** Write in your exercise books how you would explain to a friend who did not understand how to draw the linear inequality for $x < 5$ on a number line.
2. Allow pupils time to write an explanation in their exercise books.
3. Select a volunteer pupil to read out their explanation. They can get help from the class. (Example answer:
    - Draw a number line; mark an appropriate scale on the number line
    - Draw an open circle at 5 to show the point.
    - Start at 5, draw an arrow pointing to the left. This shows that the values for x are less than but not equal to 5.

    Accept all reasonable answers.)

[QUESTIONS FOR GUIDED PRACTICE]

Write down the linear inequality shown by the number lines below:

1.



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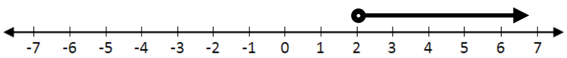

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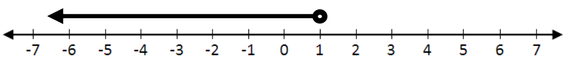

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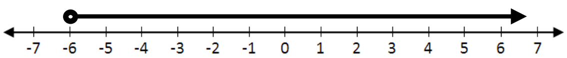

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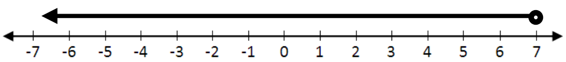

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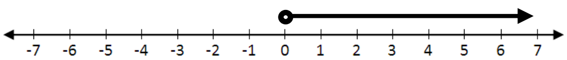


Answers: Note that the scales on the number lines may be different. The important thing is that the point is at the correct place and the arrow points in the correct direction.

1. $x < -4$; 2. $x > 2$; 3.. $x < 1$; 4. $x > -6$; 5. $x < 7$; 6. $x > 0$

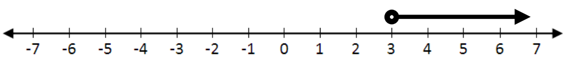
[QUESTIONS FOR INDEPENDENT PRACTICE]

Show the inequalities below on a number line:

1. $x > 3$ 2. $x < -2$ 3. $x > -1$ 4. $x < 0$

Answers:

1.



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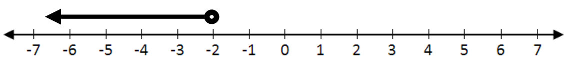

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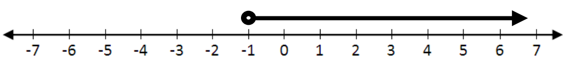

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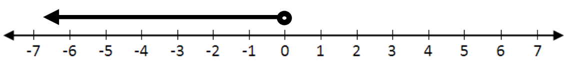
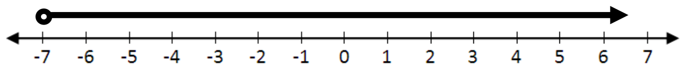


Lesson Title: Solving Linear Inequalities in One Variable I	Theme: Algebra	
Lesson Number: M-09-109	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to solve linear inequalities in one variable using addition and subtraction.	None	1. Draw 4 number lines from $-7\ to\ +7$ in the centre of the board. 2. Write the equations from Introduction to the New Material on the board.

Opening *(3 minutes)*

1. **Write** on the board: Show on a number line the linear inequality $\mathrm{x} > -7$.
2. Allow pupils time to do this. Select a volunteer pupil to show the answer on the board. (Answer: Shown below)



3. **Say**: Today we are going to solve linear inequalities in one variable using addition and subtraction.

Introduction to the New Material *(10 minutes)*

1. Point to these equations on the board:

$$x + 2 = 7 \quad (1)$$

$$x + 2 < 7 \quad (2)$$

$$x + 2 > 7 \quad (3)$$

2. **Say**: Show step by step in your exercise books how you would solve for x in the equation in line (1). Raise your hand when you finish.
3. Allow time for pupils to solve the equation.
4. Select a pupil with a raised hand to explain their answer on the board. An example solution is shown below.

$$x + 2 = 7$$

$$x + 2 - 2 = 7 - 2 \quad \longleftarrow \textit{Subtract 2 from both sides}$$

$$x = 5 \quad \longleftarrow \textit{Solution for } x$$

5. **Say**: We check that our solution for x is correct by substituting $x = 5$ and solving.
6. Have another pupil volunteer to check the solution to the equation.

<u>*Check*</u> *when* $x = 5$

$$5 + 2 = 7$$

$$7 = 7$$

$$\text{LHS} = \text{RHS}$$

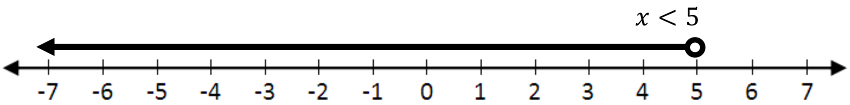
7. **Say:** We use the exact same method to solve for x in the linear inequality in line (2)

$$x + 2 < 7$$

$$x + 2 - 2 < 7 - 2 \quad \longleftarrow \quad \textit{Subtract 2 from both sides}$$

$$x < 5 \quad \longleftarrow \quad \textit{Solution for } x$$

8. Ask the pupils to show in their exercise books the solution for x on a number line.
9. Have a pupil volunteer to show the solution on the board.



10. **Say**: We can check that our solution for x is correct just as we did for the equation. In an equation, there is only one solution to the equation. In a linear inequality, there are an infinite number of solutions. We check at least 2 values. We first check our solution for the related linear equation.
11. Show again that LHS=RHS of the linear equation when $x = 5$.

<u>Check</u> *when* $x = 5$

$$5 + 2 = 7$$
$$7 = 7$$
$$\text{LHS} = \text{RHS}$$

12. **Say**: We check for at least one other value which we choose according to the solution. We choose 3 here because our solution says all the values of x are less than 5.

<u>Check</u> *when* $x = 3$

$$3 + 2 < 7$$
$$5 < 7$$

13. **Say**: The checks show that our solution is correct and all numbers less than 5, denoted by $x < 5$, are solutions to the linear inequality. We call 5 the boundary point. Everything to the left of 5, denoted by $x < 5$, is part of the solution for this inequality. Everything to the right of 5, denoted by $x > 5$ is not part of the solution for this inequality.

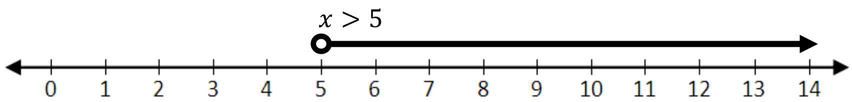
Guided Practice *(10 minutes)*

1. **Say**: Let us now look at the linear inequality $x + 2 > 7$. Work with your neighbour to solve for x. Draw the solution on a number line.
2. Allow time for pupils to solve the equation and draw the number line.
3. Walk around, if possible, to check answers and correct any misconceptions.
4. Have a pupil volunteer to show their solution on the board.

$$x + 2 > 7$$

$$x + 2 - 2 > 7 - 2 \quad \longleftarrow \quad \textit{Subtract 2 from both sides}$$

$$x > 5 \quad \longleftarrow \quad \textit{Solution for } x$$



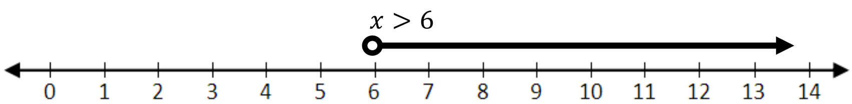
5. **Say:** We need to check for at least 2 values of x. We already checked for the linear equation $x + 2 = 7$. Let us check instead for values on either side of 5 on the number line. Work in your pairs. One of you check for $x = 4$ and the other for $x = 6$.
6. Allow time for pupils to check. Have 2 pupils volunteer to do the checks on the board.

<u>Check</u> *when* $x = 4$	<u>Check</u> *when* $x = 6$
$4 + 2 \quad > \quad 7$	$6 + 2 \quad > \quad 7$
$6 \quad \ngtr \quad 7$	$8 \quad > \quad 7$

7. **Ask**: What is the boundary point for this inequality? (Answer: 5)
8. **Say**: This time, everything to the right of 5, denoted by $x > 5$, is part of the solution for this inequality. Everything to the left of 5, denoted by $x < 5$ is not part of the solution for this inequality. Equally, 5 itself is not part of the solution.

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to solve for x in the linear inequality $x - 4 > 2$. They should check for 2 values of x and show the solution on a number line.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Have a pupil volunteer to show their solution on the board.
4. Correct any errors in the calculation on the board. Ask pupils to check their work.
   (Answer: $\mathrm{x} - 4 > 2$, $\mathrm{x} - 4 + 4 > 2 + 4$, $\mathrm{x} > 6$; Check the linear equation $\mathrm{x} - 4 = 2$, when $\mathrm{x} = 6$: $6 - 4 = 2$, $2 = 2$, LHS = RHS;
   Check the linear inequality $\mathrm{x} - 4 > 2$, when $\mathrm{x} = 9$: $9 - 4 > 2$, $5 > 2$, the solution $\mathrm{x} > 6$ is correct; solution shown on number line below)



Closing *(2 minutes)*

1. **Say:** Write down in your pairs 2 different things you learnt today.
2. Allow pupils 1 minute to discuss and share ideas.
3. Have 2 pupils volunteer to answer.
   (Example answers: How to solve linear inequalities involving addition and subtraction; How to show the solution to linear inequalities on a number line; How to check that a solution to a linear inequality is correct; That the solution to the inequality is called the boundary point. Accept all reasonable answers.)

Lesson Title: Solving Linear Inequalities in One Variable II	Theme: Algebra	
Lesson Number: M-09-110	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to solve linear inequalities in one variable using multiplication and division.	None	1. Draw 4 unmarked number lines in the centre of the board. 2. Write the question for the different sections of this lesson on the right (or left) hand side of the board (see the end of the lesson plan).

Opening *(3 minutes)*

1. **Ask**: Who can remind the class of what we did last lesson*?* (Example answers: Solved linear inequalities using addition and subtraction; Showed the solutions for linear inequalities on a number line. Accept all reasonable answers.)
2. **Ask**: Check that $x < 2$ is the solution to $x + 3 < 5$.
3. Allow pupils time to do this. Have a pupil volunteer to show the answer on the board. (Answer: $\mathrm{x} < 2$; Check the equation $\mathrm{x} + 3 = 5$, when $\mathrm{x} = 2$: $2 + 3 = 5$, $5 = 5$, LHS = RHS; Check the inequality $\mathrm{x} + 3 < 5$, when $\mathrm{x} = 0$: $0 + 3 < 5$, $3 < 5$, the solution $\mathrm{x} < 2$ is correct).
4. **Say**: Today we are going to solve linear inequalities in one variable using multiplication and division.

Introduction to the New Material *(10 minutes)*

1. **Write** on the board: Solve for x when $3\mathrm{x} > 12$
2. **Say**: Let us start by looking at a linear inequality involving multiplication. We use the exact same method to solve for x in the linear inequality as if it was a linear equation.
3. Show the solution for x on the board.

$$3x > 12$$

$$\frac{3x}{3} > \frac{12}{3} \quad \longleftarrow \textit{Divide both sides by 3}$$

$$x > 4 \quad \longleftarrow \textit{Solution for } x$$

4. **Say**: We can check that our solution for x is correct just as we do for a linear equation. We check at least 2 values. First check for the related linear equation, $3x = 12$.

<u>*Check*</u> *when* $x = 4$

$$3 \times 4 = 12$$

$$12 = 12$$

$$\text{LHS} = \text{RHS}$$

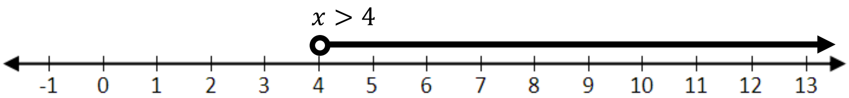
We next check for at least one other value which we choose according to the solution.
We choose 8 here because our solution says all the values of x are greater than 4.

$$\underline{Check}\ when\ x = 8$$

$$3 \times 8 > 12$$

$$24 > 12$$

5. **Say**: The checks show that our solution is correct. 4 is the boundary point for this inequality and all numbers greater than 4, denoted by $x > 4$, are solutions to the linear inequality $3x > 12$.
6. Ask the pupils to show in their exercise books the solution for x on a number line.
7. Have a pupil volunteer to show the solution on the board. Allow flexibility in the scale of the number line.



8. **Say**: Let us now look at a linear inequality involving division.
9. **Write** on the board: Solve for x when $\frac{x}{2} < 3$
10. **Say:** We solve for x in the linear inequality as if it was a linear equation.
11. Show the solution for x on the board.

$$\frac{x}{2} < 3$$

$$\frac{2x}{2} < 3 \times 2 \longleftarrow \textit{Multiply both sides by 2}$$

$$x < 6 \longleftarrow \textit{Solution for } x$$

12. **Say**: We do our usual checks for at least 2 values. We first check our solution for the related linear equation, $\frac{x}{2} = 3$.

$$\underline{Check}\ when\ x = 6$$

$$\frac{6}{2} = 3$$

$$3 = 3$$

$$\text{LHS} = \text{RHS}$$

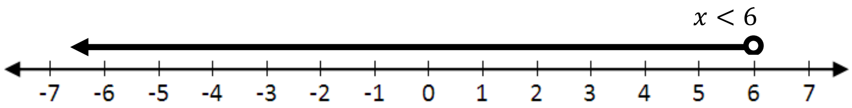
13. **Say**: We next check for at least one other value which we choose according to the solution. We choose 4 here because our solution says all the values of x are less than 6.

$$\underline{Check}\ when\ x = 4$$

$$\frac{4}{2} < 3$$

$$2 < 3$$

14. **Say**: The checks show that our solution is correct and all numbers less than 6, denoted by $x < 6$, are solutions to the linear inequality $\frac{x}{2} < 3$.
15. Ask the pupils to show in their exercise books the solution for x on a number line.
16. Have a pupil volunteer to show the solution on the board.



Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs to answer the questions for Guided Practice.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to present their answers to the class.
4. Correct any errors. (Answers: Shown below the questions at the end of this lesson plan.)

Independent Practice *(10 minutes)*

1. Ask the pupils to work *independently* to answer the questions for Independent Practice.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Select pupils from around the classroom to present their answers to the class.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

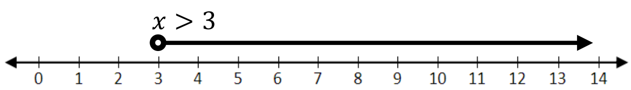
Closing *(2 minutes)*

1. **Say:** Write down one new thing you learnt today.
2. Allow pupils time to write down their answer.
3. Have 2 pupils volunteer to answer.
   (Example answer: How to solve linear inequalities using multiplication and division; How to draw the solutions to 2 linear inequalities on the same number line. Accept all reasonable answers.)

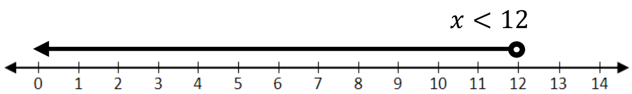
[QUESTIONS FOR GUIDED PRACTICE]

1. Solve for x in the following linear inequalities.
   Check your answer. Draw the solution on a number line.
   a. $5x > 15$ b. $\frac{x}{3} < 4$

Answers: a. $5x > 15$, $x > 3$, Check the linear equation $5x = 15$, when $x = 3$: $5 \times 3 = 15$, $15 = 15$, LHS = RHS; Check the linear inequality $5x > 15$ when $x = 6$, $5 \times 6 = 30$, $30 > 15$, the solution $x > 3$ is correct; solution shown on number line).



b.. $\frac{x}{3} < 4$, $x < 12$, Check the linear equation $\frac{x}{3} = 4$ when $x = 12$: $\frac{12}{3} = 4$, $4 = 4$, LHS = RHS; Check the linear inequality $\frac{x}{3} < 4$ when $x = 9$, $\frac{9}{3} < 4$, $3 < 4$, the solution $x < 12$ is correct; solution shown on number line).

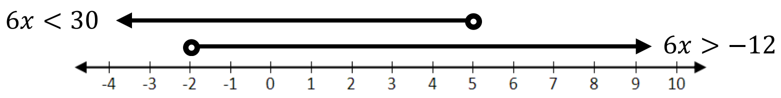


[QUESTIONS FOR INDEPENDENT PRACTICE]

Solve for x in the following linear inequalities.
Check your answers. Draw the solutions on the same number line.

a. $6x < 30$ b. $6x > -12$

Answers: a. $6x < 30$, $x < 5$, Check the linear equation $6x = 30$, when $x = 5$: $6 \times 5 = 30$, $30 = 30$, LHS = RHS; Check the linear inequality $6x < 30$ when $x = 4$, $6 \times 4 = 24$, $24 < 30$, the solution $x < 5$ is correct; solution shown on number line below).
b. $6x > -12$, $x > -2$, Check the linear equation $6x = -12$, when $x = -2$: $6 \times (-2) = -12$, $-12 = -12$, LHS = RHS; Check the linear inequality $6x > -12$ when $x = -1$, $6 \times (-1) = -6$, $-6 > -12$, the solution $x > -2$ is correct; solution shown on number line below).



Lesson Title: Solving Linear Inequalities in One Variable III	**Theme**: Algebra	
Lesson Number: M-09-111	**Class/Level**: JSS 3	**Time**: 35 minutes

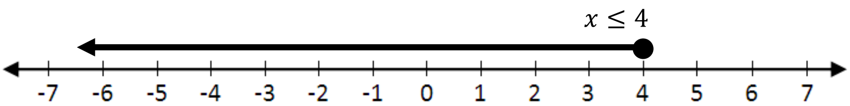
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to apply algebraic principles to solve a linear inequality and illustrate the answer on the number line.	None	Draw 4 unmarked number lines in the centre of the board.

Opening *(3 minutes)*

1. **Ask**: Who can remind the class of what we did last lesson? (Example answers: Solved linear inequalities using multiplication and division; Showed the solutions for linear inequalities on the same number line. Accept all reasonable answers.)
2. **Ask**: Solve for x when $\frac{x}{4} < 2.5$. Check your solution to the inequality.
3. Allow pupils time to do this. Have a pupil volunteer to show the answer on the board. (Answer: $\frac{x}{4} < 2.5, x < 10$; *Check the equation* $\frac{x}{4} = 2.5$, *when* $x = 10$: $\frac{10}{4} = 2.5,\ 2.5 = 2.5$, LHS = RHS; *Check the inequality* $\frac{x}{4} < 2.5$, *when* $= 8$: $\frac{8}{4} < 2.5,\ 2 < 2.5$; *the solution* $x < 10$ *is correct*).
4. **Say**: Today we are going to apply algebraic principles to solve a linear inequality and illustrate the answer on the number line.

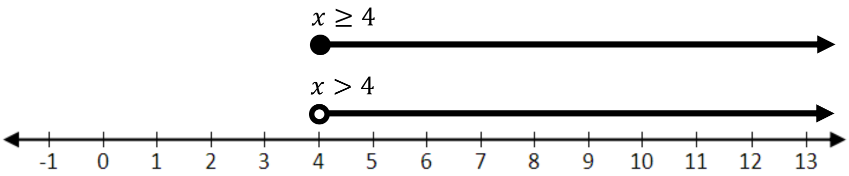
Introduction to the New Material *(10 minutes)*

1. **Write** on the board: $\leq, \geq$
2. **Ask**: Who can tell the class what these symbols are?
3. Guide a volunteer pupil to say: Less than or equal to, and greater than or equal to.
4. **Say:** $\leq$ and $\geq$ are 2 more symbols which show inequality.
5. **Ask:** What are the 3 inequality symbols we know already? (Answer: $\neq, <, >$)
6. **Write** on the board: $x \leq 4$.
7. **Ask**: Who can read this for the class? (Answer: x *is less than or equal to 4*).
8. **Say**: We can show this relationship on a number line.



9. **Ask**: Do you notice anything different from how you drew inequalities before?
10. Guide a volunteer pupil to say: The circle is shaded; The circle is filled in.
11. **Say:** We call the shaded or filled in circle a closed circle. It means that $x = 4$ is a possible solution to the linear inequality $x \leq 4$. The arrow shows that all numbers from 4 and to the left of 4, going to negative infinity are answers or solutions to the linear inequality $x \leq 4$. 4 is the boundary point for this inequality.
12. **Ask**: Work with your neighbour to show the relationship for both $x > 4$ and $x \geq 4$on the same number line.
13. Allow time for pupils to discuss and share ideas.

14. Have a pupil volunteer to show their answer on the board. (Answer: Shown below)



15. **Say:** Take note of the difference between the 2 inequalities. We will use them both in future problems.
16. **Write** on the board: Solve for x when $3(x+2) \geq 18$. Show your solution on a number line.
17. **Say:** We know already that we solve for x in the linear inequality as if it was a linear equation.
18. Work through the solution with the pupils. Ask them how to solve each step and write down their answers. An example solution is shown below.

$$
\begin{aligned}
3(x+2) &\geq 18 \\
3x+6 &\geq 18 && \longleftarrow \textit{Expand the brackets} \\
3x+6-6 &\geq 18-6 && \longleftarrow \textit{Subtract 6 from both sides} \\
3x &\geq 12 \\
\frac{3x}{3} &\geq \frac{12}{3} && \longleftarrow \textit{Divide both sides by 3} \\
x &\geq 4 && \longleftarrow \textit{Solution for } x
\end{aligned}
$$

19. **Say**: Let us check our solution for the related linear equation, $3(x+2) = 18$.
20. Allow time for pupils to check. Have a pupil volunteer to show their check on the board.

<u>*Check*</u> *when* $x = 4$

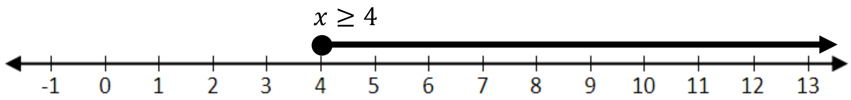
$$
\begin{aligned}
3 \times (4+2) &= 18 \\
3 \times 6 &= 18 \\
18 &= 18 \\
\text{LHS} &= \text{RHS}
\end{aligned}
$$

21. **Say**: Choose a value for x to check our solution for the inequality.
22. Allow time for pupils to check. Select a pupil to show their check on the board. An example is shown for $x = 5$.

<u>*Check*</u> *when* $x = 5$

$$
\begin{aligned}
3 \times (5+2) &\geq 18 \\
3 \times 7 &\geq 18 \\
21 &\geq 18
\end{aligned}
$$

23. **Say**: The checks show that our solution is correct and all numbers greater than or equal to 4, denoted by $x \geq 4$, are solutions to the linear inequality $3(x+2) \geq 18$.
24. Ask the pupils to show in their exercise books the solution for x on a number line.
25. Have a pupil volunteer to show the solution on the board.



Allow flexibility in the scale of the number line.

Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs to answer the questions for Guided Practice.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to present their answers to the class.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer Questions 3 and 4.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Closing *(2 minutes)*

1. **Say**: Write on a piece of paper one thing you do not understand about linear inequalities.
2. Allow time for pupils to answer. Collect papers from pupils at the end of the lesson.
3. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be creating inequalities from story problems.

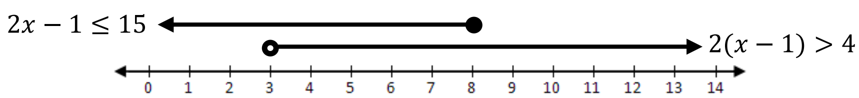
[QUESTIONS FOR GUIDED PRACTICE]

Solve for x in the following linear inequalities.
Check your answers. Draw the solution on the same number line.

1. $2x - 1 \leq 15$ 2. $2(x - 1) > 4$

Answers: 1. $2x - 1 \leq 15$, $x \leq 8$, Check the linear equation $2x - 1 \leq 15$ when $x = 8$: $(2 \times 8) - 1 = 15$, $15 = 15$, LHS = RHS; Check the linear inequality $2x - 1 \leq 15$ when $x = 4$, $(2 \times 4) - 1 \leq 15$, $7 \leq 15$, the solution $x > 3$ is correct; solution shown on number line).

2. $2(x - 1) > 4, 2x - 2 > 4$, $2x > 4 + 2$, $2x > 6, x > 3$, Check the linear equation $2(x - 1) = 4$ when $x = 3$: $2 \times (3 - 1) = 4$, $2 \times 2 = 4$, $4 = 4$, LHS = RHS; Check the linear inequality $2(x - 1) > 4$ when $x = 5$, $2 \times (5 - 1) > 4, 2 \times 4 > 4$, $8 > 4$, the solution $x > 3$ is correct; solution shown on number line).



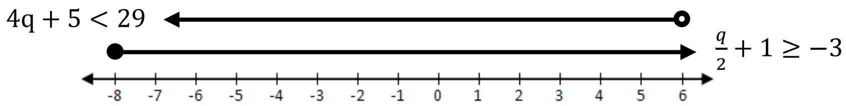
[QUESTIONS FOR INDEPENDENT PRACTICE]

Solve for x in the following linear inequalities.
Check your answers and draw the solutions on the same number line.

3. $4q + 5 < 29$ 4. $\frac{q}{2} + 1 \geq -3$

Answers: 3. $4q + 5 < 29$, $4q < 24$, $q < 6$, Check the linear equation $4q + 5 = 29$ when $q = 6$: $(4 \times 6) + 5 = 29$, $29 = 29$, LHS = RHS; Check the linear inequality $4q + 5 < 29$ when $q = 5$, $(4 \times 5) + 5 < 29$, $25 < 29$, the solution $q < 6$ is correct; solution shown on number line).

4. $\frac{q}{2} + 1 \geq -3$, $\frac{q}{2} \geq -4$, $q \geq -8$, Check the linear equation $\frac{q}{2} + 1 = -3$ when $q = -8$: $\frac{-8}{2} + 1 = -3, -4 + 1 = -3, -3 = -3$, LHS = RHS; Check the linear inequality $\frac{q}{2} + 1 \geq -3$ when $q = 0$, $\frac{0}{2} + 1 \geq -3$, $1 \geq -3$, the solution $q \geq -8$ is correct; solution shown on number line).



Lesson Title: Creating Inequalities from Story Problems	**Theme**: Algebra	
Lesson Number: M-09-112	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify the unknown variable in a story problem. 2. Identify an inequality problem and apply the appropriate inequality symbol.	None	On the board, write the questions for this lesson found at the end of the lesson plan.

Opening *(3 minutes)*

1. **Say**: Listen carefully to the question. Write the answer in your exercise books.
2. **Ask**: On a number line, why is an open circle used for the symbols $<$ and $>$, and a closed circle for the symbols $\leq$ and $\geq$?
3. Repeat the question if necessary.
4. Allow time for pupils to answer.
5. Have 2-3 pupils volunteer to read out their answers. (Example answer: An open circle is used to show the number is not part of the solution. A closed circle is used to show the number is part of the solution. Accept all reasonable answers.)
6. **Say**: Today we are going to identify the unknown variable in a story problem. We will also identify an inequality problem and apply the appropriate inequality symbol.

Introduction to the New Material *(10 minutes)*

1. Ask a pupil to read Question 1 on the board.
2. **Ask:** What is the variable in this question? What changes? Raise your hand if you think you know.
3. Guide a volunteer pupil to say that the number of passengers changes.
4. **Ask**: How do we represent the number of passengers as a variable?
5. Guide a volunteer pupil to say: We represent the variable by a letter, such as p.
6. **Write** on the board: Let $\mathrm{p} =$ number of passengers.
7. **Ask**: We want to write the statement as an inequality. What words can help us to do that?
8. Guide a pupil to say: At most 30 passengers.
9. **Ask:** Is this more than 30 passengers, or less than 30 passengers? Can we have 30 passengers? Discuss this with your neighbour.
10. Allow pupils to discuss and share ideas.
11. Have a pupil volunteer to share with the class. (Answer: 30 passengers or fewer.)
12. **Say**: The words 'at most 30' means we can have up to 30 but no more. The number of passengers should be 30 or fewer.
13. **Ask**: Which of the inequality symbols will show this?
14. Guide a volunteer pupil to say: $\leq$, less than or equal to.

15. **Say**: We now have all the information we need to write the inequality. Work with your neighbour to write it down. Raise your hand when you finish.
16. Allow time for pupils to discuss and share ideas.
17. Have a pupil volunteer to write the inequality on the board. (Answer: $p \leq 30$)
18. Ask: What is the boundary point for this inequality? (Answer: 30)
19. Allow a few moments for pupils to think over how the inequality was derived.
20. **Say**: We will always have clues in the question to help us write the inequality.
21. Ask a pupil to read Question 2.
22. **Ask**: What is the variable in this statement? (Answer: Boxes of chicken pieces)
23. **Ask**: How do we represent the boxes of chicken pieces? (Example answer: With a letter, e.g. b)
24. **Write** on the board: Let b $=$number of boxes of chicken pieces.
25. **Say**: Read the statement again. Work with your neighbour to write down the words and the inequality symbol which you think represent the words.
26. Allow pupils time to share and discuss their ideas.
27. Have a pupil volunteer to give the words and the inequality symbol. (Answer: At least 10 kg. $\geq$)
28. **Say**: The words 'at least 10' means we cannot have fewer than 10. 10 kg is the smallest weight we can have. The weight of the chicken pieces must be greater than 10.
29. Ask pupils to write the inequality in their exercise books. Allow time for them to do this.
30. Have a pupil volunteer to write the inequality on the board. (Answer: $b \geq 10$).
31. Allow pupils time to continue to work together to discuss and share ideas for Question 3.
32. Have a pupil volunteer to share their answer with the class. (Answer: Let t = the income tax, then t > 500000)
33. Some pupils may have written $t \geq 500000$. However, the question says 'income <u>exceeds</u> 500,000' so the inequality symbol is just 'greater than', $>$.
34. Ask a pupil who volunteers to read Question 4.
35. **Ask**: How many statements do we have for this question?
36. Guide a volunteer pupil to say: 2 statements.
37. **Ask**: How many inequalities can we write?
38. Guide a volunteer pupil to say: 2 inequalities.
39. **Ask:** What is the variable in the statement? With what should we represent it? (Answer: Variable is the speed, represent by s)
40. Write on the board: Let s = speed.
41. Ask pupils to write the 2 inequalities in their exercise books. Allow time for pupils to do this.
42. Have a pupil volunteer to write one of the inequalities on the board. (Answer: $s \geq 40$)
43. Have another pupil volunteer to write the other inequality. (Answer: $s \leq 65$)
44. **Ask**: How many boundary points do you think we have for this inequality?
45. Guide a volunteer pupil to say: 2 boundary points, 40 and 65.
46. **Say**: This inequality says the speed goes from 40 to 65. It cannot be less than 40 or greater than 65. We have written 2 inequalities for this statement. We will learn in a future lesson how to write this as one inequality and draw the solution on a number line.

Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.

4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer the questions.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Do not do the answers for Question 12. Use it to check pupils' understanding of the work.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Closing *(2 minutes)*

1. **Say:** Write your name on a piece of paper. Write your answer for Question 12 on the paper. Hand the paper in at the end of the lesson.
2. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be creating and solving inequality story problems.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

State the variable in each of the statements below. Write the statements as linear inequalities:

1. A poda-poda can accommodate at most 30 passengers.
2. Boxes of chicken pieces will be sold by a shop only if the weight is at least 10 kg.
3. Workers start paying income tax when their income exceeds Le 500,000 a month.
4. The minimum speed on a road is 40 kph. The maximum speed on the same road is 65 kph.

[QUESTIONS FOR GUIDED PRACTICE]

5. To enter a talent show, contestants must be over 14 years old.
6. Ada saves at least Le 3,000 every week from the money her parents give her.
7. The number of passengers a ferry can hold must not exceed 400.
8. A car can travel at least 320 km on a full tank of petrol. It cannot travel more than 500 km on a full tank of petrol. (Hint: Write this as 2 inequalities.)

   (Answers: for the answers below, the variable is given first, then the inequality).
   v. Let c = talent show contestant, $c > 14$; vi. Let s = savings, $s \geq 3000$; vii. Let n = number of passengers, $p \leq 400$; viii. Let d = distance car travels, $d \geq 320$, $d \leq 500$).

[QUESTIONS FOR INDEPENDENT PRACTICE]

9. To enter a talent show, contestants must be at least 14 years old.
10. The temperature in Freetown on a particular day is 27 ^{o}C. It is warmer in Kono on that day. (Hint: the temperature in Kono is the variable).
11. The maximum speed limit on a road is 80 km/hr.
12. The maximum mark in a test is 10. Rugiatu always scores at least half of the marks. She also never scores maximum marks. (Hint: write this as 2 inequalities).

(Answers: for the answers below, the variable is given first, then the inequality).
ix. Let c = talent show contestant, $c \geq 14$; x. Let t = temperature in Kono, $t > 27$; xi. Let s = speed limit, $s \leq 80$; xii. Let m = test marks, $m \geq 5$, $m < 10$).

Lesson Title: Solving Inequality Story Problems I	**Theme**: Algebra	
Lesson Number: M-09-113	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to create and solve an inequality from a story problem.	None	Write the questions for this lesson, found at the end of the lesson plan, on the board.

Opening *(3 minutes)*

1. **Say**: Read Question 1 on the board. Write the inequality for the statement.
2. Allow time for pupils to answer. Have a pupil volunteer to answer. (Answer: $c \leq 18000$).
3. **Say**: Remember 'at most' means the maximum amount we are interested in. We want to spend that amount or less than that amount. Today we are going to create and solve an inequality from a story problem.

Introduction to the New Material *(10 minutes)*

1. Have a pupil volunteer to read Question 2 on the board.
2. **Ask**: What do you notice about Question 2?
3. Guide pupils to say: It is building on Question 1; It is asking us to solve for how many weeks Mohammed can save before he can buy the shoes.
4. **Ask**: What is the variable in this situation?
5. Guide a volunteer pupil to say: Number of weeks.
6. **Write** on the board: Let w = number of weeks.
7. **Ask**: Let us think about what the inequality should be. Imagine that he saves Le 2,000 for 3 weeks. How much would he have saved?
8. Guide a volunteer pupil to say: $2000 \times 3 = 6000$.
9. **Ask**: Has he saved enough to buy the shoes? (Answer: No)
10. **Ask**: What about after 4 weeks? Would he have saved the Le 18,000 that he needs? (Answer: No, because $2000 \times 4 = 8000$)
11. **Say**: It is only after $2000 \times w$ weeks that he will have enough money to buy the shoes.
12. **Ask:** How can we write this as a linear inequality?
13. Allow pupils to discuss and share ideas with their neighbour.
14. Have a pupil volunteer to answer. (Answer: $2000w \leq 18000$)
15. **Ask**: What do we get when we simplify? (Answer: $2w \leq 18$)
16. **Ask**: So for how many weeks does Mohammed need to save?
17. Allow time for pupils to solve the inequality. (Answer: $w \leq 9$ *weeks*)
18. **Say**: Do checks in your exercise books to verify your solution.
19. Allow time for pupils to check and verify that their solution is correct.
    (Answer: Shown right, checks verify solution is correct)
20. **Say**: Let us look at another question.
21. Have a pupil volunteer to read Question 3.

Check when $w = 9$

$$2 \times 9 = 18$$
$$18 = 18$$
$$\text{LHS} = \text{RHS}$$

Check when $w = 8$

$$2 \times 8 \leq 18$$
$$16 \leq 18$$

22. **Ask**: What is the variable in this situation? Write it using your chosen letter. (Example answer: Let d = number of days)
23. **Say:** Binta has Le 120,000 at the start and spends Le 5,000 every day.
24. **Ask:** After the first day, what calculation tells us how much she has remaining?
25. Allow pupils to discuss and share ideas with their neighbour.
26. Have a pupil volunteer to answer. (Answer: $120000 - 5000$, do not accept a value, just the calculation).
27. **Ask**: What calculation tells us how much remains after 2 days? (Answer: $120000 - 5000 \times 2$)
28. **Ask:** Think how much this will be after 3 days, 4 days and so on. How do we write how much remains after d days? (Answer: $120000 - 5000d$)
29. **Say:** She wants at least Le 10,000 to remain at the end of the month. Work with your neighbour to write the inequality.
30. Have a pupil volunteer to answer. (Answer: $120000 - 5000d \geq 10000, 24 - d \geq 2$)
31. **Ask**: We want to find the number of days before she has at least Le 10,000.
32. **Say**: Let us do this on the board together.

$$24 - d \geq 2$$

$$24 - 24 - d \geq 2 - 24 \longleftarrow \textit{Subtract 24 from both sides}$$

$$-d \geq -22$$

$$d \leq 22 \longleftarrow \textit{Divide both sides by } -1$$

33. **Ask**: Does anyone notice anything about the solution?
34. Allow pupils to discuss and share ideas with their neighbour.
35. Guide a volunteer pupil to say: The inequality symbol switches from $\geq$ *to* $\leq$.
36. **Say**: Whenever we divide or multiply an inequality by a negative number we have to reverse the direction of the inequality symbol. Let us check our solution to see why this is the case. Let us leave the inequality as it was and check for a value of $d > 22$.

Check when $d = 23$

$$24 - 23 \geq 2 \longleftarrow \textit{Using original inequality}$$

$$1 \ngeq 2$$

37. **Ask**: What do you notice?
38. Guide a volunteer pupil to say: The check fails because 1 is not greater than 2.
39. **Say**: The only way the inequality will hold is if we switch the symbols from $\geq$ *to* $\leq$.
40. **Ask:** What will our check then give us? (Answer: $1 \leq 2$, which is obviously correct.)
41. **Say:** This shows how important it is to always check our solution. We must remember to always reverse the direction of the inequality symbols when multiplying or dividing by a negative number.

Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs to answer the question for Guided Practice.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.

4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer the question for Independent Practice.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Closing *(2 minutes)*

1. **Say:** Write down in your pairs 2 different things you learnt today.
2. Allow pupils time to discuss and share ideas.
3. Have a pair volunteer to share their answer. (Example answers: How to create and solve a linear inequality; That when we multiply or divide a linear inequality by a negative number we must reverse the direction of the inequality symbol. Accept all reasonable answers.)

[QUESTION FOR OPENING]

1. Mohammed plans to spend at most Le 18,000 on a pair of shoes. If c represents the cost of the shoes, write this as an inequality.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

2. Mohammed plans to spend at most Le 18,000 on a pair of shoes. He saves Le 2,000 a week towards the cost of the shoes. Write this information as a linear inequality. What is the least amount of time for which he needs to save?

3. Binta has Le 120,000 at the start of the month. She wants to have at least Le 10,000 at the end of the month. She spends Le 5,000 every day. Write this information as a linear inequality. How many days can she continue spending before she reaches her limit?

[QUESTIONS FOR GUIDED PRACTICE]

4. A market trader wants to buy some boxes of tomato paste to sell. Each box costs Le 50,000 and she wants to spend no more than Le 300,000. Write this information as a linear inequality. How many boxes of tomato paste can she buy?
   (Answer: Let b = number of boxes of tomato paste, $50000\text{b} \leq 300000,\ \text{b} \leq 6$ boxes).

5. A school club is organising an outing. The minimum cost for the outing is Le 250,000. They pay Le 75,000 for the bus and the ticket cost Le 5,000 per person. How many people need to buy a ticket for them to make a profit? Write this information as a linear inequality.
   (Answer: Let p = number of people buying tickets, $75000 + 5000\text{p} > 250000, \text{p} > 35$).

[QUESTION FOR INDEPENDENT PRACTICE]

6. Fatmata wants to spend less than Le 525,000 to build a security post. A builder charges her Le 75,000 per square metre to build the post. Write this information as a linear inequality. How large can the security post be in square metres?
(Answer: Let a = size of security post, $75000\text{a} < 525000$, $\text{a} < 6.25$ square metres).

7. Kemi owes her mother Le 200,000. She wants to reduce this to at most Le 80,000 after 4 months. Write this information as a linear inequality. What is the minimum amount she should pay every month? (Answer: Let p = amount to pay every month, $200000 - 4\text{p} \leq 80000$, $200000 - 200000 - 4\text{p} \leq 80000 - 200000, -4\text{p} \leq -120000$, $\text{p} \geq 30000$).

Lesson Title: Solving Inequality Story Problems II	**Theme**: Algebra	
Lesson Number: M-09-114	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to create and solve an inequality from more complicated story problems.	None	On the board, write the questions for this lesson found at the end of the lesson plan.

Opening *(3 minutes)*

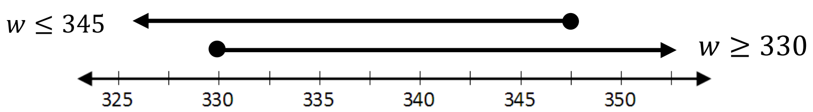
1. **Ask**: Show a different way to use another inequality symbol to write the solution $\underline{x \geq 7}$. (Answer: $7 \leq x$)
2. Have a pupil volunteer to give the answer.
3. **Say:** Remember the point of the inequality symbol is always towards the smaller value. Today we are going to create and solve an inequality from more complicated story problems.

Introduction to the New Material *(10 minutes)*

Wait a few moments after each question for pupils to discuss and share ideas, before having a pupil volunteer to answer.

1. Ask a pupil to read Question 2 on the board.
2. **Ask**: What is the variable in this situation?
3. Guide a volunteer pupil to say: w, for the number of litres of water used.
4. **Write** on the board: Let w = the number of litres of water used.
5. Ask: How many inequalities can we write? (Answer: 2 inequalities)
6. **Say**: Work with your neighbour to write the 2 inequalities.
7. Have 2 pupils volunteer to give the inequalities. (Answer in any order: $w \geq 330, w \leq 345$)
8. **Ask**: The question asks for a compound inequality. This means one inequality instead of the 2 we currently have.
9. **Say**: Show a different way to use another inequality symbol to write the solution $w \geq 330$. (Answer: $330 \leq w$) So now we have the 2 inequalities written as: $330 \leq w$ and $w \leq 345$. We can write the solution as a compound inequality like this: $330 \leq w \leq 345$. We write one inequality combining the 2 separate inequalities. We use the same inequality symbol in a compound inequality. We read it as: w is greater than or equal to 330 and less than or equal to 345.
   Remember, the point of the inequality symbol is always towards the smaller value.
   In simple English this is the same as saying 'the water use goes from 330 litres to 345 litres'.

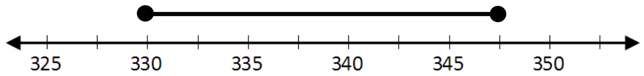
   We are also asked to show this solution on a number line. Let us start by drawing the 2 separate inequalities. Do this in your exercise books using a suitable scale.
10. Have a pupil volunteer to draw their solution on the board. Allow flexibility on the scale.



11. **Ask**: How many boundary points do we have? (Answer: 2 boundary points, at 330 and 345)

12. **Say**: Read the question again and discuss with your partner where you think the values lie.
13. Guide a volunteer pupil to say: All the values are between 330 and 345.
14. **Say**: We can show this on the number line like so:

$$330 \leq w \leq 345$$



15. **Say**: As we can see from the number line, for both of the separate inequalities to be true, then the compound inequality is the overlap of the 2 solutions. We do not have to keep drawing 2 separate inequalities. We can simply draw the appropriate circles at the boundary points and join them with a straight line to give the final solution.
16. Ask a pupil to read Question 2 a.
17. **Ask**: What is the variable in this situation?
18. Guide a volunteer pupil to say: m, for the amount of money in Bami's pocket.
19. **Write** on the board: Let m = the amount of money in Bami's pocket.
20. **Ask**: How do we represent this situation as a compound linear inequality?
21. **Say**: Think of which inequality symbol you will use and how you will write the inequality.
22. Have a pupil volunteer to write the linear inequality on the board. (Answer: $8000 \leq m \leq 10000$)
23. **Ask:** Now let us look at Question *ii.b.* Do we need a new variable? (Answer: No, we are still talking about the money in Bami's pocket.)
24. **Ask:** What has changed?
25. Guide a volunteer pupil to say: It is now his original money and the money given to him by his brother which is between Le 8,000 and 10,000.
26. **Ask**: How can we represent his original money and the money given to him by his brother? (Answer: $m + 2000$).
27. **Say:** Write the new inequality to represent this situation. (Answer: $8000 \leq m + 2000 \leq 10000$)
28. **Say:** If this was just a simple inequality, e.g. $m + 2000 \leq 10000$, how would you solve for m?
29. Guide a volunteer pupil to say: Subtract 2000 from both sides.
30. **Say**: We do exactly the same here, except we subtract 2000 from all sides.
31. Show the solution on the board:

$$\begin{array}{rcccl} 8000 & \leq & m + 2000 & \leq & 10000 \\ 8000 - 2000 & \leq & m + 2000 - 2000 & \leq & 10000 - 2000 \quad \longleftarrow \textit{Subtract 6 from both sides} \\ 6000 & \leq & m & \leq & 8000 \quad \longleftarrow \textit{Solution for } m \end{array}$$

32. **Say**: We can verify each part of the inequality by checking for appropriate values of m.

Guided Practice *(10 minutes)*

1. Ask pupils to work in pairs to answer the questions for Guided Practice.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer the questions for Independent Practice.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Do not do the answers for Questions 6. Use it to check pupils' understanding of the work.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Closing *(2 minutes)*

1. **Say:** Write your name on a piece of paper. Write your work and answer for Questions 6 on the paper. Hand the paper in at the end of the lesson.
2. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be practising solving problems with algebraic expressions.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

1. A study shows that on average every person uses from 330 litres to 345 litres of water a day. Write this statement as a compound inequality. Show the solution on a number line.
2. a. Bami knows he has from Le 8,000 to Le 10,000 in his pocket. Write this information as a linear inequality.
   b. Bami has some money in his pocket. After he was given Le 2000 by his brother, he has from Le 8,000 to Le 10,000. Write this information as a linear inequality. How much money did he have originally in his pocket?

[QUESTIONS FOR GUIDED PRACTICE]

3. The temperature in a certain town ranged from 25°C to 28°C on a particular day. Write this information as a linear inequality.
   (Answer: iii. Let t = temperature, $25 \leq \mathrm{t} \leq 28$)
4. John wants to spend from Le 15,000 to Le 25,000 on a present. He has already saved Le 5,000. If a is the amount he still has to save, write this information as a linear inequality. Simplify your answer.
   (Answer: Let a = amount John still has to save, $15000 \leq \mathrm{a} + 5000 \leq 25000, 15000 - 5000 \leq \mathrm{a} + 5000 - 5000 \leq 25000 - 5000, 10000 \leq \mathrm{a} \leq 20000$)

[QUESTIONS FOR INDEPENDENT PRACTICE]

5. A farmer says the minimum number of cows he can keep on his farm is 22 and the maximum is 35. Write this information as a linear inequality.
   (Answer: v. Let c = number of cows, $22 \leq \mathrm{c} \leq 35$).
6. Binta has Le 120,000 at the start of the month. She wants to have from Le 10,000 to Le 15000 at the end of the month. She spends Le 5,000 every day. Write this information as a linear inequality. Simplify your answer.
   (Answer: Let d = number of days, $10000 \leq 120000 - 5000\mathrm{d} \leq 15000, 10 \leq 120 - 5\mathrm{d} \leq 15,\ 2 \leq 24 - \mathrm{d} \leq 3, -22 \leq -\mathrm{d} \leq -21, 22 \geq \mathrm{d} \geq 21$, can be written as: $21 \leq \mathrm{d} \leq 22$).

Lesson Title: Practice with Inequalities	Theme: Algebra	
Lesson Number: M-09-115	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to solve various linear inequality problems and represent answers on the number line.	None	On the board, write the questions for this lesson found at the end of the lesson plan.

Opening *(3 minutes)*

1. **Say**: Copy and complete the sentences on the board.
2. Allow time for pupils to complete the sentences.
3. Have a pupil volunteer to give the answer to Question 1.
4. Ask the class if they agree. (Answer: *open*).
5. Do the same for Question 2. (Answer: *closed*).
6. **Say**: Today we are going to solve various linear inequality problems and represent answers on the number line.

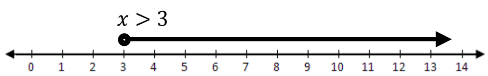
Introduction to the New Material *(10 minutes)*

1. Go through the procedure to solve Question 3 on the board.
2. Ask pupils from around the classroom to tell you what to do for each step as you go through the solution.
3. The solution is shown below:

$$\begin{array}{rcll} 5-3x & < & -4 & \\ -3x & < & -9 & \longleftarrow \textit{Subtract 5 from both sides} \\ \frac{-3x}{-3} & < & \frac{-9}{-3} & \longleftarrow \textit{Divide both sides by -3} \\ x & > & 3 & \longleftarrow \textit{Solution for } x \end{array}$$

4. **Ask**: Why have we reversed the direction of the inequality symbol? (Answer: Because we divided by -3)
5. Let pupils volunteer to answer.
6. **Ask**: What checks do we need to verify our solution? (Answer: Check the related equation and a suitable value based on the solution.)
7. Have 3 pupils volunteer to come to the board one at a time to do the checks and draw the number line. (Answers: shown below)

$$\begin{array}{rcl} \multicolumn{3}{c}{\underline{\textit{Check}}\textit{ when } x=3} \\ 5-(3\times 3) & = & -4 \\ 5-9 & = & -4 \\ -4 & = & -4 \\ \text{LHS} & = & \text{RHS} \end{array} \qquad \begin{array}{rcl} \multicolumn{3}{c}{\underline{\textit{Check}}\textit{ when } x=4} \\ 5-(3\times 4) & < & -4 \\ 5-12 & < & -4 \\ -7 & < & -4 \end{array}$$

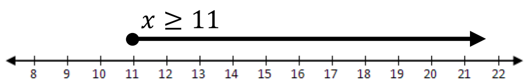


8. **Say**: The checks verify our solution that $x > 3$.
9. Ask a pupil to read Question 4 on the board.

10. **Say**: Work with the pupil sitting beside you to answer this question.
11. Allow time for pupils to discuss and share ideas. Encourage them to check their answers.
12. Have 2 pupils volunteer to come to the board one at a time to do the solution and number line. (Answers: shown below)

Solution

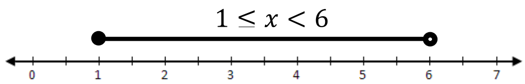
$$\begin{aligned} \frac{2x-4}{3} &\geq 6 \\ 2x - &\geq 18 \\ 2x &\geq 22 \\ x &\geq 11 \end{aligned}$$



13. Ask pupils to continue working together to solve Question 5.
14. Provide guidance to help them complete the solution. Encourage them to check their answers.
15. Have 2 pupils to explain answers as before. The answers are shown below.

Solution

$$\begin{array}{rcccl} 1 & \leq & \frac{6x-1}{5} & < & 7 \\ 1 \times 5 & \leq & 6x-1 & < & 7 \times 5 \\ 5 & \leq & 6x-1 & < & 35 \\ 6 & \leq & 6x & < & 36 \\ 1 & \leq & x & < & 6 \end{array}$$



Guided Practice *(10 minutes)*

1. Ask pupils to continue to work in pairs to answer the questions for Guided Practice.
2. Walk around, if possible, to check answers and correct any misconceptions.
3. Encourage pupils to check their answers.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan).

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer the questions for Independent Practice.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Encourage pupils to check their answers.
4. Do not do the answers for Questions 10. Use it to check pupils' understanding of the work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: Shown below the questions at the end of this lesson plan)

Closing *(2 minutes)*

1. **Say:** Write your name on a piece of paper. Write your working out and answer for Questions 10. on the paper. Hand the paper in at the end of the lesson.
2. Check the work done by the pupils after the lesson. This will help you tell how well this topic is understood by pupils.

[QUESTION FOR OPENING]

1. On a number line, a(n) _______ circle shows that the boundary point is not a possible solution.
2. On a number line, a(n) _______ circle shows that the boundary point is a possible solution.

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Solve for x in the following linear inequalities.

3. $5 - 3x < -4$ 4. $\frac{2x-4}{3} \geq 3$ 5. $1 \leq \frac{6x-1}{5} < 7$

[QUESTIONS FOR GUIDED PRACTICE]

Solve for x in the following linear inequalities.

6. $3(x+1) < 9$ 7. $\frac{4x+5}{2} \geq \frac{1}{2}$ 8. $-4 \leq -2(2x+3) < 4$

(Answers:
6. $3(x+1) < 9,\ 3x+3 < 9,\ 3x < 9-3,\ 3x < 6,\ x < 2;$
7. $\frac{4x+5}{2} \geq \frac{1}{2},\ 4x+5 \geq 1,\ 4x \geq 1-5,\ 4x \geq -4, x \geq -1;$
8. $-4 \leq -2(2x+3) < 4,\ -4 \leq -4x-6 < 4,\ -4+6 \leq -4x < 4+6,\ 2 \leq -4x < 10,$
$\frac{2}{-4} \leq \frac{-4x}{-4} < \frac{10}{-4},\ -\frac{1}{2} \geq x > -\frac{5}{2}$, *can be written as* $-\frac{5}{2} < x \leq -\frac{1}{2}$).

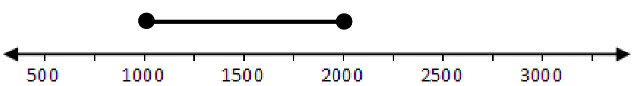
[QUESTIONS FOR INDEPENDENT PRACTICE]

Solve for x in the following linear inequalities.
Draw the solution for xi on a number line.

e. $6 - 2x < 4$ f. $\frac{2x}{3} + 5 \geq 3$

g. David gets Le 4,000 per day for lunch. He spends from one-quarter to one-half of it every day. Write the amount he spends every day as a linear inequality.

$1000 \leq x < 2000$



9. $6 - 2x < 4, -2x < 4 - 6, -2x < -2,\ x > 1;$

10. $\frac{2x}{3} + 5 \geq 3, \frac{2x}{3} \geq 3 - 5, \frac{2x}{3} \geq -2,\ 2x \geq (-2) \times 3,\ 2x \geq -6,\ x \geq -3$

11. David spends from Le $\frac{4000}{4}$ to Le $\frac{4000}{2}$ every day. That is, he spends from Le 1000 to Le 2000. Let x = amount of money David spends every day, then $1000 \leq x \leq 2000$

Lesson Title: Data Collection	**Theme**: Statistics and Probability	
Lesson Number: M-09-116	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to collect data from class members and display it in lists and pictograms.	None	Write the information shown for the Opening activity, at the end of this lesson plan, on the board.

Opening *(3 minutes)*

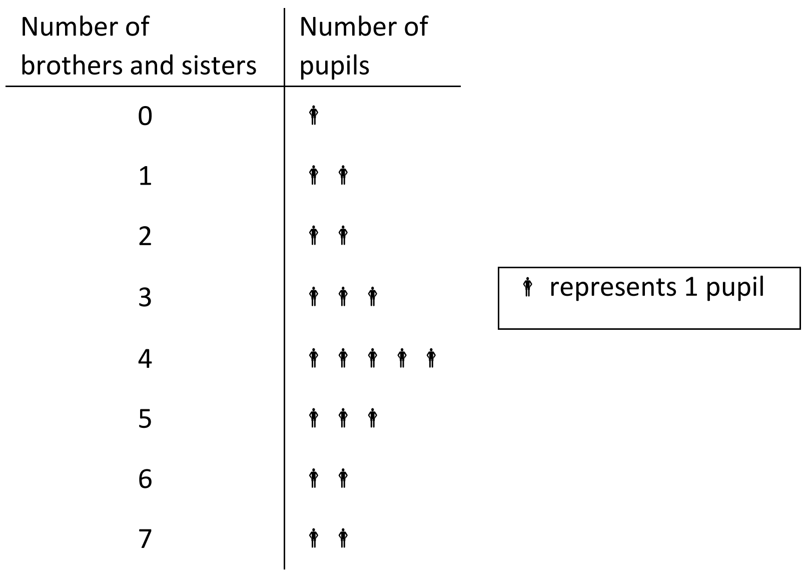
1. **Say**: Please look at the board. We can see different groups of items.
2. **Ask**: Can you think of any similar groups that you can add to the list?
3. Allow pupils to discuss and share ideas with their partners.
4. **Write** the additional groups on the board. (Example answers: Ages of pupils in a JSS 3 class, Favourite fruits, Favourite subjects, Favourite colours. Accept all reasonable answers.)
5. **Ask**: How do you think we can find out about each group?
6. Guide a volunteer pupil to say we can collect data for each group.
7. **Say**: Today we are going to collect data from class members and display it in lists and pictograms.

Introduction to the New Material *(10 minutes)*

1. **Say:** Let us look at one of the groups on the board that we can collect data for in class. We will collect data from 20 pupils in our JSS 3 class. We will write down the number of brothers and sisters they have.
2. Have 20 pupils around the classroom volunteer to give you the number of siblings they have. Write them as an unordered list. (Example answer: 3, 1, 4, 6, 3, 7, 1, 4, 6, 5, 5, 4, 7, 0, 2, 3, 4, 4, 2, 5)
3. **Say**: On the board, we have a list of the number of brothers and sisters which 20 JSS 3 pupils have. Writing a list is one form of collecting data. Our list is very basic and does not give us much information. We can organise our list by writing the numbers in rank order. This means we will write the list in ascending or descending order. Write the list on the board in ascending rank order. Please raise your hand when you finish.
4. Allow time for pupils to rank the numbers. Have a pupil volunteer to give the ordered list. (Answer: 0, 1, 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 6, 6, 7, 7)
5. **Say**: Notice that we write down all the numbers in our original list even though most of them appear more than once. The ordered list gives us better information than what we had before. We can more easily find information that we want.
6. **Ask**: What is the smallest number of brothers and sisters the 20 pupils we asked have? (Example answer: 0)
7. Have 2 pupils volunteer to answer. Note the data from your class may give different information.
8. **Ask**: What is the largest number? (Example answer: 7)
9. **Ask**: How many pupils have 4 brothers and sisters? (Example answer: 5)
10. Let pupils raise their hands to answer these questions.
11. **Say**: We can look at the ordered list and find other types of information. The ordered list is still not a very organised way of displaying the data. One other way to display the data is in a pictogram.

12. **Ask**: Who can remind the class of what a pictogram is?
    (Example answer: A diagram that uses pictures or drawings to display the data collected.)
13. Invite 2 pupils to raise their hands to answer.
14. **Say**: There are 2 main decisions to make when drawing a pictogram.
    a. What type of picture or drawing we will use to display the data.
    b. How much of the data will be represented by 1 picture or drawing.
    This is called the key of the pictogram. It allows anyone looking at the pictogram to understand the information it shows.
15. **Say**: For the data we collected, we will use one stick figure to represent the number of pupils who have that number of brothers and sisters.
16. An example pictogram is shown below. It may be different for the data from your class.

Pictogram showing number of brothers and sisters in JSS 3 class



17. **Say**: The pictogram is easy to read and shows the data in an attractive format. It is also much easier to find information we want straight away from the pictogram than it is from the ordered list.
18. **Ask** pupils the following questions and let them raise their hands to answer:
    a. What can you see straight away? (Example answer: The largest and smallest number of brothers and sisters. Accept all reasonable answers.)
    b. What is the largest number? (Example answer: 4 brothers and sisters)
    c. How many pupils have that number of brothers or sisters? (Example answer: 5)
    d. What is the smallest number of brothers and sisters? (Example answer: 0)
    e. How many pupils have that number of brothers or sisters? (Example answer: 1)

Guided Practice *(10 minutes)*

1. Ask 20 volunteer pupils to stand up. You may decide on more or fewer pupils if appropriate.
2. **Say**: Please say any letter from A to F. Sit down once you have given your letter.
3. Write the letters given by the pupils on the board as an unordered list.
4. Ask the pupils to write the list in ascending rank order.
5. Walk around, if possible, to check their answers and clear up any misconceptions.
6. Ask pupils to exchange exercise books and check each other's work.

7. Have a pupil volunteer to write their ordered list on the board.
8. Correct any errors in the calculation on the board. Ask pupils to check their work. (Example answer: A, A, A, A, A, A, B, B, B, B, B, C, C, C, C, E, E, F, F, F)

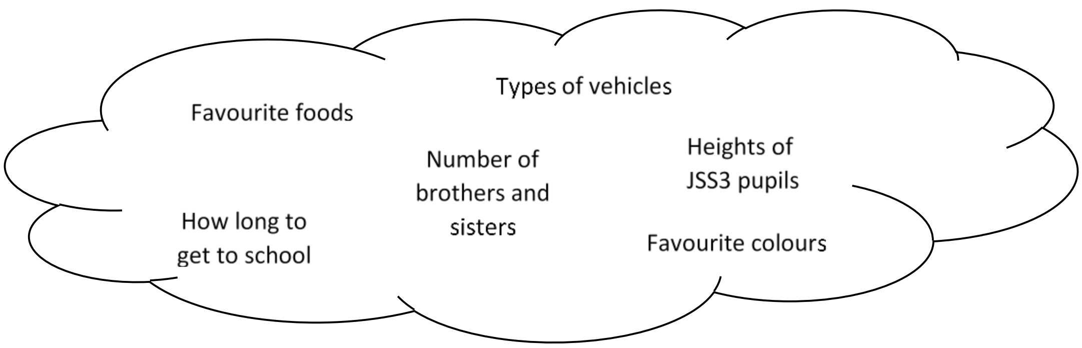
Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to draw a pictogram of the data collected in Guided Practice.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have a pupil volunteer to draw their pictogram on the board.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Example answer: Shown at the end of the lesson plan)

Closing *(2 minutes)*

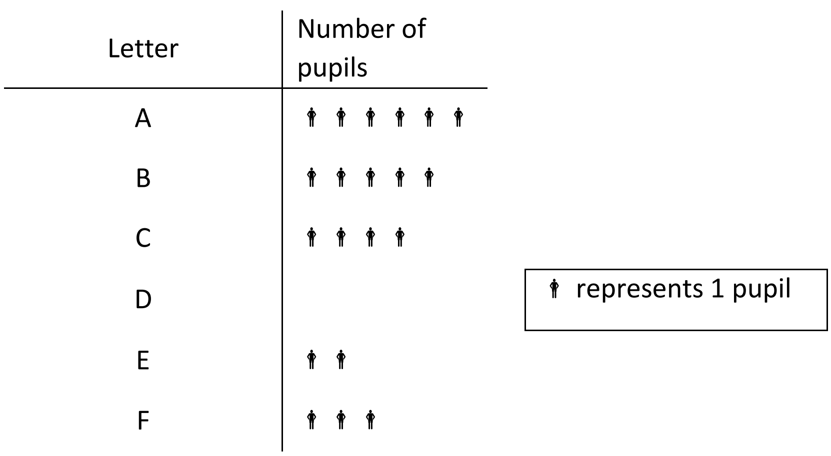
1. **Say:** Please write down one advantage a pictogram has over an ordered list.
2. Allow time for pupils to answer the question.
3. Have a pupil volunteer to answer. (Example answers: It is easy to read; It looks attractive; We can see the smallest and largest data immediately. Accept all reasonable answers.)

[INFORMATION FOR THE OPENING ACTIVITY]



[ANSWER FOR INDEPENDENT PRACTICE]

Pictogram showing letters chosen by JSS 3 class



Lesson Title: Frequency Tables	**Theme**: Statistics and Probability	
Lesson Number: M-09-117	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to organise and display collected data in a frequency table.	None	Draw the frequency table shown at the end of the lesson plan for Introduction to the New Material on the board.

Opening *(3 minutes)*

1. **Ask**: Who can remind the class of what we did in the last lesson?
2. Have a pupil volunteer to answer. (Answer: Collect data and display it as lists and pictograms.)
3. **Ask:** Please raise your hand if you can give one advantage a pictogram has over a list. (Example answer: A pictogram is easy to read; It gives information straight away on the smallest and largest data. Accept all reasonable answers.)
4. **Say**: Today we are going to organise and display collected data in a frequency table.

Introduction to the New Material *(10 minutes)*

1. **Say**: To get more information out of data, we need to collect it in a more organised way. Once we have collected it we need to display it in a way that is easy for people to understand. We want to collect data from this class on the day of the week they were born. One way to collect the data is by using a frequency table.
2. Point to the table below already on the board.
3. **Say**: To help with collecting the data, we will tally up the number of pupils born on a particular day and count in steps of 5.
4. Show how to tally on the board: vertical lines are drawn for each count; 4 vertical lines with a fifth line crossing them denotes 5 pupils. (shown at right) ~~||||~~
5. **Say**: Let us go through the procedure of how we collect the data. We will need 2 volunteers – one to count and the other to record the tally on the board.
6. **Say:**
   - Pupils born on Sunday stand up first.
   - The pupil counting points to one pupil at a time and counts in steps of 5.
   - The pupil at the board records the count as tallies in steps of 5.
   - Pupils who have been counted can sit down.
   - Any pupils left at the end are recorded as individual tally marks.
   - Repeat the procedure for pupils born on Tuesday to Saturday.
7. Guide pupils to carry out this procedure. Make sure the volunteers are counted as well. Also make sure that the total matches the number of pupils in the class. An example of a completed table is shown below.
8. **Say**: The tally marks are counted and recorded as the frequency in the table. It is the number of pupils who were born on a particular day.
9. Ask pupils to leave enough space in their exercise books to draw the bar chart of the data at the next lesson.

Day of the week	**Tally**	**Frequency**

Sunday	𝍸 ///	8
Monday	𝍸 ////	9
Tuesday	𝍸 //	7
Wednesday	𝍸 𝍸 //	12
Thursday	𝍸 ///	8
Friday	𝍸 //	7
Saturday	𝍸 ////	9

Guided Practice *(10 minutes)*

1. Have 20 pupils volunteer to stand up. You may decide on more or fewer pupils if appropriate.
2. Ask each pupil in which month is their birthday. Ask them to sit down once they have answered.
3. Write the months given by the pupils on the board as an unordered list.
   You can use simple abbreviations for the months. (e.g. J for January, Ju for June, Jul for July)
4. Ask pupils to draw a frequency table and put the months in order from January to December.
5. Ask pupils to complete the table with the data using a tally mark for each pupil.
6. Walk around, if possible, to check their answers and clear up any misconceptions.
   - Check that pupils are recording the tallies as groups of 5 for easy counting.
   - Check that pupils include months in which no pupils were born in their tables.
7. Have a pupil volunteer to write their frequency table on the board.
8. Correct any errors in the calculation on the board. Ask pupils to check their work. (Example answer: Shown at the end of this lesson plan)

Independent Practice *(10 minutes)*

1. Ask a different set of 20 pupils to stand up. You may decide on more or fewer pupils if appropriate.
2. Ask each pupil in which year they were born. Ask them to sit down once they have answered.
3. Write the years given by the pupils on the board as an unordered list.
4. Ask pupils to draw a frequency table and put the years in ascending order.
5. Ask pupils to complete the table with the data using a tally mark for each pupil.
6. Walk around, if possible, to check their answers and clear up any misconceptions.
   Pupils should record a 0 for any year where there is a gap in the data.
7. Have a pupil volunteer to write their frequency table on the board.
8. Correct any errors on the board. Ask pupils to check their work. (Example answer: Shown at the end of this lesson plan)

Closing *(2 minutes)*

1. **Say:** Organising the data we collect in a frequency table ensures that we are able to display our data in a way that is easy to use and understand. We use the information in frequency tables to draw pictograms, bar charts, pie charts and other types of diagrams. We will use the information we have collected today to draw bar charts in our next lesson.

[INTRODUCTION TO THE NEW MATERIAL – FREQUENCY TABLE]

Day of the week	**Tally**	**Frequency**
Sunday		
Monday		
Tuesday		
Wednesday		
Thursday		
Friday		
Saturday		

Keep a record of the frequency tables to use to draw bar charts in the next lesson. Ask pupils to leave enough space underneath the tables to draw their bar charts.

[GUIDED PRACTICE – EXAMPLE ANSWER]

Month	**Tally**	**Frequency**
January	///	3
February	//	2
March	////	4
April		0
May	//	2
June	/	1
July		0
August	///	3
September	/	1
October	/	1
November	/	1
December	//	2
		20

[INDEPENDENT PRACTICE – EXAMPLE ANSWER]

Year	Tally	Frequency
1998	///	3
1999	//	1
2000	///	3
2001	~~////~~ ~~////~~ /	11
2002	//	2
		20

Lesson Title: Bar Charts	**Theme**: Statistics and Probability	
Lesson Number: M-09-118	**Class/Level**: JSS 3	**Time:** 35 minutes

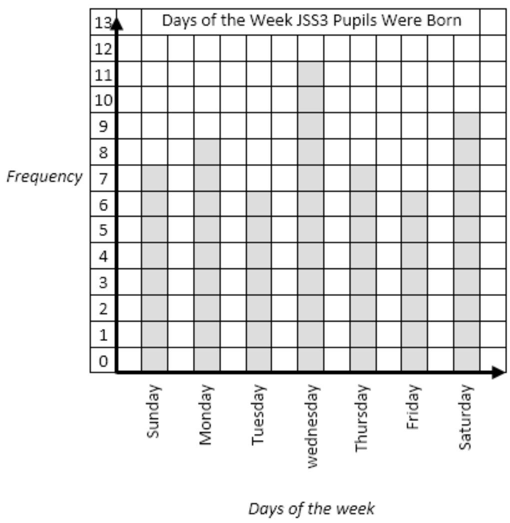
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to display collected data in a bar chart.	None	Draw the completed tables of data collected from the last lesson on the board. Examples are given in the Introduction and at the end of this lesson plan.

Opening *(3 minutes)*

1. **Say**: A classmate was absent from our last lesson. Please write a short description of what we did to help them catch up.
2. Allow time for pupils to write their answer.
3. Have a pupil volunteer to give a summary of the previous lesson. (Example answer: We collected data as an unordered list; We organised and displayed data in a frequency table; We used tally marks to record data. Accept all reasonable answers.)
4. **Say**: Today we are going to display collected data in a bar chart.

Introduction to the New Material *(10 minutes)*

Day of the week	Tally	Frequency
Sunday	~~////~~ ///	8
Monday	~~////~~ ////	9
Tuesday	~~////~~ //	7
Wednesday	~~////~~ ~~////~~ //	12
Thursday	~~////~~ ///	8
Friday	~~////~~ //	7
Saturday	~~////~~ ////	9
		60



1. **Say:** We will use the data we collected in our last lesson to draw the bar charts. Who can remind the class of the set of data we collected for the whole class? (Answer: We collected data on the day we were born.)
2. Let a pupil volunteer to answer.
3. **Say:** We are going to use the data for the day we were born to draw our first bar chart.
4. Draw the axes for the graph.
5. **Ask**: What scale do we use to represent the frequency on the y-axis? (Example answer: 2 cm represents 1 pupil)
6. Mark the scale for the y-axis as a pupil volunteers to answer.
7. **Say**: We need an appropriate scale for the x-axis. Since we will draw 7 bars in our chart, we will use 2 cm to represent 1 bar. We need to label the axes. Our table already has headings for 'Day of the week' and 'Frequency'. We will use them as our labels.
8. **As**k. Which label goes on which axis?
9. Guide a volunteer pupil to say the 'Days of the week' goes on the x-axis and 'Frequency' goes on the y-axis.
10. Label the axes.
11. **Say**: We now have all the information we need to draw our bar chart.
12. Draw the bar for Sunday. Show that the bar is drawn as a rectangle from 0 to 8.

13. Ask the pupils to complete the bar chart for the rest of the week with the information on the table. Ask them to leave a gap between each bar.
14. **Say**: We give every bar chart a title which describes what it is displaying. Who can think of a title for this graph?
15. Allow pupils to discuss and share their ideas.
16. Have pupils volunteer to share their ideas with the class. Select the most appropriate title suggestion.
    (Example answer: Day of the Week JSS 3 Pupils Were Born)
17. The completed bar chart is shown on the previous page.
18. **Say**: Let us look at our graph. We can answer questions on the bar chart about the data we have just drawn.
19. Give pupils some time to think before having 1 pupils volunteering to answer each question.
    a. On which day were the most number of pupils born? (Answer: Wednesday)
    b. How many pupils were born on that day? (Answer: 12)
    c. On which day were the least number of pupils born? (Answer: Tuesday and Friday)
    d. How many pupils were born on those days? (Answer: 7)
    e. How many people were born on Monday? (Answer: 9)
    f. How many people were born altogether on the weekend? (Answer: 17)
    g. How many days were 9 people born? (Answer: 2 days)

Guided Practice *(10 minutes)*

1. Ask the pupils to work in pairs to draw the bar chart of the information on the months of the year they were born.
2. Tell pupils they should each draw their own individual bar charts.
3. **Say**: Work together to select the scales of the axes, appropriate labels and a title for the bar chart.
4. Walk around, if possible, to check their answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to draw a bar each on the chart.
6. Correct any errors on the board. Ask pupils to check their work. Answers are shown next to the frequency table at the end of the lesson plan.

Independent Practice *(10 minutes)*

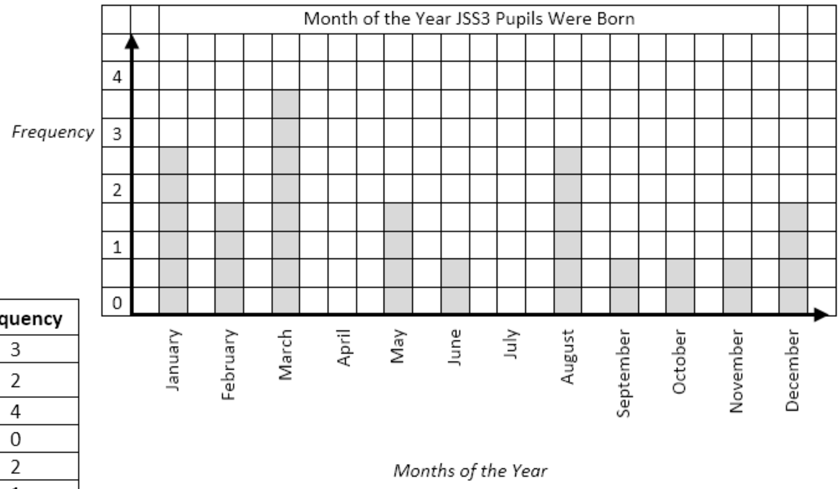
1. Ask the pupils to work independently to draw the bar chart of the information on the year they were born.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. Have pupils from around the classroom volunteer to draw a bar each on the chart.
5. Correct any errors on the board. Ask pupils to check their work. Answers are shown next to the frequency table at the end of the lesson plan.

Closing *(2 minutes)*

1. **Say:** We have now used 4 different ways to organise and display data. Who can tell the class what these are? (Answer: lists, pictograms, frequency tables and bar charts)
2. **Ask**: Please look at the frequency tables and the bar charts we have been drawing today. Which one do you think best shows the data? (Example answers: Pupils may differ in their opinions on whether the frequency table or the bar chart is better. The frequency table is easy to use to do calculations; A bar chart shows a large amount of data in a format that is easy to understand. It

is easier to visually compare differences in the data using a bar chart. Accept all reasonable answers.)

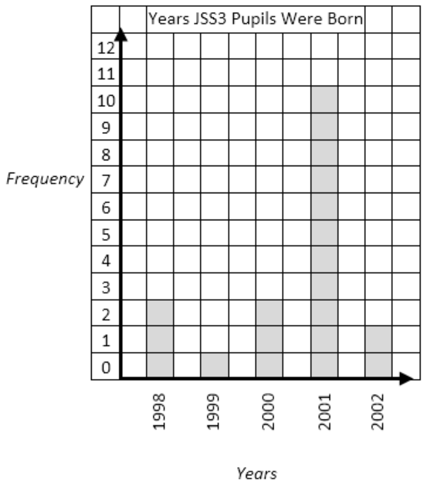
[GUIDED PRACTICE – EXAMPLE ANSWER]



Month	Tally	Frequency
January	///	3
February	//	2
March	////	4
April		0
May	//	2
June	/	1
July		0
August	///	3
September	/	1
October	/	1
November	/	1
December	//	2
		20

[INDEPENDENT PRACTICE – EXAMPLE ANSWER]

Year	Tally	Frequency
1998	///	3
1999	//	1
2000	///	3
2001	~~////~~ ~~////~~ /	11
2002	//	2
		20

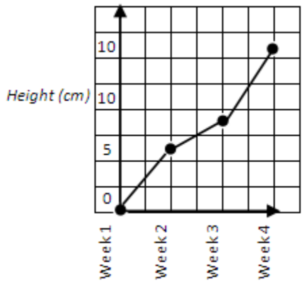


Lesson Title: Line Graphs	**Theme**: Statistics and Probability	
Lesson Number: M-09-119	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes By the end of the lesson, pupils will be able to display collected data in a line graph.	**Teaching Aids** None	**Preparation** 1. Draw a sketch of the line graph in the Opening section on the board. 2. Write the questions for each section of this lesson shown at the end of the lesson plan.

Opening *(3 minutes)*

1. **Say**: So far we have been working with data which do not change. For example, the day or year we were born. We drew bar charts which compared the number of pupils born in every day of the week. We found which day had the most or the least number of pupils.
2. Point to the sketch of the line graph on the board.
3. **Say**: Raise your hand if you know what kind of graph this is.
4. Guide a pupil with a raised hand to give the answer. (Answer: Line graph)
5. **Ask**: What do you think the graph is showing?
6. Guide a volunteer pupil to say: The height of an item over a period of 4 weeks.
7. **Say**: Today we are going to display collected data in a line graph.



Introduction to the New Material *(10 minutes)*

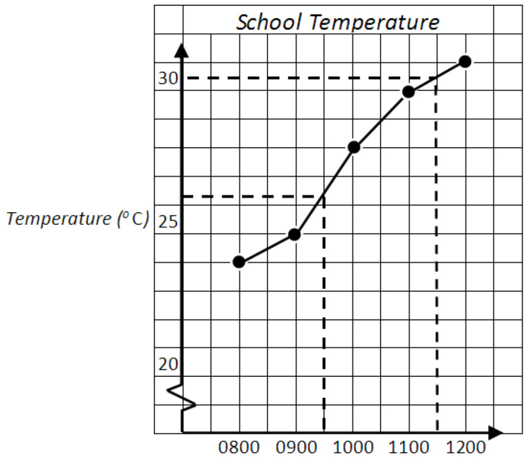
1. **Say**: When we want to record data that changes over time we use a line graph. For example, the mass of a baby changes very quickly over the first few days after birth. This change can be shown on a line graph.
2. **Ask**: What other types of data change over time?
3. Guide pupils to give data involving measurements. (Example answers: distance, height, weight, price of goods, temperature, population, etc.)
4. **Say:** Bar charts also show data on height, weights, etc. but it usually displays the information at a particular point in time. For example, we can measure all of your heights right now and make a bar chart to display the data. We can use it to compare the difference between the tallest and shortest heights. But if we wanted to display how one of your heights changes over a year, we would use a line graph. Since we do not have data on any of you that covers a time period, we will use previously collected data.
5. Ask a pupil to read Question 1.
6. **Ask:** Look at the data in the table. What is it showing?
7. Have a pupil volunteer to answer. (Answer: Temperature at various times of the day)
8. Draw the axes for the graph.
9. **Say:** We have 2 variables in the table. Which one do you think is the independent variable?
10. Guide pupils to say: Time.
11. **Ask**: Which axis do we use to represent Time? (Answer: x-axis).

12. **Ask:** Look at the data for Temperature. It goes from *24 °C* to *31 ° C.* How do you think the scale should look?

13. Guide a volunteer pupil to say: We start the scale close to the value for the lowest temperature. Since that is *24 ° C,* we start our scale at *20 ° C.*
14. **Say:** We show on the axis that we are not starting from 0.
15. Amend the axis to show that the scale is now starting from *20* (shown by the zig-zag on the y-axis).
16. **Ask:** What title can we give the graph?
17. Guide pupils to think of a title based on the question. (Example: *School Temperature*)

1.

Time	0800	0900	1000	1100	1200
Temperature (° C)	24	25	28	30	31



18. Complete the labelling of the graph.
19. Plot the point for *0800*.
20. Ask pupils to copy and complete the graph in their exercise books.
21. Allow time for pupils to complete the graph.
22. Have pupils from around the classroom volunteer to plot and join the points for the line graph on the board.
23. The completed graph is shown above.
24. **Say**: We can use our graph to estimate the temperatures for which we do not have values
25. Draw a vertical line from a requested time to the graph; draw a horizontal line from the point the vertical line meets the graph to the y-axis. Read the temperature (*see graph*).
26. **Ask**: What is the temperature at *0930*? (Accept answers in the range: *26.1 – 26.3 °C.*)
27. **Ask**: What is the temperature at *1130*? (Accept answers in the range: *30.3 – 30.5 °C.*)

Guided Practice *(10 minutes)*

1. Ask the pupils to work in pairs to answer Questions for Guided Practice.
2. Tell pupils they should each draw their own individual line graphs.
3. **Say**: Work together to decide on appropriate scales for the axes.
4. Walk around, if possible, to check answers and correct any misconceptions.
5. Have pupils from around the classroom volunteer to plot and join the points for the line graph.
6. Correct any errors in the calculation on the board. Ask pupils to check their work. Answer is shown next to question.

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer Questions for Independent Practice.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to plot and join the points for the line graph.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. Answer shown next to question.

Closing *(2 minutes)*

1. **Ask:** What type of graph do we use to display data which compares information? (Answer: bar chart)
2. **Ask**: What sort of graph do we use to display data which changes over time? (Answer: line graph)
3. Let pupils raise their hands to answer these questions.
4. **Say**: We will apply everything we have learnt so far in our next lesson. We will make comparisons and draw conclusions from charts and graphs.

[QUESTION FOR INTRODUCTION TO THE NEW MATERIAL]

1. The temperature in a school one morning is shown in the table below. Draw a line graph of the data.

Time	0800	0900	1000	1100	1200
Temperature (° C)	24	25	28	30	31

[QUESTIONS FOR GUIDED PRACTICE]

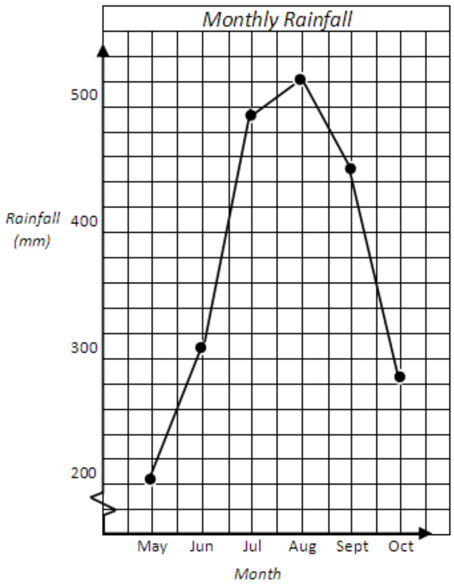
2. The amount of rainfall in a town in Sierra Leone over a 6 month period is shown in the table below.

Month	May	Jun	Jul	Aug	Sept	Oct
Rainfall (mm)	205	309	493	520	450	282

a. Draw a line graph of the data.
b. Which month had the most rainfall?

Answers:

a. graph shown right – allow some flexibility in choice of scale
b. August



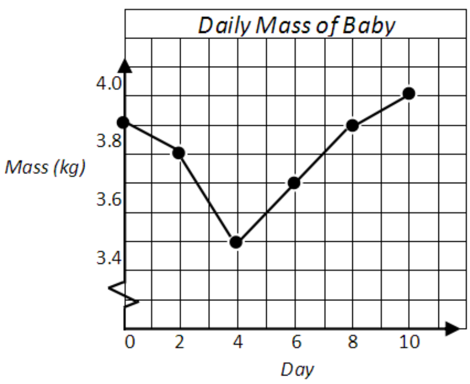
[QUESTION FOR INDEPENDENT PRACTICE]

3. The mass of a baby for the first few days of its life is shown in the table below.

Day	0	2	4	6	8	10
Mass (kg)	3.9	3.8	3.5	3.7	3.9	4.0

a. Draw a line graph of the data.
b. Estimate the mass on day 9.

Answers:



a. graph shown right – allow some flexibility in choice of scale
b. accept 3.95 – 3.97 kg

Lesson Title: Interpreting Charts and Graphs	**Theme**: Statistics and Probability	
Lesson Number: M-09-120	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Make comparisons using pictograms, bar charts, and line graphs. 2. Draw conclusions from charts and graphs.	None	None

Opening *(3 minutes)*

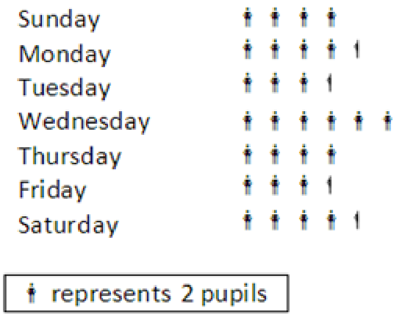
1. **Ask**: Who can remind the class of the different ways we use to collect, organise and display data?
2. Have 3 pupils volunteer to answer each, this and the next 2 questions. (Answer: list, pictogram, bar chart, line graph)
3. **Ask**: Which one of these is best used to compare data? (Answer: bar chart)
4. **Ask**: Which one is best used to display data which changes over time? (Answer: line graph)
5. **Say**: Today we are going to make comparisons using pictograms, bar charts, and line graphs. We will also draw conclusions from charts and graphs.

Introduction to the New Material *(10 minutes)*

1. **Say**: Look back in your exercise books. We collected data on the day of the week we were born.
2. **Ask**: How did we display the data? (Answer: Frequency table and bar chart)
3. Have a pupil volunteer to answer.
4. **Say**: We can display the same data as a pictogram.
5. **Ask**: What can we use to represent the data? (Example answer: Stick figure)
6. **Ask**: How many pupils will 1 stick figure represent?
7. Let pupils volunteer to answer. This answer will depend on your data. For the data given as an example, we will use 1 stick figure to represent 2 pupils. (Example answer: 1 stick figure represents 2 pupils)
8. **Say**: The pictogram does not usually show labels for the data it is representing. The title and the key is enough to understand the information.
9. Ask pupils to draw the pictogram using this information.
10. Have pupils volunteer to complete the pictogram on the board. (Example answer: Shown right)

Day of the week	**Tally**	**Frequency**
Sunday	𝍸 ///	8
Monday	𝍸 ////	9
Tuesday	𝍸 //	7
Wednesday	𝍸 𝍸 //	12
Thursday	𝍸 ///	8
Friday	𝍸 //	7
Saturday	𝍸 ////	9
		60

Day of the week pupils in JSS3 class were born



11. **Ask**: Who can tell some of the similarities and differences between the pictogram and the bar chart we drew previously?
12. Guide pupils who volunteer to say:
    - they both compare data, but the pictogram shows the data in a nicer way than the bar chart
    - in the pictogram, we have to count using the key for each piece of information; the information from the bar chart can be read off the axis.
    - It is difficult to represent and read some of the data in the pictogram when they are not multiples of the chosen key
13. **Ask:** What conclusion can we draw from the pictogram and bar chart?
14. Guide volunteer pupils to say what they conclude from the 2 diagrams.
15. Your data may lead to different conclusions. (Example answers: *Shown below*)
    a. The number of pupils born on most days is very similar to each other – 7, 8 or 9 pupils.
    b. On average 8 pupils are born per day except Wednesday.
    c. There are an unusual number of pupils born on Wednesday compared to the other 6 days.
    d. The difference between Wednesday and the other days is shown more clearly in the bar chart than in the pictogram. For this data, the bar chart is a better representation of the data than the pictogram.
    e. Accept all other reasonable conclusions from the data.

Guided Practice *(10 minutes)*

1. **Say**: There are times when we want to use the data collected for different purposes. We want to both compare the data and show how it changes over time. We will draw a bar chart of the information on *Rainfall* that we drew as a line graph.
2. Ask pupils to work in pairs to answer Questions for Guided Practice. Tell pupils they should each draw their own individual bar charts.
3. **Say**: Work together to decide on appropriate scales for the axes.
4. Walk around, if possible, to check answers and correct any misconceptions.
5. Have pupils from around the classroom volunteer to draw the bars for the chart.
6. Correct any errors in the calculation on the board. Ask pupils to check their work. Answers are shown next to questions.

Independent Practice *(10 minutes)*

1. Ask the pupils to work independently to answer Questions for Independent Practice.
2. Walk around, if possible, to check answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to draw the bars for the chart.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. Answers are shown next to questions.

Closing *(2 minutes)*

1. **Say:** Listen carefully to this question, then write down the answer in your exercise books.
2. **Ask**: Musa noticed his height was changing very quickly. He decided to record his height once a month for 6 months. How should he display the data? Give a reason for your answer.
3. Allow time for pupils to write the answer in their exercise books.
4. This question is a test of whether pupils can give a good reason for their choice of chart or graph.

5. Have pupils from around the classroom volunteer to answer. (Example answers: Pictogram to give a visual representation of the data (use a picture such as a ruler to represent the heights); Bar chart to compare his height in one month with that of another month; Line graph to show how his height changes over the 6 months. Accept all reasonable answers.)

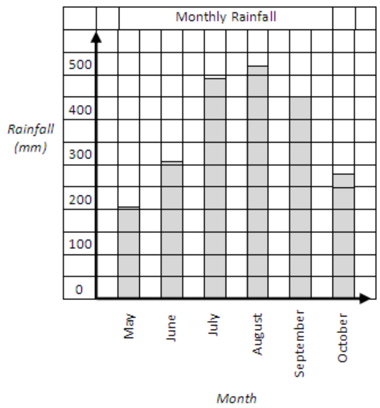
[QUESTIONS FOR GUIDED PRACTICE]

1. The amount of rainfall in a town in Sierra Leone over a 6 month period is shown in the table below.

Month	May	Jun	Jul	Aug	Sept	Oct
Rainfall (*mm*)	205	309	493	520	450	282

a. Draw a bar chart of the data.
b. Look at the line graph previously drawn. What comparisons can you make between the bar chart and the line graph?
c. Can you draw the same conclusions from both the bar chart and the line graph?

Answers:
a. Bar chart shown right
b. A different scale is used for the bar chart as compared to the line graph; The line graph shows the change over time whilst the bar graph compares the rainfall month by month. Accept all reasonable answers.
c. Yes, both the bar chart and the line graph show that: rainfall increases from May to August; It reaches a maximum in August; The rainfall then decreases from August to October; There is a sharp increase from June to July. Accept all reasonable answers.



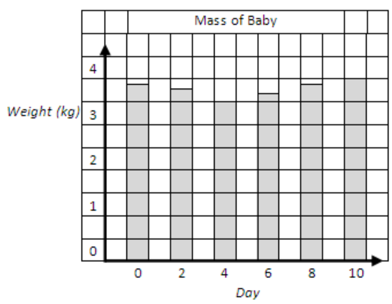
[QUESTIONS FOR INDEPENDENT PRACTICE]

2. The mass of a baby for the first few days of its life is shown in the table below.

Day	0	2	4	6	8	10
Mass (kg)	3.9	3.8	3.5	3.7	3.9	4.0

a. Draw a bar chart of the data.
b. Is it possible to estimate the mass of the baby on day 9? Give a reason for your answer.

Answers
a. Bar chart shown right.
b. It is not possible to estimate the mass of the baby on day 9. We cannot estimate the mass for the days not shown in the chart. We can compare the mass for



the days we have plotted (i.e. highest, lowest, differences in mass etc.), but the bar chart is not suitable to estimate changes in mass over time.

The line graph previously drawn is a better graph for this data as it tells us the changes over time.

Accept all reasonable answers.

Lesson Title: Interpreting Pie Charts	**Theme**: Statistics and Probability	
Lesson Number: M-09-121	**Class/Level**: JSS 3	**Time**: 35 minutes

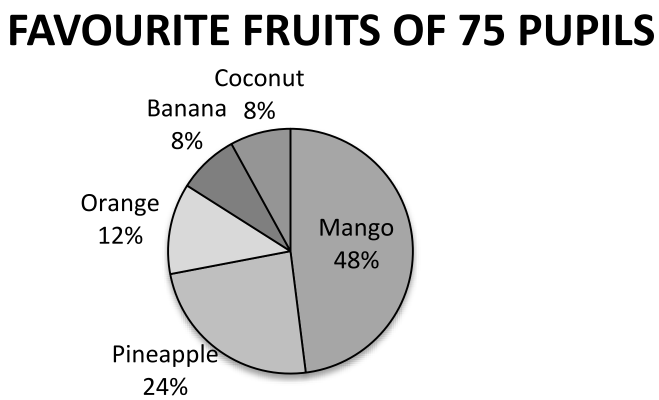
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Interpret information from a pie chart. 2. Find the sectoral angles of a pie chart and relate them to the whole($360°$).	None	1. Draw the pie charts and write the questions from Introduction to the New Material and Guided Practice on the board. 2. Write the frequency table and the questions from Independent Practice on the board.

Opening *(2 minutes)*

1. Ask pupils the following questions and have pupils volunteer to answer:
   a. What does percentage (%) mean? (Answer: Percentage means a part out of 100)
   b. What is a pie chart? (Answer: A pie chart is a type of graph in which a circle is divided into sectors that each represent a portion of the whole.)
   c. What do the percentages in a pie chart add up to? (Answer: 100%)
   d. What do you call each part of a pie chart? (Answer: A sector)
2. **Say:** Today, we are learning how to interpret information from a pie chart, and calculate the sectoral angles of a pie chart.

Introduction to the New Material *(14 minutes)*

1. Point to the following pie cha
   the board:
2. **Say**: This pie chart shows the
   fruits for a group of 75 pupils
3. Ask pupils questions to check
   the pie chart. Discuss each or
   a. Which fruit has the lowes
      and Banana)
   b. Which fruit has the highe
   c. What percentage of pupi
      (Answer: 12%)



4. **Ask:** How many pupils in this group of 75 say mango is their favourite fruit?
5. Have 2 pupils volunteer to answer.
6. **Say:** This question asks us 'how many', not what percentage. We will need to find the number of pupils out of the 75 that said mango. That means we need to find 48% of 75 pupils.
7. Solve the problem on the board with pupils' participation. (Answer: $\frac{48}{100} \times 75 = 36$ pupils)
8. **Ask:** How many pupils in this group prefer a fruit other than pineapple as their favourite?
9. Have a pupil volunteer to answer.
10. **Ask:** What is the first thing we need to do to solve this problem?
11. Allow pupils to brainstorm. Guide them to understand that we first need to find the *percentage* of pupils who did not choose pineapple.

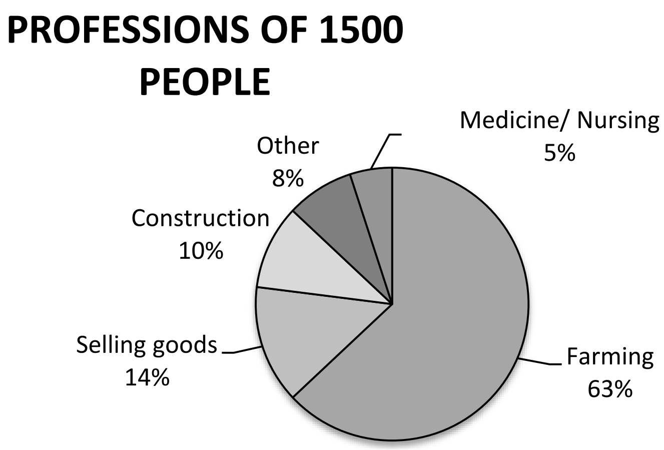
12. **Say:** If the percentage of pupils who chose pineapple is 24%, then we need to subtract this number from 100%. This will tell us the percentage of pupils who did not choose pineapple.
13. **Write** on the board: $100\% - 24\% = 76\%$
14. **Say:** 76% of pupils did not choose pineapple.
15. Ask pupils to find 76% of the 75 pupils in their exercise books. (Answer: $\frac{76}{100} \times 75 = 57$ pupils)
16. Write the answer on the board and ask pupils to check the answers in their exercise books.
17. **Say**: When you know the percentages for each sector, you can also figure out how many degrees each angle at the centre of each sector is. Being able to calculate the angles allows us to construct our own pie charts.
18. **Ask**: How many degrees are in a full circle? (Answer: 360°)
19. **Write** on the board: $Mango = \frac{48}{100} \times 360° = 172.8°$
20. **Say:** To find the degrees for the angle at the centre of each sector we can multiply the percentage for each fruit by 360. This is the same as multiplying the ratio by $360°$, if you have a frequency table.
21. **Say**: Depending on what information you have, you can calculate the angle measurement either way. For example, we calculated that 36 pupils said mangos were their favourite.
22. **Write** on the board: $Mango = \frac{36}{75} \times 360° = 172.8°$
23. Ask pupils to calculate the rest of the degrees for the other fruits in their exercise books.
24. Calculate the rest of the sectors on the board:

    $Pineapple = \frac{24}{100} \times 360° = 86.4°$

    $Orange = \frac{12}{100} \times 360° = 43.2°$

    $Banana = \frac{8}{100} \times 360° = 28.8°$

25. **Say**: Since Bananas and Coconuts have the same percentage we know they have the same angle measurement too.
26. **Say**: We can check that we did our Maths correctly because all the sectors must add up to 360°
27. Ask pupils to add up all the sector angle measurements from their exercise books.
28. **Write** on the board: $172.8° + 86.4° + 43.2° + 28.8° + 28.8 = 360°$



Guided Practice *(7 minutes)*

1. Point to the following pie chart and write on the board: This pie chart shows the professions of 1500 people surveyed in Sierra Leone.
2. Ask pupils to work in pairs.
3. Point to the following questions on the board:
    a. Which profession has the lowest percentage? (Answer: Medicine/Nursing – 5%)
    b. Which profession has the highest percentage? (Answer: Farming – 63%)
    c. What percentage of people surveyed work in construction? (Answer: 10%)

d. What percentage of people surveyed sell goods? (Answer: 14%)
e. How many people surveyed work as farmers? (Answer: $\frac{63}{100} \times 1500 = 945$)
f. How many people surveyed work in medicine or nursing? (Answer: $\frac{5}{100} \times 1500 = 75$)

4. Move around the classroom and check pupils as they work, clear misconceptions.
5. Have 6 pairs of pupil's volunteer to share their answers to the questions about the pie chart.

Independent Practice *(10 minutes)*

1. Ask pupils to work independently.
2. Point to the following frequency table and questions on the board: Use the pie chart and frequency table to calculate the inside angle measures for each of the sectors of the chart.
3. Remind pupils they can calculate the angle using the percentage or ratio
4. Move around the classroom to check for understanding, clear misconceptions.
5. Ask pupils to turn to their partner and compare their answers when they are finished.
6. Have 5 pupils volunteer to share their answers, if time allows ask for pupils to solve using both methods, the percentage, and the ratio.

Profession	**Frequency**
Farming	**945**
Selling goods	**210**
Construction	**150**
Other	**120**
Medicine/Nursing	**75**
TOTAL	**1,500**

Answers:

$Farming = \frac{945}{1500} \times 360° = 226.8°$; $Farming = \frac{63}{100} \times 360° = 226.8°$; $Selling\ goods = \frac{210}{1500} \times 360° = 50.4°$; $Selling\ goods = \frac{14}{100} \times 360° = 50.4°$; $Construction = \frac{150}{1500} \times 360° = 36°$; $Construction = \frac{10}{100} \times 360° = 36°$; $Other = \frac{120}{1500} \times 360° = 28.8°$; $Other = \frac{8}{100} \times 360° = 28.8°$; $Medicine/Nursing = \frac{75}{1500} \times 360° = 18°$; $Medicine/Nursing = \frac{5}{100} \times 360° = 18°$;

Closing *(2 minutes)*

1. Ask pupils the following questions to check understanding:
   a. What do the angles at the center of a pie chart add up to? (Answer: 360°)
   b. What does it mean when one sector is larger than another? (Answer: A larger sector means a larger quantity of the data is in that category.)
2. Allow pupils to share their ideas and discuss them.

Lesson Title: Creating Pie Charts	**Theme**: Statistics and Probability	
Lesson Number: M-09-122	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to display data collected from the class in a pie chart.	Protractors (see the attached page at the end of the lesson.)	Ask pupils to bring mathematical sets to class if they have them. For pupils who do not have them, see the following pages on how to make them.

Opening *(2 minutes)*

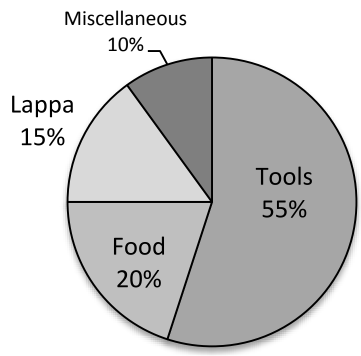
1. **Say**: We are going to collect data from the class and use it to create a pie chart.
2. Ask pupils to raise their hands to vote for their favourite football team from the following selection; Barcelona, Manchester United, Real Madrid, or Arsenal.
3. Write the frequency chart on the board and fill it out as pupils raise their hands to vote.
4. **Say:** Today, we are learning how to create pie charts.

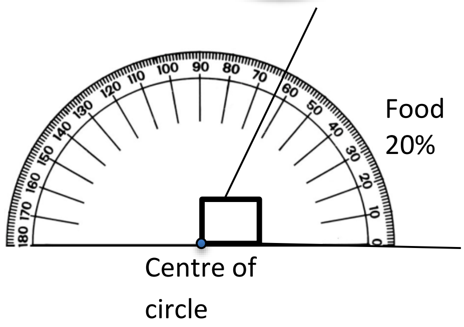
Football Team	**Frequency**
Barcelona	**16**
Manchester U.	**10**
Real Madrid	**6**
Arsenal	**8**
TOTAL	**40**

Introduction to the New Material *(13 minutes)*

1. Draw the following pie chart on the board.
2. **Say**: This pie chart shows the percentages of goods the market. A protractor is used to measure angles measured in degrees, and this protractor can meas angle less than 180 degrees. Look at the numbers c protractor. They count by tens from 0 to 180. This i but instead of measuring length we use it to measu an angle opens.
   We will use this protractor to measure the angles a each sector of this pie chart.
   You place the centre of the protractor on the centr the pie chart and place the bottom of the protractor exactly along one line of one sector of the pie chart.
3. Follow the instructions as you explain, for example ➔
4. **Say:** Now look at the other line of the angle. It crossed the protractor at a number. That number is the angle's measurement in degrees. Let's count the degrees between the 2 lines of this angle.
5. Count by tens from 0° to measure the angle measure.
6. Follow the same steps for each angle in the pie chart on the board to demonstrate for pupils.
7. **Say**: Now we will create our own pie chart with data collected from our class. First we must calculate the degree measures for the centre angles of our pie chart using our data collected in the beginning of class.

GOODS PRINCESS SELLS



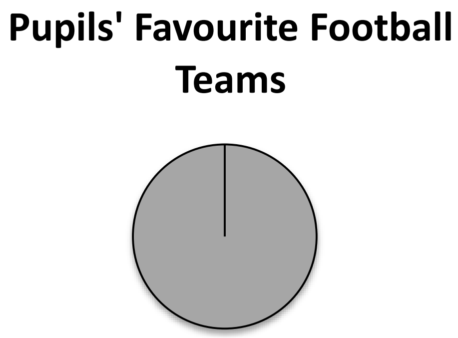


8. **Ask**: How do we calculate the degree measurements? (Answer: Multiply the ratio for each team by 360°.)
9. Write on the board: $Barcelona = \frac{16}{40} \times 360° = 144°$

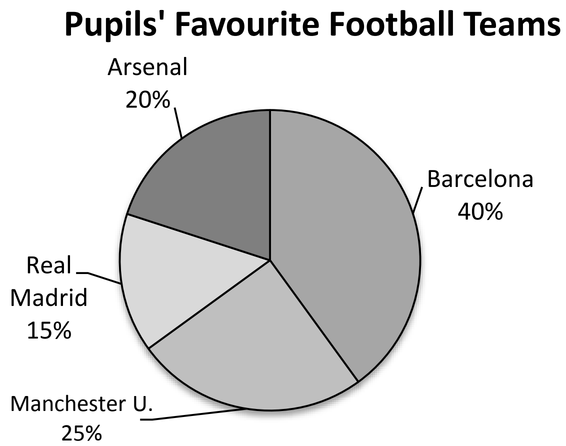
   $Manchester\ U. = \frac{10}{40} \times 360° = 90°$

   $Real\ Madrid = \frac{6}{40} \times 360° = 54°$

   $Arsenal = \frac{8}{40} \times 360° = 72°$
10. **Say**: Using these measurements, we can create a pie chart representing our class.
11. Draw an empty pie chart on the board.



12. **Say:** Place the centre of the protractor on the centre of the pie chart and place the bottom of the protractor exactly along one radius of the circle. Now find the angle measurement for the first team.
13. Make a mark at 144°. Use a straightedge to draw another radius from the centre at 144°.
14. **Say**: This sector represents the number of people who prefer Barcelona.
15. Repeat these steps for each of the sectors until the pie chart is complete.
16. **Say**: We must always label the sectors of the pie chart.
17. Write the labels for percentages and football teams on each of the sections.



Guided Practice *(8 minutes)*

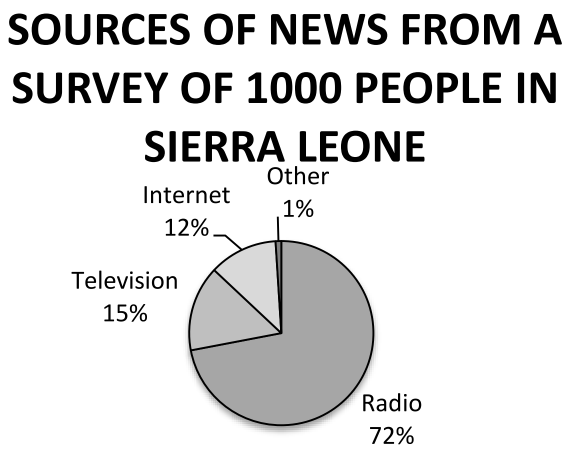
1. Write the following question on the board: This frequency table contains the sources where people said they got their news in a survey of 1000 people in Sierra Leone.
2. Ask pupils to work in pairs to calculate the inside angle measurements at the centre for each of the sectors.
3. Move around the classroom and check pupils as they work, clear misconceptions.
4. Have 4 pupils volunteer to come to the board and share their answers on the board while the rest of the class checks their work.

   (Answers: $Radio = \frac{72}{100} \times 360° = 259.2°$; $Television = \frac{15}{100} \times 360° = 54°$; $Internet = \frac{15}{100} \times 360° = 43.2°$; $Other = \frac{1}{100} \times 360° = 3.6°$

SOURCES OF NEWS FROM A SURVEY OF 1000 PEOPLE IN SIERRA LEONE	
Source	**Percent of Pop.**
Radio	**72%**
Television	**15%**
Internet	**12%**
Other	**1%**

Independent Practice *(10 minutes)*

1. Ask pupils to use the inside angle measurements for each of the sectors that they have just found to construct their own pie charts showing the sources of news from a survey of 1000 people in Sierra Leone.
2. Ask pupils to work independently.
3. Move around the classroom to check for understanding and clear misconceptions.
4. If pupils are having difficulties, allow them to work with their partners.
5. When pupils are finished, they should compare their answers with their partners.
6. Have 2 pairs volunteer to come to the board and share their pie charts.
7. Allow pupils to discuss and ask questions. (Answer: see left)

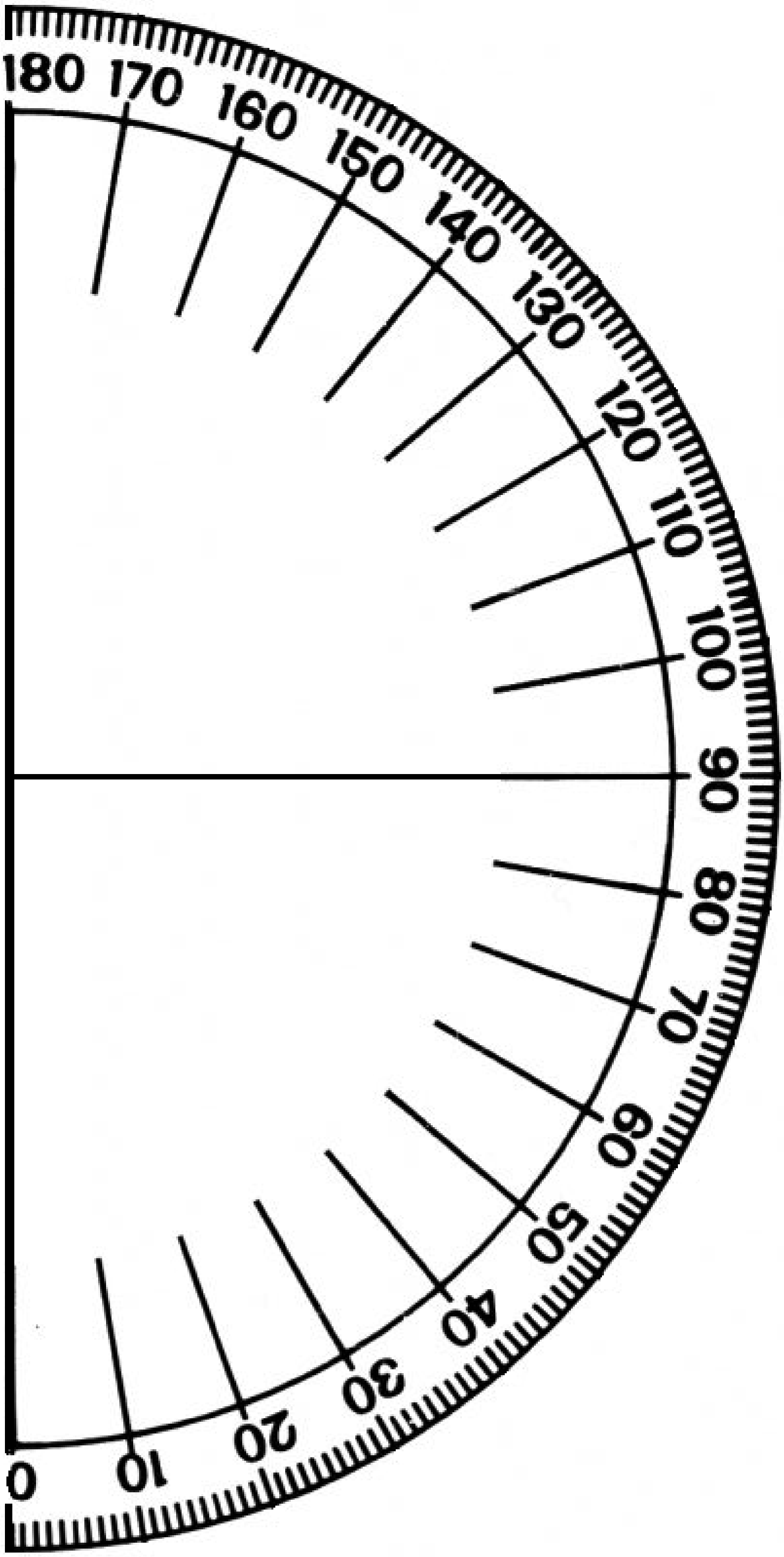


Closing *(2 minutes)*

1. Ask pupils the following questions to check understanding, and let them volunteer to answer:
   a. What do you measure angles in? (Answer: degrees)
   b. How many degrees are in a circle? (Answer: 360°)
   c. What is a protractor used for? (Answer: It is used for measuring angles in degrees.)
2. Allow pupils to share their ideas and discuss them.

[MAKING TEACHING AIDS: PROTRACTORS]

Teachers can use the large protractor below to show pupils how to measure angles on the board. Pupils can use the small protractors on the next page to measure angles in their exercise books. Teachers do not need to cut out the protractors from this book. These can be traced with a pen onto a sheet of paper, and then cut out with scissors. Teachers do not need to trace each of the 180 lines. If you trace the tens lines (0, 10, 20, etc.) it will be enough to estimate the measurement of angles. The page with small protractors can also be photocopied to provide protractors to more pupils.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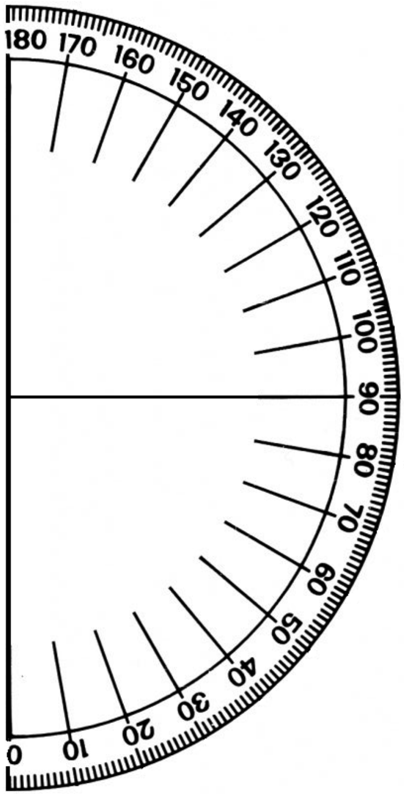
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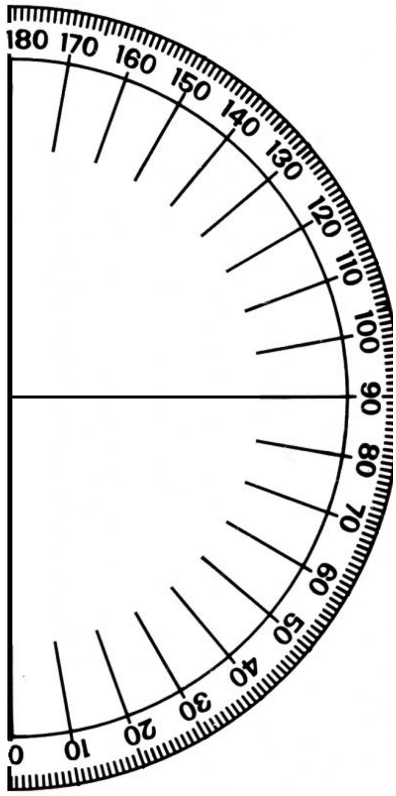
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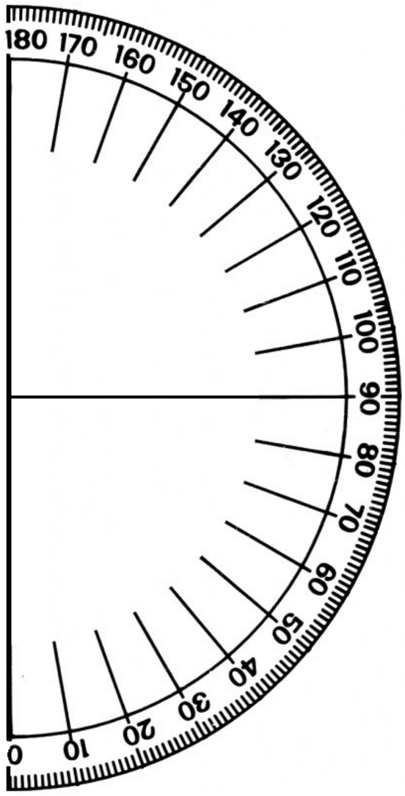
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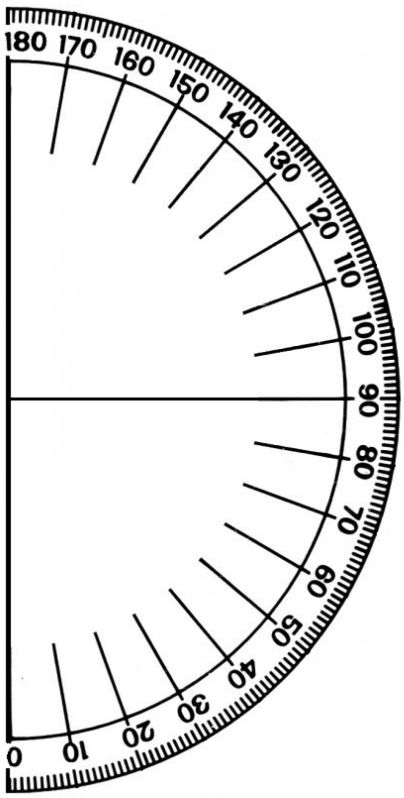
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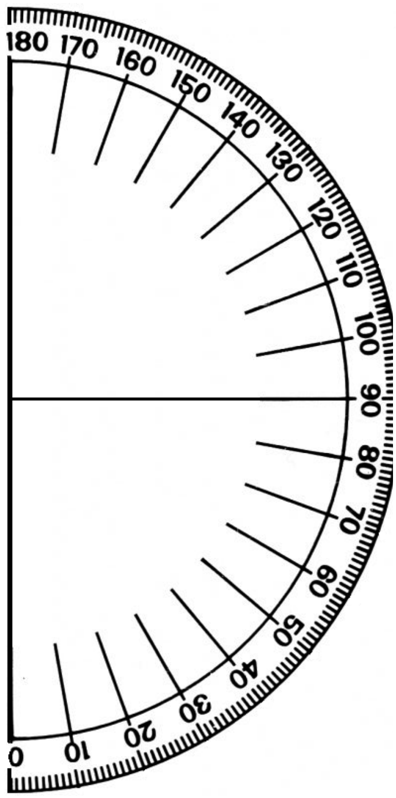
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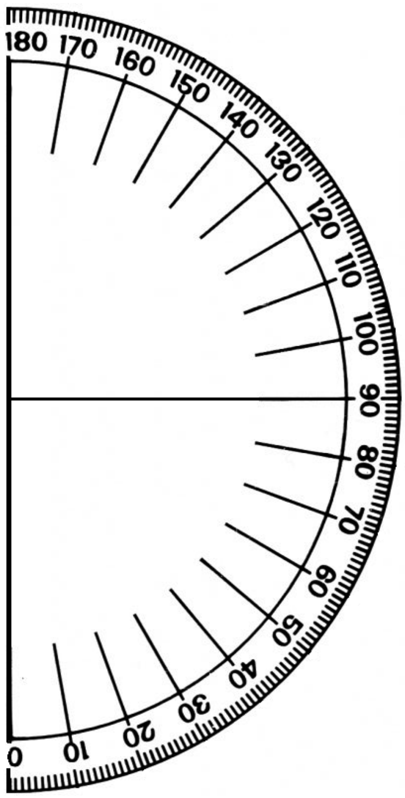
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Lesson Title: Choosing a Graph or Chart	Theme: Statistics and Probability	
Lesson Number: M-09-123	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to collect data and decide on the best type of graph or chart to represent it.	None	Draw the tables and graphs from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

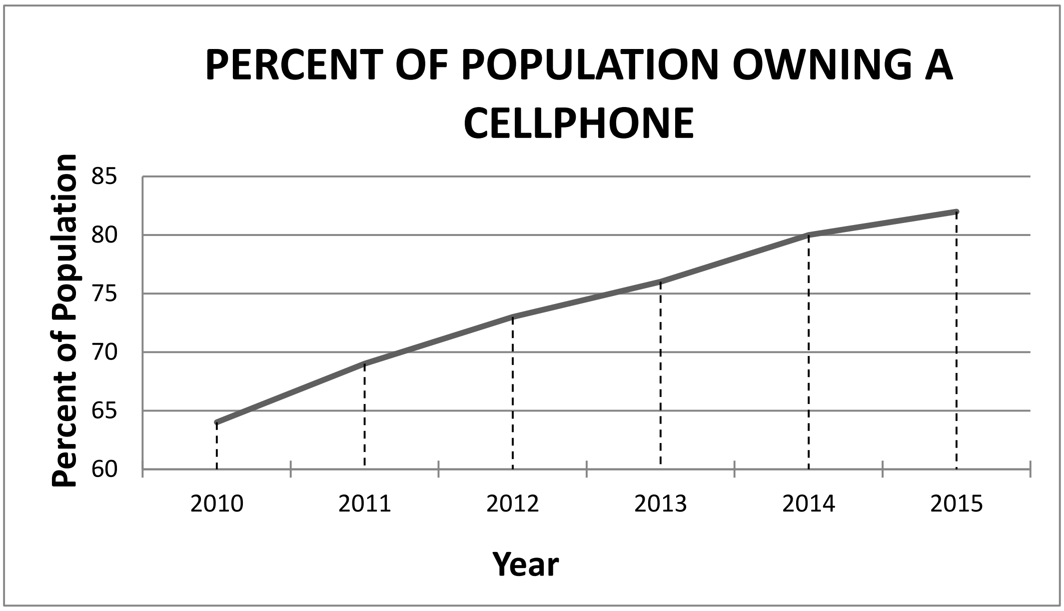
1. Ask pupils to describe the following types of charts and graphs in their own words:
   (i) Bar chart
   (ii) Line graph
   (iii) Pie chart
   (iv) Stem Diagram
2. Allow pupils to discuss these ideas and have 4 pupils volunteer to share them with the class. (Example answers: (i) A chart with rectangular bars of equal width that interpret statistical information; (ii) A graph used to display data or information that changes continuously over time; (iii) A type of graph in which a circle is divided into sectors that each represents a proportion of the whole; (iv) A chart showing frequency with columns with first digit(s) in the left column, and last digit in the right column)
3. **Say:** Today we are going to learn how decide on the best type of graph or chart to represent data we collect.

Introduction to the New Material *(12 minutes)*

1. **Say:** Bar charts and line graphs are used to compare different amounts. Pie charts (circle graphs) are use when you are trying to compare parts of a whole. They do not show changes over time.
2. **Say:** Stem diagrams show the frequency with which certain numbers occur.
3. Point to the table on the board. **Say**: The table below shows the percentage of the population of Sierra Leone who have owned a cell phone over the past 6 years. Construct a graph, which best demonstrates the percentage over the years.

PERCENT OF POPULATION OWNING A CELL PHONE	
Year	**Frequency**
2010	64%
2011	69%
2012	73%
2013	76%
2014	80%
2015	82%

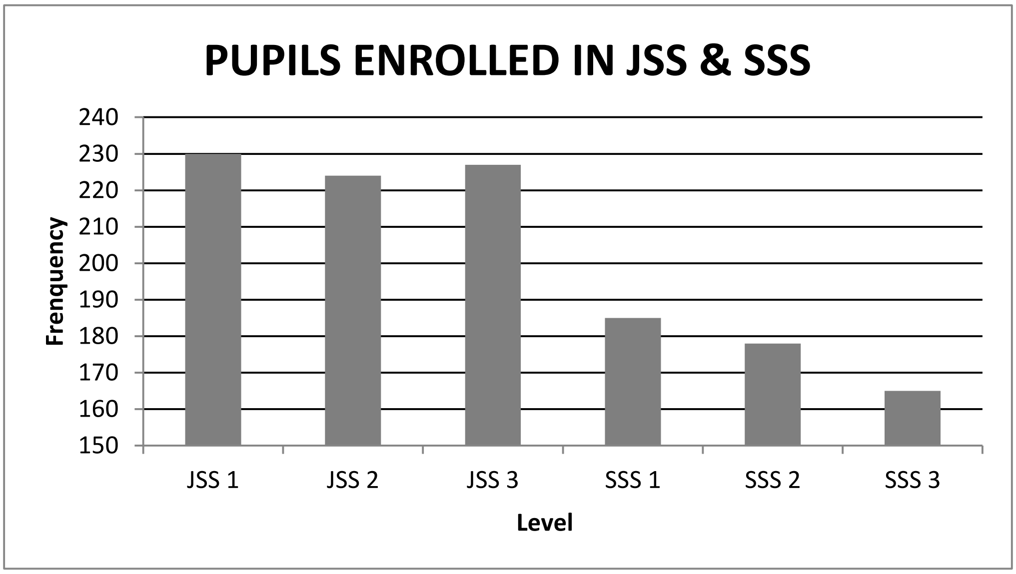
4. **Ask:** What is the appropriate graph for this data?
5. Have 2-3 pupils volunteer to answer. (Example: Bar chart, Pie chart and line graph)
6. **Say:** The table does not indicate any parts in relation to a whole, so a circle graph is not the right choice. We want to compare different values, and we can see that the data is changing over time. A line graph would be the best choice for displaying this data.
7. Together with pupils, draw the line graph on the board as shown below.
8. Ask pupils to help by calling out labels for the x- and y-axis, the title, and scale.
9. Allow pupils to discuss and answer any questions.



Guided Practice *(8 minutes)*

1. Point to the following table on the board. →
2. **Say**: This table shows the numbers of pupils enrolled in each of the grade levels in school.
3. Ask pupils to work in pairs.
4. **Say:** Draw the graph or chart that best displays this data.
5. Move around the classroom to check for understanding and clear misconceptions.
6. If they have difficulty deciding which graph or chart to use, guide them to use a bar chart. Encourage them to start the y-axis around 150.
7. Draw the empty graph on the board, and have different pairs volunteer to come and draw the 6 bars on the board.

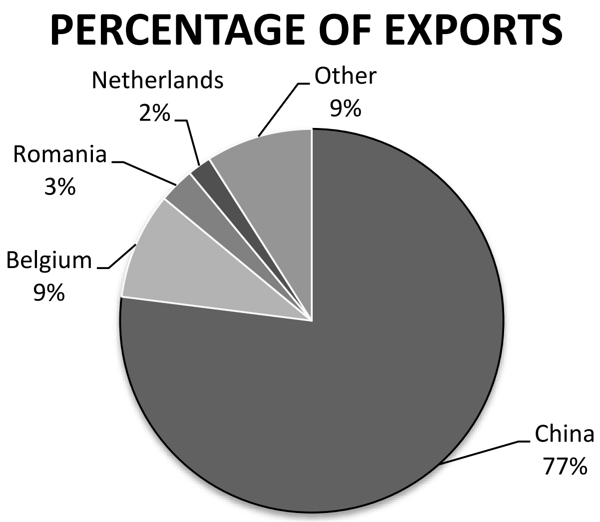
PUPILS ENROLLED IN JSS & SSS	
Level	**Frequency**
JSS 1	230
JSS 2	224
JSS 3	227
SSS 1	185
SSS 2	178
SSS 3	165



Independent Practice *(10 minutes)*

PERCENTAGE OF EXPORTS	
Country	**Percentage of Exports**
China	**77%**
Belgium	**9%**
Romania	**3%**
Netherlands	**2%**
Other	**9%**

1. Point to the table of exports on the board. **Say**: The table below shows the approximate percentages of exports from Sierra Leone to its major trading partners.
2. Draw the graph or chart that best displays the data.
3. Ask pupils to work independently to solve the problem.
4. Move around the classroom to check for understanding. For example, make sure pupils understand that a pie chart is the best way to represent the data because it is percentages of a whole.
5. Ask pupils to turn to a partner and compare answers when they have finished working.
6. Have a pupil volunteer to come to the board and present his/her answer. Do corrections where necessary.
   (Answers: $\text{China} = \frac{77}{100} \times 360° = 277.2°$; $\text{Belgium} = \frac{9}{100} \times 360° = 32.4$; $\text{Romania} = \frac{3}{100} \times 360° = 10.8°$; $\text{Netherlands} = \frac{2}{100} \times 360° = 7.2°$; $\text{Other} = \frac{9}{100} \times 360° = 32.4°$)



Closing *(2 minutes)*

1. Ask the following questions about the graphs drawn throughout the lesson and let pupils volunteer to answer:
   a. How many pupils are enrolled in JSS? (Answer: 681)
   b. What percentage of the Sierra Leone's exports go to Belgium? (Answer: 9%)
   c. What year did cell phone ownership rise above 75% (Answer: 2013)
   d. What country receives most of Sierra Leone's exports? (Answer: China)
2. Allow pupils to discuss and share ideas with the class.

Lesson Title: Mean	Theme: Statistics and Probability	
Lesson Number: M-09-124	Class/Level: JSS 3	Time: 35 minutes

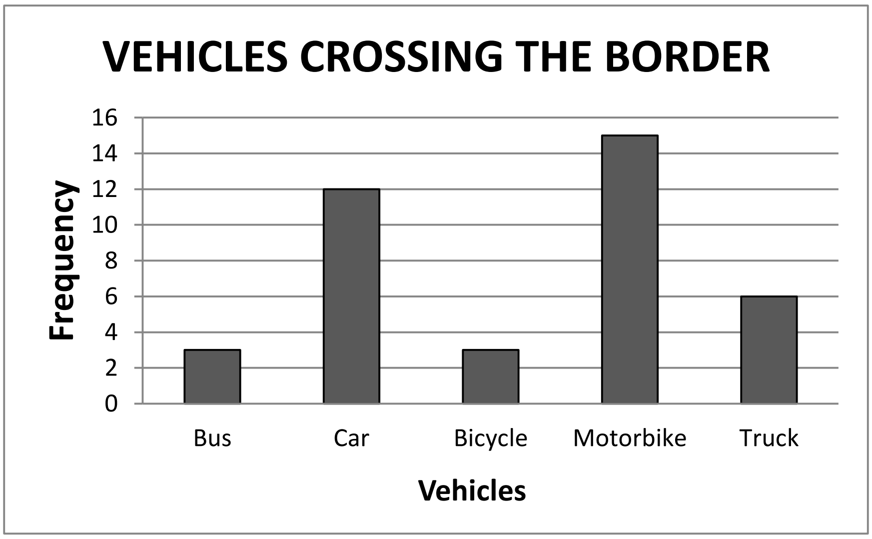
Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to: 1. Calculate the mean of a set of data from a list, chart, or graph. 2. Interpret mean.	None	Draw the tables and graphs from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils to name the types of charts and graphs we have studied so far and what kind of data they display. (Answers: (i) bar charts show quantity; (ii) line graphs show change over time; (iii) pie charts show parts of a whole)
2. Let 2-3 pupils volunteer to answer.
3. **Say:** Today we will calculate the mean of a set of data from a list, chart, or graph, and interpret what it means.

Introduction to the New Material *(15 minutes)*

1. Write the following question on the board: There are 6 pupils with the following scores on their exam: 87, 100, 76, 92, 90, and 95. What is the mean of their scores?
2. **Say**: The *mean* is a number that can tell us where the middle of the data is. It is also commonly known as the 'average'. To find the mean of a set of data, add the numbers together and divide the total by the number of items. The quotient is the mean.
3. **Write** on the board: $87 + 100 + 76 + 92 + 90 + 95$
4. **Say**: Please add the numbers in your exercise books.
5. Have a pupil volunteer to write the sum on the board while the rest work in their exercise books. (Answer: $87 + 100 + 76 + 92 + 90 + 95 = 540$)
6. **Say**: Now we will divide the sum, 540, by the number of items, 6.
7. Write on the board: $540 \div 6$
8. Have a pupil volunteer to give the answer. (Answer: $540 \div 6 = 90$)
9. **Say**: 90 is the mean score of the 6 pupils. This is the average score of the pupils, and one way to find the approximate middle of their scores.
10. Point to the following chart and on the board and **ask:** This chart shows the number of vehicles crossing the border during 1 day. Calculate the mean of the data.



11. **Say**: To calculate the mean the first thing we need to do is create a list of the frequency for each vehicle by reading the chart.
12. Ask pupils to help you create a frequency table (as below) by calling out the numbers for each of the vehicle types.
13. **Say**: Now we must calculate the mean of the data.
14. **Write** on the board: $3 + 12 + 3 + 15 + 6 =$
15. **Say**: Please add the numbers in your exercise books.
16. Have one pupil volunteer to write the sum on the board. (Answer: $3 + 12 + 3 + 15 + 6 = 39$)
17. **Say**: Now we will divide the sum, 39, by the number of items, 5.
18. **Write** on the board: $39 \div 5$
19. Have one pupil volunteer to give the answer. (Answer: $39 \div 5 = 7.8$)
20. **Say**: 7.8 is the mean number of vehicles of each type that crossed the border. It is the average out of the numbers.

Vehicle	**Frequency**
Bus	**3**
Car	**12**
Bicycle	**3**
Motorbike	**15**
Truck	**6**
TOTAL	**39**

Guided Practice *(5 minutes)*

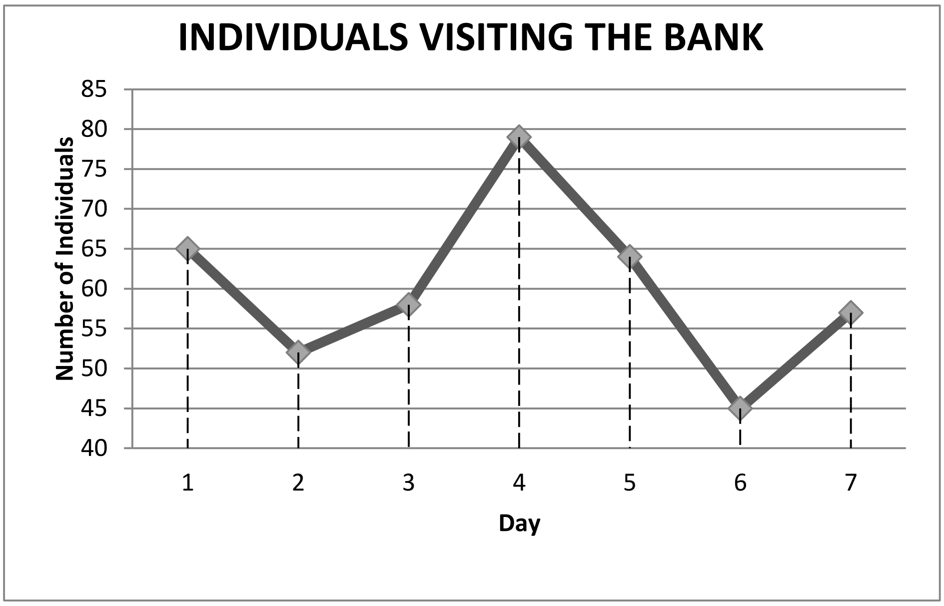
1. **Write** the following on the board:
   There are 10 individuals earning the following monthly incomes: Le 200,000; Le 175,000; Le 250,000; Le 195,000; Le 300,000; Le 410,000; Le 380,000; Le 220,000; Le 180,500; and Le 245,000. Calculate the mean income for the group.
2. Ask pupils to work with their partners to solve the problem in their exercise books.
3. Move around the room and clear misconceptions.
4. Have one pair volunteer to share their answer on the board while the rest of the class check their work. (Answer: $\text{Le } 200{,}000 + \text{Le } 175{,}000 + \text{Le } 250{,}000 + \text{Le } 195{,}000 + \text{Le } 300{,}000 + \text{Le } 410{,}000 + \text{Le } 380{,}000 + \text{Le } 220{,}000 + \text{Le } 180{,}500 + \text{Le } 245{,}000 = \text{Le } 2{,}555{,}500$; $\text{Le } 2{,}555{,}500 \div 10 = \text{Le } 255{,}550$)

Independent Practice *(10 minutes)*

1. Write the following question on the board and point to the table on the board:
   The table contains data showing the number of individuals visiting the bank every day for a week. Create a line graph and then calculate the mean number of individuals.

Day	1	2	3	4	5	6	7
Number of Individuals	65	52	58	79	64	45	57

2. Ask pupils to work independently to create a line graph and find the mean of the list of data.
3. Walk around to check for understanding and clear misconceptions.
4. Ask pupils to compare their answers with their seatmates.
5. Have 2 pupils volunteer to share their line graphs on the board, while another presents the mean. The rest of the class should checks their work.



(Answers: $65 + 52 + 58 + 79 + 64 + 45 + 57 = 420;\ 420 \div 7 = 60$ individuals)

Closing *(2 minutes)*

1. **Write** the following 5 numbers on the board and ask pupils to quickly calculate the mean: 6, 10, 15, 7, 13.
2. Ask pupils to write down their work and answer in their exercise books and have them show it to you at the end of the lesson. (Answer: $6 + 10 + 15 + 7 + 13 = 51 \div 5 = 10.2$)

Lesson Title: Median	**Theme**: Statistics and Probability	
Lesson Number: M-09-125	**Class/Level**: JSS 3	**Time**: 35 minutes

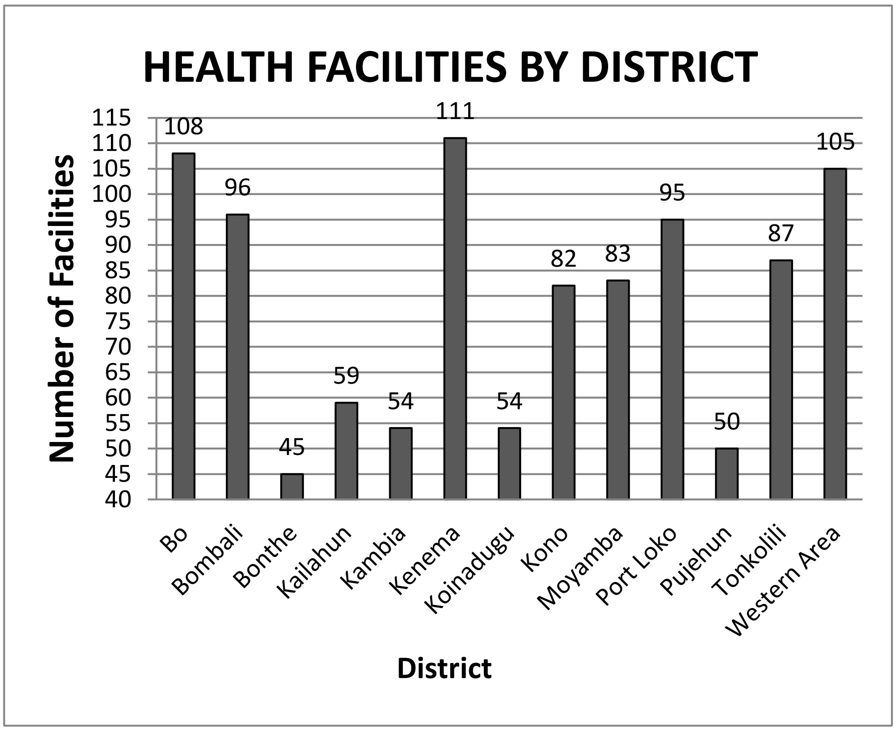
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Calculate the median of a set of data from a list, chart, or graph. 2. Interpret median.	None	Draw the tables and graphs from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils to volunteer to answer the following questions in their own words:
   a. Definition of mean. (Example answer: Mean is the average of a set of data, one way to determine the middle of a set of numbers.)
   b. When would it be useful to know the mean? (Example answers: to figure out the average grade of pupils, or the average amount of money earned or spent)
2. **Say:** Today we will calculate the median of a set of data from a list, chart, or graph, and interpret what it means.

Introduction to the New Material *(15 minutes)*

1. Point to the following chart and write the following question on the board:
   The chart contains data showing the number of health facilities in each of the 12 districts of Sierra Leone. Calculate the median number of health facilities.



2. **Say**: The number in the middle is called the median. The median is another number that can tell us where the middle of the data is.
3. Remind pupils that the mean is the other number that tells us where the middle of the data is. It is also commonly known as the 'average'.
4. **Say**: To find the median of a set of data, we must arrange the numbers in order of size and then find the middle one.
5. Ask pupils to read the chart and call out the numbers to create a list.

6. Have a pupil volunteer to come to the board and put the numbers in order of size while the rest of the class does so in their exercise books. (Answer: 45, 50, 54, 54, 59, 82, 83, 87, 95, 96, 105, 108, 111)
7. **Ask**: What is the middle number? (Answer: 83)
8. **Say**: 83 is the median number of health facilities. Most of the time the mean and median are not equal, but they are often close together.
9. Ask pupils to calculate the mean of scores, to see if the mean and median are similar. (Answer: $45 + 50 + 54 + 54 + 59 + 82 + 83 + 87 + 95 + 96 + 105 + 108 + 111 = 1{,}029;\ 1{,}029 \div 13 \approx 79.2$)
10. Remind pupils to add up the numbers in the list and divide by the number of items in the list.
11. **Say**: The mean is 79.2 health facilities per district. Median and mean are 2 ways to look at the middle of a set of data.

Guided Practice *(5 minutes)*

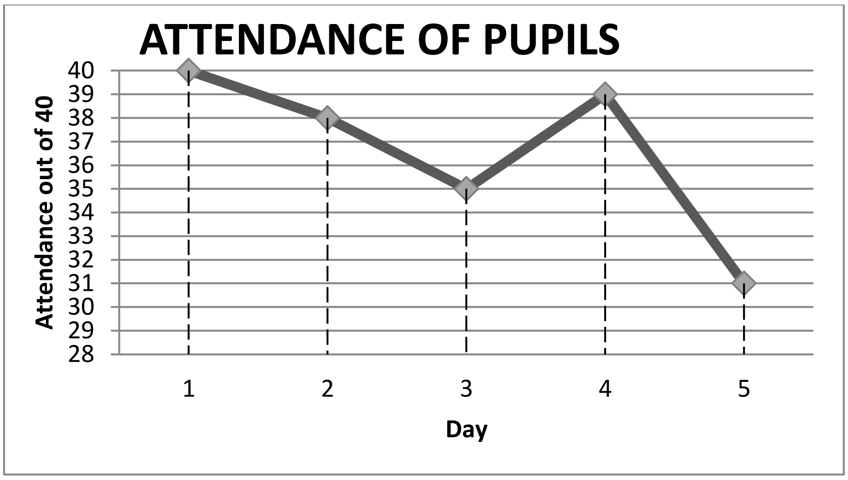
1. Write the following question on the board:
   8 pupils received the following marks on their homework assignments: 15, 17, 15, 18, 16, 20, 20, and 15. What is the median of their scores? Compare it to the mean of the scores.
2. Ask pupils to work with their partners to solve the problem in their exercise books.
3. Remind pupils that when we have an even number of items, there is no single middle number. We look at the 2 middle numbers and find their mean. We add the 2 numbers and divide by 2.
4. Move around the room and help pupils if needed.
5. Have a pair volunteer to share their answer on the board while the rest of the class checks their work. (Answers: Median: $16 + 17 = 33 \div 16.5$; Mean: $15 + 15 + 15 + 16 + 17 + 18 + 20 + 20 = 37; 119 \div 8 = 17$)

Independent Practice *(10 minutes)*

1. Point to the following question and frequency table on the board:
   The teacher recorded attendance for 5 days. Create a line graph and find the median of the data.

Day	1	2	3	4	5
Attendance of Pupils out of 40	40	38	35	39	31

2. Ask pupils to work independently to create a line graph and find the median of the list of data.
3. Walk around to check for understanding and clear misconceptions.
4. Ask pupils to compare their answers with their partners.
5. Have one pair volunteer to share their line graph on the board, while another presents the median. The rest of the class should check their work.
   (Answers: 31, 35, 38, 39, 40; The median is 38 pupils.)



Closing *(2 minutes)*

1. Ask pupils to think about the following questions:
   (i) What is the difference between mean and median? (Example answer: The mean is the average. The median is the middle number when the numbers are put in order.)
   (ii) Give an example of a time when the median and mean would be different. (Example answer: The average could be much lower or much higher than the middle number.)
2. Allow pupils to share their ideas and discuss with their partners.
3. Have a pair volunteer to share their ideas.

Lesson Title: Mode and Range	**Theme**: Statistics and Probability	
Lesson Number: M-09-126	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Calculate the mode and range of a set of data from a list, chart, or graph. 2. Interpret mode and range.	None	Draw the tables and graphs from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils the following questions to review the statistics measures they have learned. Let them volunteer to answer:
    a. Which statistical measure is the middle number in the list in order from lowest to highest? (Answer: Median)
    b. Which statistical measure is the average? (Answer: Mean)
2. **Say:** Today, we will learn about 2 more statistical calculations, called mode and range.

Introduction to the New Material *(14 minutes)*

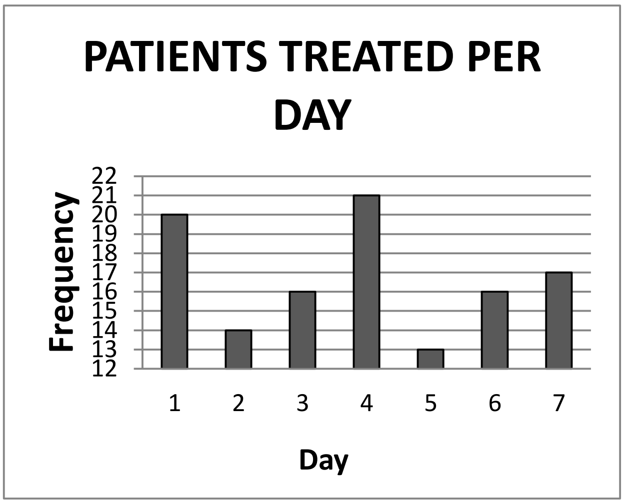
1. Write the following list of numbers on the board: 102, 100, 110, 111, 110, 105
2. **Say:** The range is the difference between the highest and lowest numbers. It tells us how spread apart our numbers are.
3. **Ask:** What is the lowest number? (Answer: 100)
4. Let a pupil volunteer to answer this and the next question.
5. **Ask:** What is the highest number? (Answer: 111)
6. **Ask:** For the numbers on the board, what numbers will we subtract to find the range? (Answer: $111-100$)
7. Write on the board: Range $= 111 - 100 = 11$
8. **Say**: The range of these numbers is 11. The mode is the number that appears most often in a list.
9. **Ask**: Which number appears more often than the others? (Answer: 110)
10. Have a pupil volunteer to answer.
11. **Say**: 110 is the mode for this data.
12. **Write** on the board: Mode $= 110$
13. Point to the following question and frequency table on the board:
    The frequency table shows the number of patients treated at the clinic each day for a week. Display the data in a bar chart and then find the mode and range of the data.

Day	1	2	3	4	5	6	7
Number of patients treated	20	14	16	21	13	16	17

14. Ask pupils to help draw a bar chart by suggesting the title, x- and y-axis, and height of the bars.
15. Ask pupils to write these in order from lowest to highest in their exercise books. (Answer: 13, 14, 16, 16, 17, 10, 21)

16. **Ask** pupils the following questions and have them volunteer to answer :

    a. What is the highest number of people treated on one day? (Answer: 21)
    b. What is the lowest number of people treated on one day? (Answer: 13)
    c. What is the range of the number people treated? Explain. (Answer: the range is 8 because we subtract $21 - 13 = 8$)



    d. What is the mode of the numbers? Explain. (Answer: The mode is 16, because 16 people were treated on 2 different days.)

3. **Say**: If a number is not repeated in a list, then that list of numbers does not have a mode. There can also be 2 or more modes. If 2 different numbers are repeated the same number of times, they are both called the mode.

Guided Practice *(5 minutes)*

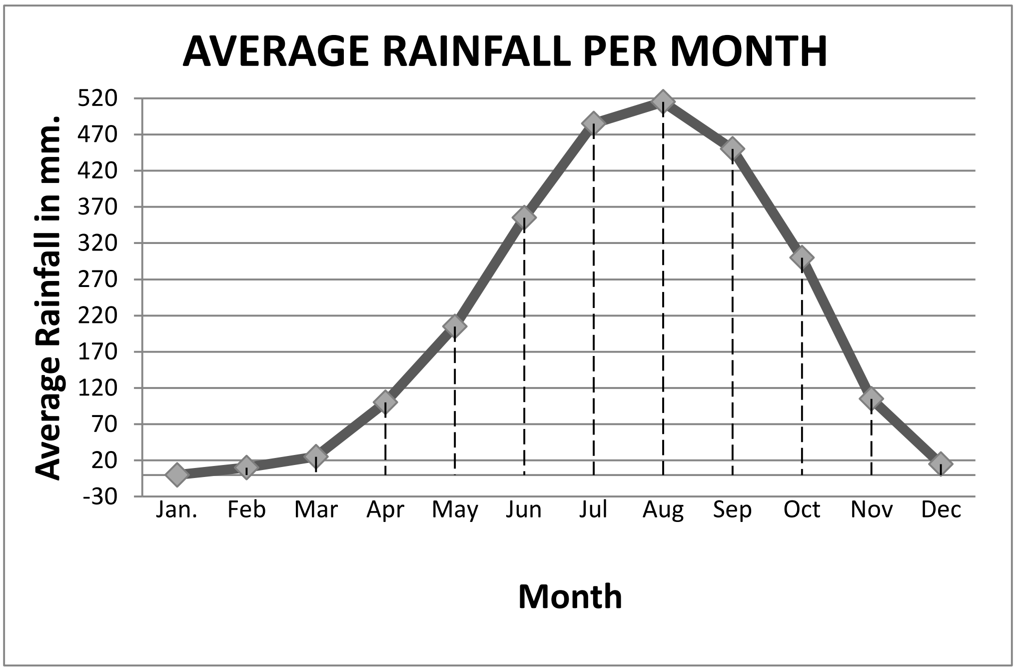
1. Ask pupils to work with their partners.
2. **Write** the following question on the board.
   The following is the list of heights of 10 pupils: 143 cm., 145 cm., 136 cm., 151 cm., 145 cm., 147 cm., 145 cm., 149 cm., 150 cm., and 152 cm. Find the mode and range of the data.
3. Walk around to check for understanding and clear misconceptions.
    a. If necessary, remind pupils to arrange the numbers from lowest to highest first.
4. Have 2 pairs volunteer to share their answers for the different parts of the question. (Answers: List in order: 136 cm., 143 cm., 145 cm., 145 cm., 145 cm., 147 cm., 149 cm., 150 cm., 151 cm. 152 cm.; the range is 16 because we subtract $152 - 136 = 16$; Mode = 145 cm.)

Independent Practice *(10 minutes)*

1. Point to the following question and frequency table on the board.
   The table shows the average rainfall per month measured in mm. Draw a line graph counting by 50's on the y-axis, and then calculate the mode and range of the data.

Month	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Rainfall in mm	0	10	25	100	205	355	485	515	450	300	100	15

2. Walk around to check for understanding and clear misconceptions. If necessary, remind pupils to arrange the numbers from lowest to highest first.
3. Allow pupils to compare answers with their seatmates when they are finished.
4. Have 2 pairs volunteer to share their answers for the different parts of the question. One group should draw the line graph, and another should find the mode and range. (Answers: List in order: 0, 10, 15, 25, 100, 100, 205, 300, 355, 450, 485, 515; the range is 515 because we subtract $515 - 0 = 515$; Mode = 100)



Closing *(3 minutes)*

1. Ask pupils to name and define the 4 statistical measures they have learned now in their exercise books. Let them raise their hands to answer.
   a. Mean (Answer: the average of a set of data)
   b. Median (Answer: the middle number in the list in order from lowest to highest)
   c. Mode (Answer: the number that appears most often in a list)
   d. Range (Answer: the difference between the highest and lowest numbers?)
2. Check that pupils have the definitions in their exercise books as they leave the classroom.

Lesson Title: Introduction to Grouped Data	**Theme**: Statistics and Probability	
Lesson Number: M-09-127	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify that 'grouped data' involves dividing a set of data into groups, or 'class intervals'. 2. Create a frequency table for grouped data.	None	Draw the tables from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils the following questions and have them volunteer to answer:
   a. What does frequency mean in statistics? (Answer: the number of times something happens)
   b. What does the word interval mean? (Answer: the distance or time between something)
2. Allow pupils to discuss and share answers.
3. **Say:** Today we will learn about 'grouped data' and how to create frequency tables for grouped data.

Introduction to the New Material *(15 minutes)*

1. **Say**: If we have a lot of data, sometimes it can be helpful to divide the data into groups. This is called 'grouped data.'
2. **Write** on the board: 23 cm., 17 cm., 16 cm., 17 cm., 18 cm., 18 cm., 21 cm., 22 cm., 19 cm., 23 cm., 17 cm., 26 cm., 17 cm., 18 cm., 18 cm., 21 cm., 22 cm., 19 cm., 26 cm., 17 cm., 29 cm., 24 cm., 21 cm., 25 cm., and 25 cm.
3. **Say:** These are the lengths in cm. of 25 individuals' feet, which they measured to buy shoes.
4. Ask pupils to discuss how we could group this data into smaller groups. Help them to understand that they can divide it into groups based on the number of cm.
5. **Say**: The first thing we must do is order the data from smallest to largest.
6. Ask pupils to do so in their exercise books, and then ask one pupil to come to the board rewrite the numbers in the correct order. (Answer: 16 cm., 17 cm., 17 cm., 17 cm., 17 cm., 17 cm., 18 cm., 18 cm., 18 cm., 18 cm., 19 cm., 19 cm., 21 cm., 21 cm., 21 cm., 22 cm., 22 cm., 23 cm., 23 cm., 24 cm., 25 cm., 25 cm., 26 cm., 26 cm., 29 cm.)
7. **Ask**: What is the range of the data? (Answer: $29\text{ cm.} - 16\text{ cm.} = 13$)
8. **Say**: If we divide the range by the number of groups we want to put our data into, it can help us decide how large our groups will be.
9. **Say**: Let us divide this data into 3 groups. We can also call our groups 'class intervals.'
10. Remind pupils of the definition of the word 'interval.'
11. **Ask**: What is $13 \div 3 \approx 4.33$, let us round up to 5.
12. **Say:** Now we can make our groups. The starting point should be smaller than or equal to the smallest number in the data set. Let us start with our smallest number: 16 cm.
13. **Say:** Now we count by 5's.
14. **Ask:** What is 16 + 5? (Answer: 21)

15. **Say:** So our first group is 16 cm. – 21 cm.
16. Continue this adding process until you have completed creating the intervals.
17. **Say**: It is important to check that all of the values are included. Since our highest value is 27 cm., the last group we need to have is 26 cm. – 31 cm.
18. **Say**: Now that we have our intervals, we must determine what number of people falls into each group.
19. **Ask:** How many measurements fit in the group 16 cm. – 21 cm.? (Answer: 15)
20. **Write** 15 in the frequency table.
21. **Ask:** How many measurements fit in the group 22 cm. – 27 cm.? (Answer: 9)
22. Write 9 in the frequency table.
23. **Ask:** How many measurements fit in the group 27 cm. – 32 cm.? (Answer: 1)
24. **Write** 1 in the frequency table.
25. **Say**: Now we have created a grouped frequency table. Remember, the groups or intervals must always be equal in size, even though they will have different numbers of frequencies.

Measurement of Feet in cm.	
Measurement	**Frequency**
16 cm. – 21 cm.	
22 cm. – 27 cm.	
27 cm. – 32 cm.	
Total	

Measurement of Feet in cm.	
Measurement	**Frequency**
16 cm. – 20 cm.	15
22 cm. – 27 cm.	9
27 cm. – 32 cm.	1
Total	25

Guided Practice *(5 minutes)*

1. Point to the following table and question on the board.
2. This table contains data for the frequency of pupils born during each month. Group the data and create a frequency chart.
3. Tell pupils to work in pairs to solve the question in their exercise books.
4. Remind pupils that the intervals or groups must be even.
5. Move around the room to clear misconceptions.
6. Ask one pair or have a pair volunteer to come to the board and write their grouped frequency table on the board. (Answer: below)

MONTHS PUPILS WERE BORN IN	
MONTH	**NUMBER OF PUPILS**
January	4
February	3
March	5
April	1
May	0
June	6
July	7
August	5
September	4
October	3
November	4
December	3
Total	45

MONTHS PUPILS WERE BORN IN	
Month	**Frequency**
Jan. – Mar.	12
April – June	7
July – Sept.	16
Oct. – Dec.	10
Total	45

Independent Practice *(10 minutes)*

1. Write the following problem on the board:
   20 pupils took a 100-point math exam. They received the following marks: 90, 92, 100, 73, 61, 85, 72, 75, 64, 80, 95, 82, 64, 85, 98, 70, 100, 90, 95, and 88. Organise the data into a grouped frequency table with 4 groups.
2. Ask pupils to solve the problem independently in their exercise books.
3. If pupils are struggling, help them decide to use groups of 10 each, starting from 61.
4. Move around the room to clear misconceptions.
5. After pupils have been working for 3 minutes, allow them to discuss with their neighbours.
6. Have one pupil volunteer to come to the board to share his/her grouped data frequency table, while the other pupils check their work in their exercise books. (Answer: below)

PUPILS' SCORES ON MATH EXAM	
Scores	**Frequency**
61 – 70	4
71 – 80	4
81 – 90	6
91 – 100	6
TOTAL	**20**

Closing *(2 minutes)*

1. **Write** the following questions on the board:
   a. Why is it called grouped data? (Answer: Because we divide the data up into groups or class intervals)
   b. How do you decide on the intervals? (Answer: Calculate the range and divide by the number of groups you want to have, then round up.)
2. Allow pupils to discuss. Clear any misconceptions.

Lesson Title: Mean of Grouped Data	**Theme**: Statistics and Probability	
Lesson Number: M-09-128	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to estimate the mean of grouped data from a frequency table using the formula: $\bar{\bar{x}} = \frac{\Sigma fx}{\Sigma f}$.	None	Draw the tables from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils the following questions:
   a. What is grouped data? (Answer: Data arranged into equal groups)
   b. How do you calculate the mean from data? (Answer: Add up all the values in a data set and divide by the number of items)
2. Allow pupils to discuss and share answers.
3. **Say:** Today we will learn how to estimate the mean of grouped data from a frequency table.

Introduction to the New Material *(15 minutes)*

1. **Write** the following question on the board:
   The following list tells how many pumpkins 15 different farmers harvested: 0, 3, 5, 6, 6, 8, 9, 11, 11, 13, 14, 15, 17, 19, and 22. Create a grouped data frequency table with 5 groups and then estimate the mean of the data.
2. Ask pupils to help decide how large each group should be.
3. **Ask**: What do we do to figure out the size of the groups? (Answer: Find the range of the data and divide by the number of groups.)
4. Ask pupils to do the calculations in their exercise books. Call on one pupil to share their answer with the class. (Answer: $22 - 0 = 22;\ 22 \div 5 = 4.4 \rightarrow round\ to\ 5$)
5. **Ask**: What should our first interval be? (Answer: 0 – 4)
6. Have pupils continue calculating intervals and fill in the chart as below:
7. **Say**: When we have a grouped data frequency chart, we cannot calculate the mean exactly because we do not know the exact numbers in the data set, we only know the frequency for each interval.
   We can estimate the mean using the following formula:
   $\bar{\bar{x}} = \frac{\Sigma fx}{\Sigma f}$
8. **Say:** Sigma (Σ) frequency (*f)* times the midpoint (*x)* all divided by sigma (Σ) frequency (*f)*. Point to each of the parts as you read them.
9. Remind pupils that Sigma (Σ) means to add all the results.
10. Ask pupils to repeat the formula back to you, allow them to practice saying the formula.

HARVEST OF 15 FARMERS	
Harvest of Pumpkins	**Frequency**
0 – 4	2
5 – 9	5
10 – 14	4
15 – 19	3
20 – 24	1
Total	15

HARVEST OF 15 FARMERS		
Harvest of Pumpkins	**Frequency**	**Midpoint**
0 – 4	2	2
5 – 9	5	7
10 – 14	4	12
15 – 19	3	17
20 – 24	1	22
Total	15	

11. **Say**: To estimate the mean we must first find the midpoint of each of the intervals our data is grouped into.

To find the midpoint you find the average of the 2 numbers making up the interval. For example: $0 + 4 = 4;\ 4 \div 2 = 2$

12. **Ask**: What is the midpoint of 0 – 4? (Answer: 2)
13. **Ask**: What is the midpoint of 5 – 9? (Answer: 7)
14. Let pupils raise their hands to answer these questions.
15. Continue until you have determined the midpoints of all the intervals.
16. **Say**: Now we can use the formula and substitute the values.
17. **Write** on the board: $\frac{\sum fx}{\sum f} = \frac{\sum(2\times2)+(5\times7)+(4\times12)+(3\times17)+(1\times22)}{\sum(2+5+4+3+1)}$

Frequency x	Midpoint =	Product
2	2	4
5	7	35
4	12	48
3	17	51
1	22	22
15		

18. **Say**: First we must multiply the midpoint of the interval by the frequency of numbers in that interval. For example, $fx = 5 \times 7 = 35$ and add up all of those products. Then we must divide it by the total frequency.
19. Ask pupils to do the calculations in their exercise books while you do so on the board. $\frac{\sum fx}{\sum f} = \frac{\sum 4+35+48+51+22}{\sum(15)}$
20. Have a pupil volunteer to come to the board to share the answer: $4 + 35 + 48 + 51 + 22 = 160$
21. **Write** on the board $160 \div 15 =$
22. Ask pupils to calculate the answer and ask one to share it with the class. (Answer: 10.667)
23. **Say**: Now let us go back to the original data and calculate the mean so we can see how close the estimate is to the actual mean.

a. Remind pupils that if they are estimating the mean for grouped frequency data they will not have all of the original data; we are only doing this to see the difference.

BABIES BORN IN THE CLINIC	
Number of Babies	**Frequency**
0 – 4	5
5 – 9	7
10 – 14	9
15 – 19	14
Total	35

24. Invite a pupil to come to the board and do the calculations while the rest of the class calculates the mean in their exercise books: $0 + 3 + 5 + 6 + 6 + 8 + 9 + 11 + 11 + 13 + 14 + 15 + 17 + 19 + 22 = 159;\ 159 \div 15 = 10.6$
25. **Say**: The estimate is very close to the real mean.

BABIES BORN IN THE CLINIC		
Number of Babies	**Frequency**	**Midpoint**
0 – 4	5	2
5 – 9	7	7
10 – 14	9	12
15 – 19	14	17
Total	35	

Guided Practice *(5 minutes)*

1. Point to the following table and question on the board:

This grouped frequency table contains data showing the number of babies born in the clinic over a 2 week period. Estimate the mean of the data.

2. Tell pupils to work in pairs to solve the question in their exercise books.
3. Move around the room to clear misconceptions.
4. Have one pair volunteer to come to the board and write their grouped frequency table on the board. Answers: $\frac{\sum fx}{\sum f} = \frac{\sum(5\times2)+(7\times7)+(9\times12)+(14\times17)}{\sum(5+7+9+14)} = \frac{\sum(10)+(49)+(108)+(238)}{\sum(35)} = \frac{405}{35} = 11.57$

Independent Practice *(10 minutes)*

1. Point to the following problem on the board:
   This is the frequency table we created in the previous lesson, showing pupils' scores on their math exams. Estimate the mean of the data.

PUPILS' SCORES ON MATH EXAM	
Scores	**Frequency**
61 – 70	4
71 – 80	4
81 – 90	6
91 – 100	6
TOTAL	**20**

2. Ask pupils to solve the problem independently in their exercise books.
3. Move around the room to clear misconceptions.
4. After pupils have been working for 3 minutes, allow them to discuss with their neighbours.
5. Have one pupil volunteer to come to the board to share his/her answer, while the other pupils check their work in their exercise books. Answers: $\frac{\sum fx}{\sum f} = \frac{\sum(4\times65.5)+(4\times75.5)+(6\times85.5)+(6\times95.5)}{\sum(4+4+6+6)} = \frac{\sum(262)+(302)+(513)+(573)}{\sum(20)} = \frac{1650}{20} = 82.5$

PUPILS' SCORES ON MATH EXAM		
Scores	**Frequency**	**Midpoint**
61 – 70	4	65.5
71 – 80	4	75.5
81 – 90	6	85.5
91 – 100	6	95.5
TOTAL	**20**	

Closing *(2 minutes)*

1. **Write** the following questions on the board:
   a. What is the formula to calculate the mean of grouped data? (Answer: $\frac{\sum fx}{\sum f}$)
   b. Why is the formula different from calculating mean of a regular data set? (Answer: We cannot calculate the exact mean, because we do not have all the data, so we must estimate.)
2. Allow pupils to discuss. Clear any misconceptions.

Lesson Title: Median and Modal Class of Grouped Data	**Theme:** Statistics and Probability	
Lesson Number: M-09-129	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify the modal class as the class interval with the highest frequency. 2. Estimate the median of grouped data from a frequency table using the formula: $L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m}\right] \times c$	None	Draw the tables from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils the following questions:
   a. What is grouped data? (Answer: data arranged into equal groups)
   b. How do you estimate the mean of grouped data? (Answer: $\bar{\bar{x}} = \frac{\sum fx}{\sum f}$)
2. Allow pupils to discuss and volunteer to share their answers.
3. **Say:** Today we will learn how to identify the modal class as the class interval with the highest frequency, and estimate the median of grouped data from a frequency table.

Introduction to the New Material *(15 minutes)*

1. **Write** the following question on the board:
   The following list tells us how many pineapples 15 different market sellers brought to sell: 6, 6, 7, 8, 8, 11, 11, 12, 13, 15, 16, 17, 18, 20 and 22. Create a grouped data frequency table with 5 groups and then identify the modal class and estimate the median of the data.
2. Ask pupils to help decide how large each group should be.
3. **Ask**: What do we do to figure out the size of the groups? (Answer: Find the range of the data and divide by the number of groups.)
4. Ask pupils to do the calculations in their exercise books. Call on one pupil to share their answer with the class. (Answer: $22 - 6 = 16;\ 16 \div 5 = 3.2 \rightarrow round\ up\ to\ 4$)
5. **Ask**: What should our first interval be? (Answer: 5 – 8)
6. Have pupils volunteer to answer and continue calculating intervals and fill in the chart as below:
7. **Ask**: What is the mode? (The most common number in a data set.)
8. Have a pupil volunteer to answer.
9. **Say**: In a grouped frequency table we do not know the exact mode, since we do not have the exact data. The modal class is the interval with the highest frequency.
10. Remind pupils that we also call the groups in data 'class intervals.'
11. **Ask**: Which group in our table has the highest frequency? (Answer: 5 – 8)
12. Allow pupils to volunteer their answers, and to discuss and ask questions.

PINEAPPLES FROM 15 FARMERS	
Pineapples to Sell	**Frequency**
5 – 8	5
9 – 12	3
13 – 16	3
17 – 20	3
21 – 24	1
Total	15

13. **Say**: When we have a grouped data frequency chart, just like we cannot calculate the mean exactly because we do not know the exact numbers in the data set, so we cannot calculate the exact median, we only know the frequency for each interval. Since we know the number of market sellers we know the middle number.
14. **Ask**: What is the middle value out of the 15 sellers? (Answer: the 8th seller)
15. Have pupils volunteer to answer this question and the next one.
16. **Ask**: Which interval contains the 8th seller? (Answer: 9 – 12)
17. **Say**: We can estimate the mean using a special formula.
18. **Write** the formula on the board:

$$L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m}\right] \times c$$

19. Read the parts of formula out loud and write definitions:
    a. L_m is the lower class boundary of the group containing the median.
    b. $\frac{n}{2}$ is the total number of values divided by 2
    c. F_{m-1} means the total frequency for the groups before the median group.
    d. f_m is the frequency of the median group.
    e. c is the group width.
20. Point to each of the parts as you read them.
21. Remind pupils that m is referring to the median group.
22. Ask pupils to repeat the formula back to you, allow them to practice saying the formula.
23. **Say**: Now we must begin to substitute the values into the formula.
24. Ask pupils to help by calling out the numbers for you to substitute.

$$L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m}\right] \times c = 9 + \left[\frac{\frac{15}{2} - 5}{3}\right] \times 4 = 9 + \left[\frac{\frac{5}{2}}{3}\right] \times 4 = 9 + \frac{10}{3} = 12.33$$

25. Ask pupils to calculate the answer and ask one to share it with the class. (Answer: 12.33)
26. **Say**: Now let us go back to the original data and find the median so we can see how close the estimate is to the actual median.
27. Remind pupils that if they are estimating the median for grouped frequency data they will not have all of the original data; we are only doing this to see the difference.
28. Have a pupil volunteer to share their answer: 12
29. **Say**: The estimate is very close to the real mean.

Guided Practice *(5 minutes)*

1. Point to the following table and question on the board:

   This grouped frequency table contains data showing the number of books read by pupils. Identify the modal class and estimate the median of the data.

NUMBER OF BOOKS READ BY PUPILS	
Number of Books	**Frequency**
0 – 4	9
5 – 9	11
10 – 14	8
15 – 19	12
Total	40

2. Tell pupils to work in pairs to solve the question in their exercise books.
3. Move around the room to clear misconceptions.
   Ask one pair or group to come to the board and write their grouped frequency table on the board. (Answers: Modal class = 15 – 19)

$$L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m}\right] \times c = 5 + \left[\frac{\frac{40}{2} - 9}{11}\right] \times 5 = 5 + \left[\frac{11}{11}\right] \times 5 = 5 + 5 = 10$$

Independent Practice *(10 minutes)*

1. Point to the following problem on the board:
   This is the frequency table we created in the previous lesson, showing pupils' scores on their math exams. Find the modal class and estimate the median of the data.
2. Ask pupils to solve the problem independently in their exercise books.
3. Move around the room to clear misconceptions.
4. After pupils have been working for 3 minutes, allow them to discuss with their partners.
   Have one pupil volunteer to come to the board to share his/her answer, while the other pupils check their work in their exercise books. Answers: Modal class = 81 – 90 and 91 – 100

$$L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m}\right] \times c = 81 + \left[\frac{\frac{20}{2} - 8}{6}\right] \times 10 = 81 + \left[\frac{2}{6}\right] \times 10 = 81 + \frac{10}{3} = 84.33$$

PUPILS' SCORES ON MATH EXAM	
Scores	**Frequency**
61 – 70	4
71 – 80	4
81 – 90	6
91 – 100	6
TOTAL	**20**

Closing *(2 minutes)*

1. **Write** the formula to estimate the median of grouped data on the board.
2. **Say**: Review your notes for 1 minute and then I will call on pupils at random to define the terms.
3. Have pupils volunteer to explain each of the terms in the formula:
   a. L_m (Answer: the lower class boundary of the group containing the median.)
   b. $\frac{n}{2}$ (Answer: the total number of values divided by 2)
   c. F_{m-1} (Answer: the total frequency for the groups before the median group.)
   d. f_m (Answer: the frequency of the median group.)
   e. c (Answer: the group width.)
4. Allow pupils to discuss. Clear any misconceptions.

Lesson Title: Practice with Grouped Data	Theme: Statistics and Probability	
Lesson Number: M-09-130	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to estimate the mean, median, and modal class for grouped data.	None	Draw the tables from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils the following questions:
   a. What is grouped data? (Answer: Data arranged into equal groups)
   b. How are mean, median, and mode different when you are working with grouped data? (Answer: You must estimate them, since you do not have the exact data you cannot calculate exactly)
2. Allow pupils to discuss and share answers.
3. **Say:** Today we will practice estimating the mean, median, and modal class for grouped data from a frequency table.

Introduction to the New Material *(15 minutes)*

1. **Write** the following question on the board:
   The following grouped data frequency table displays the number of customers at the bank across a period of 11 days. Estimate the mean and median, and identify the modal class of the data.

CUSTOMERS AT THE BANK	
Number of Customers	**Frequency**
20 – 29	2
30 – 39	4
40 – 49	3
50 – 59	2
Total	11

2. **Ask**: What is the formula to estimate the mean of grouped data? (Answer: $\bar{\bar{x}} = \frac{\sum fx}{\sum f}$)
3. **Say:** Sigma (Σ) frequency (*f)* times the midpoint (*x)* all divided by sigma (Σ) frequency (*f)*.
4. Point to each of the parts as you read them. Remind pupils Sigma (Σ) means to add all the results.
5. Ask pupils to repeat the formula back to you, allow them to practice saying the formula.
6. **Say**: First we must calculate the midpoints of each group.
7. Ask pupils to do so in their exercise books while you do so on the board.
8. **Say**: Now we can use the formula and substitute the values.

CUSTOMERS AT THE BANK		
Number of Customers	**Frequency**	**Midpoint**
20 – 29	2	24.5
30 – 39	4	34.5
40 – 49	3	44.5
50 – 59	2	55.5
Total	11	

9. **Write** on the board: $\frac{\sum fx}{\sum f} = \frac{\sum(2\times24.5)+(4\times34.5)+(3\times44.5)+(2\times55.5)}{\sum(2+4+3+2)} = \frac{\sum(49)+(138)+(133.5)+(111)}{\sum(11)} = 39.23$
10. **Say**: Next let us estimate the median.

11. **Ask**: What is the formula to estimate the median of grouped data? (Answer: $L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m}\right] \times c$)
12. Read the formula out loud. Remind pupils of each of the terms.
    a. L_m is the lower class boundary of the group containing the median.
    b. $\frac{n}{2}$ is the total number of values divided by 2
    c. F_{m-1} means the total frequency for the groups before the median group.
    d. f_m is the frequency of the median group.
    e. *c* is the group width.
13. Ask pupils to repeat the formula back to you, allow them to practice saying the formula.
14. **Ask**: What is the median group? (Answer: 30-39)
15. Ask pupils to substitute the values into the formula in their exercise books, do so on the board to guide them.

$$L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m}\right] \times c = 30 + \left[\frac{\frac{11}{2} - 2}{4}\right] \times 10 = 38.75$$

16. **Say**: Now we can identify the modal group.
17. **Ask**: How can we identify the modal group? (Answer: it is the group with the largest frequency)
18. Allow pupils to discuss and ask 1 pupil to share the answer: 30 – 39 is the modal group.

Guided Practice *(5 minutes)*

1. Point to the following table and question on the board:
   This grouped frequency table contains data showing the number of mosquito nets distributed over 3 weeks by the Ministry of Health. Estimate the mean of the data.

NUMBER OF MOSQUITO NETS DISTRIBUTED	
Number of Nets	**Frequency**
5 – 9	1
10 – 14	1
15 – 19	4
20 – 24	9
Total	15

2. Tell pupils to work in pairs to solve the question in their exercise books.
3. Move around the room to clear misconceptions.
   Ask one pair or group to come to the board and write their grouped frequency table on the board.
   (Answers: $\frac{\sum fx}{\sum f} = \frac{\sum(1\times7)+(1\times12)+(4\times17)+(9\times22)}{\sum(1+1+4+9)} = \frac{\sum(7)+(12)+(68)+(198)}{\sum(15)} = \frac{285}{15} = 19$)

NUMBER OF MOSQUITO NETS DISTRIBUTED		
Number of Nets	**Frequency**	**Midpoint**
5 – 9	1	7
10 – 14	1	12
15 – 19	4	17
20 – 24	9	22
Total	15	

Independent Practice *(10 minutes)*

1. Point to the following problem on the board:
   Using the same grouped frequency table, estimate the median and identify the modal class.
2. Ask pupils to solve the problem independently in their exercise books.
3. Move around the room to clear misconceptions.

4. After pupils have been working for 3 minutes, allow them to discuss with their partners.
   Have a pupil volunteer to come to the board to share his/her answer, while the other pupils check their work in their exercise books. (Answers: Modal class = 20 – 24
   $$L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m}\right] \times c = 20 + \left[\frac{\frac{15}{2} - 6}{9}\right] \times 5 = 20 + \left[\frac{\frac{3}{2}}{9}\right] \times 10 = 21.67)$$

Closing *(2 minutes)*

1. Ask the following questions for review and let pupils volunteer to answer:
   a. How can you identify the median class? (Answer: It is the one with the middle value.)
   b. How can you identify the modal class? (Answer: It is the one with the largest frequency.)
   c. What does Σ (sigma) mean? (Answer: sum, add up)
2. Allow pupils to discuss. Clear any misconceptions.

Lesson Title: Probability	Theme: Statistics and Probability	
Lesson Number: M-9-131	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify that probability describes the chance of something happening. 2. Discuss the probability of an event happening in words.	None	Write the questions and statements from Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils the following questions and have them volunteer to answer:
   a. How likely is it that it will be sunny tomorrow? (Example answers: it will probably happen; it may not happen)
   b. How likely is it that it will rain tomorrow? (Example answers: it will probably rain tomorrow; it will probably not rain tomorrow)
   c. How likely is it that a person's first-born child is a daughter? (Example answers: It's possible; it might happen, it might not happen)
   d. How likely is it that a person's first-born child is a son? (Example answers: It's possible; it might happen, it might not happen)
2. Allow pupils to discuss and share their ideas with the class.
3. **Say:** Today, we will learn that probability describes the chance of something happening. We will discuss the probability of different events happening.

Introduction to the New Material *(12 minutes)*

1. **Say:** Consider the question about rain. There are 2 possible outcomes for tomorrow: It will rain, or it will not rain.
2. **Ask:** In the rainy season, what are the chances that it would rain on any given day?
3. Allow pupils to volunteer to share their answers. (Example answers: It is likely to rain; It rains most days; It would probably rain)
4. **Ask:** In the dry season, what are the chances that it would rain on any given day?
5. Allow pupils to volunteer to share their answers. (Example answers: It is unlikely that it would rain; It doesn't rain often; It probably wouldn't rain)
6. **Ask**: What are some words we use to describe the chances something will happen? (Example answers: Impossible, unlikely, likely, certain, possibly, probably, maybe, might, definite)
7. Write on the board as pupils volunteer to share their answers: Impossible, unlikely, likely, certain, possibly, probably, maybe, might, definite
8. **Say**: These are some words we use to talk about the chances of something happening.
9. Ask pupils to help define the words, for example:
   a. Impossible – something will not happen
   b. Unlikely – it probably won't happen
   c. Likely – it probably will happen
   d. Certain – it will definitely happen

e. Possibly – something could happen
f. Probably – something will likely happen
g. Maybe – it is unknown
h. Definite – something will certainly happen

10. Ask pupils questions and encourage them to talk about probability in everyday language (they do not need to use numbers or Maths language yet). For example:
    a. What is the probability that you will study tonight? (Example answers: It is likely; it is unlikely because I have other work to do)
    b. What is the probability you will eat rice today? (Example answers: It is possible; it is certain)
    c. What is the probability you will speak to a cousin today? (Example answers: It is likely; it is impossible because they live in another town)
    d. What is the probability you will enroll in JSS1 next year? (Example answers: It is impossible, because you are already in JSS 3)
11. **Say**: In math, we can study the possibility of something happening. We call this 'probability'. In probability, we use numbers and math language to show the chances of something happening.

Guided Practice *(8 minutes)*

1. Point to the following statements on the board:
    a. It is _______ that medicine will help you get better.
    b. It is ________ that I will be able to fly home today.
    c. It is _______ that you will pass the test if you study.
    d. It is ________ that my mother is older than me.
    e. It is ________ you will get malaria if you properly use a mosquito net.
    f. It is ________ that medicine will help you get better.
2. Ask pupils to work with their partners by getting into pairs.
3. **Say**: Write the following questions in your exercise book. Work with your partners to use the words 'impossible', 'unlikely', 'likely', and 'certain' to fill in the blank in each of the sentences.
4. After reading the questions and allowing pairs to decide their own answers, ask pupils to volunteer to share their answers with the class.
5. Encourage pupils to discuss when they disagree. For example, some pupils might think it is impossible to cure a disease without drugs, and other pupils might think it's unlikely. Either answer is okay. The discussion about probability is important. (Answers: (a) likely; (b) impossible; (c) likely; (d) certain; (e) unlikely; (f) likely)

Independent Practice *(10 minutes)*

1. Point to the following statements on the board:
    a. It will rain during dry season.
    b. The teacher will take attendance.
    c. Tuesday will come after Monday in the week.
    d. The cows will be able to fly.
    e. It will be dark at nighttime.
    f. The sellers will be in the market.
2. Ask pupils to choose whether the statements are impossible, unlikely, likely or certain and write your answer in your exercise books.

3. Move around the room and check on pupils. Clear any misconceptions.
4. Ask pupils to discuss their answers with their partners when they have finished.
5. Have 5 pupils volunteer to each read a statement to the class, and share their answer. (Answers: (a) unlikely; (b) likely; (c) certain; (d) impossible; (e) certain; (f) likely)

Closing *(2 minutes)*

1. Ask pupils to come up with their own examples of likely, unlikely, certain, or impossible events. (For example, it is certain they will go home and study because they have a quiz tomorrow.)
2. Allow pupils to discuss with their partners.
3. Have 1-2 pairs volunteer to share their ideas with the class.

Lesson Title: Probability Experiments with 1 Event	**Theme**: Statistics and Probability	
Lesson Number: M-09-132	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Conduct simple probability experiments 2. Use probability terms such as 'experiment,' 'outcome' and 'event'.	A plastic or paper die (see last page of lesson plan), or if a die is unavailable, 6 squares of paper with the numbers 1-6 written on them.	Locate or make die, or prepare numbered squares of paper.

Opening *(3 minutes)*

1. Ask pupils to raise their hands to answer the following questions:
    a. What is probability? (Example answer: Probability is the chance that something will happen or how likely it is that it will happen.)
    b. Write a statement about something that is likely to happen. (Example answers: I will study when I get home.)
    c. Write a statement about something that is unlikely to happen. (Example answers: I will see a snake on my way home from school.)
2. **Say:** Today, we will learn how we use experiments to understand probability in mathematics.

Introduction to the New Material *(11 minutes)*

1. **Ask:** What is an experiment?
2. Have 3-4 pupils volunteer to answer. (Example answers: It's when we try something to understand how it works; Scientists do experiments to understand new ideas.)
3. **Say**: In probability, an experiment is a situation involving chance. An experiment leads to results called outcomes. For example, I can toss a dice as an experiment. There is a chance that the dice will land on any number between 1 and 6. This is an experiment involving chance.
4. **Ask:** What is an outcome when we talk about experiments?
5. Allow pupils to raise their hands to answer.
6. **Say:** An outcome is a single result of an experiment. It is something that could possibly happen.
7. **Ask:** If I roll a die, what are the possible outcomes?
8. Have 2-3 pupils volunteer to answer. (Answers: You roll a 1, 2, 3, 4, 5, or 6.)
9. **Say**: I will now conduct an experiment. I will close my eyes and roll this die.
10. **Ask**: What are the possible outcomes of my experiment? Turn to the pupil beside you and discuss.
11. Allow pupils to discuss. Guide them to see that they already answered the question; it was just asked in a mathematical way this time.
12. **Say**: There are 6 possible outcomes. I could roll a 1, 2, 3, 4, 5 or 6. Those are the only possible outcomes for my experiment of rolling a die.
13. **Write** on the board: Outcomes:
14. Do the experiment a few times. Roll the die and write the number on the board. (Example on the board: Outcomes: 3, 4, 1, 6, 5, 3)
15. **Say**: Another word we use in probability is 'event'.
16. **Ask**: What is the meaning of event in our everyday lives?

17. Have pupils volunteer to share their answers. (Example answers: Events are things that happen, often at certain times. A party or football match can be called an event.)
18. **Say**: In probability, the outcomes of an experiment can also be called events. Events can involve 1 outcome or more than 1 outcome. 1 event of our experiment is rolling a 4 with the die. Rolling the 4 is the event. Events can also involve more than 1 outcome. For example, rolling an odd number on the die is an event. We could roll a 1, 3 or 5. They are 3 different outcomes, but they are all odd numbers, so in this case we can describe this as an event. Before we can do calculations with probability, we must understand experiments. It is important to learn the correct words for talking about probability.

Guided Practice *(9 minutes)*

1. **Write** on the board:
   Experiment: Selecting pieces of paper
   Outcomes:
2. Ask pupils to work with their partners.
3. **Say**: I want you to make your own experiment. Tear 10 small pieces of paper, from 1 page of your exercise book or any other sheet. You can share 10 pieces for the whole group. I want you to write on them. You can write letters, numbers, or words, such as the number 1-10 or letters A- J. All 10 pieces of paper should be different.
   After you make your papers, I want you to conduct the experiment with your group. Fold the pieces of paper and put them in a pile. Everyone should take turns to randomly choose a piece of paper. Write down the outcomes of your experiment in a list.
4. Walk around to check for understanding. Make sure each pupil is involved in the experiment. Make sure pupils are writing the list of outcomes correctly. (Example outcomes: Depending on what pupils wrote on their papers and the number of pupils per group: 1, 7, 2, 9; or B, F, E, D, A; Bird, Cow, Dog, Cat; etc.)
5. Have a pair volunteer to come forward and explain their experiment to the class. Ask them to give the outcomes of their experiment. (Example: In our experiment, we wrote letters on the 10 pieces of paper. The outcomes of our experiment were B, F, E, D, A.)
6. Ask pupils to save these pieces of paper to do more experiments during later lessons.

Independent Practice *(10 minutes)*

1. **Say**: I am going to write down some experiments and outcomes on the board. I will not tell you what type they are. I want you to choose whether each is an experiment or outcome.
2. Show an example first. **Write** on the board: 1) Rolling a die
3. **Ask**: Is this an experiment or outcome? (Answer: experiment)
4. Write the answer on the board after the statement: 1) Rolling a die - *Experiment*
5. **Say**: I want you to do the same thing in your exercise books.
6. Continue the list on the board:
   a. Rolling a die - *Experiment*
   b. Randomly choosing an orange from a bag of oranges and mangos
   c. Drawing a red 10 from a deck of cards
   d. Flipping a coin
   e. A die landing on 6

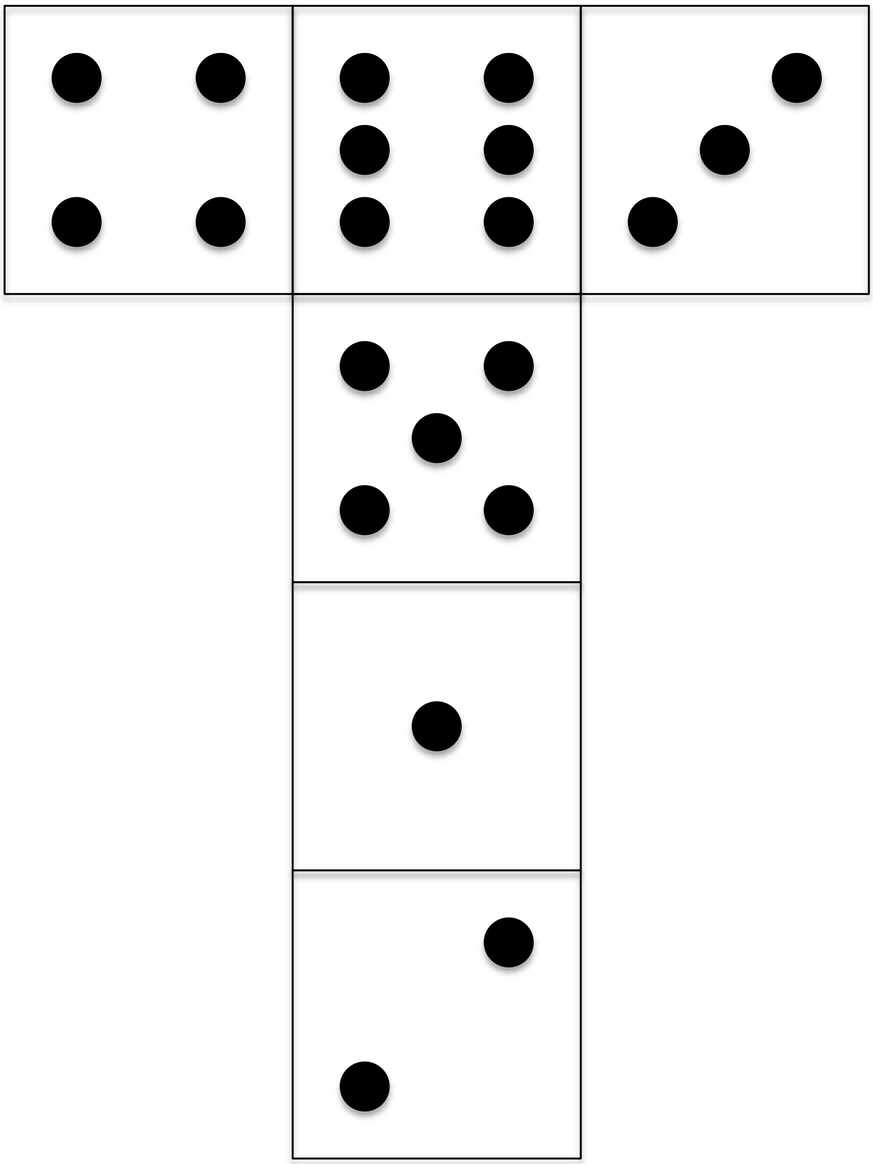
f. Putting pupils' names in a hat to draw from
g. Randomly selecting a blue pen from a cup

7. Have 6 different pupils volunteer to answer 1 question each. (Answers: (ii) outcome; (iii) outcome; (iv) experiment; (v) outcome; (vi) experiment; (vii) outcome)
8. Make corrections and discuss where necessary.

Closing *(2 minutes)*

1. Ask pupils to define the following words in statistics.
   a. What is an experiment? (Answer: A situation involving chance or probability that leads to results, or outcomes.)
   b. What is an outcome? (Answer: It is a single result of an experiment.)
   c. What is an event? (Answer: It is 1 or more outcomes of an experiment.)
2. Have 3 pupils volunteer to share their answers with the class.

Note: If necessary, a die can be created using this pattern and taping it together.



Lesson Title: Expressing Probability with Numbers	**Theme**: Statistics and Probability	
Lesson Number: M-09-133	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Express the probability of an event happening as a fraction. 2. Express the probability of an event happening as a percentage.	A plastic or paper die (see last page of lesson plan) or, if unavailable, 6 squares of paper with the numbers 1-6 written on them.	1. Locate or make die, or prepare numbered squares of paper. 2. Draw the diagram from Introduction to the New Material on the board.

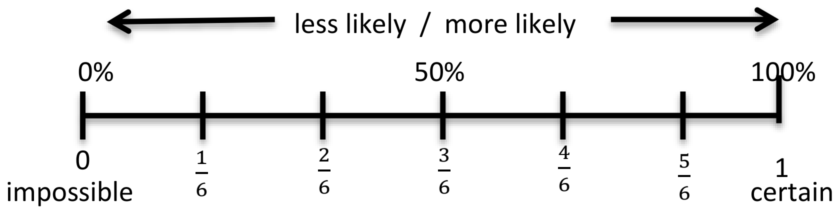
Opening *(2 minutes)*

1. **Write** the following questions on the board to review fractions and percentages.
    a. Simplify $\frac{4}{12}$ and $\frac{8}{16}$
    b. Write $\frac{1}{4}$ and $\frac{1}{5}$ as percentages.
2. Allow pupils work in their exercise books.
3. Invite 2 volunteers to come to the board and write their answers. (Answer: (i) $\frac{1}{3}$ and $\frac{1}{2}$; (ii) 25% and 20%)
4. **Say**: Today we will learn how to express the probability of an event happening as a fraction and as a percentage.

Introduction to the New Material *(13 minutes)*

1. **Say**: The sides of this die are labeled 1 to 6.
2. (If no die is available, use paper die or individual pieces of paper.)
3. **Ask:** What are the chances that I will randomly roll or select an 8?
4. Have 2 pupils volunteer to share their answers. (Answer: It is impossible; we cannot get an 8 because it's not on the die.)
5. **Say**: If the statement is impossible, the probability is 0.
6. **Write** on the board: Probability of rolling 8 = 0
7. **Ask**: What are the chances that I will randomly select a number less than 7?
8. Have 2-3 pupils volunteer to share their answers with the class. (Answer: It is certain; the number selected will definitely be less than 7 because all numbers 1 to 6 are less than 7.)
9. **Write** on the board: Probability of drawing a number less than 7 = 1
10. **Say**: If an event is impossible, then it definitely will not happen and the probability of it happening is 0. If an event is certain, then it definitely will happen and the probability of it happening is 1.
11. **Say**: We have also discussed likely and unlikely events. Those events might happen, or they might not. If there is a chance of them happening, the probability is between 0 and 1. That is, it is a fraction, or part of a whole.
12. **Ask**: If I roll this die, what is the probability that I will roll a 3?
13. Have 3 pupils volunteer to share their answers with the class. (Example answer: It is unlikely, because only 1 side has a 3.)

14. **Say**: Today we will learn how to answer this type of question with numbers. Remember that in probability, 0 is impossible and 1 is certain. That means that all other likely and unlikely events are expressed with fractions between 0 and 1.
15. **Write** on the board: $\text{probability} = \frac{\text{the number of ways an event can occur}}{\text{total number of possible outcomes}}$
16. Remind pupils of the question: If I roll this die, what is the probability that I will roll a 3?
17. **Say:** To find the numerator, we need to decide how many ways it could possibly happen that we draw a 3.
18. **Ask**: How many sides of this die have a 3 on them? (Answer: 1 side)
19. Have a pupil volunteer to answer.
20. **Say**: Yes, there is 1 possible way this event can outcome.
21. **Ask**: How many total possible outcomes are there on the die? That is, how many different sides are there? (Answer: 6)
22. **Write** on the board: $\text{probability of rolling } 3 = \frac{1}{6}$
23. **Say:** The fraction $\frac{1}{6}$ represents the probability of choosing a 3. We can say 'there is a 1-in-6 chance of choosing 3'.
24. **Write** on the board: There is a 1-in-6 chance of choosing 3.
25. **Ask**: How can we write this as a percentage? (Answer: 16.67%; remind pupils how to convert to percentages if necessary; multiply the fraction by 100%)
26. **Say**: There is a 16.67% chance that we will roll a 3.
27. Point to the diagram on the board:



28. **Say**: This diagram helps us to compare likely and unlikely events. The bigger a fraction or percent is, the more likely the event is to occur. The smaller the fraction or percent, the less likely.
29. Allow pupils to ask questions to make sure they understand the fractions and percentages are equivalent.

Guided Practice *(7 minutes)*

1. Ask pupils to work in pairs.
2. **Write** the following questions on the board: There are 2 yellow balls, 4 red balls and 6 blue balls in a sack. What is the probability that I will draw:
   a. A yellow ball
   b. A blue ball
   c. A red ball
   d. A yellow or blue ball
3. Ask 1 person in each pair to calculate the fraction in simplest form and ask the other pupil in each pair to calculate the percentage.
4. Allow pupils to discuss and exchange ideas.
5. Move around to check for pupils understanding and clear misconceptions.

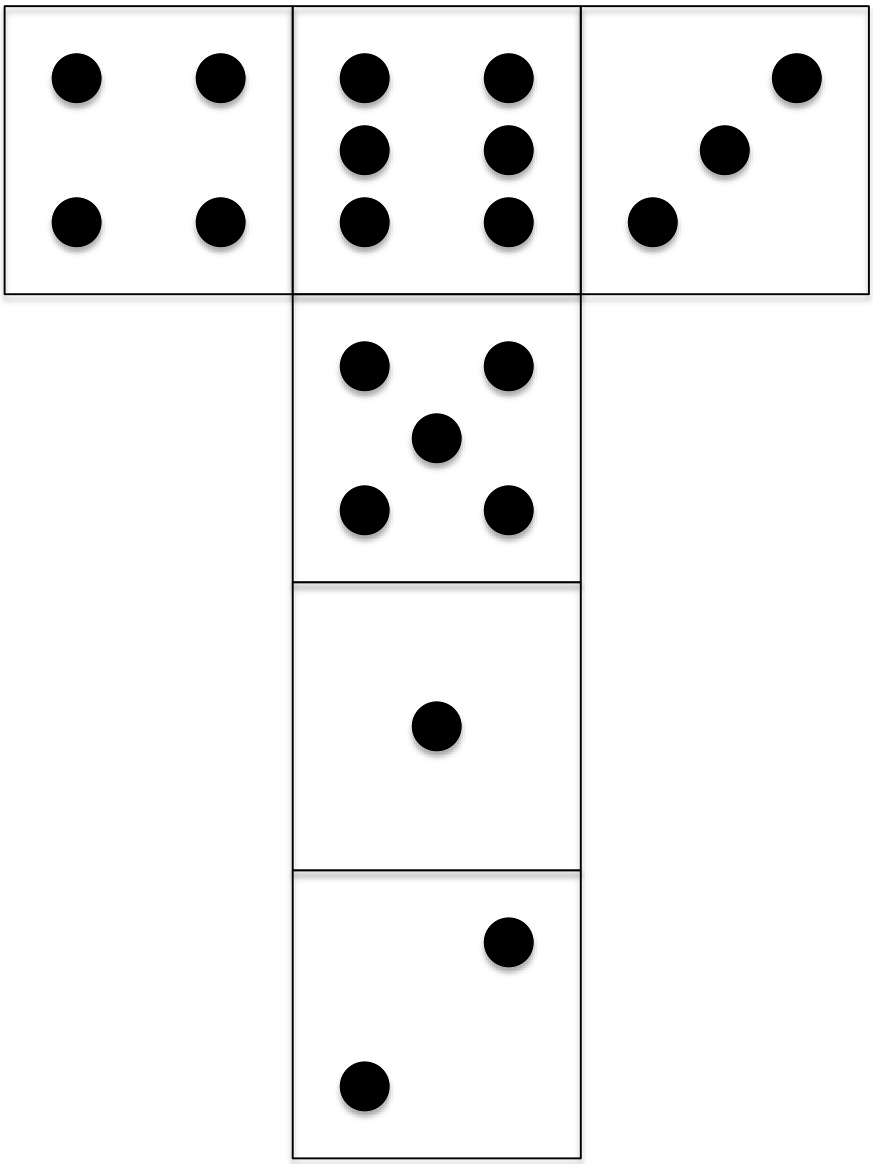
6. Have 4 different pairs volunteer to give the 4 answers and explain. (Answers: (a) $\frac{2}{12} = \frac{1}{6}$ or 16.67%; (b) $\frac{6}{12} = \frac{1}{2}$ or 50%; (c) $\frac{4}{12} = \frac{1}{3}$ or 33.3%; (d) $\frac{8}{12} = \frac{2}{3}$ or 66.67%)

Independent Practice *(10 minutes)*

1. **Write** on the board: There are 5 bananas, 2 oranges, and 3 mangos in a box. If 1 is selected at random, what is the probability of choosing:
   a. a banana
   b. an orange
   c. a mango
2. **Say**: Remember to simplify the fraction in your answer to its lowest form and express the fraction as a percent too.
3. Ask pupils to work independently.
4. Walk around to check for understanding and clear misconceptions.
5. If they finish, ask them to turn to a neighbor to compare and discuss answers.
6. Have 3 different pupils volunteer to write the 3 answers on the board and explain. (Answers: (a) $\frac{5}{10} = \frac{1}{2}$ or 50%; (b) $\frac{2}{10} = \frac{1}{5}$ or 20%; (c) $\frac{3}{10}$ or 30%)
7. Check for errors in the answers on the board and correct them if needed. Ask all pupils to check their work.

Closing *(3 minutes)*

1. Ask pupils to look at the answers of the Independent Practice section.
2. Ask the following questions to check their understanding of probability:
   a. Is it more likely that you will choose a banana or a mango? Why? (Answer: It is more likely to choose a banana because the fraction or percentage is larger ($\frac{1}{2}$ or $50\% > \frac{3}{10}$ or 30%); it has a higher probability)
   b. Is it more likely that you will choose an orange or a mango? Why? (Answer: It is more likely to choose a mango because the fraction or percentage is larger ($\frac{3}{10}$ or $30\% > \frac{1}{5}$ or 20%); it has a higher probability)



Lesson Title: Likelihood of Events	**Theme**: Statistics and Probability	
Lesson Number: M-09-134	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to compare whether events are impossible, unlikely, likely, or certain.	None	Write the questions from Introduction to the New Material and Guided Practice on the board.

Opening *(3 minutes)*

1. Ask the following questions to review certain and impossible events, and let pupils volunteer to answer:
   a. What does it mean if probability is 0? (Answer: It means the event is impossible)
   b. What does it mean if probability is 1? (Answer: It means the event is certain to happen)
   c. How do you calculate probability? (Answer: $probability = \frac{the\ number\ of\ ways\ an\ event\ can\ occur}{total\ number\ of\ possible\ outcomes}$)
2. **Say:** Today we will continue to discuss probability and compare whether events are impossible, unlikely, likely or certain.

Introduction to the New Material *(13 minutes)*

1. Allow pupils to discuss the following questions with their partners and raise their hands to answer:
   a. What does it mean if an event is *likely* to happen? (Example answer: A likely event has a greater chance of occurring; it will probably happen but we are not certain.)
   b. What does it mean if an event is *unlikely* to happen? (Example answer: An unlikely event is an event that is not sure of occurring; it is not impossible but it will probably not happen.)
   c. What does it mean if an event is *certain* to happen? (Example answer: A certain event will occur.)
   d. What does it mean if it is *impossible* for an event to happen? (Example answer: An impossible event will not occur.)
3. Have 5 pupils volunteer to come stand at the front of the room. This should consist of 4 girls and 1 boy.
4. **Ask**: If I select a pupil at random, how likely am I to select a girl? (Answer: likely)
5. Ask pupils to discuss why it is likely that I would choose a girl; guide them to understand it is because there are more girls than boys in the group.
6. Ask pupils to calculate the probability of randomly selecting a girl in their exercise books.
7. Have a pupil volunteer to share their answer: $\frac{4}{5}$
8. **Ask**: If I select a pupil at random, how likely am I to select a boy? (Answer: unlikely)
9. Ask pupils to discuss why it is unlikely that I would choose a boy; guide them to understand it is because there are fewer boys than girls in the group, but since there is 1 boy it is not impossible.
10. Ask pupils to calculate the probability of randomly selecting a girl in their exercise books.
11. Have a pupil volunteer to share their answer: $\frac{1}{5}$
12. **Say**: $\frac{4}{5}$ is greater than $\frac{1}{5}$, so we know it has a higher probability.
13. **Ask**: If I select a person from this group, how likely is it that the person will be a pupil at this school? (Answer: certain)

14. Have a pupil volunteer to answer.
15. **Ask**: Calculate the probability of randomly selecting a pupil at this school from the group. (Answer: 1)
16. Allow pupils to discuss that the whole group is enrolled in the school, thus it is certain.
17. **Ask**: If I select a person from this group, how likely is it that the person will be a teacher at this school? (Answer: impossible)
18. **Ask**: Calculate the probability of randomly selecting a teacher from the group. (Answer: 0)
19. Allow pupils to discuss that the group is all pupils, not teachers, thus it is impossible to randomly select a teacher from the group.
20. Point to the following question on the board and ask them to calculate the probability for each situation:
    (i) There are 8 red balls and 2 blue balls in a bag. If I randomly select 1, is it likely that I will select a red ball? (Answer: Yes, likely, the probability is $\frac{8}{10} = \frac{4}{5}$)
    (ii) If I randomly select 1, is it likely that I will select a blue ball? (Answer: No, unlikely, the probability is $\frac{2}{10} = \frac{1}{5}$)
    (iii) If I randomly select 1, is it likely that I will select a green ball? (Answer: No, impossible, the probability is $\frac{0}{10} = 0$)

Guided Practice *(7 minutes)*

1. Point to the following on the board: There are 8 pencils, 4 blue pens, and 5 black pens in a cup. I will randomly select 1. Rank the following events from most likely (or certain) to least likely (or impossible) and calculate the probability:
   (i) I will select a pencil.
   (ii) I will select a pen.
   (iii) I will select a green pen.
   (iv) I will select a black pen.
   (v) I will select a blue pen.
2. Ask pupils to work in pairs to list the statements in the correct order. Allow them to discuss.
3. Remind pupils that they calculate the probabilities and compare the fractions to help them figure out which is the most likely.
4. Move around the room to check for understanding and clear misconceptions.
5. Have different pairs volunteer to answer each question:
   a. Which event was most likely or certain? (Answer: (ii) – there are 9 pens so that is the highest probability $- \frac{9}{17}$)
   b. Which event is next most likely? (Answer: (i) – there are 8 pencils so that is the second highest probability $- \frac{8}{17}$)
   c. Which event is next most likely? (Answer: (iv) – there are 5 black pens so it is the next highest probability $- \frac{5}{17}$)
   d. Which event is next most likely? (Answer: (v) – there are 4 blue pens so it is even less likely I will select a blue pen; probability $- \frac{4}{17}$)
   e. Which event is least likely or impossible? (Answer: (iii) Selecting a green pen is impossible because there are not any in the cup; probability $= \frac{0}{17} = 0$)

Independent Practice *(10 minutes)*

1. Point to the following question on the board: There are 20 biscuits in a bag, 12 of them are chocolate, 5 of them are vanilla and the rest of them are strawberry. I will randomly select 1 to eat. Rank the following events from most likely (or certain) to least likely (or impossible):
   a. I will eat a chocolate biscuit.
   b. I will eat a vanilla biscuit.
   c. I will eat a ginger biscuit.
   d. I will eat a biscuit that is not chocolate.
2. Ask pupils to work individually to list statements in the correct order.
3. If they finish, ask them to compare answers and discuss with seatmates.
4. Ask different pupils to volunteer to give the answers in order from mostly likely to least likely and explain.
   - Eating a chocolate biscuit is most likely with the highest probability $-\frac{12}{20}=\frac{3}{5}$
   - Eating a biscuit that is not chocolate is next most likely with the probability $-\frac{8}{20}=\frac{2}{5}$
   - Eating a vanilla biscuit is next likely with the probability $-\frac{5}{20}=\frac{1}{5}$
   - Eating a ginger biscuit is impossible with the probability $-\frac{0}{20}=0$

Closing *(2 minutes)*

1. Ask pupils the following questions and allow them to explain in their own words:
   a. What is a likely event? (Example answer: An event that will probably occur, but it is not certain.)
   b. What is an unlikely event? (Example answer: An event that will probably not occur, but it is not impossible.)
   c. How can you tell if 1 probability is higher than another? (Answer: A higher probability will have a higher fraction or percentage.)

Lesson Title: Probability Experiments with 2 Independent Events	**Theme**: Statistics and Probability	
Lesson Number: M-09-135	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Conduct simple probability experiments with 2 independent events. 2. Identify that if 2 events are independent, the outcome of 1 does not affect the outcome of the other.	None	None

Opening *(3 minutes)*

1. Ask pupils the following questions to review probability experiments.
   a. What is an experiment in probability? (Answer: An experiment is a situation involving chance. An experiment leads to results called outcomes.)
   b. What is an outcome in probability? (Answer: An outcome is a single result of an experiment.)
   c. What is an event in probability? (Answer: The outcomes of an experiment can also be called events. Events can involve 1 outcome or more than 1 outcome.)
2. Allow pupils to discuss answers and volunteer to share their ideas.
3. **Say:** Today, we will learn about probability experiments with 2 independent events.

Introduction to the New Material *(11 minutes)*

1. **Ask:** What does the word 'independent' mean?
2. Allow pupils to volunteer their answers. (Example answer: It means when something happens on its own, without anything else causing it.)
3. **Say**: In probability, independent means that the outcome of 1 event does not affect the outcome of the other event. We have discussed several types of probability experiments. Let us say I have a die and a bag with red and blue balls in it.
4. **Ask**: If I roll a 2, does that mean that I will draw a red ball from the bag? (Answer: No)
5. Let pupils raise their hands to answer.
6. **Say**: Those 2 events are independent. The outcome of rolling a die does not affect the outcome of drawing a ball from a bag at random. Let us consider a die with 6 sides (or 6 squares of paper numbered 1-6). I can toss a die as an experiment. There is a chance that the die will land on any number between 1 and 6.
7. **Ask**: What are the possible outcomes of my experiment?
8. Allow pupils to volunteer to answer.
9. **Say**: There are 6 possible outcomes. I could roll a 1, 2, 3, 4, 5, or 6. Those are the only possible outcomes for my experiment of rolling a die.
10. Do the experiment once. Roll the die and write the number on the board. (Example on the board: Outcome: 3)
11. **Ask**: If I roll the die a second time, what are the possible outcomes of my experiment?
12. Allow pupils to discuss their ideas. Have a pupil volunteer to share.

13. **Say**: There are 6 possible outcomes. I could roll a 1, 2, 3, 4, 5, or 6.
14. **Ask**: Since I rolled a 3 before, what is the probability that I will roll a 3 now? (Answer: $\frac{1}{6}$)
15. **Say**: Each roll is an independent event. The outcome of 1 event does not affect the outcome of the next event. Even though I rolled a 3 before, the probability that I will roll 1 again is the same.

Guided Practice *(9 minutes)*

1. **Write** on the board:
   Experiment: Selecting 2 pupils at random from a group
   Outcomes:
2. Ask pupils to work with the2-3 pupils sitting beside them.
3. **Say**: You will make your own experiment. Count the number of seatmates or pupils in your group and tear that number of small pieces of paper, from 1 page of your exercise book or any other sheet. You can share the pieces for the whole group. I want you to write the name of each member in your group on them. All members of the group should have their name on 1 piece of paper.
4. **Say**: Using your papers, I want you to conduct the experiment with your group. Fold the pieces of paper and put them in a pile. Everyone should take turns to randomly choose 1 name, put it back, and choose another name. Write down the outcomes of your experiment in a list. Each pupil should draw a pair of names 2 times.
5. Remind pupils that each time they draw it is 1 event. Make sure they are returning the name after each draw.
6. Move around the room and check for understanding. Clear any misconceptions. Make sure each pupil is involved in the experiment. Make sure pupils are writing the list of outcomes correctly.
7. **Say**: There should be 2 outcomes for each pupils turn. (Example outcomes: depending on the pupils in the group: Momo, Gbessay; Gbessay, Maima; Momo, Momo; Gbessay, Momo, etc.)
8. Ask a group to volunteer to stand and explain their experiment to the class. Ask them to give the outcomes of their experiment. (Example: In our experiment, we wrote our names on the sheets and the outcomes were Momo, Gbessay; Gbessay, Maima; Momo, Momo; Gbessay, Momo.)

Independent Practice *(10 minutes)*

1. **Write** on the board:
   Experiment: Selecting 2 pieces of paper from 2 different piles
   Outcomes:
2. Ask pupils to work independently.
3. **Say**: I want you to make your own experiment. Tear 10 small pieces of paper, from 1 page of your exercise book or any other sheet. I want you to write on them. You can write letters, numbers, or words, such as the number 1-10 or letters A- J. All 10 pieces of paper should be different. (If pupils have made these pieces before, they are free to reuse, but most pupils will not have made them because it was a group activity before.)
   Using your papers, I want you to conduct the experiment. Fold the pieces of paper and put them in 2 piles of 5 each. Randomly choose a piece of paper from each pile. Write down the outcomes of your experiment in a list. Conduct the experiment at least 5 times.
4. Remind pupils that each time they draw it is 1 event.

5. Move around the room and check for understanding, clear any misconceptions. Make sure each pupil is conducting his/her own experiment. Make sure pupils are writing the list of outcomes correctly.
6. **Say**: There should be 2 outcomes for each turn. (Example outcomes: depending on what pupils wrote on their papers: 1 and 7; 2 and 7; 1 and 6; 3 and 9; 4 and 8)
7. Have a pupil volunteer to explain his/her experiment to the class. Ask them to give the outcomes of their experiment. (Example: In my experiment, I wrote numbers on the 10 pieces of paper and made 2 piles. The outcomes of my experiment were 1 and 7; 2 and 7; 1 and 6; 3 and 9; 4 and 8)

Closing *(2 minutes)*

1. Ask pupils the following questions and allow them to volunteer to explain in their own words:
   a. What is an experiment? (Example answer: a situation involving chance or probability that leads to results, or outcomes)
   b. What is an outcome? (Example answer: It is a single result of an experiment.)
   c. What is an event? (Example answer: It is 1 or more outcomes of an experiment.)
   d. What is an independent event? (Example answer: The outcome of 1 event does not affect the outcome of the other event.)
2. Have 4 pupils volunteer to share their answers.

Lesson Title: Probability Experiments Independent Events I	**Theme**: Statistics and Probability	
Lesson Number: M-09-136	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Solve simple probability problems with 2 independent events. 2. Interpret the word 'and' in probability problems as multiplication.	None	Write the problems from the Opening, Introduction to the New Material, Guided Practice, and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils to multiply the following fractions as review:
   a. $\frac{1}{4} \times \frac{3}{4}$
   b. $\frac{2}{5} \times \frac{1}{3}$
   c. $\frac{3}{5} \times \frac{2}{3}$
2. Have 3 pupils volunteer to answer the questions. (Answers: (i) $\frac{3}{16}$; (ii) $\frac{2}{15}$; (iii) $\frac{6}{15}$)
3. **Say:** Today, we will learn how to solve simple probability problems with 2 independent events, and interpret the word 'and' in probability problems.

Introduction to the New Material *(14 minutes)*

1. **Ask:** What does the word independent mean in probability? (Answer: In probability, independent means that the outcome of 1 event does not affect the outcome of the other event.)
2. Have 1-2 pupils volunteer to share their answers.
3. **Write** the following question on the board:
   I have 2 dice. What is the probability I will roll a 3 with both dice?
4. **Ask**: Are these 2 events independent? (Answer: Yes)
5. Let pupils raise their hands to answer this and the next 3 questions.
6. **Ask**: How do we calculate probability? (Answer: $probability = \frac{the\ number\ of\ ways\ an\ event\ can\ occur}{total\ number\ of\ possible\ outcomes}$)
7. **Ask**: What is the probability of rolling a 3 on 1 die? (Answer: $probability = \frac{1}{6}$)
8. Remind pupils of the question: What is the probability I will roll a 3 with both dice?
9. **Ask**: What is the probability of rolling a 3 on the other die? (Answer: $probability = \frac{1}{6}$)
10. Guide the pupils to understand they each have the same probability because they are independent of each other.
11. **Say**: If we have 2 independent events and we want to calculate the probability of both of them happening, a 3 on 1 die *and* a 3 on the other die, we must multiply the 2 probabilities.
12. **Write** on the board:
    $$Probability\ of\ 2\ independent\ events = Probability\ Event\ A \times Probability\ Event\ B$$
    $$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

13. **Say**: There is a 1 in 36 chance that I will roll a 3 on both dice. To calculate the probability of 1 independent event *and* another independent event happening, we multiply the probability of the first and the second.
14. Point to the following question on the board: There are 2 bags; Bag A has 6 blue balls and 2 red balls, Bag B has 4 blue balls and 4 red balls. What is the probability I will randomly draw a blue ball from both bags?
15. Give pupils 1 minute to work on this question. Then let them raise their hands to answer the next questions.
16. **Ask**: How do we calculate the probability of independent events? (Answer: $probability\ 2\ independent\ events = \ Probability\ Event\ A \times Probability\ Event\ B$)
17. **Ask**: What is the probability of drawing a blue ball from Bag A? (Answer: $probability = \frac{6}{8} = \frac{3}{4}$)
18. **Ask**: What is the probability of drawing a blue ball from Bag B? (Answer: $probability = \frac{4}{8} = \frac{1}{2}$)
19. **Say**: If we have 2 separate events and we want to calculate the probability of both of them happening, a blue ball from Bag A and Bag B, we must multiply the 2 probabilities.
20. **Write** on the board:
    $probability = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$
21. **Say**: There is a 3 in 8 chance that I will draw a blue ball from both bags.

Guided Practice *(6 minutes)*

1. Point to the following question on the board: I have 2 dice. What is the probability I will roll a 1 with 1 die and a 6 with the other die?
2. Ask pupils to work with their partners.
3. Walk around to check for understanding and clear misconceptions.
4. Ask a pair to volunteer to come to the board and solve the problem, while the rest of the class checks their work in their exercise books.
   (Answers: $probability = probability\ for\ 1\ \times\ probability\ for\ 6 = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$)
5. Correct any errors in the calculation on the board. Ask pupils to check their work.

Independent Practice *(10 minutes)*

1. Point to the following question on the board:
   There are 2 cups. Cup A has 8 pens and 2 pencils; Cup B has 4 pens and 6 pencils. If 1 is selected at random, what is the probability of the following events?
   a. A pen will be selected from Cup A and a pencil from Cup B?
   b. Pens will be selected from both cups?
   c. Pencils will be selected from both cups?
2. Ask pupils to work independently in their exercise books.
3. Walk around to check for understanding, clear misconceptions.
4. Have a pupil volunteer to come to the board and solve the problem, while the rest of the class checks their work in their exercise books.
   (Answers: (a) $selecting\ a\ pen\ from\ cup\ A = \frac{8}{10} = \ \frac{4}{5}; selecting\ a\ pencil\ from\ cup\ B = \frac{6}{10} = \frac{3}{5};\ \frac{4}{5} \times \frac{3}{5} = \frac{12}{25}\ for\ both.$ (b) $selecting\ a\ pen\ from\ cup\ A = \frac{8}{10} =$

$\frac{4}{5}$; $selecting\ a\ pen\ from\ cup\ B = \frac{4}{10} = \frac{2}{5}$; $\frac{4}{5} \times \frac{2}{5} = \frac{8}{25}\ for\ both.$;

(c) $selecting\ a\ pencil\ from\ cup\ A = \frac{2}{10} = \frac{1}{5}$; $selecting\ a\ pencil\ from\ cup\ B = \frac{6}{10} = \frac{3}{5}$; $\frac{1}{5} \times \frac{3}{5} = \frac{3}{25}\ for\ both.$

Closing *(2 minutes)*

1. Ask pupils the following questions and allow them to explain in their own words:
   (i) If you are looking for the probability of 2 independent events occurring, what are the steps? (Example answer: Find the probability of each individually and multiply)
   (ii) Why do you multiply 2 probabilities? (Example answer: since we want both events to happen it is less likely than just 1. Multiplying fractions gives us smaller fractions, and therefore lower probability.)
2. Allow pupils to discuss with their partners
3. Have 1-2 pairs volunteer to share their ideas with the class.

Lesson Title: Probability Experiments Independent Events II	Theme: Statistics and Probability	
Lesson Number: M-09-137	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to: 1. Identify whether 2 given events are independent or dependent. 2. Solve more difficult probability problems with 2 independent events.	None	Write the questions from Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils the following questions to review.
   a. What is an independent event in probability? (Answer: The outcome of 1 event does not affect the outcome of the other event.)
   b. Give an example of an independent event. (Example answer: rolling 2 dice)
2. Allow pupils to volunteer to share their answers.
3. **Say:** Today, we will learn to identify whether events are independent or dependent, and we will practise solving probability problems.

Introduction to the New Material *(11 minutes)*

1. **Write** the following question on the board:
   I have 1 bag with 4 oranges and 6 mangos. What is the probability that I will randomly select a mango to eat?
2. **Ask:** How many pieces of fruit are in my bag at the beginning of this problem? (Answer: 10)
3. Let pupils raise their hands to answer.
4. **Ask:** What is the probability that I will randomly select a mango? (Answer:$\frac{6}{10} = \frac{3}{5}$)
5. Let pupils raise their hands to answer.
6. **Say**: Now the outcome of my random selection was a mango, and I ate it. Now I want to eat another piece of fruit.
7. **Ask:** After I eat the first mango, how many pieces of fruit are in my bag then? (Answer: 9)
8. Invite pupils to raise their hands to answer this question and for the next 2 questions as well.
9. **Ask**: What are the possible outcomes if I randomly select a piece of fruit from the bag now? (Answers: 4 oranges and 5 mangos)
10. **Ask:** Now, what is the probability that I will randomly select a mango? (Answer:$\frac{5}{9}$)
11. **Say**: The probabilities we have been looking at so far have been for independent events.
12. Remind pupils that independent events meant that the outcome of 1 event does not affect the other event.
13. **Say**: This question is an example of a 'dependent' event.
14. Ask pupils to discuss the meaning of dependent events in probability with their partners. Allow them to discuss until they reach the conclusion that dependent events are affected by other events.
15. **Say**: Dependent events are the opposite of independent. The outcome of a dependent event is affected by the outcome of the event it is dependent on.

16. **Ask**: In this example, did my probability of eating a mango for the second piece of fruit increase or decrease? Why? (Answer: decrease, there were less mangos as possible outcomes in the bag, so the likelihood of randomly selecting 1 decreased)
17. **Write** another question on the board:
    There is a cup with 4 blue pens and 1 black pen. What is the probability that the first pen Momo randomly selects will be black?
18. Have a pupil volunteer to share the answer: $\frac{1}{5}$
19. **Say**: Let us say that Momo takes his pen and leaves.
20. **Write** on the board:
    Jebbeh now randomly selects a second pen. What is the probability that Jebbeh will select a black pen?
21. Ask pupils to discuss, guide them to the conclusion that the probability depends on what color pen Momo took.
22. **Say**: If Momo selected a black pen, then the probability that Jebbeh will select a black pen is $\frac{0}{4}$ or 0. However, if Momo selected a blue pen, then the probability that Jebbeh will select a black pen is $\frac{1}{4}$.
23. **Say**: The probability of the second outcome depends on the first outcome.

Guided Practice *(9 minutes)*

1. Write the following question on the board:
    Identify whether the following events are independent or dependent:
    a. Rolling 2 dice and getting 5s on both of them.
    b. Randomly selecting a red ball out of a bag, and then holding it while randomly selecting another ball.
    c. Picking 1 colored pen each at random from 2 different cups.
    d. Selecting 1 pupil at random from the front row, and 1 from the back row.
    e. Asking 1 friend to draw a card at random from a deck of cards and hold it while you have another friend draw another card at random from the deck.
2. Ask pupils to work with their partners.
3. Walk around to check for understanding, clear misconceptions.
4. Have 5 pairs volunteer to share their answers while the rest of the class checks their work in their exercise books. (Answers: (a) independent; (b) dependent; (c) independent; (d) independent; (e) dependent)

Independent Practice *(10 minutes)*

1. Write the following problem on the board and ask pupils to answer the questions:
    The commissioner wants to give a prize at random. She has 2 groups of 5 people each. She selects 1 finalist at random from each group. Then she randomly selects the final winner from between the 2 finalists.
    a. Is this problem describing independent events or dependent events?
    b. What is the probability of being selected as a finalist from 1 of the groups?
    c. What is the probability of being selected as the final winner?
2. Ask pupils to work independently in their exercise books.

3. Walk around to check for understanding and clear misconceptions.
4. Have 1 pupil volunteer to come to the board and solve the problem, while the rest of the class checks their work in their exercise books. (Answers: (a) independent; (b) $\frac{1}{5}$; (c) $\frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$)
5. Correct any errors in the calculation on the board. Ask pupils to check their work.

Closing *(2 minutes)*

1. Ask pupils to think of 1 example of 2 independent events and 1 example of dependent events with their partners.
2. Give pupils 1 minute to do so.
3. Have 2 pairs volunteer to share their ideas with the class. (Example answers: The examples they come up with can be modeled after the class examples using dice, cards, and bags of fruit or balls.)

Lesson Title: Sample Space	**Theme**: Statistics and Probability	
Lesson Number: M-09-138	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify that the 'sample space' of an experiment is the set of all possible outcomes 2. Record the possible outcomes of an experiment in a sample space diagram.	None	Draw the tables from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils to define the following words:
   a. Independent event
   b. Dependent event
2. Allow pupils to volunteer to share their answers. (Answers: (a) The outcome of 1 event does not affect the outcome of the other event; (b) The outcome of 1 event is affected by the outcome of another event.)
3. **Say:** Today, we will learn to about sample space and how to record possible outcomes in a sample space.

Introduction to the New Material *(12 minutes)*

1. **Ask:** What are the possible outcomes to rolling a die? (Answer: 1, 2, 3, 4, 5, or 6)
2. **Write** on the board: $\{1, 2, 3, 4, 5, 6\}$
3. **Say**: We know the possible outcomes of rolling a die, all the possible outcomes are call the 'sample space'. For more complicated probabilities, it can be useful to record all of the possible outcomes in a sample space diagram.
4. **Write** the following problem on the board:
   Find the sample space of selecting 2 balls at random from 2 different bags, each containing 3 blue and 2 red balls. Create a sample space diagram.

5. **Say**: A sample space diagram is like a special table.
6. Point to the following empty diagram on the board:
7. **Say**: On the left side we list all the possible outcomes of the first event. On the top we list all the possible outcomes of the second event.

		Bag B				
Bag A		**Blue**	**Blue**	**Blue**	**Red**	**Red**
	Blue					
	Blue					
	Blue					
	Red					
	Red					

8. Point to the first empty row.
9. **Say**: We could select a blue ball from Bag A and then a blue, blue, blue, red or red ball from Bag B. Let us use the letters B and R to represent each of the colors.
10. Write the combinations: B, B; B, B; etc. as you are speaking.
11. Continue going through the rows asking the pupils to help you fill out each of the boxes of the diagram as below.

		Bag B				
		Blue	**Blue**	**Blue**	**Red**	**Red**
Bag A	**Blue**	B, B	B, B	B, B	B, R	B, R
	Blue	B, B	B, B	B, B	B, R	B, R
	Blue	B, B	B, B	B, B	B, R	B, R
	Red	R, B	R, B	R, B	R, R	R, R
	Red	R, B	R, B	R, B	R, R	R, R

12. **Say**: This sample space diagram shows us all the possible combinations of outcome we could end up with.
13. **Ask**: How many possible combinations of outcomes are there? (Answer: 25)
14. **Write** on the board:
15. $\begin{Bmatrix} (B,B)(B,B)(B,B)(B,R)(B,R)(B,B)(B,B)(B,B)(B,R)(B,R)(B,B)(B,B)(B,B)(B,R)(B,R)(R,B)(R,B) \\ (R,B)(R,R)(R,R)(R,B)(R,B)(R,B)(R,R)(R,R) \end{Bmatrix}$
16. **Say**: We can write the sample space as a set of all the possible outcomes. When it is a set we put it inside these brackets: { }

Guided Practice *(8 minutes)*

1. **Write** the following question on the board:
   What is the sample space of rolling 2 dice? Draw a sample space diagram.
2. Ask pupils to work in pairs.
3. Walk around to check for understanding, clear misconceptions.
4. Have 1 pair volunteer to share their answers while the rest of the class checks their work in their exercise books.
   Answer: There are 36 possible options:

$$\begin{Bmatrix} (1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)(4,1) \\ (4,2)(4,3)(4,4)(4,5)(4,6)(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)(6,1)(6,2)(6,3)(6,4)(6,5)(6,6) \end{Bmatrix}$$

		Second Roll					
		1	**2**	**3**	**4**	**5**	**6**
First Roll	**1**	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Independent Practice *(10 minutes)*

1. **Write** the following problem on the board and ask pupils to answer the questions:
   What is the sample space of rolling a die and selecting 1 ball from a bag of 4 red and 2 yellow balls? Draw a sample space diagram.
2. Ask pupils to work independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.

4. Invite 2 pupils who volunteer to come to the board and share their answers. Ask 1 to draw the diagram while the other lists the set. The rest of the class should check their work in their exercise books.

Answers: There are 36 possible options:

$$\left\{\begin{matrix} (1,\mathrm{R})(1,\mathrm{R})(1,\mathrm{R})(1,\mathrm{R})(1,\mathrm{Y})(1,\mathrm{Y})(2,\mathrm{R})(2,\mathrm{R})(2,\mathrm{R})(2,\mathrm{R})(2,\mathrm{Y})(2,\mathrm{Y}) \\ (3,\mathrm{R})(3,\mathrm{R})(3,\mathrm{R})(3,\mathrm{R})(3,\mathrm{Y})(3,\mathrm{Y})(4,\mathrm{R}) \\ (4,\mathrm{R})(4,\mathrm{R})(4,\mathrm{R})(4,\mathrm{Y})(4,\mathrm{Y})(5,\mathrm{R})(5,\mathrm{R})(5,\mathrm{R})(5,\mathrm{R})(5,\mathrm{Y})(5,\mathrm{Y})(6,\mathrm{R})(6,\mathrm{R})(6,\mathrm{R}) \\ (6,\mathrm{R})(6,\mathrm{Y})(6,\mathrm{Y}) \end{matrix}\right\}$$

		Draw a ball					
		Red	**Red**	**Red**	**Red**	**Yellow**	**Yellow**
Roll a Die	**1**	1, R	1, R	1, R	1, R	1, Y	1, Y
	2	2, R	2, R	2, R	2, R	2, Y	2, Y
	3	3, R	3, R	3, R	3, R	3, Y	3, Y
	4	4, R	4, R	4, R	4, R	4, Y	4, Y
	5	5, R	5, R	5, R	5, R	5, Y	5, Y
	6	6, R	6, R	6, R	6, R	6, Y	6, Y

Closing *(2 minutes)*

1. Ask pupils the following questions:
   a. What is sample space? (Answer: the 'sample space' of an experiment is the set of all possible outcomes)
   b. What is the sample space of randomly selecting 1 pupil from the front row? (Example answer: Pupils should name all the students sitting in the front row.)
2. Allow pupils to discuss with their partners.
3. Have 1-2 pupils volunteer to share their ideas with the class.

Lesson Title: Probability Trees	**Theme**: Statistics and Probability	
Lesson Number: M-09-139	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to use a probability tree to demonstrate the probability of different outcomes occurring.	None	Write the questions from the Guided Practice and Independent Practice sections of the board.

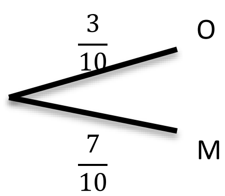
Opening *(3 minutes)*

1. Ask pupils the following questions and have them volunteer to answer:
   a. Define sample space.
   b. What is the sample space of rolling 1 die?

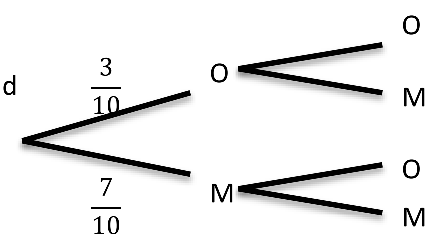
   (Answers: (i) the 'sample space' of an experiment is the set of all possible outcomes; (ii) $\{1, 2, 3, 4, 5, 6\}$)
2. **Say:** Today, we will learn to about probability trees and how to demonstrate the probability of different outcomes occurring.

Introduction to the New Material *(11 minutes)*

1. **Write** the following question on the board:
   There is a bag with 3 oranges and 7 mangoes in it. Abel will select 1 at random, replace it and select another. Draw a probability tree to demonstrate the probability of different outcomes occurring.
2. **Say**: A probability tree is a type of diagram to help up understand the possible outcomes.
3. **Ask**: If Abel randomly selects a piece of fruit, what are the options? (Answer: an orange or a mango)
4. **Say**: There are 2 options, so we must draw 2 branches.
5. **Write** the first 2 branches on the board, put an O for orange at the end of 1, and M for mango at the end of the other, as so:

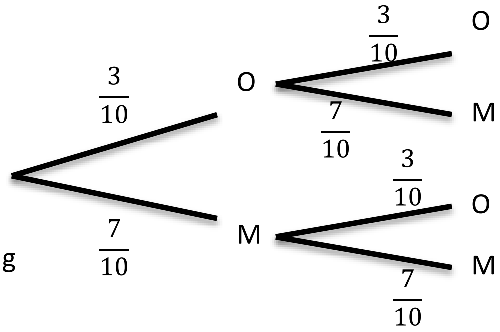


6. **Ask**: What is the probability that Abel will select an orange? (Answer: $\frac{3}{10}$)
7. **Write** the probability over the line.
8. **Ask**: What is the probability that Abel will select a mango? (Answer: $\frac{7}{10}$)
9. Write the probability under the line.
10. **Say**: This shows the probability for the first selection. Now let us show the next selection.
11. Write the next 2 branches on the board, coming from the O, put an O for orange at the end of 1, and M for mango at the end of the other, as so:

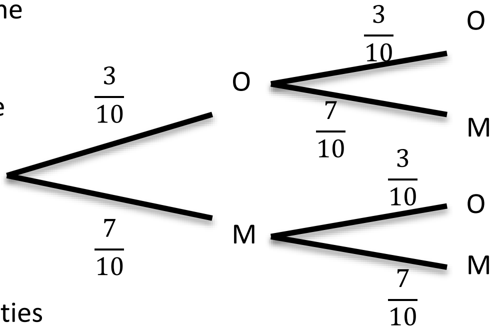


12. **Say**: Now let us write the probability for the next branches.
13. **Ask**: What is the probability that Abel will select an orange the second time? (Answer: $\frac{3}{10}$)
14. Write the probability over the line.
15. **Ask**: What is the probability that Abel will select a mango the second time? (Answer: $\frac{7}{10}$)

16. Write the probability under the line.
17. Repeat until all the probabilities are filled in as so:



18. **Say**: Now we can identify the outcomes and calculate the percentages.
19. **Ask**: What is the outcome of the top branch? (Answer: OO)
20. Point to the board and show pupils that the first choice can lead to O and then the second choice can lead to O by pointing to the top branches of both events.
21. **Say**: We know that if we want to calculate the probability of 1 event *and* another, it means that we multiply.
22. **Write** the multiplication on the board: $\frac{\mathbf{3}}{\mathbf{10}} \times \frac{\mathbf{3}}{\mathbf{10}} =$
23. Ask pupils to calculate the answer in their exercise books.
24. Have 1 pupil volunteer to give the answer: $\frac{\mathbf{9}}{\mathbf{100}}$
25. Continue moving through all the branches of the tree until you have identified all the outcomes and calculated the probabilities as so:



= OO $\frac{\mathbf{3}}{\mathbf{10}} \times \frac{\mathbf{3}}{\mathbf{10}} = \frac{\mathbf{9}}{\mathbf{100}}$

= OM $\frac{\mathbf{3}}{\mathbf{10}} \times \frac{\mathbf{7}}{\mathbf{10}} = \frac{\mathbf{21}}{\mathbf{100}}$

= MO $\frac{\mathbf{3}}{\mathbf{10}} \times \frac{\mathbf{3}}{\mathbf{10}} = \frac{\mathbf{21}}{\mathbf{100}}$

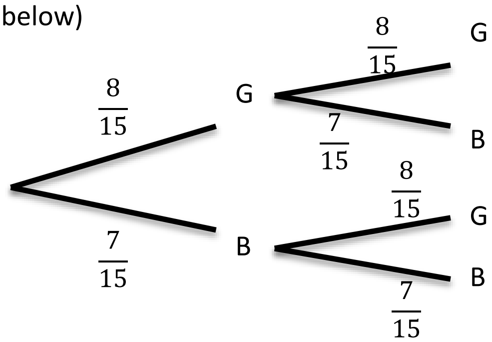
= MM $\frac{\mathbf{7}}{\mathbf{10}} \times \frac{\mathbf{7}}{\mathbf{10}} = \frac{\mathbf{49}}{\mathbf{100}}$

26. **Say**: We can check our probabilities because they should always add up to 100% or 1.
27. Ask pupils to calculate the sum of the probabilities at the end of the probability tree in their exercise books.
28. Have 1 pupil volunteer to give the answer: $\frac{9}{100} + \frac{21}{100} + \frac{21}{100} + \frac{49}{100} = \frac{100}{100}$

Guided Practice *(9 minutes)*

1. Point to the following question on the board:

   There is a class with 14 boys and 16 girls in it. 1 pupil is selected at random to answer a question. Then another is selected, but the teacher may select anyone, even the pupil who just answered. Draw a probability tree to demonstrate the probability of different outcomes occurring of selecting a boy or girl pupil.
2. Ask pupils to work with their seatmates.
3. Remind pupils to simplify their fractions.
4. Walk around to check for understanding, clear misconceptions.
5. Have a pair share their probability tree while the rest of the class checks their work in their exercise books. (Answer: below)



= GG $\frac{\mathbf{8}}{\mathbf{15}} \times \frac{\mathbf{8}}{\mathbf{15}} = \frac{\mathbf{64}}{\mathbf{225}}$

= GB $\frac{\mathbf{8}}{\mathbf{15}} \times \frac{\mathbf{7}}{\mathbf{15}} = \frac{\mathbf{56}}{\mathbf{225}}$

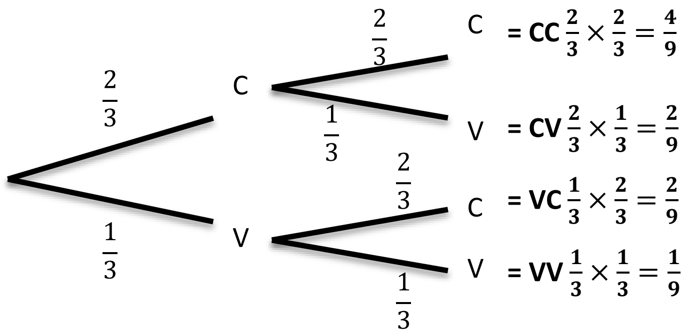
= BG $\frac{\mathbf{7}}{\mathbf{15}} \times \frac{\mathbf{8}}{\mathbf{15}} = \frac{\mathbf{56}}{\mathbf{225}}$

= BB $\frac{\mathbf{7}}{\mathbf{15}} \times \frac{\mathbf{7}}{\mathbf{15}} = \frac{\mathbf{49}}{\mathbf{225}}$

Independent Practice *(10 minutes)*

1. Point to the following question on the board:

   There is a box with 8 chocolate biscuits and 4 vanilla biscuits. 1 biscuit is selected at random, returned and another biscuit is selected. Draw a probability tree to demonstrate the probability of different outcomes occurring of selecting a vanilla or chocolate biscuit.
2. Ask pupils to work independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Have 1 pupil volunteer to come to the board and share their probability tree, while the rest of the class checks their work in their exercise books. (Answers: below)
5. **Write** the correct answers and steps on the board. Ask pupils to check their work.



Closing *(2 minutes)*

1. Ask pupils the following questions about the biscuit scenario above to check for understanding.
   a. What combination of outcomes is most likely?
   b. What is the probability of selecting 1 chocolate and 1 vanilla biscuit?
   c. What combination of outcomes is the least likely?
2. Allow pupils to discuss with their partners.
3. Have 1-2 pairs volunteer to share their ideas with the class. (Answers: (a) Chocolate, Chocolate, it has the highest probability; (b) $\frac{2}{9} + \frac{2}{9} = \frac{4}{9}$; (c) Vanilla, Vanilla, it has the lowest probabiility)

Lesson Title: Probability Story Problems	**Theme**: Statistics and Probability	
Lesson Number: M-09-140	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to solve story problems involving probability of an event happening.	None	Write the problems from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils to respond to the following questions in their own words:
   a. What does a probability of zero mean?
   b. What does a probability of 1 mean?
   c. What is an independent event?
2. Have 3 pupils volunteer to share their answers. (Answers: (a) impossible; (b) certain; (c) when the outcome is not affected by the outcome of another event)
3. **Say**: Today, we will practise solving story problems involving probability.

Introduction to the New Material *(12 minutes)*

1. Point to this problem on the board:
   There are 20 people applying to a job. 12 of them are female and the rest of them are male. The order for interviews is decided by random selection. The first interviewee is a female.
   a. What was the probability that a female would be selected first?
   b. What was the probability that a male would be selected first?
   c. What is the probability that a female will be selected second?
   d. What is the probability that a male will be selected second?
2. **Ask**: Is the first selection an independent or dependent event? (Answer: independent)
3. **Ask**: How do you find the answer to (a)? (Answer: Find the fraction of female interviewees over the total number of interviewees.)
4. Let pupils volunteer to answer.
5. Ask pupils to calculate the probability in their exercise books.
6. Have 1 pupil volunteer to share the answer: $\frac{12}{20} = \frac{3}{5}$
7. **Say**: There is a 3 in 5 chance that the first interviewee selected will be female.
8. **Ask**: How do you find the answer to (b)? (Answer: Find the fraction of male interviewees over the total number of interviewees.)
9. Ask pupils to calculate the probability in their exercise books.
10. Have a pupil volunteer to share the answer: $\frac{8}{20} = \frac{2}{5}$
11. **Say**: There is a 2 in 5 chance that the first interviewee selected will be male. Now, we know the first interviewee selected was female.
12. **Ask**: Is the second selection an independent or dependent event? (Answer: dependent)
13. **Say**: We know the second selection is dependent because the probabilities of being selected second are affected by the outcome of the first selection.
14. **Ask**: What is the probability the second person selected will be female?
15. Ask pupils to calculate the probability in their exercise books.

16. Have a pupil volunteer to share the answer: $\frac{11}{19}$
17. **Ask**: What is the probability the second person selected will be male?
18. Ask pupils to calculate the probability in their exercise books.
19. Have a pupil volunteer to share the answer: $\frac{8}{19}$

Guided Practice *(7 minutes)*

1. Point to the following question on the board:
   The school is having a competition. 1 pupil each will be randomly selected from JSS 1, JSS 2, and JSS 3. There are 150 pupils in each grade. 1 winner will be randomly selected from the 3 finalists. What is the probability of the following?
   a. A pupil from JSS 3 being selected as 1 of the final 3?
   b. A pupil from JSS 3 being the winner?
2. Ask pupils to work with their partners to find the answers in their exercise books.
3. Remind pupils to simplify their fractions.
4. Move around the room to check pupils' work and clear any misconceptions.
5. Have 2 pairs volunteer to share 1 answer each. (Answers: (a) $\frac{1}{150}$; (b) $\frac{1}{150} \times \frac{1}{3} = \frac{1}{450}$)

Independent Practice *(10 minutes)*

1. Ask pupils to work independently.
2. Point to the following question on the board:
   Joe has 5 potatoes, 8 mangos, and 7 oranges in a bag. If he randomly selects 1 from the bag, what is the probability of selecting the following?
   a. A mango
   b. A piece of fruit
   c. A potato
   d. Anything but an orange.
3. Ask pupils to do the work in their exercise books.
4. Move around the room to check for understanding and clear any misconceptions.
5. Ask 4 pupils to volunteer to stand and each give 1 of the 4 answers. (Answers: (a) $\frac{8}{20} = \frac{2}{5}$; (b) $\frac{8}{20} + \frac{7}{20} = \frac{15}{20} = \frac{3}{4}$; (c) $\frac{5}{20} = \frac{1}{4}$; (d) $\frac{8}{20} + \frac{5}{20} = \frac{13}{20}$)
6. **Write** the correct answers and steps on the board. Ask pupils to check their work.

Closing *(3 minutes)*

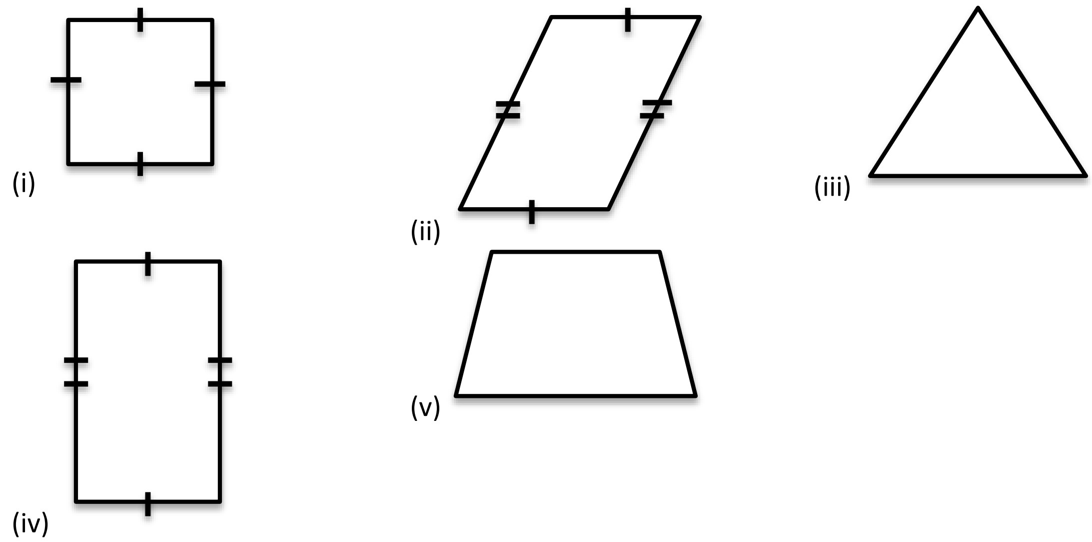
1. Ask the following questions to check for understanding of the question above.
   a. Is it more likely that Joe will select a mango or potato?
   b. Is it more likely that Joe will select a potato or an orange?
   c. If Joe wants to eat a mango, what are his chances of getting 1 on the first selection?
2. Have pupils volunteer to share their answers. (Answers: (a) a mango; (b) $\frac{8}{20} = \frac{2}{5}$; (c) an orange)

Lesson Title: Perimeter of Triangles and Quadrilaterals	Theme: Measurement and Estimation	
Lesson Number: M-09-141	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes: By the end of the lesson, pupils will be able to find the perimeter of a triangle and quadrilateral.	**Teaching Aids** None	**Preparation** 1. Draw the shapes from the Opening, New Material, Guided Practice and Independent Practice on the board. 2. Draw the tables from Independent Practice on the board.

Opening *(3 minutes)*

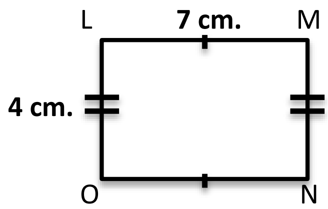
1. Ask pupils to identify the following plane shapes:



2. Have 5 pupils volunteer to answer the questions. (Answers: (i) square; (ii) parallelogram; (iii) triangle; (iv) rectangle; (v) trapezium)
3. Remind pupils that the small lines show the sides of the shape are equal lengths.
4. **Say:** Today, we will be practising finding the perimeter of triangles and quadrilaterals.

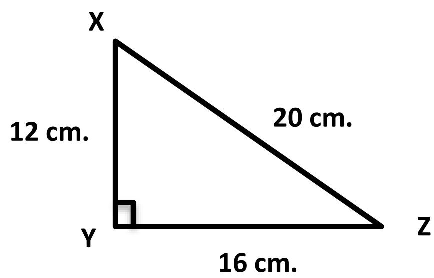
Introduction to the New Material *(15 minutes)*

1. **Ask**: How do calculate the perimeter of any plane shape? I will show you today. (Answer: Add up the length of the outside edges of the shape.)
2. Point to the rectangle LMNO on the board:



3. **Ask**: What characteristics do we know about this shape? (Answer: It is a quadrilateral, parallelogram. That means it has 4 sides and both sets of sides are parallel and equal in length.)
4. Have any 1 pupil volunteer to answer.
5. **Say**: This rectangle is also a parallelogram.

6. **Ask**: What is the formula for perimeter of a parallelogram? (Answer: $Perimeter = 2l + 2w$)
7. Remind pupils that this is the same formula as for a rectangle, and for a square or a rhombus it is $Perimeter = 4l$, which is the same but simplified.
8. Ask pupils to calculate the perimeter of the rectangle, as you do so on the board, using the formula. (Answer: $P = 2l + 2w = 2 \times 7 \text{ cm.} + 2 \times 4 \text{ cm.} = 14 \text{ cm.} + 8 \text{ cm.} = 22 \text{ cm.}$)

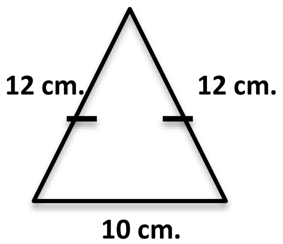


9. **Say**: The perimeter of the rectangle is 22 cm.
10. Point to the triangle XYZ on the board:
11. **Ask**: What is the formula for perimeter of a triangle? (Answer: $Perimeter = a + b + c,$ $where\ a\ and\ c\ are\ sides\ of\ the\ triangle\ and\ b\ is\ the\ base$)
12. **Write** the formula for perimeter of a triangle on the board: $Perimeter = a + b + c, where\ a\ and\ c\ are\ sides\ of\ the\ triangle\ and\ b\ is\ the\ base.$
13. **Say**: To calculate the perimeter of a triangle, you add up the 3 sides.
14. Ask pupils to calculate the perimeter of the triangle, as you do so on the board, using the formula. (Answer: $P = a + b + c = 12 \text{ cm.} + 20 \text{ cm.} + 16 \text{ cm.} = 48$ cm.)
15. **Say**: The perimeter of the triangle is 48 cm.

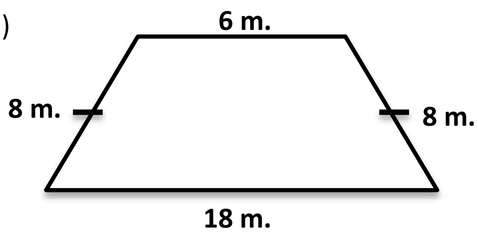
Guided Practice *(5 minutes)*

1. Point to the following plane shapes on the board:

(a)



(b)

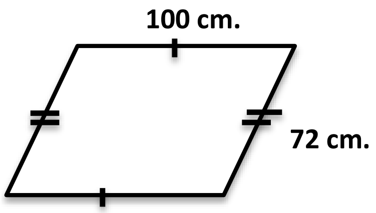


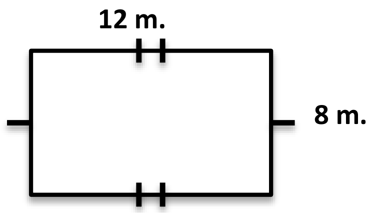
2. Ask pupils to work with their partners to find the perimeter.
3. Move around to check for understanding and clear misconceptions.
4. Remind pupils that to calculate the perimeter of any plane shape, add up the length of the outside edges of the shape.
5. Ask 2 pairs to volunteer to share their solutions on the board, while the rest of the class checks their work in their exercise books. (Answers: (i) $P = a + b + c = 12 \text{ cm.} + 12 \text{ cm.} + 10 \text{ cm.} =$ 34 cm. (ii) $P = a + b + c + d = 6 \text{ m.} + 8 \text{ m.} + 18 \text{ m.} + 8 \text{ m.} = 40 \text{ m.}$)
6. Check the work on the board for any errors. Ask pupils to check their own work.

Independent Practice *(10 minutes)*

1. Point to the following plane shapes on the board. Ask pupils to find the perimeter of each of the shapes.

(a) (b)





2. Ask pupils to work independently to find the answers.
3. Walk around to check for understanding and clear misconceptions.
4. Ask them to exchange exercise books with a partner to check their answers. Have 2 pupils volunteer to share their answers.
   Answers: (a) $P = 2l + 2w = 2 \times 100 \text{ cm.} + 2 \times 72 \text{ cm.} = 200\text{cm.} + 144 \text{ cm.} = 344 \text{ cm.}$; (b)$P = 2l + 2w = 2 \times 12 \text{ m.} + 2 \times 8 \text{ m.} = 24 \text{ m.} + 16 \text{ m.} = 40 \text{ m.}$

Closing *(2 minutes)*

1. Point to the following chart on the board. Ask pupils to copy the chart into their exercise books with enough space to add to it and fill in the boxes and formula for each of the plane shapes:

Plane Shape	**Quadrilateral**	**Parallelogram**	**Equal Sides**	**Equal Angles**	**Perimeter Formula**
Square					
Rectangle					
Rhombus					
Trapezium					
Triangle					

2. **Say**: Please keep this chart for reference. We will add more columns to it for area as well. (Answers: below)

Plane Shape	**Quadrilateral**	**Parallelogram**	**Equal Sides**	**Equal Angles**	**Perimeter Formula**
Square	Yes	Yes	Yes	Yes	P = 4l
Rectangle	Yes	Yes	No	Yes	P = 2l + 2w
Rhombus	Yes	Yes	Yes	No	P = 4l
Trapezium	Yes	No	No	No	P = a + b + c + d

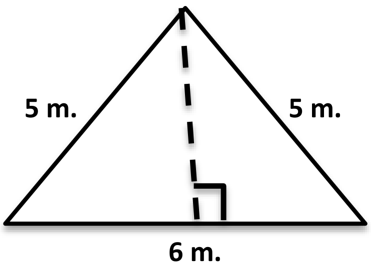
Triangle	No	No	No	No	P = a + b + c

Lesson Title: Area of Triangles	**Theme:** Measurement and Estimation	
Lesson Number: M-09-142	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes:	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to calculate the area of a triangle.	None	Draw the triangles from throughout the lesson plan on the board.

Opening *(3 minutes)*

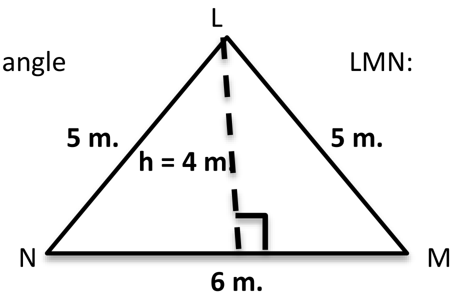
1. Point to the triangle on the board with the measurements below:



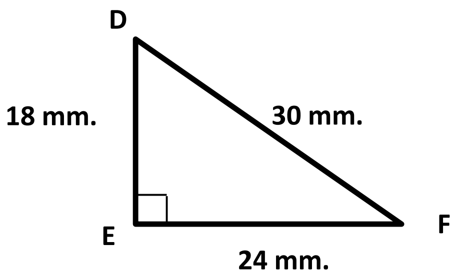
2. Ask pupils to calculate the perimeter of the triangle. (Answer: $P = a + b + c = 5\text{ m.} + 5\text{ m.} + 6\text{ m.} = 16\text{ m.}$)
3. Have any 1 pupil volunteer to share the answer.
4. **Say:** Today, we will practise calculating the area of a triangle using a formula.

Introduction to the New Material *(15 minutes)*

1. Add labels to the triangle from the opening to make it triangle LMN:

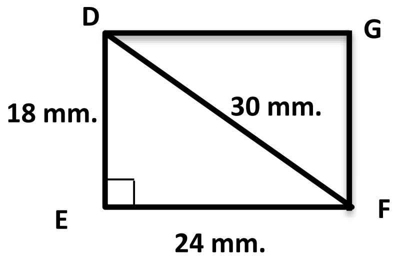


2. **Write** the formula on the board: $\text{Area of a triangle} = \frac{1}{2}\text{base} \times \text{height} = \frac{1}{2}\text{bh}$
3. **Say**: To calculate the area of a triangle, you multiply 1 half times the base and the height.
4. **Ask:** What is the base of this triangle? (Answer: Side NM, which is 6 m. in length)
5. **Ask:** What is the height of the triangle? (Answer: 4 m.)
6. Let pupils volunteer to answer.
7. **Say**: These are the 2 numbers we need to find the area of the triangle. We will substitute them in the formula. Base and height are always perpendicular to each other. You can take any side of the triangle as its base. Then you find the height of the triangle from that base. The height is a perpendicular line drawn from the base to the opposite angle of the triangle.
8. **Write** on the board: $A = \frac{1}{2}bh = \frac{1}{2} \times 6\text{ m.} \times 4\text{ m.}$
9. Solve for A by multiplying: $A = \frac{1}{2} \times 6\text{ m.} \times 4\text{ m.} = 3\text{ m.} \times 4\text{ m.} = 12\text{ m.}^2$
10. Point to triangle DEF on the board:

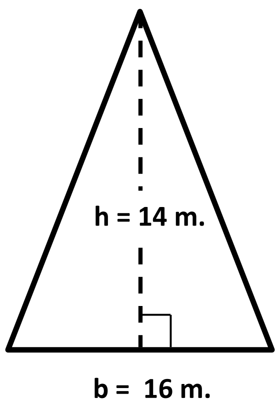


11. **Say**: Now we will calculate the area.

12. **Ask**: How long is the base? (Answer: 24 mm.)
13. **Ask**: How long is the height? (Answer: 18 mm.)
14. Allow pupils to raise their hands to answer.
15. **Say**: Notice that the height is perpendicular to the base. It reaches from the base to the opposite angle.
16. Have a pupil volunteer to come to the board and substitute the values for base and height in the formula: $A = \frac{1}{2}bh = \frac{1}{2} \times\ 24\text{ mm.} \times\ 18\text{ mm.}$
17. Have another pupil volunteer to do the multiplication while the rest of the class works in their exercise books: (Answer: $A = \frac{1}{2} \times\ 24\text{ mm.}\ \times\ 18\text{ mm.} = 12\text{ mm.}\ \times\ 18\text{ mm.} = 216\text{ mm.}^2$)
18. Discuss with pupils that the area of a triangle with known height and base is half of the area of the rectangle.
19. Remind pupils about the formula for area of rectangle: (Length x Width).
20. Make triangle DEF into a rectangle DEFG by drawing 2 lines:



21. Have a pupil volunteer to find the area of rectangle DEFG on the board while the rest of the class calculates it in their exercise books. (Answer: $A = 24\text{ mm.}\ \times\ 18\text{ mm.} = 432\text{ mm.}^2$)
22. **Say**: Notice that triangle DEF takes up half the space in rectangle DEFG. We have seen that the area of DEFG is 432 mm.^2, and the area of DEF is 216 mm.^2. This is where the $\frac{1}{2}$ in the formula for area of a triangle comes from.



Guided Practice *(5 minutes)*

1. Point to the following triangle on the board:
2. Ask pupils to work with their partners to find the area.
3. Move around to check for understanding and clear misconceptions.
4. Have 1 pair volunteer to share their solution on the board, while the rest of the class checks their work.

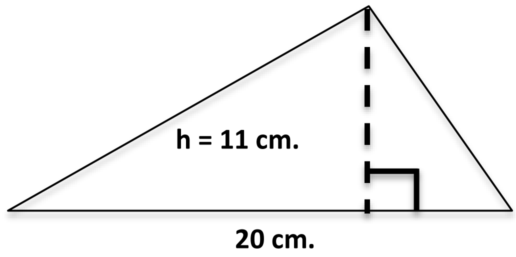
   (Answer: $\text{A} = \frac{1}{2} \times 16\text{ m.} \times\ 14\text{ m.} = 8\text{ m.}\ \times\ 14\text{ m.} = 112\text{ m.}^2$)

   5. Correct any errors in calculations on the board. Ask pupils to check their own work as well.

Independent Practice *(10 minutes)*

1. Point to the following questions on the board:
   (i) If the base of a triangle is 12 m. and the height is 7 m., what is the area of the triangle?

(ii) Find the area of the triangle below:

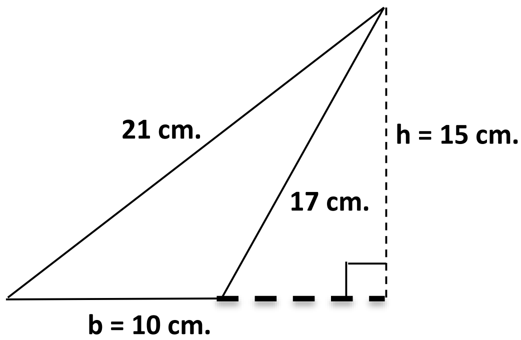


2. Ask pupils to work independently to find the answers.
3. Walk around to check for understanding and clear misconceptions.
4. Ask them to exchange exercise books with a partner to check their answers. Have 2 pupils volunteer to share their answers.

   (Answers: (i) $A = \frac{1}{2}bh \ = \frac{1}{2} \times 12 \text{ m.} \times 7 \text{ m.} = 6 \text{ m.} \times 7 \text{ m.} = 42 \text{ m.}^2$; (ii) $A = \frac{1}{2} \times 10 \text{ cm.} \times 15 \text{ cm.} = 5 \text{ cm.} \times 15 \text{ cm.} = 75 \text{ cm.}^2$)
5. **Write** the correct answers and steps on the board. Ask pupils to check their work.

Closing *(2 minutes)*

1. Draw the following triangle on the board and ask pupils to calculate the area.
2. Ask pupils to show you the answer and their work in their exercise books as they leave class.
   (Answer: $A = \frac{1}{2}bh = \frac{1}{2} \times 20 \text{ cm.} \times 11 \text{ cm.} = 10 \text{ cm.} \times 11\text{cm.} = 110 \text{ cm}^2$)

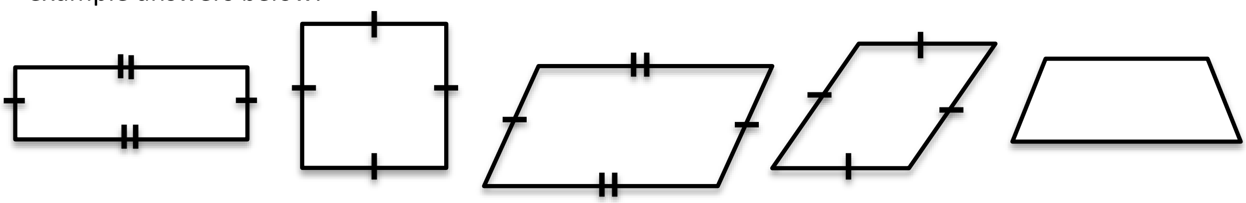


Lesson Title: Area of Quadrilaterals	**Theme**: Measurement and Estimation	
Lesson Number: M-09-143	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to calculate the area of a square, rectangle, parallelogram, and trapezium.	None	Draw the shapes from Introduction to New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

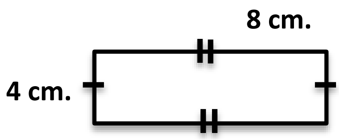
1. Ask pupils to draw the following 5 shapes in their exercise books: Rectangle, Square, Parallelogram, Rhombus, and Trapezium
2. Give pupils 1 minute to work, and then have 5 pupils volunteer to come to the board and draw each of the shapes.
3. Make sure their drawing are accurate and include markings to show equal side lengths, as the example answers below:



4. **Say**: Today we will practise calculating the area of squares, rectangles, parallelograms, and trapeziums.

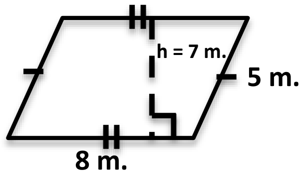
Introduction to the New Material *(12 minutes)*

1. **Ask**: What is area? (Answer: The size inside of a shape.)
2. Have 1-2 pupils volunteer to answer. Remind pupils that area is always written in units squared.
3. **Say**: All of the shapes you have drawn are quadrilaterals, meaning they have 4 sides. We can also call the first 4 shapes parallelograms. A parallelogram is a 4-sided plane figure with opposite sides parallel. Rectangle, square, and rhombus are special kinds of parallelograms.
4. **Say**: The small lines show sides that are equal in length. If the sides in a shape have 1 small line on them, they are equal. If they have 2 small lines, those are equal to each other too.
5. **Ask**: How do we find the area of squares or rectangles? (Answer: multiply length times width)
6. **Say**: To find the area of a square or rectangle, multiply the lengths of the 2 sides.
7. **Write** the formulae for area of a square and rectangle on the board:
   Square: $A = length \times length = l \times l = l^2$ **Rectangle**: $A = length \times width = l \times w$
8. Label the rectangle on the board with measurements.

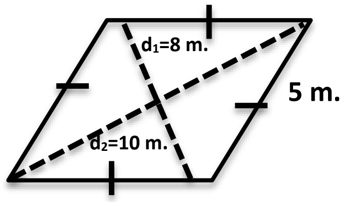


9. Calculate the area of the rectangle on the board and explain each step: $\text{Area} = \text{l} \times \text{ w} = 8 \text{ cm.} \times 4 \text{ cm.} = 32 \text{ cm.}^2$
10. **Say**: For parallelograms and rhombuses, we use different, special formulae to find their area. We need to know measurements that are inside of them.
11. Draw the height into the parallelogram, and the 2 diagonals of the rhombus:

12. **Say**: For the parallelogram, we use its height to find the area. We use a similar formula to the 1 we used for area of a rectangle.

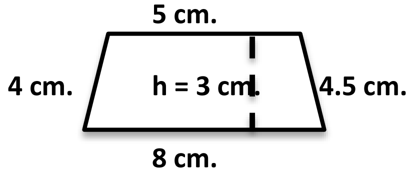


13. **Write** on the board: $A = base \times height = b \times h$
14. Ask pupils to volunteer to identify base and height of the parallelogram. (Answer: base = 8 m.; height = 7 m.)



15. Calculate the area on the board and explain each step. (Answer: $A = b \times h = 8\text{ m.} \ \times 7\text{ m.} = 56\text{ m.}^2$).
16. **Say**: For the rhombus, we use the length of its 2 diagonals to find the area.
17. **Write** on the board: $A = \frac{1}{2} \times diagonal\ 1 \times diagonal\ 2 = \frac{1}{2} d_1 \times d_2$
18. Ask pupils to identify the 2 diagonals. (Answer: $d_1 = 8$ m.; $d_2 = 10$ m.)
19. Calculate the area on the board and explain each step. (Answer: $A = \frac{1}{2} d_1 \times d_2 = \frac{1}{2} \times 8\text{ m.} \times 10\text{m.} = 40\text{m.}^2$)
20. **Say**: A trapezium is a quadrilateral, meaning it has 4 sides. However only 1 set of sides is parallel, so it is not a parallelogram like the other shapes we have been studying.
21. Label the measurements on the trapezium
22. **Write** the formula for area of a trapezium on the board:

    $Area = \frac{1}{2}(a + b)h$

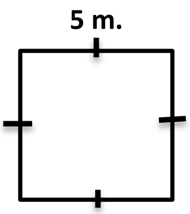


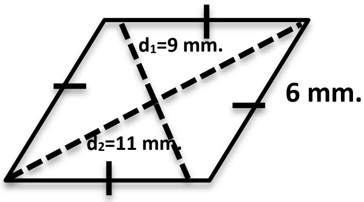
23. **Say**: To calculate the area of a trapezium you add the 2 parallel bases together (a + b) and divide the sum by 2. Then you multiply by the height (h) of the trapezium. You must be careful to always use the height, not the side of the trapezium. You can see that those may be different numbers.
24. Ask pupils to calculate the area of the trapezium in their exercise books using the formula, while you do so, on the board. (Answer: $Area = \frac{1}{2}(a + b)h = \frac{1}{2}(5\text{cm.} + 8\text{cm.})3\text{cm.} = \ 19.5\text{ cm.}^2$)
25. **Say**: Remember area is always written in units squared.

Guided Practice *(8 minutes)*

1. Ask pupils to work in pairs.
2. Sketch and label a square and a rhombus on the board:

   (i) (ii)

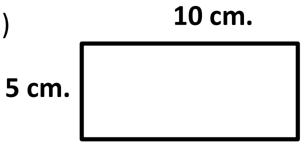


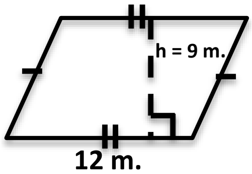


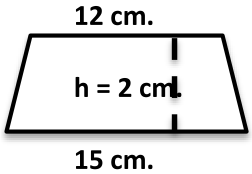
3. Ask pupils to calculate the area of the 2 shapes.
4. Move around to check for understanding and clear misconceptions. Check that their answers are in m.^2 and mm.^2
5. Ask 2 different pairs to give their answers to the 2 questions (Answers: (i) $A = l \times l = 5\text{ m.} \times 5\text{ m.} = 25\text{ m.}^2$; $(ii) A = \frac{1}{2} d_1 \times d_2 = \frac{1}{2} \times 9\text{ mm.} \times 11\text{mm.} = 49.5\text{mm.}^2$)

Independent Practice *(10 minutes)*

1. Point to the rectangle, parallelogram, and trapezium on the board.

(i) 

(iii) 

(ii) 

2. Ask pupils to work independently to solve the problems.
3. Move around to check for understanding and clear misconceptions.
4. Ask pupils to turn to their partner and compare answers for 2 minutes.
5. Have 3 pairs volunteer to share their answers.
6. (Answers: (i) $A = l \times w = 10 \text{ cm.} \times 5 \text{ cm.} = 50 \text{ cm.}^2$; (ii) $A = b \times h = 12 \text{ m.} \times 9 \text{ m.} = 108 \text{ m.}^2$; (iii) $Area = \frac{1}{2}(a + b)h = \frac{1}{2}(12 \text{ cm.} + 15 \text{ cm.})2 \text{ cm.} = 27 \text{ cm.}^2$
7. **Write** the correct answers and steps on the board. Ask pupils to check their work.

Closing *(2 minutes)*

1. Draw the following chart on the board:

Plane Shape	Area Formula
Square	
Rectangle	
Parallelogram	
Rhombus	
Trapezium	

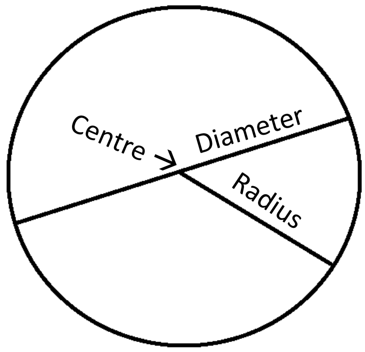
2. Ask pupils to fill in the chart with the area formulae.
3. **Say**: If you have the chart from the previous lesson with perimeter formulae, you can add a column and all the formulae will be in 1 place. (Answer: The full chart should look like below)

Plane Shape	Quadrilateral	Equal Sides	Equal Angles	Perimeter Formula	Area Formula
Square	Yes	Yes	Yes	P = 4l	$A = l^2$
Rectangle	Yes	No	Yes	P = 2l + 2w	$A = l \times w$
Parallelogram	Yes	No	No	P = 2l + 2w	$A = b \times h$
Rhombus	Yes	Yes	No	P = 4l	$A = \frac{1}{2} d_1 \times d_2$
Trapezium	Yes	No	No	P = a + b + c + d	$A = \frac{1}{2}(a + b)h$
Triangle	No	No	No	P = a + b + c	

Lesson Title: Area and Circumference of Circles	**Theme**: Measurement and Estimation	
Lesson Number: M-09-144	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to calculate the area and circumference of a circle.	None	None

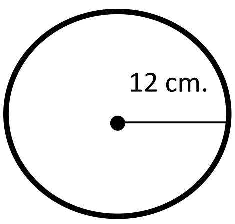
Opening *(3 minutes)*



1. Draw a blank circle on the board.
2. Have 3 pupils volunteer to come to the board and draw and label the centre, diameter, and radius of the circle, as in the diagram. →
3. Ask 3 different pupils to volunteer to explain the following terms in their own words:
   a. Centre
   b. Diameter
   c. Radius
   (Example answers: (a) Centre is the point in the middle of the circle. (b) Diameter is the distance across the circle, passing through the centre. (c) Radius is the distance from the centre to the circumference, half the diameter.)
4. **Say:** A circle is a round plane shape. Today, you will practise calculating the circumference and area of circles.

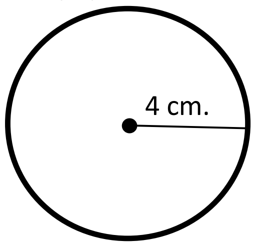
Introduction to the New Material *(13 minutes)*

1. Ask pupils to volunteer to describe circumference in their own words. (Example answers: The circumference of a circle is the same as the perimeter of the circle; It is the distance around a circle.)
2. Remind pupils about the perimeter of plane shapes as the total distance around the edges of the shape.
3. Sketch a circle of radius 12 cm. on the board.



4. **Ask:** What formula do we use to find the circumference? (Answer: $C = 2\pi r$)
5. **Write** on the board: $C = 2\pi r$
6. Read the formula out loud and ask pupils to repeat: 'C equals two pie r'
7. **Say**: The C in this formula is circumference, and r is radius. These numbers change for each circle. The other symbol in the formula is called 'pi' and it is a constant number that never changes.

8. **Say**: If we write pi as a decimal, it will be very long. We won't find the end of it. We have a fraction and a rounded off decimal that we use in this formula to give us an 'approximate' value for pi.
9. **Write** on the board: $pi = \pi \approx \frac{22}{7} \approx 3.14159$
10. **Say**: We can use either value of pi, $\frac{22}{7}$ or 3.14. We will use the decimal for this question.
11. **Say**: Now, let's calculate the circumference of the circle on the board.
12. **Ask**: What is the value for r? (Answer: 12 cm.)
13. Ask pupils to substitute the values into the formula in their exercise books.
14. Do the calculation on the board: $C = 2\pi r = 2 \times 3.14 \times 12 \text{ cm.} =$
15. Have a pupil volunteer to share the answer: 75.36 cm.
16. **Say**: The circumference of the circle is 75.36 cm.
17. **Say:** The area of a circle is the total space inside the circumference.
18. **Ask:** What is the formula to calculate the area of a circle? (Answer: $Area = \pi r^2$)
19. **Write** the formula for area of a circle on the board: $Area = \pi r^2$
20. Read the formula out loud and ask pupils to repeat: 'Area equals pie r squared'
21. **Say**: Now, we will find the area of this circle using the formula.
22. Substitute the values of *r* and π on the board: $A = \pi r^2 = 3.14 \times (12 \text{ cm.})^2$
23. Multiply and find the area: $A = 3.14 \times 12 \text{ cm.} \times 12 \text{ cm.} = 452.16 \text{ cm.}^2$
24. Remind pupils that area must be written in units squared



Guided Practice *(7 minutes)*

1. Sketch a circle with radius = 4 cm. on the board:
2. Ask pupils to work in pairs to find the circumference and area of the circle.
3. **Say**: Use 3.14 for the value of π for both questions.
4. Walk around to check for understanding and clear misconceptions.
5. Have 1 pair volunteer to present their work on the board, making sure that different pupils solve for circumference and area. Ask the rest of the class to check their work in their exercise books.
6. Allow pupils to ask questions. (Answer: $C = 2\pi r = 2 \times 3.14 \times 4 \text{ cm.} = 25.12 \text{ cm.}$; $A = \pi r^2 = 3.14 \times (4 \text{ cm.})^2 = 3.14 \times 16 = 50.24 \text{ cm.}^2$)
7. **Write** the correct answers and steps on the board. Ask pupils to check their work.

Independent Practice *(10 minutes)*

1. Ask pupils to work independently to solve the problems.
2. Write the following questions on the board:
   a. Find the circumference and area of a circle of radius 8 km.
   b. Find the circumference and area of a circle of diameter 18 m.
3. **Say**: Use 3.14 for the value of π for both questions.
4. Move around to check for understanding and clear misconceptions.

5. Have 2 pupils volunteer to draw and label the circles and solve the questions on the board while the rest of the class checks their work. (Answers: (a) $C = 2\pi r = 2 \times\ 3.14\ \times 8\ \text{km.} = 50.24\ \text{km};\ A = 3.14 \times (8\ \text{km.})^2 = 200.96\ \text{km.}^2$; (b) $r = \frac{d}{2} = \frac{18\ m.}{2} = 9\ \text{m.};\ C = 2\pi r = 2 \times 3.14\ \times 9\ \text{m.} = 56.52\ \text{m.};\ A = 3.14 \times (9\ \text{m.})^2 = 254.34\ \text{m.}^2$)
6. Correct any errors in the calculation on the board. Ask pupils to check their work.

Closing *(2 minutes)*

1. Have pupils complete solve this question in their exercise books. If a circle has a radius of 2 cm., calculate the diameter, circumference and area.
2. Ask pupils to show you the answers in their exercise books on their way out of the classroom. (Answers: $d = 2r = 2 \times 2\ \text{cm.} = 4\ \text{cm.};\ C = 2\pi r = 2 \times 3.14\ \times 2\ \text{cm.} = 12.56\ \text{cm.};\ A = 3.14 \times (2\ \text{cm.})^2 = 12.56\ \text{cm.}^2$)

Lesson Title: Practical Problems with Area and Perimeter	**Theme**: Measurement and Estimation	
Lesson Number: M-09-145	**Class/Level**: JSS 3	**Time**: 35 minutes

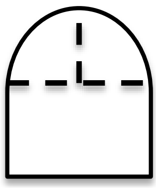
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Find the perimeter and area of composite shapes. 2. Solve multi-step word problems on perimeter and area.	None	1. Draw the diagrams from Introduction to the New Material and Guided practice on the board. 2. Write the word problem from the Guided Practice on the board.

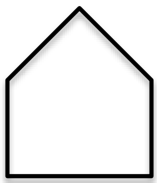
Opening *(2 minutes)*

1. Ask pupils to review the formulae for perimeter and area of the following shapes:
   a. Rectangle
   b. Triangle
   c. Trapezium
   d. Rhombus
2. Ask 4 pupils from different parts of the room to come to the board and write the formulae on the board. (Answers: (a) P = 2l + 2w; $A = l \times w$; (b) P = a + b + c; $A = \frac{1}{2}bh$; (c) P = a + b + c + d; $A = \frac{1}{2}(a + b)h$; (d) P = 4l; $A = \frac{1}{2}d_1 \times d_2$)
3. **Say:** Today, we will learn how to find the perimeter and area of composite shapes and solve story problems involving perimeter and area.

Introduction to the New Material *(12 minutes)*

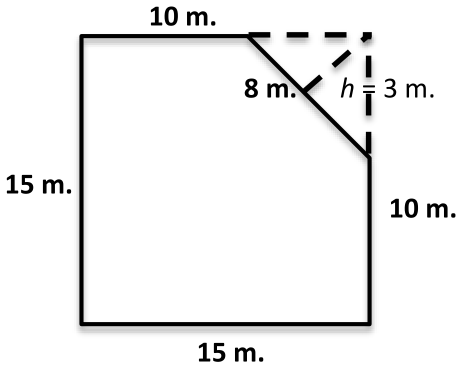
1. Point to the following composite shapes as examples on the board:





2. **Say**: Composite plane shapes are shapes that can be divided into 1 or more basic plane shapes.
3. **Say**: To find the perimeter and area of composite shapes you use the formulae for basic shapes such as square, rectangles, and circles, which we already know.
4. Ask pupils to identify basic shapes within the composite shapes you have drawn. Have pupils riase their hands to answer. (Answers: (a) rectangle + half-circle; (b) rectangle + triangle)
5. **Write** the following story problem on the board:
   Abel's farm is a square with 1 corner cut off. 2 sides measure 15 m., the other 2 sides measure 10 m. The cut-off corner forms a triangle with a base of 8 m. and height 3 m. Calculate the length of fencing Abel needs, and then find the area of his farm.
6. Have a pupil volunteer to read the question aloud to the class.
7. Discuss the meaning of this question with the pupils. Remind pupils that the distance around a shape is the perimeter.

8. **Say:** We need to find the perimeter of the farm, which will tell us the amount of fencing. To be able to understand story problems involving perimeter or area of plane shapes, we can be helpful to draw the picture of the shapes first, and then we apply the correct formula to solve the problem.
9. Draw and label the farm on the board. →

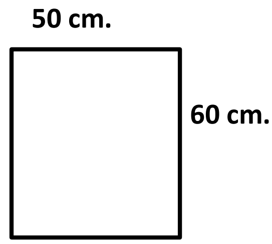


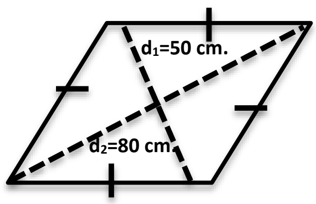
10. **Ask**: How should we calculate the perimeter of the farm? (Answer: add up all the lengths around the shape)
11. Calculate the perimeter of the farm as below: $P = 10\text{ m.} + 8\text{ m.} + 10\text{ m.} + 15\text{ m.} + 15\text{ m.} = 58\text{ m.}$
12. **Ask**: How can we calculate the area of Abel's farm? (Calculate the areas of the square and the missing triangle and subtract)
13. Allow pupils to raise their hands to answer these questions.
14. **Ask**: What is the formula for the area of a square? (Answer: $A = l^2$)
15. Ask pupils to calculate the area of the square in their exercise books, while you do so on the board: $A = l^2 = 15\text{m.}^2 = 225\text{ m.}^2$
16. **Ask**: What is the formula for the area of a triangle? (Answer: $A = \frac{1}{2}bh$)
17. Ask pupils to calculate the area of the missing triangle in their exercise books, while you do so on the board: $A = \frac{1}{2}bh = \frac{1}{2} \times 8\text{ m.} \times\ 3\text{m.} = 4\text{ m.} \times 3\text{ m.} = 12\text{ m.}^2$
18. **Ask**: How do you calculate the final area? (Answer: Subtract the area of the triangle from the area of the square.)
19. Ask pupils to calculate the final area in their exercise books: $Total\ Area = 225\text{ m.}^2 - 12\text{ m.}^2 = 213\text{ m.}^2$
20. **Say**: Abel's farm is 213 m.^2

Guided Practice (*8 minutes)*

1. Ask pupils to work with their partners to solve the following problem.
2. Point to on the board:

   Gbessay wants to buy nets for her windows to keep the mosquitos out and prevent malaria. She has 2 rectangular windows that are 60 cm. tall, and 50 cm. wide. She also has 2 rhombus-shaped windows that have diagonals of 50 cm. and 80 cm. How much net will she need to cover the 4 windows?





3. Walk around to check for understanding and clear misconceptions.
4. Make sure they are drawing the pictures, applying the correct formulae and their answers are in correct units.
5. Have 3 pairs volunteer to present their answers on the board while the rest of the class checks their work. Have the first group share the answer for the rectangle windows, the second group share the answer for the rhombus windows, and the third group give the final answer. (Answers: Rectangle: $A = lw = 60\text{ cm.} \times\ 50\text{ cm.} = 3{,}000\text{ cm.}^2$; Rhombus: $A = \frac{1}{2}d_1 \times d_2 = \frac{1}{2} \times 50\text{ cm.} \times 80\text{ cm.} = 2{,}000\text{ cm.}^2$; Total area $2 \times (3{,}000\text{ cm.}^2 + 2{,}000\text{ cm.}^2) = 10{,}000\text{ cm.}^2$
6. Correct any errors in the calculation on the board. Ask pupils to check their work.

Independent Practice *(10 minutes)*

1. Choose 1-2 problems from the question bank on the next page (based on your pupils' level of understanding) and write them on the board.
2. Ask pupils to work independently to find the answers.
3. Walk around to check for understanding and clear misconceptions.
4. Have 1-2 pupils volunteer to come solve the problems on the board.
5. **Write** the correct answers and steps on the board. Ask pupils to check their work.

Closing *(3 minutes)*

1. Ask pupils to write their own story problem using the lesson as an example.
2. Allow pupils to share their story problems with their partners, or the class if time allows.
3. Move around the room and assist pupils as necessary.
4. Remind pupils to pick 1 of the basic shapes and make sure they give the needed information – such as length and width, or radius.

[QUESTION BANK]

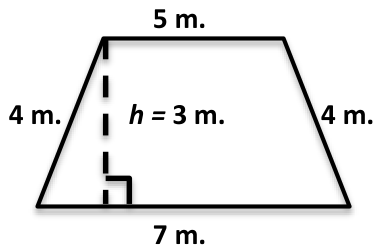
Joe wants to build a porch in the shape of a trapezium next to his house. The 2 parallel sides are 5 m. and 7 m.; the diagonals are both 4 m, and the height is 3 m. Each bag of cement can make 2 square metres of porch. How many bags will Joe need?

Answer:

$$A = \frac{1}{2}(a+b)h$$
$$= \text{ A} = \frac{1}{2}(5\text{ m.} + 7\text{ m.})3\text{ m.}$$
$$= 6\text{ m.} \times 3\text{ m}$$
$$= 18\text{ m.}^2$$

$18\text{ m.}^2 \div 2\text{m.}^2 = 9$ bags of cement

Diagram:



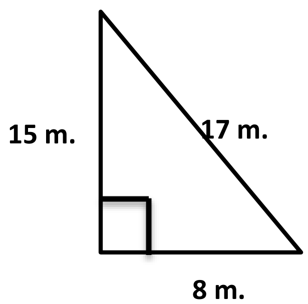
Bendu is fencing her triangular farm. The base of the triangle is 8 m. long, the height is 15 m. long and the diagonal is 17 m. long (i) How long is Bendu's fence? (ii) Fencing costs Le 250 per metre. How much will Bendu's fence cost?

Answer:

(i) $P = 15\text{ m.} + 17\text{ m.} + 8\text{ m.} = 40\text{ m.}$

(ii) $40\text{ m.} \times \text{ le } 250 = \text{Le } 10{,}000.$

Diagram:



Sam wants to put 2 mats on the floor of his room. His room is shaped like a rectangle with a small square on it. The rectangle is 4 m. long and 3 m. wide. The square part of the room is 1 m. long by 1 m. wide. (i) How many square metres of mat must he buy? (ii) If the mat is Le 200 for 1 square metre how much will it cost?

Answer:

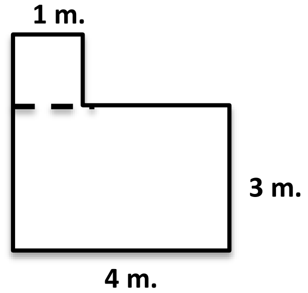
$Rectangle: A = 3 \text{ m.} \times 4 \text{ m.} = 12 \text{ m.}^2$

$Square: A = 1 \text{ m.} \times 1 \text{ m.} = 1 \text{ m.}^2$

Total = $12 \text{ m.}^2 + 1 \text{ m.}^2 = 13 \text{ m.}^2$

$Cost = 13 \text{ m.}^2 \times \text{Le } 200 = \text{Le } 2{,}600$

Diagram:



Lesson Title: Volume of Prisms	**Theme**: Measurement and Estimation	
Lesson Number: M-09-146	**Class/Level**: JSS 3	**Time**: 35 minutes

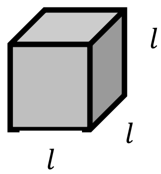
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to find the volume of cubes, rectangular prisms, and triangular prisms.	None	Draw the cubes and shapes from the Introduction to the New Material, Guided Practice and Independent Practice on the board.

Opening *(3 minutes)*

1. Ask pupils to volunteer to describe the following in their own words. :
   a. Cube (Example answer: a 3 dimensional shape where all 6 surfaces are equal squares)
   b. Rectangular prism (Example answer: a 3 dimensional shape where all 6 surfaces are rectangles, also known as a cuboid)
   c. Triangular prism (Example answer: a solid with 2 parallel surfaces that are triangles)
2. **Say:** Today we will practice finding the volume of cubes, rectangular prisms, and triangular prisms.

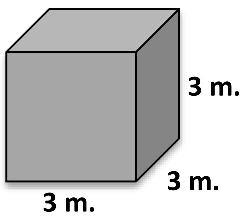
Introduction to the New Material *(13 minutes)*

1. Point to a cube on the board. **Say**: A cube is a special type of rectangular prism. In a rectangular prism, each surface is a rectangle. Because this is a cube, we know that each face is a square. Every edge is the same length, labeled as l.
2. Ask pupils to volunteer to state the formula of the volume of rectangular solid:

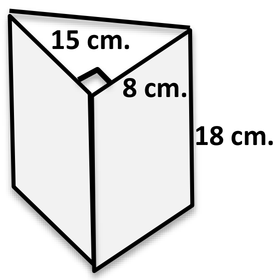


$$V = l \times w \times h = lwh$$

3. **Say**: Since all the sides are of the same length, then 1 number will represent all the sides. We can simplify the formula.
4. Ask pupils to replace the width and height with length. For a square, they are all the same.
5. **Write** on the board: Volume of a cube is $V = l \times l \times l = l^3$.
6. Remind pupils that volume is measured in units cubed.
7. **Say**: If you know the length of 1 side of the cube then you will be able to calculate its volume.



8. Ask pupils to draw and label a diagram of the cube in their exercise books. Draw it on the board to help them.
9. Remind them of the formula: $= l \times l \times l = l^3$.
10. **Write** on the board: $V = 3\text{ m.} \times 3\text{ m.} \times 3\text{ m.}$
11. Ask pupils to solve the multiplication problem and give the answer. Make sure they have the correct units. (Answer: 27 m^3)



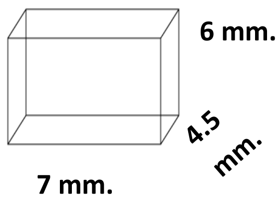
12. **Say**: Now let us look at triangular prisms.
13. **Write** the following problem on the board:
    Calculate the volume of a right triangular prism with base 8 cm., height 15 cm., and length 18 cm.

14. Ask pupils to draw the right triangular prism in their exercise books. Do the same on the board as ➔
15. **Say:** When the triangular surfaces are right triangles, like this, it is called a right triangular prism.
16. **Ask**: What is the formula for area of a triangle? ($A = \frac{1}{2}bh$)
17. **Write** the formula on the board:

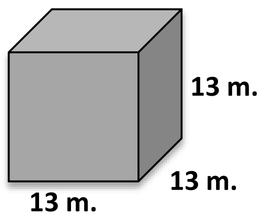
$$V = \frac{1}{2} \times \,base\, \times \,height\, \times \,length\, = \frac{1}{2}bhl$$

18. **Say**: To find the volume of a triangular prism, we multiply the area of 1 side (a triangle cross-section, $A = \frac{1}{2}bh$) by the length (*l*).
    a. Remind pupils to find volume, it is important that the sides of a solid are all given in the same units
19. Ask pupils to call out values to substitute in the formula: $V = \frac{1}{2}bhl = \frac{1}{2} \times 8 \text{ cm.} \times 15 \text{ cm.} \times 18 \text{ cm.}$
20. Ask pupils to calculate the volume of the triangular prism using the formula in their exercise books.
21. Calculate the volume on the board: $V = \frac{1}{2}bhl = \frac{1}{2} \times 8 \text{ cm.} \times 15 \text{ cm.} \times 18 \text{ cm.} = 1{,}080 \text{ cm.}^3$
22. **Say**: The volume of the triangular prism is $1{,}080 \text{ cm.}^3$

Guided Practice *(7 minutes)*



1. Write the following problems on the board and point to the solids:
    a. Calculate the volume of a rectangular prism with length 7 mm., width 4.5 mm., and height 6 mm.
    b. Calculate the volume of a cube with side 13 m.
2. Ask pupils to work in pairs.
3. Move around the room and clear any misconceptions.
4. Have 2 pairs volunteer to share their answers on the board. Have 1 pupil draw the solid while the other calculates the volume.
5. Ask the rest of the pupils to check their work in their exercise books. (Answers: (a) $V = lwh = 7 \text{ mm.} \times 4.5 \text{ mm.} \times 6 \text{ mm.} = 189 \text{ mm.}^3$ (b) $V = 13 \text{ m.} \times 13 \text{ m.} \times 13 \text{ m.} = 2{,}197 \text{ m.}^3$)
6. **Write** the correct answers and steps on the board. Ask pupils to check their work.



Independent Practice *(10 minutes)*

1. **Write** 2 problems on the board:
    a. Calculate the volume of a rectangular prism with length 15 m., width 2 m., and height 8 m.
    b. Calculate the volume of a triangular prism with base 14 cm., height 10 cm., and length 5 cm.
2. Ask pupils to solve the problems independently in their exercise books.
3. Go round and check their answers and clear any misconceptions. (For example, make sure pupils write the units for volume correctly: m.^3 and cm.^3)

4. Have2 pupils volunteer to come to the board to share their answers while the rest of the class checks their work in their exercise books. (Answers: (a) $V = lwh = 15\text{ m.} \times 2\text{ m.} \times 8\text{ m.} = 240\text{ m}^3$; (b) $V = \frac{1}{2}bhl = \frac{1}{2} \times 14\text{ cm.} \times 10\text{ cm.} \times 5\text{ cm.} = 350\text{ cm.}^3$
5. **Write** the correct answers and steps on the board. Ask pupils to check their work.

Closing *(2 minutes)*

1. Ask pupils to volunteer to say the formulae for the volume of the following solids:
   a. Cube
   b. Rectangular prism
   c. Triangular prism

   (Answers: (i) Cube: $V = l^3$; (ii) Rectangular prism: $V = lwh$; (iii) Triangular prism: $V = \frac{1}{2}bhl$)

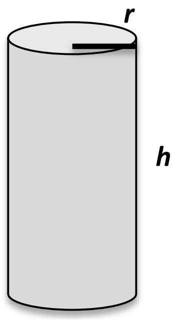
Lesson Title: Volume of Cylinders	**Theme**: Measurement and Estimation	
Lesson Number: M-09-147	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to find the volume of a cylinder.	None	Draw the cylinder diagrams from the Introduction to the New Material and Guided Practice on the board.

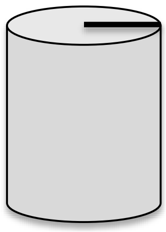
Opening *(3 minutes)*

1. **Write** the following problem on the board:
   Calculate the volume of a triangular prism with base 9 m., height 8 m., length 5 m.
2. Ask pupils to solve the problem in their exercise books.
3. Have 1 pupil volunteer to share their answer and describe the steps to finding the volume. (Answer: $V = \frac{1}{2}bhl = \frac{1}{2} \times 9\text{ m.} \times 8\text{ m.} \times 5\text{ m.} = 180\text{ m.}^3$)
4. **Say**: Today we will practice finding the volume of cylinders.

Introduction to the New Material *(14 minutes)*



1. Point to the cylinder on the board. Label the height and radius as ➔
2. **Say**: A cylinder is a solid with 2 parallel faces that are circles. A cylinder will have a height and a radius or diameter. Its volume is measured in cubic units.
3. Ask pupils to think of examples of cylinders they have seen. (Example answers: milk cans, glass bottles)
4. **Ask:** What is the formula for volume of a cylinder. (Answer: $V = \pi r^2 h$)
5. Have pupils raise their hands to answer your questions.
6. **Write** on the board:
   $V = \pi r^2 h$
7. **Say**: To find the volume of a cylinder, we multiply the area of one side (a cross-section, which is a circle with $A = \pi r^2$) by the height (h).
8. Remind pupils to find volume, it is important that the sides of a solid are all given in the same units
9. **Write** the following problem on the board: Find the volume of a cylinder with radius 2 cm. and height 8 cm.
10. Ask pupils to draw the cylinder in their exercise book. Do the same on the board as ➔



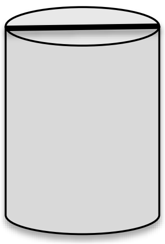
11. Have pupils volunteer to call out the value for π. (Answer $pi = \pi \approx \frac{22}{7} \approx 3.1416$)
12. **Say**: Use 3.14 as the value for π.
13. Ask pupils to calculate the volume of the cylinder using the formula.
14. Calculate the volume on the board: $V = \pi r^2 h. = 3.14 \times (2\text{ cm}).^2 \times 8\text{ cm.} = 100.48\text{ cm.}^3$
15. **Write** another problem on the board:
    What is the volume of a cylinder with radius 6 m. and height 2 m.?

16. Ask pupils to call out the steps to calculate the volume of the cylinder using the formula in their exercise books. (Answers: Write the formula; substitute the values for π, radius, and height; multiply.)
17. Calculate the volume on the board: $V = \pi r^2 h. = 3.14 \times (6 \text{ m.})^2 \times 2 \text{ m.} = 226.08 \text{ m.}^3$

Guided Practice *(5 minutes)*

1. **Write** the following problem on the board:
   a. Draw a cylinder with height 6 cm., and a diameter 10 cm.
   b. What is the volume of the cylinder?
2. Ask pupils to work in pairs and to write their answers in the exercise books.
3. Move around the room and clear misconceptions.
4. Have 1 pair volunteer to draw the cylinder on the board, and answer the question.
   Answers:

(a)



(b) $r = \frac{d}{2} = \frac{10}{2} = 5 \text{ cm.}$; $V = \pi r^2 h = 3.14 \times (5 \text{ cm.})^2 \times 6 \text{ cm.} = 471 \text{ cm}^3$

5. Correct any errors on the board and ask pupils to check their work.

Independent Practice *(10 minutes)*

1. **Write** the following problems on the board:
   a. Calculate the volume of a cylinder with radius 2.5 cm. and height 6 cm.
   b. Calculate the volume of a cylinder with radius 7 km. and height 6 km.
2. Ask pupils to work individually and write their answers in their exercise books.
3. Move around the room and clear misconceptions.
4. Have 2 pupils volunteer to come to the board and share their answers while the rest of the class checks their work in their exercise books. (Answers: (a) $V = \pi r^2 h = 3.14 \times (2.5 \text{ cm.})^2 \times 6 \text{ cm.} = 117.75 \text{ cm}^3$; (b) $V = \pi r^2 h = 3.14 \times (7 \text{ km.})^2 \times 6 \text{ km.} = 923.16 \text{ km}^3$)
5. **Write** the correct steps and answers on the board. Ask pupils to check their work.

Closing *(3 minutes)*

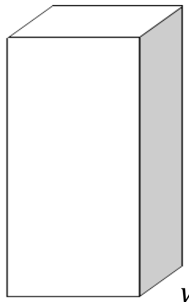
1. Ask pupils to answer the following questions by raising their hands.:
   a. Define 'cylinder'.
   b. What is the formula for the volume of a cylinder?
2. Have 2 pupils volunteer to share their ideas with the class. (Answer: (i) A cylinder is a solid with 2 parallel faces that are circles; (ii) $V = \pi r^2 h$)

Lesson Title: Surface Area of Prisms	**Theme**: Measurement and Estimation	
Lesson Number: M-09-148	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to calculate the surface area of cubes, rectangular prisms, and triangular prisms.	None	Draw the shapes and diagrams from Introduction to New Material, Guided Practice and Independent Practice on the board.

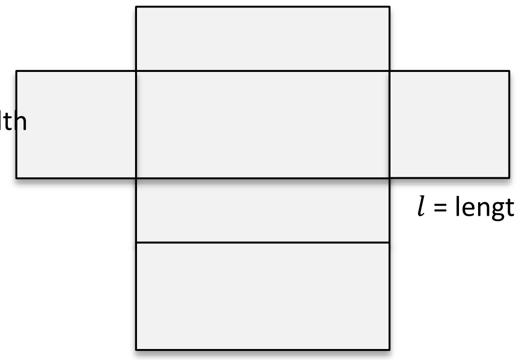
Opening *(3 minutes)*

1. **Write** the following questions on the board:
   a. What is the formula for the area of a square?
   b. What is the formula for the area of a rectangle?
   c. What is the formula for the area of a triangle?
2. Have 3 pupils volunteer to shares the answers. (Answers: (a) $A = l^2$; (b) $A = l \times w$; (c) $A = \frac{1}{2}bh$)
3. **Say**: Today we will learn to calculate the surface area of cubes, rectangular prisms, and triangular prisms.



Introduction to the New Material *(14 minutes)*

1. Point to the rectangular prism on the board:
2. **Ask**: What is surface area? (Answer: The outside layer of a solid is called the *surface area*.)
3. Remind pupils that using a net can help us. A net is like a paper version of a solid that can be opened up and laid flat.
4. Ask pupils to draw a net of the rectangular prism in their exercise books. Do the same on the board to help them.



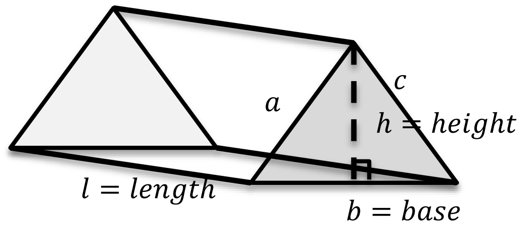
5. **Ask**: How can we calculate the surface area of the rectangular prism? (Answer: Add up the areas of each of the rectangles in the net, like a composite shape).
6. **Say**: A rectangular prism has 6 sides, just like a cube, but they are not all the same.
7. Ask pupils to discuss what rectangles in the prism are the same with their partners. (Answer: The top and bottom are the same, the front and back are the same, and the left and right are the same.)
8. **Write** the formula for the surface area of a rectangular prism on the board:

$$SA = 2lw + 2wh + 2lh$$

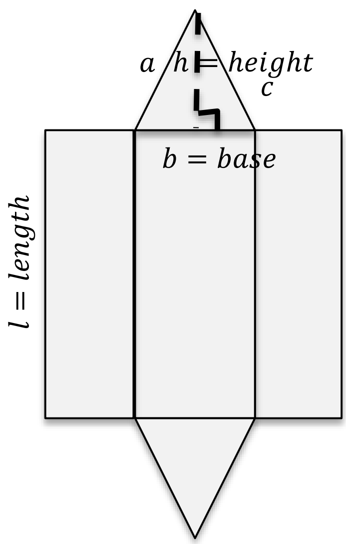
9. **Say**: To calculate the total surface area of a rectangular prism we find the area of each of the 3 different surfaces and multiply by 2. If this were a cube, the area of each surface would be the same, so we would find the area of 1 surface and multiply it by 6. ($SA = 6l^2$)
10. Label the rectangular prism with length 5 m., width 4 m., and height 9 m.
11. Ask pupils to calculate the surface area of the rectangular prism.
12. Calculate the surface area on the board:

$$\begin{aligned} SA &= 2lw + 2wh + 2lh \\ &= 2 \times 5 \text{ m.} \times 4 \text{ m.} + 2 \times 4 \text{ m.} \times 9 \text{ m.} + 2 \times 5 \text{ m.} \times 9 \text{ m.} \\ &= 40 \text{ m.}^2 + 72 \text{ m.}^2 + 90 \text{ m.}^2 = 202 \text{ m.}^2 \end{aligned}$$

13. Point to the triangular prism on the board:



14. **Say:** Now we are going to calculate the surface area of a triangular prism.
15. Have a pupil volunteer to draw a net of the triangular prism on the board:

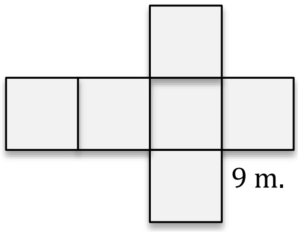


16. **Ask**: How can we calculate the surface area of the triangular prism? (Answer: Add the areas of each of the shapes in the net, like a composite shape).
17. **Write** the formula for the surface area of a triangular prism on the board:
    $SA = bh + (a + b + c)l$
18. **Say**: To calculate the total surface area of a triangular prism we add the area of the triangle surfaces to the area of the 3 rectangles made by the sides of the triangle.
19. **Ask**: Why does the formula only say bh not $\frac{1}{2}bh$? (Answer: There are 2 triangular surfaces and $\frac{1}{2} \times 2$ cancels out.)
20. Label the length of the prism 7 cm., the height 3 cm., the base 8 cm., and sides a and c = 5 cm.
21. Ask pupils to calculate the surface area of the triangular prism in their exercise books.
22. Calculate the surface area on the board:

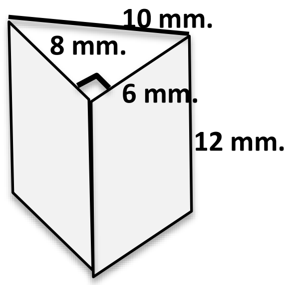
$$SA = bh + (a + b + c)l = 8 \text{ cm.} \times 3 \text{ cm.} + (5\text{cm.} + 8 \text{ cm.} + 5 \text{ cm.}) \times 7 \text{ cm.}$$
$$= 24 \text{ cm.}^2 + (18 \text{ cm.}) \times 7 \text{ cm.}$$
$$= 24 \text{ cm.}^2 + 126 \text{ cm.}^2 = 150 \text{ cm.}^2$$

Guided Practice *(5 minutes)*

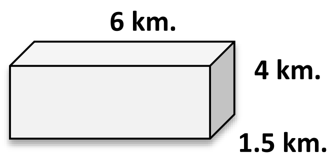
1. **Write** the following problem on the board: Draw the net of a cube with side length 9 m. Then calculate the surface area.
2. Ask pupils to work in pairs and to write their answers in their exercise books.
3. **Ask**: What is the formula for the surface area of a cube? (Answer: $SA = 6l^2$)
4. Move around the room and clear any doubts.
5. Have 1 pair volunteer to draw the solid on the board and answer the question. Correct any error in their calculations and ask other pupils to check their work.
   (Answer: $SA = 6l^2 = 6 \times (9\text{m.})^2 = 486 \text{ m.}^2$)



Independent Practice *(10 minutes)*



1. **Write** the following problems on the board:
   a. Calculate the surface area of a right triangular prism with length 12 mm., base 6 mm., height 8 mm., and the remaining side 10 mm.?
   b. Calculate the surface area of a rectangular prism with length 6 km., width 1.5 km., and height 4 km.
2. Ask pupils to work individually and write their answers in their exercise books.
3. Move around the room and clear misconceptions.
4. Have 2 pupils volunteer to come to the board and share their answers while the rest of the class checks their work in their exercise books.



Answers:

a. $SA = bh + (a + b + c)l$
$= 6 \text{ mm.} \times 8 \text{ mm.} + (6 \text{ mm.} + 8 \text{ mm.} + 10 \text{ mm.}) \times 12 \text{ mm.}$
$= 48 \text{ mm.}^2 + (24 \text{ mm.}) \times 12 \text{ mm.}$
$= 336 \text{ mm}^2$

b. $SA = 2lw + 2wh + 2lh$
$= 2 \times 6 \text{ km.} \times 1.5 \text{ km.} + 2 \times 1.5 \text{ km.} \times 4 \text{ km.} + 2 \times 6 \text{ km.} \times 4 \text{ km.}$
$= 18 \text{ km.}^2 + 12 \text{ km.}^2 + 48 \text{ km.}^2 = 78 \text{ km.}^2$

Closing *(3 minutes)*

1. Ask pupils to pick a cube, rectangular prism, or triangular prism and sketch both the prism and its matching net.

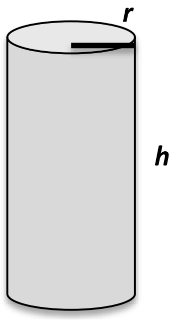
2. Give pupils one minute to work and then ask pupils to compare their answers with their partners.
3. Allow 1-2 pupils to volunteer to share their work with the class.

Lesson Title: Surface Area of Cylinders	**Theme**: Measurement and Estimation	
Lesson Number: M-09-149	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to calculate the surface area of a cylinder.	None	1. Draw the cylinders and diagrams from the Introduction to New Material, Guided Practice and Independent Practice on the board. 2. Draw the tables from the Closing section on the board.

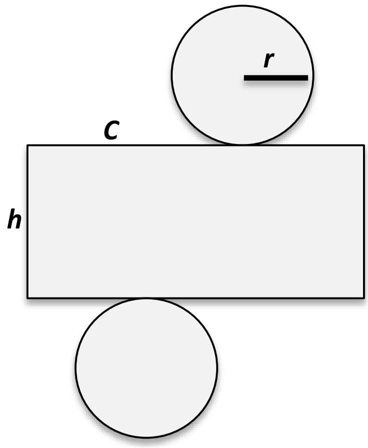
Opening *(2 minutes)*

1. **Write** the following questions on the board:
   a. What is a cylinder?
   b. What is the formula for the area of a circle?
   c. What is the formula for the circumference of a circle?
2. Ask 2 pupils to volunteer to share the answers. (Answers: (a) A cylinder is a solid with 2 parallel faces that are circles; (b) $A = \pi r^2$; (c) $C = 2\pi r$)
3. **Say**: Today we will practice calculating the surface area of a cylinder.



Introduction to the New Material *(13 minutes)*

1. Point to the cylinder on the board:
2. **Ask**: What is surface area? (Answer: The outside layer of a solid is called the *surface area*.)
3. Have pupils volunteer to answer.
4. Remind pupils that using a net can help us. A net is like a paper version of a solid that can be opened up and laid flat.
5. Have a pupil volunteer to come to the board and draw a net of the cylinder:



6. **Ask**: How can we calculate the surface area of the cylinder? (Answer: Add the areas of each of the shapes in the net, like a composite shape).
7. **Ask**: What is the formula for the surface area of a cylinder? (Answer: $SA = 2\pi r^2 + 2\pi rh$)
8. Let pupils volunteer to respond. Then **write** the formula on the board.
9. **Say**: To calculate the total surface area of a cylinder we add the area of the 2 circle surfaces on the ends to the area of the rectangle made by the circumference of the circle and the height.
10. Label the height of the cylinder 9 cm., and the radius 3 cm.
11. Ask pupils to calculate the surface area of the cylinder in their exercise books.
12. Calculate the surface area on the board:

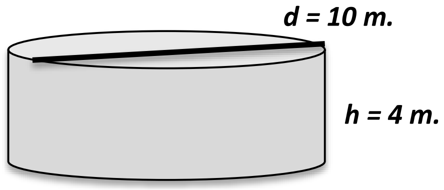
$$SA = 2\pi r^2 + 2\pi rh = 2 \times 3.14 \times (3 \text{ cm.})^2 + 2 \times 3.14 \times 3 \text{ cm.} \times 9 \text{ cm.}$$
$$= 56.52 \text{ cm.}^2 + 169.56 \text{ cm.}^2$$

$= 226.08 \text{ cm.}^2$

13. Remind pupils that surface area is measured in units squared, just like area.

Guided Practice *(6 minutes)*

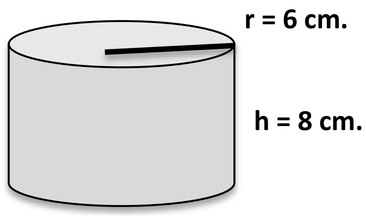
1. **Write** the following problem on the board: What is the surface area of a cylinder with height 4m., and diameter 10 m.?
2. Ask pupils to work in pairs and write their answers in their exercise books.
3. Move around the room and clear misconceptions.
4. Have one pair of pupils volunteer to come to the board and sketch and label the cylinder, while another pair solves the surface area and the class checks their work in their exercise books. Then correct any errors on the board.
   Answers: $r = \frac{d}{2} = \frac{10 \text{ m.}}{2} = 5\text{m.}$



$SA = 2\pi r^2 + 2\pi rh$
$= 2 \times 3.14 \times (5 \text{ m.})^2 + 2 \times 3.14 \times 5 \text{ m.} \times 4 \text{ m.}$
$= 157 \text{ m.}^2 + 125.6 \text{ m.}^2$
$= 282.6 \text{ m.}^2$

Independent Practice *(10 minutes)*

1. **Write** the following problem on the board: Find the surface area of a cylinder with height 8 cm., and radius 6 cm.
2. Ask pupils to work individually and write their answers in their exercise books.
3. Move around the room and clear any doubts pupils have.
4. Have a pupil volunteer to come to the board and share their answer while the rest of the class checks their work in their exercise books. (Answer: below)
5. **Write** the correct answer and steps on the board. Ask pupils to check their work.



$SA = 2\pi r^2 + 2\pi rh = 2 \times 3.14 \times (6 \text{ cm.})^2 + 2 \times 3.14 \times 6 \text{ cm.} \times 8 \text{ cm.}$
$= 226.08 \text{ cm.}^2 + 301.44 \text{ cm.}^2$
$= 527.52 \text{ cm.}^2$

Closing *(4 minutes)*

1. Point to the following chart on the board and ask pupils to copy it into their exercise books and fill it out.

Solid	Surface Area Formula
Cube	
Rectangular Prism	
Triangular Prism	
Cylinder	

2. Have 4 pupils volunteer to share the answers for each of the solids.

Solid	Surface Area Formula
Cube	$SA = 6l^2$
Rectangular Prism	$SA = 2lw + 2wh + 2l$
Triangular Prism	$SA = bh + (a + b + c)$
Cylinder	$SA = 2\pi r^2 + 2\pi rh$

Lesson Title: Practical Problems with Volume and Surface Area	Theme: Measurement and Estimation	
Lesson Number: M-09-150	**Class/Level**: JSS 3	**Time**: 35 minutes

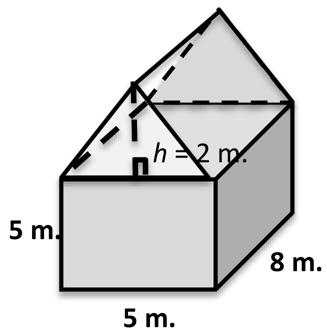
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Find the volume and surface area of composite shapes. 2. Solve multi-step word problems on volume and surface area.	None	1. Write the problems from Introduction to the New Material and Guided Practice on the board. 2. Draw the diagrams from the lesson plan on the board.

Opening *(2 minutes)*

1. Have pupils volunteer to call out the formulae for volume and surface area of the following solids:
   a. Cube
   b. Rectangular prism
   c. Triangular prism
   d. Cylinder
   (Answers: (a) Cube: $V = l^3$; $SA = 6l^2$; (b) Rectangular prism: $V = lwh$; $SA = 2lw + 2wh + 2lh$; (c) Triangular prism: $V = \frac{1}{2}bhl$; $SA = bh + (a + b + c)l$; (d) Cylinder: $V = \pi r^2 h$; $SA = 2\pi r^2 + 2\pi rh$)
2. **Say**: Today, we will learn how to find the volume and surface area of composite shapes and practice solving multi-step word problems on volume and surface area.

Introduction to the New Material *(14 minutes)*

1. Point to the following story problem on the board:
   Jebbeh is painting her storeroom. It is a rectangular prism with a triangular prism as the roof. The length is 8 m., width is 5 m., and height of the walls is 5 m.; the height of the triangular prism of the roof is 2 m. She wants to paint the outside walls so she needs to calculate how many square metres she has to cover, and find the full volume of the building.
2. Have 1 pupil volunteer to read the question aloud to the class.
3. Discuss the meaning of this question with pupils.
4. **Say:** To be able to understand story problems involving composite solids it can be helpful to draw the picture of the solids first, and then we apply the correct formulae to solve the problem.
5. Draw and label the storeroom on the board. →

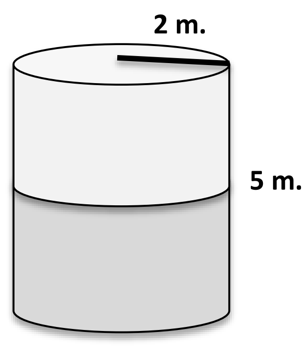


6. **Say:** First we need to find the surface area of the rectangular prism and then subtract the bottom and the top areas, since Jebbeh will not paint the ceiling or the floor. Then we need to find the volume for the 2 prisms.
7. **Ask**: What is the formula for the surface area of a rectangular prism? (Answer: $SA = 2lw + 2wh + 2lh$)
8. Let pupils raise their hands to answer.

9. **Say**: Jebbeh will only paint the walls, so we can subtract the top and bottom of the rectangular prism's surface area by leaving them out of the formula to begin with**.**
10. **Ask:** According to our diagram, what part of the solid is the ceiling and floor of the storeroom? (Answer: length x width)
11. Invite pupils to raise their hands to answer.
12. **Ask:** How can we change the formula to calculate just the parts we need? (Answer: $SA = 2wh + 2lh$)
13. Ask pupils to calculate the surface area of the walls of Jebbeh's house in their exercise books using the formula.
14. Calculate the surface area on the board:
    $SA = 2wh + 2lh = \ 2 \times 5 \text{ m.} \times 5 \text{ m.} + 2 \times 8 \text{ m.} \times 5 \text{ m.} = 50 \text{ m}^2 + 80 \text{ m}^2 = 130 \text{ m.}^2$
15. **Say:** The surface area of the house to be painted is 130 m.^2
16. **Say**: Now we must calculate the volume of the storeroom:
17. Ask pupils to discuss how do we find the volume of composite solids, guide them to understand that you find the individual volumes and then add them.
18. **Ask:** What are the solids that we must calculate the volume of to find the total volume of the storeroom? (Answer: a rectangular prism and a triangular prism)
19. Let pupils raise their hands to answer each of these questions.
20. **Ask:** What is the formula for the volume of a rectangular prism? (Answer: $V = lwh$)
21. Ask pupils to calculate the volume of the rectangular prism in their exercise books, while you do so on the board: $V = lwh = 8 \text{ m.} \times\ 5 \text{ m.} \times\ 5 \text{ m.} = 200 \text{ m.}^3$
22. **Ask:** What is the formula for volume of a triangular prism? (Answer: $V = \frac{1}{2}bhl$)
23. Ask pupils to calculate the volume of the triangular prism in their exercise books, while you do so on the board: $V = \frac{1}{2}bhl = \frac{1}{2} \times 5 \text{ m.} \times\ 2 \text{ m.}\ \ \times\ 8 \text{ m.} = 40 \text{ m.}^3$
24. **Say**: Now we must add together the 2 volumes.
25. **Ask**: What is the total volume of Jebbeh's storeroom? (Answer: $40 \text{ m.}^3 = 200 \text{ m.}^3 = 240 \text{ m.}^3$)

Guided Practice *(5 minutes)*

1. Point to the following story problem on the board:
   A water tank with a height of 5 m. is in the shape of a cylinder with radius 2 m. The tank is half full of water. Calculate the volume of the water in the tank. 1 can of paint covers 30 m.^{2}; calculate how many cans of paint would be needed to paint the whole tank.



2. Have 1 pupil volunteer to read the question aloud to the class.
3. Ask pupils to discuss how to solve the problem with their partners: calculate the volume first and divide in half; and guide them to understand that they need to calculate the surface area of the cylinder and then divide by the number of m^2 per can.
4. Ask pupils to work in pairs to solve the problems in their exercise books. Remind pupils to draw a diagram of the solids.
5. Have 2 pairs of pupils volunteer to come to the board and share their answers. Have 1 pair calculate the volume and 1 calculate the surface area. Correct any errors they make on the board and ask all pupils to check their work.

(Answers: $V = \pi r^2 h = 3.14 \times (2\text{ m.})^2 \times 5\text{ m.} = 62.8\text{ m.}^3$

$$62.8\text{ m.}^3 \div 2 = 31.4\text{ m.}^3$$

$$SA = 2\pi r^2 + 2\pi rh = 2 \times 3.14 \times (2\text{ m.})^2 + 2 \times 3.14 \times 2\text{ m.} \times 5\text{ m.}$$
$$= 25.12\text{ m.}^2 + 62.8\text{ m.}^2 = 87.92\text{ m.}^2$$
$$= 87.92\text{ m.}^2 \div 30\text{ m.}^2 = 2.93 \approx 3\text{ cans}$$

Independent Practice *(10 minutes)*

1. Choose 1-2 problems from the question bank on the next page (based on your pupils' level of understanding) and write them on the board.
2. Ask pupils to work independently to find the answers.
3. Walk around to check for understanding and clear any doubts.
4. Have 1-2 pupils volunteer to come solve the problems on the board.
5. **Write** the correct answers and steps on the board. Ask pupils to check their work.

Closing *(4 minutes)*

1. Ask pupils to write their own story problem using the lesson as an example.
2. Allow pupils to share their story problems with partners, or the class, if time allows.
3. Move around the room and assist pupils as necessary.
4. Remind pupils they can choose basic or composite solids, and to make sure they give the needed information – such as length and width, or radius.

[QUESTION BANK]

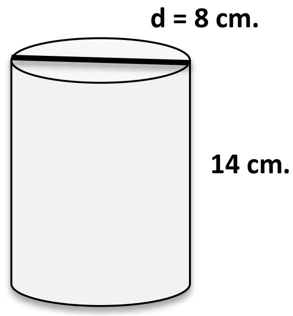
1. A soda company puts labels on their cylinder-shaped drink bottles. The drink bottles are 14 cm. tall with a diameter of 8 cm. (a) What is the surface area of the label the company needs to print so that it will cover the bottle from top to bottom? (b) What is the volume of the soda cans?

Answer:

(a) $SA = 2\pi r^2 + 2\pi rh$; but since the label will not be on the top or bottom surfaces, we use only $SA = 2\pi rh$
$= 2 \times 3.14 \times 4\text{ cm.} \times 14\text{ cm.}$
$= 351.68\text{ cm.}^2$

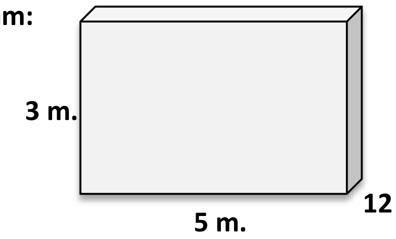
(b) $V = \pi r^2 h$
$= 3.14 \times (4\text{ cm.})^2 \times 14\text{ cm.}$
$= 703.36\text{ cm.}^3$

Diagram:



2. Gbessay is building and painting a cement wall measuring 5 m. long, 12 cm. wide, and 3 m. high. (a) What volume of cement does she need? (b) How much paint does she need to cover the surface area if 1 can covers 30 m.^{2}?

Answer: **Diagram:**



$1 \text{ m.} = 100 \text{ cm.}$

$(a)\ V = lwh = 500 \text{ cm.}\ \times\ 12 \text{ cm.}\ \times\ 300 \text{ cm.} = 1{,}800{,}000 \text{ cm.}^3$

$1{,}800{,}000 \text{ cm.}^3 \div 1{,}000{,}000 = 1.8 \text{ m}^3$

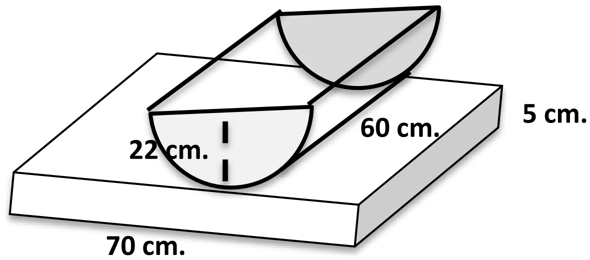
$OR: V = lwh = 5 \text{ m.}\ \times\ 0.12 \text{ m.}\ \times\ 3 \text{ m.} = 1.8 \text{ m.}^3$

(b) $SA = 2lw + 2wh + 2lh$

$= 1 \times 500 \text{ cm.} \times 12 \text{ cm.} + 2 \times 12 \text{ cm.} \times 300 \text{ cm.} + 2 \times 500 \text{ cm.} \times 300 \text{ cm.}$
$= 6{,}000 \text{ cm}^2 + 7{,}200 \text{ cm}^2 + 300{,}000 \text{ cm}^2 = 313{,}200 \text{ cm.}^2 \div 10{,}000 = 31.32 \text{ m.}^2$

$31.32 \text{ m.}^2 \div 30 \text{ m.}^2 = 1.04 \approx 2\ cans\ of\ paint\ needed$

3. Samuel has a water trough in the shape of a half cylinder that is 60 cm. long and has a radius of 22 cm. The trough stands on a block that is a rectangular prism measuring 60 cm. long, 70 cm. wide, and 5 cm. high. (a) What is the volume of the trough? (b) What is the surface area of the rectangular prism above the ground?



Answer:

Diagram

$a. V = \pi r^2 h = 3.14 \times (22 \text{ cm.})^2 \times 60 \text{ cm.} = 91{,}185.6 \text{ cm.}^3$
$91{,}185.6 \text{ cm.}^3 \div 2 = 45{,}592.8 \text{ cm.}^3$

$b. SA = 2lw + 2wh + 2lh$
$= 1 \times 60 \text{ cm.} \times 70 \text{ cm.} + 2 \times 70 \text{ cm.} \times 5 \text{ cm.} + 2 \times 60 \text{ cm.} \times 5 \text{ cm.}$
$= 4{,}200 \text{ cm}^2 + 700 \text{ cm}^2 + 600 \text{ cm}^2 = 5{,}500 \text{ cm.}^2$

Appendix I: Squares of Numbers 10-100

Squares of Numbers

$x \to x^2$

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1000	1020	1040	1061	1082	1103	1124	1145	1166	1188	2	4	6	8	10	13	15	17	19
11	1210	1232	1254	1277	1300	1323	1346	1369	1392	1416	2	5	7	9	11	14	16	18	21
12	1440	1464	1488	1513	1538	1563	1588	1613	1638	1664	2	5	7	10	12	15	17	20	22
13	1690	1716	1742	1769	1796	1823	1850	1877	1904	1932	3	5	8	11	13	16	19	22	24
14	1960	1988	2016	2045	2074	2103	2132	2161	2190	2220	3	6	9	12	14	17	20	23	26
15	2250	2280	2310	2341	2372	2403	2434	2465	2496	2528	3	6	9	12	15	19	22	25	28
16	2560	2592	2624	2657	2690	2723	2756	2789	2822	2856	3	7	10	13	16	20	23	26	30
17	2890	2924	2958	2993	3028	3063	3098	3133	3168	3204	3	7	10	14	17	21	24	28	31
18	3240	3276	3312	3349	3386	3423	3460	3497	3534	3572	4	7	11	15	18	22	26	30	33
19	3610	3648	3686	3725	3764	3803	3842	3881	3920	3960	4	8	12	16	19	23	27	31	35
20	4000	4040	4080	4121	4162	4203	4244	4285	4326	4368	4	8	12	16	20	25	29	33	37
21	4410	4452	4494	4537	4580	4623	4666	4709	4752	4796	4	9	13	17	21	26	30	34	39
22	4840	4884	4928	4973	5018	5063	5108	5153	5198	5244	4	9	13	18	22	27	31	36	40
23	5290	5336	5382	5429	5476	5523	5570	5617	5664	5712	5	9	14	19	23	28	33	38	42
24	5760	5808	5856	5905	5954	6003	6052	6101	6150	6200	5	10	15	20	24	29	34	39	44
25	6250	6300	6350	6401	6452	6503	6554	6605	6656	6708	5	10	15	20	25	31	36	41	46
26	6760	6812	6864	6917	6970	7023	7076	7129	7182	7236	5	11	16	21	26	32	37	42	48
27	7290	7344	7398	7453	7508	7563	7618	7673	7728	7784	5	11	16	22	27	33	38	44	49
28	7840	7896	7952	8009	8066	8123	8180	8237	8294	8352	6	11	17	23	28	34	40	46	51
29	8410	8486	8526	8585	8644	8703	8762	8821	8880	8940	6	12	18	24	29	35	41	47	53
30	9000	9060	9120	9181	9242	9303	9364	9425	9486	9548	6	12	18	24	30	37	43	49	55
31	9610	9672	9734	9797	9860	9923	9986	–	–	–	6	13	19	25	31	38	44	50	57
31	–	–	–	–	–	–	–	1005	1011	1018	1	1	2	3	3	4	5	5	6
32	1024	1030	1037	1043	1050	1056	1063	1069	1076	1082	1	1	2	3	3	4	5	5	6
33	1089	1096	1102	1109	1116	1122	1129	1136	1142	1149	1	1	2	3	3	4	5	5	6
34	1156	1163	1170	1176	1183	1190	1197	1204	1211	1218	1	1	2	3	3	4	5	6	6
35	1225	1232	1239	1246	1253	1260	1267	1274	1282	1289	1	1	2	3	4	4	5	6	6
36	1296	1303	1310	1318	1325	1332	1340	1347	1354	1362	1	1	2	3	4	4	5	6	7
37	1369	1376	1384	1391	1399	1406	1414	1421	1429	1436	1	2	2	3	4	5	5	6	7
38	1444	1452	1459	1467	1475	1482	1490	1498	1505	1513	1	2	2	3	4	5	5	6	7
39	1521	1529	1537	1544	1552	1560	1568	1576	1584	1592	1	2	2	3	4	5	6	6	7
40	1600	1608	1616	1624	1632	1640	1648	1656	1665	1673	1	2	2	3	4	5	6	6	7
41	1681	1689	1697	1706	1714	1722	1731	1739	1747	1756	1	2	2	3	4	5	6	7	7
42	1764	1772	1781	1789	1798	1806	1815	1823	1832	1840	1	2	3	3	4	5	6	7	8
43	1849	1858	1866	1875	1884	1892	1901	1910	1918	1927	1	2	3	3	4	5	6	7	8
44	1936	1945	1954	1962	1971	1980	1989	1998	2007	2016	1	2	3	4	5	5	6	7	8
45	2025	2034	2043	2052	2061	2070	2079	2088	2098	2107	1	2	3	4	5	5	6	7	8
46	2116	2125	2134	2144	2153	2162	2172	2181	2190	2200	1	2	3	4	5	6	7	7	8
47	2209	2218	2228	2237	2247	2256	2266	2275	2285	2294	1	2	3	4	5	6	7	8	9
48	2304	2314	2323	2333	2343	2352	2362	2372	2381	2391	1	2	3	4	5	6	7	8	9
49	2401	2411	2421	2430	2440	2450	2460	2470	2480	2490	1	2	3	4	5	6	7	8	9
50	2500	2510	2520	2530	2540	2550	2560	2570	2581	2591	1	2	3	4	5	6	7	8	9
51	2601	2611	2621	2632	2642	2652	2663	2673	2683	2694	1	2	3	4	5	6	7	8	9
52	2704	2714	2725	2735	2746	2756	2767	2777	2788	2798	1	2	3	4	5	6	7	8	9
53	2809	2820	2830	2841	2852	2862	2873	2884	2894	2905	1	2	3	4	5	6	7	9	10
54	2916	2927	2938	2948	2959	2970	2981	2992	3003	3014	1	2	3	4	5	7	8	9	10
55	3025	3036	3047	3058	3069	3080	3091	3102	3114	3125	1	2	3	4	6	7	8	9	10
56	3136	3147	3158	3170	3181	3192	3204	3215	3226	3238	1	2	3	5	6	7	8	9	10
57	3249	3260	3272	3283	3295	3306	3318	3329	3341	3352	1	2	3	5	6	7	8	9	10
58	3364	3376	3387	3399	3411	3422	3434	3446	3457	3469	1	2	4	5	6	7	8	9	10
59	3481	3493	3505	3516	3528	3540	3552	3564	3576	3588	1	2	4	5	6	7	8	10	11
60	3600	3612	3624	3636	3648	3660	3672	3684	3697	3709	1	2	4	5	6	7	8	10	11
61	3721	3733	3745	3758	3770	3782	3795	3807	3819	3832	1	2	4	5	6	7	9	10	11
62	3844	3856	3869	3881	3894	3906	3919	3931	3944	3956	1	2	4	5	6	7	9	10	11
63	3969	3982	3994	4007	4020	4032	4054	4058	4070	4083	1	3	4	5	6	8	9	10	11
64	4096	4109	4122	4134	4147	4160	4173	4186	4199	4212	1	3	4	5	6	8	9	10	12
65	4225	4238	4251	4264	4277	4290	4303	4316	4330	4343	1	3	4	5	7	8	9	10	12
66	4356	4369	4382	4396	4409	4422	4436	4449	4462	4476	1	3	4	5	7	8	9	11	12
67	4489	4502	4516	4529	4543	4556	4570	4583	4597	4610	1	3	4	5	7	8	9	11	12
68	4624	4638	4651	4665	4679	4692	4706	4720	4733	4747	1	3	4	5	7	8	10	11	12
69	4761	4775	4789	4802	4816	4830	4844	4858	4872	4886	1	3	4	6	7	8	10	11	13
70	4900	4914	4928	4942	4956	4970	4984	4998	5013	5027	1	3	4	6	7	8	10	11	13
71	5041	5055	5069	5084	5098	5112	5127	5141	5155	5170	1	3	4	6	7	9	10	11	13
72	5184	5198	5213	5227	5242	5256	5271	5285	5300	5314	1	3	4	6	7	9	10	11	13
73	5329	5344	5358	5373	5388	5402	5417	5432	5446	5461	1	3	4	6	7	9	10	12	13
74	5476	5491	5506	5520	5535	5550	5565	5580	5595	5610	1	3	4	6	7	9	10	12	13
75	5625	5640	5655	5670	5685	5700	5715	5730	5746	5761	2	3	5	6	8	9	11	12	14
76	5776	5791	5806	5822	5837	5852	5868	5883	5898	5914	2	3	5	6	8	9	11	12	14
77	5929	5944	5960	5975	5991	6006	6022	6037	6053	6068	2	3	5	6	8	9	11	12	14
78	6084	6100	6115	6131	6147	6162	6178	6194	6209	6225	2	3	5	6	8	9	11	13	14
79	6241	6257	6273	6288	6304	6320	6336	6352	6368	6384	2	3	5	6	8	10	11	13	14
80	6400	6416	6432	6448	6464	6480	6496	6512	6529	6545	2	3	5	6	8	10	11	13	14
81	6561	6577	6593	6610	6626	6642	6659	6675	6691	6708	2	3	5	7	8	10	11	13	15
82	6724	6740	6757	6773	6790	6806	6823	6839	6856	6872	2	3	5	7	8	10	12	13	15
83	6889	6906	6922	6939	6956	6972	6989	7006	7022	7039	2	3	5	7	8	10	12	13	15
84	7056	7073	7090	7106	7123	7140	7157	7174	7191	7208	2	3	5	7	8	10	12	14	15
85	7225	7242	7259	7276	7293	7310	7327	7344	7362	7379	2	3	5	7	9	10	12	14	15
86	7396	7413	7430	7448	7465	7482	7500	7517	7534	7552	2	3	5	7	9	10	12	14	16
87	7569	7586	7604	7621	7639	7656	7674	7691	7709	7726	2	3	5	7	9	10	12	14	16
88	7744	7762	7779	7797	7815	7832	7850	7868	7885	7903	2	4	5	7	9	11	12	14	16
89	7921	7939	7957	7974	7992	8010	8028	8046	8064	8082	2	4	5	7	9	11	13	14	16
90	8100	8118	8136	8154	8172	8190	8208	8226	8245	8263	2	4	5	7	9	11	13	14	16
91	8281	8299	8317	8336	8354	8372	8391	8409	8427	8446	2	4	5	7	9	11	13	15	16
92	8464	8482	8501	8519	8538	8556	8575	8593	8612	8630	2	4	6	7	9	11	13	15	17
93	8649	8668	8686	8705	8724	8742	8761	8780	8798	8817	2	4	6	7	9	11	13	15	17
94	8836	8855	8874	8892	8911	8930	8949	8968	8987	9006	2	4	6	8	9	11	13	15	17
95	9025	9044	9063	9082	9101	9120	9139	9158	9178	9197	2	4	6	8	10	11	13	15	17
96	9216	9235	9254	9274	9293	9312	9332	9351	9370	9390	2	4	6	8	10	12	14	15	17
97	9409	9428	9448	9467	9487	9506	9526	9545	9565	9584	2	4	6	8	10	12	14	16	18
98	9604	9624	9643	9663	9683	9702	9722	9742	9761	9781	2	4	6	8	10	12	14	16	18
99	9801	9821	9841	9860	9880	9900	9920	9940	9960	9980	2	4	6	8	10	12	14	16	18
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Sines of Angles (x in degrees)

$x \rightarrow \sin x$

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	2	3	5	7	9	10	12	14	16
1	0·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	2	3	5	7	9	10	12	14	16
2	0·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	2	3	5	7	9	10	12	14	16
3	0·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	2	3	5	7	9	10	12	14	16
4	0·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	2	3	5	7	9	10	12	14	16
5	0·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	2	3	5	7	9	10	12	14	16
6	0·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	2	3	5	7	9	10	12	14	16
7	0·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	2	3	5	7	9	10	12	14	16
8	0·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	2	3	5	7	9	10	12	14	16
9	0·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	2	3	5	7	9	10	12	14	15
10	0·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	2	3	5	7	9	10	12	14	15
11	0·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2	3	5	7	9	10	12	14	15
12	0·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2	3	5	7	9	10	12	14	15
13	0·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2	3	5	7	8	10	12	14	15
14	0·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2	3	5	7	8	10	12	14	15
15	0·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2	3	5	7	8	10	12	13	15
16	0·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2	3	5	7	8	10	12	13	15
17	0·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	2	3	5	7	8	10	12	13	15
18	0·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	2	3	5	7	8	10	12	13	15
19	0·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	2	3	5	7	8	10	12	13	15
20	0·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	2	3	5	7	8	10	11	13	15
21	0·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	2	3	5	6	8	10	11	13	15
22	0·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	2	3	5	6	8	10	11	13	15
23	0·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	2	3	5	6	8	10	11	13	14
24	0·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	2	3	5	6	8	10	11	13	14
25	0·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	2	3	5	6	8	9	11	13	14
26	0·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	2	3	5	5	8	9	11	12	14
27	0·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	2	3	5	6	8	9	11	12	14
28	0·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	2	3	5	6	8	9	11	12	14
29	0·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	2	3	5	6	8	9	11	12	14
30	0·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	2	3	5	6	8	9	11	12	14
31	0·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	1	3	4	6	7	9	10	12	13
32	0·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	1	3	4	6	7	9	10	12	13
33	0·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	1	3	4	6	7	9	10	12	13
34	0·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	1	3	4	6	7	9	10	12	13
35	0·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	1	3	4	6	7	9	10	11	13
36	0·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	1	3	4	6	7	8	10	11	13
37	0·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	1	3	4	6	7	8	10	11	12
38	0·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	1	3	4	5	7	8	10	11	12
39	0·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	1	3	4	5	7	8	9	11	12
40	0·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	1	3	4	5	7	8	9	11	12
41	0·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	1	3	4	5	7	8	9	10	12
42	0·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	1	3	4	5	6	8	9	10	12
43	0·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	1	3	4	5	6	8	9	10	11
44	0·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
45	0·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	1	2	4	5	6	7	9	10	11
46	0·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	1	2	4	5	6	7	8	10	11
47	0·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	1	2	4	5	6	7	8	9	11
48	0·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	1	2	3	5	6	7	8	9	10
49	0·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	1	2	3	5	6	7	8	9	10
50	0·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	1	2	3	4	6	7	8	9	10
51	0·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	1	2	3	4	5	7	8	9	10
52	0·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	1	2	3	4	5	6	7	8	10
53	0·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	1	2	3	4	5	6	7	8	9
54	0·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	1	2	3	4	5	6	7	8	9
55	0·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	1	2	3	4	5	6	7	8	9
56	0·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	1	2	3	4	5	6	7	8	9
57	0·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	1	2	3	4	5	6	7	8	8
58	0·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	1	2	3	4	5	5	6	7	8
59	0·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	2	3	4	4	5	6	7	8
60	0·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	2	3	3	4	5	6	7	8
61	0·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	2	2	3	4	5	6	7	7
62	0·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	2	2	3	4	5	6	6	7
63	0·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	2	2	3	4	5	5	6	7
64	0·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	2	2	3	4	5	5	6	7
65	0·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	1	2	3	4	4	5	6	7
66	0·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	1	2	3	3	4	5	6	6
67	0·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	1	2	3	3	4	5	5	6
68	0·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	1	2	3	3	4	4	5	6
69	0·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	1	2	2	3	4	4	5	6
70	0·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	1	2	2	3	3	4	5	5
71	0·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	1	2	2	3	3	4	4	5
72	0·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	1	2	2	3	3	4	4	5
73	0·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	0	1	1	2	2	3	3	4	4
74	0·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0	1	1	2	2	3	3	4	4
75	0·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	0	1	1	2	2	3	3	3	4
76	0·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	0	1	1	2	2	2	3	3	4
77	0·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	0	1	1	2	2	2	3	3	3
78	0·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	0	1	1	1	2	2	2	3	3
79	0·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0	1	1	1	2	2	2	3	3
80	0·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	1	1	2	2	2	3
81	0·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	1	1	2	2	2	2
82	0·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	0	1	1	1	1	2	2	2
83	0·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	0	1	1	1	1	1	2	2
84	0·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	0	1	1	1	1	1	1	2
85	0·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	0	1	1	1	1	1	1
86	0·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	0	0	1	1	1	1	1
87	0·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	0	0	0	1	1	1
88	0·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998									
89	0·9998	9999	9999	9999	9999	1·0000	1·0000	1·0000	1·0000	1·0000									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix III: Cosines of Angles

Cosines of Angles (x in degrees)

$x \rightarrow \cos x$

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	SUBTRACT Differences 1	2	3	4	5	6	7	8	9
0	1·0000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9999									
1	0·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995									
2	0·9994	·9993	·9993	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	0	0	0	1	1	1
3	0·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	0	0	1	1	1	1	1
4	0·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	0	1	1	1	1	1	1
5	0·9962	·9960	·9959	·9957	·9956	·9954	·9952	·9951	·9949	·9947	0	0	1	1	1	1	1	1	2
6	0·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	0	1	1	1	1	1	2	2
7	0·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	0	1	1	1	1	2	2	2
8	0·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	1	1	2	2	2	2
9	0·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	1	1	2	2	2	3
10	0·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	0	1	1	1	2	2	2	3	3
11	0·9816	·9813	·9810	·9806	·9803	·9799	·9796	·9792	·9789	·9785	0	1	1	1	2	2	2	3	3
12	0·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	0	1	1	2	2	2	3	3	3
13	0·9744	·9740	·9736	·9732	·9728	·9724	·9720	·9715	·9711	·9707	0	1	1	2	2	2	3	3	4
14	0·9703	·9699	·9694	·9690	·9686	·9681	·9677	·9673	·9668	·9664	0	1	1	2	2	3	3	3	4
15	0·9659	·9655	·9650	·9646	·9641	·9636	·9632	·9627	·9622	·9617	0	1	1	2	2	3	3	4	4
16	0·9613	·9608	·9603	·9598	·9593	·9588	·9583	·9578	·9573	·9568	0	1	1	2	2	3	3	4	4
17	0·9563	·9558	·9553	·9548	·9542	·9537	·9532	·9527	·9521	·9516	1	1	2	2	3	3	4	4	5
18	0·9511	·9505	·9500	·9494	·9489	·9483	·9478	·9472	·9466	·9461	1	1	2	2	3	3	4	4	5
19	0·9455	·9449	·9444	·9438	·9432	·9426	·9421	·9415	·9409	·9403	1	1	2	2	3	3	4	5	5
20	0·9397	·9391	·9385	·9379	·9373	·9367	·9361	·9354	·9348	·9342	1	1	2	2	3	4	4	5	6
21	0·9336	·9330	·9323	·9317	·9311	·9304	·9298	·9291	·9285	·9278	1	1	2	3	3	4	4	5	6
22	0·9272	·9265	·9259	·9252	·9245	·9239	·9232	·9225	·9219	·9212	1	1	2	3	3	4	5	5	6
23	0·9205	·9198	·9191	·9184	·9178	·9171	·9164	·9157	·9150	·9143	1	1	2	3	3	4	5	6	6
24	0·9135	·9128	·9121	·9114	·9107	·9100	·9092	·9085	·9078	·9070	1	1	2	3	4	4	5	6	7
25	0·9063	·9056	·9048	·9041	·9033	·9026	·9018	·9011	·9003	·8996	1	2	2	3	4	5	5	6	7
26	0·8988	·8980	·8973	·8965	·8957	·8949	·8942	·8934	·8926	·8918	1	2	2	3	4	5	5	6	7
27	0·8910	·8902	·8894	·8886	·8878	·8870	·8862	·8854	·8846	·8838	1	2	2	3	4	5	6	6	7
28	0·8829	·8821	·8813	·8805	·8796	·8788	·8780	·8771	·8763	·8755	1	2	2	3	4	5	6	7	7
29	0·8746	·8738	·8729	·8721	·8712	·8704	·8695	·8686	·8678	·8669	1	2	3	3	4	5	6	7	8
30	0·8660	·8652	·8643	·8634	·8625	·8616	·8607	·8599	·8590	·8581	1	2	3	4	4	5	6	7	8
31	0·8572	·8563	·8554	·8545	·8536	·8526	·8517	·8508	·8499	·8490	1	2	3	4	5	5	6	7	8
32	0·8480	·8471	·8462	·8453	·8443	·8434	·8425	·8415	·8406	·8396	1	2	3	4	5	6	7	8	8
33	0·8387	·8377	·8368	·8358	·8348	·8339	·8329	·8320	·8310	·8300	1	2	3	4	5	6	7	8	9
34	0·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	1	2	3	4	5	6	7	8	9
35	0·8192	·8181	·8171	·8161	·8151	·8141	·8131	·8121	·8111	·8100	1	2	3	4	5	6	7	8	9
36	0·8090	·8080	·8070	·8059	·8049	·8039	·8028	·8018	·8007	·7997	1	2	3	4	5	6	7	8	9
37	0·7986	·7976	·7965	·7955	·7944	·7934	·7923	·7912	·7902	·7891	1	2	3	4	5	6	7	8	10
38	0·7880	·7869	·7859	·7848	·7837	·7826	·7815	·7804	·7793	·7782	1	2	3	4	5	6	7	9	10
39	0·7771	·7760	·7749	·7738	·7727	·7716	·7705	·7694	·7683	·7672	1	2	3	4	6	7	8	9	10
40	0·7660	·7649	·7638	·7627	·7615	·7604	·7593	·7581	·7570	·7559	1	2	3	5	6	7	8	9	10
41	0·7547	·7536	·7524	·7513	·7501	·7490	·7478	·7466	·7455	·7443	1	2	3	5	6	7	8	9	10
42	0·7431	·7420	·7408	·7396	·7385	·7373	·7361	·7349	·7337	·7325	1	2	4	5	6	7	8	9	11
43	0·7314	·7302	·7290	·7278	·7266	·7254	·7242	·7230	·7218	·7206	1	2	4	5	6	7	8	10	11
44	0·7193	·7181	·7169	·7157	·7145	·7133	·7120	·7108	·7096	·7083	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	SUBTRACT Differences 1	2	3	4	5	6	7	8	9
45	0·7071	·7059	·7046	·7034	·7022	·7009	·6997	·6984	·6972	·6959	1	2	4	5	6	7	9	10	11
46	0·6947	·6934	·6921	·6909	·6896	·6884	·6871	·6858	·6845	·6833	1	3	4	5	6	8	9	10	11
47	0·6820	·6807	·6794	·6782	·6769	·6756	·6743	·6730	·6717	·6704	1	3	4	5	6	8	9	10	12
48	0·6691	·6678	·6665	·6652	·6639	·6626	·6613	·6600	·6587	·6574	1	3	4	5	7	8	9	10	12
49	0·6561	·6547	·6534	·6521	·6508	·6494	·6481	·6468	·6455	·6441	1	3	4	5	7	8	9	11	12
50	0·6428	·6414	·6401	·6388	·6374	·6361	·6347	·6334	·6320	·6307	1	3	4	5	7	8	9	11	12
51	0·6293	·6280	·6266	·6252	·6239	·6225	·6211	·6198	·6184	·6170	1	3	4	5	7	8	10	11	12
52	0·6157	·6143	·6129	·6115	·6101	·6088	·6074	·6060	·6046	·6032	1	3	4	5	7	8	10	11	12
53	0·6018	·6004	·5990	·5976	·5962	·5948	·5934	·5920	·5906	·5892	1	3	4	6	7	8	10	11	13
54	0·5878	·5864	·5850	·5835	·5821	·5807	·5793	·5779	·5764	·5750	1	3	4	6	7	9	10	11	13
55	0·5736	·5721	·5707	·5693	·5678	·5664	·5650	·5635	·5621	·5606	1	3	4	6	7	9	10	12	13
56	0·5592	·5577	·5563	·5548	·5534	·5519	·5505	·5490	·5476	·5461	1	3	4	6	7	9	10	12	13
57	0·5446	·5432	·5417	·5402	·5388	·5373	·5358	·5344	·5329	·5314	1	3	4	6	7	9	10	12	13
58	0·5299	·5284	·5270	·5255	·5240	·5225	·5210	·5195	·5180	·5165	1	3	4	6	7	9	10	12	13
59	0·5150	·5135	·5120	·5105	·5090	·5075	·5060	·5045	·5030	·5015	2	3	5	6	8	9	11	12	14
60	0·5000	·4985	·4970	·4955	·4939	·4924	·4909	·4894	·4879	·4863	2	3	5	6	8	9	11	12	14
61	0·4848	·4833	·4818	·4802	·4787	·4772	·4756	·4741	·4726	·4710	2	3	5	6	8	9	11	12	14
62	0·4695	·4679	·4664	·4648	·4633	·4617	·4602	·4586	·4571	·4555	2	3	5	6	8	9	11	12	14
63	0·4540	·4524	·4509	·4493	·4478	·4462	·4446	·4431	·4415	·4399	2	3	5	6	8	9	11	12	14
64	0·4384	·4368	·4352	·4337	·4321	·4305	·4289	·4274	·4258	·4242	2	3	5	6	8	9	11	13	14
65	0·4226	·4210	·4195	·4179	·4163	·4147	·4131	·4115	·4099	·4083	2	3	5	6	8	10	11	13	14
66	0·4067	·4051	·4035	·4019	·4003	·3987	·3971	·3955	·3939	·3923	2	3	5	6	8	10	11	13	14
67	0·3907	·3891	·3875	·3859	·3843	·3827	·3811	·3795	·3778	·3762	2	3	5	6	8	10	11	13	15
68	0·3746	·3730	·3714	·3697	·3681	·3665	·3649	·3633	·3616	·3600	2	3	5	6	8	10	11	13	15
69	0·3584	·3567	·3551	·3535	·3518	·3502	·3486	·3469	·3453	·3437	2	3	5	6	8	10	11	13	15
70	0·3420	·3404	·3387	·3371	·3355	·3338	·3322	·3305	·3289	·3272	2	3	5	7	8	10	12	13	15
71	0·3256	·3239	·3223	·3206	·3190	·3173	·3156	·3140	·3123	·3107	2	3	5	7	8	10	12	13	15
72	0·3090	·3074	·3057	·3040	·3024	·3007	·2990	·2974	·2957	·2940	2	3	5	7	8	10	12	13	15
73	0·2924	·2907	·2890	·2874	·2857	·2840	·2823	·2807	·2790	·2773	2	3	5	7	8	10	12	13	15
74	0·2756	·2740	·2723	·2706	·2689	·2672	·2656	·2639	·2622	·2605	2	3	5	7	8	10	12	13	15
75	0·2588	·2571	·2554	·2538	·2521	·2504	·2487	·2470	·2453	·2436	2	3	5	7	8	10	12	14	15
76	0·2419	·2402	·2385	·2368	·2351	·2334	·2317	·2300	·2284	·2267	2	3	5	7	8	10	12	14	15
77	0·2250	·2233	·2215	·2198	·2181	·2164	·2147	·2130	·2113	·2096	2	3	5	7	9	10	12	13	15
78	0·2079	·2062	·2045	·2028	·2011	·1994	·1977	·1959	·1942	·1925	2	3	5	7	9	10	12	14	15
79	0·1908	·1891	·1874	·1857	·1840	·1822	·1805	·1788	·1771	·1754	2	3	5	7	9	10	12	14	15
80	0·1736	·1719	·1702	·1685	·1668	·1650	·1633	·1616	·1599	·1582	2	3	5	7	9	10	12	14	15
81	0·1564	·1547	·1530	·1513	·1495	·1478	·1461	·1444	·1426	·1409	2	3	5	7	9	10	12	14	16
82	0·1392	·1374	·1357	·1340	·1323	·1305	·1288	·1271	·1253	·1236	2	3	5	7	9	10	12	14	16
83	0·1219	·1201	·1184	·1167	·1149	·1132	·1115	·1097	·1080	·1063	2	3	5	7	9	10	12	14	16
84	0·1045	·1028	·1011	·0993	·0976	·0958	·0941	·0924	·0906	·0889	2	3	5	7	9	10	12	14	16
85	0·0872	·0854	·0837	·0819	·0802	·0785	·0767	·0750	·0732	·0715	2	3	5	7	9	10	12	14	16
86	0·0698	·0680	·0663	·0645	·0628	·0610	·0593	·0576	·0558	·0541	2	3	5	7	9	10	12	14	16
87	0·0523	·0506	·0488	·0471	·0454	·0436	·0419	·0401	·0384	·0366	2	3	5	7	9	10	12	14	16
88	0·0349	·0332	·0314	·0297	·0279	·0262	·0244	·0227	·0209	·0192	2	3	5	7	9	10	12	14	16
89	0·0175	·0157	·0140	·0122	·0105	·0087	·0070	·0052	·0035	·0017	2	3	5	7	9	10	12	14	16
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix IV: Tangents of Angles

$x \rightarrow \tan x$

Tangents of Angles (x in degrees)

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences 1	2	3	4	5	6	7	8	9
0	0·0000	·0017	·0035	·0052	·0070	·0087	·0105	·0122	·0140	·0157	2	3	5	7	9	10	12	14	16
1	0·0175	·0192	·0209	·0227	·0244	·0262	·0279	·0297	·0314	·0332	2	3	5	7	9	10	12	14	16
2	0·0349	·0367	·0384	·0402	·0419	·0437	·0454	·0472	·0489	·0507	2	3	5	7	9	10	12	14	16
3	0·0524	·0542	·0559	·0577	·0594	·0612	·0629	·0647	·0664	·0682	2	4	5	7	9	11	12	14	16
4	0·0699	·0717	·0734	·0752	·0769	·0787	·0805	·0822	·0840	·0857	2	4	5	7	9	11	12	14	16
5	0·0875	·0892	·0910	·0928	·0945	·0963	·0981	·0998	·1016	·1033	2	4	5	7	9	11	12	14	16
6	0·1051	·1069	·1086	·1104	·1122	·1139	·1157	·1175	·1192	·1210	2	4	5	7	9	11	12	14	16
7	0·1228	·1246	·1263	·1281	·1299	·1317	·1334	·1352	·1370	·1388	2	4	5	7	9	11	12	14	16
8	0·1405	·1423	·1441	·1459	·1477	·1495	·1512	·1530	·1548	·1566	2	4	5	7	9	11	12	14	16
9	0·1584	·1602	·1620	·1638	·1655	·1673	·1691	·1709	·1727	·1745	2	4	5	7	9	11	13	14	16
10	0·1763	·1781	·1799	·1817	·1835	·1853	·1871	·1890	·1908	·1926	2	4	5	7	9	11	13	14	16
11	0·1944	·1962	·1980	·1998	·2016	·2035	·2053	·2071	·2089	·2107	2	4	5	7	9	11	13	15	16
12	0·2126	·2144	·2162	·2180	·2199	·2217	·2235	·2254	·2272	·2290	2	4	5	7	9	11	13	15	16
13	0·2309	·2327	·2345	·2364	·2382	·2401	·2419	·2438	·2456	·2475	2	4	6	7	9	11	13	15	17
14	0·2493	·2512	·2530	·2549	·2568	·2586	·2605	·2623	·2642	·2661	2	4	6	7	9	11	13	15	17
15	0·2679	·2698	·2717	·2736	·2754	·2773	·2792	·2811	·2830	·2849	2	4	6	8	9	11	13	15	17
16	0·2867	·2886	·2905	·2924	·2943	·2962	·2981	·3000	·3019	·3038	2	4	6	8	9	11	13	15	17
17	0·3057	·3076	·3096	·3115	·3134	·3153	·3172	·3191	·3211	·3230	2	4	6	8	10	12	13	15	17
18	0·3249	·3269	·3288	·3307	·3327	·3346	·3365	·3385	·3404	·3424	2	4	6	8	10	12	14	16	17
19	0·3443	·3463	·3482	·3502	·3522	·3541	·3561	·3581	·3600	·3620	2	4	6	8	10	12	14	16	18
20	0·3640	·3659	·3679	·3899	·3719	·3739	·3759	·3779	·3799	·3819	2	4	6	8	10	12	14	16	18
21	0·3839	·3859	·3879	·3899	·3919	·3939	·3959	·3979	·4000	·4020	2	4	6	8	10	12	14	16	18
22	0·4040	·4061	·4061	·4101	·4122	·4142	·4163	·4183	·4204	·4224	2	4	6	8	10	12	14	16	18
23	0·4245	·4265	·4286	·4307	·4327	·4348	·4369	·4390	·4411	·4431	2	4	6	8	10	12	15	17	19
24	0·4452	·4473	·4494	·4515	·4536	·4557	·4578	·4599	·4621	·4642	2	4	6	8	11	13	15	17	19
25	0·4663	·4684	·4706	·4727	·4748	·4770	·4791	·4813	·4834	·4856	2	4	6	9	11	13	15	17	19
26	0·4877	·4899	·4921	·4942	·4964	·4986	·5008	·5029	·5051	·5073	2	4	7	9	11	13	15	17	20
27	0·5095	·5117	·5139	·5161	·5184	·5206	·5228	·5250	·5272	·5295	2	4	7	9	11	13	16	18	20
28	0·5317	·5340	·5362	·5384	·5407	·5430	·5452	·5475	·5498	·5520	2	5	7	9	11	14	16	18	20
29	0·5543	·5566	·5589	·5612	·5635	·5658	·5681	·5704	·5727	·5750	2	5	7	9	12	14	16	18	21
30	0·5774	·5797	·5820	·5844	·5867	·5890	·5914	·5938	·5961	·5985	2	5	7	9	12	14	16	19	21
31	0·6009	·6032	·6056	·6080	·6104	·6128	·6152	·6176	·6200	·6224	2	5	7	10	12	14	17	19	22
32	0·6249	·6273	·6297	·6322	·6346	·6371	·6395	·6420	·6445	·6469	2	5	7	10	12	15	17	20	22
33	0·6494	·6519	·6544	·6569	·6594	·6619	·6644	·6669	·6694	·6720	3	5	8	10	13	15	18	20	23
34	0·6745	·6771	·6796	·6822	·6847	·6873	·6899	·6924	·6950	·6976	3	5	8	10	13	15	18	21	23
35	0·7002	·7028	·7054	·7080	·7107	·7133	·7159	·7186	·7212	·7239	3	5	8	11	13	16	18	21	24
36	0·7265	·7292	·7319	·7346	·7373	·7400	·7427	·7454	·7481	·7508	3	5	8	11	14	16	19	22	24
37	0·7536	·7563	·7590	·7618	·7646	·7673	·7701	·7729	·7757	·7785	3	6	8	11	14	17	19	22	25
38	0·7813	·7841	·7869	·7898	·7926	·7954	·7983	·8012	·8040	·8069	3	6	9	11	14	17	20	23	26
39	0·8098	·8127	·8156	·8185	·8214	·8243	·8273	·8302	·8332	·8361	3	6	9	12	15	18	21	23	26
40	0·8391	·8421	·8451	·8481	·8511	·8541	·8571	·8601	·8632	·8662	3	6	9	12	15	18	21	24	27
41	0·8693	·8724	·8754	·8785	·8816	·8847	·8878	·8910	·8941	·8972	3	6	9	12	16	19	22	25	28
42	0·9004	·9036	·9067	·9099	·9131	·9163	·9195	·9228	·9260	·9293	3	6	10	13	16	19	22	26	29
43	0·9325	·9358	·9391	·9424	·9457	·9490	·9523	·9556	·9590	·9623	3	7	10	13	17	20	23	27	30
44	0·9657	·9091	·9725	·9759	·9793	·9827	·9861	·9896	·9930	·9965	3	7	10	14	17	21	24	27	31
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	ADD Differences 1	2	3	4	5	6	7	8	9
45	1·000	1·003	1·007	1·011	1·014	1·018	1·021	1·025	1·028	1·032	0	1	1	1	2	2	2	3	3
46	1·036	1·039	1·043	1·046	1·050	1·054	1·057	1·061	1·065	1·069	0	1	1	1	2	2	3	3	3
47	1·072	1·076	1·080	1·084	1·087	1·091	1·095	1·099	1·103	1·107	0	1	1	2	2	2	3	3	3
48	1·111	1·115	1·118	1·122	1·126	1·130	1·134	1·138	1·142	1·146	0	1	1	2	2	2	3	3	4
49	1·150	1·154	1·159	1·163	1·167	1·171	1·175	1·179	1·183	1·188	0	1	1	2	2	2	3	3	4
50	1·192	1·196	1·200	1·205	1·209	1·213	1·217	1·222	1·226	1·230	0	1	1	2	2	3	3	4	4
51	1·235	1·239	1·244	1·248	1·253	1·257	1·262	1·266	1·271	1·275	0	1	1	2	2	3	3	4	4
52	1·280	1·285	1·289	1·294	1·299	1·303	1·308	1·313	1·317	1·322	0	1	1	2	2	3	3	4	4
53	1·327	1·332	1·337	1·342	1·347	1·351	1·356	1·361	1·366	1·371	0	1	1	2	2	3	3	4	4
54	1·376	1·381	1·387	1·392	1·397	1·402	1·407	1·412	1·418	1·423	1	1	2	2	3	3	4	4	5
55	1·428	1·433	1·439	1·444	1·450	1·455	1·460	1·466	1·471	1·477	1	1	2	2	3	3	4	4	5
56	1·483	1·488	1·494	1·499	1·505	1·511	1·517	1·522	1·528	1·534	1	1	2	2	3	3	4	5	5
57	1·540	1·546	1·552	1·558	1·554	1·570	1·576	1·582	1·588	1·594	1	1	2	2	3	4	4	5	5
58	1·600	1·607	1·613	1·619	1·525	1·632	1·638	1·645	1·651	1·658	1	1	2	3	3	4	4	5	6
59	1·664	1·671	1·678	1·684	1·691	1·698	1·704	1·711	1·718	1·725	1	1	2	3	3	4	5	5	6
60	1·732	1·739	1·746	1·753	1·760	1·767	1·775	1·782	1·789	1·797	1	1	2	3	4	4	5	6	6
61	1·804	1·811	1·819	1·827	1·834	1·842	1·849	1·857	1·865	1·873	1	2	2	3	4	5	5	6	7
62	1·881	1·889	1·897	1·905	1·913	1·921	1·929	1·937	1·946	1·954	1	2	2	3	4	5	6	7	7
63	1·963	1·971	1·980	1·988	1·997	2·006	2·014	2·023	2·032	2·041	1	2	3	4	4	5	6	7	8
64	2·050	2·059	2·069	2·078	2·087	2·097	2·106	2·116	2·125	2·135	1	2	3	4	5	6	7	8	8
65	2·145	2·154	2·164	2·174	2·184	2·194	2·204	2·215	2·225	2·236	1	2	3	4	5	6	7	8	9
66	2·246	2·257	2·267	2·278	2·289	2·300	2·311	2·322	2·333	2·344	1	2	3	4	5	7	8	9	10
67	2·356	2·367	2·379	2·391	2·402	2·414	2·426	2·438	2·450	2·463	1	2	4	5	6	7	8	10	11
68	2·475	2·488	2·500	2·513	2·526	2·539	2·552	2·565	2·578	2·592	1	3	4	5	7	8	9	10	12
69	2·605	2·619	2·633	2·646	2·660	2·675	2·689	2·703	2·718	2·733	1	3	4	6	7	9	10	11	13
70	2·747	2·762	2·778	2·793	2·808	2·824	2·840	2·856	2·872	2·888	2	3	5	6	8	9	11	13	14
71	2·904	2·921	2·937	2·954	2·971	2·989	3·006	3·024	3·042	3·060	2	3	5	7	9	10	12	14	16
72	3·078	3·096	3·115	3·133	3·152	3·172	3·191	3·211	3·230	3·251	2	4	6	8	10	12	14	15	17
73	3·271	3·291	3·312	3·333	3·354	3·376	3·398	3·420	3·442	3·465	2	4	6	9	11	13	15	17	19
74	3·487	3·511	3·534	3·558	3·582	3·606	3·630	3·655	3·681	3·706	2	5	7	10	12	15	17	20	22
75	3·732	3·758	3·785	3·812	3·839	3·867	3·895	3·923	3·952	3·981	3	6	8	11	14	17	20	22	25
76	4·011	4·041	4·071	4·102	4·134	4·165	4·198	4·230	4·264	4·297	3	6	10	13	16	19	22	26	29
77	4·331	4·366	4·402	4·437	4·474	4·511	4·548	4·586	4·625	4·666									
78	4·705	4·745	4·787	4·829	4·872	4·915	4·959	5·005	5·050	5·097									
79	5·145	5·193	5·242	5·292	5·343	5·396	5·449	5·503	5·558	5·614									
80	5·671	5·730	5·789	5·850	5·912	5·976	6·041	6·107	6·174	6·243	differences unreliable:								
81	6·314	6·386	6·460	6·535	6·612	6·691	6·772	6·855	6·940	7·026									
82	7·115	7·207	7·300	7·396	7·495	7·596	7·700	7·806	7·916	8·028									
83	8·144	8·264	8·386	8·513	8·643	8·777	8·915	9·058	9·205	9·357									
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20									
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95									
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46									
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27									
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08									
89	57·29	63·66	71·62	81·85	95·46	114·6	143·2	191·0	286·5	573·0									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix V: Square Roots of Numbers, 1-10

Square Roots of Numbers, 1-10

$x \to \sqrt{x}$

														Differences				
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1.0	1·000	1·005	1·010	1·015	1·020	1·025	1·030	1·034	1·039	1·044	0	1	1	2	2	3	3	4	4
1.1	1·049	1·054	1·058	1·063	1·068	1·072	1·077	1·082	1·086	1·091	0	1	1	2	2	3	3	4	4
1.2	1·095	1·100	1·105	1·109	1·114	1·118	1·123	1·127	1·131	1·136	0	1	1	2	2	3	3	4	4
1.3	1·140	1·145	1·149	1·153	1·158	1·162	1·166	1·171	1·175	1·179	0	1	1	2	2	3	3	3	4
1.4	1·183	1·187	1·192	1·196	1·200	1·204	1·208	1·212	1·217	1·221	0	1	1	2	2	3	3	3	4
1.5	1·225	1·229	1·233	1·237	1·241	1·245	1·249	1·253	1·257	1·261	0	1	1	2	2	2	3	3	4
1.6	1·265	1·269	1·273	1·277	1·281	1·285	1·288	1·292	1·296	1·300	0	1	1	2	2	2	3	3	3
1.7	1·304	1·308	1·312	1·315	1·319	1·323	1·327	1·330	1·334	1·338	0	1	1	2	2	2	3	3	3
1.8	1·342	1·345	1·349	1·353	1·357	1·360	1·364	1·368	1·371	1·375	0	1	1	1	2	2	3	3	3
1.9	1·378	1·382	1·386	1·389	1·393	1·396	1·400	1·404	1·407	1·411	0	1	1	1	2	2	3	3	3
2.0	1·414	1·418	1·421	1·425	1·428	1·432	1·435	1·439	1·442	1·446	0	1	1	1	2	2	2	3	3
2.1	1·449	1·453	1·456	1·460	1·463	1·466	1·470	1·473	1·477	1·480	0	1	1	1	2	2	2	3	3
2.2	1·483	1·487	1·490	1·493	1·497	1·500	1·503	1·507	1·510	1·513	0	1	1	1	2	2	2	3	3
2.3	1·517	1·520	1·523	1·526	1·530	1·533	1·536	1·539	1·543	1·546	0	1	1	1	2	2	2	3	3
2.4	1·549	1·552	1·556	1·559	1·562	1·565	1·568	1·572	1·575	1·578	0	1	1	1	2	2	2	3	3
2.5	1·581	1·584	1·587	1·591	1·594	1·597	1·600	1·603	1·606	1·609	0	1	1	1	2	2	2	3	3
2.6	1·612	1·616	1·619	1·622	1·625	1·628	1·631	1·634	1·637	1·640	0	1	1	1	2	2	2	2	3
2.7	1·643	1·646	1·649	1·652	1·655	1·658	1·661	1·664	1·667	1·670	0	1	1	1	2	2	2	2	3
2.8	1·673	1·676	1·679	1·682	1·685	1·688	1·691	1·694	1·697	1·700	0	1	1	1	1	2	2	2	3
2.9	1·703	1·706	1·709	1·712	1·715	1·718	1·720	1·723	1·726	1·729	0	1	1	1	1	2	2	2	3
3.0	1·732	1·735	1·738	1·741	1·744	1·746	1·749	1·752	1·755	1·758	0	1	1	1	1	2	2	2	3
3.1	1·761	1·764	1·766	1·769	1·772	1·775	1·778	1·780	1·783	1·786	0	1	1	1	1	2	2	2	3
3.2	1·789	1·792	1·794	1·797	1·800	1·803	1·806	1·808	1·811	1·814	0	1	1	1	1	2	2	2	2
3.3	1·817	1·819	1·822	1·825	1·828	1·830	1·833	1·836	1·839	1·841	0	1	1	1	1	2	2	2	2
3.4	1·844	1·847	1·849	1·852	1·855	1·857	1·860	1·863	1·866	1·868	0	1	1	1	1	2	2	2	2
3.5	1·871	1·874	1·876	1·879	1·882	1·884	1·887	1·889	1·892	1·895	0	1	1	1	1	2	2	2	2
3.6	1·897	1·900	1·903	1·905	1·908	1·911	1·913	1·916	1·918	1·921	0	1	1	1	1	2	2	2	2
3.7	1·924	1·926	1·929	1·931	1·934	1·937	1·939	1·942	1·944	1·947	0	1	1	1	1	2	2	2	2
3.8	1·949	1·952	1·955	1·957	1·960	1·962	1·965	1·967	1·970	1·972	0	1	1	1	1	2	2	2	2
3.9	1·975	1·977	1·980	1·982	1·985	1·988	1·990	1·993	1·995	1·998	0	1	1	1	1	2	2	2	2
4.0	2·000	2·003	2·005	2·008	2·010	2·013	2·015	2·017	2·020	2·022	0	0	1	1	1	1	2	2	2
4.1	2·025	2·027	2·030	2·032	2·035	2·037	2·040	2·042	2·045	2·047	0	0	1	1	1	1	2	2	2
4.2	2·049	2·052	2·054	2·057	2·059	2·062	2·064	2·066	2·069	2·071	0	0	1	1	1	1	2	2	2
4.3	2·074	2·076	2·078	2·081	2·083	2·086	2·088	2·091	2·093	2·095	0	0	1	1	1	1	2	2	2
4.4	2·098	2·100	2·102	2·105	2·107	2·110	2·112	2·114	2·117	2·119	0	0	1	1	1	1	2	2	2
4.5	2·121	2·124	2·126	2·128	2·131	2·133	2·135	2·138	2·140	2·142	0	0	1	1	1	1	2	2	2
4.6	2·145	2·147	2·149	2·152	2·154	2·156	2·159	2·161	2·163	2·166	0	0	1	1	1	1	2	2	2
4.7	2·168	2·170	2·173	2·175	2·177	2·179	2·182	2·184	2·186	2·189	0	0	1	1	1	1	2	2	2
4.8	2·191	2·193	2·195	2·198	2·200	2·202	2·205	2·207	2·209	2·211	0	0	1	1	1	1	2	2	2
4.9	2·214	2·216	2·218	2·220	2·223	2·225	2·227	2·229	2·232	2·234	0	0	1	1	1	1	2	2	2
5.0	2·236	2·238	2·241	2·243	2·245	2·247	2·249	2·252	2·254	2·256	0	0	1	1	1	1	2	2	2
5.1	2·258	2·261	2·263	2·265	2·267	2·269	2·272	2·274	2·276	2·278	0	0	1	1	1	1	2	2	2
5.2	2·280	2·283	2·285	2·287	2·289	2·291	2·294	2·296	2·298	2·300	0	0	1	1	1	1	2	2	2
5.3	2·302	2·304	2·307	2·309	2·311	2·313	2·315	2·317	2·320	2·322	0	0	1	1	1	1	2	2	2
5.4	2·324	2·326	2·328	2·330	2·332	2·335	2·337	2·339	2·341	2·343	0	0	1	1	1	1	1	2	2
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

														Differences				
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
5·5	2·345	2·347	2·350	2·352	2·354	2·356	2·358	2·360	2·362	2·364	0	0	1	1	1	1	1	2	2
5·6	2·366	2·369	2·371	2·373	2·375	2·377	2·379	2·381	2·383	2·385	0	0	1	1	1	1	1	2	2
5·7	2·388	2·390	2·392	2·394	2·396	2·398	2·400	2·402	2·404	2·406	0	0	1	1	1	1	1	2	2
5·8	2·408	2·410	2·412	2·415	2·417	2·419	2·421	2·423	2·425	2·427	0	0	1	1	1	1	1	2	2
5·9	2·429	2·431	2·433	2·435	2·437	2·439	2·441	2·443	2·445	2·447	0	0	1	1	1	1	1	2	2
6·0	2·450	2·452	2·454	2·456	2·458	2·460	2·462	2·464	2·466	2·468	0	0	1	1	1	1	1	2	2
6·1	2·470	2·472	2·474	2·476	2·478	2·480	2·482	2·484	2·486	2·488	0	0	1	1	1	1	1	2	2
6·2	2·490	2·492	2·494	2·496	2·498	2·500	2·502	2·504	2·506	2·508	0	0	1	1	1	1	1	2	2
6·3	2·510	2·512	2·514	2·516	2·518	2·520	2·522	2·524	2·526	2·528	0	0	1	1	1	1	1	2	2
6·4	2·530	2·532	2·534	2·536	2·538	2·540	2·542	2·544	2·546	2·548	0	0	1	1	1	1	1	2	2
6·5	2·550	2·551	2·553	2·555	2·557	2·559	2·561	2·563	2·565	2·567	0	0	1	1	1	1	1	2	2
6·6	2·569	2·571	2·573	2·575	2·577	2·579	2·581	2·583	2·585	2·587	0	0	1	1	1	1	1	2	2
6·7	2·588	2·590	2·592	2·594	2·596	2·598	2·600	2·602	2·604	2·606	0	0	1	1	1	1	1	2	2
6·8	2·608	2·610	2·612	2·613	2·615	2·617	2·619	2·621	2·623	2·625	0	0	1	1	1	1	1	2	2
6·9	2·627	2·629	2·631	2·632	2·634	2·636	2·638	2·640	2·642	2·644	0	0	1	1	1	1	1	2	2
7·0	2·646	2·648	2·650	2·651	2·653	2·655	2·657	2·659	2·661	2·663	0	0	1	1	1	1	1	2	2
7·1	2·665	2·667	2·668	2·670	2·672	2·674	2·676	2·678	2·680	2·681	0	0	1	1	1	1	1	1	2
7·2	2·683	2·685	2·687	2·689	2·691	2·693	2·694	2·696	2·698	2·700	0	0	1	1	1	1	1	1	2
7·3	2·702	2·704	2·706	2·707	2·709	2·711	2·713	2·715	2·717	2·719	0	0	1	1	1	1	1	1	2
7·4	2·720	2·722	2·724	2·726	2·728	2·729	2·731	2·733	2·735	2·737	0	0	1	1	1	1	1	1	2
7·5	2·739	2·740	2·742	2·744	2·746	2·748	2·750	2·751	2·753	2·755	0	0	1	1	1	1	1	1	2
7·6	2·757	2·759	2·760	2·762	2·764	2·766	2·768	2·769	2·771	2·773	0	0	1	1	1	1	1	1	2
7·7	2·775	2·777	2·778	2·780	2·782	2·784	2·786	2·787	2·789	2·791	0	0	1	1	1	1	1	1	2
7·8	2·793	2·795	2·796	2·798	2·800	2·802	2·804	2·805	2·807	2·809	0	0	1	1	1	1	1	1	2
7·9	2·811	2·812	2·814	2·816	2·818	2·820	2·821	2·823	2·825	2·827	0	0	1	1	1	1	1	1	2
8·0	2·828	2·830	2·832	2·834	2·835	2·837	2·839	2·841	2·843	2·844	0	0	1	1	1	1	1	1	2
8·1	2·846	2·848	2·850	2·851	2·853	2·855	2·857	2·858	2·860	2·862	0	0	1	1	1	1	1	1	2
8·2	2·864	2·865	2·867	2·869	2·871	2·872	2·874	2·876	2·877	2·879	0	0	1	1	1	1	1	1	2
8·3	2·881	2·883	2·884	2·886	2·888	2·890	2·891	2·893	2·895	2·897	0	0	1	1	1	1	1	1	2
8·4	2·898	2·900	2·902	2·903	2·905	2·907	2·909	2·910	2·912	2·914	0	0	1	1	1	1	1	1	2
8·5	2·915	2·917	2·919	2·921	2·922	2·924	2·926	2·927	2·929	2·931	0	0	1	1	1	1	1	1	2
8·6	2·933	2·934	2·936	2·938	2·939	2·941	2·943	2·944	2·946	2·948	0	0	1	1	1	1	1	1	2
8·7	2·950	2·951	2·953	2·955	2·956	2·958	2·960	2·961	2·963	2·965	0	0	1	1	1	1	1	1	2
8·8	2·966	2·968	2·970	2·972	2·973	2·975	2·977	2·978	2·980	2·982	0	0	1	1	1	1	1	1	2
8·9	2·983	2·985	2·987	2·988	2·990	2·992	2·993	2·995	2·997	2·998	0	0	1	1	1	1	1	1	2
9·0	3·000	3·002	3·003	3·005	3·007	3·008	3·010	3·012	3·013	3·015	0	0	0	1	1	1	1	1	1
9·1	3·017	3·018	3·020	3·022	3·023	3·025	3·027	3·028	3·030	3·032	0	0	0	1	1	1	1	1	1
9·2	3·033	3·035	3·036	3·038	3·040	3·041	3·043	3·045	3·046	3·048	0	0	0	1	1	1	1	1	1
9·3	3·050	3·051	3·053	3·055	3·056	3·058	3·059	3·061	3·063	3·064	0	0	0	1	1	1	1	1	1
9·4	3·066	3·068	3·069	3·071	3·072	3·074	3·076	3·077	3·079	3·081	0	0	0	1	1	1	1	1	1
9·5	3·082	3·084	3·085	3·087	3·089	3·090	3·092	3·094	3·095	3·097	0	0	0	1	1	1	1	1	1
9·6	3·098	3·100	3·102	3·103	3·105	3·106	3·108	3·110	3·111	3·113	0	0	0	1	1	1	1	1	1
9·7	3·115	3·116	3·118	3·119	3·121	3·123	3·124	3·126	3·127	3·129	0	0	0	1	1	1	1	1	1
9·8	3·131	3·132	3·134	3·135	3·137	3·139	3·140	3·142	3·143	2·145	0	0	0	1	1	1	1	1	1
9·9	3·146	3·148	3·150	3·151	3·153	3·154	3·156	3·158	3·159	3·161	0	0	0	1	1	1	1	1	1
x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Appendix VI: Square Roots of Numbers, 10-100

Square Roots of Numbers 10-100

$x \rightarrow \sqrt{x}$

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	3·162	3·178	3·194	3·209	3·225	3·240	3·256	3·271	3·286	3·302	2	3	5	6	8	9	11	12	14
11	3·317	3·332	3·347	3·362	3·376	3·391	3·406	3·421	3·435	3·450	1	3	4	6	7	9	10	12	13
12	3·464	3·479	3·493	3·507	3·521	3·536	3·550	3·564	3·578	3·592	1	3	4	6	7	8	10	11	13
13	3·606	3·619	3·633	3·647	3·661	3·674	3·688	3·701	3·715	3·728	1	3	4	5	7	8	10	11	12
14	3·742	3·755	3·768	3·782	3·795	3·808	3·821	3·834	3·847	3·860	1	3	4	5	7	8	9	11	12
15	3·873	3·886	3·899	3·912	3·924	3·937	3·950	3·962	3·975	3·988	1	3	4	5	6	8	9	10	11
16	4·000	4·012	4·025	4·037	4·050	4·062	4·074	4·087	4·099	4·111	1	2	4	5	6	7	9	10	11
17	4·123	4·135	4·147	4·159	4·171	4·183	4·195	4·207	4·219	4·231	1	2	4	5	6	7	8	10	11
18	4·243	4·254	4·266	4·278	4·290	4·301	4·313	4·324	4·336	4·347	1	2	3	5	6	7	8	9	10
19	4·359	4·370	4·382	4·393	4·405	4·416	4·427	4·438	4·450	4·461	1	2	3	5	6	7	8	9	10
20	4·472	4·483	4·494	4·506	4·517	4·528	4·539	4·550	4·561	4·572	1	2	3	4	6	7	8	9	10
21	4·583	4·594	4·604	4·615	4·626	4·637	4·648	4·658	4·669	4·680	1	2	3	4	5	6	8	9	10
22	4·690	4·701	4·712	4·722	4·733	4·743	4·754	4·765	4·775	4·785	1	2	3	4	5	6	7	8	9
23	4·796	4·806	4·817	4·827	4·837	4·848	4·858	4·868	4·879	4·889	1	2	3	4	5	6	7	8	9
24	4·899	4·909	4·919	4·930	4·940	4·950	4·960	4·970	4·980	4·990	1	2	3	4	5	6	7	8	9
25	5·000	5·010	5·020	5·030	5·040	5·050	5·060	5·070	5·079	5·089	1	2	3	4	5	6	7	8	9
26	5·099	5·109	5·119	5·128	5·138	5·148	5·158	5·167	5·177	5·187	1	2	3	4	5	6	7	8	9
27	5·196	5·206	5·215	5·225	5·235	5·244	5·254	5·263	5·273	5·282	1	2	3	4	5	6	7	8	9
28	5·292	5·301	5·310	5·320	5·329	5·339	5·348	5·357	5·367	5·376	1	2	3	4	5	6	7	7	8
29	5·385	5·394	5·404	5·413	5·422	5·431	5·441	5·450	5·459	5·468	1	2	3	4	5	5	6	7	8
30	5·477	5·486	5·495	5·505	5·514	5·523	5·532	5·541	5·550	5·559	1	2	3	4	4	5	6	7	8
31	5·568	5·577	5·586	5·596	5·604	5·612	5·621	5·630	5·639	5·648	1	2	3	3	4	5	6	7	8
32	5·657	5·666	5·675	5·683	5·692	5·701	5·710	5·718	5·727	5·736	1	2	3	3	4	5	6	7	8
33	5·745	5·753	5·762	5·771	5·779	5·788	5·797	5·805	5·814	5·822	1	2	3	3	4	5	6	7	8
34	5·831	5·840	5·848	5·857	5·865	5·874	5·882	5·891	5·899	5·908	1	2	3	3	4	5	6	7	8
35	5·916	5·925	5·933	5·941	5·950	5·958	5·967	5·975	5·983	5·992	1	2	2	3	4	5	6	7	8
36	6·000	6·008	6·017	6·025	6·033	6·042	6·050	6·058	6·066	6·075	1	2	2	3	4	5	6	7	7
37	6·083	6·091	6·099	6·107	6·116	6·124	6·132	6·140	6·148	6·156	1	2	2	3	4	5	6	7	7
38	6·164	6·173	6·181	6·189	6·197	6·205	6·213	6·221	6·229	6·237	1	2	2	3	4	5	6	6	7
39	6·245	6·253	6·261	6·269	6·277	6·285	6·293	6·301	6·309	6·317	1	2	2	3	4	5	6	6	7
40	6·325	6·332	6·340	6·348	6·356	6·364	6·372	6·380	6·387	6·395	1	2	2	3	4	5	6	6	7
41	6·403	6·411	6·419	6·427	6·434	6·442	6·450	6·458	6·465	6·473	1	2	2	3	4	5	5	6	7
42	6·481	6·488	6·496	6·504	6·512	6·519	6·527	6·535	6·542	6·550	1	2	2	3	4	5	5	6	7
43	6·557	6·565	6·573	6·580	6·588	6·595	6·603	6·611	6·618	6·626	1	2	2	3	4	5	5	6	7
44	6·633	6·641	6·648	6·656	6·663	6·671	6·678	6·686	6·693	6·701	1	2	2	3	4	5	5	6	7
45	6·708	6·716	6·723	6·731	6·738	6·745	6·753	6·760	6·768	6·775	1	1	2	3	4	4	5	6	7
46	6·782	6·790	6·797	6·804	6·812	6·819	6·826	6·834	6·841	6·848	1	1	2	3	4	4	5	6	7
47	6·856	6·863	6·870	6·878	6·885	6·892	6·899	6·907	6·914	6·921	1	1	2	3	4	4	5	6	7
48	6·928	6·935	6·943	6·950	6·957	6·964	6·971	6·979	6·986	6·993	1	1	2	3	4	4	5	6	6
49	7·000	7·007	7·014	7·021	7·029	7·036	7·043	7·050	7·057	7·064	1	1	2	3	4	4	5	6	6
50	7·071	7·078	7·085	7·092	7·099	7·106	7·113	7·120	7·127	7·134	1	1	2	3	4	4	5	6	6
51	7·141	7·148	7·155	7·162	7·169	7·176	7·183	7·190	7·197	7·204	1	1	2	3	4	4	5	6	6
52	7·211	7·218	7·225	7·232	7·239	7·246	7·253	7·259	7·266	7·273	1	1	2	3	3	4	5	6	6
53	7·280	7·287	7·294	7·301	7·308	7·314	7·321	7·328	7·335	7·342	1	1	2	3	3	4	5	5	6
54	7·349	7·355	7·362	7·369	7·376	7·382	7·389	7·396	7·403	7·410	1	1	2	3	3	4	5	5	6
55	7·416	7·423	7·430	7·436	7·443	7·450	7·457	7·463	7·470	7·477	1	1	2	3	3	4	5	5	6
56	7·483	7·490	7·497	7·503	7·510	7·517	7·523	7·530	7·537	7·543	1	1	2	3	3	4	5	5	6
57	7·550	7·556	7·563	7·570	7·576	7·583	7·589	7·596	7·603	7·609	1	1	2	3	3	4	5	5	6
58	7·616	7·622	7·629	7·635	7·642	7·649	7·655	7·662	7·668	7·675	1	1	2	3	3	4	5	5	6
59	7·681	7·688	7·694	7·701	7·707	7·714	7·720	7·727	7·733	7·740	1	1	2	3	3	4	4	5	6
60	7·746	7·752	7·759	7·765	7·772	7·778	7·785	7·791	7·797	7·804	1	1	2	3	3	4	4	5	6
61	7·810	7·817	7·823	7·829	7·836	7·842	7·849	7·855	7·861	7·868	1	1	2	3	3	4	4	5	6
62	7·874	7·880	7·887	7·893	7·899	7·906	7·912	7·918	7·925	7·931	1	1	2	3	3	4	4	5	6
63	7·937	7·944	7·950	7·956	7·962	7·969	7·975	7·981	7·987	7·994	1	1	2	3	3	4	4	5	6
64	8·000	8·006	8·012	8·019	8·025	8·031	8·037	8·044	8·050	8·056	1	1	2	2	3	4	4	5	6
65	8·062	8·068	8·075	8·081	8·087	8·093	8·099	8·106	8·112	8·118	1	1	2	2	3	4	4	5	6
66	8·124	8·130	8·136	8·142	8·149	8·155	8·161	8·167	8·173	8·179	1	1	2	2	3	4	4	5	5
67	8·185	8·191	8·198	8·204	8·210	8·216	8·222	8·228	8·234	8·240	1	1	2	2	3	4	4	5	5
68	8·246	8·252	8·258	8·264	8·270	8·276	8·283	8·289	8·295	8·301	1	1	2	2	3	4	4	5	5
69	8·307	8·313	8·319	8·325	8·331	8·337	8·343	8·349	8·355	8·361	1	1	2	2	3	4	4	5	5
70	8·367	8·373	8·379	8·385	8·390	8·396	8·402	8·408	8·414	8·420	1	1	2	2	3	4	4	5	5
71	8·426	8·432	8·438	8·444	8·450	8·456	8·462	8·468	8·473	8·479	1	1	2	2	3	4	4	5	5
72	8·485	8·491	8·497	8·503	8·509	8·515	8·521	8·526	8·532	8·538	1	1	2	2	3	3	4	5	5
73	8·544	8·550	8·556	8·562	8·567	8·573	8·579	8·585	8·591	8·597	1	1	2	2	3	3	4	5	5
74	8·602	8·608	8·614	8·620	8·626	8·631	8·637	8·643	8·649	8·654	1	1	2	2	3	3	4	5	5
75	8·660	8·666	8·672	8·678	8·683	8·689	8·695	8·701	8·706	8·712	1	1	2	2	3	3	4	5	5
76	8·718	8·724	8·729	8·735	8·741	8·746	8·752	8·758	8·764	8·769	1	1	2	2	3	3	4	5	5
77	8·775	8·781	8·786	8·792	8·798	8·803	8·809	8·815	8·820	8·826	1	1	2	2	3	3	4	4	5
78	8·832	8·837	8·843	8·849	8·854	8·860	8·866	8·871	8·877	8·883	1	1	2	2	3	3	4	4	5
79	8·888	8·894	8·899	8·905	8·911	8·916	8·922	8·927	8·933	8·939	1	1	2	2	3	3	4	4	5
80	8·944	8·950	8·955	8·961	8·967	8·972	8·978	8·983	8·989	8·994	1	1	2	2	3	3	4	4	5
81	9·000	9·006	9·011	9·017	9·022	9·028	9·033	9·039	9·044	9·050	1	1	2	2	3	3	4	4	5
82	9·055	9·061	9·066	9·072	9·077	9·083	9·088	9·094	9·099	9·105	1	1	2	2	3	3	4	4	5
83	9·110	9·116	9·121	9·127	9·132	9·138	9·143	9·149	9·154	9·160	1	1	2	2	3	3	4	4	5
84	9·165	9·171	9·176	9·182	9·187	9·192	9·198	9·203	9·209	9·214	1	1	2	2	3	3	4	4	5
85	9·220	9·225	9·230	9·236	9·241	9·247	9·252	9·257	9·263	9·268	1	1	2	2	3	3	4	4	5
86	9·274	9·279	9·284	9·290	9·295	9·301	9·306	9·311	9·317	9·322	1	1	2	2	3	3	4	4	5
87	9·237	9·333	9·338	9·343	9·349	9·354	9·359	9·365	9·370	9·375	1	1	2	2	3	3	4	4	5
88	9·381	9·386	9·391	9·397	9·402	9·407	9·413	9·418	9·423	9·429	1	1	2	2	3	3	4	4	5
89	9·434	9·439	9·445	9·450	9·455	9·460	9·466	9·471	9·476	9·482	1	1	2	2	3	3	4	4	5
90	9·487	9·492	9·497	9·503	9·508	9·513	9·518	9·524	9·529	9·534	1	1	2	2	3	3	4	4	5
91	9·539	9·545	9·550	9·555	9·560	9·566	9·571	9·576	9·581	9·586	1	1	2	2	3	3	4	4	5
92	9·592	9·597	9·602	9·607	9·613	9·618	9·623	9·628	9·633	9·638	1	1	2	2	3	3	4	4	5
93	9·644	9·649	9·654	9·659	9·664	9·670	9·675	9·680	9·685	9·690	1	1	2	2	3	3	4	4	5
94	9·695	9·701	9·706	9·711	9·716	9·721	9·726	9·731	9·737	9·742	1	1	2	2	3	3	4	4	5
95	9·747	9·752	9·757	9·762	9·767	9·772	9·778	9·783	9·788	9·793	1	1	2	2	3	3	4	4	5
96	9·798	9·803	9·808	9·813	9·818	9·823	9·829	9·834	9·839	9·844	1	1	2	2	3	3	4	4	5
97	9·849	9·854	9·859	9·864	9·869	9·874	9·879	9·884	9·889	9·894	1	1	2	2	3	3	4	4	5
98	9·900	9·905	9·910	9·915	9·920	9·925	9·930	9·935	9·940	9·945	0	1	1	2	2	3	3	4	4
99	9·950	9·955	9·960	9·965	9·970	9·975	9·980	9·985	9·990	9·996	0	1	1	2	2	3	3	4	4
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Appendix VII: Reciprocals of Numbers

Reciprocals of Numbers

$x \rightarrow 1/x$

											SUBTRACT Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1000	9901	9804	9709	9615	9524	9434	9346	9259	9174	9	18	28	37	46	55	64	74	83
11	9091	9009	8929	8850	8772	8696	8621	8547	8475	8403	8	15	23	31	38	46	53	61	69
12	8333	8264	8197	8130	8065	8000	7937	7874	7813	7752	7	13	20	26	33	39	46	52	59
13	7692	7634	7576	7519	7463	7407	7353	7299	7246	7194	6	11	17	22	28	33	39	44	50
14	7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5	10	14	19	24	29	33	38	43
15	6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	4	8	13	17	21	25	29	33	38
16	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	7	11	15	18	22	26	29	33
17	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	7	10	13	16	20	23	26	29
18	5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3	6	9	12	15	17	20	23	26
19	5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3	5	8	11	13	16	18	21	24
20	5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19	21
21	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	7	9	11	13	15	17	19
22	4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	12	14	16	18
23	4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2	4	5	7	9	11	13	14	16
24	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	3	5	7	8	10	12	13	15
25	4000	3981	3968	3953	3937	3922	3906	3891	3876	3861	2	3	5	6	8	9	11	12	14
26	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1	3	4	6	7	9	10	11	13
27	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	3	4	5	7	8	9	11	12
28	3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	1	2	4	5	6	7	9	10	11
29	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	2	3	5	6	7	8	9	10
30	3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1	2	3	4	5	6	8	9	10
31	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1	2	3	4	5	6	7	8	9
32	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8	9
33	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	4	5	6	7	8
34	2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	1	2	3	3	4	5	6	7	8
35	2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1	2	2	3	4	5	6	6	7
36	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	2	3	4	5	5	6	7
37	2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	1	2	3	4	4	5	6	6
38	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1	1	2	3	3	4	5	5	6
39	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	1	2	3	3	4	4	5	6
40	2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	5
41	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	1	2	2	3	3	4	5	5
42	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	4	5
43	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1	1	2	2	3	3	4	4	5
45	2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	0	1	1	2	2	3	3	4	4
46	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	1	1	2	2	3	3	4	4
47	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	3	3	4	4
48	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	0	1	1	2	2	3	3	3	4
49	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	2	2	2	3	3	4
51	1961	1957	1953	1949	1946	1941	1938	1934	1931	1927	0	1	1	2	2	2	3	3	3
52	1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0	1	1	1	2	2	2	3	3
53	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
54	1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0	1	1	1	2	2	2	3	3
55	1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0	1	1	1	2	2	2	3	3
56	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0	1	1	1	2	2	2	3	3
57	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	1	2	2	2	3
58	1724	1721	1718	1715	1712	1709	1706	1704	1701	1696	0	1	1	1	1	2	2	2	3
59	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0	1	1	1	1	2	2	2	3
60	1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	1	2	2	2	3
61	1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
62	1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0	0	1	1	1	1	2	2	2
64	1562	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	0	1	1	1	1	2	2	2
65	1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	0	0	1	1	1	1	2	2	2
66	1515	1513	1511	1508	1506	1504	1502	1499	1497	1495	0	0	1	1	1	1	2	2	2
67	0·1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	1	1	1	1	2	2	2
68	1471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	0	1	1	1	1	2	2	2
69	1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	0	1	1	1	1	2	2	2
70	1429	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	0	1	1	1	1	1	2	2
71	1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0	0	1	1	1	1	1	2	2
72	1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	0	1	1	1	1	1	2	2
73	1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	0	1	1	1	1	1	2	2
74	1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	0	1	1	1	1	1	1	2
75	1333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0	0	1	1	1	1	1	1	2
76	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	1	1	1	1	1	1	2
77	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	1	1	1	1	1	1
78	1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	0	0	1	1	1	1	1	1
79	1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	0	0	1	1	1	1	1	1
80	1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	1	1	1	1	1	1
81	1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	0	0	1	1	1	1	1	1
82	1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
83	1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0	0	0	1	1	1	1	1	1
84	1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	1	1	1	1	1	1
85	1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	0	0	1	1	1	1	1	1
86	1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
87	1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	0	0	1	1	1	1	1	1
88	1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	0	0	1	1	1	1	1	1
89	1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	0	0	1	1	1	1	1	1
90	1111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0	0	0	1	1	1	1	1	1
91	1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0	0	0	0	1	1	1	1	1
92	1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	0	0	0	1	1	1	1	1
93	1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	0	0	0	1	1	1	1	1
94	1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	1	1	1	1	1
95	1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
96	1042	1041	1039	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	1	1	1	1	1
97	1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0	0	0	0	1	1	1	1	1
98	1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	1	1	1	1	1
99	1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	0	1	1	1	1
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

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