THE PRESIDENT'S **RECOVERY** PRIORITIES

Education

Ministry of Education, Science and Technology

Lesson plans for

JSS *Mathematics*

JSS 3

TERM 1

DOI: 10.5281/zenodo.3745187
Please see final page for further information.

NOT FOR SALE

Foreword

Our country's future lies in the education of our children. The Government of Sierra Leone is committed to doing whatever it takes to secure this future.

As Minister of Education, Science and Technology since 2007, I have worked every day to improve our country's education. We have faced challenges, not least the Ebola epidemic which as we all know hit our sector hard. The Government's response to this crisis – led by our President – showed first-hand how we acted decisively in the face of those challenges, to make things better than they were in the first place.

One great success in our response was the publication of the Accelerated Teaching Syllabi in August 2015. This gave teachers the tools they needed to make up for lost time whilst ensuring pupils received an adequate level of knowledge across each part of the curriculum. The Accelerated Teaching syllabi also provided the pedagogical resource and impetus for the successful national radio and TV teaching programs during the Ebola epidemic.

It is now time to build on this success. I am pleased to issue new lesson plans across all primary and JSS school grades in Language Arts and Mathematics. These plans give teachers the support they need to cover each element of the national curriculum. In total, we are producing 2,700 lesson plans – one for each lesson, in each term, in each year for each class. This is a remarkable achievement in a matter of months.

These plans have been written by experienced Sierra Leonean educators together with international experts. They have been reviewed by officials of my Ministry to ensure they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognised techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new plans. It is really important that these Lesson Plans are used, together with any other materials you may have.

This is just the start of education transformation in Sierra Leone. I am committed to continue to strive for the changes that will make our country stronger.

I want to thank our partners for their continued support. Finally, I also want to thank you – the teachers of our country – for your hard work in securing our future.

Dr. Minkailu Bah

Minister of Education, Science and Technology

Table of Contents

JSS
Mathematics

Introduction

to the Lesson Plan Manual

These lesson plans are based on the National Curriculum
and meet the requirements established
by the Ministry of Education, Science and Technology.

1. The lesson plans will not take the whole term, so use spare time to review material or prepare for exams
2. Teachers can use other textbooks alongside or instead of these lesson plans.
3. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
4. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
5. Quickly review what you taught last time before starting each lesson.
6. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
7. Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
8. Use the board and other visual aids as you teach.
9. Interact with all pupils in the class – including the quiet ones.
10. Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Learning outcomes

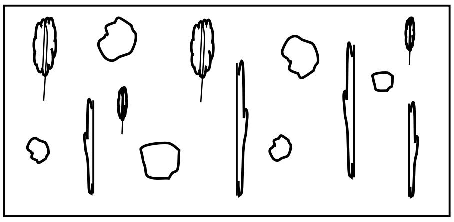
Teaching aids

Preparation

Lesson Title: Sorting Objects	Theme: Numbers and Numeration	
Lesson Number: M-09-001	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to: 1. Collect and sort objects into groups. 2. Describe groups of objects.	None	1. Draw on the board (see example below): a. 3 large, 3 small stones. b. 2 large, 2 small leaves. c. 2 large, 2 small sticks. 2. Write the list of objects shown in the box in the Independent Practice section, on the board.

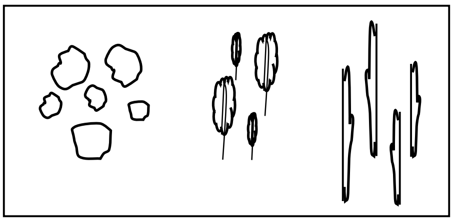
Opening *(2 minutes)*



1. **Say**: Please look at the objects on the board. What do you see? Raise your hand. (Answers: stones, leaves, sticks).
2. **Say:** Today we will learn about collecting and sorting different objects into groups. We will also describe the groups we have collected.

Introduction to the New Material *(15 minutes)*

1. **Write** this vocabulary list on the top left-hand corner of the board: object, collect, sort, group, property, describe.
2. Ask pupils to copy the list into their exercise books.
3. **Say**: Sometimes we use words differently in Maths than how we learnt in English. This can often be confusing. But don't worry as you will understand the meaning of these words better as we use them throughout the lesson.
4. Point to the objects on the board. **Say**: We want to collect and sort objects according to properties they have in common. We can sort them according to the type of object. We can group all the stones together. We can also group all the leaves together, and all the sticks together.
5. If possible, quickly draw all the objects sorted according to type as shown below.
6. **Ask**: What common properties have we used to group these objects?
7. Give a few moments for pupils to think before asking pupils to raise their hand to give an answer. (Example answers: The objects have been sorted according to the type of object; an object has been sorted according to whether it is a stone, leaf or stick)



Objects collected and sorted into groups according to the type of object

8. **Ask**: What property can we use to sort just the stones? Raise your hand. (Example answer: We can sort the stones according to size, large stones together and small stones together).
9. **Ask**: What other properties can we use to collect and sort the objects? Raise your hand.
10. Guide the pupils to give other properties. (Example answers*:* large leaves together and small leaves together; large sticks and small sticks; all large objects and all small objects)

11. **Write** the following on the board and ask pupils to copy into their exercise books:
    When we have a collection of objects, we can use a common property to sort them into groups. This property can be used to describe the sort of group we make with the objects.

Guided Practice *(6 minutes)*

1. Ask pupils raise their hand and call out the names of 8 pupils in the class beginning with the letter A, B or C.
2. **Write** the names on the board: For example, Binta, Aminata, Brima, Charles, Ahmed, Ben, Catherine, Betty.
3. Read out the following questions one by one. Ask pupils to work in pairs to discuss and answer the following in their exercise books:
    a. Look at the collection of names. Sort and list all the names of pupils beginning with B.
    b. How many pupils have names beginning with B?
    c. What common property is used to describe the group?
4. Walk around, if possible, to check the answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give the answers to the questions. Correct any errors. (Answers: a. Binta, Brima, Ben, Betty; b. 4; c. Names beginning with B)

Independent Practice *(10 minutes)*

banana, guava, bench, angry, table, pink, spoon, plum, yellow, sad, blue, knife, chair, happy, cup, orange, plate

1. Read the list of objects shown in the box on the board.
2. **Say**: We can sort the objects on the board into different groups according to a common property.
3. **Say**: One group is comprised of fruits. Please raise your hand to give me the name of another group of objects you can see on the list. (Example answers: Colours, furniture, feelings, kitchen objects, objects for eating. Accept other reasonable answers which describe the objects)
4. **Say**: Now work by yourselves to sort the objects on the board into groups.
5. Allow pupils 5 minutes to work independently.
6. Walk around, if possible, to check the answers and clear up any misconceptions.
7. **Say**: Discuss your answers with a partner. Please do not change any answers as we will check them together.
8. Allow 1 minute for pupils to discuss and share their ideas.
9. Have pupils from around the classroom volunteer to come to the board and write one group of objects. (Example answers: As shown in the table)
10. **Ask:** Who has a different idea or answer?
11. Discuss other reasonable groupings, for example orange can be put with both banana as fruit, and yellow as colour.
12. **Say**: We can put an object in more than one group. It will depend on the property they have in common with other objects.

spoon cup plate knife	*pink yellow blue orange*	*sad happy angry*	*bench table chair*	*banana plum orange guava*

Closing *(2 minutes)*

1. **Ask:** What do we do when we want to collect, sort and group objects together? Raise your hand. (Answer: We use a property or properties they have in common.)
2. **Say**: Tomorrow we will look at ‘sets’. This is the special name given to groups of objects which have been collected and sorted according to a common property.

Lesson Title: Introduction to Sets	**Theme:** Numbers and Numeration	
Lesson Number: M-09-002	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to identify a set as a well-defined collection of objects or ideas.	None	1. Write this vocabulary list on the top left-hand corner of the board: Vocabulary List set, set notation, well-defined, curly brackets, list, member, element, 'is an element of.' 2. Write on the board: Cow, book, goat, sheep, pencil, paper, pig, ruler, pen. 3. Write the sets and instructions in the Independent Practice section on the board.

Opening *(3 minutes)*

1. Read these words from the board: Cow, book, goat, sheep, pencil, paper, pig, ruler, pen.
2. **Say:** Work in pairs to sort the objects on the board into groups. Use any property they have in common. You have 1 minute.
3. After 1 minute, ask pupils to raise their hand and call out the groups of objects and state the common property.
   (Example answer: The group including cow, goat, sheep, pig – common property - farm animals; group including book, pencil, paper, ruler, pen – common property - stationery items).
4. **Ask:** How many objects are in each group? Raise your hand. (Answer: 4 farm animals; 5 stationery items).
5. **Say:** A collection of objects is called a set.
6. **Write** on the board: A collection of objects is called a set.
7. **Say:** We will be looking at many aspects of sets over the next 2 weeks. Today's lesson will be an introduction to sets.

Introduction to the New Material *(15 minutes)*

1. Point to the vocabulary list on the board.
2. **Say**: Maths has its own language which can sometimes be confusing. You will understand the meanings of these words better as we use them throughout the lesson.
3. **Say:** Please copy the list into your exercise books. You must also take notes during the lesson. We study sets so that we can collect and examine objects and ideas. These objects and ideas share a common property. This helps us to classify and count them.
4. Ask pupils to give examples of sets. Guide them to give examples from everyday life.
   (Example answers: Pupils in JSS3; school subjects; months of the year; days of the week; districts in Sierra Leone)
5. **Say:** When we want to talk about sets, we use special language and notation. We will use the set of stationery items to learn some of the basic language and set notation.
6. **Write** on the board: { stationery items }
7. **Say:** We read this as: "the set of stationery items".
8. Ask the pupils to read this out loud.

{ *stationery items* }
read as: "*the set of stationery items*"

9. **Say:** We use these brackets, { }, called **curly brackets** to show we are talking about sets.
10. **Write** on the board: S = { stationery items }
11. **Say**: We can use a capital letter to represent a set. In our example, we will use S to represent the set of stationery items. We read this as: "S is the set of stationery items." We can also list the objects of the set inside curly brackets.
12. **Ask:** What objects belong to the set of stationery items? Raise your hand to answer.
13. Write the answer on the board. (Answer: { book, pencil, paper, ruler, pen })
14. **Say:** It does not matter in what order the objects are written. We just need to list all of them. We call the objects in the set members or elements. We separate them using commas.
15. **Ask**: Who can read what the statement says? Raise your hand.
16. Guide a pupil who has raised their hand to read the statement out loud. (Answer: "The set whose elements are book, pencil, paper, ruler, pen.")
17. **Say:** We denote the members or elements of the set by this symbol: ∈
18. **Ask**: What are the elements of set S? Raise your hand. (Answers: Book, pencil, paper, ruler, pen.)
19. **Say:** We can write that the pen belongs to set S as: pen ∈ S. We read it as: "Pen is an element of set S."

pen ∈ S read as: *"pen is an element of set S"*

20. **Say***:* We can now describe a set properly.
21. **Write** on the board: A set is a <u>well-defined</u> collection of objects or ideas.
22. **Say**: If the set is not well-defined, we will not be able to identify all its elements. For example, the set of greatest football players is not well-defined because everybody has their own opinion of what makes a football player great. We cannot say with certainty who should belong to the set.
23. **Ask**: Look again at set S - how many elements are in the set? (Answer*: 5*)
24. **Write** on the board: The number of elements in a set is denoted by n(S).

Guided Practice *(5 minutes)*

1. **Write** these 2 sets on the board: A = { vowels }; B = { tall people }
2. Guide pupils to say out loud: "A is the set of vowels; B is the set of tall people."
3. **Say:** Please work in pairs. Discuss for 1 minute whether or not the sets are well-defined. Remember we should be able to identify all the members of a well-defined set.
4. Allow 2 minutes for the pupils to discuss and share their ideas.
5. **Say**: Please raise your hand if you think set A is well defined.
6. Do a visual check of pupils who have their hands raised.
7. **Say:** Now raise your hands if you think set A is **not** well defined.
8. Take note of pupils who appear unsure and provide assistance during independent practice.
9. Most of the pupils should have raised their hands to say set A is well-defined.

25. **Ask**: Why is set A well-defined? Raise your hand.

10. Select a pupil from the back of the classroom to answer. (Example answers: We can identify all the elements of the set; we can say or list them with certainty.)
11. Have a pupil volunteer to write the set in set notation. (Answer: A = { a, e, i, o, u })
12. Most of the pupils should have raised their hands to say set B is **not** well-defined.
13. **Ask:** Why is set B not well-defined? (Example answers: We cannot identify all the elements with certainty; we need to specify a minimum height; people may disagree about what tall means, someone who is tall to one person may not be tall to another).

Independent Practice *(10 minutes)*

1. Ask pupils to answer the questions in their exercise books independently.
2. Point to these sets and instructions on the board:
   Sets: A = { days of the week }; B = { months of the year }

   For each set:
   a. Describe the set in words.
   b. Is the set well-defined?

   If the set is well-defined:
   c. List the elements of the set;
   d. Select the third element of the set and use set notation to show that it belongs to the set;
   e. Write the number of elements in the set using set notation.
3. Walk around the classroom, if possible, and check pupils' work. Give particular attention to the pupils who appeared unsure during the previous activity.
4. Ask the pupils to stop after 6 minutes and to discuss their answers with a partner for 1 minute.
5. Have pupils from around the classroom volunteer to come to the board and write their answers for the questions. Correct any errors.
   (Answers: Set A - a. A is the set of days of the week; b. Yes, A is well defined; c. A = { Monday, Tuesday, Wednesday, Thursday, Friday }; d. Wednesday ∈ A; e. n(A) = 7
   Set B - a. B is the set of months of the year; b. Yes, B is well defined; c. B = { January, February, March, April, May, June, July, August, September, October, November, December }; d. March ∈ B; e. n(B) = 12)

Closing *(2 minutes)*

1. **Ask:** What is a set? Raise your hand.
2. Select a pupil from the back of the class to answer.
   (Answer: A set is a well-defined collection of objects or ideas).
3. Make sure pupils have this definition written in their exercise books.
4. **Ask:** How can we tell if a set is well-defined or not? Raise your hand.
5. Select a pupil from the middle of the class to answer. (Example answers: We can fully describe the elements of the set; we are able to identify with certainty all the elements of the set.)

Lesson Title: Sets in Real Life	**Theme:** Numbers and Numeration	
Lesson Number: M-09-003	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify sets of objects or ideas from everyday life. 2. Sort objects or ideas from everyday life into sets.	None	Write the lists of items from everyday life of a JSS 3 pupil from the box in the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say:** In the last lesson, we started looking at sets of objects and ideas. Let us see how much we remember.
2. **Write** this set on the board: { colours on the Sierra Leone flag }
3. **Say**: List the elements in the set of colours on the Sierra Leone flag using set notation in your exercise books.
4. Allow 1 minute for the pupils to answer.
5. Have a pupil volunteer to come to the board and write down the answer. (Answer: { green, white, blue })
6. **Say:** Remember to always write sets using curly brackets. You may need to practice how to write the brackets until you get used to writing them. Today we are going to investigate sets further by looking at sets from real life.

Introduction to the New Material (*15 minutes*)

1. **Say**: Sets can be made from things we can see and touch which are called objects, and things we cannot see and touch which are called ideas. We do not need to worry whether they are objects or ideas as long as we can group them using a common property. We use the general term 'element' or 'member' to identify them.
2. **Say**: Please work in pairs. Discuss and list the elements in the set of 'regions in Sierra Leone'.
3. Allow the pupils 1 minute to discuss and write down their answer.
4. Have a pupil from the front of the classroom volunteer to write their answer on the board. (Answer: { Eastern Province, Northern Province, Southern Province, Western Area })
5. **Say**: It does not matter in what order we write the elements. We just need to list all of them.
6. **Ask:** Who remembers what the definition of a set is? Raise your hand.
7. Guide the pupil to say: A set is a well-defined collection of objects or ideas.
8. **Say**: Now we will write a set about activities people do in their daily lives. One activity which people do on a daily basis is eat. We can call the set D.
9. **Write** on the board: D = { eat, }
10. **Say:** Work with your partner to discuss and add 4 other similar activities from daily life to set D.
11. After 2 minutes, have a pupil from the middle of the classroom volunteer to come to the board.
12. **Say:** Add 2 daily activities to set D. (Example answer: D = { eat, sleep, wash })
13. **Say** (to the pupil at the board)**:** Ask the class to give you 2 more activities. Please add them to the set. (Example answer: D = { eat, sleep, wash, cook, clean }).
14. **Say**: Look at the answers on the board.
    a. Did anyone have different ideas for the activities in D?

b. Is D well-defined?

15. Ask pupils to discuss their ideas in their pairs for 1 minute.
16. Ask pupils to raise their hand to share their answers. Discuss all the ideas that are presented. (Pupils may have opposing answers: Daily activities such as play, fetch water, go to the farm, etc., are not on the list so D is not well-defined; D is well-defined, but there are too many elements to list).
17. **Say**: We have a lot of ideas and many of you make good points. D is actually well-defined. However, there are a lot of activities people do on a daily basis, and we cannot list them all. D is a special type of set called an 'infinite set'. We will talk about this type of set next week. For now, we will specify the number of elements we want to list in our sets.

Guided Practice (*5 minutes*)

1. Ask pupils to continue working in pairs.
2. **Write** on the board:
   a. Write this list as a set of sauces - S: potato leaves, cassava leaves, groundnut soup, crain-crain.
   b. List 4 elements from the set of soft-drinks, D.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from the left- and right-hand side of the classroom volunteer to write the sets on the board.
   (Answer: a. S = { potato leaves, cassava leaves, groundnut soup, crain-crain }
   Example answer: b. D = { Fanta, Sprite, Coca-Cola, Vimto })

Independent Practice (*10 minutes*)

1. Read the lists of items from everyday life of a JSS3 pupil from the board (see box below).
2. Ask the pupils to sort the items into sets and to identify each set in their exercise books.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Stop pupils after 6-7 minutes to allow them to discuss their answers with a partner.
5. Have pupils raise their hand to share answers.
   (Example answers: { relatives } = { Aunty Memuna, Uncle Saidu, Aunty Fatu };
   { school subjects } = { French, English, Mathematics, Social Studies, Science };
   { bedroom furniture } = { mirror, clock, lamp, table, chair, bed })

English	*Aunty Memuna*	*Science*
chair	*Mathematics*	*Aunty Fatu*
mirror	*table*	*bed*
French	*lamp*	*Uncle Saidu*
clock	*Social Studies*	

Closing (*2 minutes*)

1. **Ask:** What did you learn in this lesson? Raise your hand to answer. (Example answers: About sets in real life; listing elements of a set; sets from real life can have a lot of elements.)

Lesson Title: Describe Sets of Objects	**Theme:** Numbers and Numeration	
Lesson Number: M-09-004	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Describe sets using words. 2. Define the properties of a set of objects or ideas.	None	1. Write this vocabulary list on the top left-hand corner of the board: <u>Vocabulary List</u> describe, define, list, such that 2. Write the exercise from the Independent Practice section on the board.

Opening (*2 minutes*)

1. Write these 2 sets on the board: { beautiful people }, { pupils in JSS3 }
2. **Ask**: Which of these 2 sets is well-defined?
3. Allow a few moments for thinking, and then have pupils raise their hand to answer. (Answer: { pupils in JSS3 })
4. **Ask**: Why is the set of beautiful people not well-defined?
5. Have 2-3 pupils from around the classroom volunteer to answer. (Answer: We cannot identify with certainty all the elements of the set; someone who may appear beautiful to one person may not to another; we do not know all the beautiful people in the world.)
6. **Say**: Today we are going to describe sets using words. We will also define the properties of a set of objects or ideas. We will consider only well-defined sets.

Introduction to the New Material (*15 minutes*)

1. **Say**: Write the vocabulary list into your exercise books. We will be using the terms throughout the lesson.
2. Give pupils 2 minutes to copy.
3. **Say**: There are several ways we can identify a set of objects.
4. **Say**: Let us use the example of the continents of the world. One way of describing the set is by using words. We can say: C is the set of continents of the world.
5. **Write**: C is the set of continents of the world.
6. **Say**: Another way to describe a set is by listing the elements of the set.
7. **Ask**: What are the 5 continents of the world? Raise your hand. (Answer: C = { Africa, America, Australia, Asia, Europe })
8. **Ask**: Does it matter in what order we list elements? Raise your hand. (Answer: No, as long as we list all of them.)

> <u>Describing a Set</u>
> C is the set of continents of the world
> C = { Africa, America, Australia, Asia, Europe }
> C = { continents in the world }

9. **Write**: C = { Africa, America, Australia, Asia, Europe }
10. **Say**: If there are a lot of elements, we can simply describe the set like this:
    C = { continents of the world }
11. **Write:** C = { continents of the world }
12. Ask the pupils to write the different ways of writing the set C in their exercise books. Give them 2 minutes.

13. **Say**: Another way we can identify a set is by defining the properties of the set. We use a special notation to do this.

Defining the Properties of a Set
C = { x : x is a continent of the world }
C is the set of all x, such that x is a continent of the world

14. **Write**:
    C = { x : x is a continent of the world }
    C is the set of all x, such that x is a continent of the world
15. **Say**: We use a small letter to represent the elements. For this set we have used 'x'. We use the colon symbol : and we read it as "such that".
16. Let us look at another example. We will write the set of JSS3 pupils by defining its property.
17. **Say**: Let us call the set J. We will use a small 'p' to represent the elements.
18. Guide a pupil to write the set J on the board: J = { p : p is a pupil of JSS3 }
19. **Ask**: How should we read the set written on the board? Answer together.
20. Guide pupils to say aloud: "J is the set of all p such that p is a pupil of JSS3."
21. **Say**: Because you are all pupils in JSS3, you are all elements of this set.
22. Ask pupils to write the set and how to read it in their exercise books.
23. **Say:** This method is very useful when we are describing sets of numbers. We shall do so in tomorrow's lesson. If we are given one of the descriptions of a set, we should be able to write the sets in the other different ways.

Guided Practice (*5 minutes*)

1. **Write** on the board: O is the set of oceans of the world. Use *x* for the elements of O.
2. **Say**: Identify O, the set of oceans of the world, using all the ways we have learned today.
3. Ask pupils to work in pairs. Allow 3 minutes to answer the question.
4. Walk around to check the answers and clear any misconceptions.
5. Have pupils from around the classroom volunteer to write their answers on the board. Correct any errors. (Answers: As shown in the box).

O is the set of oceans of the world
O = { Arctic, Pacific, Atlantic, Indian, Antarctic }
O = { oceans in the world }
O = { x : x is an ocean of the world }

Independent Practice (*10 minutes*)

1. Point to and explain the exercise on the board:

The following are sets. Use the first letter to represent the set and the second to represent the elements.

a. Vowels in the English alphabet (V, x)
b. Colours in a rainbow (C, r)
c. Letters in the word Maths (L, m)

Identify the sets on the board by:

i. Defining its properties ii. Listing its elements.

2. Walk around to check the answers and clear up any misconceptions.
3. Ask pupils to exchange exercise books and check each other's work.
4. **Write** the answers on the board. (Answer: a. i . V = { x : x is a vowel }, ii. { a, e, i, o, u };
   b. i. C = { r : r is a colour in a rainbow }, ii. { red, orange, yellow, green, blue, indigo, violet };
   c. i. L = { m : m is a letter in Maths }, ii. { m, a, t, h, s }).

Closing (*3 minutes*)

1. **Say:** Define the properties of the set of countries in the Mano River Union. Use M for the set and c for the elements of M. Please write your answer on a piece of paper with your name on it. You have 2 minutes.
2. Collect the work from pupils at end of the lesson to check their understanding of the topic. (Answer: M = { c : c is a country in the Mano River Union })

Lesson Title: Write Sets of Numbers	**Theme:** Numbers and Numeration	
Lesson Number: M-09-005	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. List the numbers in a set using brackets. 2. Identify and interpret set notation.	None	1. Write this vocabulary list on the top left-hand corner of the board: <u>Vocabulary List</u> Venn diagram, universal, 'is not an element of,' complement, less than, greater. 2. Write the exercise from the Guided Practice section on the board. 3. Write the exercise from the Independent Practice section on the board.

Opening (2 minutes)

1. **Write** on the board: A = { x : x is a prime number less than 10 }
2. Ask the class to read the set out loud together. (Answer: A is the set of all x such that x is a prime number less than 10.)
3. **Say**: Today we are going to list the numbers in a set using brackets. We will also identify and interpret set notation.

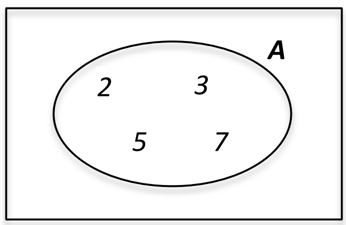
Introduction to the New Material (15 minutes)

- *Write all relevant information on the board as you give its explanation.*
- *Ask the pupils to copy the information on the board in their exercise books.*
- *After each question, wait a few moments before selecting pupils from different parts of the classroom to either call out the answer or come to the board and write the answer.*

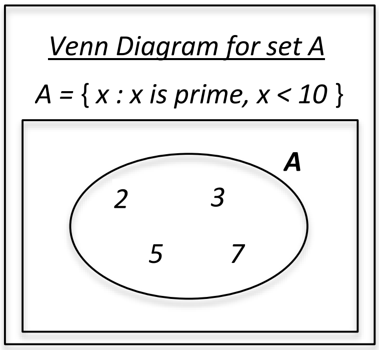
1. **Say**: Write the vocabulary list into your exercise books. We will be using the terms throughout the lesson. You have 2 minutes.
2. **Say:** We can identify set A another way using set notation: A = { x : x is prime, x < 10 }
3. **Write:** A = { x : x is prime, x < 10 }
4. Ask pupils to copy it in their exercise books.
5. **Say**: We read it the same way as before: A is the set of all x such that x is a prime number less than 10.

<u>*Venn Diagram for set A*</u>

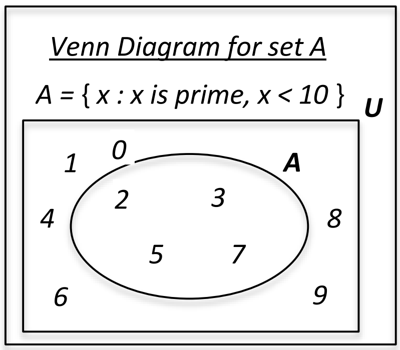
A = { x : x is prime, x < 10 }



6. **Ask**: What is the property that defines set A? Raise your hand. (Answer: The elements are all prime numbers less than 10.)



7. **Say**: List the elements of the set in your exercise books. Remember to use curly brackets.
8. After 2 minutes ask pupils to raise their hand to answer. **Write** the answer on the board. (Answer: A = { 2, 3, 5, 7 }
9. **Say**: Write in your exercise books how you would show that 7 belongs to set A.
10. After 1 minute ask pupils to raise their hand to answer. **Write** the answer on the board. (Answer: 7 ∈ A)
11. **Say**: We have one more way to represent a set. We can use a diagram called a ‘Venn diagram’.
12. Draw the Venn diagram shown on the board and **write** ‘Venn Diagram’ under it.
13. **Say**: The oval represents the set A. We list all of the elements of set A inside the oval. We sometimes use circles instead of ovals.
14. Ask pupils to copy the Venn diagram in their exercise books.
15. **Say**: Please look at the Venn diagram. We have drawn set A inside a rectangle.
16. **Say**: This rectangle is the set from which we have taken the elements of set A.
17. Point to the definition of set A.
18. **Ask**: Can anyone look at the definition of set A and tell the set that we have taken its elements from?
19. Guide the pupils to the part which shows that the elements have been taken from the set of numbers less than 10. (Answer: A = { x : x is prime, <u>x < 10</u> })
20. **Say**: This set is called the ‘universal set’. We define it as the set of all the elements under consideration. We represent it with the letter U. We will assume that the set has no fractions or decimals less than 10, just whole numbers.
21. **Write** on the board: U = { x : x < 10 }
22. Ask the pupils to read this out loud. (Answer: U is the set of all x such that x is less than 10.)
23. Write ‘U’ on the Venn diagram next to the rectangle. See the diagram below.
24. **Say:** Please copy the definition of set U in your exercise books. List all its elements in set notation.
25. Allow time for pupils to copy and write down the answers.
26. Ask pupils to raise their hand to answer. **Write** the answer on the board. (Answer: U = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 })
27. **Ask**: What numbers can you see in set U which are not in set A?



28. Ask pupils to think for a minutes and then raise their hand to answer. (Answers: 0, 1, 4, 6, 8, 9.)
29. **Say**: We use this symbol ∉ to show that a number does not belong to set A.
30. **Write** on the board: 6 ∉ A.
31. **Say**: This is read as "6 is not an element of A".
32. **Say**: We refer to the set of elements in set U which are not in set A as the <u>complement of A</u>. We denote this by the symbol A^C.
33. **Say**: Note the spelling of the word 'complement'.
34. **Write** on the board: A^c = { 0, 1, 4, 6, 8, 9 }
35. **Say**: We can show A^C on the Venn diagram.
36. **Write** the numbers in A^C in the Venn diagram (as show in the box). Ask the pupils to copy this in their exercise books.

37. **Say**: It is not necessary to put A^C in its own oval as this can become confusing when we have more than one set on the same Venn diagram.

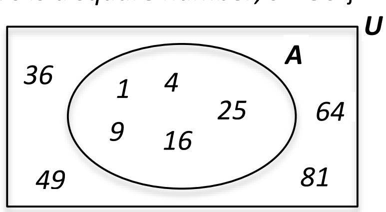
Guided Practice (*5 minutes*)

1. Point to and read the exercise on the board:
   U = { x : x is a counting number, x < 10 }
   a. Define B, the set of all even numbers y less than 10.
   b. List all the elements in set B using set notation.
   c. Write using set notation:
      i) an element in set B.
      ii) an element in set B^C.
2. Ask pupils to work in pairs to discuss and write the answers.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to share answers.
   (Answer: a. B = { y : y is even, y < 10 }; b. B = { 2, 4, 6, 8 }; c. i) Any one of these: 2 ∈ B, 4 ∈ B, 6 ∈ B, 8 ∈ B); ii) Any one of these: 1 ∈ B^C, 3 ∈ B^C, 5 ∈ B^C, 7 ∈ B^C, 9 ∈ B^C)

Independent Practice (*10 minutes*)

1. Point to and read out the exercise on the board:
   U = { 1, 4, 9, 16, 25, 36, 49, 64, 81 }
   a. Describe U, the universal set in words.
   b. Define the set of square numbers less than 30. Represent the set by A, the elements by s.
   c. List the elements of set A.
   d. Draw the Venn diagram for the set.
   e. Write the elements of A^C in the Venn diagram.
2. Ask the pupils to answer independently in their exercise books.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to write the answers on the board. Correct any errors.
   (Answer: a. U = { square numbers less than 100 }; b. A = { s : s is a square number, s < 30 }; c. A = { 1, 4, 9, 16, 25 }; d. see box; e. see box)

A = { s : s is a square number, s < 30 }



Closing (*3 minutes*)

1. **Ask:** What is the universal set, U? Raise your hand. (Answer: The universal set, U is the set of all elements under consideration.
2. **Ask:** What is the complement of a set, A^C? Raise your hand. (Answer: The complement of a set, A^C is the set of elements in set U which are not in set A).

Lesson Title: Finite Sets	Theme: Number and Numeration	
Lesson Number: M-09-006	**Class/Level:** JSS 3	**Time:** 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils, will be able to identify sets of objects, things, ideas and numbers that are finite.	None	1. Write this set in the centre of the board: { Sunday, Monday, Tuesday, Wednesday, Thursday, Friday } 2. Write this vocabulary list on the top left-hand corner of the board: Vocabulary List: finite, ellipsis 3. Write the exercise from the Guided Practice section on the board. 4. Write the exercise from the Independent Practice section on the board.

Opening (*2 minutes*)

1. **Say**: Please raise your hands if you recognise the set on the board. (Answer: The set of the days of the week.)
2. **Say**: Today we are going to identify sets of objects, things, ideas and numbers that are finite.

Introduction to the New Material (*10 minutes*)

1. **Say:** Let us call the set on the board, D.
2. **Ask:** How many elements are in set D? Raise your hand. (Answer: 7)
3. **Say:** There are 7 days in a week. We will always be able to list the days and count how many there are. We call a set where we can list and count all the elements or members a 'finite' set.
4. **Write** the definition of a finite set on the board: A set where we can list and count all the elements or members.
5. Ask the pupils to copy the information on the board in their exercise books.
6. Have a pupil from the left-hand side of the classroom volunteer to remind the class how to write the number of elements in set D using set notation. (Answer: n(D) = 7)
7. **Write** on the board: L = { letters in the English alphabet }
8. **Ask**: How can we tell if this is a finite set?
9. Give a few moments for the pupils to discuss this with a partner.
10. Have a pupil from the front of the classroom volunteer to answer. (Answer: We can list and count all its elements.)
11. **Ask:** How many elements are in set L? Raise your hand. (Answer: n(L) = 26)
12. **Say:** Because we have so many elements in set L, we may not want to list all of them. We can use 'ellipsis' in the middle of the list to show that some elements are missing.
13. **Write** on the board: L = { a, b, c, d, . . ., x, y, z }
14. **Say**: Please list enough elements so the ones missed out are clearly understood.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to and read the exercise on the board:
   a. A = { factors of 10 }

b. B = { whole numbers less than 100 }
c. C = { multiples of 3 between 30 and 50 }
   i. State whether the set is finite or not.
   If the set is finite:
   ii. List its elements. Use ellipsis if the set has more than 6 elements.
   iii. Write the number of elements in the set using set notation.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give the various answers to the problem.
5. Correct any errors.

Answers:			
	a.i. finite	ii. A = { 1, 2, 5, 10 }	iii. n(A) = 4
	b.i. finite	ii. B = { 0, 1, 2, . . ., 97, 98, 99}	iii. n(B) = 100
	c.i. finite	ii. C = { 33, 36, 39, 42, 45, 48 }	iii. n(C) = 6

Independent Practice (*10 minutes*)

1. Point to and read the following exercise from the board for pupils to complete independently.

   U = { whole numbers from 1 to 50 }

   List the elements in the following sets and give the number of elements in each set. Use ellipsis if there are more than 8 elements in the set.
   a. A = { prime numbers }
   b. B = { square numbers }
   c. C = { numbers divisible by 5 }
   d. D = { multiples of 4 }
2. Give pupils 7 minutes to complete the exercise.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from the front, back, left-hand side and right-hand side of the classroom volunteer to give the answers. Write the correct answers on the board. Ask pupils to check their work

Answers:		
	a. A = { 2, 3, 5, 7, . . ., 37, 41, 43, 47 }	n(A) = 15
	b. B = { 1, 4, 9, 16, 25,36, 49 }	n(B) = 7
	c. C = { 5, 10, 15, 20, . . ., 35, 40, 45, 50 }	n(C) = 10
	d. D = { 4, 8, 12, 16, . . ., 36, 40, 44, 48 }	n(D) = 12

Closing (*3 minutes*)

1. **Ask**: What is a finite set? Raise your hand. (*Answer*: A set where we can list and count all the elements or members.)
2. **Say**: Great work! In the next lesson, we will look at infinite sets.

Lesson Title: Infinite Sets	Theme: Number and Numeration	
Lesson Number: M-09-007	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to identify sets of objects, things, ideas and numbers that are infinite.	None	1. Write this vocabulary list on the top left-hand corner of the board: <u>Vocabulary List:</u> infinite 2. Write the exercise from the Independent Practice section on the board.

Opening (*5 minutes*)

1. **Write** on the board: { factors of 2 }
2. **Say**: Please list the element of the set of factors of 2.
3. Allow a few moments for the pupils to list the elements of the set.
4. **Ask:** How many elements do we have in the set? Raise your hand. (Answer: 2 elements, { 1, 2 })
5. **Say**: Raise your hand if you know the name of this type of set.
6. Select a pupil who has raised their hand to answer. (Answer: finite set.)
7. **Ask**: What is a finite set?
8. Have a pupil at the front of the classroom volunteer to answer. (Answer: A finite set is a set where we can list and count all the elements or members.)
9. **Say**: Today we are going to identify sets of objects, things ideas and numbers that are infinite.

Introduction to the New Material (*13 minutes*)

1. **Write** on the board: { multiples of 2 }
2. **Say:** Give me examples of members of this set.
3. Allow pupils from around the classroom to give examples of multiples of 2.
4. **Write** them on the board as they call them out. Allow this to continue until you have 8-10 examples of multiples of 2. (Example answers: 2, 4, 6, 8, 10, 12, 14, 16)
5. **Ask:** How many elements do we have so far?
6. Have a pupil volunteer to count the examples on the board and give the answer.
7. **Ask:** What other elements can you think of? (Example answers: 18, 20, 22)
8. **Say:** The set of multiples of 2 is an example of an infinite set.
9. **Ask:** From what we have done so far, who can explain what an infinite set is?
10. Have 2-3 pupils volunteer to give explanations. (Example answers: An infinite set is a set where there are many elements; we cannot list all the elements; there are too many elements to list)
11. **Write** on the board: An infinite set is a set where we <u>cannot</u> list or count all the elements or members. No matter how many elements we list, there will always be more.
12. Enclose the set of numbers on the board with curly brackets and add ellipsis at the end: { 2, 4, 6, 8, 10. 12, . . . })
13. **Say:** We add ellipses at the end of infinite sets to show that the elements go on forever. This is different from when we added ellipsis to finite sets. In a finite set, we know the number of elements. We add ellipsis to show some elements have been left out. In an infinite set, we do not know the number of elements. We add ellipsis to show that there are infinitely more elements to come.
14. Ask pupils to copy the information on the board in their exercise books.

Guided Practice (*5 minutes*)

1. **Write** on the board: A = { 1, 2, 3, 4, . . . 18, 19, 20 }, B = { 1, 2, 3, 4, . . . }
2. **Say**: Please write down which set you think is a finite set and which one is an infinite set.
3. Allow pupils 1 minute to answer.
4. **Say**: Turn to your neighbour and discuss your answers with them. You have 1 minute.
5. **Say**: Raise your hand if you think set A is a finite set.
6. Do a visual check of hands raised and take note of pupils who seem unsure.
7. **Say**: Raise your hand if you think set A is an infinite set.
8. Do a visual check of hands raised and again take note of pupils who seem unsure.
9. **Ask**: Do you think set B is finite or infinite?
10. Allow pupils to call out answers.
11. Have 2-3 pupils volunteer to give a reason for their answers for sets A and B.
12. **Say**: A is a finite set because the elements go from 1 to 20. The ellipsis shows we have not listed some elements. B is an infinite set because the elements start from 1 and continue for ever. The ellipsis shows that we can continue writing elements until infinity.
13. Write this on the board and ask pupils to copy this in their exercise books.

Independent Practice (*10 minutes*)

1. Point to and read the following exercise:

Sets:	
a. { even numbers from 20 to 30 }.	b. { prime numbers}
c. { odd numbers between 10 and 100 }	d. (whole numbers greater than 2000 }

   For each set:
   i. Say whether the set is finite or infinite.
   ii. List the elements of the set using set notation.
2. Ask the pupils to answer independently in their exercise books.
3. Walk around, if possible, to check the answers and clear up any misconceptions, particularly with the pupils who seemed unsure during the previous exercise.
4. Have pupils from around the classroom volunteer to give their answers to the questions. (Answers: a. i. finite, ii. { 20, 22, 24, 26, 28, 30 }; b. i. infinite, ii. { 2, 3, 5, 7, . . . }; c. i. finite, ii. { 11, 13, 15, 17, . . . 95, 97, 99 }; d. i. infinite ii. { 2001, 2002, 2003, 2004, . . . }).

Closing (*2 minutes*)

1. **Ask:** What did you learn in this lesson? (Example answers: About infinite sets; the difference between finite and infinite sets; the difference between using ellipsis for finite and infinite sets)

Lesson Title: Unit and Empty Sets	**Theme**: Number and Numeration	
Lesson Number: M-09-008	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to identify a unit set as one with one element, and an empty set as one with no elements.	None	1. Write this vocabulary list on the top left-hand corner of the board: Vocabulary List: null 2. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Write** on the board: C = { even prime numbers }, D = { even prime numbers greater than 5 }
2. **Say**: List the element of sets C and D.
3. Allow 1 minute for the pupils to list the elements of the sets.
4. **Ask:** How many elements do we have in each set? Raise your hand to answer. (Answers: C = { 2 }, n(C) = 1; D = { }, n(D) = 0)
5. **Say**: Today we are going to be looking at unit and empty sets.

Introduction to the New Material (*12 minutes*)

1. **Write**:
   C = { 2 }, n(C) = 1 – Unit set (has only 1 element)
   D = { }, n(D) = 0 – Empty or null set (has no elements). Denoted by **:** { } or ∅
2. **Say:** There are special names given to sets C and D. Because there is only one element in set C, it is called a 'unit set'. Because set D has no elements, it is called an 'empty set' or a 'null set'. It is written as { } or ∅.
3. Ask pupils to copy the information in their exercise books.
4. **Say:** An example of a set with only one element from everyday life is a set of months with 28 days.
5. **Ask:** What month has 28 days?
6. Have a pupil volunteer to answer. (Answer: February)
7. **Say:** Let us call the set of months with 28 days E.
8. **Write** on the board: E = { February }, n(E) = 1.
9. **Say**: An example of an empty set from everyday life is a set of months with 32 days.
10. **Write** on the board: { months with 32 days } = ∅, n(∅) = 0.
11. **Say**: Remember the set written as { } is an empty set. It contains nothing. The set written as { ∅ } is not an empty set. It is a unit set containing one element which is the empty set.
12. Discuss this for 1 minute. Make sure the pupils understand the difference between { } and { ∅ }.

Guided Practice (*5 minutes*)

1. **Write** on the board:
   a. Is an empty set a finite or infinite set?
   b. Is a unit set a finite or infinite set?
2. Ask pupils to work in pairs to answer the questions.
3. Walk around, if possible, to check answers and clear any misconceptions about what makes a set finite or infinite.

4. Have one pupil from the left-hand side and one from the right-hand side of the classroom volunteer to give their answers to the questions. (Answers: a. finite, it has 0 elements; b. finite, it has 1 element.)

Independent Practice (*10 minutes*)

1. Read the questions below from the board.
   i. Give one example from everyday life of: a) an empty set b) a unit set
   ii. Give one example from Maths of: a) an empty set b) a unit set
2. Ask the pupils to work independently to answer the questions in their exercise books.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Ask the opinion of the class after each answer and discuss the answer for a few moments. (Example answers:
   i. a) { dogs with 5 legs }; { year with 13 months }
      b) { months with 29 days }, { February }; { days starting with the letter M }, { Monday }
   ii. a) { whole numbers which are both even and odd }; { squares with unequal sides }
      b) { whole numbers which have only one factor}, { 1 }; { whole numbers with 4 as their square }, { 2 })
6. Clear up any misconceptions. For example, the fact that 2 is not the same as { 2 }. Explain that 2 is the number 2, while { 2 } is the set which has the number 2 as its only element.

Closing (*5 minutes*)

1. **Say:** Please say 2 things you learned during this lesson.
2. Have 2-3 pupils volunteer to say 2 things they learned. (Example answers: I learned about unit and empty sets; I learned that both unit and empty sets are finite; February can have both 28 and 29 days)
3. Discuss and ask follow-up questions. Example follow-up questions: What did you learn about unit sets? What did you learn about empty sets? How many elements are in unit and empty sets? Is there any other month where the number of days changes like February?

Lesson Title: Equal Sets	**Theme**: Number and Numeration	
Lesson Number: M-09-009	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to identify sets with the same elements.	None	1. Write on the board: { 4, 2, 1, 3 }, { 3, 2, 1, 4 }, { 2, 1, 2, 3, 4 } 2. Write the questions from the Independent Practice section on the board.

Opening (*2 minutes*)

1. Point to the sets on the board: { 4, 2, 1, 3 }, { 3, 2, 1, 4 }, { 2, 1, 2, 3, 4 }
2. **Ask**: Look at the sets on the board. What do you notice?
3. Give pupils 1 minute to discuss the sets with their partner.
4. Guide pupils to say that the elements of all 3 sets are 1, 2, 3, 4.
5. **Say**: In today's lesson, we will be looking at equal sets.

Introduction to the New Material (*15 minutes*)

1. **Say** and **write** on the board**:** 2 sets are equal when they have **exactly** the same elements.
2. **Say:** We can also compare more than 2 sets.
3. **Write** on the board: { 4, 2, 1, 3 } = { 3, 2, 1, 4 } = {2 , 1, 2, 3, 4 }.
4. **Say** and **write** on the board: It does not matter what order we write the elements or if we write the same element more than once. When counting the number of elements in a set, we only count the unique elements. This means we do not count any element more than once.
5. **Ask**: How many unique elements do the 3 sets have exactly? Raise your hand. (Answer: All 3 sets have exactly 4 unique elements – 1, 2, 3, 4.
6. **Say**: We count the '2' in the third set only once. It is also good practice to put the elements in ascending or descending order. We can now write one set to represent all 3 sets on the board.
7. **Write** on the board: All 3 sets can be represented by { 1, 2, 3, 4 }.
8. Ask pupils to write down all the information on the board in their exercise books.
9. **Write** on the board: A = { 2, 4, 6, 8 }, B = { 10, 20, 30, 40, 50 }.
10. **Say**: Look at the sets on the board. What can you say about the elements in the sets?
11. Give pupils 2-3 minutes to discuss and share ideas with their partner.
12. **Say:** Describe the elements in sets A and B. Raise your hand to answer. (Example answers: The elements in set A are even numbers less than 10 (or multiples of 2 less than 10), the elements in set B are multiples of 10 from 10 to 50.)
13. **Ask**: Do the sets have exactly the same elements? Raise your hand to answer. (Answer: No, the elements in the sets are all different.)
14. **Say**: 2 sets are **not** equal when they have different elements.
15. **Write** on the board: { 2, 4, 6, 8 } $\neq$ { 10, 20, 30, 40, 50 }
16. **Say** and **write :** This can also be written as A $\neq$ B.
17. Ask the pupils to copy all the new information on the board in their exercise books.

Guided Practice (*5 minutes*)

1. **Write** on the board: C = { 1, 3, 5, 7 }, D = { 3, 5 }

2. **Say**: Discuss with your partner if you think sets C and D are equal.
3. Allow 1-2 minutes for the discussion.
4. Select 2-3 pupils who have their hands raised to answer. (Answers: Set C is not equal to set D; they do not have the same number of unique elements; n(C) = 4, n(D) = 2; the 2 sets do not have exactly the same elements)
5. Clear up any misconceptions.
6. **Say**: The most important thing for sets to be equal is that they must have 'exactly' the same elements.

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the following questions on the board:
   i. Put $=$ or $\neq$ between the 2 sets to make the statement true.
      a. { prime numbers less than 10 } ☐ { 2, 3, 5, 7 }
      b. { 1, 3, 5 } ☐ { 1, 2, 3 }
      c. { a, a, b } ☐ { a, b }
   ii. Are A and B equal?
      a. A = { first 4 positive whole numbers }; B = { 4, 2, 1, 3 }
      b. A = { s : s is a square number, s < 20 }; B = { 1, 4, 9, 16, 25 }
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions. (Answers: i. a. $=$; b. $\neq$; c. $=$; ii. a. Yes ; b. No).

Closing *(3 minutes)*

1. **Ask:** What is the most important thing for sets to be equal?
2. Ask pupils to write their names and answers on a piece of paper and to hand them in. (Answer: Equal sets must have exactly the same elements.)
3. Use this to determine whether pupils achieved the main learning outcome of the lesson. Review at start of the next lesson.

Lesson Title: Equivalent Sets	**Theme**: Numbers and Numeration	
Lesson Number: M-09-010	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify that an equivalent set has the same number of elements. 2. Distinguish between equal and equivalent sets.	None	1. Write this vocabulary list on the top left-hand corner of the board: Vocabulary List: equivalent, unique 2. Write the questions from the Independent Practice section on the board. 3. Write on the board: { a, b, c }, { b, a, b, c, a, c, b }

Opening (*2 minutes*)

1. Point to the sets on the board: { a, b, c }, { b, a, b, c, a, c, b }
2. **Ask**: Are these 2 sets equal? Raise your hand.
3. Select a pupil from the back of the classroom to answer. (Answer: Yes).
4. **Ask:** What makes them equal? Raise your hand. (Example answers: They have exactly the same elements. The elements for both sets are a, b, c).
5. **Say**: Today we will be looking at equivalent sets and learning how they differ from equal sets.

Introduction to the New Material (*15 minutes*)

1. **Write** on the board: A = { a, b, c }, B = { 1, 2, 3 }.
2. **Ask**: What do you notice about the 2 sets on the board?
3. Guide the pupils to say the 2 sets have the same number of elements.
4. **Write** on the board: n(A) = n(B).
5. **Say** and **write** on the board: When 2 sets have the same number of elements they are called 'equivalent' sets. We write this using a double-headed arrow as: A ↔ B.
6. **Say:** We read it as set A is equivalent to set B.
7. **Write**: C = { 5, 10, 15 }, D = { 5, 15, 5, 10 }.
8. **Say**: Please look at sets C and D. Can you say if they are equal sets, equivalent sets, neither or both?
9. Ask pupils to think about this for 2 minutes and write down any ideas they have.
10. Guide them to think about the number and type of unique elements in each set.
11. Ask pupils to discuss their ideas with their partner for 2 more minutes.
12. Have pupils from around the classroom volunteer to share their ideas with the class. Discuss these ideas. (Example answers: Set C has 3 unique elements 5, 10 and 15. Set D also has 3 unique elements and can be written as { 5, 10, 15 }. Therefore the 2 sets are equal because they have exactly the same elements. Sets C and D are equivalent because they have the same number of elements n(C) = n(D) = 3).
13. **Write** on the board: All equal sets are equivalent but not all equivalent sets are equal.
14. Discuss the above statement using open-ended questions for 2 minutes using sets A, B, C and D.
    a. Which sets are equivalent? Why?
    b. Which sets are equal? Why?
    (Answers: Sets A, B, C and D all have 3 unique elements so they are all equivalent with each other. Sets A and B have exactly the same 3 unique elements, so they are equal).

Guided Practice (*5 minutes*)

1. **Write** on the board: A = { w, x, y, z }, B = { cow, goat, sheep, pig }, C = { w, x, y, x. w, y, z }.
2. Ask the pupils to discuss in pairs which of these sets are equal and which are equivalent to each other.
3. Have pupils from around the classroom volunteer to give their answers.
   (Answers: A ↔ B ↔ C because they all have 4 unique elements, n(A) = n(B) = n(C) = 4; A = C because they have exactly the same 4 unique elements, w, x, y, z).

Independent Practice (*10 minutes*)

1. Point to the following questions on the board for pupils to answer independently:
   The 4 sets M, N, O and P are described as shown:
   M = { kite, rectangle, rhombus, square, trapezium }, N = { odd numbers between 1 and 10 },
   O = { square numbers between 1 and 30 }, P = { 1, 3, 5, 7, 9 },
   i. List the elements for the sets N and O .
   ii. Use ↔ (is equivalent to) or = (is equal to) between the 2 sets to make the statements true.
   a. M ☐ N b. O ☐ P c. N ☐ P d. M ☐ P
2. Have pupils from around the classroom volunteer to give their answers to the questions.
3. Discuss the answers and clear any misconceptions (in particular ii. c.).
   (Answers: i. N = **{** 1, 3, 5, 7, 9 }, O = { 1, 4, 9, 16, 25 };
   ii. a. M ↔ N b. O ↔ P c. N = P (accept also N ↔ P) d. M ↔ P)

Closing (*3 minutes*)

1. **Ask:** What is the difference equal and equivalent sets?
2. Have a pupil from the back of the classroom volunteer to answer.
3. Have another pupil from the front of the classroom volunteer to repeat the answer given.
   (Answer: Equal sets have exactly the same elements while equivalent sets have the same number of elements.)

Lesson Title: Introduction to Subsets	**Theme**: Numbers and Numeration	
Lesson Number: M-09-011	**Class/Level**: JSS 3	**Time:** 35 minutes

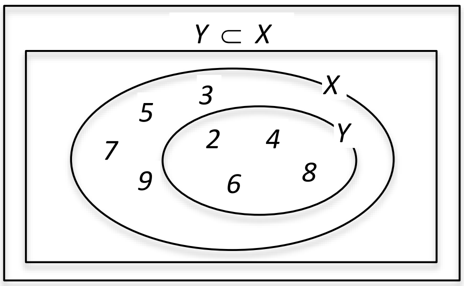
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to identify subsets as a collection of objects within a set.	None	1. Write this vocabulary list on the top left-hand corner of the board: <u>Vocabulary List - </u>subset 2. Write on the board: X = { whole numbers between 1 and 10 }, Y = { even numbers between 1 and 10 } 3. Write the exercise from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: List the elements of the following sets on the board: X = { whole numbers between 1 and 10 }, Y = { even numbers between 1 and 10 }.
2. Allow time for the pupils to list the elements of the 2 sets.
3. Ask pupils to raise their hand to share answers. **Write** the answers on the board. (Answer: X = { 2, 3, 4, 5, 6, 7, 8, 9 }, Y = { 2, 4, 6, 8 }
4. **Ask:** Which elements in set Y are present in set X? Raise your hand. (Answer: { 2, 4, 6, 8 })
5. **Say**: Today we will introduce subsets where all the elements in one set are present in another.

Introduction to the New Material (*15 minutes*)

1. **Say:** If 'every' element in set Y is present in set X, then Y is a subset of X. We use a special notation for this relationship between the 2 set.
2. **Write** on the board:
   $\{ 2, 4, 6, 8 \} \subset \{ 2, 3, 4, 5, 6, 7, 8, 9 \}$
   $Y \subset X$
   This is read as set Y "is a subset of" set X
3. **Say:** If 'every' element in a set, e.g. W, is not present in set X, then W is not a subset of X.
4. **Write** on the board.
   $W = \{ 5, 6, 7, 8, 9, 10 \}$
   $W \not\subset X$
   This is read as set W "is not a subset of" set X
5. **Ask**: There is one element in set W which is not present in set X. What element is that?
6. Allow pupils to call out the answer. (Answer: 10).
7. **Say**: For set W to be a subset of set X, 'every' element in W must be present in X. We can draw a Venn diagram showing the relationship between sets X and Y.
8. Draw the Venn diagram as shown in the box.
9. **Say**: Note how all the even numbers are shown in both X and Y. If we remove the oval shape for set Y, all the elements in X will still be shown. The set X has many subsets, not just Y. One other example is { 2, 3, 9 }.
10. **Say:** Write down 3 more subsets of set X in your exercise books.
11. After 2 minutes ask pupils to raise their hand to answer. **Write** some example answers on the board. (Example answers: { 3, 5, 7, 9 }, { 8, 9 }, { 7 }, { 2 }, { 2, 3 }, { })



12. **Write** on the board: { } $\subset$ X.
13. **Ask**: How can you explain the statement, "the empty set is a subset of set X?"
14. Discuss this statement with the pupils. (Discussion point: The empty set has nothing in it, and one of the subsets of set X also has nothing in it. This set with nothing in it is an empty set. Therefore the empty set is a subset of X.)
15. **Say** and **write** on the board: The empty set is a subset of every set, because there will always be one subset which has nothing in it.
16. Ask pupils to copy the above statement in their exercise books.

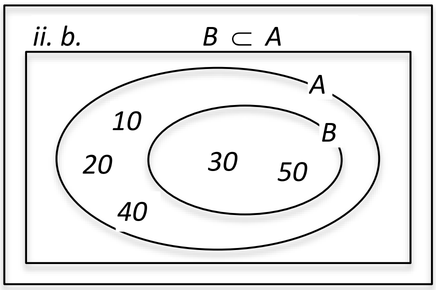
Guided Practice (*5 minutes*)

1. **Write** on the board: S = { 6, 12, 18, 24, 30 }, T = { 6, 9, 12 }
2. **Ask**: Look at the sets on the board - is set T a subset of set S?
3. Allow the pupils some time (less than a minute) to think about the question.
4. **Say**: Please discuss your answer with your neighbour.
5. Allow 1 minute for the pupils to discuss their answer with their neighbour.
6. Have a pupil from the front of the classroom volunteer to give their answer.
7. Have 1-2 other pupils from around the classroom volunteer to say if they agree/disagree with the answer and why.
8. Allow the class to discuss the answer for 1 minute and come to an agreement about the answer.
9. Share the answer with the class. (Answer: T $\not\subset$ S, because the element 9, which is in set T, is not in set S).

Independent Practice (*10 minutes*)

1. Point to the following exercise on the board:
   i. A = { 21, 22, 23, 24, 25 }, B = { 31, 32, 33, 34, 35 }
   ii. A = { 10, 20, 30, 40, 50 }, B = { 30, 50 }
   iii. A = { 1, 3, 6, 10, 15 } B = { 21, 28, 36, 45, 55 }
      a. Use set notation to write whether B is a subset of A.
      b. Draw the Venn diagram if B is a subset of A.
   iv. Write all the subsets you can find for the set { a, b, c }.
2. Ask the pupils to answer the questions independently in 6 minutes.
3. Walk around the classroom to check on their answers and clear up any misconceptions.

(Answers: i. a. B $\not\subset$ A; ii. a. B $\subset$ A iii. a. B $\not\subset$ A
only 1 Venn diagram required for ii. b. shown right.
iv. { }, { a }, { b }, { c }, { a, b } { a, c }, { b, c }, { a, b, c }).



4. Discuss the answers to Question iv, especially why { } and { a, b, c } are subsets of { a, b, c }. (Discussion point: Empty set { } is discussed above in the introduction. In the case of { a, b, c }, one of the subsets of any set, A, is the set itself because every element of set A is an element of A).

Closing (*2 minutes*)

1. **Ask:** What is the condition for set P to be a subset of set Q? Raise your hand.
   (Answer: If 'every' element in set P is present in set Q, then P is a subset of Q).

Lesson Title: Identifying Subsets of a Set of Real Numbers	**Theme**: Numbers and Numeration	
Lesson Number: M-09-012	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to identify subsets of real numbers: natural numbers, whole numbers, rational numbers (integers, fractions and decimals).	None	1. Write this vocabulary list on the top left-hand corner of the board: Vocabulary List: natural numbers, integers, rational numbers, irrational numbers. 2. Write this set of numbers on the board: { 5, -2, 0, $\frac{3}{4}$, 0.5, $\sqrt{3}$, π }. 3. Write these instructions on the board: a. Draw each subset according to the relationship found in the previous exercise. b. Label each subset with the correct symbol. c. List a few examples of elements which belong in each subset.

Opening (*2 minutes*)

1. Read this set of numbers from the board: { 5, -2, 0, $\frac{3}{4}$, 0.5, $\sqrt{3}$, π }.
   (Leave the list on the board to use throughout the lesson).
2. **Say**: Each number on the board has got a name that we identify it by. For instance, $\frac{3}{4}$ is called a fraction.
3. **Ask**: What name do we give the number 0.5? Raise your hand. (Answer: Decimal).
4. **Say**: In today's lesson, we will learn how to identify subsets of real numbers.

Introduction to the New Material (*15 minutes*)

1. **Say:** All the numbers we will look at today are subsets of real numbers. The set of real numbers is the universal set for the sets we are considering. It is denoted by R.
2. **Write**: R is the universal set for all real numbers.
   R = { real numbers }
3. **Say**: All the subsets of R are infinite sets so we will only list a few elements for each. We will use ellipsis to show how infinite sets go on forever.
4. **Ask**: How is an ellipsis written? Raise your hand. (Answer; as 3 dots . . .)
5. **Say**: The first subset is the set of natural numbers. These are the counting numbers we learnt in primary school.
6. **Ask**: What are the first 5 elements of this set? Raise your hand. (Answer: 1, 2, 3, 4, 5).
7. If a pupil gives 0 as an example, **ask**: Why is 0 not a counting number? (Example answers: We always start counting objects from 1; we never start counting from 0)
8. **Write**: N = { natural numbers }
   = { 1, 2, 3, 4, 5, . . . }
9. **Say:** The next subset is the set of whole numbers, W. These are the natural numbers plus 0.
10. **Write**: W = { whole numbers }
    = { 0, 1, 2, 3, 4, 5, . . . }
11. Point to the number -2 from the list at the start of the lesson.
12. **Ask**: What is this type of number called? Raise your hand. (Answer: negative integer).

13. **Say**: The set of integers is made up of positive natural numbers, 0, and negative natural numbers. It is denoted by Z.
14. **Write:** Z = { integers }
    = { . . ., −3, −2, −1, 0, 1, 2, 3, . . . }
15. Explain to pupils how the ellipsis at both ends of the set shows the numbers carry on forever in both positive and negative directions.
16. **Ask**: What numbers can you see on the board that we have not discussed as yet? Raise your hand. (Answers: $\frac{3}{4}$; 0.5; $\sqrt{3}$; π).
17. **Say**: We already know that $\frac{3}{4}$ is a fraction and 0.25 is a decimal number. Both these numbers belong to the set of rational numbers, Q.
18. **Write**: Q = { rational numbers }
    = { . . ., −2, −11/2, −0.5, 0, 1, 33/4, . . . }
    All integers, fractions and decimals are rational numbers. However, he denominator cannot be 0.
19. **Say**: 'Rational' comes from the word 'ratio' and describes how all the numbers can be written as ratios or fractions of integers.
20. **Write**: $\frac{3}{4}$ is a ratio of the integers 3 and 4.
21. **Write:** 0.5 can be written as a fraction $\frac{5}{10}$ which is the same as $\frac{1}{2}$.
22. **Ask**: What 2 integers make up this fraction? (Answer: 1 and 2).
23. Explain that the denominator of rational numbers cannot be 0 as we cannot divide by 0.
24. **Ask**: What about the integers themselves? Can they be rational numbers as well? Raise your hand.
25. Guide the pupils to see how an integer, e.g. 5, can be written as the fraction $\frac{5}{1}$.
26. **Say**: All integers, fractions and decimals are rational numbers. However, the denominator cannot be 0. We have just listed a few in the example.
27. **Say:** The last 2 numbers on the list are $\sqrt{3}$ and π. They belong to a set of numbers which cannot be written as fractions.
28. Explain to pupils that π written as $\frac{22}{7}$ or 3.142 is just an approximate value. In reality, its value goes on forever. It is $3.141592653\ldots$ to 9 decimal places.
29. **Write**: $3.141592653\ldots$ to 9 decimal places.
30. Point out the ellipsis.
31. **Say**: Numbers whose values go on forever like that are called irrational numbers. You will learn more about irrational numbers in senior secondary school.

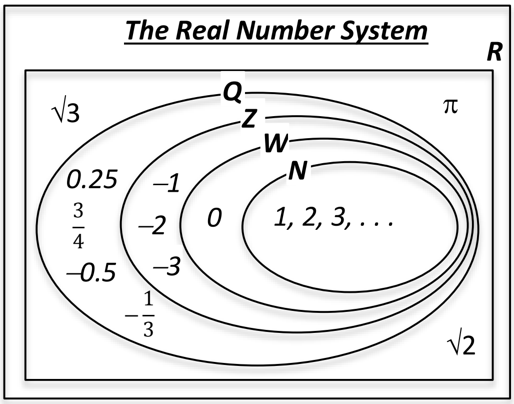
Guided Practice (*5 minutes*)

1. **Say**: Look at all the sets on the board. We can show in set notation the relationship between each of them. All the elements in the set of natural numbers N also belong to the set of whole numbers W.
2. **Ask**: How would we write this? Raise your hand. (Answer: N $\subset$ W).
3. **Say:** Work in pairs to write the relationships between the other subsets of R.
4. Walk around the class to check on answers and clear any misconceptions.
5. Have a pupil from the left-hand side of the classroom volunteer to come to the board to write the relationship between the sets W and Z. (Answer: W $\subset$ Z).
6. **Ask:** Who can write the complete relationship between R and all its subsets? Raise your hand.
7. Select a pupil with their hand raised to do so on the board.
8. Ask the pupil at the board to explain how they arrived at their answer.

9. Ask the other pupils if they agree. Ask them to make comments and any corrections to what is on the board.
   (Answer: N ⊂ W ⊂ Z ⊂ Q ⊂ R; the sets all form subsets as shown; all the elements in N belong to W; all the elements in W belong to Z; all the elements in Z belong to Q; all the elements in Q belong to R.
10. **Write**: N ⊂ W ⊂ Z ⊂ Q ⊂ R
    Natural numbers are whole numbers, whole numbers are integers and integers are rational numbers. They are all part of the real number system which we use everyday.
11. **Say**: Natural numbers are whole numbers, whole numbers are integers and integers are rational numbers. These all form the real number system which we use in Mathematics and in everyday life.

Independent Practice (*10 minutes*)

1. **Say**: A good way to remember these subsets of R is by representing them all on the same Venn diagram.
2. **Say**: Draw R, the universal set as a rectangle.
3. Show this on the board.
4. Read these instructions from the board and ask the pupils to complete them independently in their exercise books:
   a. Draw each subset according to the relationship found in the previous exercise.
   b. Label each subset with the correct symbol.
   c. List a few examples of elements which belong in each subset.
5. Give pupils 7 minutes to complete the activity.
6. Walk around to check answers and clear misconceptions.
7. Have pupils from around the classroom volunteer to give their answers to the questions.
8. **Write** the correct answer on the board. Ask pupils to correct their work.
   (Answer: See box - shown with a few elements listed as examples. If the pupil puts an irrational number in any of the subsets, show how we can represent it within R for now as shown).



Closing (*3 minutes*)

1. **Say:** List all the subsets of R to which '4' and '–4' belong. You can use the definitions or the Venn diagram to help.
2. Ask pupils to raise their hand to answer. (Answer: 4 belongs to the sets of natural numbers, whole numbers, integers and rational numbers. –4 belongs to the sets of integers and rational numbers).

Lesson Title: Comparing Sets of Real Numbers	Theme: Number and Numeration	
Lesson Number: M-09-013	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to: 1. Compare sets of real numbers. 2. Use a Venn diagram to compare sets of real numbers.	None	1. Write this vocabulary list on the top left-hand corner of the board: Vocabulary List: data, intersection, union 2. Write the exercise from the Guided Practice section on the board. 3. Write the exercise from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say:** In the last lesson, we looked at the subsets of the set of real numbers. Who can name one of the subsets? Raise your hand.
2. Select pupils with their hands raised to give the answers. (Answers: Counting or natural numbers, whole numbers, integers, rational numbers, irrational numbers (optional)).
3. **Say**: Give me an example of a rational number.
4. Have a pupil from the back of the classroom volunteer to answer. (Answer: 0.3, 7, $-\frac{1}{5}$)
5. Discuss incorrect answers.
6. **Say**: Today we are going to compare sets of real numbers. This will include how to use Venn diagrams to compare sets of real numbers.

Introduction to the New Material (15 minutes)

1. **Say**: We compare sets of numbers to determine the similarities and differences between them. This is useful in fields such as science, banking, and accounting where large amounts of data are processed. Data are 'collections' of objects and ideas that have been sorted into groups.
2. **Ask**: What does the definition of data remind you of? Raise your hand.
3. Give the pupils some time to think and then select a pupil who has raised their hand. (Answer: It is similar to that of a well-defined set. So, data are well-defined sets).
4. **Say**: One way to compare sets of data is to examine the lists of elements from the sets.
5. **Write** on the board: A= { 1, 3, 5, 7, 9 }, B = { 1, 2, 3, 4, 5 }
6. **Say**: There are special symbols and vocabulary we use in set notation to show the similarities and differences between sets.
7. **Ask**: Which elements are in set A that are also in set B? Raise your hand.
8. Give the pupils some time to think and then select a pupil who has raised their hand. (Answer: 1, 3, 5).
9. **Write** on the board:

{ elements in both sets A and B } = { 1, 3, 5 }

$A \cap B$ = { 1, 3, 5 } read as "A intersection B" or the "intersection of sets A and B"

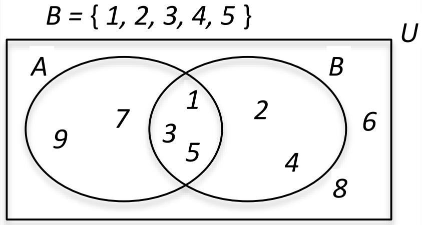
10. **Ask**: Which elements are in either set A or set B? Raise your hand. (Answer: 1, 2, 3, 4, 5, 7, 9)
11. **Write** on the board:

{ elements in either set A or set B } = { 1, 2, 3, 4, 5, 7, 9 }

$A \cup B$ = { 1, 2, 3, 4, 5, 7, 9 } read as "A union B" or the "union of sets A and B"

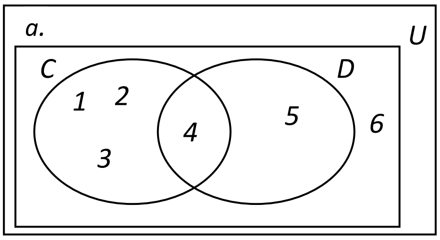
12. **Say**: Look at the union of sets A and B. Discuss with your partner what you think the universal set might be. (Example answer: U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }.
13. **Say**: Discuss with your partner what the elements of the complement of set A are.
14. Have 2-3 pupils from around the classroom volunteer to give their answers.
15. Ask the class if they agree. You may get 2 answers to this question: A^c = { 2, 4 } and A^C = { 2, 4, 6, 8 }. Discuss the difference between the 2 answers. (Discussion point: A^C = { 2, 4 } is the set of numbers which are in B but not in A. The complement of a set means 'all' the elements not in the set – we need to consider the elements in the universal set as well. Correct answer: A^C = { 2, 4, 6, 8 }).
16. **Ask**: What is the complement of set B? Raise your hand. (Answer: B^C = { 6, 7, 8, 9 }).
17. **Say**: We can show all this information on a Venn diagram.
18. Draw the Venn diagram shown.
19. **Say**: It is now clear from the Venn diagram what the elements of sets A^C and B^C are.

U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }
A= { 1, 3, 5, 7, 9 }
B = { 1, 2, 3, 4, 5 }



Guided Practice (*5 minutes*)

1. Point to the following exercise on the board: U = { 1, 2, 3, 4, 5, 6 }, C = { 1, 2, 3, 4 }, D = { 4, 5 }.
   a. Draw a Venn diagram of the given information.
   b. Write in set notation:
   i. $C \cap D$ ii. $C \cup D$ iii. C^C iv. D^C
2. Ask pupils to work in pairs to discuss and share ideas. Give them 3 minutes.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions. (Answers: a. see Venn diagram in box;
5. Ask pupils to correct their work as needed.
   b. ii. $C \cap D$ = { 4 } iii. $C \cup D$ = { 1, 2, 3, 4, 5 } iv. C^C = { 5, 6 } v. D^C= { 1, 2, 3, 6 })



Independent Practice (*10 minutes*)

1. Point to the following exercise on the board:

U = { natural numbers less than 10 } R = { squares less than 10 }

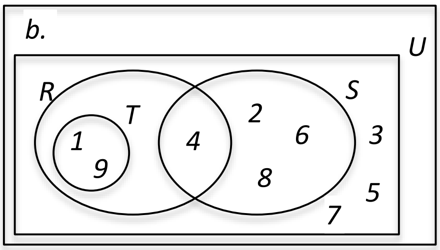
S = { even numbers less than 10 } T = { odd squares less than 10 }

   a. List the elements of the given sets.
   b. Draw the sets on a Venn diagram.
   c. Find $R \cap S$ d. Find $R \cup S$

e. Is $R \subset T$? f. Is $T \subset R$?

2. Ask the pupils to answer independently in their exercise books. Give them 7 minutes.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.

(Answers: *a.* $U = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$ $R = \{ 1, 4, 9 \}$ $S = \{ 2, 4. 6, 8 \}$ $T = \{ 1, 9 \}$



c. $R \cap S = \{ 4 \}$ *d.* $R \cup S = \{ 1, 2, 4, 6, 8, 9 \}$ *e. No,* $R \not\subset T$ *f. Yes,* $T \subset R$

Closing (*2 minutes*)

1. Ask pupils the questions below. Allow them some time to think and then have a pupil volunteer to answer.
2. **Ask:** What does the intersection of the 2 sets A and B show?
   (Answer: { elements in both sets A and B })
3. **Ask**: What does the union of 2 sets show?
   (Answer: { elements in either set A or set B })

Lesson Title: Ordering Sets of Real Numbers	**Theme**: Number and Numeration	
Lesson Number: M-09-014	**Class/Level**: JSS 3	**Time:** 35 minutes

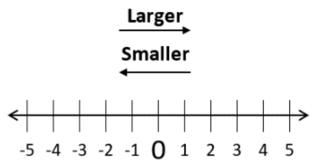
Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to order sets of real numbers.	None	None

Opening (*3 minutes*)

1. **Ask:** Can someone tell the class what we did in the last lesson?
2. Select a pupil who has their hand raised to answer. (Example answers: Compared sets of numbers; found intersection and union of sets; drew Venn diagrams)
3. **Ask**: What is the intersection of 2 sets?
4. Have a pupil from the back of the classroom volunteer to answer. (Answer: It is the set of elements common to both sets.)
5. **Ask**: What is the union of 2 sets?
6. Have a pupil from the front of the classroom volunteer to answer. (Answer: It is the set which has all the elements from both sets.)
7. **Say**: In the last lesson, we only compared sets from natural numbers. We can compare other subsets of the real numbers in a similar way. Today we are going to order sets of real numbers.

Introduction to the New Material (*10 minutes*)

1. **Write** on the board: { 4, -2, 0, 9, -4 }
2. **Ask:** What set do these numbers belong to? Raise your hand. (Answer: Integers).
3. **Say:** Please put the set of integers in ascending order. Raise your hand. (Answer: { -4, -2, 0, 4, 9 }).
4. Remind pupils that negative integers get smaller the further away from 0 they are. Positive integers get larger the further away from 0 they are. Draw a quick number line to show this (see example).



5. **Write** on the board: <, >, ≤, ≥
6. **Say**: We can use these symbols to compare 2 numbers. You know them from previous years.
7. Ask pupils to raise their hand to identify the symbols. (Answer: less than, greater than, less than or equal to, greater than or equal to).
8. **Write** on the board: $\frac{3}{5} \square \frac{4}{7}$.
9. **Say:** Put one of these symbols between these 2 numbers to make the statement true.
10. Have a pupil volunteer to put the right symbol on the board and explain how they got their answer. (Answer: >; possible explanation – use LCM to make a common denominator and order according to numerator, $\frac{3}{5}=\frac{21}{35}$, $\frac{4}{7}=\frac{20}{35}$, $\frac{21}{35}>\frac{20}{35}$)
11. **Write** on the board: { 0.3, 1.35, 0.25, 1.257 }.
12. Ask pupils to work in pairs to put this set of decimal numbers in ascending order.
13. Allow them 2 minutes to do this work.
14. **Ask**: Who can explain to the class what to do?

1	·	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	·	2	5	
0	·	3		
1	·	2	5	7
1	·	3	5	

15. Select a pupil who has raised their hand to answer. (Example answer: Look at the individual digits of each number one at a time and compare them with each other. Smaller digits are less than larger digits.)
16. Use a place value chart to help explain the process.
17. Ask the pairs to check their answers and make any corrections. (Answer: { 0.25, 0.3, 1.257, 1.35}).
18. **Say:** We can also order a mixture of real numbers.
19. **Write** on the board: {1.5, $\frac{2}{5}$, 6, 0.35 }
20. **Say**: Put this set of numbers in descending order. Raise your hand. (Answer: { 6, 1.5, $\frac{2}{5}$, 0.35 }. Change the fraction $\frac{2}{5}$ to a decimal 0.4).

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to put the sets of real numbers in ascending order.
2. **Write** on the board:

   a. { 3, -5, 1, -3 } b. { $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{4}{7}$ } c. { 3.4, -3.4, 3.04, 0.34 }
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Ask pupils to correct any errors. (Answers: a. { -5, -3, 1, 3 }; b. { $\frac{2}{5}$, $\frac{4}{7}$, $\frac{1}{2}$, $\frac{3}{5}$ }; c. { -3.4, 0.34, 3.04, 3.4 }.

Independent Practice (*10 minutes*)

1. Ask the pupils to answer independently.
2. **Write** on the board:

   Put the following sets of numbers in ascending order:

   a. { -2, $\frac{2}{5}$, 3, 2.5 } b. { 0.35, -0.25, $\frac{1}{2}$, -0.5 }

   Put the following sets of numbers in descending order:

   c. { 2.4, -2.4, 2.04, 0.24 } d. { 4, 0, -3, 2, -6 }
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Ask pupils to correct any errors. (Answers: a. { -2, $\frac{2}{5}$, 2.5, 3 }; b. { -0.5, -0.25, 0.35, $\frac{1}{2}$ }; c. { 2.4, 2.04, 0.24, -2.4 }; d.{ 4, 2, 0, -3, -6 }).

Closing (*2 minutes*)

1. **Say:** Please tell the class one new thing you learned today.
2. Have pupils from around the class volunteer to answer. (Example answers: How to order integers (or fractions or decimals or mixture of real numbers.)

Lesson Title: Real Numbers on a Number Line	Theme: Numbers and Numeration	
Lesson Number: M-09-015	**Class/Level**: JSS 3	**Time:** 35 minutes

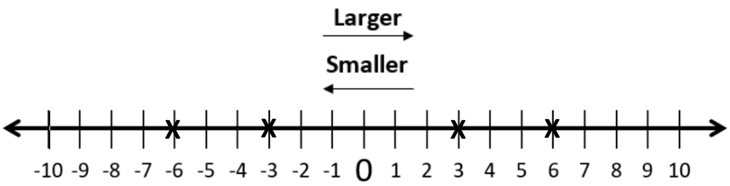
Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to locate real numbers on a number line.	None	1. Write on the board: { -6.7, 6, -0.67, -$\frac{6}{7}$ } Leave on the board as it will be used throughout lesson. 2. Draw a number line from -10 to +10 (see example below).

Opening (*3 minutes*)

1. **Say**: Please put the elements of the set in ascending order.
2. Have a pupil from the front of the classroom volunteer to explain their solution to the class. (Answer: { -6.7, -$\frac{6}{7}$,- 0.67, 6 }).
3. **Ask:** What do you notice about the negative numbers? Raise your hand.
4. Have a pupil from the back of the classroom volunteer to answer. (Answer: Negative numbers are smaller the further they are from 0).
5. **Say**: Today we are going to locate real numbers on a number line.

Introduction to the New Material (*10 minutes*)

1. **Say**: All real numbers can be located on a number line. We can draw a number line to show only integers, only decimals or fractions or a mixture of numbers.
2. **Write** on the board: { -6, 3, 6, -3 }
3. **Say**: Can someone show the class where these numbers fit on the number line?
4. Have a pupil volunteer to show these with an **X** on the number line on the board.

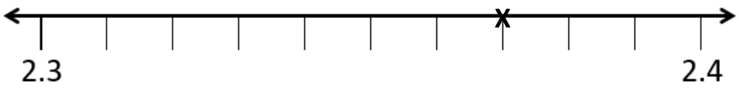


6. **Ask**: What do you notice about the positions of the 3 and -3 in relation to 0? Raise your hand. (Answer: They are the same distance away from 0).
5. **Say**: Every real number has positive and negative counterparts. They are the same distance away from zero. They will always equal to 0 when we add them together.
6. **Write**:

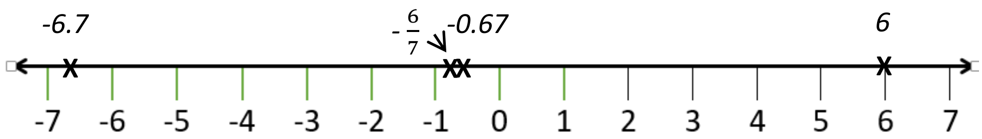
3 + (-3)	=	0	three plus negative three equals zero
$\frac{1}{2}+\left(-\frac{1}{2}\right)$	=	0	half plus negative half equals zero

7. **Say**: Also notice that numbers get smaller and smaller as we move left on the number line. They get larger as we move towards the right.
8. Show this on the number line as above.
9. **Say**: Let us now look at how to put decimal numbers on a number line.
10. **Write** on the board: Show 2.37 on a number line.
11. **Say**: It is not easy to put this on the number line on the board.

12. Show how difficult it is to accurately put this number on the number line because we have to guess where it fits between 2 and 3.
13. **Say**: We need a number line with a different scale to show this.
14. Draw the number line below. Ask a pupil to show where to mark 2.37 on it. (Answer: As shown.)



15. **Say:** It is not always practical to have different scales, especially if we want to fit different types of numbers on the same number line. Most times we can only try to be as accurate as possible. Let us try to locate our first set of numbers on the number line. { -6.7, 6, -0.67, $-\frac{6}{7}$ }
16. Draw a number line as shown. Select pupils to mark the numbers on the number line.



Guided Practice (*10 minutes*)

1. **Say**: Please work in pairs to answer the questions on the board.
2. **Write** on the board:

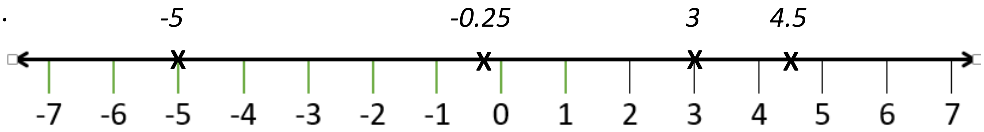
Draw number lines marked from -7 to +7.
Locate the set of real numbers on the number line

a. { 3, -0.25, -5, 4.5 }

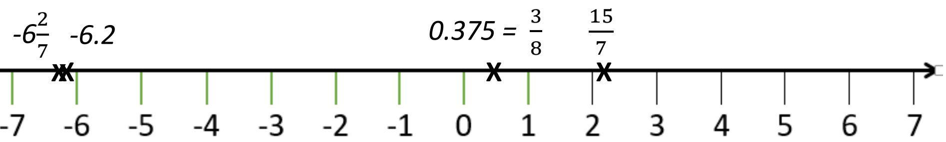
b. { $\frac{3}{8}$, 0.375, $\frac{15}{7}$, -6.2, $-6\frac{2}{7}$ }

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to show their answers on the number lines.
5. **Ask:** How did you decide where the fractions fitted on the number line. Raise your hand. (Example answers: By changing them to decimal numbers; changing them to mixed numbers; finding common denominators.)
6. Ask pupils to correct any errors. (Answers: As shown below)

a.



b.

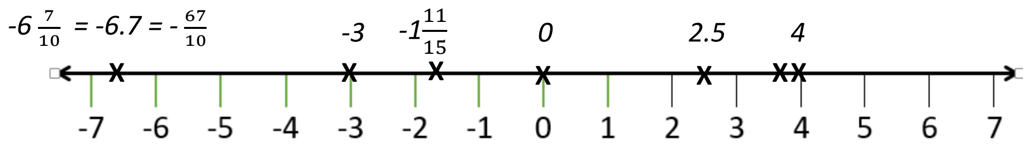


Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. **Write** on the board: Mark the set of real numbers on a number line

$\{ 4, -6\frac{7}{10}, -1\frac{11}{15}, 3\frac{3}{4}, -6.7, -3, 0, 2.5, -\frac{67}{10} \}$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions. (Answer: As shown below)
6. Ask pupils to correct any errors.



Closing (*2 minutes*)

1. **Say:** Please listen to this statement. Raise your left hand if you think it is true, raise your right hand if you think it is false. The number to the right on the number line is always greater than the one on the left.
2. Allow a few moments for pupils to think, and raise their hands.
3. Have a pupil to volunteer give their reason for their choice. Ask the class if they agree.
4. Show on a number line that a number on the right of another is bigger, e.g. 6 > 4, -3 > -5.

Lesson Title: The Roman Numeral System	Theme: Number and Numeration	
Lesson Number: M-09-016	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify the symbols used to show a Roman numeral. 2. Read, write and count numbers up to 20.	None	1. If possible, bring an object such as a clock which shows Roman numerals. 2. Write on the board: (I, II, III, . . . }

Opening (*2 minutes*)

1. Point to the Roman numerals on the board. **Say**: Please raise your hand if you have seen numbers written using these symbols on the board before.
2. If no one raises their hands, **ask:** What do the symbols look like? Raise your hand. (Example answers: Roman numerals; symbols for counting.)
3. **Say**: Today we are going to identify the symbols used to show a Roman numeral. You will be able to read, write and count numbers up to 20. Romans are people who lived in Rome in the country of Italy.

Introduction to the New Material (*15 minutes*)

1. **Say**: The Romans wrote their numbers in a different way from what we are used to. Their numbers are used on clocks, watches and similar objects. Roman numerals were invented by the ancient Romans between the years 900 and 800 B.C.
2. Show the object with Roman numerals to the class, if available.
3. **Say**: The Romans used 7 basic symbols to represent their numbers. We can match these numbers to our base 10 numbers. Our base 10 numbers are derived from the Hindu-Arabic numbers.
4. **Write** the information in the box on the board.
5. **Say**: These symbols can be written in capital or small letters. They mean the same thing.
6. **Write** on the board: i, ii, iii
7. **Ask**: Where have you seen these numbers before in the classroom? (Answer: For questions on the board).
8. **Say:** We can write or work out other numbers from these 7 Roman numerals.
9. **Say**: Look at the numbers on the table. Work in pairs and see if you can write the first 5 numbers of the Roman number system.
10. Allow pupils 1 minute to answer the question.
11. Have a pupil from the right-hand side of the classroom volunteer to come to the board to write their answer.
12. The pupil may write 4 = IIII, or 4 = IV. Ask if anyone has a different idea.
13. Allow pupils to share their ideas.
14. **Say**: The Romans never wrote more than 3 of the same symbols together. They always tried to find a different way to show the number instead.
15. **Write** on the board: 4 = 5 – 1 - IV
16. **Say**: The 'I' in front of 'V' means subtraction. Whenever a smaller numeral is in front of a larger numeral we subtract.

Symbols used in Roman Numerals	
Hindu-Arabic	Roman
1	I
5	V
10	X
50	L
100	C
500	D
1000	M

17. Have pupils raise their hand to share the first 5 numbers. (Answers: 1 = I, 2 = II, 3 = III, 4 = IV, V = 5).
18. **Say**: The Romans made numbers by adding or subtracting like this.
19. **Write** the box on the right, on the board.
20. **Say:** Work with your partner. Follow the same method to calculate the numbers 8 to 11. Remember, we cannot have more than 3 of the same symbols together.
21. Allow the pupils 3 minutes to discuss and share their ideas on writing the numbers.
22. Have pupils from around the classroom volunteer to come to the board to complete the answers.
23. Discuss the answers given. Point out the 'I' in front of the 'X,' which means subtract. (Answers: 8 = 5+1+1+1 = VIII; 9 = 10-1 = IX; 10 = X; 11 = 10+1 = XI).

4	=	*5 – 1*	=	*IV*
5	=	*5*	=	*V*
6	=	*5 + 1*	=	*VI*
7	=	*5 + 1 + 1*	=	*VII*

Guided Practice (*5 minutes*)

1. **Say**: Continue to work in your pairs. Show how you would calculate the Roman numerals for the numbers 12-14.
2. Walk around, if possible, to check answers and clear misconceptions.
3. Have pupils from around the classroom volunteer to come to the board to complete the answers.
4. Discuss the answers given and ask pupils to correct any errors in their work.
   (Answers: 12 = 10+1+1 = XII; 13 = 10+1+1+1 = XIII; 14 = 10 + 5 – 1 = XIV).

Independent Practice (*10 minutes*)

1. **Write** the table on the right on the board.
2. As you write, ask the pupils to work independently to copy and complete the table.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers.
5. **Write** the correct answers and ask pupils to correct any errors in their work.
   (Answers: 5 = V; 10 = X; 11 = 10+1 = XI; 12 = 10+2 = XII;
   13 = 10+3 = XIII; 15 = 10+5 = XV; 16 = 10+6 = XVI:
   17 = 10+7 = XVII; 18 = 10+8 = XVIII; 19 = 10+9 = XIX).

Numerals from 1-20

Hindu-Arabic	*Calculation*	*Roman*
1	*1*	*I*
2	*1 + 1*	*II*
3	*1 + 1 + 1*	*III*
4	*5 – 1*	*IV*
5		
6	*5 + 1*	*VI*
7	*5 + 1 + 1 = 5 + 2*	*VII*
8	*5 + 3*	*VIII*
9	*10 – 1*	*IX*
10		
11		
12		
13		
14	*10 + (5 – 1) = 10 + 4*	*XIV*
15		
16		
17		
18		
19		
20	*10 + 10*	*XX*

Closing (*3 minutes*)

1. **Ask:** How did the Romans work out how to write the number 19?
2. Have a pupil from the back of the classroom volunteer to answer. (Answer: 19 = 10 + (10 – 1) = 10 + 9 = XIX).
3. **Say**: Tomorrow, we will look at how to write larger Roman numerals. Great job, class!

Lesson Title: Converting between Base 10 and Roman Numerals	Theme: Numbers and Numeration	
Lesson Number: M-09-017	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to convert numbers up to 100 from base 10 to Roman numerals and vice versa.	None	Write the number 22 on the board.

Opening (*3 minutes*)

1. Ask the pupils to convert the number on the board to Roman numerals.
2. **Say**: Please raise your hand as soon as you finish.
3. After 1 minute, have a pupil volunteer to show their answer and how they got it, on the board. (Example answers: 22 = 10 + 10 + I + I = XXII; 22 = 10 + 10 + 2 = XXII; 22 = 20 + 2 = XXII)
4. **Say**: Today we are going to convert numbers up to 100 from base 10 to Roman numerals, and from Roman numerals to base 10.

Introduction to the New Material (*15 minutes*)

1. **Say:** In the Hindu-Arabic system, which numerals are used to make the numbers from 1 to 100?
2. After a few moments, allow pupils to call out the answers. (Answers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
3. **Ask**: Which symbols are used to make the Roman numerals from 1 to 100?
4. After a few moments, allow pupils to call out the answer. (Answer: 1 = I; 5 = V; 10 = X; 50 = L, 100 = C)
5. **Say:** All the numbers from 1 to 100 have to be made from just these 5 numerals.
6. Write on the board: 10, 20, 30, 40, 50, 60, 70, 80, 90.
7. **Say**: Work in pairs to write the Roman numbers for the Tens numbers.
8. Allow 3 minutes for the pupils to calculate the Tens numbers.
9. Stop pupils after 3 minutes. Discuss their answers by asking open-ended questions. For example:
   a. How did you calculate the number 40?
   b. Does anyone have a different idea or method?
   c. Did you have difficulties with calculating 60? What were they?
   d. Explain how the calculation for the number 90 is similar to that for the number 9.
   (Example answers: a. 40 = 50 – 10 = XL; b. Allow pupils to share any different ideas or methods. Check and clear any misconceptions, (e.g. that 40 is equal to XXXX);
   c. Discuss any difficulties pupils may have, clear up any misconceptions.
   60 = 50 + 10 = LX;
   d. 9 = 10 – 1 and 90 = 100 – 10. If we multiply the calculation for 9 with 10 we get the calculation for 90. The answer is XC. Ask pupils to see if this works for all the other Tens numbers. (Yes, it does.) Explain to pupils that this is clear from how numbers are written in base 10, i.e. 9 and 90. This shows that 90 is 10 times bigger than 9. This is not so clear in the way numbers are written in the Roman system, i.e. IX and XC)).
10. **Write** the answers for 10, 20, 30, 40 and 50 on the board. (Answers; 10 = X; 20 = XX; 30 = XXX; 40 = XL; 50 = L).

11. Have pupils from around the classroom volunteer to write the answers for 60, 70, 80 and 90 on the board.
    (Answers: 60 = LX; 70 = LXX; 80 = LXXX; 90 = XC).
12. **Say**: Now call out any number from 20 to 40. We will convert it to a Roman numeral together.
    a. Allow pupils to call out numbers.
    b. Write a few on the board.
    c. Select one to work through together on the board. Use the table from the last lesson.
    d. Discuss any problems.
       (Example answer: 28 = 20 + 8 = XX + VIII = XXVIII; another possibility is: X+X+V+I+I+I)
13. **Write** on the board: Convert this number to base 10: XLI
14. Allow 1 minute for pupils to carry out their calculation.
15. Have one pupil from the front volunteer to come to the board and show their calculation.
16. Ask if there are any different ideas or methods. Check and clear up any misconceptions.
    (Example answers: XLI = 40 + 1 = 41; XLI = 50 – 10 + 1 = 41; XLI = (50 – 10) + 1 = 41).
17. **Say**: Please remember to use brackets as it will help to make the calculation clear.

Guided Practice (*5 minutes*)

1. **Write** on the board:
   a. Convert 44 to Roman numerals. b. Convert XCII to Hindu-Arabic (base 10) numbers.
   Show your calculations.
2. Ask the pupils to work in pairs to convert the numbers.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have one pupil from the left- and one from the right-hand side of the classroom volunteer to give their answers.
5. Write the correct answers on the board. (Answers: a. 44 = 40 + 4 = XLIV; accept other calculations,
   e.g. 44 = (50 – 10) + (5 – 1); b. XCII = 90 + 2 = 92; accept other calculations, e.g. (100-10) + 2 = 92)

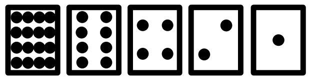
Independent Practice (*10 minutes*)

1. **Write** the following numbers on the board:
   a. Convert these base 10 numbers to Roman numerals: 49, 74, 99
   b. Convert these Roman numerals to base 10: XXXIV, LIV, LXXXVIII.
      Show your calculations.
2. Ask the pupils to work independently to answer the questions.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to show their calculations on the board.
5. Accept all accurate calculations. Correct any errors. (Example answers: 49 = 40+9 = XLIX;
   74 = 70+4 = LXXIV; 99 = 90-9 = XCIX; XXXIV = 30+4 = 34; LIV = 50+4 = 54; LXXXVIII = 80+8 = 88)

Closing (*2 minutes*)

1. Ask pupils to write their name on a piece of paper.
2. Ask them to write the conversions for the numbers 48 and XCIV on the piece of paper.
3. Collect the paper from each pupil at the end of the lesson to check their understanding.

Lesson Title: Introduction to Base 2	**Theme**: Number and Numeration	
Lesson Number: M-09-018	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify the numerals used to read and write in base 2. 2. Count up to 20 in base 2 and work out the pattern of numbers.	None	1. Draw the dot pattern below, in the centre of the board:  2. Write the vocabulary list on the top right-hand corner of the board - Vocabulary List: binary, device

Opening (*3 minutes*)

1. Write these numbers on the board: { 1, 2, 4, 8, 16, . . . }
2. **Say**: Please look at the pattern on the board. Discuss the pattern with your seatmate. Raise your hand if you can tell the next number in the pattern.
3. Allow a few moments for discussion and for pupils to raise their hands.
4. If no hands are raised, **say:** The numbers form a pattern based on doubling. Look at the first 3 numbers. Use it to help you find the next number.
5. Have a pupil from the back of the classroom volunteer to answer. (Answer: 32)
6. **Say**: Today we are going to learn how to identify the numerals used to read and write in base 2. We will also learn how to count up to 20 in base 2 and work out the pattern of numbers.

Introduction to the New Material (*15 minutes*)

1. **Ask**: Which numerals do we use when we count in base 10?
2. Allow pupils to call out the answers. (Answers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
3. **Say**: There are 10 numerals in base 10. These are 0 to 9.
4. **Ask**: How many numerals do you think will be in base 2? Raise your hand. (Answer: 2)
5. **Ask**: Which numerals do you think we will use to count in base 2?
6. Guide pupils to say 0 and 1.
7. **Say**: Base 2 numbers are also called 'binary' numbers. They are called binary numbers because 2 different digits are used. All base 2 numbers are made using the 2 numerals, 0 and 1.
8. **Say:** We use base 2 or binary numbers in all our mobile phones, laptops, computers, DVDs, etc. All the information in these devices is stored and sent in 0s and 1s.
9. **Say**: When we learned how to count in base 10, we used Units, Tens, Hundreds, Thousands and so on. These are all powers of 10.
10. **Say**: To count with base 2, we will use the powers of 2. Work in pairs. Show that these numbers, { 1, 2, 4, 8, 16, . . . } are powers of 2
11. Allow pupils time to work out the powers of 2. Have pupils from around the classroom volunteer to call out the answers. (Answers: $1 = 2^0$; $2 = 2^1$; $4 = 2^2$; $8 = 2^3$; $16 = 2^4$).
12. **Say**: To help us understand how base 2 numbers work, we are going to make the set of numbers on the board. We need 5 pieces of paper – tear them out from your exercise books. Mark the dots on your paper as shown on the board.

16	*8*	*4*	*2*	*1*		
					=	***1***
					=	***2***
					=	***3***
					=	***4***
					=	***5***

13. **Say**: Let us make the first 5 numbers using our dot paper. We will use base 10 numbers to help us keep track.
14. **Say:** Draw a table as shown on the board. Arrange your dot papers in the order shown.
15. **Say**: When we use a number, we will turn the dot paper face up. We will write 1 underneath the number.
16. **Say**: When we are not using a number, we will turn the dot paper face down. We will write 0 underneath the number.
17. **Say**: Let us start with 1. Which dot papers should be face up?
18. Guide pupils to see that to make 1, they need to turn up the dot paper with one dot. All the others should be turned over. See the guide at the end of the lesson plan for the first 5 numbers.
19. Ask pupils to fill in the first row of their table as shown below.
20. **Say**: In base 2, we can read this as "no 16s, no 8s, no 4s, no 2s, one 1. So, 1 = 1. Let us now use our dot papers to make 2.
21. Allow the pupils to discuss and share ideas with their partner.
22. Have a pupil volunteer to come to the board and show how they made 2 using their dot papers. This is shown in row 2 of the table.
23. **Say**: In base 2 we do not read this number as ten. Instead we say "one zero".
24. **Write** on the board:
    2 = 10 one zero
25. Repeat these steps for the numbers 3 to 5.
26. **Write** on the board how to say each number shown by the pupil:
    3 = 11 one one
    4 = 100 one zero zero
    5 = 101 one zero one

16	***8***	***4***	***2***	***1***		
0	*0*	*0*	*0*	*1*	=	***1***
0	*0*	*0*	*1*	*0*	=	***2***
0	*0*	*0*	*1*	*1*	=	***3***
0	*0*	*1*	*0*	*0*	=	***4***
0	*0*	*1*	*0*	*1*	=	***5***

27. **Say:** Note how you are adding the powers of 2 to make the numbers from 1 to 5.

Guided Practice (*5 minutes*)

1. Ask the pupils to work in pairs.
2. Ask them to use their dot papers to make the numbers 6 to 10 in base 2.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions. They can ignore the extra zeros in front of the numbers.

16	***8***	***4***	***2***	***1***		
		1	*1*	*0*	=	***6***
		1	*1*	*1*	=	***7***
	1	*0*	*0*	*0*	=	***8***
	1	*0*	*0*	*1*	=	***9***
	1	*0*	*1*	*0*	=	***10***

5. **Say**: Please look at the numbers on the table. Can you see a pattern in the numbers?
6. Guide pupils to say that the numbers on the table go up in the pattern:
   1 number has 1 digit; 2 numbers have 2 digits, 4 numbers have 3 digits.
   This is the same as the original set on the board { 1, 2, 4, . . . }.
7. **Ask**: How many numbers will you expect to have 4 digits if we continue the table? (Answer: 8)
8. **Say**: We already have 3. Let us see how many more we will find.

16	***8***	***4***	***2***	***1***		
	1	*0*	*1*	*1*	=	***11***
	1	*1*	*0*	*0*	=	***12***
	1	*1*	*0*	*1*	=	***13***
	1	*1*	*1*	*0*	=	***14***
	1	*1*	*1*	*1*	=	***15***
1	*0*	*0*	*0*	*0*	=	***16***
1	*0*	*0*	*0*	*1*	=	***17***
1	*0*	*0*	*1*	*0*	=	***18***
1	*0*	*0*	*1*	*1*	=	***19***
1	*0*	*1*	*0*	*0*	=	***20***

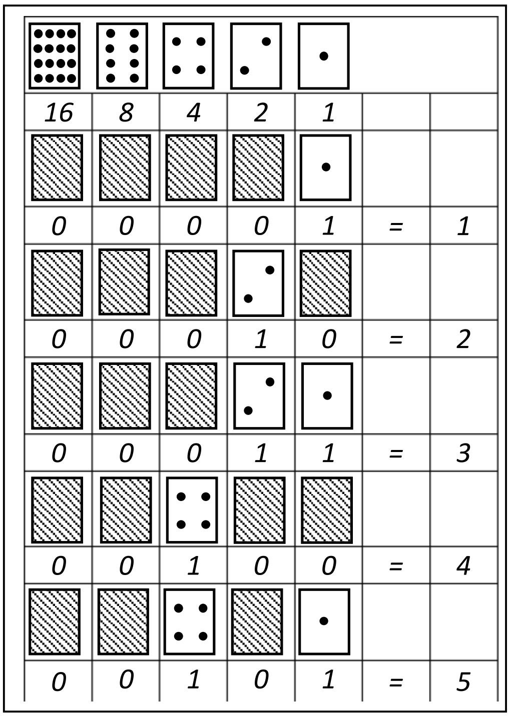
Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to complete the table from 11 to 20.
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions. (Answers: as shown in the table.)
4. **Ask**: Did we get the numbers we expected to have 5 digits?
5. Allow the pupils some time to count the numbers with 5 digits and then raise their hand to answer. (Answer: Yes, there are 8 numbers with 5 digits.)

Closing (*2 minutes)*

1. **Say:** Please raise your hand if you know the pattern in making base 2 numbers.
2. Have a pupil who has raised their hand to answer the question.
3. Guide the pupil to explain that there is only one number with one digit, 2 numbers with 2 digits, 4 with 3, 8 with 4 and so on. This follows the same pattern as the numbers we get with the set of powers of 2 = { 1, 2, 4, 8… }. There will be 16 numbers with 5 digits.
4. Note the pupils who did not raise their hand.

[GUIDE TO USING DOT PAPER TO MAKE BASE 2 NUMBERS]



Lesson Title: Ordering and Comparing Numbers in Base 2	**Theme**: Number and Numeration	
Lesson Number: M-09-019	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. List sets of base 2 numbers up to 20 in ascending order. 2. Compare base 2 numbers up to 20.	None	Write on the board: How do you make the base 2 number for 12?

Opening (*3 minutes*)

1. **Ask**: How do you make the base 2 number for 12?
2. Have a pupil from the front of the classroom volunteer to show how to make 12 in base 2. (Answer: As shown).

16	***8***	***4***	***2***	***1***		
	1	*1*	*0*	*0*	*=*	***12***

3. **Ask:** How do you read this number?
4. Have a pupil from the back of the classroom volunteer to answer. (Answer: one one zero zero.)
5. **Say**: Today we are going to learn how to list sets of base 2 numbers. We will also learn how to compare base 2 numbers up to 20.

Introduction to the New Material (*15 minutes*)

1. **Write** on the board: { 11, 10 }.
2. **Say**: Look at the set of base 2 numbers on the board. How would you work out which number was the smaller one?
3. Guide the pupils to say that the number 10 is smaller than the number 11 because when counting, 0 comes before 1.
4. **Say**: We can use the symbol $<$ which means "is less than" to compare the 2 numbers.
5. **Write** on the board 10 $<$ 11.
6. **Say:** We use the symbol $>$ if one number "is greater than" another.
7. **Write** on the board: { 110, 111, 101 100 }
8. **Ask**: How would we put this set of base 2 numbers in ascending order?
9. Discuss as a class how best to work out how to put the numbers in ascending order. (Discussion point: Take the digits one at a time and compare them with each other).
10. **Say**: Let us compare the digits step by step. All the numbers start with 1, so let us look at the second digit.
11. **Ask**: What numbers do we have in the second digit? (Answer: 0 or 1).
12. **Say**: There are 2 numbers with 0 as the second digit. 101 and 100. What should we do?
13. Guide pupils to say to we should look at the third digit.
14. **Say**: Look at the third digit of the 2 numbers. Which number is smaller?
15. Allow a few moments for pupils to think.
16. Have a pupil from the left-hand side of the classroom volunteer to call out the answer. (Answer: 100).
17. **Say**: Look at the 2 remaining numbers and use the same method to put them in order. Write down all 4 numbers in ascending order.
18. Allow the pupils 2 minutes to work in pairs to put the numbers in order.

19. Have a pupil volunteer to read out the full set of numbers in ascending order. (Answer: { 100, 101, 110, 111 }).
20. Remind pupils to read each digit individually, i.e. 100 is one zero zero.

Guided Practice (*5 minutes*)

1. **Write** on the board:

   Put $<$ or $>$ to make the following statements true

   a. 1101 ☐ 1001 b. 10011 ☐ 10010
2. Ask pupils to work in pairs to answer the question.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from the front and back of the classroom volunteer to give their answers to the questions.
5. Write the correct answers on the board. (Answers: a. 1101 $>$ 1001; b. 10011 $>$ 10010).

Independent Practice (*10 minutes*)

1. **Write** the following on the board:

   a. Put the set of base 2 numbers in ascending order { 1101, 10110, 101, 10100, 111 }

   b. Put the set of base 2 numbers in descending order { 110, 1001, 10100, 1110 }
2. Ask the pupils to work independently to answer the questions.
3. Walk around, if possible, to check answers and clear misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Write the correct answers on the board. (Answers: a. { 101, 111, 1101, 10100, 10110 }, b. { 10100, 1110, 1001, 110 }).

Closing (*2 minutes*)

1. **Ask:** How do we compare base 2 numbers?
2. Have a pupil from the right-hand side of the classroom volunteer to give the answer. (Answer: We should take the digits of the numbers one at a time and compare them with each other).

Lesson Title: Converting between Base 10 and Base 2	**Theme**: Number and Numeration	
Lesson Number: M-09-020	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to convert numbers up to 50 from base 10 to base 2 and vice versa.	None	Write the table in Step 19 of the Introduction to New Material section on the board.

Opening (*2 minutes*)

1. **Write** on the board: Which is smaller: 10011 or 10001?
2. **Say**: Please raise your hand when you have the answer.
3. Wait a few moments or until most of the pupils have their hands raised.
4. Have a pupil from the back of the classroom volunteer to call out their answer. (Answer: 10001).
5. **Say**: Today we are going to convert numbers up to 50 from base 10 to base 2 and from base 2 to base 10.

Introduction to the New Material (*15 minutes*)

1. Ask pupils to look at their table from the last lesson for the base 2 numbers from 1 to 20.
2. **Say**: The table has powers of 2 going up to 16. What is the next power of 2 we must add to the table to make larger numbers? Raise your hand. (Answer: 32).
3. Ask the pupils to add 32 to their tables.
4. **Write** on the board: Convert 34_{ten} to base 2.
5. Point to the number 34_{ten}.
6. **Say**: When we are working with different bases, it is good practice to write the base of the numbers as shown. This tells us the base of the number we are working with. When we use the table, please remember to write 1 whenever we use a power of 2 to make our number. We can write 0 when we do not use a power of 2.
7. Have a pupil volunteer to show how to do this for 34_{ten} on the board. (Answer: As shown below).
8. **Say**: We can also convert from base 10 to base 2.
9. **Write** on the board:
   Convert 110010_2 to base 10.
10. **Say**: We will start by filling in the base 2 number in the table from right to left.
11. Complete the table.
12. **Say:** Next we will add the powers of base 2 wherever we have a 1.
13. **Write** on the board: $32 + 16 + 2 = 50_{ten}$
14. Allow the pupils a few moments to write the information on the board into their exercise books.

32	***16***	***8***	***4***	***2***	***1***		
1	*0*	*0*	*0*	*1*	*0*	=	34_{ten}
write as 100010_2							
32	***16***	***8***	***4***	***2***	***1***		
1	*1*	*0*	*0*	*1*	*0*	=	50_{ten}
write as 50_{ten}							

15. **Ask**: Who remembers where binary or base 2 numbers are used?
16. Guide the pupils to answer that they are used in mobile phones, DVDs and other similar devices.
17. **Say**: If we want to send a text message to a friend, the mobile phone converts the message to 1s and 0s. It sends these numbers very quickly so that a message is received almost as soon as it is sent. Let us see how it uses binary numbers to do so.
18. **Say**: We will start by using the numbers 1 to 26 for the letters of the alphabet.

19. Point to the table below on the board. Ask the pupils to copy it in their exercise books. Note: the numbers have been written without the base$_{ten}$.

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
n	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*	*w*	*x*	*y*	*z*
14	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*

20. **Say**: We are going to do it a little easier than a mobile phone would. The basic method is still the same.

Guided Practice (5 minutes)

1. **Write** on the board: Convert the text "Call me" to base 2
2. Guide the pupils to use the table to first assign a base 10 number to each letter of the text.

c	a	l	l	m	e
3	1	12	12	13	5

3. Ask the pupils to work in pairs. They should each convert 3 of the base 10 numbers to base 2. (Both pupils are to convert the letter l).
4. Ask pupils to check each other's conversions.
5. Walk around, if possible, to check answers and clear misconceptions.
6. Have pupils from around the classroom volunteer to give their answers.
7. **Write** the correct answers on the board. (Answers: c- 11; a- 1; l- 1100; m- 1101; e- 101.
   The complete text is: 11 1 1100 1100 1101 101).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions below.
2. **Write** the following on the board:
   Write the following words in base 2
   a. your name
   b. the day of the week you were born
   c. the month you were born
3. **Say**: You will have to convert any letter you use from 21 to 26 to base 2.
4. Walk around, if possible, to check the answers and clear up any misconceptions.
5. While pupils work , write the complete table with base 2 numbers for each letter, on the board.
6. Have pupils from around the classroom volunteer to give their answers to the questions. Check their answers using the table on the board.
7. Ask pupils to exchange exercise books and check each other's work using the table on the board. Ask pupils to make any corrections.

Closing (*3 minutes*)

1. **Say:** Please convert 44_{ten} to base 2. Raise your hand when you are finished.
2. Select a pupil who has raised their hand to call out the answer. (Answer: 101100_2)

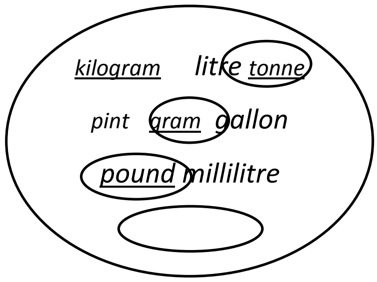
[GUIDE TO BASE 2 NUMBERS FOR THE ALPHABET]

Letter	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*
Base 10 number	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
Base 2 number	*1*	*10*	*11*	*100*	*101*	*110*	*111*	*1000*	*1001*	*1010*	*1011*	*1100*	*1101*
Letter	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*	*w*	*x*	*y*	*z*
Base 10 number	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*
Base 2 number	*1110*	*1111*	*10000*	*10001*	*10010*	*10011*	*10100*	*10101*	*10110*	*10111*	*11000*	*11001*	*11010*

Lesson Title: Capacity and Mass	**Theme**: Everyday Arithmetic	
Lesson Number: M-09-021	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Differentiate between mass and capacity. 2. Solve problems with masses and problems with capacities.	None	1. Write on the board: *kilogram litre* *tonne pint gram* *gallon pound* *millilitre* The mass of an object is the amount of matter it contains. The volume is the amount of space the object takes up. 2. Write the questions in the Guided Practice section on the board. 3. Write the questions in the Independent Practice section on the board.

Opening (*3 minutes*)



1. **Say**: Look at the words on the board. Please copy them in your exercise books. Underline the units for mass and circle the units for capacity. Raise your hand when you finish.
2. Allow 1 minute for the activity. Select a pupil who has raised their hand to show the answer on the board. (Answer: As shown).
3. **Say**: Today we are going to learn how to differentiate between mass and capacity. We will also solve problems with both mass and capacity.

Introduction to the New Material (*10 minutes*)

1. **Say**: Imagine you have 2 bottles. Each takes 1 litre of liquid. One is full of water, the other one is full of palm oil. One litre of water has a mass of 1000 g. One litre of palm oil has a mass of 890 g. The mass of an object is the amount of matter it contains. The volume is the amount of space the object takes up.
2. Ask pupils to copy this information already on the board into their exercise books.
3. **Say**: We use the metric system of measurement for mass and capacity.
4. **Ask**: Which units on the board are the metric ones for mass. Which ones are those for volume? Raise your hand.
5. Select a pupil who has raised their hand. (Answer: Mass – gram, kilogram, tonne; volume – litre, millilitre).
6. **Say**: In problems involving mass and volume, we may need to convert between units.

7. **Write** on the board: 1 kg = 1000 g
   1 litre = 1000 ml
8. **Say**: We will now look at problems with both mass and volume.
9. **Write** a question on mass on the board:
   Morlai buys a 15 kg bag of rice for a party. He uses 13.5 kg for the party. How much rice does he have left: a. in kg b. in g?
10. **Ask**: What do we need to do in this problem?
11. Allow pupils to think about and write down their ideas for 1 minute.
12. After 1 minute, ask them to pair up with their neighbour and discuss how to solve the problem for another minute.
13. **Ask**: Who would like to share their ideas with the class? Raise your hand.
14. Select a pupil who has raised their hand. Ask them to explain their method to the class.
    (Answer: a. [Subtract the 2 masses] 15 kg – 13.5 kg = 1.5 kg [answer in kg];
    b. [convert kg to g by multiplying by 1000] 1.5 kg $\times$ 1000 = 1500 g [answer in g])
15. **Write** a question on capacity on the board:
    A bottle contains 2 litres of ginger beer. Adama drinks 350 ml of the ginger beer. Foday drinks 550 ml. How much of the ginger beer is left? Give your answer in:
    a. ml b. litres
16. Allow pupils to think and pair up as before. Have a pupil volunteer to share their ideas with the class.
    (Answer:
    [add the 2 capacities] 350 ml + 550 ml = 900 ml [amount drunk by Adama and Foday];
    [convert litres to ml by multiplying by 1000] 2 litres $\times$ 1000 = 2000 ml;
    [subtract amount drunk from the original amount] a. 2000 ml – 900 ml = 1100 ml [answer in ml];
    b. [convert answer to litres by dividing by 1000] = 1100 ml $\div$ 1000 = 1.1 litres [answer in litres]).

Guided Practice (*10 minutes*)

1. Point to the questions on the board:
   a. A baker needs 125 g of flour to make a loaf of bread. How much flour does he need to make 12 loaves? Please give your answer in kg.
   b. A bottle contains 3 litres of water. It is used to fill glasses which hold 150 ml each. How many glasses does it fill?
2. Walk around, if possible, to check the answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.
4. Write the correct answers with the calculation on the board. (Answers: a. 125 $\times$ 12 = 1500 g = 1.5 kg; b. 3 litres = 3 $\times$ 1000 = 3000 ml, $\frac{3000}{150}$ = 20 glasses).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the following questions on the board:
   a. A can containing 5 litres of palm oil has a mass of 4.45 kg. It is used to fill bottles which hold 1.25 litres of palm-oil each.
      i. How many bottles of palm-oil does it fill?
      ii. What is the mass of each bottle of palm-oil? Please give your answer in g.
   b. A bag contains 3.5 kg of rice. A second bag contains 3 times as much.
      i. How much rice does the second bag contain?
      ii. What is the difference in mass between the 2 bags?

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Write the correct answers with the calculation on the board. (Answers: a. i. $\frac{5}{1.25} = \frac{500}{125}$ = 4 bottles, ii. 4.45 kg = 4450 g, $\frac{4450}{4}$ = 1112.5 g;
   b. i. 3.5 × 3 = 10.5 kg, ii. 10.5 – 3.5 = 7 kg).

Closing (*2 minutes*)

1. **Ask:** Who can tell the class what mass is? Raise your hand.
2. Select a pupil who has raised their hand to answer. (Answer: Mass is the amount of matter an object contains.)
3. **Ask:** Who can tell the class what volume is? Raise your hand.
4. Select a different pupil who has raised their hand to answer. (Answer: The amount of space an object takes up.)

Lesson Title: Percentages of Quantities	**Theme**: Everyday Arithmetic	
Lesson Number: M-09-022	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to find percentages of quantities.	None	Write the question in the Independent Practice on the board.

Opening (*4 minutes*)

1. **Say:** Percentages are used all the time in everyday life. Please give me one example where percentages are used. Raise your hand. (Example answer: To find by how much a quantity has increased or decreased)
2. **Say**: When we write a percentage, e.g. 60%, we mean 60 parts out of 100. We write this as $\frac{60}{100}$. This means that if we divide the whole of a quantity into 100 parts, then we are interested in 60 of the parts. Today we are going to calculate percentages of quantities.

Introduction to the New Material (*14 minutes*)

1. **Write** on the board: Find 60% of 150 kg.
2. **Say**: We want to find out if we divide 150 kg into 100 equal parts, how many kg will 60 of the parts be equal to?
3. Show on the board how the above calculation is done:

$$60\% \text{ of } 150$$

$$= \frac{60}{100} \times 150$$

$$= \frac{60 \times 150}{100}$$ cancel the 0s on the numerator and denominator

$$= 6 \times 15$$

$$= 90 \text{ kg}$$

4. **Say**: Always simplify the calculation by cancelling zeros or dividing by a common factor.
5. **Write** on the board: Find 25% of 80 m.
6. **Say**: Now we want to find out if we divide 80 m into 100 equal parts, how many m will 25 of the parts be equal to?
7. Guide a pupil to show the calculation on the board:

$$25\% \text{ of } 80$$

$$= \frac{25}{100} \times 80$$

$$= \frac{25 \times 80}{100}$$ cancel the 0s on the numerator and denominator, divide by common factor 2

$$= \frac{25 \times 4}{5}$$

$$= 20 \text{ m}$$

8. **Say**: Percentages are found everywhere in everyday life.

9. **Write** on the board: A farmer owns 360 acres of land. He plants rice on 30% of his farm. How many acres of land are planted with rice?
10. **Ask**: What calculation should we do for this problem? Give pupils 1 minute to think and then raise their hand to answer. (Answer: 30% of 360).
11. Have a pupil volunteer to show the calculation on the board. (Answer: 30% of 360 = $\frac{30}{100} \times 360$ = $\frac{30 \times 360}{100}$ = 3×36 = 108 acres)

Guided Practice (*5 minutes*)

1. **Write** on the board: Fatu scored 75% on a test. The total marks for the test was 56. How many marks did Fatu score?
2. **Ask**: What calculation should we do for this problem? (Answer: 75% of 56).
3. Ask pupils to work in pairs to solve the problem.
4. Walk around, if possible, to check the answers and clear up any misconceptions.
5. Have a pupil volunteer to show the calculation on the board.
6. Correct any errors on the pupil's calculation on the board. Ask pupils to correct their work. (Answer: 75% of 56 = $\frac{75}{100} \times 56$ = $\frac{75 \times 56}{100}$ = $\frac{3 \times 56}{4}$ = 42 marks).

Independent Practice (*10 minutes*)

1. Point to the following problems on the board:
   a. There are 250 JSS 3 pupils in a school. 40% of them are girls. How many are girls?
   b. Aisha had Le 150,000. She spent 25% of it on books. How much did she have left?
   c. Kemi and Yemi received Le 50,000 from their grandfather. Kemi's share of the money was 45%. How much did each receive?
2. Ask the pupils to work independently to solve the problems.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Select pupils from around the classroom to give their answers to the questions.
5. Correct any errors. (Answers: a. 40% of 250 = $\frac{40}{100} \times 250$ = $\frac{40 \times 250}{100}$ = 4×25 = 100 girls;

   b. 25% of 150,000 = $\frac{25}{100} \times 150{,}000$ = $\frac{25 \times 150}{100}$ = $\frac{1 \times 150}{4}$ = 37,500,
   Aisha had Le 150,000 – 37,500 = Le 112,500 left;

   c. Kemi's share: 45% of 50,000 = $\frac{45}{100} \times 50000$ = 45×500 = Le 22,500,
   Yemi's share was Le 50,000 – 22,500 = Le 27,500).

Closing (*2 minutes*)

1. **Ask:** How do we simplify a percentage calculation? Raise your hand.
2. Have a pupil from the back of the classroom volunteer to answer. (Answer: By cancelling zeros. Sometimes we also have to divide by a common factor.)

Lesson Title: Percentage Increase and Decrease	Theme: Everyday Arithmetic	
Lesson Number: M-09-023	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Calculate the percentage increase or decrease, given 2 numbers. 2. Increase and decrease quantities by a percentage.	None	1. Write the following on the board: a. percentage increase given 2 numbers b. percentage decrease, given 2 numbers c. increasing quantities by a percentage d. decreasing quantities by a percentage 2. Write the problems in the Independent Practice section on the board

Opening (*3 minutes*)

1. **Ask:** Who can tell the class what we did in the last lesson? Raise your hand.
2. Have a pupil from the back of the classroom answer. (Answer: Percentages of quantities)
3. **Say**: In this lesson, we are going to learn how to calculate the percentage increase or decrease given 2 numbers. We will also do calculations on increasing and decreasing quantities by a percentage.

Introduction to the New Material (*10 minutes*)

1. **Say**: When we calculate with percentages, it is important to be clear on the type of calculation the question requires us to do. There are 2 main types of percentage problems shown on the board. One deals with percentage increase and decrease given 2 numbers. The other type deals with increasing or decreasing quantities by a given percentage. We will go through each type of problem step by step.
2. **Write** Question a. on the board: The price of a pen increased from Le 1,000 to Le 1,200. What is the percentage increase?
3. Remind pupils of the formula used for a percentage increase given 2 numbers.
4. Explain each step of the problem. See the problem worked out in the example below.
5. At the point marked *, pause for a moment. **Ask:** What do we do next?
6. Guide a pupil to give the next step. (Answer: Substitute the values for the increase and original amounts in the formula.)
7. Continue with working out the problem. Calculate the percentage increase.
8. **Write** Question b. on the board: The price of a bag of cement decreased from Le 50,000 to Le 45,000. What is the percentage decrease?
9. Have a pupil volunteer to solve the problem on the board with your help.

a. Percentage increase given 2 numbers

The price of a pen increased from Le 1000 to Le 1,200. What is the percentage increase?

Percentage increase	=	$\frac{\text{increase}}{\text{original amount}} \times 100$
Increase	=	1,200 – 1,000
	=	200

b. Percentage decrease given 2 numbers

The price of a bag of cement decreased from Le 50,000 to Le 45,000. What is the percentage decrease?

Percentage decrease	=	$\frac{\text{decrease}}{\text{original amount}} \times 100$
Decrease	=	50,000 – 45,000
	=	5,000

*Percentage increase	=	$\frac{200}{1,000} \times 100$	Percentage decrease	=	$\frac{5,000}{50,000} \times 100$
	=	$\frac{200 \times 100}{1,000}$		=	$\frac{5,000 \times 100}{50,000}$
	=	20%		=	10%

10. Check that pupils recognise that to calculate both an increase and decrease, they always have to subtract the higher number from the lower number.
11. If pupils make a mistake, help them make the proper corrections.
12. **Say:** Now let us look at the other type of percentage problems.
13. **Write** Question c. on the board: Increase 40 kg by 50%.
14. **Ask**: Will our answer be in % or kg?
15. Give pupils a moment to think.
16. Have a pupil volunteer to answer the question. (Answer: 'kg')
17. **Say**: In Question c., we need to increase a quantity. This quantity can be a measurement in metres, kilograms, Leones, etc. The answer will be in the same unit.
18. **Ask**: How is this different from the previous 2 questions we just answered?
19. Help pupils understand that questions a. and b. were on increasing or decreasing 'percentages' so the answers were also in percentages.
    This question is about increasing a 'quantity.' The answer will be in the unit of measurement in the question.
20. **Say:** Maybe you already learned a way to solve increasing or decreasing quantities. We will look at 2 different methods and you can choose the one you prefer.
21. Solve the problem using 2 different methods (as shown below). Explain each step as you do it.

c. <u>Increase quantities by a percentage</u>

Increase 40 kg by 5%

<u>Method 1:</u>			<u>Method 2</u>:		
5% of 40 kg	=	$\frac{5}{100} \times 40$	100% + 5%	=	105%
	=	$\frac{5 \times 40}{100}$			
	=	2 kg	New quantity	=	$\frac{105}{100} \times 40$
New quantity	=	40 + 2		=	$\frac{105 \times 40}{100}$ or 1.05×40
	=	42 kg		=	42 kg

22. **Say**: Either of these methods can be used to find the new quantity after a percentage decrease.

Guided Practice (10 minutes)

1. Point on the board to d. <u>Decrease quantities by a percentage</u>
2. **Write** this question: Decrease Le 60,000 by 10%
3. Ask the class to work in pairs.
4. **Say**: This question asks us to decrease the quantity. What calculation should we use?
5. Give pupils time to think about the question then have a pupil volunteer to answer. (Answer: Subtraction.)
6. **Say**: In your pairs, one of you will use Method 1 and the other will use Method 2 to calculate the answer. Choose which one you want to use now.
7. **Say**: Please raise your hand when you finish.
8. Allow pupils 2 minutes to answer the question.
9. Walk around, if possible, to check the answers and clear up any misconceptions.

10. Ask for one pupil who chose Method 1 and another who chose Method 2 to volunteer to come to the board.
11. Ask the pupils at the board to write down their solutions.
12. Whilst they are doing this, ask the rest of the class for each pair to show their partner their calculation.
13. **Say**: Please show your partner the calculation you did to find the answer. Discuss your methods together.
14. Allow a further minute for the pupils to discuss and share ideas.
15. **Ask**: Do we agree with the calculations on the board?
16. Ask for views from several pupils.
17. Correct any errors on the calculation on the board. Ask pupils to correct their work. (Answer: Method 1: $\frac{10}{100} \times 60{,}000$ = $\frac{10 \times 60{,}000}{100}$ = 6,000, new amount = 60,000 – 6,000 = Le 54,000; Method 2: 100% - 10% = 90% = 0.9, new amount = 0.9 × 60,000 = Le 54,000).
18. **Say:** Most people prefer Method 2 because you can do the calculation for the increase or decrease in your head, and easily divide by 100. You can then simply multiply that figure by the original quantity to get the new quantity.

Independent Practice (*10 minutes*)

1. Point to the following problems on the board:
   a. The number of pupils in a Junior Secondary School increased from 820 to 861
      Calculate the percentage increase

   b. Alusine's tax decreased from Le 6,000 to Le 3,900
      Calculate the percentage decrease

   c. Increase Le 150,000 by 8%

   d. Decrease 900 m by 30%

2. Ask the pupils to work independently to answer the questions. They can choose method 1 or method 2 for questions c. and d.
3. Walk around, if possible, to check the answers and correct any mistakes.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Write the correct answers on the board.
   (Answers:
   a. Increase = 861 – 820 = 41, percentage increase = $\frac{41}{820} \times 100$ = 5%;
   b. Decrease = 6,000 – 3,900 = 2,100; percentage decrease = $\frac{2100}{6000} \times 100$ = 35%.
   c. Method 1: $\frac{8}{100} \times 150{,}000$ = $\frac{8 \times 150{,}000}{100}$ = 12,000, new amount = 150,000 + 12,000 = Le 162,000;
      Method 2: 100% + 8% = 108% = 1.08, new amount = 1.08 × 150,000 = Le 162,000.
   d. Method 1: $\frac{30}{100} \times 900$ = $\frac{30 \times 900}{100}$ = 270, new amount = 900 – 270 = 630 m;
      Method 2: 100% - 30% = 70% = 0.7, new amount = 0.7 × 900 = L630 m).

Closing (*2 minutes*)

1. Ask pupils what they learned during the lesson.
2. Allow pupils to raise their hand and give their views. (Example answers: Different calculations for solving a percentage increase or decrease from an increase or decrease of a quantity. Accept all reasonable answers.)

Lesson Title: Ratio	**Theme**: Everyday Arithmetic	
Lesson Number: M-09-024	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Review the forms of ratio: m : n and $\frac{\mathrm{m}}{\mathrm{n}}$. 2. Divide a number into a given ratio. 3. Solve ratio problems and simplify answers to the lowest terms.	None	1. Write on the board: A JSS 3 class has 35 girls and 30 boys. Find: a. The ratio of girls to boys b. The ratio of boys to girls Please give your answers in the lowest terms. 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Discuss with your seatmate everything you know about ratios.
2. Allow the pupils to work in pairs for 2 minutes to discuss all they remember about ratios.
3. Have 2-3 pupils volunteer to tell the class what they know. Accept all reasonable answers. (Example answers: A ratio is used to compare 2 or more quantities. The quantities must be measured in the same unit. We can write a ratio using a colon, :, or as a fraction).
4. **Say**: Today we are going to review the forms of ratios, and how to divide a number into a given ratio. We will also solve problems and simplify our answers to the lowest terms.

Introduction to the New Material (*10 minutes*)

1. **Say**: We use ratios to compare quantities of the same type, e.g. length, weight, people, etc. We change the units if they are different e.g. metres to centimetres, or grams to kilograms. We describe ratios in 2 different ways. In a JSS 3 class, there are 24 girls and 36 boys.
2. **Write** on the board: 24 girls is to 36 boys.
3. **Ask**: How do we write this as a ratio? Raise your hand. (Answer: 24 : 36)
4. **Ask**: What other way can we write this ratio? Raise your hand. (Answer: as a fraction, $\frac{24}{36}$)
5. **Say:** It does not matter which of the 2 ways we write a ratio, we should always simplify it to its lowest terms. Simplify the ratio on the board by dividing with the common factor, 12.
6. Give pupils 1 minute to work out the answer in their exercise**.**
7. Have one pupils volunteer share the answer and explain what it means. (Answer: 2 : 3, $\frac{2}{3}$. This means that for every 2 girls there are 3 boys).
8. **Say**: It is very important to keep the order of the ratio as given in the problem. A ratio written as 2 : 3 means $\frac{2}{3}$ while a ratio written as 3 : 2 means $\frac{3}{2}$. The fractions are different and give different answers. A ratio compares 2 quantities. If we know the ratio and the total number of quantities, we can calculate how many of one quantity we have compared to the other.
9. **Write** on the board: Share 75 kg in the ratio 2 : 3
10. **Say:** Use any method you know to solve the problem.
11. Allow 1 minute for pupils to answer the question.
12. Have a pupil volunteer to come to the board. Ask them to explain their steps to the class.

13. It is possible to solve the problem in several ways. One possibility is shown below.

$$2 + 3 = 5 \longleftarrow \text{Work out the total number of shares}$$

$$\frac{75}{5} \times 2 = 15 \times 2 = 30 \text{ kg} \longleftarrow \text{Work out the quantity according to its share (2)}$$

$$\frac{75}{5} \times 3 = 15 \times 3 = 45 \text{ kg} \longleftarrow \text{Work out the quantity according its share (3)}$$

14. Check if others in the class have used a different method from the one on the board.
15. Check that the methods give the same answer. Tell them to use the method they prefer.
16. **Say**: We can check our calculations by adding the 2 quantities to see if we get the original amount. You will know that you made a mistake if you get a different amount.
17. **Write** on the board: 30 kg + 40 kg = 75 kg
18. **Say:** We know our calculation is correct as we get the amount we started with. We can also solve problems with ratios. We will compare the girls and boys in the class.
19. Refer to the problem already written on the board. You can replace the numbers in the problem by those from the actual counts of boys and girls in the class.
20. **Say:** We are going to count the number of girls and boys in the class.
21. Ask all the girls to stand up. Ask a boy to count the girls. Write the number in the problem.
22. Ask all the boys to stand up. Ask a girl to count the boys. Write the number in the problem.
23. Allow 1 minute for pupils to answer the questions.
24. Have 2 pupils volunteer to come to the board at the same time. Ask each to show how they write one of the ratios.
25. Use the actual numbers for your JSS 3 class. Depending on the numbers, you may not be able to simplify your answers. (Example answers: 35 : 30 = 7 : 6; 30 : 35 = 6 : 7).

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:
   a. Share 48 litres in the ratio 3 : 5. Please check your answer.
   b. The shape of a room is a rectangle. One side is 3.5 m long and the other is 5 m
      Find: i. the ratio of width to length ii. the ratio of length to width
      Give your answers in the lowest term.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Allow pupils to call out the answers.
5. Write the correct answers on the board. Ask pupils to check their work. (Answers: a. $\frac{48}{8} \times 3 = 6 \times 3 = 18$ litres; $\frac{48}{8} \times 5 = 6 \times 5 = 30$ litres,
   Check: 18 litres + 30 litres = 48 litres; b. i. 3.5 : 5 = 7 : 10, ii. 5 : 3.5 = 10 : 7)

Independent Practice (*10 minutes*)

1. Ask the pupils to continue to work independently to answer the questions.
2. Point to the questions on the board:
   a. Share 450 m in the ratio 5 : 4. Please check your answer.
   b. A Junior Secondary School has 7 teachers and 420 pupils.
      Find: i. the ratio of pupils to teachers ii. the ratio of teachers to pupils
      Give your answers in the lowest term.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.

5. Write the correct answers on the board. Ask pupils to check their work. (Answers: a. $\frac{450}{9} \times 5 = 50 \times 5 = 250$ m; $\frac{450}{9} \times 4 = 50 \times 4 = 200$ m,
   Check 250 m + 200 m = 450 m; b. i. 420 : 7 = 60 : 1, ii. 7 : 420 = 1 : 60)

Closing (*2 minutes*)

1. **Say**: Please tell the class something you learned today about ratios. Raise your hand.
2. Allow a pupil one minute to explain. (Example answer: How to write ratios using a colon; how to use a fraction to write a ratio; sharing an amount in a given ratio; solve ratio problems.)
3. **Say**: Excellent work class! In the next lesson, we will learn about a special type of ratio called 'rates'.

Lesson Title: Rates	**Theme**: Everyday Arithmetic	
Lesson Number: M-09-025	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify that rate is a special ratio that compares 2 units of measurement. 2. Solve problems involving rate.	None	1. Write on the board: Write the ratio 250 cm to 2m in its lowest terms. 2. Write the problem from the Guided Practice section on the board. 3. Write the problems from the Independent Practice section on the board.

Opening (*3 minutes*)

1. Ask pupils to solve the ratio problem on the board. Allow 1 minute.
2. Have a pupil volunteer to come to the board and explain how to solve the problem.
3. Ask the class if they agree. Discuss and correct any misconceptions.
4. **Write** the answer with the steps on the board.

   (Answer: 2 m = 200 cm ← Change m to cm

   250 : 200 ← Write ratio

   5 : 4 ← Simplify to lowest terms)

5. **Say**: Today we are going to identify that rate is a special ratio that compares 2 units of measurement. We will also solve problems involving rate.

Introduction to the New Material (*10 minutes*)

1. **Ask**: Who can tell the class what we did just now before comparing the 2 quantities? Raise your hand.
2. Guide a pupil to think about the first step in the calculation above. (Answer: The quantities were changed to the same unit.)
3. **Say**: Sometimes we want to compare 2 quantities that are not alike and cannot be changed to the same unit. An example is when we want to work out how fast a car is travelling.
4. **Ask**: Who can give the 2 quantities we compare?
5. Guide the pupils to think of the formula and units of speed. (Answer: Distance and time, metres per second, kilometres per hour.)
6. **Say**: When we compare quantities that are 2 different units of measurement we use a special type of ratio called a rate. Let us see if we can tell which of the following is a ratio and which is a rate. Think about the units of measurement of the quantities.
7. Allow a few moments after each statement for pupils to work in pairs to discuss and share their ideas. **Write** each answer on the board.
   a. **Ask:** What is the length of a rectangle compared to its width? Raise your hand. (Answer: Ratio, units of length, e.g. m, cm)
   b. **Ask**: What is the number of metres you travel in one second? Raise your hand. (Answer: Rate, units are metres and second (metres per second))
   c. **Ask**: What is the area of a square compared to the area of a triangle? Raise your hand. (Answer: Ratio, units of area, e.g. cm^2, m^2)
   d. **Ask:** What is the percentage of interest you pay in one year? Raise your hand. (Answer: Rate, actually called ‘interest rate’, units are percentage and year (percentage per annum))

8. **Ask:** What do you notice about the answers on the board? Raise your hand.
9. Guide pupils to look at the units of measurement of the quantities. (Example answer: The quantities in the ratio are measured with one unit, the quantities in the rate are measured with 2 units.)
10. **Say**: The units in a ratio cancel each other because they are the same. The units in a rate take on the unit from the numerator and the unit from the denominator, e.g. $\frac{\text{km}}{\text{hr}}$.
11. **Write** on the board: A car travels a distance of 240 km in 3 hours. What is the average speed in kilometres per hour (km/hr)?
12. Show how to solve this problem on the board. Explain each step as shown.

Speed	=	240 km for every 3 hours	
	=	240 : 3	Write as a ratio
	=	$\frac{240}{3}$	Write as a fraction
	=	$\frac{80}{1}$	Divide both numbers by 3
	=	80 km/hr	Write as a rate in km/hr

13. **Say**: The speed in our example is the unit rate at which the car is travelling. A unit rate is a ratio that tells us how many units of one quantity there are for every one unit of the second quantity.
14. **Write** the definition of the unit rate on the board: A ratio that tells us how many units of one quantity there are for every one unit of the second quantity.
15. **Say**: In our example, this is '80' kilometres for every '1' hour, i.e. 80 km/hr. Rates use words and symbols such as 'per' (/), 'each' (ea) and 'at' (@).
16. **Write**: Rates use words and symbols such as 'per' (/), 'each' (ea) and 'at' (@).

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the question on the board: Mabinti spends Le 20,000 to buy 4 pints of palm oil. How much did it cost her per pint?
3. Walk around, if possible, to check answers and clear misconceptions.
4. Have a pupil volunteer to come to the board and explain how to solve the problem.
5. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: $\frac{20000}{4} =$ Le 5,000/pint)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:
   a. A bunch of 4 plantains costs Le 12,000. How much does it cost per plantain?
   b. A car travels at 560 km in 7 hours. What is its speed?
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have a pupil volunteer to call out their answer to the problem.
6. Write the correct answer on the board. Ask pupils to correct their work. (Answers: a. $\frac{12000}{4} =$ Le 3,000/plantain; b. $\frac{560}{7} = 80$ km/hr)

Closing (*2 minutes*)

1. **Ask:** What is unit rate? Raise your hand.
2. Have a pupil at the back of the classroom to answer. (Answer: A unit rate is a ratio that tells us how many units of one quantity there are for every one unit of the second quantity.)
3. **Say**: In the next lesson, we will learn how to use the unit rate to calculate with direct proportions.

Lesson Title: Direct Proportion	**Theme**: Everyday Arithmetic	
Lesson Number: M-09-026	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Identify the symbol for proportionality ($\propto$), the means and extremes.
2. Solve direct proportion problems.

Teaching Aids
None

Preparation
1. Write the following problem on the board:
A lorry travels at 180 km in 3 hours. What is its average speed in km/h?
2. Draw the table below:

Distance, d (km)			180	
Time, t (h)	1	2	3	4

3. Write the problem in the Guided Practice section on the board.
4. Write the problem in the Independent Practice section on the board.

Opening (*3 minutes*)

1. Point to this problem on the board: A lorry travels at 180 km in 3 hours. What is its average speed in km/h?
2. **Say**: Please look at the problem on the board. You have 2 minutes to solve it.
3. Allow pupils to call out the answer. (Answer: $\frac{180}{3} = 60$ km/h)
4. **Say**: Today we are going to identify the symbol for proportionality ($\propto$), the means and extremes. We will also solve problems on direct proportion.

Introduction to the New Material (*10 minutes*)

1. **Ask:** How far does the car in our problem travel after 1 hour? Raise your hand. (Answer: 60 km)
2. Ask the pupils to put the information in the table. Ask them to use the information to complete the table for the remaining times. (Answers: As shown.)
3. **Ask:** Who can tell the class what they notice in the table? Raise your hand.
4. Guide a pupil to explain the relationship between distance and time. (Example answers: When the distance doubles from 60 km to 120 km the time also doubles from 1 hour to 2 hours. The unit rate stays the same – it is always 60 km/h).
5. Complete the third row of the table showing that the unit rate is always 60.
6. **Say**: From the table, we can see that all the rates are equal to each other. We can take any 2.
7. **Write** on the board:
8. **Say**: We can write the rates as equivalent fractions:

Distance, d (km)	60	120	180	240
Time, t (h)	1	2	3	4
Speed (unit rate) (km/h)	60	60	60	60

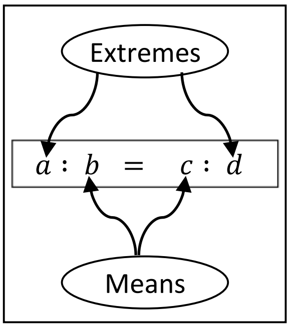
$$60 : 1 = 120 : 2$$

$$\frac{60}{1} = \frac{120}{2}$$

$$60 = 60$$

9. **Say**: In any rates (or ratio) problem, where we have equal ratios, $a : b = c : d$ we can write this as equivalent fractions:

$$\frac{a}{b} = \frac{c}{d}$$



10. **Say**: Cross-multiplying gives us: $a \times d = b \times c$

11. **Say**: The quantities have special names, ′a′ and ′d′ are called ‘extremes’;
′b′ and ′c′ are called ‘means’.
12. **Write** on the board: The product of the extremes equals the product of the means.
13. **Say**: In our example we can write:

$$60 \times 2 = 120 \times 1$$
$$120 = 120$$

14. **Say**: Writing the ratios like this is very useful. In a lot of ratio or rates problem, we usually know 3 of the quantities and we need to find the fourth. We can see how this works in our problem.
15. **Say**: Find the distance, x, travelled by the lorry after 6 hours. We will start by writing:

$60 : 1 = d : 6$

16. Ask the pupils to work in pairs to continue to discuss and solve the problem.
17. Have a pupil from the left-hand side of the classroom volunteer to come to the board and explain how to solve the problem.

$\frac{60}{1} = \frac{d}{6}$	⟵ Write ratios as equivalent fractions
$60 \times 6 = d \times 1$	⟵ Cross-multiply (product of extremes = product of means)
$360 = d$	⟵ Solve for x by multiplying
$d = 360$ m	⟵ Distance travelled in 6 hours

18. **Say**: This shows that when the distance goes up 6 times from 60 km to 360 km, the time also goes up 6 times from 1 hour to 6 hours. This means the distance travelled, d, is directly proportional to the time, t.

19. **Write** on the board: $d \propto t$ read as d 'is directly proportional to' t

20. **Say**: This can be written as an equation: $d = kt$ where k is called the constant of <u>proportionality</u>

21. **Say**: In our example the unit rate, 60 km/h, is the constant of proportionality. It is the constant (steady or average) speed at which the car travels. We can write the equation as:

$$d = 60t$$

22. **Say**: If d and t are known, we can find k using this form of the equation:

$$\frac{d}{t} = k$$ k will always be the same no matter the values of d and t

23. **Say**: We can use this fact to solve similar problems on direct proportionality.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to solve the problem.
2. Point to this problem on the board:

   A car travels at a steady speed of 60 km/h. Using the equation from our example, find:

   a. the distance, d, it travels after 5 hours.

   b. the time it takes to travel 360 km

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have a pupil from the right-hand side of the classroom volunteer to explain how to solve the problem on the board.
5. Write the correct answers and steps on the board. Ask pupils to check their work.
   (Answers: a. d = 60 t, t = 5 hours, d = 60 $\times$ 5 = 300 km;
   b. d = 60 t, d = 360 m, 360 = 60 $\times$ t, t = $\frac{360}{60}$ = 6 hours)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the following ratio problem on the board:

   The distance a car travels, d, is directly proportional to the time, t.
   If it travels 150 km at a steady speed for 2 hours:

   a. Find the constant of proportionality, k in km/h.
   b. Use k to find the time it takes to travel 300 km.

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from the front and back of the classroom volunteer to give their answers to the questions.
6. Write the correct answers and steps on the board. Ask pupils to check their work.
   (Answers: i. d $\propto$ t; d = k t; $k = \frac{d}{t}$; $k = \frac{150}{2}$ = 75 km/h;
   ii. 75 = $\frac{300}{t}$, t = $\frac{300}{75}$, t = 4 hours)

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper. You will hand it in at the end of the lesson.
2. **Say:** A quantity, y, is directly proportional to another quantity, x. Write on your paper how you show this as an equation if the constant of proportionality is k.
3. Ask the pupils to hand in their papers.
4. Go through these papers after the lesson to check their understanding of the topic. Use a similar question at the start of the next lesson to clear up any misconceptions.
   (Answer: $y = kx$).

Lesson Title: Indirect Proportion	**Theme**: Everyday Arithmetic	
Lesson Number: M-09-027	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify the form of an indirectly proportional relationship ($t \propto \frac{1}{s}$). 2. Solve indirect proportion problems.	None	1. Write on the board: The cost, C, is directly proportional to the number, n, of items bought. Write this relationship as an equation. 2. Write the problems from the Guided Practice section on the board. 3. Write the problems from the Independent Practice section on the board.

Opening (*3 minutes*)

1. Point to the following question: The cost, C, is directly proportional to the number, n, of items bought. Write this relationship as an equation.
2. **Say**: Please answer the question on direct proportions on the board. You have 2 minutes.
3. Have a pupil volunteer to come to the board to explain how to write the relationship. (Answer: $C = kn$ where k is the constant of proportionality)
4. Ask the class if they agree. Discuss any misconceptions – for example, some pupils may have written, $C \propto n$. Explain that this is the correct expression for the relationship between C and n.
5. Explain that an equation always has an equal sign, so $C = kn$ is the correct answer. We must define k if it is not done in the question.
6. **Say**: Today we are going to identify the form of an indirectly proportional relationship ($t \propto \frac{1}{s}$). We will also solve problems with indirect proportion.

Introduction to the New Material (*10 minutes*)

1. **Say**: In direct proportion problems, when one quantity gets bigger, the other quantity also gets bigger. We see we can write this as either a ratio (C : n) or as a formula using a constant of proportionality ($C = kn$).
2. **Write**: (C : n) or ($C = kn$).
3. **Say**: There are other times when one quantity gets smaller as another gets bigger. Think about a situation when a car is travelling at different speeds. It will take the car less time to cover a particular distance if it travels fast than if it travels slowly.
4. **Say**: Suppose we are going on a 200 km journey.
5. Draw this table on the board. Complete the first 2 rows.

Speed, s (km/h)	50	40	25
Time, t (hours)	4	5	8
Constant of proportionality, k			

6. **Ask:** Who can tell the class what they notice in the table? Raise your hand.

7. Guide a pupil to explain the relationship between the speed and time. (Example answers: When the speed halves from 50 km/h to 25 km/h the time doubles from 4 hours to 8 hours. When you multiply any speed by its time we get 200.)
8. **Say**: We can see from the table that multiplying each speed by its related time gives the same answer, 200. We can also see that as the speed increases, the time decreases. This means the time it takes to travel, t, is 'indirectly proportional' to the speed, s.
9. **Write** on the board:

$$t \propto \frac{1}{s}$$ read as t 'is indirectly proportional to' s

10. **Say**: This can be written as an equation:

$$t = \frac{k}{s}$$ where k is called the constant of proportionality.

11. Therefore, the constant of proportionality, k = s $\times$ t = 200
12. **Ask:** What is the unit of the constant of proportionality? Raise your hand. (Answer: km)
13. **Say:** This is the distance for the journey. It remains constant.
14. Ask pupils to use the information to complete the row for the constant of proportionality.
15. **Say**: In our example we can write the equation as:

$$t = \frac{200}{s}$$

16. **Ask:** How long does it take the car to travel at a speed of 20 km/h?
17. Allow the pupils to work in pairs for 2 minutes to solve the problem.
18. Have a pupil volunteer to come to the board to explain how to solve the problem.

$t = \frac{200}{s}$	⟵	equation for relationship, k = 200 km
$= \frac{200}{20}$	⟵	substitute 20 km/h for speed
= 10 hours	⟵	time taken at 20 km/h

19. Correct any errors on the calculation on the board. Ask pupils to check their work.
20. **Say**: If s and t are known, we can find k using this form of the equation: $s \text{ x } t = k$. k will always be the same no matter the values of s and t.
21. **Say**: We can use this fact to solve similar problems on indirect proportionality.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs to solve the problem.
2. Point to the question on the board:

    A car travels a distance of 300 km at a steady speed of 50 km/h. How long does it take to complete the journey?

3. Walk around, if possible, to check answers and clear misconceptions.
4. Have a pupil from the left-hand side of the classroom volunteer to give their answer to the questions.
5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: k = 300 km, s = 50 km/h, t = $\frac{k}{s}$ = $\frac{300}{50}$= 6 hours).

Independent Practice (10 minutes)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

   The time t, it takes a van to travel on a journey is indirectly proportional to its speed, s.
   It takes a van 3 hours to complete its journey travelling at a steady speed of 50 km/h.

   a. Find the constant of proportionality, k in km.
   b. Use k to find the time it takes if it travels at a steady speed of 75 km/h.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from the front and back of the classroom volunteer to give their answers to the questions.
5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: i. $t \propto \frac{1}{s}$, $t = \frac{k}{s}$, k = t s = 3 × 50 = 150 km; ii. $t = \frac{150}{75}$ = 2 hours).

Closing (*2 minutes*)

1. Allow a few moments after each question for pupils to answer. **Write** the answers on the board.
   a. **Say:** A quantity, y, is directly proportional to another quantity, x.
   Write how to show this as an equation if the constant of proportionality is k. (Answer: $y = kx$).
   b. **Say:** A quantity, y, is indirectly proportional to another quantity, x.
   Write how to show this as an equation if the constant of proportionality is k. (Answer: $y = \frac{k}{x}$).
2. **Say**: In our next lesson, we will use both these equations to answer problems on direct and indirect proportionality.

Lesson Title: Proportion Problem Solving	**Theme**: Everyday Arithmetic	
Lesson Number: M-09-028	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to solve direct and indirect proportion problems.	None	1. Write the following problems on the board: a. The cost, C, of putting a roof on a building is directly proportional to the area, A, of the roof. A roof costs Le 18,000,000 and covers an area of 36 m^2. i. Find the relationship between C and A. ii. Find out how much it costs to cover a roof with an area of 24 m^2. iii. A roof costs Le 15,000,000. What area does it cover? b. The time, T, it takes to dig a well is indirectly proportional to the number of men, P, digging the well. It takes 2 days for 5 men to dig a well. i. Find the relationship between T and N. ii. Find out how long it will take 4 men working at the same rate to dig the well. 2. Write the problems from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: We have 2 quantities, y and x, which are directly proportional. How do we write this as a ratio?
2. Allow pupils to call out their answers. (Answer: $y : x$)
3. **Ask**: How do we write this as a proportion? Raise your hand. (Answer: $y \propto x$)
4. **Say**: What if the quantities are indirectly proportional. What do we write? Raise your hand. (Answer: $y : \frac{1}{x}$ and $y \propto \frac{1}{x}$)
5. **Say**: Today we are going to solve direct and indirect proportion problems.

Introduction to the New Material (*10 minutes*)

1. **Say**: Who can explain to the class how to solve the first part of Question a.? Raise your hand.

2. Guide a pupil who has raised their hand to come to the board and solve Question a., part i. (Answer: As shown below)

$$\begin{array}{llcll}
\text{i.} & C & \propto & A & \longleftarrow \text{Cost is directly proportional to area} \\
 & C & = & kA & \longleftarrow \text{where k is the constant of proportionality} \\
 & 18000000 & = & k \times 36 & \longleftarrow \text{Using C = Le 18,000,000 when A = 36 m}^2 \\
 & k & = & \frac{18000000}{36} & \longleftarrow \text{Divide cost by area to find k} \\
 & & & \frac{500000}{1} & \longleftarrow \text{Divide by a common factor of 36} \\
 & & & \text{Le } 500{,}000/\text{m}^2 & \longleftarrow \text{Cost per square metre of roof} \\
 & C & = & 500000A & \longleftarrow \text{Equation relating C and A}
\end{array}$$

3. Ask pupils to work in pairs to solve parts ii and iii. Give them 3 minutes.
4. Ask pupils to raise their hand to answer. Guide pupils to explain how to solve parts ii and iii. on the board.

$$\begin{array}{llcll}
\text{ii.} & C & = & 500000A & \\
 & & = & 500000 \times 24 & \longleftarrow \text{Using A = 24 m}^2 \\
 & & = & \text{Le } 12000000 & \longleftarrow \text{Cost of roof with area 24 m}^2
\end{array}$$

$$\begin{array}{llcll}
\text{iii.} & C & = & 500000A & \\
 & 15000000 & = & 500000A & \longleftarrow \text{Using C = Le 15,000,000} \\
 & A & = & \frac{15000000}{500000} & \longleftarrow \text{Divide cost by k to find area} \\
 & & & 30 \text{ m}^2 & \longleftarrow \text{Divide by 500000 to find area}
\end{array}$$

5. Stop here and check that pupils understand everything up to this point.
6. **Ask:** What did we do in part i of the question to find k? Raise your hand.
7. Guide pupils to look at the initial information given. (Answer: We used the information given that the cost of Le 18,000,000 was used to cover an area of 36 m^2.)
8. **Ask**: What did we do next? Raise your hand.
9. Guide pupils to look at the next part where k can be used to find the cost for a given area.
10. **Say**: Once we know k, the constant of proportionality, we can find any of the other missing quantities. You can see how that works in part iii. of the question when we have cost and we are asked to find the area.
11. **Say**: This is a standard method when solving problems on direct and indirect proportion. The initial information allows us to find k, the constant of proportionality. We then use k to solve for any quantities required. Let us see how it works for a question on indirect proportionality.

Guided Practice (*10 minutes*)

1. Have a pupil from the front of the classroom volunteer to read Question b.
2. **Ask**: What is our first step? Raise your hand.
3. Have a pupil from the back of the class answer. (Answer: Use the initial information given to find k. Then find the equation relating time and the number of men.)
4. Ask pupils to continue working in pairs to solve Question b. part i.

5. Give pupils 2 minutes. Stop to check that all of them have the correct relationship connecting T and N.
6. Have a pupil from the front of the classroom volunteer to show on the board how to solve part i. (Answer: As shown below)

i.	T	$\propto$	$\frac{1}{N}$	$\longleftarrow$	*Time is indirectly proportional to men*
	T	$=$	$\frac{k}{N}$	$\longleftarrow$	*where k is the constant of proportionality*
	2	$=$	$\frac{k}{5}$	$\longleftarrow$	*Using T = 2 days when N = 5 men*
	k	$=$	5×2	$\longleftarrow$	*Multiply T by N to find k*
		$=$	10	$\longleftarrow$	*Number of man days to dig a well*
	T	$=$	$\frac{10}{N}$	$\longleftarrow$	*Equation relating T and N*

7. Correct any errors on the calculation on the board. Ask pupils to check their work.
8. **Say:** Now use the equation to answer part ii.
9. Allow a further 1 minute. Walk around, if possible, to check the answers and clear up any misconceptions.
10. Have a volunteer come to the board to solve part ii.
11. Correct any errors on the calculation on the board. Ask pupils to check their work. (Answer: ii. T = $\frac{10}{N}$ = $\frac{10}{4} = 2.5$ days)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to solve the problems.
2. Point to the following problems on the board:
    a. The amount of sugar, S, required to make ginger beer is directly proportional to the amount, G, of ginger beer being made. It requires 280 g of sugar to make 2 litres of ginger beer.
        i. Find the constant of proportionality, k.
        ii. How much sugar is required to make 6 litres of ginger beer?
    b. The time, t, it takes to plant a field is indirectly proportional to the number of people, p, available. It takes 5 days for 15 people to plant the field.
        i. Find the constant of proportionality, k.
        ii. How long will it take 25 people, working at the same rate, to plant the field?
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to some to the board and show their answers to the questions. Ask other pupils to observe to check whether they agree or not.
5. Correct any errors on the calculation on the board. Ask pupils to check their work.
   (Answers: c. i. $S \propto G$, $S = k G$, $280 = 2k$, $k = \frac{280}{2} = 140$ g/l; ii. $S = k G = 140 \times 6 = 840$ g
   d. i. $t \propto \frac{1}{p}$, $t = \frac{k}{p}$, $5 = \frac{k}{15}$, $k = 5 \times 15 = 75$; ii. $t = \frac{75}{p} = \frac{75}{25} = 3$ days)

Closing (*2 minutes*)

1. **Ask:** How do we find the constant of proportionality in a standard proportion problem?
2. Have a pupil from the back of the classroom volunteer to answer. (Answer: Use the initial information given in the problem.)

Lesson Title: Financial Literacy I	Theme: Everyday Arithmetic	
Lesson Number: M-09-029	**Class/Level**: JSS 3	**Time:** 35 minutes

<table>
<tr>
<td>Learning Outcomes
By the end of the lesson, pupils will be able to solve problems with wages, salaries, and income tax.</td>
<td>Teaching Aids
None</td>
<td>Preparation
1. Write on the board:
<table>
<tr><th colspan="2">Tax Rate</th></tr>
<tr><td>Not over Le 500,000 per month</td><td>Nil</td></tr>
<tr><td>Next Le 500,000 per month</td><td>15%</td></tr>
<tr><td>Next Le 500,000 per month</td><td>20%</td></tr>
<tr><td>Next Le 500,000 per month</td><td>30%</td></tr>
<tr><td>Above Le 2 million per month:</td><td>35%</td></tr>
</table>
2. Write the problems from the Guided Practice section on the board.
3. Write the problems from the Independent Practice section on the board.</td>
</tr>
</table>

Opening (*3 minutes*)

1. **Say**: The money that people earn is taxed by the government. It is used to provide services to the country such as education, health and social welfare.
2. **Ask**: What is the name of the tax that is paid on the money people earn? Raise your hand. (Answer: Income tax)
3. Change the year in the next question from 2015 to the current year. The income tax rates table also changes to the current rates. Use the current income tax rates in the questions.
4. Point to the table on the board.
5. **Ask**: Who can tell the class, what the tax-free income is? This is the income below which you do not have to pay any income tax? (Answer: Le 500,000 per month)
6. **Say**: Today we are going to solve problems with wages, salaries, and income tax.

Introduction to the New Material (*10 minutes*)

1. **Say:** Some of you may be wondering what the difference is between a wage and a salary. Wages refer to the money people earn who get paid by the hour or by the day. They get paid at the end of the month only for the hours or days they work. Salaries do not depend on the number of hours or days someone works. It is a fixed amount paid at the end of the month regardless of the number of hours or days someone works.
2. **Say:** No matter whether you earn a wage or a salary, you still have to pay income tax on it. We will now do some simple problems on wages and salaries. We will also see how the income tax rates are applied to wages and salaries.
3. **Write** on the board:

   Adama earns Le 950,000 per month. She is given a 5% rise in salary. How much is her new monthly salary?
4. **Ask:** Who remembers doing questions like this when we were doing percentages? Raise your hand.
5. Select a pupil who has raised their hand to carry out the calculations on the board. Guide as needed.

$100\% + 5\% \quad = \quad 105\%$ ⟵ Add the percentage rise to 100%

	=	$\frac{105}{100} = 1.05$	← Convert to a decimal
New salary	=	1.05 × 950000	← Multiply by original income
	=	Le 997,500	← New salary

6. **Write** on the board: How much income tax does Adama pay per month?
7. **Say**: Look at the table of income tax rates on the board.
8. **Ask:** What is the first thing we need to do to find the income tax? Raise your hand.
9. Guide a pupil to think about what tax-free income means in practice. (Answer: We need to find the taxable income. We need to subtract tax-free income from new income).
10. Ask pupils to use the table and work in pairs for 3 minutes to discuss and share their ideas.
11. **Say**: Please raise your hand when you finish.
12. Select one of the last pupils who raised their hand to explain on the board how to solve the problem. Ask other pupils to observe carefully to see if they agree with the calculation.
13. Correct any errors in the calculation on the board. Ask pupils to check their work.
    Example calculation:

Taxable income	= 997,500 – 500.000	← Subtract tax-free from new income
	= 497,500	← Taxable income
Tax on Le 497,500	= 15% of Le 497,500	← Tax rate from table, Adama pays only 15% as her taxable income is less than Le 500,000
	= 0.15 × Le 497.500	← Multiply the tax rate and the taxable income
	= Le 74,625	← Adama pays Le74,625 in tax

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:
   Sallieu earns a daily wage of Le 50,000.
   a. How much does he earn in 22 days?
   b. How much tax does he pay on his wages?
3. Guide pupils to break down the taxable income to Le 500,000 @ 15% and Le 100,000 @ 20%.
4. Walk around, if possible, to check the answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Write the correct answers and steps on the board. Ask pupils to check their work.
   (Answers: a. $22 \times Le\ 50{,}000 = Le\ 1{,}100{,}000$;
   b. Taxable income = Le 1,100,000 – 50,0000 = Le 600,000
   Tax on Le 600,000 = 15% of Le 500,000 + 20% of Le 100,000
   = 0.15 × 500,000 + 0.2 × 100,000 = 75,000 + 20,000 = Le 95,000).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:
   a. A manager earns Le 4,000,000 per month. Please calculate the income tax paid by the manager per year.
   b. An office worker earns Le 1,200,000 per month. His salary was increased by 10%. What is his new salary? How much <u>extra</u> income tax does he pay on his new salary?
3. Walk around, if possible, to check the answers and clear up any misconceptions.

4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Write the correct answers and steps on the board. Ask pupils to check their work.
   (Answers: a. Taxable income in one month = Le 4,000,000 – 500.000 = 3,500,000,
   Tax on Le 3,500,000 = $0.15 \times 500{,}000 + 0.2 \times 500{,}000 + 0.3 \times 500{,}000 + 0.35 \times 2{,}000{,}000$
   $= 75{,}000 + 100{,}000 + 150{,}000 + 700{,}000 =$ Le 1,025,000;
   Income tax per year = $12 \times$ Le1,025,000 = Le 12,300,000
   b. New salary = $1.1 \times$ Le 1,200,000 = Le 1,320,000;
   Additional income = Le 1,320,000 – 1,200,000 = 120,000. From the table, the tax rate for the additional Le 120,000 = $0.2 \times 120{,}000$ = Le 24,000.)

Closing (*2 minutes*)

1. **Say:** Please write down 2 different things that you learned today.
2. Allow pupils one minute to discuss and share their ideas.
3. Have one pupil from the front, and one from the back of the classroom volunteer to answer. (Example answers: The difference between a wage and salary; how to calculate income tax; how to find a percentage increase)

Lesson Title: Financial Literacy 2	Theme: Everyday Arithmetic	
Lesson Number: M-09-030	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson, pupils will be able to solve simple interest problems.	None	1. Write on the board: a. Find 10% of 500,000. b. Find the simple interest on Le 500,000 for one-year at an interest rate of 10% per annum. c. Find the simple interest on Le 400,000 for 3 years at an interest rate of 5% per annum. d. Find the total amount to repay if a loan of Le 200,000 is taken out at an interest rate of 11% for 1 year. 2. Write the problems from the Guided Practice section on the board. 3. Write the problems from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please calculate 10% of 500,000.
2. Have a pupil from the back of the classroom to write the answer. Correct any errors. (Answer: $10\% = \frac{10}{100} = 0.1;\ 0.1 \times 500{,}000 \ = 50{,}000$)
3. **Say**: Today we are going to solve simple interest problems.

Introduction to the New Material (*10 minutes*)

1. **Say**: When someone puts money in a bank, the bank pays them an additional amount of money. This money depends on the amount put in by the person and how long they keep the money in the bank. Simple interest is calculated as a percentage of the money put in the bank on an annual, or yearly, rate. You do not need to memorise a formula. Just remember how to calculate with percentages.
2. **Say**. Please look at Question b. on the board. Who can tell the class, in their own words, what the question is actually asking? Raise your hand.
3. Select a pupil who has raised their hand. (Example answer: Find 10% of Le 500,000.)
4. **Say**: The example on the board gives the amount paid out if Le 500,000 is kept at a bank at an interest of 10% per annum.
5. **Say:** We already calculated the answer for Question b. It is a simple percentage calculation.
6. **Say**: If the question requires you to find the simple interest for more than one year, simply multiply your answer by the number of years asked.
7. **Ask**: What simple calculation is Question c. asking us to do? Raise your hand.
8. Select a pupil who has raised their hand. (Answer: Find 5% of Le 400,000, 3 times.)
9. Ask the class to calculate the answer. (Answer: $0.05 \times 400{,}000$ = Le 20,000; $20{,}000 \times 3 =$ Le 60,000)
10. **Say**: Simple interest calculations also apply with loan repayments. Work with a partner to answer Question d.

11. Have a pupil volunteer to explain on the board how to solve Question d. Ask other pupils to observe carefully to see if they agree with the calculation.
12. Ask if anyone has used a different method to explain their method on the board.
13. **Say**: This is a percentage increase problem. Le 200,000 has been increased by 11%. (Answer: d. $0.11 \times 200{,}000 = 22{,}000$; $200{,}000 + 22{,}000$ = Le222,000).
14. **Say**: If the loan is for more than one-year long, multiply the interest rate by the number of years the loan is for.

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:
   a. Find the simple interest on:
      i. Le 600,000 for 5 years at an interest rate of 8% per annum.
      ii. Le 100,000 for 4 years at an interest rate of 4½% per annum.

   b. Find the total amount to repay if a loan of:
      i. Le 45,000 is taken out at an interest rate of 9% for 1 year.
      ii. Le 120,000 is taken out at an interest rate of 7% for 3 years.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. i. $(0.08 \times 600{,}000) \times 5 = 48{,}000 \times 5 =$ Le 240,000;
   ii. $(0.045 \times 100{,}000) \times 4 = 4{,}500 \times 4 =$ Le 18,000;
   b. i. $0.09 \times 45{,}000 = 4{,}050$, 45,000 + 4,050 = Le 49,050;
   ii. $(0.07 \times 120{,}000) \times 3 = 8{,}400 \times 3 =$ Le 25,200; 120,000 + 25,200 = Le 145,200)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:
   a. Find the simple interest on:
      i. Le 750,000 for 3 years at an interest rate of 6% per annum.
      ii. Le 300,000 for 5 years at an interest rate of 2½% per annum.

   b. Find the total amount to repay if a loan of:
      i. Le 165,000 is taken out at an interest rate of 12% for 2 years. How much is repaid each month?
      ii. Le 250,000 is taken out at an interest rate of 9% for 5 years.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Write the correct answers and steps on the board. Ask pupils to check their work.
   (Answers: a. i. $(0.06 \times 750{,}000) \times 3 = 45{,}000 \times 3 =$ Le 135,000;
   ii. $(0.025 \times 300{,}000) \times 5 = 7{,}500 \times 5 =$ Le 37,500;
   b. i. $(0.12 \times 165{,}000) \times 2 = 19{,}800 \times 2 =$ Le 39,600; 165,000 + 39,600 = Le 204,600;
   $204{,}600 \div 24 =$ Le 8,525 per month;
   ii. $(0.09 \times 250{,}000) \times 5 = 22{,}500 \times 5 =$ Le 112,500, 250,000 + 112,500 = Le 362,500).

Closing (*2 minutes*)

1. **Ask:** What type of calculation should we think about when we see simple interest problems? Raise your hand.
2. Select a pupil who has raised their hand. (Example answers: Percentages, percentage increases, percentages of quantities)
3. **Say:** Simple interest problems are percentage problems. If we know how to calculate percentages of quantities and percentage increases, we can solve problems involving simple interest.

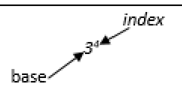
Lesson Title: Index Notation and the Laws of Indices	**Theme**: Numbers and Numeration	
Lesson Number: M-09-031	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Interpret numbers in index notation. 2. State the 6 laws of indices and solve simple examples for each.	None	1. Write the problems from the Guided Practice section on the board. 2. Write the problems from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Write** on the board: $3^{4.}$
2. **Say**: Please raise your hand if you can expand the number on the board.
3. Select a pupil who has raised their hand to write the expansion on the board. (Answer: $3^4 = 3 \times 3 \times 3 \times 3 = 81$)
4. **Ask**: How do we read the number on the board? (Answer: Read as "three to the power four".)
5. **Say**: In today's lesson, we are going to interpret numbers in index notation. We will also look at the 6 laws of indices and solve simple examples for each.

Introduction to the New Material (*15 minutes*)



The index (or power) tells us how many of the base number to multiply together.

Note: The plural of index is 'indices'

1. Point to the number 3^4 on the board.
2. **Say**: We call the 3 the base. The 4 is the index. The index tells us how many of the base numbers to multiply together. It is also called the 'power'. Note that the plural of index is 'indices'.
3. **Write** this on the board as shown.
4. **Say**: When we write numbers in this way, we call it 'index notation'. Questions often say to leave our answers in index notation. So, we do not need to calculate what the answer is for 3^4.
5. **Say**: Let us now look at the rules we use when we do calculations with numbers written with indices. We call these rules the laws of indices.
6. **Write** on the board: First law of indices
   Write on the board: $a^m \times a^n = a^{m+n}$
7. **Ask:** Who can explain to the class what the first law of indices states?
8. Guide a pupil to say that: When we multiply expressions (or numbers) with the same base, we add the indices.
9. **Write** on the board: $2^5 \times 2^3$
10. Ask the class to use the law to work out the answer.
11. Have a pupil volunteer to solve the question on the board. (Answer: As shown in the table under 'First law of indices'.)
12. **Write** the heading and the second law of indices on the board.
13. Guide a pupil to say that: When we divide expressions (or numbers) with the same base, we subtract the indices.

First Law of Indices $a^m \times a^n = a^{m+n}$ $2^5 \times 2^3 = 2^{5+3}$ $= 2^8$	*Second Law of Indices* $\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$ $\frac{6^5}{6^3} = 6^5 \div 6^3 = 6^{5-3}$ $= 6^2$
Third Law of Indices $a^0 = 1$ $\frac{6^5}{6^3} = 6^5 \div 6^5 = 6^{5-5}$ $= 6^0$ $= 1$	*Fourth Law of Indices* $(a^m)^n = a^{mn}$ $(3^3)^2 = 3^{3 \times 2}$ $= 3^6$
Fifth Law of Indices $(a \times b)^n = a^n \times b^n$ $(2 \times 3)^4 = 2^4 \times 3^4$	*Sixth Law of Indices* $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ $\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3}$

14. **Write** on the board: $\frac{6^5}{6^3}$
15. Ask pupils to work in pairs to solve the third law of indices.
16. Have pupils volunteer to call out the answer. **Write** it on the board. (Answer: As shown in the table under 'Third law of indices').
17. **Say**: Sometimes when we subtract the indices we end up with nothing or zero. The third law of indices explains what to do when this happens.
18. **Write** the information for the third law on the board.
19. **Say**: 6^0 gives 1 because when we divide a number by itself we get 1.
20. Point out on the example where we are dividing 6^5 by itself.
21. **Say**: In the fourth law, we have an index raised to another index. In this case, we multiply the 2 indices as shown on the board. The fifth and sixth laws deal with bases that are different.
22. **Write** the fifth law and ask pupils to work out the example: $(2 \times 3)^4$. (Answer: As shown in the table).
23. **Write** the sixth law and ask pupils to work out the example: $\left(\frac{3}{5}\right)^3$. (Answer: As shown in the table).

Guided Practice (*5 minutes*)

1. Ask the pupils to work in pairs to answer the questions on the board.
2. Point to the questions on the board:
   a. Expand:
      i. 2^3 ii. 2^4
   b. Find $2^3 \times 2^4$
      i. using expansion ii. using the first law of indices
   c. Explain your answers to Question b.
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. i. $2^3 = 2 \times 2 \times 2$; ii. $2^4 = 2 \times 2 \times 2 \times 2$; b. i. $2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$; ii. $2^3 \times 2^4 = 2^{3+4} = 2^7$; c. We get the same answer.)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions on the board.
2. Point to the questions on the board:

   Please give the answers to the following expressions in index notation:

   a. $(5^2)^3$ b. $7^5 \div 7^3$
   c. $4^3 \times 4^4$ d. $(4 \times 6)^5$
   e. $\frac{8^5}{8^2 \times 8^3}$ f. $\left(\frac{4}{7}\right)^3$
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Write the correct answers and steps on the board. Ask pupils to check their work.

7. (Answers: a. $(5^2)^3 = 5^{2\times3} = 5^6$; b. $7^5 \div 7^3 = 7^{5-3} = 7^2$; c. $4^3 \times 4^4 = 4^{3+4} = 4^7$; d. $(4 \times 6)^5 = 4^5\times6^5$
e. $\frac{8^5}{8^2 \times 3^3} = \frac{8^5}{8^{2+3}} = \frac{8^5}{8^5} = 8^{5-5} = 8^0 = 1$; f. $\left(\frac{4}{7}\right)^3 = \frac{4^3}{7^3}$).

Closing (*2 minutes*)

1. **Say:** Look at the sixth law of indices. There is something important in this law we need to be careful about.
2. **Ask:** Who can tell me what it is? Raise your hand.
3. Select a pupil who has raised their hand to answer.
4. If no hand is raised, guide a pupil to say that 'the denominator must not be zero'.
5. **Ask**: Why can't we have zero in the denominator? (Answer: We cannot divide by zero; we will not get a real answer)

[LAWS OF INDICES]

First Law: multiplication of indices	$a^m \times a^n = a^{m+n}$
Second Law: division of indices	$a^m \div a^n = a^{m-n}$
Third Law: power of zero	$(a^0 = 1)$
Fourth Law: powers of indices	$(a^m)^n = a^{mn}$
Fifth Law: power of a product	$(a \times b)^n = a^n \times b^n$
Sixth Law: power of a quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = b \neq 0$

Lesson Title: Application of the Laws of Indices	**Theme**: Numbers and Numeration	
Lesson Number: M-09-032	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to apply the 6 laws of indices to simplify problems.	None	1. Write the questions from the Guided Practice section on the board. 2. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Ask:** Who can remind the class of what we were doing in the last lesson? Raise your hand.
2. Select a pupil who has raised their hand. (Example answer: Index notation and the 6 laws of indices. Accept other reasonable answers).
3. **Say**: Today we are going to apply the laws of indices to simplify problems.

Introduction to the New Material (*10 minutes*)

1. **Say**: We can use the laws of indices to simplify expressions with variables.
2. Remind the pupils to copy the information on the board throughout the lesson.
3. **Write** on the board: Simplify: $x^2 \times x^5$
4. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
5. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$x^2 \times x^5$	=	x^{2+5}	Use the first law $a^m \times a^n = a^{m+n}$
	=	x^7	Add the indices

6. **Write** on the board: Simplify: $\frac{p^7}{p^4}$
7. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
8. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$\frac{p^7}{p^4}$	=	p^{7-4}	Use the second law $a^m \div a^n = a^{m-n}$
	=	p^3	Subtract the indices

9. **Write** on the board: Simplify: $\frac{x^2 \times x^3}{x^5}$
10. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
11. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$\frac{x^2 \times x^3}{x^5} = \frac{x^{2+3}}{x^5} = x^{(2+3)-5} = x^0 = 1$$ Use the first, second, and third laws ($a^0 = 1$)

12. **Write** on the board: Simplify: $(u^3)^2$
13. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
14. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$(u^3)^2$	=	$u^{3\times2}$	Use the fourth law $(a^m)^n = a^{mn}$
	=	u^6	Multiply the indices

15. **Write** on the board: Simplify: $(a \times b)^3$
16. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
17. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$(a \times b)^3$	=	$a^3 \times b^3$	Use the fifth law $(a \times b)^n = a^n \times b^n$
	=	a^3b^3	Simplify

18. **Write** on the board: Simplify $\left(\frac{c}{d}\right)^2$
19. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
20. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$\left(\frac{c}{d}\right)^2 = \frac{c^2}{d^2}$$ Use the sixth law $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = b \neq 0$

21. **Say**: We will now apply the laws to more complex expressions.

Guided Practice (*10 minutes*)

1. Ask the pupils to work in pairs to solve the problems on the laws of indices.
2. Point to the questions on the board:
   Please give the answers to the following expressions in index notation:

   a. $(c^3)^2$ b. $q^5 \div q^3$
   c. $z^3 \times z^4$ d. $(a^2 \times b)^3$
   e. $\frac{s^6}{s^2 \times s^4}$ f. $\left(\frac{u^2}{v}\right)^3$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom to give their answers to the questions.
6. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. $(c^3)^2 = c^{3\times2} = c^6$; b. $q^5 \div q^3 = q^{5-3} = q^2$; c. $z^3 \times z^4 = z^{3+4} = z^7$;

d. $(a^2 \times b)^3 = a^{2\times3} \times b^3 = a^6 \times b^3 = a^6b^3$; e. $\frac{s^6}{s^2 \times s^4} = \frac{s^6}{s^{2+4}} = s^{6-(2+4)} = s^{6-6} = s^0 = 1$

f. $\left(\frac{u^2}{v}\right)^3 = \frac{u^{2\times3}}{v^3} = \frac{u^6}{v^3}$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions on the board.
2. Point to the questions on the board:

   Simplify the following expressions giving your answers in index notation:

   a. $\frac{4p^2q^4}{2pq^2}$ b. $xy^2 \times x^2y$

   c. $(p \times q \times r)^3$ d. $(5r^3s^2)^2$

   e. $\left(\frac{a^2b}{ba^2}\right)^0$ f. $\left(\frac{a^4b^2}{a^2b}\right)^2$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. Write the correct answers and steps on the board. Ask pupils to check their work.

   (Answers: a. $\frac{4p^2q^4}{2pq^2} = \frac{4}{2}p^{2-1}q^{4-2} = 2pq^2$; b. $xy^2 \times x^2y = x^{1+2}y^{2+1} = x^3y^3$

   c. $(p \times q \times r)^3 = p^3 \times q^3 \times r^3 = p^3q^3r^3$; d. $(5r^3s^2)^2 = 25r^{3\times2}s^{2\times2} = 25r^6s^4$

   e. $\left(\frac{a^2b}{ba^2}\right)^0 = 1$; f. $\left(\frac{a^4b^2}{a^2b}\right)^2 = (a^{4-2}b^{2-1})^2 = (a^2b)^2 = a^4b^2$).

Closing (*2 minutes*)

1. **Write** on the board: Simplify $\frac{s^3}{s^4}$
2. Give pupils 2 minutes to solve the question.
3. **Ask:** What answer do you get? Raise your hand.
4. Pupils might be unsure of their answer. Encourage them to raise their hands to share their answers.
5. Select a pupil who has raised their hand to explain their answer. (Answer: $\frac{s^3}{s^4} = s^{3-4} = s^{-1}$)
6. **Say:** We will look at these types of expressions in more detail in our next lesson.

Lesson Title: Indices with Negative Powers	**Theme**: Numbers and Numeration	
Lesson Number: M-09-033	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify that a number with a negative index can be rewritten as a fraction ($a^{-n} = \frac{1}{a^n}$). 2. Apply the laws for multiplying and dividing indices to those with positive and negative powers.	None	1. Write on the board: Simplify $\frac{a^4}{a^6}$ a. using the laws of indices b. using expansion 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Write** on the board: Simplify $\frac{a^4}{a^5}$
2. **Say:** Please raise your hand when you have the answer.
3. Select a pupil who has raised their hand to explain their answer. (Answer: $\frac{a^4}{a^5} = a^{4-5} = a^{-1}$)
4. **Say**: Today we are going to identify that a number with a negative index can be rewritten as a fraction. We will apply the laws to positive and negative powers.

Introduction to the New Material (*10 minutes*)

1. **Ask**: Who can explain to the class how to expand $\frac{a^4}{a^5}$? Raise your hand.
2. Select a pupil who has raised their hand to show the expansion on the board. Ask the others to solve the question in their exercise books.
3. Correct any errors in the calculation on the board. Ask pupils to check their work.
   (Answer: $\frac{a^4}{a^5} = \frac{a \times a \times a \times a}{a \times a \times a \times a \times a} = \frac{1}{a}$)
4. **Ask**: Can anyone tell the class what they notice about the 2 answers? Raise your hand.
5. Select a pupil who has raised their hand to make the connection between the 2 answers.
   (Answer: They are the same, $a^{-1} = \frac{1}{a}$).
6. **Say**: Answer the question on the board: $\frac{a^4}{a^6}$ .You have 1 minute to think and write down your ideas.
7. After 1 minute, ask them to pair up with their neighbour and discuss how to solve the problem for another minute.
8. **Ask**: Who would like to share their ideas with the class? Raise your hand.
9. Select a pupil who has raised their hand to show the solution for part a. on the board. Select another pupil to show part b.
   (Answer: i. $\frac{a^4}{a^6} = a^{4-6} = a^{-2}$; ii. $\frac{a^4}{a^6} = \frac{a \times a \times a \times a}{a \times a \times a \times a \times a \times a} = \frac{1}{a \times a} = \frac{1}{a^2}$).
10. Correct any errors in the calculation on the board. Ask pupils to check their work.
11. **Write** on the board:

$$a^{-2} = \frac{1}{a^2} = \frac{1}{a \times a}$$

$$a^{-3} = \frac{1}{a^3} = \frac{1}{a \times a \times a}$$

12. **Say**: In general: $a^{-n} = \frac{1}{a^n}$, where n is a positive integer.
13. Ask the pupils to copy the information on the board. Remind them to take notes throughout the lesson.

14. **Say:** All the laws of indices apply to negative indices as well.
15. Show on the board how the expression $4^2 \times 4^{-3}$ is simplified.

$$4^2 \times 4^{-3} = 4^{2+(-3)} = 4^{-1} = \frac{1}{4}$$

16. Have a pupil volunteer to come to the board to simplify $\frac{5^7}{5^9}$. Ask other pupils to solve it in their exercise books.
17. Correct any errors in the calculation on the board. Ask pupils to check their work.

$$\begin{aligned} \frac{5^7}{5^9} &= 5^{7-9} \\ &= 5^{7-9} \\ &= 5^{-2} \\ &= \frac{1}{5^2} = \frac{1}{5 \times 5} = \frac{1}{25} \end{aligned}$$

18. **Write**: $(2^3)^{-2}$
19. Ask pupils to work in pairs to simplify $(2^3)^{-2}$
20. Have a pupil volunteer to explain how to simplify the expression on the board. Ask other pupils to solve it in their exercise books.
21. Correct any errors in the calculation on the board. Ask pupils to check their work.

$$\begin{aligned} (2^3)^{-2} &= 2^{-6} \\ &= \frac{1}{2^6} \\ &= \frac{1}{2 \times 2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{64} \end{aligned}$$

22. Explain to pupils that it is not usually necessary to find the value of the expression. It can be left in index notation.
23. **Say:** We can also use negative indices with variables.

Guided Practice (*10 minutes*)

1. Ask the pupils to continue to work in pairs.
2. **Say:** Apply the laws of indices so the expressions do not contain any negative indices.
3. Point to the questions on the board:

Please give the answers to the following expressions in index notation:

g. $(c^{-3})^2$ h. $q^3 \div q^5$
i. $z^3 \times z^{-4}$ j. $(a \times b)^{-2}$
k. $\left(\frac{1}{w}\right)^{-3}$ l. $\left(\frac{u^2}{v}\right)^{-3}$

4. Walk around, if possible, to check the answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work. Pay particular attention to questions e. and f.

(Answers: a. $(c^{-3})^2 = c^{-3\times2} = c^{-6} = \left(\frac{1}{c^6}\right)$; b. $q^3 \div q^5 = q^{3-5} = q^{-2} = \left(\frac{1}{q^2}\right)$;
c. $z^3 \times z^{-4} = z^{3+(-4)} = z^{-1} = \left(\frac{1}{z}\right)$; d. $(a \times b)^{-2} = a^{-2} \times b^{-2} = a^{-2}b^{-2} = \left(\frac{1}{a^2b^2}\right)$;
e. $\left(\frac{1}{w}\right)^{-3} = \frac{1}{w^{-3}} = w^3$; f. $\left(\frac{u^2}{v}\right)^{-3} = \frac{u^{2\times(-3)}}{v^{-3}} = \frac{u^{-6}}{v^{-3}} = \frac{u^3}{v^6}$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:
   Apply the laws of indices so the expressions do not contain any negative indices:

   g. $\frac{p^2q^4}{p^3q^2}$ h. $x^{-2}y^2 \times x^2y^{-2}$
   i. $(p \times q \times r)^{-3}$ j. $(5r^3s^2)^{-2}$
   k. $\left(\frac{u^2}{v^2}\right)^{-4}$
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. $\frac{p^2q^4}{p^3q^2} = p^{2-3}q^{4-2} = p^{-1}q^2 = \frac{q^2}{p}$; b. $x^{-2}y^2 \times x^2y^{-2} = x^{-2+2}y^{2+(-2)} = x^0y^0 = 1$
   c. $(p \times q \times r)^{-3} = (pqr)^{-3} = \frac{1}{(pqr)^3} = \frac{1}{p^3q^3r^3}$;
   d. $(5r^3s^2)^{-2} = \frac{1}{(5r^3s^2)^2} = \frac{1}{25r^{3\times2}s^{2\times2}} = \frac{1}{25r^6s^4}$; e. $\left(\frac{u^2}{v}\right)^{-4} = \frac{u^{-8}}{v^{-4}} = \frac{v^4}{u^8}$

Closing (*2 minutes*)

1. **Ask:** Who can tell the class what they learned today? Raise your hand.
2. Select a pupil who raised their hand to answer. (Example answer: Learnt about negative indices; how to apply the laws of indices to negative powers)
3. **Say:** Index notation is very useful when simplifying expressions and equations in algebra. We will look at fractional indices in the next lesson.

Lesson Title: Indices with Fractional Powers	**Theme**: Numbers and Numeration	
Lesson Number: M-09-034	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify that a number with a fractional power can be rewritten as a root $\left(a^{\frac{1}{n}} = \sqrt[n]{a}\right)$. 2. Simplify simple indices with fractional powers.	None	1. Write in the centre of the board: Simplify with no fractional powers: $\left(\frac{xyz^2}{x^2y^3z}\right)^{-2}$ 2. Draw this table on one side of the board: $a^{\frac{1}{2}} = \sqrt{a}$ $\quad 9^{\frac{1}{2}} = 3$ $a^{\frac{1}{3}} = \sqrt[3]{a}$ $\quad 8^{\frac{1}{3}} = 2$ $a^{\frac{1}{4}} = \sqrt[4]{a}$ $\quad 256^{\frac{1}{4}} = 4$ $a^{\frac{1}{n}} = \sqrt[n]{a}$ 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Simplify the expression on the board: $\left(\frac{xyz^2}{x^2y^3z}\right)^{-2}$
2. Allow 2 minutes or until most of the pupils have raised their hands.
3. Have a pupil from the back of the classroom give their answer. (Answer: $\frac{x^2y^4}{z^2}$)
4. **Say**: Today we are going to identify that a number with a fractional power can be rewritten as a root. We will also solve simple indices with fractional powers.

Introduction to the New Material (*10 minutes*)

1. **Say**: We sometimes have indices that are fractions. They are used to represent square roots, cube roots and other roots of numbers.
2. Point to the table written on the board.
3. **Say**: Take a look at how square roots, cube roots and other roots are represented with indices.
4. Point to each line in turn.
5. **Say:** a raised to the power of half is the square root of a.
6. **Say:** a raised to the power of one-third is the cube root of a.
7. **Say**: The general rule is a raised to the power of one over n and it is the n^{th} root of a.
8. Ask the pupils to copy the information on the board.
9. **Say**: Let us look at one more example of a square and a cube root.
10. **Write** on the board:

$$25^{-\frac{1}{2}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$$
$$27^{\frac{1}{3}} = \sqrt[3]{27} =$$

11. **Ask**: What is the cube root of 27?
12. Allow a few moments for pupils to think.

13. Have a pupil from the left-hand side of the classroom volunteer to answer. (Answer: 3)
14. **Say**: Let us simplify some simple numbers with fractional indices.
15. Show on the board how to simplify $(4^2 \times 4^4)^{\frac{1}{2}}$

$$\begin{aligned}(4^2 \times 4^4)^{\frac{1}{2}} &= (4^{2+4})^{\frac{1}{2}} \\ &= (4^6)^{\frac{1}{2}} \\ &= 4^{6\times\frac{1}{2}} \\ &= 4^3\end{aligned}$$

16. **Say**: This is exactly the same method we used for positive and negative indices. We just need to apply the law and do the calculation.
17. **Write** on the board: Simplify: $\left(\frac{3^4}{3^2}\right)^{\frac{1}{2}}$
18. Allow pupils to think and write down ideas for 1 minute.
19. After 1 minute, ask them to pair up with their neighbour and discuss how to solve the problem for another minute.
20. **Ask**: Who would like to share their ideas with the class?
21. Select a pupil who has raised their hand to explain on the board how to solve the problem.
22. Correct any errors in the calculation on the board. Ask pupils to check their work.

(Answer: $\left(\frac{3^4}{3^2}\right)^{\frac{1}{2}}$ = $(3^{4-2})^{\frac{1}{2}}$ = $(3^2)^{\frac{1}{2}}$ = $3^{2\times\frac{1}{2}}$ = 3^1 = 3)

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

   Calculate the following roots:

   m. $49^{\frac{1}{2}}$ n. $49^{-\frac{1}{2}}$
   o. $64^{\frac{1}{3}}$ p. $32^{-\frac{1}{5}}$
   q. $(5^3 \times 5^5)^{\frac{1}{2}}$ r. $\left(\frac{9\times 27}{3}\right)^{\frac{1}{4}}$
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions on the board.
5. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: a. $49^{\frac{1}{2}} = \sqrt{49} = 7$; b. $49^{-\frac{1}{2}} = \frac{1}{\sqrt{49}} = \frac{1}{7}$; c. $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$;
   d. $32^{-\frac{1}{5}} = \frac{1}{\sqrt[5]{32}} = \frac{1}{2}$; e. $(5^3 \times 5^5)^{\frac{1}{2}} = (5^{3+5})^{\frac{1}{2}} = (5^8)^{\frac{1}{2}} = 5^{8\times\frac{1}{2}} = 5^4$;
   f. $\left(\frac{9\times 27}{3}\right)^{\frac{1}{4}}$ = $(3 \times 27)^{\frac{1}{4}} = (3 \times 3^3)^{\frac{1}{4}} = (3^{1+3})^{\frac{1}{4}} = (3^4)^{\frac{1}{4}} = 3^{4\times\frac{1}{4}} = 3$).

Independent Practice (*10 minutes*)

1. Ask the pupils to continue to work in pairs to answer the questions.
2. Tell each pupil to take one question to answer. After they have solved their question, pupil 1 should explain how they worked out the answer to their question to pupil 2. Pupil 2 will check

that their method and answer are correct. Then pupil 2 will share the solution for their question and pupil 1 will check the method and answer.

3. Point to the questions on the board:

   Please calculate the following roots:

   a. $4^{-\frac{1}{2}} + 4^{\frac{1}{2}}$ b. $8^{\frac{1}{3}} - 8^{-\frac{1}{3}}$

4. Walk around, if possible, to check the answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. $4^{-\frac{1}{2}} + 4^{\frac{1}{2}} = \frac{1}{\sqrt{4}} + \sqrt{4} = \frac{1}{2} + 2 = 2\frac{1}{2}$

   b. $8^{\frac{1}{3}} - 8^{-\frac{1}{3}} = \sqrt[3]{8} - \frac{1}{\sqrt[3]{8}} = 2 - \frac{1}{2} = 1\frac{1}{2}$

Closing (*2 minutes*)

1. **Say**: Please write your name on a piece of paper. Solve the following problem on your paper.
2. **Write** on the board: Simplify $(3^4 \times 3^6)^{\frac{1}{2}}$
3. Collect the papers at the end of the lesson.
4. Check the pupils' work to get an idea of their level of understanding. (Answer: $(3^4 \times 3^6)^{\frac{1}{2}} = (3^{4+6})^{\frac{1}{2}} = (3^{10})^{\frac{1}{2}} = 3^{10\times\frac{1}{2}} = 3^5$)
5. Address any issues in the next lesson when pupils will be practicing more problems on fractional indices.

Lesson Title: Multiplying and Dividing Indices with Fractional Powers	**Theme**: Numbers and Numeration	
Lesson Number: M-09-035	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes By the end of the lesson, pupils will be able to apply the laws for multiplying and dividing indices to those with positive and negative fractional powers.	**Teaching Aids** None	**Preparation** 1. Write in the centre of the board: Simplify with no fractional powers: $\left(\frac{2^6}{2^2}\right)^{\frac{1}{2}}$ 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: In the last lesson, we were looking at expressions which have fractional powers.
2. **Say**: Please simplify the expression on the board: $\left(\frac{2^6}{2^2}\right)^{\frac{1}{2}}$
3. After 1 minute ask pupils to raise their hand to share the answer. Select a pupil from the back of the classroom to give the answer to the problem.

   (Answer: $\left(\frac{2^6}{2^2}\right)^{\frac{1}{2}} = (2^{6-2})^{\frac{1}{2}} = (2^4)^{\frac{1}{2}} = 2^{4\times\frac{1}{2}} = 2^2$)
4. **Say**: Today we are going to apply the laws for multiplying and dividing indices to those with positive and negative fractional powers.

Introduction to the New Material (*10 minutes*)

1. Remind the pupils to copy the information on the board throughout the lesson.
2. **Write** on the board: Simplify: $(x^3 \times x^6)^{\frac{1}{3}}$
3. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$\begin{aligned}(x^3 \times x^6)^{\frac{1}{3}} &= (x^{3+6})^{\frac{1}{3}} \\ &= x^{9\times\frac{1}{3}} \\ &= x^3\end{aligned}$$

5. **Write** on the board: Simplify: $\left(\frac{p^8}{p^4}\right)^{\frac{1}{2}}$
6. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
7. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$\left(\frac{p^8}{p^4}\right)^{\frac{1}{2}} = (p^{8-4})^{\frac{1}{2}}$$

$$= (p^4)^{\frac{1}{2}}$$

$$= p^{4\times\frac{1}{2}}$$

$$= p^2$$

8. **Write** on the board: Simplify: $(u^{10})^{-\frac{1}{2}}$
9. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
10. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$(u^{10})^{-\frac{1}{2}} = u^{10\times(-\frac{1}{2})}$$

$$= u^{-5}$$

$$(u^{10})^{-\frac{1}{2}} = \frac{1}{u^5}$$

11. **Write** on the board: Simplify: $\left(\frac{c}{d}\right)^{-\frac{1}{3}}$
12. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
13. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$\left(\frac{c}{d}\right)^{-\frac{1}{3}} = \left(\frac{d}{c}\right)^{\frac{1}{3}}$$

$$= \frac{d^{\frac{1}{3}}}{c^{\frac{1}{3}}}$$

14. **Write** on the board: Simplify: $(a \times b)^{\frac{1}{4}}$
15. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
16. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$(a \times b)^{\frac{1}{4}} = a^{\frac{1}{4}} \times b^{\frac{1}{4}}$$

$$= a^{\frac{1}{4}}b^{\frac{1}{4}}$$

17. **Say**: We can apply the laws to even more complex expressions.
18. **Write** on the board: Simplify: $\left(\frac{a^2b^4}{c^6}\right)^{-\frac{1}{2}}$
19. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
20. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$\left(\frac{a^2b^4}{c^6}\right)^{-\frac{1}{2}} = \left(\frac{c^6}{a^2b^4}\right)^{\frac{1}{2}}$$

$$= \frac{c^{6\times\frac{1}{2}}}{a^{2\times\frac{1}{2}}b^{4\times\frac{1}{2}}}$$

$$= \frac{c^3}{ab^2}$$

Guided Practice (*10 minutes*)

1. Ask the pupils to work in pairs.
2. Point to the questions on the board:

   Please simplify the following expressions:

   s. $(x^5 \times x^7)^{\frac{1}{3}}$ t. $\frac{a}{a^{\frac{1}{2}}}$

   u. $\frac{a^{\frac{1}{2}}}{a}$ v. $(t^9)^{-\frac{1}{3}}$

   w. $(ab)^{-\frac{1}{4}}$ x. $\left(\frac{u}{v}\right)^{-\frac{1}{4}}$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. $(x^5 \times x^7)^{\frac{1}{3}} = (x^{5+7})^{\frac{1}{3}} = (x^{12})^{\frac{1}{3}} = x^{12\times\frac{1}{3}} = x^4$;
   b. $\frac{a}{a^{\frac{1}{2}}} = a^{1-\frac{1}{2}} = a^{\frac{1}{2}}$; c. $\frac{a^{\frac{1}{2}}}{a} = a^{\frac{1}{2}-1} = a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}}$; d. $(t^9)^{-\frac{1}{3}} = t^{9\times(-\frac{1}{3})} = t^{-3} = \frac{1}{t^3}$;
   e. $(ab)^{-\frac{1}{4}} = \frac{1}{(ab)^{\frac{1}{4}}} = \frac{1}{a^{\frac{1}{4}}b^{\frac{1}{4}}}$; f. $\left(\frac{u}{v}\right)^{-\frac{1}{4}} = \left(\frac{v}{u}\right)^{\frac{1}{4}} = \frac{v^{\frac{1}{4}}}{u^{\frac{1}{4}}}$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

   Simplify the following expressions:

   a. $\frac{p^{\frac{3}{2}}}{p}$ b. $\frac{p}{p^{\frac{3}{2}}}$

   c. $(t^{12})^{-\frac{1}{4}}$ d. $\frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}}$

   e. $\left(\frac{1}{y}\right)^{-\frac{1}{5}}$ f. $\left(\frac{a^6b^3}{c^9}\right)^{-\frac{1}{3}}$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. $\frac{p^{\frac{3}{2}}}{p} = p^{\frac{3}{2}-1} = p^{\frac{1}{2}}$; b. $\frac{p}{p^{\frac{3}{2}}} = p^{1-\frac{3}{2}} = p^{-\frac{1}{2}} = \frac{1}{p^{\frac{1}{2}}}$;

c. $(t^{12})^{-\frac{1}{4}} = t^{12\times\left(-\frac{1}{4}\right)} = t^{-3} = \frac{1}{t^3}$; d. $\frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{\frac{1}{3}-\frac{1}{2}} = x^{-\frac{1}{6}} = \frac{1}{x^{\frac{1}{6}}}$; e. $\left(\frac{1}{y}\right)^{-\frac{1}{5}} = y^{\frac{1}{5}}$;

f. $\left(\frac{a^6b^3}{c^9}\right)^{-\frac{1}{3}} = \left(\frac{c^9}{a^6b^3}\right)^{\frac{1}{3}} = \frac{c^{9\times\frac{1}{3}}}{a^{6\times\frac{1}{3}}b^{3\times\frac{1}{3}}} = \frac{c^3}{a^2b}$)

Closing (*2 minutes*)

1. **Say:** Discuss in your pairs some of the things you learned about index notation.
2. Allow one minute for discussion.
3. **Say:** Raise your hand to tell the class one thing you learned.
4. Select as many pupils in one minute to say what they learned about index notation.
5. Tell pupils if they did not get a chance to speak now, they will be asked to speak on another topic in the future.
   (Example answers: Learned about index and base; how to expand numbers written in powers; laws of indices; applying laws to positive, negative, fractional powers)

Lesson Title: Multiplying and Dividing by Powers of 10	**Theme**: Numbers and Numeration	
Lesson Number: M-09-036	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to multiply and divide whole numbers and decimals by powers of 10.	None	1. Write this set on the board: { 1, 10, 100, 1000, . . . } 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please raise your hand if you can tell the next number in the set on the board.
2. Select a pupil who has raised their hand to answer. (Answer: 10000)
3. **Say**: Today we are going to multiply and divide whole numbers and decimals by powers of 10.

Introduction to the New Material (*10 minutes*)

1. **Say**: Think back to our lessons on index notation. Who can write 1 as a power of 10? Raise your hand to answer. (Answer: 10^0)
2. **Say**: Write a new set showing all the numbers as powers of 10.
3. Have a pupil volunteer to come to the board and write the set in powers of 10. (Answer: { 10^0, 10^1, 10^2, 10^3, . . . })
4. **Ask:** What do you notice about the indices in the powers of 10? Raise your hand to answer. (Answer: The number of zeros in the number and the index are the same.)
5. **Ask**: How would you multiply and divide by powers of 10. Raise your hand to answer. (Answer: To multiply, move the decimal point to the right by the index (or power) of 10. To divide, move the decimal point to the left by the power of 10.)
6. Explain to the pupils that if the number is a whole number, then a decimal point is placed at the end before moving the correct number of places.
7. Emphasise the relationship between the power of 10 and the number of places to move the decimal. For example, when the power of 10 is 2 for 100, the decimal point is moved 2 places.
8. **Write** on the board: 58×100.
9. **Ask**: Where is the decimal point in 58? Raise your hand. (Answer: At the end of the number, after the 8.)
10. **Ask**: What is the power of 10 in 100? Raise your hand to answer. (Answer: 2)
11. **Ask**: How many places should the decimal point be moved? Raise your hand to answer. (Answer: 2)
12. **Say:** To multiply by 100, move the decimal point 2 places to the right. To do this we will add 2 zeros to the number.
13. Show the following explanation on the board:

$$\begin{aligned} 58 \times 100 &= 58 \times 10^2 \\ &= 58.00 \times 10^2 \\ &= 5800 \end{aligned}$$

14. **Write** on the board:

a. 457×100 b. 3.42×10 c. $978 \div 10$ d. $6.7 \div 1000$

15. Have pupils from around the classroom volunteer to explain their answers to the questions.
16. Ask each pupil to give the power of 10 (underlined) and therefore how many places to move the decimal point. (Answer: a. $\underline{2}$, 45700; b. $\underline{1}$ 34.2; c. $\underline{1}$, 97.8; d. $\underline{3}$, 6700).
17. **Say:** We are now going to start thinking of numbers in a slightly different way to what we have been used to. We often have to multiply and divide with numbers such as 20, 30, 300 and other multiples of powers of 10. It is very useful to know a different way to do this sort of calculation. This helps when we have to calculate with very large and very small numbers.
18. **Write** on the board: a. 1.32×20 b. $24.6 \div 20$
19. Explain each calculation as shown below.

a.
$$\begin{aligned} 1.32 \times 20 &= 1.32 \times 2 \times 10 \\ &= 2.64 \times 10 \\ &= 26.4 \end{aligned}$$

b.
$$\begin{aligned} 24.6 \div 20 &= \frac{24.6}{20} \\ &= \frac{24.6}{2 \times 10} \\ &= \frac{12.3}{10} \\ & \quad 1.23 \end{aligned}$$

20. **Say**: We will start learning how to calculate with very large and very small numbers in the next lesson.
21. For today, we will practise calculating with numbers less than 1000.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

Give the power of 10 for each of the calculations below.
Calculate:

a. 7.12×10	b. 2.8×100	c. $23.2 \div 100$
d. $84.5 \div 10$	e. $3.4 \div 1000$	f. $36 \div 100$
g. 2.9×20	h. 52×200	i. $3.63 \div 30$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: The powers of 10 are underlined a. $\underline{1}$, 71.2; b. $\underline{2}$, 280; c. $\underline{2}$, 0.232; d. $\underline{1}$, 8.45; e. $\underline{3}$, 0.0034; f. $\underline{2}$, 0.36; g. $\underline{1}$ 58; h. $\underline{2}$, 10,400; i. $\underline{1}$, 0.12)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Give the power of 10 for each of the calculations below.
Calculate:

a. 9.34×10	b. 289×1000	c. $0.042 \div 10$
d. 1.5×9000	e. $45.3 \div 30$	f. $42 \div 3000$
g. 25×700	h. $615 \div 500$	i. 4×6

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: The powers of 10 are underlined. a. <u>1</u>, 93.4; b. <u>3</u>, 289,000; c.<u>1</u>, 0.0042; d. <u>3</u>, 13,500; e. <u>1</u>, 1.51; f. <u>3</u>, 0.014; g. 17,500; h. <u>2</u>, 1.23; i. <u>0</u>, 24)

Closing (*2 minutes*)

1. **Ask:** Why is it useful to learn how to multiply and divide with powers of 10? Raise your hand. (Answer: To help with calculating very large and very small numbers).
2. **Say:** Tomorrow we will learn about 'standard form' which is a way of writing very large numbers using powers of 10.

Lesson Title: Standard Form of Large Numbers	**Theme**: Numbers and Numeration	
Lesson Number: M-09-037	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to interpret and write large numbers in standard form (scientific notation): $a \times 10^{n}$ where $1 \leq a < 10$ and n is an integer.	None	1. Write on the board: Write the power of 10 and calculate: a. 5×1000 b. 5.25×100 c. 4×10000 d. 5.37×10 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please look at the questions on the board. You have 1 minute to write the answers:

   Write the power of 10 and calculate:

   a. 5×1000 b. 5.25×100
   c. 4×10000 d. 5.37×10

2. Have pupils from around the classroom volunteer to give their answers. (Answers: The powers of 10 are underlined. a. 3, 5000; b. 2, 525; c. 4, 40000; d. 1, 53.7)
3. **Say**: Today we are going to interpret and write large numbers in standard form, also called scientific notation.

Introduction to the New Material (*10 minutes*)

1. **Say**: Think back to our lessons on index notation. Write the questions on the board as powers of 10. For example, 5×1000 will become 5×10^{3}
2. Allow 1 minute for the pupils to answer.
3. Have pupils from around the classroom volunteer to answer.
   (Answers: ii. 5.25×10^{2}; iii. 4×10^{4}; iv. 5.37×10^{1})
4. **Write** on the board:

$$5 \times 10^{3}$$

$$5.25 \times 10^{2}$$

$$4 \times 10^{4}$$

$$5.37 \times 10^{1}$$

5. **Say**: There are 2 main points we need to recognise in the way we write the numbers. The first one is that the numbers we are multiplying or dividing are all between 1 and 10. Let us check if that is true.
6. Guide a pupil to read the numbers. (Answer: 5, 5.25, 4, 5.37)
7. **Ask:** Is there any number which does not follow our rule? Raise your hand if you say 'yes'. (Answer: No)
8. **Say:** The second thing to recognise is that all the powers of 10 are integers. Let us check if that is true.
9. Guide another pupil to read the powers. (Answer: 3, 2, 4, 1)

10. **Ask:** Is there any power which does not follow our rule? Raise your hand if you say ‘yes’ (Answer: No)
11. **Say**: This method of writing a number is called ‘standard form.’
12. **Write** on the board:

$$\underline{a \times 10^{n}} \qquad \text{where } 1 \leq a < 10 \text{ and n is an integer}$$

13. **Say:** We read this as "a times 10 to the power n" where ‘a’ is greater than or equal to 1 and less than 10; ‘a’ is also an integer’. This simply means that ‘a’ can be a whole number or a decimal number, but ‘n’ has to be an integer; no fractions or decimals. If ‘a’ is a fraction, we will change it to a decimal.
14. **Say:** We write numbers in standard form when we want a short way of writing very large or very small numbers. For example, we can write Le 25, 000 as Le 2.5×10^{4}. This is very useful, especially for very large sums of money.
15. **Say:** Standard form is also called scientific notation. Science deals with very large numbers such as the distance between the earth and the sun. Science also deals with very small numbers such as the distance between atoms in a molecule.
16. **Say:** Today, we are looking at very large numbers. We call any number greater than 0 a large number and we can write it in standard form. We will look at very small numbers in the next lesson. These will be numbers smaller than zero.
17. Show on the board how to change another number to standard form.
18. **Say**: Let us change 45,100 to standard form.

$$\begin{aligned} 45100 &= 4.51 \times 10000 \\ &= 4.51 \times 10^{4} \end{aligned}$$

19. **Ask**: Is ‘a’ equal to or greater than 1, and also less than 10? Raise your hand. (Answer: Yes, it is 4.51.)
20. **Ask**: Is the power of 10 an integer? Raise your hand. (Answer: Yes, it is 4.)
21. **Say**: Then we have written the number in standard form.
22. **Ask**: If we change back from standard form to the ordinary number, do we get the number we started with? Raise your hand. (Answer: Yes, then we have converted the number correctly to standard form.)
23. **Say:** If we have a number in standard form, we already know how to write it as an ordinary number.
24. Show the following explanation on the board.

$$\begin{aligned} 4.51 \times 10^{4} &= 4.51 \times 10000 \\ &= 45100 \end{aligned}$$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

   Mark with a ✓ if the number is in standard form.

   Mark with a × if the number is not in standard form. Write the number in standard form.

   a. 31.7×10^{2} b. 2.834×10^{4}
   c. 1875 d. 4×10
   e. 243×10^{2} f. 987

Please change these numbers from standard form to ordinary numbers.

g. 1.25×10^{6} h. 1.89×10^{4}

i. 8.5×10^{7} j. 3.927×10^{2}

3. Walk around, if possible, to check the answers and clear up any misconceptions.
   For example, some pupils may think that Question d. is not in standard form. Ask them the questions we used to check above.
   - ✓ **Ask**: Is 4 equal to or greater than 1, and also less than 10. (Answer: Yes)
   - ✓ **Ask**: Is the power of 10 an integer? (Answer: Yes, the integer is 1)
   - ✓ Then the number is in standard form.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. ✗, 3.17×10^{3}; b. ✓; c. ✗, 1.875×10^{3}; d ✓ ; e. ✗, 2.43×10^{4}; f. ✗, 9.87×10^{2}; g. 1,250,000; h. 18,900; i. 85,000,000; j. 392.7)

Independent Practice (*10 minutes)*

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:
   Change these numbers to standard form.

   a. 872.7×10^{2} b. 2846×10^{3}
   c. 318,972 d. 4387×10
   e. 27×10^{4} f. 2

   Change these numbers from standard form to ordinary numbers.

   g. 7.25×10^{6} h. 9×10^{4}
   i. 2.173×10^{7} j. 3.0001×10^{2}
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. 8.727×10^{4}; b. 2.846×10^{6}; c. 3.18972×10^{5}; d. 4.387×10^{4}; e. 2.7×10^{5}; f. 2×10^{0} (or 2×1 or 2); g. 7,250,000; h. 90,000; i. 21,730,000; j. 300.01)

Closing (*2 minutes*)

1. **Ask:** What 2 facts show that a number is in standard form? Raise your hand.
2. Have a pupil from the back of the classroom volunteer to answer. (Answer: The number must be greater than or equal to 1 and also less than 10, the power of 10 must be an integer.)
3. **Say**: In the next lesson, we will look at the standard form of small numbers. Great work class!

Lesson Title: Standard Form of Small Numbers	**Theme**: Numbers and Numeration	
Lesson Number: M-09-038	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to interpret and write small numbers in standard form (scientific notation): $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.	None	1. Write on the board: Write the power of 10, then calculate: a. 5 x 100000 b. 5 ÷ 1000 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Ask**: Who can remind the class of what we did in the last lesson? Raise your hand.
2. Select a pupil who has raised their hand to answer. (Example answers: We learned about standard form; how to write numbers with multiples of 10; using index notation to write numbers)
3. **Say**: Calculate the expression on the board for Question a. 5×100000. Please raise your hand when you finish.
4. Select a pupil who has raised their hand to answer. (Answer: $5 \times 100000 = 5 \times 10^5$)
5. **Ask**: What do we call numbers we write in this way? Raise your hand. (Answer: Standard form)
6. **Say**: Today we are going to interpret and write small numbers in standard form.

Introduction to the New Material (*10 minutes*)

1. **Say**: Calculate Question b. $5 \div 1000$. What is the answer? Raise your hand.
2. Select a pupil who has raised their hand to answer. (Answer: 0.005)
3. **Say:** Write Question b. on the board as a power of 10.
4. Have a pupil from the back of the classroom volunteer to answer. (Answer: $5. \div 10^3$)
5. **Write** on the board:

$$5 \div 1000 = 5 \div 10^3$$

Say: We write this as:

$$= \frac{5}{10^3}$$

$$=$$ ← complete later

6. **Say**: We want to write this so we do not have a fraction in our answer. Think back to writing numbers using index notation.
7. **Ask:** What should we do?
8. Allow the pupils a few moments to think (less than a minute).
9. Ask pupils to pair up with their neighbour. Allow a few more moments for them to discuss how to write the number in index notation.
10. **Ask**: Who would like to share their ideas with the class? Raise your hand.
11. Have 2-3 pupils to tell their ideas to the class.
12. If no one gives the right answer, guide a pupil to say we use a negative index. (Answer: 5×10^{-3})
13. Complete the answer on the board.

14. Do the check for standard form:
    - ✓ **Ask**: Is 5 equal to or greater than 1? Raise your hand. (Answer: Yes)
    - ✓ **Ask**: Is 5 less than 10? Raise your hand. (Answer: Yes)
    - ✓ **Ask**: Is the power of 10 an integer? Raise your hand. (Answer: Yes, it is a negative integer, –3)
    - ✓ Then the number is in standard form.
15. **Say**: We can change the number from standard form back to an ordinary number.
16. **Ask**: Can someone remind the class what we do to the decimal point when we divide by a power of 10? Raise your hand. (Answer: To divide, move the decimal point to the left by the power of 10. We add zeros in front of the number if necessary.)
17. Have a pupil from the front of the classroom volunteer to show this on the board. (Answer: $5 \times 10^{-3} = \frac{5}{10^3} = \frac{5}{1000} = 0.005$. Other correct methods are acceptable.)
18. **Ask**: If we change back from standard form to the ordinary number, do we get the number we started with? Raise your hand. (Answer: Yes, then we have converted the number correctly to standard form.)
19. **Write** on the board: Change 0.000525 to standard form.
20. Work through the question on the board. Have pupils volunteer to answer open-ended questions.

$$0.000525 = \frac{5.25}{10000}$$

21. **Ask:** What do we write this as? $= \frac{5.25}{10^4}$
22. **Ask:** What answer do we get? $= 5.25 \times 10^{-4}$
23. **Say**: In standard form, we write numbers smaller than zero with powers of 10 that are negative integers.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. Point to the questions on the board:

   Mark with a ✓ if the number is in standard form.

   Mark with a × if the number is not in standard form. Write the number in standard form.

   a. 35.8×10^{-2} b. 7.934×10^{-4}
   c. 0.5278 d. 0.07×10
   e. 43.2×10^{-3} f. 0.000975

   Change these numbers from standard form to ordinary numbers.

   g. 1.25×10^{-6} h. 1.89×10^{-4}
   i. 8.5×10^{-7} j. 3.927×10^{-3}
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. ×, 3.58×10^{-1}; b. ✓; c. ×, 5.278×10^{-1}; d. 7×10^{-1}; e. ×, 4.32×10^{-2}; f. ×, 9.75×10^{-4}; g. 0.00000125; h. 0.000189; i. 0.00000085; j. 0.003927)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

    Please change these numbers to standard form:

    a. 87.5×10^{-3} b. 24587×10^{-6}
    c. 0.000008756 d. 0.00000005486
    e. 125×10^{-6} f. 0.02

    Change these numbers from standard form to ordinary numbers:

    g. 7.25×10^{-6} h. 9×10^{-4}
    i. 2.173×10^{-3} j. 3.0001×10^{-5}
3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. 8.75×10^{-2}; b. 2.4587×10^{-2}; c.8.756×10^{-6}; d. 5.486×10^{-8}; e.1.25×10^{-4}; f. 2×10^{-2}; g. 0.00000725; h. 0.0009; i. 0.002173; j. 0.000030001)

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper. Now write your answers to the questions on the board. Your work will be collected at the end of the lesson.
2. **Write** on the board:
    Convert: a. 0.000058 to standard form b. 6.47×10^{6} to an ordinary number
3. Collect the work at the end of the lesson. (Answer: a. 5.8×10^{-5}; b. 6,470,000)
4. Check the work to see how much the pupils have understood so far. Assist pupils in the next lesson when they will be practising converting numbers from standard form to ordinary numbers and vice versa.

Lesson Title: Conversion to and from Standard Form	**Theme**: Numbers and Numeration	
Lesson Number: M-09-039	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to convert from whole numbers and decimals to standard form and vice versa.	None	1. Write on the board: Convert to standard form: a. 5,254 b. 0.00589 c. 0.00004781 d. 0.0006 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please convert the expressions on the board to standard form:
   a. 5,254 b. 0.00589
   c. 0.00004781 d. 0.0006
2. **Say:** Raise your hand when you finish.
3. Allow 2 minutes for pupils to answer the questions.
4. Select pupils who have raised their hands to answer. (Answers: a. 5.254×10^3; b. 5.89×10^{-3};
   c. 4.781×10^{-5}; d. 6×10^{-4})
5. **Say**: Today we are going to convert from whole numbers and decimals to standard form and vice versa.

Introduction to the New Material (*10 minutes*)

1. **Ask:** Do you notice any pattern between the number of zeros and the power of 10 for the numbers smaller than 0? Raise your hand.
2. Select a pupil who raised their hand to answer. (Example answers: The number of zeros is one less than the power of 10; the power of 10 is one more than the number of zeros.)
3. **Say:** You can use that fact to help when converting to and from standard form.
4. Point out to the pupils that we ignore the negative sign when we describe the pattern.
5. **Say:** Any number can be written in standard form.
6. **Ask:** Who can remind the class the main reason we write numbers in standard form? Raise your hand.
7. Select a pupil who has raised their hand. (Answer: They are used to write very large and very small numbers in Maths, Science and everyday life.)
6. **Say:** We use standard form if the number is smaller than 0.001 or if it is equal to or greater than 10,000. Otherwise, we write them as ordinary whole numbers or decimal numbers.
7. **Say**: Let us look at a practical example. The distance of the Earth from the Sun is approximately 149,600,000,000 metres.
8. **Write** the distance on the board: The distance of the Earth from the Sun is approximately 149,600,000,000 metres.
9. Have a pupil volunteer to come to the board to write this distance in standard form. Ask other pupils to work in pairs to write this distance in standard form.
10. Check the answer. (Answer: 1.496×10^{11}m)
11. **Say:** The mass of a particle of dust is 7.53×10^{-10} kg.

12. **Write** the mass on the board and ask the pupils to write it as an ordinary number.
13. Have a pupil volunteer to come to the board to write the number in standard form on the board. Ask other pupils to work in pairs to do the same.
14. Check the answer. (Answer: 0.000000000753 kg)
15. **Say**: Remember when converting from standard form to an ordinary number, the power of 10 gives the number of places to the left or right to move the decimal point.

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:

Change these numbers from standard form to ordinary numbers:

a. 2.5×10^{6} b. 3.98×10^{-4}
c. 5.874×10^{7} d. 9.927×10^{-2}

Convert these from ordinary numbers to standard form:

e. 0.0087 f. 256,599
g. 8,875,235 h. 0.0000001483

i. The distance that light travels in a year is 5,870,000,000,000 metres. Write this in standard form.

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: i. 2,500,000; ii. 0.000398; iii. 58,740,000; iv. 0.09927; v. 8.7×10^{-3}; vi. 2.56599×10^{5}; vii. 8.875235×10^{6}; viii. 1.483×10^{-7}; ix. 5.87×10^{12})

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Convert these numbers from standard form to ordinary numbers:

a. 6.741×10^{9} b. 5.62×10^{-10}

Convert these from ordinary numbers to standard form

c. 0.0000000000254 d. 698,478,000,000,000

e. The distance of the Earth from the moon is 384,400,000 metres. Write this in standard form:

f. Which number below has the same value as 4.8×10^{4}?

4.8^{3} 4.8^{4} $(4.8 \times 10)^{4}$ 0.48×10^{3} 0.48×10^{5}

g. Which numbers below has the same value as 8.7×10^{-3}?

$$8700 \qquad 0.00087 \qquad 87 \times 10^{-4} \qquad -8.7 \times 10^{3} \qquad 8.7 \times 10^{3}$$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $6{,}741{,}000{,}000$; b. 0.000000000562; c. 2.54×10^{-11}; d. 6.98×10^{14}; e. $3.844 \times 10^{8}\ m$; f. 0.48×10^{5}; g. 87×10^{-4})

Closing (*2 minutes*)

1. **Say:** Write one thing or fact you know about converting numbers into standard form in your exercise books. Also write one fact you know about converting from standard form to ordinary number.
2. Have pupils from around the classroom volunteer to share one or both facts with the class. (Example answers: Standard form is used to write very large and very small numbers; standard form is written with a whole number or decimal number equal to or larger than 1, but smaller than 10; the index in a number written in standard form is an integer)

Lesson Title: Multiplying and Dividing Small and Large Numbers	**Theme**: Numbers and Numeration	
Lesson Number: M-09-040	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Do simple multiplication and division problems with whole numbers, decimals and fractions. 2. Give answers to problems in standard form.	None	1. Write on the board: Calculate: a. $10^2 \times 10^3$ b. $\frac{10^5}{10^2}$ c. $(5.1 \times 10^2) \times (7 \times 10^3)$ d. $(9 \times 10^5) \div (3 \times 10^3)$ e. $(3 \times 10^{-4}) \times (8 \times 10^2)$ f. $(5.6 \times 10^6) \div (7 \times 10^2)$ 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

Opening (*3 minutes*)

1. **Say**: Please write down the answers to Questions a. and b. from the board:
   a. $10^2 \times 10^3$ b. $\frac{10^5}{10^2}$
2. Allow 1 minute for pupils to answer the question.
3. Have pupils volunteer to explain their answer on the board. (Answers: a. $10^2 \times 10^3 = 10^{2+3} = 10^5$; b. $\frac{10^5}{10^2} = 10^{5-2} = 10^3$).
4. Correct any errors in the calculation on the board. Ask pupils to check their work.
5. **Say**: We use the laws of indices to calculate with numbers written in standard form. Today we are going to do simple multiplication and division problems with whole numbers, decimals and fractions. We will give our answers in standard form.

Introduction to the New Material (*10 minutes*)

1. **Say:** Write down the answer to Question c: $(5.1 \times 10^2) \times (7 \times 10^3)$. Raise your hand when you finish.
2. Select a pupil who raised their hand to explain their answer step-by-step on the board.
3. Correct any errors in the calculation on the board. Ask pupils to check their work.
   An example calculation is shown below.

$$\begin{aligned} (5.1 \times 10^2) \times (7 \times 10^3) &= 5.1 \times 7 \times 10^2 \times 10^3 && \longleftarrow \text{Collect like terms} \\ &= 5.1 \times 7 \times 10^{2+3} && \longleftarrow \text{Apply 1}^{\text{st}} \text{ law of indices} \\ &= 35.7 \times 10^5 && \longleftarrow \text{Multiply 5.1 and 7, add the indices} \\ &= 3.57 \times 10^6 && \longleftarrow \text{Convert answer to standard form} \end{aligned}$$

4. **Say:** We use the same method for dividing numbers in standard form.
5. Show how to calculate the answer to Question d:

$(9 \times 10^5) \div (3 \times 10^3)$	$= \dfrac{9 \times 10^2}{3 \times 10^3}$	Write as a fraction
	$= \dfrac{9}{3} \times \dfrac{10^5}{10^3}$	Collect like terms
	$\dfrac{9}{3} \times 10^{5-3}$	Apply 2nd law of indices
	$= 3 \times 10^2$	Divide 9 by 3, subtract the indices Answer already in standard form

6. Ask pupils to work in pairs for 3 minutes to discuss and share ideas for Questions e. and f:
   e. $(3 \times 10^{-4}) \times (8 \times 10^{2})$

   f. $(5.6 \times 10^6) \div (7 \times 10^{2})$
7. **Say**: Please raise your hand when you finish.
8. Select a pupil who has raised their hand to explain their answer to Question e. on the board.
9. Correct any errors in the calculation on the board. Ask pupils to check their work.

$(3 \times 10^{-4}) \times (8 \times 10^{2})$	$= 3 \times 8 \times 10^{-4} \times 10^{2}$	Collect like terms
	$= 3 \times 8 \times 10^{-4+2}$	Apply 1st law of indices
	$= 24 \times 10^{-2}$	Multiply 3 and 8, add the indices
	$= 2.4 \times 10^{-1}$	Convert answer to standard form

10. Select another pupil who has raised their hand to explain their answer to Question f. on the board.
11. Correct any errors in the calculation on the board. Ask pupils to check their work.

$(5.6 \times 10^6) \div (7 \times 10^{2})$	$= \dfrac{5.6 \times 10^6}{7 \times 10^2}$	Write as a fraction
	$= \dfrac{5.6}{7} \times \dfrac{10^6}{10^2}$	Collect like terms
	$= \dfrac{5.6}{7} \times 10^{6-2}$	Apply 2nd law of indices
	$= 0.8 \times 10^4$	Divide 5.6 by 7, subtract the indices
	$= 8 \times 10^3$	Convert answer to standard form

12. **Say:** We can skip some of the steps once we are confident in answering questions like these.

Guided Practice (*10 minutes*)

1. Ask pupils to continue to work in pairs.
2. Point to the questions on the board:

Please calculate the following. Give your answers in standard form.

y. $(6 \times 10^{7}) \times (5 \times 10^{6})$
z. $(4 \times 10^{-4}) \times (8.2 \times 10^{2})$
aa. $(3 \times 10^{3}) \times (2.1 \times 10^{-2})$
bb. $(3 \times 10^{4}) \div (6 \times 10^{2})$
cc. $(6.2 \times 10^{5}) \div (2 \times 10^{-8})$
dd. $(3 \times 10^{-4}) \div (4 \times 10^{-4})$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. $(6 \times 10^{7}) \times (5 \times 10^{6}) = 6 \times 5 \times 10^{7+6} = 30 \times 10^{13} = 3 \times 10^{14}$;
b. $(4 \times 10^{-4}) \times (8.2 \times 10^{2}) = 4 \times 8.2 \times 10^{-4+2} = 32.8 \times 10^{-2} = 3.28 \times 10^{-1}$;
c. $(3 \times 10^{3}) \times (2.1 \times 10^{-2}) = 3 \times 2.1 \times 10^{3+(-2)} = 6.3 \times 10^{1}$ or 6.3×10;
d. $(3 \times 10^{4}) \div (6 \times 10^{3}) = 0.5 \times 10^{4-3} = 0.5 \times 10^{1} = 5 \times 10^{0} = 5$;
e. $(6.2 \times 10^{5}) \div (2 \times 10^{-8}) = 12.4 \times 10^{5-(-8)} = 12.4 \times 10^{13} = 1.24 \times 10^{14}$;
f. $(3 \times 10^{-4}) \div (4 \times 10^{-4}) = 0.75 \times 10^{-4-(-4)} = 0.75 \times 10^{0} = 0.75 = 7.5 \times 10^{-1}$).

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer the questions.
2. Point to the questions on the board:

Calculate the following. Give your answers in standard form.

a. $(3 \times 10^{7}) \times (6 \times 10^{-6})$
b. $(3.3 \times 10^{-4}) \times (1.2 \times 10^{-2})$
c. $(4.2 \times 10^{3}) \div (6 \times 10^{-2})$
d. $(1.32 \times 10^{4}) \div (4 \times 10^{-5})$
e. $(6.2 \times 10^{2}) + (2 \times 10^{3})$
f. $(3 \times 10^{6})^2$

3. Walk around, if possible, to check the answers and clear up any misconceptions.
4. Ask pupils to exchange their exercise books and check each other's work.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work.
(Answers: a. $(3 \times 10^{7}) \times (6 \times 10^{-6}) = 3 \times 6 \times 10^{7+(-6)} = 18 \times 10^{-1} = 1.8 \times 10$;
b. $(3.3 \times 10^{-4}) \times (1.2 \times 10^{-2}) = 3.96 \times 10^{-4+(-2)} = 3.96 \times 10^{-6}$;
c. $(4.2 \times 10^{3}) \div (6 \times 10^{-2}) = 0.7 \times 10^{3-(-2)} = 0.7 \times 10^{5} = 7 \times 10^{4}$;
d. $(1.32 \times 10^{4}) \div (4 \times 10^{-5}) = 0.33 \times 10^{4-(-5)} = 0.33 \times 10^{9} = 3.3 \times 10^{8}$;
e. $(6.2 \times 10^{2}) + (2 \times 10^{3}) = 620 + 2000 = 2620 = 2.6 \times 10^{3}$;
f. $(3 \times 10^{6})^2 = (3 \times 10^{6}) \times (3 \times 10^{6}) \times = 3 \times 3 \times 10^{6+6} = 9 \times 10^{12}$)

Closing (*2 minutes*)

1. **Ask:** What do you notice about the way we calculated Question e.? Raise your hand.
2. Allow pupils a few moments to look at the calculation for Question e. (Answer: Numbers were changed to ordinary numbers, the calculation was done then changed back to standard form.)
3. **Ask:** Who can think of a way to calculate $(3 \times 10^{6})^2$ in our heads? Raise your hand. (Answer: Multiply $3 \times 3 = 9$ *and* $6 \times 2 = 12$ to get 9×10^{12}).
4. **Say:** The way we write numbers in standard form makes it easy to do calculations like this in our heads.

Lesson Title: Right-angled Triangles (Revision)	**Theme**: Geometry	
Lesson Number: M-09-041	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to: 1. Identify the parts of a right-angled triangle. 2. Identify the properties of a right-angled triangle.	A poster on large-sized paper, (vanguard if available) of the information in the table titled: Parts and Properties of a Right-angled Triangle. If poster paper is not available, use the board.	1. Make a poster on large-sized paper, (vanguard if available) of the information in the table titled: Parts and Properties of a Right-angled Triangle (see end of lesson plan). If poster paper is not available, use the board. 2. Draw the triangles at the end of this lesson plan on the board. 3. Write on the board: <u>Vocabulary List</u> – hypotenuse, acute, complementary, right-angle 4. Write the questions from the Guided Practice section on the board.

Opening (*3 minutes*)

1. **Ask**: What shapes do we have on the board? (Answer: Triangles)
2. **Say:** Work in pairs. Write down the triangles that have right angles as one set and those that do not have right angles as another set. (Answer: right angles: { a, b, f }; not right angles: { c, d, e })
3. **Say**: Today we are going to identify the parts and properties of a right-angled triangle.

Introduction to the New Material (*10 minutes*)

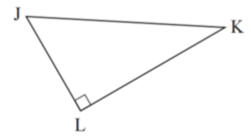
1. **Say:** Please work in pairs. Draw a right-angled triangle. Write down everything you know about the triangle.
2. Allow pupils 3 minutes to write down everything they know.
3. **Ask**: Who can tell the class what is a right-angled triangle? Raise your hand.
4. Select 2-4 pupils to share their ideas of what a right-angled triangle is.
5. **Say:** Very good. A right-angled triangle is a triangle which has got a 90^{o} angle. We call 90^{o} a right angle.
6. **Ask**: Who would like to come to the board and draw a right-angled triangle? Raise your hand.
7. Select a pupil who has raised their hand to draw a right-angled triangle on the board.
8. **Say** (to the pupil at the board): The class is now going to tell you what they know about right-angled triangles. You will show this on your triangle.
9. **Say**: Raise your hand and tell (name of pupil) one thing you know about a right-angled triangle.
10. Select pupils from around the classroom to share their ideas about right-angled triangles.
11. Guide the pupil at the board to write the information down briefly next to the triangle.
12. **Say**: Thank you for all your contributions. Let us put all this information together in a table.
13. Show the poster titled Parts and Properties of a Right-angled Triangle/ point to the table on the board.
14. Go through the details of the parts and properties of a right-angled triangle. Point to each information as you say it.
15. Do not rush through the information. Allow time for the pupils to absorb it. Repeat any information you feel necessary.
16. **Say**: Please copy the table in your exercise books.

Guided Practice (*10 minutes*)

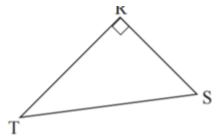
1. Ask pupils to work in pairs to answer the question on the board.
2. Point to the questions on the board:

   Which side is the hypotenuse in each of the following right-angled triangles?

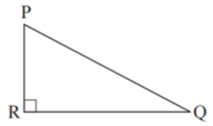
   a.



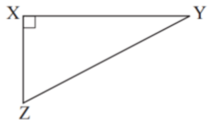
   b.



   c.



   d.



3. Walk around, if possible, to check their answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. JK; b. TS; c. PQ; d. YZ)

Independent Practice (10 minutes)

1. Ask the pupils to work independently to answer the questions.
2. **Write** the following questions on the board:

   Write 2 similarities and 2 differences between a scalene right-angled triangle and an isosceles right-angled triangle.

3. Walk around, if possible, to check their answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give one similarity or difference.
5. Correct any errors:

(Answers:

Scalene and Isosceles Right-angled Triangles

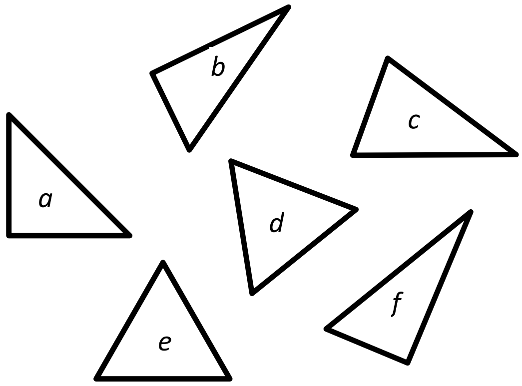
Similarities	Differences
Both have a 90° angle	Scalene has 3 different angles, isosceles has 2 angles the same size (45°)
Both have one long side and 2 shorter sides	Scalene has 3 sides of different lengths; Isosceles has 2 sides of same length
Both have 2 acute angles which are complementary angles)	

Closing (*2 minutes*)

1. **Say:** Write down 2 different things you learned today in your pairs.
2. Allow pupils 1 minute to discuss and share their ideas.
3. Have one pupil from the front, and one from the back of the classroom volunteer to answer. (Example answers: That there are 2 types of right-angled triangles: scalene and isosceles; an

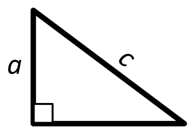
equilateral triangle cannot be a right-angled triangle; the 2 acute angles in a right-angled triangle are complementary angles. Accept all reasonable answers.)

[TRIANGLES FOR OPENING ACTIVITY]

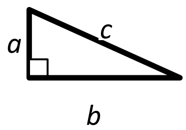
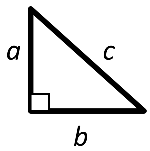


[PARTS AND PROPERTIES OF A RIGHT-ANGLED TRIANGLE]

Parts of a right-angled triangle	

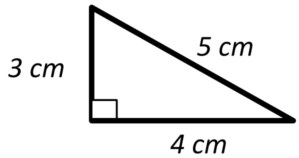
hypotenuse

 	
	• 3 sides
	  ○ longest side, c, is called the hypotenuse
	  ○ 2 shorter sides, a and b
	• 3 angles
	  ○ one angle equal to 90°, marked with a square
	  ○ 2 other angles smaller than 90° (acute angles)

Properties of a Right-Angled Triangle	

scalene right-angled triangle

 	
isosceles right-angled triangle

 	
	• all 3 angles add up to 180°
	• the 2 acute angles are also complementary angles – they add up to 90°
	• the 2 short sides, a and b form the 90° angle
	• the hypotenuse, c, is the side opposite the 90° angle
	• either one of the short sides is the base, the other will be the height
	• there are 2 types of right-angled triangles: scalene and isosceles
	• <u>scalene right-angled triangle</u>
	  ○ all 3 sides are different from each other
	  ○ one 90° angle, 2 other angles different from each other
	• <u>isosceles right-angled triangle</u>
	  ○ have 2 equal sides which form the 90° angle
	  ○ the 2 angles are 45° each
	• a right angle can never be equilateral as the hypotenuse will always be longer than the other 2 sides

Pupils can be asked to create their own versions of this poster as a class activity. The best posters can be put up on the wall of the classroom.

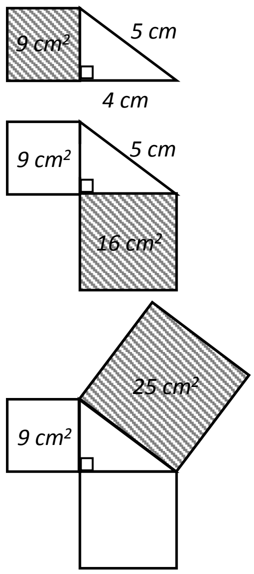
Lesson Title: Introduction to Pythagoras’ Theorem	**Theme**: Geometry	
Lesson Number: M-09-042	**Class/Level**: JSS 3	**Time**: 35 minutes

<table>
<tr>
<td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. State Pythagoras’ theorem.
2. Identify that the formula $a^2 + b^2 = c^2$ can be used to find the sides of a right-angled triangle.</td>
<td>Teaching Aids
1. Table of Squares and Square Roots found at the end of this lesson plan. This will be used for all lessons on Pythagoras’ Theorem.
2. A poster showing the simplified statement of Pythagoras’ Theorem on vanguard paper, if available. Not, use the board (see end of lesson plan)</td>
<td>Preparation
1. Create a poster showing the simplified statement of Pythagoras’ Theorem on vanguard paper, if available. Not, use the board (see end of lesson plan)
2. Draw the right-angled triangle below on the board.


3. Draw the table of Squares and Square Roots found at the end of this lesson plan, on the board (select appropriate sections to write as needed in the lesson).</td>
</tr>
</table>

Opening (3 minutes)

1. **Ask:** What best describes the type of triangle on the board? Raise your hand. (Answer: scalene right-angled triangle)
2. **Say**: Please copy the triangle into your exercise books. Use your ruler to measure the sides accurately. Mark the hypotenuse and the right angle on the triangle.
3. Allow a few moments for pupils to do so.
4. **Say**: Today we are going to state and use Pythagoras’ Theorem to find the sides of a right-angled triangle.



Introduction to the New Material (10 minutes)

1. **Say:** Pythagoras’ Theorem links the length of the hypotenuse of a right-angled triangle to the lengths of the other 2 sides.
2. Ask pupils to verify that the 3 sides of their triangles measure 3, 4 and 5 centimetres each.
   This will be the case depending on how accurately they drew the triangle.
3. **Write** on the board: $3^2 + 4^2 = 5^2$
4. **Say**: We are going to verify that the sides of our triangle are linked by this equation.
5. The following explanation shows Pythagoras’ Theorem on a practical level that will help pupils understand the definition better.
6. Draw the squares and complete the calculations as they give the answers to your questions.

7. Ask pupils to draw a square on the side measuring 3 cm.
8. **Ask:** Can someone tell us what is the area of this square? Raise your hand. (Answer: $3 \times 3 = 9$ cm^2)
9. Ask pupils to draw a square on the side measuring 4 cm. Ask them to calculate its area. (Answer: $4 \times 4 = 16$ cm^2)
10. **Say:** Add the 2 areas, what do we get? (Answer: $9 + 16 = 25$ cm^2)
11. Ask pupils to draw a square on the side measuring 5 cm (hypotenuse). Ask them to calculate its area. (Answer: $5 \times 5 = 25$ cm^2)
12. The full calculation is shown below:

$$\begin{aligned} 3^2 + 4^2 &= 9 + 16 \\ &= 25 \\ &= 5^2 \end{aligned}$$

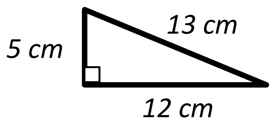
13. **Say**: We have just proved Pythagoras' Theorem.
14. **Say**: The full statement of Pythagoras' Theorem is: It is true for all right-angled triangles. 'Pythagoras' Theorem states that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other 2 sides.'
15. Explain how this statement applies to our triangle.
16. Point to the square on the hypotenuse of our triangle and show how its area is equal to the sum of the areas of the other 2 sides. You may need to repeat this explanation more than once.
17. **Say**: The full statement of Pythagoras' Theorem can be simplified so it is easier to remember.
18. Put up the poster or point to the Pythagoras' Theorem on the board.
19. **Say**: If we know the length of any 2 sides of a right-angled triangle, we can find the length of the missing side using Pythagoras' Theorem.
20. Ask pupils to copy the simplified statement in their exercise books.

Guided Practice (*10 minutes*)

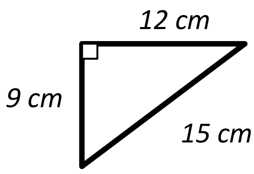
1. Ask pupils to work in pairs.
2. **Write** on the board:

Please verify Pythagoras' Theorem for the triangles below:

ee.



ff.



3. Point to the squares and square roots tables found at the back of this book.
4. Walk around, if possible, to check their answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work.

Answers: $a^2 + b^2 = c^2$

a. $a = 5, b = 12, c = 13$

$$\begin{aligned} a^2 + b^2 &= 5^2 + 12^2 \\ &= 25 + 144 \\ &= 169 \end{aligned}$$

b. $a = 9, b = 12, c = 15$

$$\begin{aligned} a^2 + b^2 &= 9^2 + 12^2 \\ &= 81 + 144 \\ & 225 \end{aligned}$$

$$\begin{aligned} c^2 &= 13^2 \\ &= 169 \\ 5^2 + 12^2 &= 13^2 \\ \text{LHS} &= \text{RHS} \end{aligned}$$

right-angled triangle

$$\begin{aligned} c^2 &= 15^2 \\ &= 225 \\ 9^2 + 12^2 &= 15^2 \\ \text{LHS} &= \text{RHS} \end{aligned}$$

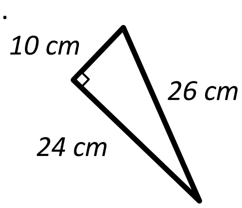
right-angled triangle

Independent Practice (*10 minutes*)

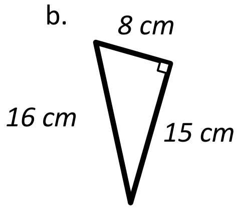
1. Ask pupils to work independently to answer the questions.
2. **Write** the following questions on the board:

   Verify Pythagoras' Theorem for the triangles below:

   a.



   b.



3. Point to the squares and square roots tables on the board.
4. Walk around, if possible, to check their answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers and steps on the board. Ask pupils to check their work.
   Answers:

$$a^2 + b^2 = c^2$$

a. $a = 10, b = 24, c = 26$

$$\begin{aligned} a^2 + b^2 &= 10^2 + 24^2 \\ &= 100 + 576 \\ &= 676 \\ c^2 &= 26^2 \\ &= 676 \\ 10^2 + 24^2 &= 26^2 \\ \text{LHS} &= \text{RHS} \end{aligned}$$

right-angled triangle

b. $a = 8, b = 15, c = 16$

$$\begin{aligned} a^2 + b^2 &= 8^2 + 15^2 \\ &= 64 + 225 \\ & 289 \\ c^2 &= 16^2 \\ &= 256 \\ 8^2 + 15^2 &\neq 16^2 \\ \text{LHS} &\neq \text{RHS} \end{aligned}$$

not a right-angled triangle,
c should be equal to 17

Closing (*2 minutes*)

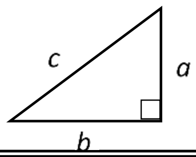
1. **Say:** Please write down one new thing you learned today.
2. Allow pupils 1 minute to discuss and share their ideas.
3. Have one pupil from around the classroom volunteer to answer. (Example answers: How to find the link between the length of the hypotenuse of a right-angled triangle and the lengths of the other 2 sides; how to verify Pythagoras' Theorem)

[SIMPLIFIED STATEMENT OF PYTHAGORAS' THEOREM]

Pythagoras' Theorem states that for any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other 2 sides.

If we use *a, b* and *c* for the sides of the triangle as shown, then:

$$a^2 + b^2 = c^2$$



Lesson Title: Finding the Hypotenuse of a Right-Angled Triangle	**Theme**: Geometry	
Lesson Number: M-09-043	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
1. By the end of the lesson, pupils will be able to find the hypotenuse of a right-angled triangle using Pythagoras' theorem.	Table of Squares and Square Roots found at the end of this lesson plan. This will be used for all lessons on Pythagoras' Theorem.	1. Draw the triangles for this lesson plan on the board (see end of this lesson plan). Note: Do not write the answers until after pupils have answered the questions. 2. Draw the table of Squares and Square Roots found at the end of this lesson plan, on the board (select appropriate sections to write as needed in the lesson). 3. Write the questions at the end of the lesson plan, on the board.

Opening (*3 minutes*)

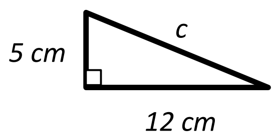
1. **Ask**: Which equation gives Pythagoras' Theorem for the triangles in Question 1?
2. Allow pupils to work in pairs for 2 minutes to answer the question.
3. Have a pupil volunteer to explain their answer on the board.
4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: see end of this lesson plan)
5. **Say**: Today we are going to find the hypotenuse of a right-angled triangle using Pythagoras' theorem.

Introduction to the New Material (*10 minutes*)

1. **Say:** In the last lesson, we used Pythagoras' Theorem to verify if a triangle is a right-angled triangle. Its main use is to find the lengths of the hypotenuse or one of the shorter sides.
2. **Ask:** Who can remind the class of what Pythagoras' Theorem states? Raise your hand. (Answer: $a^2 + b^2 = c^2$)
3. **Say:** Let us use an adjusted version to answer Question 2. We should always draw the triangle in questions such as these so we can be sure we are finding the length of the correct side. Please draw the triangle in your exercise books.
4. Allow time for pupils to draw the triangles in their exercise books.
5. Show the calculation on the board. Explain each line of the calculation.
6. Ask pupils to give answers for the steps in the calculations.
7. **Say**: By Pythagoras' Theorem: $c^2 = a^2 + b^2$
8. **Ask:** What are the values for a and b? (Answer: $a = 6$ cm, $b = 8$ cm) $= 6^2 + 8^2$
9. **Ask:** What is 6^2? (Answer: 36)
10. **Ask:** What is 8^2? (Answer: 64) $= 36 + 64$
11. **Ask**: What do we get when we add 36 to 64? $= 100$
12. **Ask:** How do we find c? (Answer: Take the square root of 100) $c = \sqrt{100}$
13. **Ask**: What is the square root of 100? (Answer: 10) $c = 10$ cm

14. **Say**: Some of you may be thinking that $\sqrt{100} = \pm 10$. That is correct. Since we cannot have a negative measurement for length we only take the positive square root. Please do Question 3 in your exercise books. Check your answer with your neighbour when you are finished.
15. Allow pupils time to do Question 3.
16. **Ask**: Who would like to explain Question 3 on the board? Raise your hand.
17. Select a pupil who has raised their hand to explain their answer step-by-step on the board.
18. Correct any errors in the calculation on the board. Ask pupils to check their work.
    An example calculation is shown below.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 12^2 \\ &= 25 + 144 \\ &= 169 \\ c &= \sqrt{169} \\ c &= 13 \text{ cm} \end{aligned}$$



Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. **Say**: Please calculate the length of the hypotenuse of the triangles for Questions 4 and 5 on the board.
3. Point to the squares and square roots tables on the board.
4. Walk around, if possible, to check their answers and clear up any misconceptions.
5. Have pupils from around the classroom volunteer to give their answers to the questions.
6. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)

Independent Practice (*10 minutes*)

1. Ask the pupils to work independently to answer Questions 6, 7, 8 and 9.
2. Point to the squares and square roots tables on the board.
3. Walk around, if possible, to check their answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to Questions 6 and 9.
5. Do not provide the answers for Questions 7 and 9. Use them to check pupils' understanding of the work.
6. **Write** the correct answers for Questions 6 and 9 on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)

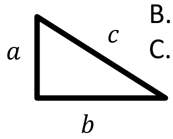
Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper. Now write your working out and answer for Questions 7 and 9 on the paper. Hand in the paper in at the end of the lesson.
2. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be applying Pythagoras' theorem to find the length of the other 2 sides of a right-angled triangle.

[QUESTIONS FOR OPENING ACTIVITY]

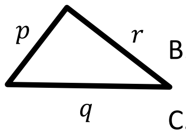
1. Which of these equations show Pythagoras Theorem?

a.



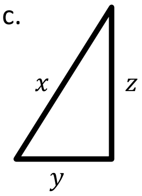
A. $a^2 = b^2 + c^2$
B. $b^2 = a^2 + c^2$
C. $c^2 = b^2 + a^2$

b.



A. $p^2 = q^2 + r^2$
B. $q^2 = p^2 + r^2$
C. $r^2 = p^2 + q^2$

c.



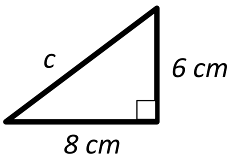
A. $x^2 = y^2 + z^2$
B. $y^2 = x^2 + z^2$
C. $z^2 = x^2 + y^2$

(Answers: a. C; b. B; c. A)

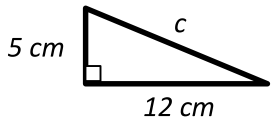
[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Calculate the length of the hypotenuse in the following triangles:

2.



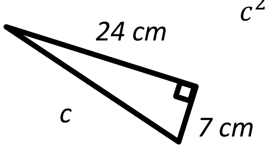
3.



[QUESTIONS FOR GUIDED PRACTICE]

Calculate the length of the hypotenuse in the following triangles:

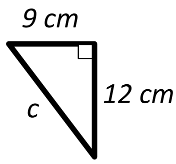
4.



Answer

$$c^2 = a^2 + b^2$$
$$= 7^2 + 24^2$$
$$= 49 + 576$$
$$= 625$$
$$c = \sqrt{625}$$
$$c = 25 \text{ cm}$$

5.



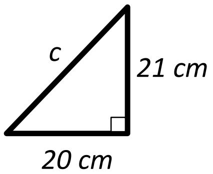
Answer

$$c^2 = a^2 + b^2$$
$$= 9^2 + 12^2$$
$$= 81 + 144$$
$$= 225$$
$$c = \sqrt{225}$$
$$c = 15 \text{ cm}$$

[QUESTIONS FOR INDEPENDENT PRACTICE]

Calculate the length of the hypotenuse in the following triangles. Give answers, if required, to 1 decimal place.

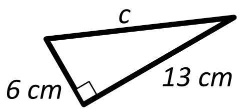
6.



Answer

$$c^2 = a^2 + b^2$$
$$= 20^2 + 21^2$$
$$= 400 + 441$$
$$= 841$$
$$c = \sqrt{841}$$
$$c = 29 \text{ cm}$$

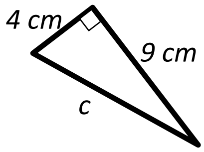
7.



Answer

$$c^2 = a^2 + b^2$$
$$= 6^2 + 13^2$$
$$= 36 + 169$$
$$= 205$$
$$c = \sqrt{205}$$
$$c = 14.3 \text{ cm}$$

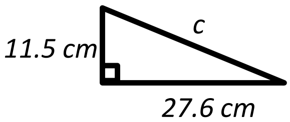
8.



Answer

$$c^2 = a^2 + b^2$$
$$= 4^2 + 9^2$$
$$= 16 + 81$$
$$= 97$$
$$c = 97$$
$$c = 9.8 \text{ cm}$$

9.



Answer

$$c^2 = a^2 + b^2$$
$$= 11.5^2 + 27.6^2$$
$$= 132.3 + 761.8$$
$$= 894.1$$
$$c = \sqrt{894.1}$$
$$c = 29.9 \text{ cm}$$

Squares - Cubes - Square Root (

number n	square n^2	cube n^3	square root $\sqrt{n}$
1	1	1	1.0000
2	4	8	1.4142
3	9	27	1.7321
4	16	64	2.0000
5	25	125	2.2361
6	36	216	2.4495
7	49	343	2.6458
8	64	512	2.8284
9	81	729	3.0000
10	100	1000	3.1623
11	121	1331	3.3166
12	144	1728	3.4641
13	169	2197	3.6056
14	196	2744	3.7417
15	225	3375	3.8730
16	256	4096	4.0000
17	289	4913	4.1231
18	324	5832	4.2426
19	361	6859	4.3589
20	400	8000	4.4721
21	441	9261	4.5826
22	484	10648	4.6904
23	529	12167	4.7958
24	576	13824	4.8990
25	625	15625	5.0000
26	676	17576	5.0990
27	729	19683	5.1962
28	784	21952	5.2915
29	841	24389	5.3852
30	900	27000	5.4772
31	961	29791	5.5678
32	1024	32768	5.6569
33	1089	35937	5.7446
34	1156	39304	5.8310
35	1225	42875	5.9161
36	1296	46656	6.0000
37	1369	50653	6.0828
38	1444	54872	6.1644
39	1521	59319	6.2450
40	1600	64000	6.3246

number n	square n^2	cube n^3	square root $\sqrt{n}$
41	1681	68921	6.4031
42	1764	74088	6.4807
43	1849	79507	6.5574
44	1936	85184	6.6332
45	2025	91125	6.7082
46	2116	97336	6.7823
47	2209	103823	6.8557
48	2304	110592	6.9282
49	2401	117649	7.0000
50	2500	125000	7.0711
51	2601	132651	7.1414
52	2704	140608	7.2111
53	2809	148877	7.2801
54	2916	157464	7.3485
55	3025	166375	7.4162
56	3136	175616	7.4833
57	3249	185193	7.5498
58	3364	195112	7.6158
59	3481	205379	7.6811
60	3600	216000	7.7460
61	3721	226981	7.8102
62	3844	238328	7.8740
63	3969	250047	7.9373
64	4096	262144	8.0000
65	4225	274625	8.0623
66	4356	287496	8.1240
67	4489	300763	8.1854
68	4624	314432	8.2462
69	4761	328509	8.3066
70	4900	343000	8.3666
71	5041	357911	8.4261
72	5184	373248	8.4853
73	5329	389017	8.5440
74	5476	405224	8.6023
75	5625	421875	8.6603
76	5776	438976	8.7178
77	5929	456533	8.7750
78	6084	474552	8.8318
79	6241	493039	8.8882
80	6400	512000	8.9443

Squares - Cubes - Square Root (

number n	square n^2	cube n^3	square root $\sqrt{n}$
81	6561	531441	9.0000
82	6724	551368	9.0554
83	6889	571787	9.1104
84	7056	592704	9.1652
85	7225	614125	9.2195
86	7396	636056	9.2736
87	7569	658503	9.3274
88	7744	681472	9.3808
89	7921	704969	9.4340
90	8100	729000	9.4868
91	8281	753571	9.5394
92	8464	778688	9.5917
93	8649	804357	9.6437
94	8836	830584	9.6954
95	9025	857375	9.7468
96	9216	884736	9.7980
97	9409	912673	9.8489
98	9604	941192	9.8995
99	9801	970299	9.9499
100	10000	1000000	10.0000
101	10201	1030301	10.0499
102	10404	1061208	10.0995
103	10609	1092727	10.1489
104	10816	1124864	10.1980
105	11025	1157625	10.2470
106	11236	1191016	10.2956
107	11449	1225043	10.3441
108	11664	1259712	10.3923
109	11881	1295029	10.4403
110	12100	1331000	10.4881
111	12321	1367631	10.5357
112	12544	1404928	10.5830
113	12769	1442897	10.6301
114	12996	1481544	10.6771
115	13225	1520875	10.7238
116	13456	1560896	10.7703
117	13689	1601613	10.8167
118	13924	1643032	10.8628
119	14161	1685159	10.9087
120	14400	1728000	10.9545

number n	square n^2	cube n^3	square root $\sqrt{n}$
121	14641	1771561	11.0000
122	14884	1815848	11.0454
123	15129	1860867	11.0905
124	15376	1906624	11.1355
125	15625	1953125	11.1803
126	15876	2000376	11.2250
127	16129	2048383	11.2694
128	16384	2097152	11.3137
129	16641	2146689	11.3578
130	16900	2197000	11.4018
131	17161	2248091	11.4455
132	17424	2299968	11.4891
133	17689	2352637	11.5326
134	17956	2406104	11.5758
135	18225	2460375	11.6190
136	18496	2515456	11.6619
137	18769	2571353	11.7047
138	19044	2628072	11.7473
139	19321	2685619	11.7898
140	19600	2744000	11.8322
141	19881	2803221	11.8743
142	20164	2863288	11.9164
143	20449	2924207	11.9583
144	20736	2985984	12.0000
145	21025	3048625	12.0416
146	21316	3112136	12.0830
147	21609	3176523	12.1244
148	21904	3241792	12.1655
149	22201	3307949	12.2066
150	22500	3375000	12.2474
151	22801	3442951	12.2882
152	23104	3511808	12.3288
153	23409	3581577	12.3693
154	23716	3652264	12.4097
155	24025	3723875	12.4499
156	24336	3796416	12.4900
157	24649	3869893	12.5300
158	24964	3944312	12.5698
159	25281	4019679	12.6095
160	25600	4096000	12.6491

Lesson Title: Finding the Other Sides of a Right-Angled Triangle	**Theme**: Geometry	
Lesson Number: M-09-044	**Class/Level**: JSS 3	**Time**: 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to apply Pythagoras' theorem to find the length of the other 2 sides of a right-angled triangle.	Table of Squares and Square Roots (see end of lesson plan). This will be used for all lessons on Pythagoras' Theorem.	1. Draw the triangles for this lesson plan on the board (see end of lesson plan). Note: Do not write the answers until after pupils have answered the questions. 2. Draw the Table of Squares and Square Roots on the board. 3. Write the questions at the end of the lesson plan, on the board.

Opening (*3 minutes*)

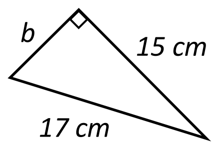
1. **Say**: Find the hypotenuse for the triangle in Question A. Raise your hand when you have the answer.
2. Allow 2 minute for pupils to answer the question.
3. Have pupils volunteer to explain their answer on the board. (Answers: see end of this lesson plan)
4. **Say**: Today we are going to apply Pythagoras' theorem to find the length of the other 2 sides of a right-angled triangle.

Introduction to the New Material (*10 minutes*)

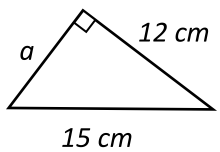
1. Ask a pupil to read Question B on the board.
2. **Say**: We will use Pythagoras' Theorem to find the length of the side.
3. Show on the board how to use Pythagoras' Theorem to find the area of the side. Ask pupils to raise their hand to answer questions.

4. **Say**: By Pythagoras' Theorem:
5. **Ask:** What do we know about this triangle? (Answer: $a = 15$ cm, $c = 17$ cm)
6. **Ask:** What is 15^2? (Answer: 225)
7. **Ask:** What is 17^2? (Answer: 289)
8. **Say**: We now subtract 225 from both sides.
9. **Ask**: What do we get when we do that?
10. **Ask**: What do we do next? (Answer: Take the square root of 64)
11. **Ask**: What does that give us? (Answer: 8)

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 15^2 + b^2 &= 17^2 \\ 225 + b^2 &= 289 \\ b^2 &= 289 - 225 \\ &= 64 \\ b &= \sqrt{64} \\ &= 8 \text{ cm} \end{aligned}$$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 12^2 &= 15^2 \\ a^2 + 144 &= 225 \\ a^2 &= 225 - 144 \\ a^2 &= 81 \end{aligned}$$



12. **Say:** Please do Question C in your exercise books. Check your answer with your neighbour when you finish.

$$a = \sqrt{81}$$
$$a = 9 \text{ cm}$$

13. Allow pupils time to do Question C
14. **Ask**: Who would like to explain Question C on the board? Raise your hand.
15. Select a pupil who has raised their hand to explain their answer step-by-step on the board.
16. Correct any errors in the calculation on the board. Ask pupils to check their work. An example calculation is shown above.

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. **Say**: Please calculate the length of the missing side of the triangles for Questions D and E. You will need the table which gives the squares and square roots.
3. Walk around, if possible, to check their answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan).

Independent Practice (*10 minutes*)

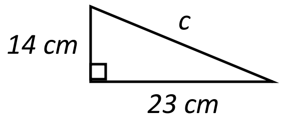
1. Ask the pupils to work independently to answer Questions F and G.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answer to the question F.
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)
5. Do not discuss the answer for Question G. Use them to check pupils' understanding of the work.

Closing (*2 minutes*)

1. **Say:** Please write your name on a piece of paper. Now write your working out and answer for Questions G. on the paper. Hand the paper in at the end of the lesson.
2. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be solving diagram and word problems involving Pythagoras' theorem.

[QUESTION FOR THE OPENING ACTIVITY]

A. Calculate the length of the hypotenuse in the following triangle. Give your answer to 1 decimal place.



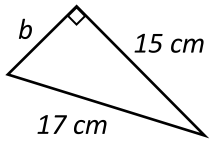
<u>Answer</u>

$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
&= 14^2 + 23^2 \\
&= 196 + 529 \\
&= 725 \\
c &= \sqrt{725} \\
c &= 26.9 \text{ cm}
\end{aligned}
$$

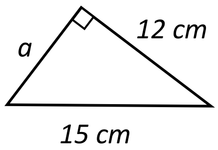
[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Calculate the length of the missing side in the following triangles:

B.



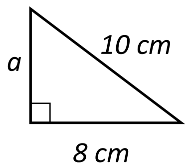
C.



[QUESTIONS FOR GUIDED PRACTICE]

Calculate the length of the missing side in the following triangles. Give your answer to 1 decimal place, if required.

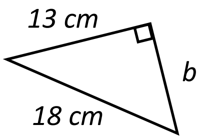
D.



<u>Answer</u>

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
a^2 + 8^2 &= 10^2 \\
a^2 + 64 &= 100 \\
a^2 &= 100 - 64 \\
a^2 &= 36 \\
a &= \sqrt{36} \\
a &= 6 \text{ cm}
\end{aligned}
$$

E.



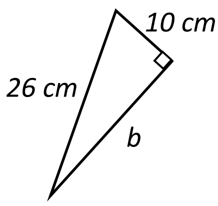
<u>Answer</u>

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
13^2 + b^2 &= 18^2 \\
169 + b^2 &= 324 \\
b^2 &= 324 - 169 \\
b^2 &= 155 \\
b &= \sqrt{155} \\
b &= 12.4 \text{ cm}
\end{aligned}
$$

[QUESTIONS FOR INDEPENDENT PRACTICE]

Calculate the length of the missing side in the following triangles. Give your answer to 1 decimal place, if required.

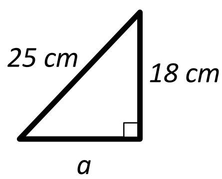
F.



<u>Answer</u>

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
10^2 + b^2 &= 26^2 \\
100 + b^2 &= 676 \\
b^2 &= 676 - 100 \\
b^2 &= 576 \\
b &= \sqrt{576} \\
b &= 24 \text{ cm}
\end{aligned}
$$

G.



<u>Answer</u>

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
a^2 + 18^2 &= 25^2 \\
a^2 + 324 &= 625 \\
a^2 &= 625 - 324 \\
a^2 &= 301 \\
a &= \sqrt{301} \\
a &= 17.3 \text{ cm}
\end{aligned}
$$

Lesson Title: Applying Pythagoras' Theorem	**Theme**: Geometry	
Lesson Number: M-09-045	**Class/Level**: JSS 3	**Time:** 35 minutes

Learning Outcomes	**Teaching Aids**	**Preparation**
By the end of the lesson, pupils will be able to solve diagram and word problems involving Pythagoras' theorem.	Table of Squares and Square Roots. This will be used for all lessons on Pythagoras' Theorem.	1. Draw the triangles for this lesson plan on the board (see end of this lesson plan). 2. Write the questions at the end of the lesson plan on the board.

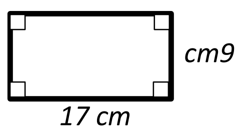
Opening (*3 minutes*)

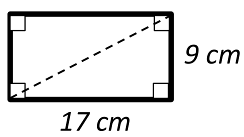
1. **Say**: We have been solving problems on Pythagoras' Theorem for the last few lessons. We have been using fairly simple diagrams so far. Before we do any more problems, let us check that we can all identify the parts of a right-angled triangle.
2. **Ask**: How do we identify the hypotenuse in a right-angled triangle? Raise your hand. (Answer: It is the side opposite the right angle.)
3. **Ask**: What do we call a right-angled triangle which has 2 equal sides and angles? Raise your hand. (Answer: isosceles right-angled triangles)
4. **Ask**: What size is the angle in an isosceles right-angled triangle? Raise your hand. (Answer: 45^{o})
5. **Say**: Today we are going to solve diagram and word problems involving Pythagoras' theorem.

Introduction to the New Material (*10 minutes*)

1. Have a pupil volunteer to read Question a. on the board.
2. **Ask**: Who can tell the class what the question is asking us to do? Raise your hand.
3. Select a pupil who has raised their hand to answer. (Answer: To find the length of the diagonal in the rectangle.)
4. **Say:** We want to find the diagonal of the rectangle. We know the length of the sides. Let us draw the diagonal to see if that helps us.
5. Draw the diagonal on the rectangle.
6. **Ask**: Do you recognise the shape we now have?
7. Guide a pupil to say what the shape now looks like. (Answer: 2 right angled triangles)
8. **Ask**: How can we find the length of the diagonal? Raise your hand. (Answer: By Pythagoras' Theorem)
9. **Ask**: Who would like to show the calculation on the board? Raise your hand.
10. Select a pupil who raised their hand to explain the calculation. Ask other pupils to solve it in their exercise books.
11. Correct any errors in the calculation on the board. Ask pupils to check their work. An example calculation is shown below.

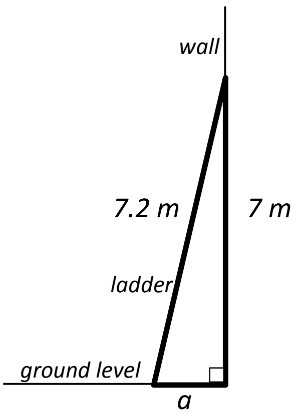
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 9^2 + 17^2 &= c^2 \\ 81 + 289 &= c^2 \\ c^2 &= 370 \\ c &= \sqrt{370} \\ c &= 19.24 \text{ from the Square root tables} \end{aligned}$$





$$= \quad 19.2 \text{ cm to 1 decimal place}$$

12. **Say**: You see from this problem how important it is to draw a diagram to help with solving a problem. The diagram showed us that we needed to use Pythagoras' Theorem to solve the problem.
13. Ask a pupil to read Question b. from the board.
14. **Say:** We do not have a diagram for this problem. Who would like to explain what our first step should be?
15. Guide a pupil to say we should draw a diagram.
16. If a pupil gives the first step as a calculation using Pythagoras' Theorem, **say**: There is a step before that one which will help us decide whether we are calculating the hypotenuse or one of the shorter sides.
17. Show how to interpret the question as a diagram (shown below).
18. Have a pupil volunteer to show this on the board and solve the problem. Ask other pupils to solve it in their exercise books.
19. Correct any errors in the calculation on the board. Ask pupils to check their work. An example calculation is shown below.



$$a^2 + b^2 = c^2$$
$$a^2 + 7 = 7.2^2$$
$$a^2 + 49 = 51.84$$
$$a^2 = 51.84 - 49$$
$$a^2 = 2.84$$
$$a = \sqrt{2.84}$$
$$a = 1.685$$
$$a = 1.7 \text{ m from the wall}$$

Guided Practice (*10 minutes*)

1. Ask pupils to work in pairs.
2. **Say**: Solve for the missing side in Questions c. and d. on the board. You will need the table which gives the squares and square roots.
3. Walk around, if possible, to check their answers and clear up any misconceptions.
4. Have pupils from around the classroom volunteer to give their answers to the questions.
5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)

Independent Practice (*10 minutes)*

1. Ask the pupils to work independently to answer Questions e. and f. to find the missing sides of the triangles.
2. Walk around, if possible, to check their answers and clear up any misconceptions.
3. Have pupils from around the classroom volunteer to give their answers to the questions.

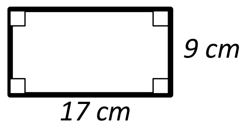
4. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)

Closing (*2 minutes*)

1. **Say:** Please write down in pairs 2 different things you learned today.
2. Allow pupils 1 minute to discuss and share their ideas.
3. Have one pupil from the front, and one from the back of the classroom volunteer to answer. (Example answers: How to solve problems with Pythagoras' theorem; solving word problems; it is important to draw a diagram to help with solving a problem)

[QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

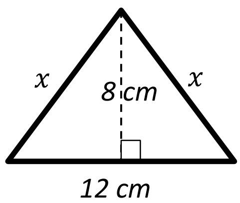
1. Find the length of the diagonal on a rectangle with sides using Pythagoras' Theorem. Give your answer to 1 decimal place.



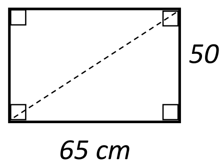
2. A ladder of length 7.2 m leans against a wall so that the top of the ladder is 7 m above ground level. How far is the bottom of the ladder from the wall? Give your answer to 1 decimal place.

[QUESTIONS FOR GUIDED PRACTICE]

3. An isosceles triangle has a base of 12 cm and a height of 8 cm. Calculate the length, x, of one of its equal sides.
(Answer: $a^2 + b^2 = x^2, a = \frac{12}{2}, b = 8,\ 6^2 + 8^2 = x^2,$
$36 + 64 = x^2 = 100, x = \sqrt{100} = 10$ cm

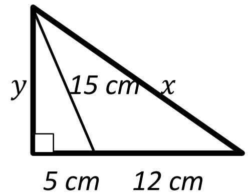


4. A box measures 60 cm long and 45 cm high. What is the diagonal length of the box to the nearest cm?
(Answer: $a^2 + b^2 = c^2, a = 65\ , b = 50, 65^2 + 50^2 = c^2,$
$4225 + 2500 = c^2, c^2 = 6725, x = \sqrt{6725} = 82.0$ cm

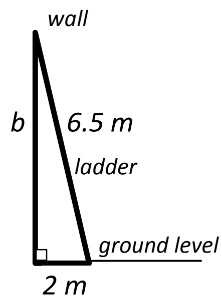


[QUESTIONS FOR INDEPENDENT PRACTICE]

5. Calculate the length of the missing sides in the diagram shown. Give your answers to 1 decimal place if required.
(Answer: $y^2 + 5^2 = 15^2, y^2 + 25 = 225, y^2 = 225 - 25,$
$y^2 = 200, y = \sqrt{200} = 14.1$ cm;
$15^2 + 12^2 = x^2, 225 + 144 = x^2, x^2 = 369, x = \sqrt{369} = 19.2$ cm



6. A ladder of length 6.5 m leans against a wall so that the bottom of the ladder is 2 m away from the wall. How far is the top of the ladder from ground level? Give your answer to 1 decimal place. (Answer: $b^2 + 2^2 = 6.5^2, b^2 + 4 = 42.25, b^2 = 42.25 - 4,$ $b^2 = 38.25, b = \sqrt{38.25} = 6.2$ m



Appendix I: Squares of Numbers 10-100

Squares of Numbers

$x \to x^2$

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1000	1020	1040	1061	1082	1103	1124	1145	1166	1188	2	4	6	8	10	13	15	17	19
11	1210	1232	1254	1277	1300	1323	1346	1369	1392	1416	2	5	7	9	11	14	16	18	21
12	1440	1464	1488	1513	1538	1563	1588	1613	1638	1664	2	5	7	10	12	15	17	20	22
13	1690	1716	1742	1769	1796	1823	1850	1877	1904	1932	3	5	8	11	13	16	19	22	24
14	1960	1988	2016	2045	2074	2103	2132	2161	2190	2220	3	6	9	12	14	17	20	23	26
15	2250	2280	2310	2341	2372	2403	2434	2465	2496	2528	3	6	9	12	15	19	22	25	28
16	2560	2592	2624	2657	2690	2723	2756	2789	2822	2856	3	7	10	13	16	20	23	26	30
17	2890	2924	2958	2993	3028	3063	3098	3133	3168	3204	3	7	10	14	17	21	24	28	31
18	3240	3276	3312	3349	3386	3423	3460	3497	3534	3572	4	7	11	15	18	22	26	30	33
19	3610	3648	3686	3725	3764	3803	3842	3881	3920	3960	4	8	12	16	19	23	27	31	35
20	4000	4040	4080	4121	4162	4203	4244	4285	4326	4368	4	8	12	16	20	25	29	33	37
21	4410	4452	4494	4537	4580	4623	4666	4709	4752	4796	4	9	13	17	21	26	30	34	39
22	4840	4884	4928	4973	5018	5063	5108	5153	5198	5244	4	9	13	18	22	27	31	36	40
23	5290	5336	5382	5429	5476	5523	5570	5617	5664	5712	5	9	14	19	23	28	33	38	42
24	5760	5808	5856	5905	5954	6003	6052	6101	6150	6200	5	10	15	20	24	29	34	39	44
25	6250	6300	6350	6401	6452	6503	6554	6605	6656	6708	5	10	15	20	25	31	36	41	46
26	6760	6812	6864	6917	6970	7023	7076	7129	7182	7236	5	11	16	21	26	32	37	42	48
27	7290	7344	7398	7453	7508	7563	7618	7673	7728	7784	5	11	16	22	27	33	38	44	49
28	7840	7896	7952	8009	8066	8123	8180	8237	8294	8352	6	11	17	23	28	34	40	46	51
29	8410	8486	8526	8585	8644	8703	8762	8821	8880	8940	6	12	18	24	29	35	41	47	53
30	9000	9060	9120	9181	9242	9303	9364	9425	9486	9548	6	12	18	24	30	37	43	49	55
31	9610	9672	9734	9797	9860	9923	9986	–	–	–	6	13	19	25	31	38	44	50	57
31	–	–	–	–	–	–	–	1005	1011	1018	1	1	2	3	3	4	5	5	6
32	1024	1030	1037	1043	1050	1056	1063	1069	1076	1082	1	1	2	3	3	4	5	5	6
33	1089	1096	1102	1109	1116	1122	1129	1136	1142	1149	1	1	2	3	3	4	5	5	6
34	1156	1163	1170	1176	1183	1190	1197	1204	1211	1218	1	1	2	3	3	4	5	6	6
35	1225	1232	1239	1246	1253	1260	1267	1274	1282	1289	1	1	2	3	4	4	5	6	6
36	1296	1303	1310	1318	1325	1332	1340	1347	1354	1362	1	1	2	3	4	4	5	6	7
37	1369	1376	1384	1391	1399	1406	1414	1421	1429	1436	1	2	2	3	4	5	5	6	7
38	1444	1452	1459	1467	1475	1482	1490	1498	1505	1513	1	2	2	3	4	5	5	6	7
39	1521	1529	1537	1544	1552	1560	1568	1576	1584	1592	1	2	2	3	4	5	6	6	7
40	1600	1608	1616	1624	1632	1640	1648	1656	1665	1673	1	2	2	3	4	5	6	6	7
41	1681	1689	1697	1706	1714	1722	1731	1739	1747	1756	1	2	2	3	4	5	6	7	7
42	1764	1772	1781	1789	1798	1806	1815	1823	1832	1840	1	2	3	3	4	5	6	7	8
43	1849	1858	1866	1875	1884	1892	1901	1910	1918	1927	1	2	3	3	4	5	6	7	8
44	1936	1945	1954	1962	1971	1980	1989	1998	2007	2016	1	2	3	4	5	5	6	7	8
45	2025	2034	2043	2052	2061	2070	2079	2088	2098	2107	1	2	3	4	5	5	6	7	8
46	2116	2125	2134	2144	2153	2162	2172	2181	2190	2200	1	2	3	4	5	6	7	7	8
47	2209	2218	2228	2237	2247	2256	2266	2275	2285	2294	1	2	3	4	5	6	7	8	9
48	2304	2314	2323	2333	2343	2352	2362	2372	2381	2391	1	2	3	4	5	6	7	8	9
49	2401	2411	2421	2430	2440	2450	2460	2470	2480	2490	1	2	3	4	5	6	7	8	9
50	2500	2510	2520	2530	2540	2550	2560	2570	2581	2591	1	2	3	4	5	6	7	8	9
51	2601	2611	2621	2632	2642	2652	2663	2673	2683	2694	1	2	3	4	5	6	7	8	9
52	2704	2714	2725	2735	2746	2756	2767	2777	2788	2798	1	2	3	4	5	6	7	8	9
53	2809	2820	2830	2841	2852	2862	2873	2884	2894	2905	1	2	3	4	5	6	7	9	10
54	2916	2927	2938	2948	2959	2970	2981	2992	3003	3014	1	2	3	4	5	7	8	9	10
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	3025	3036	3047	3058	3069	3080	3091	3102	3114	3125	1	2	3	4	6	7	8	9	10
56	3136	3147	3158	3170	3181	3192	3204	3215	3226	3238	1	2	3	5	6	7	8	9	10
57	3249	3260	3272	3283	3295	3306	3318	3329	3341	3352	1	2	3	5	6	7	8	9	10
58	3364	3376	3387	3399	3411	3422	3434	3446	3457	3469	1	2	4	5	6	7	8	9	10
59	3481	3493	3505	3516	3528	3540	3552	3564	3576	3588	1	2	4	5	6	7	8	10	11
60	3600	3612	3624	3636	3648	3660	3672	3684	3697	3709	1	2	4	5	6	7	8	10	11
61	3721	3733	3745	3758	3770	3782	3795	3807	3819	3832	1	2	4	5	6	7	9	10	11
62	3844	3856	3869	3881	3894	3906	3919	3931	3944	3956	1	2	4	5	6	7	9	10	11
63	3969	3982	3994	4007	4020	4032	4054	4058	4070	4083	1	3	4	5	6	8	9	10	11
64	4096	4109	4122	4134	4147	4160	4173	4186	4199	4212	1	3	4	5	6	8	9	10	12
65	4225	4238	4251	4264	4277	4290	4303	4316	4330	4343	1	3	4	5	7	8	9	10	12
66	4356	4369	4382	4396	4409	4422	4436	4449	4462	4476	1	3	4	5	7	8	9	11	12
67	4489	4502	4516	4529	4543	4556	4570	4583	4597	4610	1	3	4	5	7	8	9	11	12
68	4624	4638	4651	4665	4679	4692	4706	4720	4733	4747	1	3	4	5	7	8	10	11	12
69	4761	4775	4789	4802	4816	4830	4844	4858	4872	4886	1	3	4	6	7	8	10	11	13
70	4900	4914	4928	4942	4956	4970	4984	4998	5013	5027	1	3	4	6	7	8	10	11	13
71	5041	5055	5069	5084	5098	5112	5127	5141	5155	5170	1	3	4	6	7	9	10	11	13
72	5184	5198	5213	5227	5242	5256	5271	5285	5300	5314	1	3	4	6	7	9	10	11	13
73	5329	5344	5358	5373	5388	5402	5417	5432	5446	5461	1	3	4	6	7	9	10	12	13
74	5476	5491	5506	5520	5535	5550	5565	5580	5595	5610	1	3	4	6	7	9	10	12	13
75	5625	5640	5655	5670	5685	5700	5715	5730	5746	5761	2	3	5	6	8	9	11	12	14
76	5776	5791	5806	5822	5837	5852	5868	5883	5898	5914	2	3	5	6	8	9	11	12	14
77	5929	5944	5960	5975	5991	6006	6022	6037	6053	6068	2	3	5	6	8	9	11	12	14
78	6084	6100	6115	6131	6147	6162	6178	6194	6209	6225	2	3	5	6	8	9	11	13	14
79	6241	6257	6273	6288	6304	6320	6336	6352	6368	6384	2	3	5	6	8	10	11	13	14
80	6400	6416	6432	6448	6464	6480	6496	6512	6529	6545	2	3	5	6	8	10	11	13	14
81	6561	6577	6593	6610	6626	6642	6659	6675	6691	6708	2	3	5	7	8	10	11	13	15
82	6724	6740	6757	6773	6790	6806	6823	6839	6856	6872	2	3	5	7	8	10	12	13	15
83	6889	6906	6922	6939	6956	6972	6989	7006	7022	7039	2	3	5	7	8	10	12	13	15
84	7056	7073	7090	7106	7123	7140	7157	7174	7191	7208	2	3	5	7	8	10	12	14	15
85	7225	7242	7259	7276	7293	7310	7327	7344	7362	7379	2	3	5	7	9	10	12	14	15
86	7396	7413	7430	7448	7465	7482	7500	7517	7534	7552	2	3	5	7	9	10	12	14	16
87	7569	7586	7604	7621	7639	7656	7674	7691	7709	7726	2	3	5	7	9	10	12	14	16
88	7744	7762	7779	7797	7815	7832	7850	7868	7885	7903	2	4	5	7	9	11	12	14	16
89	7921	7939	7957	7974	7992	8010	8028	8046	8064	8082	2	4	5	7	9	11	13	14	16
90	8100	8118	8136	8154	8172	8190	8208	8226	8245	8263	2	4	5	7	9	11	13	14	16
91	8281	8299	8317	8336	8354	8372	8391	8409	8427	8446	2	4	5	7	9	11	13	15	16
92	8464	8482	8501	8519	8538	8556	8575	8593	8612	8630	2	4	6	7	9	11	13	15	17
93	8649	8668	8686	8705	8724	8742	8761	8780	8798	8817	2	4	6	7	9	11	13	15	17
94	8836	8855	8874	8892	8911	8930	8949	8968	8987	9006	2	4	6	8	9	11	13	15	17
95	9025	9044	9063	9082	9101	9120	9139	9158	9178	9197	2	4	6	8	10	11	13	15	17
96	9216	9235	9254	9274	9293	9312	9332	9351	9370	9390	2	4	6	8	10	12	14	15	17
97	9409	9428	9448	9467	9487	9506	9526	9545	9565	9584	2	4	6	8	10	12	14	16	18
98	9604	9624	9643	9663	9683	9702	9722	9742	9761	9781	2	4	6	8	10	12	14	16	18
99	9801	9821	9841	9860	9880	9900	9920	9940	9960	9980	2	4	6	8	10	12	14	16	18
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Appendix II: Sines of Angles

Sines of Angles (*x* in degrees)

$x \rightarrow \sin x$

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	2	3	5	7	9	10	12	14	16
1	0·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	2	3	5	7	9	10	12	14	16
2	0·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	2	3	5	7	9	10	12	14	16
3	0·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	2	3	5	7	9	10	12	14	16
4	0·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	2	3	5	7	9	10	12	14	16
5	0·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	2	3	5	7	9	10	12	14	16
6	0·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	2	3	5	7	9	10	12	14	16
7	0·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	2	3	5	7	9	10	12	14	16
8	0·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	2	3	5	7	9	10	12	14	16
9	0·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	2	3	5	7	9	10	12	14	15
10	0·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	2	3	5	7	9	10	12	14	15
11	0·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2	3	5	7	9	10	12	14	15
12	0·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2	3	5	7	9	10	12	14	15
13	0·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2	3	5	7	8	10	12	14	15
14	0·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2	3	5	7	8	10	12	14	15
15	0·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2	3	5	7	8	10	12	13	15
16	0·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2	3	5	7	8	10	12	13	15
17	0·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	2	3	5	7	8	10	12	13	15
18	0·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	2	3	5	7	8	10	12	13	15
19	0·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	2	3	5	7	8	10	12	13	15
20	0·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	2	3	5	7	8	10	11	13	15
21	0·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	2	3	5	6	8	10	11	13	15
22	0·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	2	3	5	6	8	10	11	13	15
23	0·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	2	3	5	6	8	10	11	13	14
24	0·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	2	3	5	6	8	10	11	13	14
25	0·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	2	3	5	6	8	9	11	13	14
26	0·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	2	3	5	5	8	9	11	12	14
27	0·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	2	3	5	6	8	9	11	12	14
28	0·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	2	3	5	6	8	9	11	12	14
29	0·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	2	3	5	6	8	9	11	12	14
30	0·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	2	3	5	6	8	9	11	12	14
31	0·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	1	3	4	6	7	9	10	12	13
32	0·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	1	3	4	6	7	9	10	12	13
33	0·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	1	3	4	6	7	9	10	12	13
34	0·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	1	3	4	6	7	9	10	12	13
35	0·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	1	3	4	6	7	9	10	11	13
36	0·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	1	3	4	6	7	8	10	11	13
37	0·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	1	3	4	6	7	8	10	11	12
38	0·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	1	3	4	5	7	8	10	11	12
39	0·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	1	3	4	5	7	8	9	11	12
40	0·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	1	3	4	5	7	8	9	11	12
41	0·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	1	3	4	5	7	8	9	10	12
42	0·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	1	3	4	5	6	8	9	10	12
43	0·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	1	3	4	5	6	8	9	10	11
44	0·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
45	0·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	1	2	4	5	6	7	9	10	11
46	0·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	1	2	4	5	6	7	8	10	11
47	0·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	1	2	4	5	6	7	8	9	11
48	0·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	1	2	3	5	6	7	8	9	10
49	0·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	1	2	3	5	6	7	8	9	10
50	0·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	1	2	3	4	6	7	8	9	10
51	0·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	1	2	3	4	5	7	8	9	10
52	0·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	1	2	3	4	5	6	7	8	10
53	0·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	1	2	3	4	5	6	7	8	9
54	0·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	1	2	3	4	5	6	7	8	9
55	0·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	1	2	3	4	5	6	7	8	9
56	0·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	1	2	3	4	5	6	7	8	9
57	0·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	1	2	3	4	5	6	7	8	8
58	0·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	1	2	3	4	5	5	6	7	8
59	0·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	2	3	4	4	5	6	7	8
60	0·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	2	3	3	4	5	6	7	8
61	0·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	2	2	3	4	5	6	7	7
62	0·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	2	2	3	4	5	6	6	7
63	0·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	2	2	3	4	5	5	6	7
64	0·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	2	2	3	4	5	5	6	7
65	0·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	1	2	3	4	4	5	6	7
66	0·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	1	2	3	3	4	5	6	6
67	0·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	1	2	3	3	4	5	5	6
68	0·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	1	2	3	3	4	4	5	6
69	0·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	1	2	2	3	4	4	5	6
70	0·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	1	2	2	3	3	4	5	5
71	0·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	1	2	2	3	3	4	4	5
72	0·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	1	2	2	3	3	4	4	5
73	0·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	0	1	1	2	2	3	3	4	4
74	0·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0	1	1	2	2	3	3	4	4
75	0·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	0	1	1	2	2	3	3	3	4
76	0·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	0	1	1	2	2	2	3	3	4
77	0·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	0	1	1	2	2	2	3	3	3
78	0·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	0	1	1	1	2	2	2	3	3
79	0·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0	1	1	1	2	2	2	3	3
80	0·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	1	1	2	2	2	3
81	0·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	1	1	2	2	2	2
82	0·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	0	1	1	1	1	2	2	2
83	0·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	0	1	1	1	1	1	2	2
84	0·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	0	1	1	1	1	1	1	2
85	0·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	0	1	1	1	1	1	1
86	0·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	0	0	1	1	1	1	1
87	0·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	0	0	0	1	1	1
88	0·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998									
89	0·9998	9999	9999	9999	9999	1·0000	1·0000	1·0000	1·0000	1·0000									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix III: Cosines of Angles

Cosines of Angles (x in degrees)

$x \rightarrow \cos x$

											SUBTRACT Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	1·0000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9999									
1	0·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995									
2	0·9994	·9993	·9993	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	0	0	0	1	1	1
3	0·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	0	0	1	1	1	1	1
4	0·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	0	1	1	1	1	1	1
5	0·9962	·9960	·9959	·9957	·9956	·9954	·9952	·9951	·9949	·9947	0	0	1	1	1	1	1	1	2
6	0·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	0	1	1	1	1	1	2	2
7	0·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	0	1	1	1	1	2	2	2
8	0·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	1	1	2	2	2	2
9	0·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	1	1	2	2	2	3
10	0·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	0	1	1	1	2	2	2	3	3
11	0·9816	·9813	·9810	·9806	·9803	·9799	·9796	·9792	·9789	·9785	0	1	1	1	2	2	2	3	3
12	0·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	0	1	1	2	2	2	3	3	3
13	0·9744	·9740	·9736	·9732	·9728	·9724	·9720	·9715	·9711	·9707	0	1	1	2	2	2	3	3	4
14	0·9703	·9699	·9694	·9690	·9686	·9681	·9677	·9673	·9668	·9664	0	1	1	2	2	3	3	3	4
15	0·9659	·9655	·9650	·9646	·9641	·9636	·9632	·9627	·9622	·9617	0	1	1	2	2	3	3	4	4
16	0·9613	·9608	·9603	·9598	·9593	·9588	·9583	·9578	·9573	·9568	0	1	1	2	2	3	3	4	4
17	0·9563	·9558	·9553	·9548	·9542	·9537	·9532	·9527	·9521	·9516	1	1	2	2	3	3	4	4	5
18	0·9511	·9505	·9500	·9494	·9489	·9483	·9478	·9472	·9466	·9461	1	1	2	2	3	3	4	4	5
19	0·9455	·9449	·9444	·9438	·9432	·9426	·9421	·9415	·9409	·9403	1	1	2	2	3	3	4	5	5
20	0·9397	·9391	·9385	·9379	·9373	·9367	·9361	·9354	·9348	·9342	1	1	2	2	3	4	4	5	6
21	0·9336	·9330	·9323	·9317	·9311	·9304	·9298	·9291	·9285	·9278	1	1	2	3	3	4	4	5	6
22	0·9272	·9265	·9259	·9252	·9245	·9239	·9232	·9225	·9219	·9212	1	1	2	3	3	4	5	5	6
23	0·9205	·9198	·9191	·9184	·9178	·9171	·9164	·9157	·9150	·9143	1	1	2	3	3	4	5	6	6
24	0·9135	·9128	·9121	·9114	·9107	·9100	·9092	·9085	·9078	·9070	1	1	2	3	4	4	5	6	7
25	0·9063	·9056	·9048	·9041	·9033	·9026	·9018	·9011	·9003	·8996	1	2	2	3	4	5	5	6	7
26	0·8988	·8980	·8973	·8965	·8957	·8949	·8942	·8934	·8926	·8918	1	2	2	3	4	5	5	6	7
27	0·8910	·8902	·8894	·8886	·8878	·8870	·8862	·8854	·8846	·8838	1	2	2	3	4	5	6	6	7
28	0·8829	·8821	·8813	·8805	·8796	·8788	·8780	·8771	·8763	·8755	1	2	2	3	4	5	6	7	7
29	0·8746	·8738	·8729	·8721	·8712	·8704	·8695	·8686	·8678	·8669	1	2	3	3	4	5	6	7	8
30	0·8660	·8652	·8643	·8634	·8625	·8616	·8607	·8599	·8590	·8581	1	2	3	4	4	5	6	7	8
31	0·8572	·8563	·8554	·8545	·8536	·8526	·8517	·8508	·8499	·8490	1	2	3	4	5	5	6	7	8
32	0·8480	·8471	·8462	·8453	·8443	·8434	·8425	·8415	·8406	·8396	1	2	3	4	5	6	7	8	8
33	0·8387	·8377	·8368	·8358	·8348	·8339	·8329	·8320	·8310	·8300	1	2	3	4	5	6	7	8	9
34	0·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	1	2	3	4	5	6	7	8	9
35	0·8192	·8181	·8171	·8161	·8151	·8141	·8131	·8121	·8111	·8100	1	2	3	4	5	6	7	8	9
36	0·8090	·8080	·8070	·8059	·8049	·8039	·8028	·8018	·8007	·7997	1	2	3	4	5	6	7	8	9
37	0·7986	·7976	·7965	·7955	·7944	·7934	·7923	·7912	·7902	·7891	1	2	3	4	5	6	7	8	10
38	0·7880	·7869	·7859	·7848	·7837	·7826	·7815	·7804	·7793	·7782	1	2	3	4	5	6	7	9	10
39	0·7771	·7760	·7749	·7738	·7727	·7716	·7705	·7694	·7683	·7672	1	2	3	4	6	7	8	9	10
40	0·7660	·7649	·7638	·7627	·7615	·7604	·7593	·7581	·7570	·7559	1	2	3	5	6	7	8	9	10
41	0·7547	·7536	·7524	·7513	·7501	·7490	·7478	·7466	·7455	·7443	1	2	3	5	6	7	8	9	10
42	0·7431	·7420	·7408	·7396	·7385	·7373	·7361	·7349	·7337	·7325	1	2	4	5	6	7	8	9	11
43	0·7314	·7302	·7290	·7278	·7266	·7254	·7242	·7230	·7218	·7206	1	2	4	5	6	7	8	10	11
44	0·7193	·7181	·7169	·7157	·7145	·7133	·7120	·7108	·7096	·7083	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

											SUBTRACT Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
45	0·7071	·7059	·7046	·7034	·7022	·7009	·6997	·6984	·6972	·6959	1	2	4	5	6	7	9	10	11
46	0·6947	·6934	·6921	·6909	·6896	·6884	·6871	·6858	·6845	·6833	1	3	4	5	6	8	9	10	11
47	0·6820	·6807	·6794	·6782	·6769	·6756	·6743	·6730	·6717	·6704	1	3	4	5	6	8	9	10	12
48	0·6691	·6678	·6665	·6652	·6639	·6626	·6613	·6600	·6587	·6574	1	3	4	5	7	8	9	10	12
49	0·6561	·6547	·6534	·6521	·6508	·6494	·6481	·6468	·6455	·6441	1	3	4	5	7	8	9	11	12
50	0·6428	·6414	·6401	·6388	·6374	·6361	·6347	·6334	·6320	·6307	1	3	4	5	7	8	9	11	12
51	0·6293	·6280	·6266	·6252	·6239	·6225	·6211	·6198	·6184	·6170	1	3	4	5	7	8	10	11	12
52	0·6157	·6143	·6129	·6115	·6101	·6088	·6074	·6060	·6046	·6032	1	3	4	5	7	8	10	11	12
53	0·6018	·6004	·5990	·5976	·5962	·5948	·5934	·5920	·5906	·5892	1	3	4	6	7	8	10	11	13
54	0·5878	·5864	·5850	·5835	·5821	·5807	·5793	·5779	·5764	·5750	1	3	4	6	7	9	10	11	13
55	0·5736	·5721	·5707	·5693	·5678	·5664	·5650	·5635	·5621	·5606	1	3	4	6	7	9	10	12	13
56	0·5592	·5577	·5563	·5548	·5534	·5519	·5505	·5490	·5476	·5461	1	3	4	6	7	9	10	12	13
57	0·5446	·5432	·5417	·5402	·5388	·5373	·5358	·5344	·5329	·5314	1	3	4	6	7	9	10	12	13
58	0·5299	·5284	·5270	·5255	·5240	·5225	·5210	·5195	·5180	·5165	1	3	4	6	7	9	10	12	13
59	0·5150	·5135	·5120	·5105	·5090	·5075	·5060	·5045	·5030	·5015	2	3	5	6	8	9	11	12	14
60	0·5000	·4985	·4970	·4955	·4939	·4924	·4909	·4894	·4879	·4863	2	3	5	6	8	9	11	12	14
61	0·4848	·4833	·4818	·4802	·4787	·4772	·4756	·4741	·4726	·4710	2	3	5	6	8	9	11	12	14
62	0·4695	·4679	·4664	·4648	·4633	·4617	·4602	·4586	·4571	·4555	2	3	5	6	8	9	11	12	14
63	0·4540	·4524	·4509	·4493	·4478	·4462	·4446	·4431	·4415	·4399	2	3	5	6	8	9	11	12	14
64	0·4384	·4368	·4352	·4337	·4321	·4305	·4289	·4274	·4258	·4242	2	3	5	6	8	9	11	13	14
65	0·4226	·4210	·4195	·4179	·4163	·4147	·4131	·4115	·4099	·4083	2	3	5	6	8	10	11	13	14
66	0·4067	·4051	·4035	·4019	·4003	·3987	·3971	·3955	·3939	·3923	2	3	5	6	8	10	11	13	14
67	0·3907	·3891	·3875	·3859	·3843	·3827	·3811	·3795	·3778	·3762	2	3	5	6	8	10	11	13	15
68	0·3746	·3730	·3714	·3697	·3681	·3665	·3649	·3633	·3616	·3600	2	3	5	6	8	10	11	13	15
69	0·3584	·3567	·3551	·3535	·3518	·3502	·3486	·3469	·3453	·3437	2	3	5	6	8	10	11	13	15
70	0·3420	·3404	·3387	·3371	·3355	·3338	·3322	·3305	·3289	·3272	2	3	5	7	8	10	12	13	15
71	0·3256	·3239	·3223	·3206	·3190	·3173	·3156	·3140	·3123	·3107	2	3	5	7	8	10	12	13	15
72	0·3090	·3074	·3057	·3040	·3024	·3007	·2990	·2974	·2957	·2940	2	3	5	7	8	10	12	13	15
73	0·2924	·2907	·2890	·2874	·2857	·2840	·2823	·2807	·2790	·2773	2	3	5	7	8	10	12	13	15
74	0·2756	·2740	·2723	·2706	·2689	·2672	·2656	·2639	·2622	·2605	2	3	5	7	8	10	12	13	15
75	0·2588	·2571	·2554	·2538	·2521	·2504	·2487	·2470	·2453	·2436	2	3	5	7	8	10	12	14	15
76	0·2419	·2402	·2385	·2368	·2351	·2334	·2317	·2300	·2284	·2267	2	3	5	7	8	10	12	14	15
77	0·2250	·2233	·2215	·2198	·2181	·2164	·2147	·2130	·2113	·2096	2	3	5	7	9	10	12	13	15
78	0·2079	·2062	·2045	·2028	·2011	·1994	·1977	·1959	·1942	·1925	2	3	5	7	9	10	12	14	15
79	0·1908	·1891	·1874	·1857	·1840	·1822	·1805	·1788	·1771	·1754	2	3	5	7	9	10	12	14	15
80	0·1736	·1719	·1702	·1685	·1668	·1650	·1633	·1616	·1599	·1582	2	3	5	7	9	10	12	14	15
81	0·1564	·1547	·1530	·1513	·1495	·1478	·1461	·1444	·1426	·1409	2	3	5	7	9	10	12	14	16
82	0·1392	·1374	·1357	·1340	·1323	·1305	·1288	·1271	·1253	·1236	2	3	5	7	9	10	12	14	16
83	0·1219	·1201	·1184	·1167	·1149	·1132	·1115	·1097	·1080	·1063	2	3	5	7	9	10	12	14	16
84	0·1045	·1028	·1011	·0993	·0976	·0958	·0941	·0924	·0906	·0889	2	3	5	7	9	10	12	14	16
85	0·0872	·0854	·0837	·0819	·0802	·0785	·0767	·0750	·0732	·0715	2	3	5	7	9	10	12	14	16
86	0·0698	·0680	·0663	·0645	·0628	·0610	·0593	·0576	·0558	·0541	2	3	5	7	9	10	12	14	16
87	0·0523	·0506	·0488	·0471	·0454	·0436	·0419	·0401	·0384	·0366	2	3	5	7	9	10	12	14	16
88	0·0349	·0332	·0314	·0297	·0279	·0262	·0244	·0227	·0209	·0192	2	3	5	7	9	10	12	14	16
89	0·0175	·0157	·0140	·0122	·0105	·0087	·0070	·0052	·0035	·0017	2	3	5	7	9	10	12	14	16
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix IV: Tangents of Angles

Tangents of Angles (x in degrees)

$x \to \tan x$

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	0·0000	·0017	·0035	·0052	·0070	·0087	·0105	·0122	·0140	·0157	2	3	5	7	9	10	12	14	16
1	0·0175	·0192	·0209	·0227	·0244	·0262	·0279	·0297	·0314	·0332	2	3	5	7	9	10	12	14	16
2	0·0349	·0367	·0384	·0402	·0419	·0437	·0454	·0472	·0489	·0507	2	3	5	7	9	10	12	14	16
3	0·0524	·0542	·0559	·0577	·0594	·0612	·0629	·0647	·0664	·0682	2	4	5	7	9	11	12	14	16
4	0·0699	·0717	·0734	·0752	·0769	·0787	·0805	·0822	·0840	·0857	2	4	5	7	9	11	12	14	16
5	0·0875	·0892	·0910	·0928	·0945	·0963	·0981	·0998	·1016	·1033	2	4	5	7	9	11	12	14	16
6	0·1051	·1069	·1086	·1104	·1122	·1139	·1157	·1175	·1192	·1210	2	4	5	7	9	11	12	14	16
7	0·1228	·1246	·1263	·1281	·1299	·1317	·1334	·1352	·1370	·1388	2	4	5	7	9	11	12	14	16
8	0·1405	·1423	·1441	·1459	·1477	·1495	·1512	·1530	·1548	·1566	2	4	5	7	9	11	12	14	16
9	0·1584	·1602	·1620	·1638	·1655	·1673	·1691	·1709	·1727	·1745	2	4	5	7	9	11	13	14	16
10	0·1763	·1781	·1799	·1817	·1835	·1853	·1871	·1890	·1908	·1926	2	4	5	7	9	11	13	14	16
11	0·1944	·1962	·1980	·1998	·2016	·2035	·2053	·2071	·2089	·2107	2	4	5	7	9	11	13	15	16
12	0·2126	·2144	·2162	·2180	·2199	·2217	·2235	·2254	·2272	·2290	2	4	5	7	9	11	13	15	16
13	0·2309	·2327	·2345	·2364	·2382	·2401	·2419	·2438	·2456	·2475	2	4	6	7	9	11	13	15	17
14	0·2493	·2512	·2530	·2549	·2568	·2586	·2605	·2623	·2642	·2661	2	4	6	7	9	11	13	15	17
15	0·2679	·2698	·2717	·2736	·2754	·2773	·2792	·2811	·2830	·2849	2	4	6	8	9	11	13	15	17
16	0·2867	·2886	·2905	·2924	·2943	·2962	·2981	·3000	·3019	·3038	2	4	6	8	9	11	13	15	17
17	0·3057	·3076	·3096	·3115	·3134	·3153	·3172	·3191	·3211	·3230	2	4	6	8	10	12	13	15	17
18	0·3249	·3269	·3288	·3307	·3327	·3346	·3365	·3385	·3404	·3424	2	4	6	8	10	12	14	16	17
19	0·3443	·3463	·3482	·3502	·3522	·3541	·3561	·3581	·3600	·3620	2	4	6	8	10	12	14	16	18
20	0·3640	·3659	·3679	·3699	·3719	·3739	·3759	·3779	·3799	·3819	2	4	6	8	10	12	14	16	18
21	0·3839	·3859	·3879	·3899	·3919	·3939	·3959	·3979	·4000	·4020	2	4	6	8	10	12	14	16	18
22	0·4040	·4061	·4081	·4101	·4122	·4142	·4163	·4183	·4204	·4224	2	4	6	8	10	12	14	16	18
23	0·4245	·4265	·4286	·4307	·4327	·4348	·4369	·4390	·4411	·4431	2	4	6	8	10	12	15	17	19
24	0·4452	·4473	·4494	·4515	·4536	·4557	·4578	·4599	·4621	·4642	2	4	6	8	11	13	15	17	19
25	0·4663	·4684	·4706	·4727	·4748	·4770	·4791	·4813	·4834	·4856	2	4	6	9	11	13	15	17	19
26	0·4877	·4899	·4921	·4942	·4964	·4986	·5008	·5029	·5051	·5073	2	4	7	9	11	13	15	17	20
27	0·5095	·5117	·5139	·5161	·5184	·5206	·5228	·5250	·5272	·5295	2	4	7	9	11	13	16	18	20
28	0·5317	·5340	·5362	·5384	·5407	·5430	·5452	·5475	·5498	·5520	2	5	7	9	11	14	16	18	20
29	0·5543	·5566	·5589	·5612	·5635	·5658	·5681	·5704	·5727	·5750	2	5	7	9	12	14	16	18	21
30	0·5774	·5797	·5820	·5844	·5867	·5890	·5914	·5938	·5961	·5985	2	5	7	9	12	14	16	19	21
31	0·6009	·6032	·6056	·6080	·6104	·6128	·6152	·6176	·6200	·6224	2	5	7	10	12	14	17	19	22
32	0·6249	·6273	·6297	·6322	·6346	·6371	·6395	·6420	·6445	·6469	2	5	7	10	12	15	17	20	22
33	0·6494	·6519	·6544	·6569	·6594	·6619	·6644	·6669	·6694	·6720	3	5	8	10	13	15	18	20	23
34	0·6745	·6771	·6796	·6822	·6847	·6873	·6899	·6924	·6950	·6976	3	5	8	10	13	15	18	21	23
35	0·7002	·7028	·7054	·7080	·7107	·7133	·7159	·7186	·7212	·7239	3	5	8	11	13	16	18	21	24
36	0·7265	·7292	·7319	·7346	·7373	·7400	·7427	·7454	·7481	·7508	3	5	8	11	14	16	19	22	24
37	0·7536	·7563	·7590	·7618	·7646	·7673	·7701	·7729	·7757	·7785	3	6	8	11	14	17	19	22	25
38	0·7813	·7841	·7869	·7898	·7926	·7954	·7983	·8012	·8040	·8069	3	6	9	11	14	17	20	23	26
39	0·8098	·8127	·8156	·8185	·8214	·8243	·8273	·8302	·8332	·8361	3	6	9	12	15	18	21	23	26
40	0·8391	·8421	·8451	·8481	·8511	·8541	·8571	·8601	·8632	·8662	3	6	9	12	15	18	21	24	27
41	0·8693	·8724	·8754	·8785	·8816	·8847	·8878	·8910	·8941	·8972	3	6	9	12	16	19	22	25	28
42	0·9004	·9036	·9067	·9099	·9131	·9163	·9195	·9228	·9260	·9293	3	6	10	13	16	19	22	26	29
43	0·9325	·9358	·9391	·9424	·9457	·9490	·9523	·9556	·9590	·9623	3	7	10	13	17	20	23	27	30
44	0·9657	·9691	·9725	·9759	·9793	·9827	·9861	·9896	·9930	·9965	3	7	10	14	17	21	24	27	31
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
45	1·000	1·003	1·007	1·011	1·014	1·018	1·021	1·025	1·028	1·032	0	1	1	1	2	2	2	3	3
46	1·036	1·039	1·043	1·046	1·050	1·054	1·057	1·061	1·065	1·069	0	1	1	1	2	2	3	3	3
47	1·072	1·076	1·080	1·084	1·087	1·091	1·095	1·099	1·103	1·107	0	1	1	2	2	2	3	3	3
48	1·111	1·115	1·118	1·122	1·126	1·130	1·134	1·138	1·142	1·146	0	1	1	2	2	2	3	3	4
49	1·150	1·154	1·159	1·163	1·167	1·171	1·175	1·179	1·183	1·188	0	1	1	2	2	2	3	3	4
50	1·192	1·196	1·200	1·205	1·209	1·213	1·217	1·222	1·226	1·230	0	1	1	2	2	3	3	4	4
51	1·235	1·239	1·244	1·248	1·253	1·257	1·262	1·266	1·271	1·275	0	1	1	2	2	3	3	4	4
52	1·280	1·285	1·289	1·294	1·299	1·303	1·308	1·313	1·317	1·322	0	1	1	2	2	3	3	4	4
53	1·327	1·332	1·337	1·342	1·347	1·351	1·356	1·361	1·366	1·371	0	1	1	2	2	3	3	4	4
54	1·376	1·381	1·387	1·392	1·397	1·402	1·407	1·412	1·418	1·423	1	1	2	2	3	3	4	4	5
55	1·428	1·433	1·439	1·444	1·450	1·455	1·460	1·466	1·471	1·477	1	1	2	2	3	3	4	4	5
56	1·483	1·488	1·494	1·499	1·505	1·511	1·517	1·522	1·528	1·534	1	1	2	2	3	3	4	5	5
57	1·540	1·546	1·552	1·558	1·564	1·570	1·576	1·582	1·588	1·594	1	1	2	2	3	4	4	5	5
58	1·600	1·607	1·613	1·619	1·625	1·632	1·638	1·645	1·651	1·658	1	1	2	3	3	4	4	5	6
59	1·664	1·671	1·678	1·684	1·691	1·698	1·704	1·711	1·718	1·725	1	1	2	3	3	4	5	5	6
60	1·732	1·739	1·746	1·753	1·760	1·767	1·775	1·782	1·789	1·797	1	1	2	3	4	4	5	6	6
61	1·804	1·811	1·819	1·827	1·834	1·842	1·849	1·857	1·865	1·873	1	2	2	3	4	5	5	6	7
62	1·881	1·889	1·897	1·905	1·913	1·921	1·929	1·937	1·946	1·954	1	2	2	3	4	5	6	7	7
63	1·963	1·971	1·980	1·988	1·997	2·006	2·014	2·023	2·032	2·041	1	2	3	4	4	5	6	7	8
64	2·050	2·059	2·069	2·078	2·087	2·097	2·106	2·116	2·125	2·135	1	2	3	4	5	6	7	8	8
65	2·145	2·154	2·164	2·174	2·184	2·194	2·204	2·215	2·225	2·236	1	2	3	4	5	6	7	8	9
66	2·246	2·257	2·267	2·278	2·289	2·300	2·311	2·322	2·333	2·344	1	2	3	4	5	7	8	9	10
67	2·356	2·367	2·379	2·391	2·402	2·414	2·426	2·438	2·450	2·463	1	2	4	5	6	7	8	10	11
68	2·475	2·488	2·500	2·513	2·526	2·539	2·552	2·565	2·578	2·592	1	3	4	5	7	8	9	10	12
69	2·605	2·619	2·633	2·646	2·660	2·675	2·689	2·703	2·718	2·733	1	3	4	6	7	9	10	11	13
70	2·747	2·762	2·778	2·793	2·808	2·824	2·840	2·856	2·872	2·888	2	3	5	6	8	9	11	13	14
71	2·904	2·921	2·937	2·954	2·971	2·989	3·006	3·024	3·042	3·060	2	3	5	7	9	10	12	14	16
72	3·078	3·096	3·115	3·133	3·152	3·172	3·191	3·211	3·230	3·251	2	4	6	8	10	12	14	15	17
73	3·271	3·291	3·312	3·333	3·354	3·376	3·398	3·420	3·442	3·465	2	4	6	9	11	13	15	17	19
74	3·487	3·511	3·534	3·558	3·582	3·606	3·630	3·655	3·681	3·706	2	5	7	10	12	15	17	20	22
75	3·732	3·758	3·785	3·812	3·839	3·867	3·895	3·923	3·952	3·981	3	6	8	11	14	17	20	22	25
76	4·011	4·041	4·071	4·102	4·134	4·165	4·198	4·230	4·264	4·297	3	6	10	13	16	19	22	26	29
77	4·331	4·366	4·402	4·437	4·474	4·511	4·548	4·586	4·625	4·666									
78	4·705	4·745	4·787	4·829	4·872	4·915	4·959	5·005	5·050	5·097									
79	5·145	5·193	5·242	5·292	5·343	5·396	5·449	5·503	5·558	5·614									
80	5·671	5·730	5·789	5·850	5·912	5·976	6·041	6·107	6·174	6·243	differences unreliable:								
81	6·314	6·386	6·460	6·535	6·612	6·691	6·772	6·855	6·940	7·026									
82	7·115	7·207	7·300	7·396	7·495	7·596	7·700	7·806	7·916	8·028									
83	8·144	8·264	8·386	8·513	8·643	8·777	8·915	9·058	9·205	9·357									
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20									
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95									
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46									
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27									
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08									
89	57·29	63·66	71·62	81·85	95·46	114·6	143·2	191·0	286·5	573·0									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix V: Square Roots of Numbers, 1-10

Square Roots of Numbers, 1-10

$x \rightarrow \sqrt{x}$

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	1·000	1·005	1·010	1·015	1·020	1·025	1·030	1·034	1·039	1·044	0	1	1	2	2	3	3	4	4
1.1	1·049	1·054	1·058	1·063	1·068	1·072	1·077	1·082	1·086	1·091	0	1	1	2	2	3	3	4	4
1.2	1·095	1·100	1·105	1·109	1·114	1·118	1·123	1·127	1·131	1·136	0	1	1	2	2	3	3	4	4
1.3	1·140	1·145	1·149	1·153	1·158	1·162	1·166	1·171	1·175	1·179	0	1	1	2	2	3	3	3	4
1.4	1·183	1·187	1·192	1·196	1·200	1·204	1·208	1·212	1·217	1·221	0	1	1	2	2	3	3	3	4
1.5	1·225	1·229	1·233	1·237	1·241	1·245	1·249	1·253	1·257	1·261	0	1	1	2	2	2	3	3	4
1.6	1·265	1·269	1·273	1·277	1·281	1·285	1·288	1·292	1·296	1·300	0	1	1	2	2	2	3	3	3
1.7	1·304	1·308	1·312	1·315	1·319	1·323	1·327	1·330	1·334	1·338	0	1	1	2	2	2	3	3	3
1.8	1·342	1·345	1·349	1·353	1·357	1·360	1·364	1·368	1·371	1·375	0	1	1	1	2	2	3	3	3
1.9	1·378	1·382	1·386	1·389	1·393	1·396	1·400	1·404	1·407	1·411	0	1	1	1	2	2	3	3	3
2.0	1·414	1·418	1·421	1·425	1·428	1·432	1·435	1·439	1·442	1·446	0	1	1	1	2	2	2	3	3
2.1	1·449	1·453	1·456	1·460	1·463	1·466	1·470	1·473	1·477	1·480	0	1	1	1	2	2	2	3	3
2.2	1·483	1·487	1·490	1·493	1·497	1·500	1·503	1·507	1·510	1·513	0	1	1	1	2	2	2	3	3
2.3	1·517	1·520	1·523	1·526	1·530	1·533	1·536	1·539	1·543	1·546	0	1	1	1	2	2	2	3	3
2.4	1·549	1·552	1·556	1·559	1·562	1·565	1·568	1·572	1·575	1·578	0	1	1	1	2	2	2	3	3
2.5	1·581	1·584	1·587	1·591	1·594	1·597	1·600	1·603	1·606	1·609	0	1	1	1	2	2	2	3	3
2.6	1·612	1·616	1·619	1·622	1·625	1·628	1·631	1·634	1·637	1·640	0	1	1	1	2	2	2	2	3
2.7	1·643	1·646	1·649	1·652	1·655	1·658	1·661	1·664	1·667	1·670	0	1	1	1	2	2	2	2	3
2.8	1·673	1·676	1·679	1·682	1·685	1·688	1·691	1·694	1·697	1·700	0	1	1	1	1	2	2	2	3
2.9	1·703	1·706	1·709	1·712	1·715	1·718	1·720	1·723	1·726	1·729	0	1	1	1	1	2	2	2	3
3.0	1·732	1·735	1·738	1·741	1·744	1·746	1·749	1·752	1·755	1·758	0	1	1	1	1	2	2	2	3
3.1	1·761	1·764	1·766	1·769	1·772	1·775	1·778	1·780	1·783	1·786	0	1	1	1	1	2	2	2	3
3.2	1·789	1·792	1·794	1·797	1·800	1·803	1·806	1·808	1·811	1·814	0	1	1	1	1	2	2	2	2
3.3	1·817	1·819	1·822	1·825	1·828	1·830	1·833	1·836	1·839	1·841	0	1	1	1	1	2	2	2	2
3.4	1·844	1·847	1·849	1·852	1·855	1·857	1·860	1·863	1·866	1·868	0	1	1	1	1	2	2	2	2
3.5	1·871	1·874	1·876	1·879	1·882	1·884	1·887	1·889	1·892	1·895	0	1	1	1	1	2	2	2	2
3.6	1·897	1·900	1·903	1·905	1·908	1·911	1·913	1·916	1·918	1·921	0	1	1	1	1	2	2	2	2
3.7	1·924	1·926	1·929	1·931	1·934	1·937	1·939	1·942	1·944	1·947	0	1	1	1	1	2	2	2	2
3.8	1·949	1·952	1·955	1·957	1·960	1·962	1·965	1·967	1·970	1·972	0	1	1	1	1	2	2	2	2
3.9	1·975	1·977	1·980	1·982	1·985	1·988	1·990	1·993	1·995	1·998	0	1	1	1	1	2	2	2	2
4.0	2·000	2·003	2·005	2·008	2·010	2·013	2·015	2·017	2·020	2·022	0	0	1	1	1	1	2	2	2
4.1	2·025	2·027	2·030	2·032	2·035	2·037	2·040	2·042	2·045	2·047	0	0	1	1	1	1	2	2	2
4.2	2·049	2·052	2·054	2·057	2·059	2·062	2·064	2·066	2·069	2·071	0	0	1	1	1	1	2	2	2
4.3	2·074	2·076	2·078	2·081	2·083	2·086	2·088	2·091	2·093	2·095	0	0	1	1	1	1	2	2	2
4.4	2·098	2·100	2·102	2·105	2·107	2·110	2·112	2·114	2·117	2·119	0	0	1	1	1	1	2	2	2
4.5	2·121	2·124	2·126	2·128	2·131	2·133	2·135	2·138	2·140	2·142	0	0	1	1	1	1	2	2	2
4.6	2·145	2·147	2·149	2·152	2·154	2·156	2·159	2·161	2·163	2·166	0	0	1	1	1	1	2	2	2
4.7	2·168	2·170	2·173	2·175	2·177	2·179	2·182	2·184	2·186	2·189	0	0	1	1	1	1	2	2	2
4.8	2·191	2·193	2·195	2·198	2·200	2·202	2·205	2·207	2·209	2·211	0	0	1	1	1	1	2	2	2
4.9	2·214	2·216	2·218	2·220	2·223	2·225	2·227	2·229	2·232	2·234	0	0	1	1	1	1	2	2	2
5.0	2·236	2·238	2·241	2·243	2·245	2·247	2·249	2·252	2·254	2·256	0	0	1	1	1	1	2	2	2
5.1	2·258	2·261	2·263	2·265	2·267	2·269	2·272	2·274	2·276	2·278	0	0	1	1	1	1	2	2	2
5.2	2·280	2·283	2·285	2·287	2·289	2·291	2·294	2·296	2·298	2·300	0	0	1	1	1	1	2	2	2
5.3	2·302	2·304	2·307	2·309	2·311	2·313	2·315	2·317	2·320	2·322	0	0	1	1	1	1	2	2	2
5.4	2·324	2·326	2·328	2·330	2·332	2·335	2·337	2·339	2·341	2·343	0	0	1	1	1	1	1	2	2
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5·5	2·345	2·347	2·350	2·352	2·354	2·356	2·358	2·360	2·362	2·364	0	0	1	1	1	1	1	2	2
5·6	2·366	2·369	2·371	2·373	2·375	2·377	2·379	2·381	2·383	2·385	0	0	1	1	1	1	1	2	2
5·7	2·388	2·390	2·392	2·394	2·396	2·398	2·400	2·402	2·404	2·406	0	0	1	1	1	1	1	2	2
5·8	2·408	2·410	2·412	2·415	2·417	2·419	2·421	2·423	2·425	2·427	0	0	1	1	1	1	1	2	2
5·9	2·429	2·431	2·433	2·435	2·437	2·439	2·441	2·443	2·445	2·447	0	0	1	1	1	1	1	2	2
6·0	2·450	2·452	2·454	2·456	2·458	2·460	2·462	2·464	2·466	2·468	0	0	1	1	1	1	1	2	2
6·1	2·470	2·472	2·474	2·476	2·478	2·480	2·482	2·484	2·486	2·488	0	0	1	1	1	1	1	2	2
6·2	2·490	2·492	2·494	2·496	2·498	2·500	2·502	2·504	2·506	2·508	0	0	1	1	1	1	1	2	2
6·3	2·510	2·512	2·514	2·516	2·518	2·520	2·522	2·524	2·526	2·528	0	0	1	1	1	1	1	2	2
6·4	2·530	2·532	2·534	2·536	2·538	2·540	2·542	2·544	2·546	2·548	0	0	1	1	1	1	1	2	2
6·5	2·550	2·551	2·653	2·555	2·557	2·559	2·561	2·563	2·565	2·567	0	0	1	1	1	1	1	2	2
6·6	2·569	2·571	2·573	2·575	2·577	2·579	2·581	2·583	2·585	2·587	0	0	1	1	1	1	1	2	2
6·7	2·588	2·590	2·592	2·594	2·596	2·598	2·600	2·602	2·604	2·606	0	0	1	1	1	1	1	2	2
6·8	2·608	2·610	2·612	2·613	2·615	2·617	2·619	2·621	2·623	2·625	0	0	1	1	1	1	1	2	2
6·9	2·627	2·629	2·631	2·632	2·634	2·636	2·638	2·640	2·642	2·644	0	0	1	1	1	1	1	2	2
7·0	2·646	2·648	2·650	2·651	2·653	2·655	2·657	2·659	2·661	2·663	0	0	1	1	1	1	1	2	2
7·1	2·665	2·667	2·668	2·670	2·672	2·674	2·676	2·678	2·680	2·681	0	0	1	1	1	1	1	1	2
7·2	2·683	2·685	2·687	2·689	2·691	2·693	2·694	2·696	2·698	2·700	0	0	1	1	1	1	1	1	2
7·3	2·702	2·704	2·706	2·707	2·709	2·711	2·713	2·715	2·717	2·719	0	0	1	1	1	1	1	1	2
7·4	2·720	2·722	2·724	2·726	2·728	2·729	2·731	2·733	2·735	2·737	0	0	1	1	1	1	1	1	2
7·5	2·739	2·740	2·742	2·744	2·746	2·748	2·750	2·751	2·753	2·755	0	0	1	1	1	1	1	1	2
7·6	2·757	2·759	2·760	2·762	2·764	2·766	2·768	2·769	2·771	2·773	0	0	1	1	1	1	1	1	2
7·7	2·775	2·777	2·778	2·780	2·782	2·784	2·786	2·787	2·789	2·791	0	0	1	1	1	1	1	1	2
7·8	2·793	2·795	2·796	2·798	2·800	2·802	2·804	2·805	2·807	2·809	0	0	1	1	1	1	1	1	2
7·9	2·811	2·812	2·814	2·816	2·818	2·820	2·821	2·823	2·825	2·827	0	0	1	1	1	1	1	1	2
8·0	2·828	2·830	2·832	2·834	2·635	2·837	2·839	2·841	2·843	2·844	0	0	1	1	1	1	1	1	2
8·1	2·846	2·848	2·850	2·851	2·853	2·855	2·857	2·858	2·860	2·862	0	0	1	1	1	1	1	1	2
8·2	2·864	2·865	2·867	2·869	2·871	2·872	2·874	2·876	2·877	2·879	0	0	1	1	1	1	1	1	2
8·3	2·881	2·883	2·884	2·886	2·888	2·890	2·891	2·893	2·895	2·897	0	0	1	1	1	1	1	1	2
8·4	2·898	2·900	2·902	2·903	2·905	2·907	2·909	2·910	2·912	2·914	0	0	1	1	1	1	1	1	2
8·5	2·915	2·917	2·919	2·921	2·922	2·924	2·926	2·927	2·929	2·931	0	0	1	1	1	1	1	1	2
8·6	2·933	2·934	2·936	2·938	2·939	2·941	2·943	2·944	2·946	2·948	0	0	1	1	1	1	1	1	2
8·7	2·950	2·951	2·953	2·955	2·956	2·958	2·960	2·961	2·963	2·965	0	0	1	1	1	1	1	1	2
8·8	2·966	2·968	2·970	2·972	2·973	2·975	2·977	2·978	2·980	2·982	0	0	1	1	1	1	1	1	2
8·9	2·983	2·985	2·987	2·988	2·990	2·992	2·993	2·995	2·997	2·998	0	0	1	1	1	1	1	1	2
9·0	3·000	3·002	3·003	3·005	3·007	3·008	3·010	3·012	3·013	3·015	0	0	0	1	1	1	1	1	1
9·1	3·017	3·018	3·020	3·022	3·023	3·025	3·027	3·028	3·030	3·032	0	0	0	1	1	1	1	1	1
9·2	3·033	3·035	3·036	3·038	3·040	3·041	3·043	3·045	3·046	3·048	0	0	0	1	1	1	1	1	1
9·3	3·050	3·051	3·053	3·055	3·056	3·058	3·059	3·061	3·063	3·064	0	0	0	1	1	1	1	1	1
9·4	3·066	3·068	3·069	3·071	3·072	3·074	3·076	3·077	3·079	3·081	0	0	0	1	1	1	1	1	1
9·5	3·082	3·084	3·085	3·087	3·089	3·090	3·092	3·094	3·095	3·097	0	0	0	1	1	1	1	1	1
9·6	3·098	3·100	3·102	3·103	3·105	3·106	3·108	3·110	3·111	3·113	0	0	0	1	1	1	1	1	1
9·7	3·115	3·116	3·118	3·119	3·121	3·123	3·124	3·126	3·127	3·129	0	0	0	1	1	1	1	1	1
9·8	3·131	3·132	3·134	3·135	3·137	3·139	3·140	3·142	3·143	2·145	0	0	0	1	1	1	1	1	1
9·9	3·146	3·148	3·150	3·151	3·153	3·154	3·156	3·158	3·159	3·161	0	0	0	1	1	1	1	1	1
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Appendix VI: Square Roots of Numbers, 10-100

Square Roots of Numbers 10-100

$x \rightarrow \sqrt{x}$

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	3·162	3·178	3·194	3·209	3·225	3·240	3·256	3·271	3·286	3·302	2	3	5	6	8	9	11	12	14
11	3·317	3·332	3·347	3·362	3·376	3·391	3·406	3·421	3·435	3·450	1	3	4	6	7	9	10	12	13
12	3·464	3·479	3·493	3·507	3·521	3·536	3·550	3·564	3·578	3·592	1	3	4	6	7	8	10	11	13
13	3·606	3·619	3·633	3·647	3·661	3·674	3·688	3·701	3·715	3·728	1	3	4	5	7	8	10	11	12
14	3·742	3·755	3·768	3·782	3·795	3·808	3·821	3·834	3·847	3·860	1	3	4	5	7	8	9	11	12
15	3·873	3·886	3·899	3·912	3·924	3·937	3·950	3·962	3·975	3·988	1	3	4	5	6	8	9	10	11
16	4·000	4·012	4·025	4·037	4·050	4·062	4·074	4·087	4·099	4·111	1	2	4	5	6	7	9	10	11
17	4·123	4·135	4·147	4·159	4·171	4·183	4·195	4·207	4·219	4·231	1	2	4	5	6	7	8	10	11
18	4·243	4·254	4·266	4·278	4·290	4·301	4·313	4·324	4·336	4·347	1	2	3	5	6	7	8	9	10
19	4·359	4·370	4·382	4·393	4·405	4·416	4·427	4·438	4·450	4·461	1	2	3	5	6	7	8	9	10
20	4·472	4·483	4·494	4·506	4·517	4·528	4·539	4·550	4·561	4·572	1	2	3	4	6	7	8	9	10
21	4·583	4·594	4·604	4·615	4·626	4·637	4·648	4·658	4·669	4·680	1	2	3	4	5	6	8	9	10
22	4·690	4·701	4·712	4·722	4·733	4·743	4·754	4·765	4·775	4·785	1	2	3	4	5	6	7	8	9
23	4·796	4·806	4·817	4·827	4·837	4·848	4·858	4·868	4·879	4·889	1	2	3	4	5	6	7	8	9
24	4·899	4·909	4·919	4·930	4·940	4·950	4·960	4·970	4·980	4·990	1	2	3	4	5	6	7	8	9
25	5·000	5·010	5·020	5·030	5·040	5·050	5·060	5·070	5·079	5·089	1	2	3	4	5	6	7	8	9
26	5·099	5·109	5·119	5·128	5·138	5·148	5·158	5·167	5·177	5·187	1	2	3	4	5	6	7	8	9
27	5·196	5·206	5·215	5·225	5·235	5·244	5·254	5·263	5·273	5·282	1	2	3	4	5	6	7	8	9
28	5·292	5·301	5·310	5·320	5·329	5·339	5·348	5·357	5·367	5·376	1	2	3	4	5	6	7	7	8
29	5·385	5·394	5·404	5·413	5·422	5·431	5·441	5·450	5·459	5·468	1	2	3	4	5	5	6	7	8
30	5·477	5·486	5·495	5·505	5·514	5·523	5·532	5·541	5·550	5·559	1	2	3	4	4	5	6	7	8
31	5·568	5·577	5·586	5·596	5·604	5·612	5·621	5·630	5·639	5·648	1	2	3	3	4	5	6	7	8
32	5·657	5·666	5·675	5·683	5·692	5·701	5·710	5·718	5·727	5·736	1	2	3	3	4	5	6	7	8
33	5·745	5·753	5·762	5·771	5·779	5·788	5·797	5·805	5·814	5·822	1	2	3	3	4	5	6	7	8
34	5·831	5·840	5·848	5·857	5·865	5·874	5·882	5·891	5·899	5·908	1	2	3	3	4	5	6	7	8
35	5·916	5·925	5·933	5·941	5·950	5·958	5·967	5·975	5·983	5·992	1	2	2	3	4	5	6	7	8
36	6·000	6·008	6·017	6·025	6·033	6·042	6·050	6·058	6·066	6·075	1	2	2	3	4	5	6	7	7
37	6·083	6·091	6·099	6·107	6·116	6·124	6·132	6·140	6·148	6·156	1	2	2	3	4	5	6	7	7
38	6·164	6·173	6·181	6·189	6·197	6·205	6·213	6·221	6·229	6·237	1	2	2	3	4	5	6	6	7
39	6·245	6·253	6·261	6·269	6·277	6·285	6·293	6·301	6·309	6·317	1	2	2	3	4	5	6	6	7
40	6·325	6·332	6·340	6·348	6·356	6·364	6·372	6·380	6·387	6·395	1	2	2	3	4	5	6	6	7
41	6·403	6·411	6·419	6·427	6·434	6·442	6·450	6·458	6·465	6·473	1	2	2	3	4	5	5	6	7
42	6·481	6·488	6·496	6·504	6·512	6·519	6·527	6·535	6·542	6·550	1	2	2	3	4	5	5	6	7
43	6·557	6·565	6·573	6·580	6·588	6·595	6·603	6·611	6·618	6·626	1	2	2	3	4	5	5	6	7
44	6·633	6·641	6·648	6·656	6·663	6·671	6·678	6·686	6·693	6·701	1	2	2	3	4	5	5	6	7
45	6·708	6·716	6·723	6·731	6·738	6·745	6·753	6·760	6·768	6·775	1	1	2	3	4	4	5	6	7
46	6·782	6·790	6·797	6·804	6·812	6·819	6·826	6·834	6·841	6·848	1	1	2	3	4	4	5	6	7
47	6·856	6·863	6·870	6·878	6·885	6·892	6·899	6·907	6·914	6·921	1	1	2	3	4	4	5	6	7
48	6·928	6·935	6·943	6·950	6·957	6·964	6·971	6·979	6·986	6·993	1	1	2	3	4	4	5	6	6
49	7·000	7·007	7·014	7·021	7·029	7·036	7·043	7·050	7·057	7·064	1	1	2	3	4	4	5	6	6
50	7·071	7·078	7·085	7·092	7·099	7·106	7·113	7·120	7·127	7·134	1	1	2	3	4	4	5	6	6
51	7·141	7·148	7·155	7·162	7·169	7·176	7·183	7·190	7·197	7·204	1	1	2	3	4	4	5	6	6
52	7·211	7·218	7·225	7·232	7·239	7·246	7·253	7·259	7·266	7·273	1	1	2	3	3	4	5	6	6
53	7·280	7·287	7·294	7·301	7·308	7·314	7·321	7·328	7·335	7·342	1	1	2	3	3	4	5	5	6
54	7·349	7·355	7·362	7·369	7·376	7·382	7·389	7·396	7·403	7·410	1	1	2	3	3	4	5	5	6
55	7·416	7·423	7·430	7·436	7·443	7·450	7·457	7·463	7·470	7·477	1	1	2	3	3	4	5	5	6
56	7·483	7·490	7·497	7·503	7·510	7·517	7·523	7·530	7·537	7·543	1	1	2	3	3	4	5	5	6
57	7·550	7·556	7·563	7·570	7·576	7·583	7·589	7·596	7·603	7·609	1	1	2	3	3	4	5	5	6
58	7·616	7·622	7·629	7·635	7·642	7·649	7·655	7·662	7·668	7·675	1	1	2	3	3	4	5	5	6
59	7·681	7·688	7·694	7·701	7·707	7·714	7·720	7·727	7·733	7·740	1	1	2	3	3	4	4	5	6
60	7·746	7·752	7·759	7·765	7·772	7·778	7·785	7·791	7·797	7·804	1	1	2	3	3	4	4	5	6
61	7·810	7·817	7·823	7·829	7·836	7·842	7·849	7·855	7·861	7·868	1	1	2	3	3	4	4	5	6
62	7·874	7·880	7·887	7·893	7·899	7·906	7·912	7·918	7·925	7·931	1	1	2	3	3	4	4	5	6
63	7·937	7·944	7·950	7·956	7·962	7·969	7·975	7·981	7·987	7·994	1	1	2	3	3	4	4	5	6
64	8·000	8·006	8·012	8·019	8·025	8·031	8·037	8·044	8·050	8·056	1	1	2	2	3	4	4	5	6
65	8·062	8·068	8·075	8·081	8·087	8·093	8·099	8·106	8·112	8·118	1	1	2	2	3	4	4	5	6
66	8·124	8·130	8·136	8·142	8·149	8·155	8·161	8·167	8·173	8·179	1	1	2	2	3	4	4	5	5
67	8·185	8·191	8·198	8·204	8·210	8·216	8·222	8·228	8·234	8·240	1	1	2	2	3	4	4	5	5
68	8·246	8·252	8·258	8·264	8·270	8·276	8·283	8·289	8·295	8·301	1	1	2	2	3	4	4	5	5
69	8·307	8·313	8·319	8·325	8·331	8·337	8·343	8·349	8·355	8·361	1	1	2	2	3	4	4	5	5
70	8·367	8·373	8·379	8·385	8·390	8·396	8·402	8·408	8·414	8·420	1	1	2	2	3	4	4	5	5
71	8·426	8·432	8·438	8·444	8·450	8·456	8·462	8·468	8·473	8·479	1	1	2	2	3	4	4	5	5
72	8·485	8·491	8·497	8·503	8·509	8·515	8·521	8·526	8·532	8·538	1	1	2	2	3	3	4	5	5
73	8·544	8·550	8·556	8·562	8·567	8·573	8·579	8·585	8·591	8·597	1	1	2	2	3	3	4	5	5
74	8·602	8·608	8·614	8·620	8·626	8·631	8·637	8·643	8·649	8·654	1	1	2	2	3	3	4	5	5
75	8·660	8·666	8·672	8·678	8·683	8·689	8·695	8·701	8·706	8·712	1	1	2	2	3	3	4	5	5
76	8·718	8·724	8·729	8·735	8·741	8·746	8·752	8·758	8·764	8·769	1	1	2	2	3	3	4	5	5
77	8·775	8·781	8·786	8·792	8·798	8·803	8·809	8·815	8·820	8·826	1	1	2	2	3	3	4	4	5
78	8·832	8·837	8·843	8·849	8·854	8·860	8·866	8·871	8·877	8·883	1	1	2	2	3	3	4	4	5
79	8·888	8·894	8·899	8·905	8·911	8·916	8·922	8·927	8·933	8·939	1	1	2	2	3	3	4	4	5
80	8·944	8·950	8·955	8·961	8·967	8·972	8·978	8·983	8·989	8·994	1	1	2	2	3	3	4	4	5
81	9·000	9·006	9·011	9·017	9·022	9·028	9·033	9·039	9·044	9·050	1	1	2	2	3	3	4	4	5
82	9·055	9·061	9·066	9·072	9·077	9·083	9·088	9·094	9·099	9·105	1	1	2	2	3	3	4	4	5
83	9·110	9·116	9·121	9·127	9·132	9·138	9·143	9·149	9·154	9·160	1	1	2	2	3	3	4	4	5
84	9·165	9·171	9·176	9·182	9·187	9·192	9·198	9·203	9·209	9·214	1	1	2	2	3	3	4	4	5
85	9·220	9·225	9·230	9·236	9·241	9·247	9·252	9·257	9·263	9·268	1	1	2	2	3	3	4	4	5
86	9·274	9·279	9·284	9·290	9·295	9·301	9·306	9·311	9·317	9·322	1	1	2	2	3	3	4	4	5
87	9·237	9·333	9·338	9·343	9·349	9·354	9·359	9·365	9·370	9·375	1	1	2	2	3	3	4	4	5
88	9·381	9·386	9·391	9·397	9·402	9·407	9·413	9·418	9·423	9·429	1	1	2	2	3	3	4	4	5
89	9·434	9·439	9·445	9·450	9·455	9·460	9·466	9·471	9·476	9·482	1	1	2	2	3	3	4	4	5
90	9·487	9·492	9·497	9·503	9·508	9·513	9·518	9·524	9·529	9·534	1	1	2	2	3	3	4	4	5
91	9·539	9·545	9·550	9·555	9·560	9·566	9·571	9·576	9·581	9·586	1	1	2	2	3	3	4	4	5
92	9·592	9·597	9·602	9·607	9·613	9·618	9·623	9·628	9·633	9·638	1	1	2	2	3	3	4	4	5
93	9·644	9·649	9·654	9·659	9·664	9·670	9·675	9·680	9·685	9·690	1	1	2	2	3	3	4	4	5
94	9·695	9·701	9·706	9·711	9·716	9·721	9·726	9·731	9·737	9·742	1	1	2	2	3	3	4	4	5
95	9·747	9·752	9·757	9·762	9·767	9·772	9·778	9·783	9·788	9·793	1	1	2	2	3	3	4	4	5
96	9·798	9·803	9·808	9·813	9·818	9·823	9·829	9·834	9·839	9·844	1	1	2	2	3	3	4	4	5
97	9·849	9·854	9·859	9·864	9·869	9·874	9·879	9·884	9·889	9·894	1	1	2	2	3	3	4	4	5
98	9·900	9·905	9·910	9·915	9·920	9·925	9·930	9·935	9·940	9·945	0	1	1	2	2	3	3	4	4
99	9·950	9·955	9·960	9·965	9·870	9·975	9·980	9·985	9·990	9·996	0	1	1	2	2	3	3	4	4
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Appendix VII: Reciprocals of Numbers

Reciprocals of Numbers

$x \rightarrow 1/x$

											SUBTRACT Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	1000	9901	9804	9709	9615	9524	9434	9346	9259	9174	9	18	28	37	46	55	64	74	83
11	9091	9009	8929	8850	8772	8696	8621	8547	8475	8403	8	15	23	31	38	46	53	61	69
12	8333	8264	8197	8130	8065	8000	7937	7874	7813	7752	7	13	20	26	33	39	46	52	59
13	7692	7634	7576	7519	7463	7407	7353	7299	7246	7194	6	11	17	22	28	33	39	44	50
14	7143	7092	7042	6993	6944	6897	6849	6803	6757	6711	5	10	14	19	24	29	33	38	43
15	6667	6623	6579	6536	6494	6452	6410	6369	6329	6289	4	8	13	17	21	25	29	33	38
16	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917	4	7	11	15	18	22	26	29	33
17	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587	3	7	10	13	16	20	23	26	29
18	5556	5525	5495	5464	5435	5405	5376	5348	5319	5291	3	6	9	12	15	17	20	23	26
19	5263	5236	5208	5181	5155	5128	5102	5076	5051	5025	3	5	8	11	13	16	18	21	24
20	5000	4975	4950	4926	4902	4878	4854	4831	4808	4785	2	5	7	10	12	14	17	19	21
21	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566	2	4	7	9	11	13	15	17	19
22	4545	4525	4505	4484	4464	4444	4425	4405	4386	4367	2	4	6	8	10	12	14	16	18
23	4348	4329	4310	4292	4274	4255	4237	4219	4202	4184	2	4	5	7	9	11	13	14	16
24	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016	2	3	5	7	8	10	12	13	15
25	4000	3981	3968	3953	3937	3922	3906	3891	3876	3861	2	3	5	6	8	9	11	12	14
26	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717	1	3	4	6	7	9	10	11	13
27	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584	1	3	4	5	7	8	9	11	12
28	3571	3559	3546	3534	3521	3509	3497	3484	3472	3460	1	2	4	5	6	7	9	10	11
29	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344	1	2	3	5	6	7	8	9	10
30	3333	3322	3311	3300	3289	3279	3268	3257	3247	3236	1	2	3	4	5	6	8	9	10
31	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135	1	2	3	4	5	6	7	8	9
32	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040	1	2	3	4	5	6	7	8	9
33	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950	1	2	3	4	4	5	6	7	8
34	2941	2933	2924	2915	2907	2899	2890	2882	2874	2865	1	2	3	3	4	5	6	7	8
35	2857	2849	2841	2833	2825	2817	2809	2801	2793	2786	1	2	2	3	4	5	6	6	7
36	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710	1	2	2	3	4	5	5	6	7
37	2703	2695	2688	2681	2674	2667	2660	2653	2646	2639	1	1	2	3	4	4	5	6	6
38	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571	1	1	2	3	3	4	5	5	6
39	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506	1	1	2	3	3	4	4	5	6
40	2500	2494	2488	2481	2475	2469	2463	2457	2451	2445	1	1	2	2	3	4	4	5	5
41	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387	1	1	2	2	3	3	4	5	5
42	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331	1	1	2	2	3	3	4	4	5
43	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278	1	1	2	2	3	3	4	4	5
44	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227	1	1	2	2	3	3	4	4	5
45	2222	2217	2212	2208	2203	2198	2193	2188	2183	2179	0	1	1	2	2	3	3	4	4
46	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132	0	1	1	2	2	3	3	4	4
47	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088	0	1	1	2	2	3	3	4	4
48	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045	0	1	1	2	2	3	3	3	4
49	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004	0	1	1	2	2	2	3	3	4
50	2000	1996	1992	1988	1984	1980	1976	1972	1969	1965	0	1	1	2	2	2	3	3	4
51	1961	1957	1953	1949	1946	1941	1938	1934	1931	1927	0	1	1	2	2	2	3	3	3
52	1923	1919	1916	1912	1908	1905	1901	1898	1894	1890	0	1	1	1	2	2	3	3	3
53	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855	0	1	1	1	2	2	2	3	3
54	1852	1848	1845	1842	1838	1835	1832	1828	1825	1821	0	1	1	1	2	2	2	3	3
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

											SUBTRACT Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	1818	1815	1812	1808	1805	1802	1799	1795	1792	1789	0	1	1	1	2	2	2	3	3
56	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757	0	1	1	1	2	2	2	3	3
57	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727	0	1	1	1	1	2	2	2	3
58	1724	1721	1718	1715	1712	1709	1706	1704	1701	1696	0	1	1	1	1	2	2	2	3
59	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669	0	1	1	1	1	2	2	2	3
60	1667	1664	1661	1658	1656	1653	1650	1647	1645	1642	0	1	1	1	1	2	2	2	3
61	1639	1637	1634	1631	1629	1626	1623	1621	1618	1616	0	1	1	1	1	2	2	2	2
62	1613	1610	1608	1605	1603	1600	1597	1595	1592	1590	0	1	1	1	1	2	2	2	2
63	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565	0	0	1	1	1	1	2	2	2
64	1562	1560	1558	1555	1553	1550	1548	1546	1543	1541	0	0	1	1	1	1	2	2	2
65	1538	1536	1534	1531	1529	1527	1524	1522	1520	1517	0	0	1	1	1	1	2	2	2
66	1515	1513	1511	1508	1506	1504	1502	1499	1497	1495	0	0	1	1	1	1	2	2	2
67	0·1493	1490	1488	1486	1484	1481	1479	1477	1475	1473	0	0	1	1	1	1	2	2	2
68	1471	1468	1466	1464	1462	1460	1458	1456	1453	1451	0	0	1	1	1	1	2	2	2
69	1449	1447	1445	1443	1441	1439	1437	1435	1433	1431	0	0	1	1	1	1	2	2	2
70	1429	1427	1425	1422	1420	1418	1416	1414	1412	1410	0	0	1	1	1	1	1	2	2
71	1408	1406	1404	1403	1401	1399	1397	1395	1393	1391	0	0	1	1	1	1	1	2	2
72	1389	1387	1385	1383	1381	1379	1377	1376	1374	1372	0	0	1	1	1	1	1	2	2
73	1370	1368	1366	1364	1362	1361	1359	1357	1355	1353	0	0	1	1	1	1	1	2	2
74	1351	1350	1348	1346	1344	1342	1340	1339	1337	1335	0	0	1	1	1	1	1	1	2
75	1333	1332	1330	1328	1326	1325	1323	1321	1319	1318	0	0	1	1	1	1	1	1	2
76	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300	0	0	1	1	1	1	1	1	2
77	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284	0	0	0	1	1	1	1	1	1
78	1282	1280	1279	1277	1276	1274	1272	1271	1269	1267	0	0	0	1	1	1	1	1	1
79	1266	1264	1263	1261	1259	1258	1256	1255	1253	1252	0	0	0	1	1	1	1	1	1
80	1250	1248	1247	1245	1244	1242	1241	1239	1238	1236	0	0	0	1	1	1	1	1	1
81	1235	1233	1232	1230	1229	1227	1225	1224	1222	1221	0	0	0	1	1	1	1	1	1
82	1220	1218	1217	1215	1214	1212	1211	1209	1208	1206	0	0	0	1	1	1	1	1	1
83	1205	1203	1202	1200	1199	1198	1196	1195	1193	1192	0	0	0	1	1	1	1	1	1
84	1190	1189	1188	1186	1185	1183	1182	1181	1179	1178	0	0	0	1	1	1	1	1	1
85	1176	1175	1174	1172	1171	1170	1168	1167	1166	1164	0	0	0	1	1	1	1	1	1
86	1163	1161	1160	1159	1157	1156	1155	1153	1152	1151	0	0	0	1	1	1	1	1	1
87	1149	1148	1147	1145	1144	1143	1142	1140	1139	1138	0	0	0	1	1	1	1	1	1
88	1136	1135	1134	1133	1131	1130	1129	1127	1126	1125	0	0	0	1	1	1	1	1	1
89	1124	1122	1121	1120	1119	1117	1116	1115	1114	1112	0	0	0	1	1	1	1	1	1
90	1111	1110	1109	1107	1106	1105	1104	1103	1101	1100	0	0	0	1	1	1	1	1	1
91	1099	1098	1096	1095	1094	1093	1092	1090	1089	1088	0	0	0	0	1	1	1	1	1
92	1087	1086	1085	1083	1082	1081	1080	1079	1078	1076	0	0	0	0	1	1	1	1	1
93	1075	1074	1073	1072	1071	1070	1068	1067	1066	1065	0	0	0	0	1	1	1	1	1
94	1064	1063	1062	1060	1059	1058	1057	1056	1055	1054	0	0	0	0	1	1	1	1	1
95	1053	1052	1050	1049	1048	1047	1046	1045	1044	1043	0	0	0	0	1	1	1	1	1
96	1042	1041	1039	1038	1037	1036	1035	1034	1033	1032	0	0	0	0	1	1	1	1	1
97	1031	1030	1029	1028	1027	1026	1025	1024	1022	1021	0	0	0	0	1	1	1	1	1
98	1020	1019	1018	1017	1016	1015	1014	1013	1012	1011	0	0	0	0	1	1	1	1	1
99	1010	1009	1008	1007	1006	1005	1004	1003	1002	1001	0	0	0	0	0	1	1	1	1
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

FUNDED BY

IN PARTNERSHIP WITH