INVESTIGATION OF THE MATHEMATICAL MODEL OF A SINGLE PENDULUM UNDER THE ACTION OF THE FOLLOWER FORCE

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Abstract: One of the main constructive elements of road-makers, railway bridge supports and structures is a compressed rod, to the end of which a follower force is applied. Recently, the most frequently used model of such rod in the form of an inverted mathematical pendulum under the influence of an asymmetric follower force. Asymmetry is due to the simultaneous presence of both angular and linear eccentricities. The work is devoted to the study of vertical and non-vertical states of equilibrium of a single pendulum. The reduced mathematical model of a single inverted mathematical pendulum is generalized, since it takes into account both the angular eccentricity and the linear eccentricity of the follower force. In addition, the coefficients of influence allow to consider all types of elastic elements (rigid, soft or linear). In this case, both elements can have characteristics of the same type or of different types. For direct integration of the differential equation of the pendulum motion, and also the decoupling of the corresponding Cauchy problem, the authors use the method of extending the parameter of the outstanding Japanese scientist Y.A. Shinohara. Varying of the angular eccentricity of the follower force at zero linear eccentricity results in the inverted pendulum having one or three non-vertical equilibrium positions. The type of characteristics of the elastic elements affects the maximum possible deviation from the vertical, at which the pendulum will be in a state of equilibrium. Analysis of the results of computer simulation shows that the orientation of the follower force for fixed values of other parameters of the pendulum has a significant effect on the configuration of the equilibrium curve.

Keywords: single pendulum, equilibrium states, follower force, bifurcations, mathematical model, orientation parameter, catastrophe, phase space, mechanical system, eccentricity.

1. Introduction

The concept of bifurcations of stationary states is widely used in various fields of science. Interest that manifests itself to them recently in many countries is explained by the fact that the task of identifying the transformation of quantitative changes into qualitative changes is key in revealing the fundamental secrets of nature and the underlying processes and phenomena occurring in it.

The phenomenon of bifurcation (Latin Bifurcus – bifurcation, branching) in the phase space or in the space of states of the dynamic system corresponds to a qualitative change in the character of the motion of the output mechanical system.

Mathematical pendulums with an arbitrary number of links are constructed in [9]. An analysis of the dynamics development of pendulum systems is given in [10]. The research results of bifurcations of equilibrium states of an inverted double pendulum loaded at the upper end by an asymmetric follower force are presented in [1–10]. The equilibrium states of a single inverted pendulum are considered in this paper.

2. Methods

The design scheme of the inverted single pendulum is shown in **Fig. 1**, consists of a weightless rod OA_1 1 of length l_1 and a material point A_1 of mass m_1. At the point of support O there is visco-elastic hinge (realized, for example, by means of a spiral spring and a hydraulic damper). Let c_1 be the stiffness of the spiral spring, μ_1 – the viscosity coefficient in the hinge O, and also takes into account the effect of external friction. The upper end of the pendulum is resiliently fixed by means of a horizontal spring of rigidity c. Similarly to [2], let's assume that in the vertical position of the pendulum all elastic elements are deformed. The follower force $\vec{P}$ is also applied to the upper end of the pendulum. The angle between this force and the vertical will be denoted by $a = \delta + k\varphi_1$. Here $\delta = \text{const}$ – the angular eccentricity; $k = \text{const}$ – the parameter of the follower force orientation; φ_1 – angle of the pendulum deviation from the vertical.

The equations of the perturbed motion of the pendulum have the form:

$$m_1 l_1^2 \ddot{\varphi}_1 = m_1 g l_1 \sin\varphi_1 + P\{l_1 \sin[(1-k)\varphi_1 - \delta] + \varepsilon\cos[(1-k)\varphi_1 - \delta]\} - (q_1 Q_c' + q_2 Q_c'' + q_3 Q_c''')\, l_1 \cos\varphi_1 - (q_{11} M_1' + q_{12} M_1'' + q_{13} M_1'''). \quad (1)$$

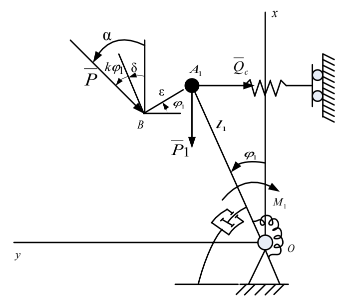

Fig. 1. The computational model of a single inverted pendulum

This mathematical model of the inverted single pendulum is generalized, since in equation (1) the angular eccentricity δ and the linear eccentricity ε and in addition, the influence factors q_i, q_{ij} $(i, j = 1, 2)$ allow to consider all possible types of elastic elements (soft, rigid, linear).

In order to reduce the number of parameters, let's proceed to dimensionless quantities by taking the values of m_1, l_1 and c_1 as units of measurement, and all other quantities will be taken to these three (basic) quantities. Dimensionless quantities are denoted by a bar above:

$$\begin{aligned}
&\varphi_1'' + (q_{11} + q_{12} + q_{13})\bar{\mu}_1 \varphi_1' = \\
&= \bar{g}\sin\varphi_1 + \bar{P}\{\sin\left[(1-k)\varphi_1 - \delta\right] + \\
&+ \bar{\varepsilon}\cos\left[(1-k)\varphi_1 - \delta\right]\} - \\
&- \bar{c}\left(\begin{array}{l} q_1 \dfrac{2\bar{a}}{\pi}\,\mathrm{tg}\,\dfrac{\pi \bar{y}_1}{2\bar{a}} + \\ + q_2 \dfrac{\bar{y}_1}{\sqrt{1 + \dfrac{\bar{c}^2 \bar{y}_1^2}{\bar{b}^2}}} + q_3 \bar{y}_1 \end{array}\right)\cos\varphi_1 - \\
&- q_{11}\frac{2a_1}{\pi}\,\mathrm{tg}\,\frac{\pi\varphi_1}{2a_1} - \\
&- q_{12}\frac{\varphi_1}{\sqrt{1 + \dfrac{\varphi_1^2}{b_1^2}}} - q_{13}\varphi_1. \qquad (2)
\end{aligned}$$

Here the prime denotes differentiation with respect to the dimensionless time $\bar{t}$.

Equations (2) at

$$\delta = 0 \text{ and } \varepsilon = 0$$

admit a solution $\varphi_1 = 0$, which corresponds to the vertical position of equilibum of the pendulum. Let's consider the problem of the existence of equilibrium positions of the pendulum for

$$\delta \neq 0 \text{ and } \varepsilon \neq 0.$$

$$f_1(\varphi_1, \delta, \bar{\varepsilon}) = 0, \quad (3)$$

where

$$f_1(\varphi_1,\delta,\bar{\varepsilon})=\bar{g}\sin\varphi_1+$$
$$+\bar{P}\left\{\sin\left[(1-k)\varphi_1-\delta\right]+\bar{\varepsilon}\cos\left[(1-k)\varphi_1-\delta\right]\right\}-$$
$$-\bar{c}\left(q_1\frac{2\bar{a}}{\pi}\operatorname{tg}\frac{\pi\bar{y}_1}{2\bar{a}}+q_2\frac{y_1}{\sqrt{1+\frac{\bar{c}^2\bar{y}_1^2}{\bar{b}^2}}}+q_3\bar{y}_1\right)\cos\varphi_1-$$
$$-q_{11}\frac{2a_1}{\pi}\operatorname{tg}\frac{\pi\varphi_1}{2a_1}-q_{12}\frac{\varphi_1}{\sqrt{1+\frac{\varphi_1^2}{b_1^2}}}-q_{13}\varphi_1. \tag{4}$$

It is logical to assume that there exists a non-vertical state of equilibrium that can be found by direct integration of the differential equation of motion of the pendulum, solved the corresponding Cauchy problem for fixed values of the parameters.

To do this, we use the parameter extension method developed by the Japanese scientist Y. A. Shinohara.

3. Results

The results of the solution of the Cauchy problem showed that only in the case of springs with soft characteristics ($q_2=q_{12}=1$ and $q_1=q_{11}=q_3=q_{13}=0$ in equation (2)) bifurcations of pendulum equilibrium states occur. In the cases of rigid ($q_1=q_{11}=1$ and $q_2=q_{12}=q_3=q_{13}=0$) and linear ($q_3=q_{13}=1$ and $q_1=q_{11}=q_2=q_{12}=0$) characteristics of the elastic elements of the pendulum, the dependences are single-valued.

Unique non-vertical (for $\bar{\varepsilon}\neq 0$) equilibrium position of the pendulum is corresponded to each value of the linear eccentricity of the follower force. In this case, the larger the value $\bar{\varepsilon}$, the greater the angle of pendulum deviation φ_1^* from the vertical.

A comparative analysis of the pendulum shows that for linear characteristics this angle is larger than for stiff characteristics, consistent with intuitive considerations. By varying the parameter b_1, characterizing the helical spring in the hinge O, it is found that increasing or decreasing b_1 leads to a change in the configuration of the equilibrium state curve. For sufficiently small values of $\bar{\varepsilon}$ for b_1=0,05, there are three equilibrium states of the pendulum, for b_1=0,25 there are five of them, and for b_1=0,5 – only one ($\varphi_1^*=0$).

The parameter b characterizing the horizontal spring at the upper end of the pendulum does not affect the configuration of the equilibrium state curve.

For a pendulum with rigid characteristics of elastic elements, a change in the orientation parameter of the follower force leads to the fact that for small angular eccentricity the pendulum has only one non-vertical equilibrium state. The effect of this eccentricity is visible only for values of δ close to π.

For a pendulum with soft characteristics of elastic elements, a similar effect of the follower force orientation parameter on the equilibrium curves is observed.

4. Discussions of results

The theoretical and practical significance of the results is the development of analytical-numerical methods for constructing the dependencies of the equilibrium values of the generalized coordinates of single and double inverted mathematical pendulums on the parameters of the linear and angular eccentricities of the follower force, as well as its orientation parameter. The obtained results for different types of characteristics of elastic elements deepen the scientific base of design engineers on the influence of the parameters of pendulum systems on their dynamic behavior and can be used in research and design organizations in the modeling of hinges of control surfaces of aircrafts, railway trackers, in calculations of dynamic vibration dampers of building structures, in predicting the dynamic behavior of one-dimensional pipelines, in the mechanics of the number of crews, while predicting the functional capabilities of machine elements and mechanisms with pendulum systems.

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