2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
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2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16

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Abstract

*This work brings **universal** and **upside-down magic squares** of order 3 to 16. The work is for two digits as {1,8}, {2,5} and {6,9}. It can easily to extended for the digits {0,1} and {0,8}. In case of orders 8, 9 and 16 the bimagic squares are also constructed. The block-wise constructions for the orders 8, 9, 12, 15 and 16 are also done. In case of order 15, two ways are given. One as blocks of pandiagonal magic squares of order 5 and another as semi-magic squares of order 3. In both the cases, the magic square of order 15 is semi-magic. The extension of this work to magic squares of orders 17 to 32 is given in another paper [38]. The whole work is without use of any programming language. It is just based on the combination of numbers.*

Contents

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1 Introduction

Author worked with digital type numbers in different papers. This work brings, magic square of order 3 to 16 just with two digits in such a way that the magic squares are **upside-down** and/or mirror looking. In each case three cases are considered with numbers $\{1,8\}$, $\{2,5\}$ and $\{6,9\}$. In the first case, i.e., for the digits $\{1,8\}$ the magic squares are always **universal** i.e., **upside-down** and **mirror looking** with same magic sums. In the second case, the numbers 2 and 5 are written in digital form. In this case the magic squares are **universal** i.e., **upside-down** and **mirror looking** with different magic sums. This is because in the **mirror looking** case 2 becomes 5 and 5 as 2. We can bring magic squares and **mirror looking** case with same magic sums but in this case the **upside-down** sum is different. The third case of $\{6,9\}$ the magic

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
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squares are only **upside-down**. It due to the fact that 6 and 9 are only **upside-down**. In case of orders 8, 9 and 16 the **bimagic squares** are also constructed. The **block-wise** constructions for the orders 8, 9, 12, 15 and 16 are also done. In case of order 15, two ways are given. One as blocks of pandiagonal magic squares of order 5 and another as semi-magic squares of order 3. In both the cases, the magic square of order 15 is semi-magic. In each case, the **palindromic magic** are also given. The extension of this work to magic squares of orders 17 to 32 is given in another paper [38]. The whole work is without use of any programming language. It is just based on the combination of numbers.

Through out we have used the double combinations of using 2-digits, 3-digits and 4-digits resulting in 4-digits, 6-digits and 8-digits respectively in each cell. Four digits in each cell lead us to magic squares of orders 3 and 4. 6-digits in each cell lead us to magic squares of orders 5 to 8. 8-digits in each cell lead us to magic squares of orders 9 to 16. Further 10-digits in each cell lead us to magic squares of orders 17 to 32. This is done in another work. Just for knowledge, below is given these combinations just with two letters *a* and *b*.

$2 - digits : 2^2 := 4$ aa, ab, ba, bb

$3 - digits : 2^3 := 8$ $aaa, aab, aba, abb, baa, bab, bba, bbb$

$4 - digits : 2^4 := 16$ $aaaa, aaab, aaba, aabb, abaa, abab, abba, abbb, baaa, baab, baba, babb, bbaa, bbab, bbba, bbbb$

$5 - digits : 2^5 := 32$ $aaaaa, aaaab, aaaba, aaabb, aabaa, aabab, aabba, aabbb, abaaa, abaab, ababa, ababb, abbaa, abbab, abbba, abbbb, baaaa, baaab, baaba, baabb, babaa, babab, babba, babbb, bbaaa, bbaab, bbaba, bbabb, bbbaa, bbbab, bbbba, bbbbb$

$6 - digits : 2^6 := 64$ $aaaaaa, aaaaab, aaaaba, aaaabb, aaabaa, aaabab, aaabba, aaabbb, aabaaa, aabaab, aababa, aababb, aabbaa, aabbab, aabbba, aabbbb, abaaaa, abaaab, abaaba, abaabb, ababaa, ababab, ababba, ababbb, abbaaa, abbaab, abbaba, abbabb, abbbaa, abbbab, abbbba, abbbbb, baaaaa, baaaab, baaaba, baaabb, baabaa, baabab, baabba, baabbb, babaaa, babaab, bababa, bababb, babbaa, babbab, babbba, babbbb, bbaaaa, bbaaab, bbaaba, bbaabb, bbabaa, bbabab, bbabba, bbabbb, bbbaaa, bbbaab, bbbaba, bbbabb,$

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

bbbbaa, bbbbab, bbbbba, bbbbbb

$7 - digits : 2^7 := 128$ *aaaaaaa, aaaaaab, aaaaaba, aaaaabb, aaaabaa, aaaabab, aaaabba, aaaabbb, aaabaaa, aaabaab, aaababa, aaababb, aaabbaa, aaabbab, aaabbba, aaabbbb, aabaaaa, aabaaab, aabaaba, aabaabb, aababaa, aababab, aababba, aababbb, aabbaaa, aabbaab, aabbaba, aabbabb, aabbbaa, aabbbab, aabbbba, aabbbbb, abaaaaa, abaaaab, abaaaba, abaaabb, abaabaa, abaabab, abaabba, abaabbb, ababaaa, ababaab, abababa, abababb, ababbaa, ababbab, ababbba, ababbbb, abbaaaa, abbaaab, abbaaba, abbaabb, abbabaa, abbabab, abbabba, abbabbb, abbbaaa, abbbaab, abbbaba, abbbabb, abbbbaa, abbbbab, abbbbba, abbbbbb, baaaaaa, baaaaab, baaaaba, baaaabb, baaabaa, baaabab, baaabba, baaabbb, baabaaa, baabaab, baababa, baababb, baabbaa, baabbab, baabbba, baabbbb, babaaaa, babaaab, babaaba, babaabb, bababaa, bababab, bababba, bababbb, babbaaa, babbaab, babbaba, babbabb, babbbaa, babbbab, babbbba, babbbbb, bbaaaaa, bbaaaab, bbaaaba, bbaaabb, bbaabaa, bbaabab, bbaabba, bbaabbb, bbabaaa, bbabaab, bbababa, bbababb, bbabbaa, bbabbab, bbabbba, bbabbbb, bbbaaaa, bbbaaab, bbbaaba, bbbaabb, bbbabaa, bbbabab, bbbabba, bbbabbb, bbbbaaa, bbbbaab, bbbbaba, bbbbabb, bbbbbaa, bbbbbab, bbbbbba, bbbbbbb*

$8 - digits : 2^8 := 256$ *aaaaaaaa, aaaaaaab, aaaaaaba, aaaaaabb, aaaaabaa, aaaaabab, aaaaabba, aaaaabbb, aaaabaaa, aaaabaab, aaaababa, aaaababb, aaaabbaa, aaaabbab, aaaabbba, aaaabbbb, aaabaaaa, aaabaaab, aaabaaba, aaabaabb, aaababaa, aaababab, aaababba, aaababbb, aaabbaaa, aaabbaab, aaabbaba, aaabbabb, aaabbbaa, aaabbbab, aaabbbba, aaabbbbb, aabaaaaa, aabaaaab, aabaaaba, aabaaabb, aabaabaa, aabaabab, aabaabba, aabaabbb, aababaaa, aababaab, aabababa, aabababb, aababbaa, aababbab, aababbba, aababbbb, aabbaaaa, aabbaaab, aabbaaba, aabbaabb, aabbabaa, aabbabab, aabbabba, aabbabbb, aabbbaaa, aabbbaab, aabbbaba, aabbbabb, aabbbbaa, aabbbbab, aabbbbba,*

$aabbbbbb, abaaaaaa, abaaaaab, abaaaaba, abaaaabb, abaaabaa, abaaabab,$
$abaaabba, abaaabbb, abaabaaa, abaabaab, abaababa, abaababb, abaabbaa,$
$abaabbab, abaabbba, abaabbbb, ababaaaa, ababaaab, ababaaba, ababaabb,$
$abababaa, abababab, abababba, abababbb, ababbaaa, ababbaab, ababbaba,$
$ababbabb, ababbbaa, ababbbab, ababbbba, ababbbbb, abbaaaaa, abbaaaab,$
$abbaaaba, abbaaabb, abbaabaa, abbaabab, abbaabba, abbaabbb, abbabaaa,$
$abbabaab, abbababa, abbababb, abbabbaa, abbabbab, abbabbba, abbabbbb,$
$abbbaaaa, abbbaaab, abbbaaba, abbbaabb, abbbabaa, abbbabab, abbbabba,$
$abbbabbb, abbbbaaa, abbbbaab, abbbbaba, abbbbabb, abbbbbaa, abbbbbab,$
$abbbbbba, abbbbbbb, baaaaaaa, baaaaaab, baaaaaba, baaaaabb, baaaabaa,$
$baaaabab, baaaabba, baaaabbb, baaabaaa, baaabaab, baaababa, baaababb,$
$baaabbaa, baaabbab, baaabbba, baaabbbb, baabaaaa, baabaaab, baabaaba,$
$baabaabb, baababaa, baababab, baababba, baababbb, baabbaaa, baabbaab,$
$baabbaba, baabbabb, baabbbaa, baabbbab, baabbbba, baabbbbb, babaaaaa,$
$babaaaab, babaaaba, babaaabb, babaabaa, babaabab, babaabba, babaabbb,$
$bababaaa, bababaab, babababa, babababb, bababbaa, bababbab, bababbba,$
$bababbbb, babbaaaa, babbaaab, babbaaba, babbaabb, babbabaa, babbabab,$
$babbabba, babbabbb, babbbaaa, babbbaab, babbbaba, babbbabb, babbbbaa,$
$babbbbab, babbbbba, babbbbbb, bbaaaaaa, bbaaaaab, bbaaaaba, bbaaaabb,$
$bbaaabaa, bbaaabab, bbaaabba, bbaaabbb, bbaabaaa, bbaabaab, bbaababa,$
$bbaababb, bbaabbaa, bbaabbab, bbaabbba, bbaabbbb, bbabaaaa, bbabaaab,$
$bbabaaba, bbabaabb, bbababaa, bbababab, bbababba, bbababbb, bbabbaaa,$
$bbabbaab, bbabbaba, bbabbabb, bbabbbaa, bbabbbab, bbabbbba, bbabbbbb,$
$bbbaaaaa, bbbaaaab, bbbaaaba, bbbaaabb, bbbaabaa, bbbaabab, bbbaabba,$
$bbbaabbb, bbbabaaa, bbbabaab, bbbababa, bbbababb, bbbabbaa, bbbabbab,$
$bbbabbba, bbbabbbb, bbbbaaaa, bbbbaaab, bbbbaaba, bbbbaabb, bbbbabaa,$
$bbbbabab, bbbbabba, bbbbabbb, bbbbbaaa, bbbbbaab, bbbbbaba, bbbbbabb,$
$bbbbbbaa, bbbbbbab, bbbbbbba, bbbbbbbb$

The section below give magic squares of order 3 to 16, where in each case the three types are considered, i.e., for the digits $\{1,8\}$, $\{2,5\}$ and $\{6,9\}$

2 Universal Magic Squares With Digits 1 and 8

2.1 Semi-Magic Square of Order 3

2.1.1 Semi-Magic Square

Example 1. *The **semi-magic square** of order 3 for the digits 1 and 8 is given by*

1811	8181	1118
1181	1818	8111
8118	1111	1881

*In this case it is **semi-magic square**. It has same sum in lines and rows, while principal diagonals sums are different. It is based on the 2 by 2 combinations of three numbers $\{11,18,81\}$. These three numbers are **upside-down** and **mirror looking** in pair $\{18,18\}$, while for 11 it always hold. It is **universal semi-magic square**. Its **semi-magic sum** is*

$$S_{3\times3} := 11110 = 100 \times 110 + 110,$$

where $110 = 11 + 18 + 81$.

2.1.2 Palindromic Semi-Magic Square

Example 2. *The **palindromic semi-magic square** of order 3 for the digits 1 and 8 is given by*

1811181	8181818	1118111
1181811	1818181	8111118
8118118	1111111	1881881

*It is a **palindromic semi-magic square** of order 3 with **semi-magic sum** $S_{3\times3} := 11111110$.*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

2.2 Magic Square of Order 4

We observe that the 2-digits combinations considering 2 by 2 maximum goes up to 16 possibilities resulting in magic squares of orders 3 and 4. The following magic square of order 4 is with all 16 possible combinations.

2.2.1 Magic Square

Example 3. *The **pandiagonal magic square** of order 4 for the digits 1 and 8 is given by*

1881	8188	1111	8818
1118	8811	1888	8181
8888	1181	8118	1811
8111	1818	8881	1188

It is based on the 2 by 2 combinations of four numbers $\{11, 18, 81, 88\}$. *These four numbers are **upside-down** and **mirror looking** in pairs* $\{18, 18\}$, *while for the numbers 11 and 88, it always holds. The above magic square is **universal pandiagonal** with **magic sum**:*

$$S_{4\times 4} := 19998 = 100 \times 198 + 198,$$

where $198 = 11 + 18 + 81 + 88$.

2.2.2 Palindromic Magic Square

Example 4. *The **palindromic pandiagonal magic square** of order 11 for the digits 1 and 8 is given by*

81188118	18111181	88888888	11811811
88811888	11888811	81111118	18188181
11111111	88188188	18811881	81888818
18888881	81811818	11188111	88111188

*It is a **palindromic pandiagonal magic square** of order 4 with **magic sum*** $S_{4\times 4} := 19999998$.

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

2.3 Magic Square of Order 5

We observe that the two digits combinations for two numbers maximum goes up to 16. This lead us to magic squares of orders 3 and 4. From now onwards, we shall work with three digits combinations for two numbers. It will go up to 8^{th} order magic squares.

2.3.1 Magic Square

Example 5. *The **pandiagonal magic square** of order 5 for the digits 1 and 8 is given by*

111111	118811	181118	811818	818181
811118	818818	111181	118111	181811
118181	181111	811811	818118	111818
818811	111118	118818	181181	811111
181818	811181	818111	111811	118118

It is based on the 2 by 2 combinations of five numbers $\{111, 118, 181, 811, 818\}$. *These five numbers are **upside-down** and **mirror looking**. It is **universal pandiagonal square** with different magic sums. The **magic sums** are*

$$S_{5\times5} := 2041039 = 1000 \times 2039 + 2039,$$

where

$$2039 = 111 + 118 + 181 + 811 + 818.$$

2.3.2 Palindromic Magic Square

Example 6. *The **palindromic pandiagonal magic square** of order 5 for the digits 1 and 8 is given by*

11111111111	11881118811	18111811181	81181818118	81818181818
81111811118	81881818818	11118181111	11811111811	18181118181
11818181811	18111111181	81181118118	81811811818	11181818111
81881118818	11111811111	11881818811	18118181181	81111111118
18181818181	81118181118	81811111818	11181118111	11811811811

It is a ***palindromic pandiagonal magic square*** *of order 5 with* ***magic sum*** $S_{5\times5} := 204104041039$.

2.4 Magic Square of Order 6

2.4.1 Magic Square

Example 7. *The* ***magic square*** *of order 6 for the digits 1 and 8 is given by*

888888	111818	111881	111118	888181	888111
181111	818181	181881	818118	818818	181888
188111	188818	811118	811881	188181	811888
811111	811181	188118	188881	811818	188888
818888	181181	818881	181118	181818	818111
111888	888818	888118	888881	111181	111111

In this case, the construction is little different. It is considered as $T := 1000 \times A + B$, *where* $A := \{111, 181, 188, 811, 818, 888\}$ *and* $B := \{111, 118, 181, 818, 881, 888\}$. *Thus, there are total, 36 entries given in above magic square of order 6. Since the sum of members of A and B are 2997, this gives* ***magic sum*** *of order 6 as* $S_{6\times6} := 2999997 = 1000 \times 2997 + 2997$. *It is* ***upside-down*** *and* ***mirror looking*** *magic square. Thus, we have a* ***universal magic square*** *of order 6 with* ***magic sum*** $S_{6\times6} := 2999997$.

2.4.2 Palindromic Magic Square

Example 8. *The* ***palindromic magic square*** *of order 6 for the digits 1 and 8 is given by*

88888888888	11181818111	11188188111	11111811111	88818181888	88811111888
18111111181	81818181818	18188188181	81811811818	81881818818	18188888181
18811111881	18881818881	81111811118	81188188118	18818181881	81188888118
81111111118	81118181118	18811811881	18888188881	81181818118	18888888881
81888888818	18118181181	81888188818	18111811181	18181818181	81811111818
11188888111	88881818888	88811811888	88888188888	11118181111	11111111111

*It is a **palindromic magic square** of order 6 with **magic sum** $S_{6\times 6} := 299999999997$.*

2.5 Magic Square of Order 7

2.5.1 Magic Square

Example 9. *The **magic square** of order 7 for the digits 1 and 8 is given by*

111111	118118	181181	188188	811811	818818	881881
818811	881818	111881	118111	181118	188181	811188
188118	811181	818188	881811	111818	118881	181111
118818	181881	188111	811118	818181	881188	111811
881181	111188	118811	181818	188881	811111	818118
811881	818111	881118	111181	118188	181811	188818
181188	188811	811818	818881	881111	111118	118181

*It is based on the 2 by 2 combinations of seven numbers $\{111, 118, 181, 188, 811, 818, 881\}$. These seven numbers are **upside-down** and **mirror looking**. It is **universal pandiagonal magic square**. Its **magic sum** is*

$$S_{7\times 7} := 3111108 = 1000 \times 3108 + 3108,$$

where

$$3108 = 111 + 118 + 181 + 188 + 811 + 818 + 881.$$

2.5.2 Palindromic Magic Square

Example 10. *The* ***palindromic pandiagonal magic square*** *of order 7 for the digits 1 and 8 is given by*

11111111111	11811811811	18118181181	18818881881	81181118118	81881818818	88188188188
81881118818	88181818188	11188188111	11811111811	18111811181	18818181881	81118881118
18811811881	81118181118	81818881818	88181118188	11181818111	11888188811	18111111181
11881818811	18188188181	18811111881	81111811118	81818181818	88118881188	11181118111
88118181188	11118881111	11881118811	18181818181	18888188881	81111111118	81811811818
81188188118	81811111818	88111811188	11118181111	11818881811	18181118181	18881818881
18118881181	18881118881	81181818118	81888188818	88111111188	11111811111	11818181811

It is a ***palindromic pandiagonal magic square*** *of order 7 with* ***magic sum*** $S_{7\times 7} := 311111111108$.

2.6 Magic Square of Order 8

We observe that the 3-digits combinations considering 2 by 2 maximum goes up to 64 possibilities resulting in magic squares of orders 5 to 8. The following magic squares of order 8 are with all 64 possible combinations.

2.6.1 Magic Square

Example 11. *The* ***pandiagonal magic square*** *of order 8 for the digits 1 and 8 is given by*

111111	888811	188188	811888	118111	881811	181188	818888
188888	811188	111811	888111	181888	818188	118811	881111
811811	188111	888888	111188	818811	181111	881888	118188
888188	111888	811111	188811	881188	118888	818111	181811
111118	888818	188181	811881	118118	881818	181181	818881
188881	811181	111818	888118	181881	818181	118818	881118
811818	188118	888881	111181	818818	181118	881881	118181
888181	111881	811118	188818	881181	118881	818118	181818

It is based on the 2 by 2 combinations of eight numbers $\{111, 118, 181, 188, 811, 818, 881, 888\}$. *These eight numbers are **upside-down** and **mirror looking**. It is **universal pandiagonal magic square**. Its **magic sum** is*

$$S_{8\times8} := 3999996 = 1000 \times 3996 + 3996,$$

where

$$3996 = 111 + 118 + 181 + 188 + 811 + 818 + 881 + 888.$$

*It is **block-wise pandigonal magic square** of order 8, where each block of order 4 is a **pandiagonal magic square** with equal **magic sums** given by*

$$S_{4\times4} := \frac{3999996}{2} = 1999998.$$

There are much more possibilities of magic square of order 4 with same magic square sum. This we left for the reader to find.

2.6.2 Palindromic Magic Square

Example 12. *The **palindromic pandiagonal magic square** of order 8 for the digits 1 and 8 is given by*

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11111111111	88881118888	18818881881	81188888118	11811111811	88181118188	18118881181	81888888818
18888888881	81118881118	11181118111	88811111888	18188888181	81818881818	11881118811	88111111188
81181118118	18811111881	88888888888	11118881111	81881118818	18111111181	88188888188	11818881811
88818881888	11188888111	81111111118	18881118881	88118881188	11888888811	81811111818	18181118181
11111811111	88881818888	18818181881	81188188118	11811811811	88181818188	18118181181	81888188818
18888188881	81118181118	11181818111	88811811888	18188188181	81818181818	11881818811	88111811188
81181818118	18811811881	88888188888	11118181111	81881818818	18111811181	88188188188	11818181811
88818181888	11188188111	81111811118	18881818881	88118181188	11888188811	81811811818	18181818181

*It is a **block-wise palindromic pandigonal magic square** of order 8, where each block of order 4 is a **palindromic pandiagonal magic square** with equal **magic sums**. The magic sums are given by*

$$S_{8\times 8} := 399999999996 \qquad \text{and} \qquad S_{4\times 4} := \frac{399999999996}{2} = 199999999998.$$

2.6.3 Bimagic Square

Example 13. *The **pandiagonal bimagic magic square** of order 8 for the digits 1 and 8 is given by*

118888	818111	811188	111811	188181	888818	881881	181118
188118	888881	881818	181181	118811	818188	811111	111888
111111	811888	818811	118188	181818	881181	888118	188881
181881	881118	888181	188818	111188	811811	818888	118111
811818	111181	118118	818881	881111	181888	188811	888188
881188	181811	188888	888111	811881	111118	118181	818818
818181	118818	111881	811118	888888	188111	181188	881811
888811	188188	181111	881888	818118	118881	111818	811181

It is also based on the 2 by 2 combinations of eight numbers $\{111, 118, 181, 188, 811, 818, 881, 888\}$. *These eight numbers are **upside-down** and **mirror looking**. It is **universal pandiagonal bimagic magic square**. Its **magic sum** is*

$$S_{8\times 8} := 3999996 = 1000 \times 3996 + 3996,$$

where

$$3996 = 111 + 118 + 181 + 188 + 811 + 818 + 881 + 888.$$

*It is also **bimagic square** of order 8 with **bimagic sum** given by*

$$Sb_{8\times 8} := 2989894989900.$$

Moreover, as specified in the figure, the sum of 2×4 *entries are of same sum as of magic square, i.e.,* 3999996.

2.6.4 Palindromic Bimagic Square

Example 14. *The **palindromic bimagic square** of order 8 for the digits 1 and 8 is given by*

11888888811	81811111818	81118881118	11181118111	18818181881	88881818888	88188188188	18111811181
18811811881	88888188888	88181818188	18118181181	11881118811	81818881818	81111111118	11188888111
11111111111	81188888118	81881118818	11818881811	18181818181	88118181188	88811811888	18888188881
18188188181	88111811188	88818181888	18881818881	11118881111	81181118118	81888888818	11811111811
81181818118	11118181111	11811811811	81888188818	88111111188	18188888181	18881118881	88818881888
88118881188	18181118181	18888888881	88811111888	81188188118	11111811111	11818181811	81881818818
81818181818	11881818811	11188188111	81111811118	88888888888	18811111881	18118881181	88181118188
88881118888	18818881881	18111111181	88188888188	81811811818	11888188811	11181818111	81118181118

*It is a **palindromic bimagic square** of order 8 with **magic sums:***

$$S_{8\times 8} := 399999999996 \qquad \text{and} \qquad Sb_{8\times 8} := 29898989908389898989900.$$

Moreover, as specified in the figure, the sum of 2×4 *entries are of same sum as of magic square, i.e.,* 399999999996.

2.7 Magic Square of Order 9

We observe that the three digits combinations considering 2 by 2 maximum goes up to 64 possibilities resulting in magic squares of orders 8. From now onwards, we shall work with four digits combinations for two numbers. It will go up to 16^{th} order magic squares.

2.7.1 Magic Square

Example 15. *The **pandiagonal magic square** of order 9 for the digits 1 and 8 is given by*

81811118	18111881	11188111	11811181	81111811	18188118	18811111	11111818	81188181
18188181	11811111	81111818	81188111	18811118	11111881	11188118	81811181	18111811
11111811	81188118	18811181	18111818	11188181	81811111	81111881	18188111	11811118
81181818	18818181	11111111	11181881	81818111	18111118	18181811	11818118	81111181
18111181	11181811	81818118	81111111	18181818	11818181	11111118	81181881	18818111
11818111	81111118	18181881	18818118	11111181	81181811	81818181	18111111	11181818
81118118	18181181	11811811	11118181	81181111	18811818	18118111	11181118	81811881
18811881	11118111	81181118	81811811	18118118	11181181	11811818	81118181	18181111
11181111	81811818	18118181	18181118	11811881	81118111	81181181	18811811	11118118

It is based on the 2 by 2 combinations of nine numbers $\{1111, 1118, 1181, 1811, 1818, 1881, 8111, 8118, 8181\}$. *These nine numbers are **upside-down** and **mirror looking**. It can be checked by combining in pairs as* $\{1118, 8111\}$, $\{1181, 1811\}$, $\{1811, 1181\}$, $\{1818, 8181\}$, *and* 1111. *It is **universal pandiagonal magic square**. Its **magic sum** is*

$$S_{9\times 9} := 333333330 = 10000 \times 33330 + 33330,$$

where

$$33330 = 1111 + 1118 + 1181 + 1811 + 1818 + 1881 + 8111 + 8118 + 8181.$$

Moreover, Each block of order 3×3 *as specified in the figure is a **semi-magic square** of order 3 with **semi-magic sum**:* $S_{3\times 3} := 111111110$.

2.7.2 Palindromic Magic Square

Example 16. *The **palindromic pandiagonal magic square** of order 9 for the digits 1 and 8 is given by*

818111181111818	181118818811181	111881111188111	118111818111811	811118111811118	181881181188181	188111111111881	111118181811111	811881818188118
181881818188181	118111111111811	811118181811118	811881111188118	188111181111881	111118818811111	111881181188111	818111818111818	181118111811181
111118111811111	811881181188118	188111818111881	181118181811181	111881818188111	818111111111818	811118818811118	181881111188181	118111181111811
811818181818118	188181818181881	111111111111111	111818818818111	818181111181818	181111181111181	181818111818181	118181181181811	811111818111118
181111818111181	111818111818111	818181181181818	811111111111118	181818181818181	118181818181811	111111181111111	811818818818118	188181111181881
118181111181811	811111181111118	181818818818181	188181181181881	111111818111111	811818111818118	818181818181818	181111111111181	111818181818111
811181181181118	181811818118181	118118111811811	111181818181111	811811111118118	188118181811881	181181111181181	111811181118111	818118818811818
188118818811881	111181111181111	811811181118118	818118111811818	181181181181181	111811818118111	118118181811811	811181818181118	181811111118181
111811111118111	818118181811818	181181818181181	181811181118181	118118818811811	811181111181118	811811818118118	188118111811881	111181181181111

*It is a **block-wise palindromic pandigonal magic square** of order 9, where each block of order 3 is a **palindromic semi-magic square** with equal **magic sums**. The magic sums are given by*

$$S_{9\times 9} := 3333333333333330 \qquad \textit{and} \qquad S_{3\times 3} := \frac{3333333333333330}{3} = 1111111111111110 \quad \textit{(semi-magic sum)}.$$

2.7.3 Bimagic Square

Example 17. *The **bimagic square** of order 9 for the digits 1 and 8 is given by*

81818181	81181111	81111818	18811118	18181881	18118111	11811811	11188118	11111181
18811811	18188118	18111181	11818181	11181111	11111818	81811118	81181881	81118111
11811118	11181881	11118111	81811811	81188118	81111181	18818181	18181111	18111818
81111111	81811818	81188181	18111881	18818111	18181118	11118118	11811181	11181811
18118118	18811181	18181811	11111111	11811818	11188181	81111881	81818111	81181118
11111881	11818111	11181118	81118118	81811181	81181811	18111111	18811818	18188181
81181818	81118181	81811111	18188111	18111118	18811881	11181181	11111811	11818118
18181181	18111811	18818118	11181818	11118181	11811111	81188111	81111118	81811881
11188111	11111118	11811881	81181181	81111811	81818118	18181818	18118181	18811111

*It is constructed with same nine digits used in previous example. According to previous example explanations, it is **universal bimagic square** with **magic** and **bimagic sums** respectively given by*

$$S_{9\times 9} := 333333330 \quad \textit{and} \quad Sb_{9\times 9} := 212645706546210 18.$$

The sum of 9 members of each block of order 3 (specified in the figure) is the same as of magic square, i.e., 333333330.

2.7.4 Palindromic Bimagic Square

Example 18. *The **palindromic bimagic square** of order 9 for the digits 1 and 8 is given by*

111111111111111	111881818188111	118118181811811	181181181181181	181818111818181	188111818111881	811118818811118	811811181118118	818181111181818
181118818811181	181811181118181	188181111181881	811111111111118	811881818188118	818118181811818	111181181181111	111818111818111	118111818111811
811181181181118	811818111818118	818111818111818	111118818811111	111811181118111	118181111181811	181111111111181	181881818188181	188118181811881
118181818181811	111118181811111	111811111118111	188118111811881	181111818111181	181881181188181	818111181111818	811181111181118	811818818818118
188111181111881	181181111181181	181818818818181	818181818181818	811118181811118	811811111118118	118118111811811	111111818111111	111881181188111
818118111811818	811111818111118	811881181188118	118111181111811	111181111181111	111818818818111	188181818181881	181118181811181	181811111118181
111818181818111	118111111111811	111181818181111	181811818118181	188181181181881	181118111811181	811881111188118	818118818811818	811111181111118
181881111188181	188118818811881	181111181111181	811818181818118	818111111111818	811181818181118	111811818118111	118181181181811	111118111811111
811811818118118	818181181181818	811118111811118	111881111188111	118118818811811	111111181111111	181818181818181	188111111111881	181181818181181

*It is a **palindromic bimagic square** of order 9 with **magic sums** given by*

$$S_{9\times 9} := 333333333333330 \qquad \textit{and} \qquad Sb_{9\times 9} := 21264570901532966109906546210 18.$$

2.8 Magic Square of Order 10

2.8.1 Magic Square

Example 19. *The **magic square** of order 10 for the digits 1 and 8 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

88888888	18118111	11181881	81818118	11111118	11818181	81111818	18811111	81181181	18181811
18818118	81818181	81111811	88881118	18188111	11188888	11111181	81181818	11811111	18111881
18111111	11111818	81188118	18188888	11181811	18811181	11811881	81818111	88888181	81111118
11111881	11181181	88881111	81118111	81188888	81811811	18818181	18181118	18111818	11818118
81111181	11818888	81811818	11181111	18811881	18111118	88888111	11111811	18188118	81188181
81811118	81181111	18118181	11811811	88881181	18181818	11188118	81111881	11118111	18818888
11818111	18811118	18181181	81181881	81811111	11118118	18111811	11188181	81118888	88881818
18188181	88881811	11118888	18811818	18118118	81111111	81181118	11811181	81811881	11188111
81181811	81118118	18818111	11118181	11811818	88881881	18181111	18118888	11181118	81811181
11181818	18181881	11811118	18111181	81118181	81188111	81818888	88888118	18811811	11111111

It is based on the 2 by 2 combinations of ten numbers

$$\{1111, 1118, 1181, 1811, 1818, 1881, 8111, 8118, 8181, 8888\}.$$

*These ten numbers are **upside-down** and **mirror looking** combining in pairs as* $\{1118, 8111\}$, $\{1181, 1811\}$, $\{1811, 1181\}$, $\{1818, 8181\}$, 1111 *and* 8888. *It is **universal magic square**. Its **magic sum** is*

$$S_{10\times 10} := 422222218 = 10000 \times 42218 + 42218,$$

where

$$42218 = 1111 + 1118 + 1181 + 1811 + 1818 + 1881 + 8111 + 8118 + 8181 + 8888.$$

2.8.2 Palindromic Magic Square

Example 20. *The **palindromic magic square** of order 10 for the digits 1 and 8 is given by*

888888888888888	181181111181181	111818818818111	818181181181818	111111181111111	118181818181811	811118181811118	188111111111881	811811818118118	181818111818181
188181181181881	818181818181818	811118111811118	888811181118888	181881111188181	111888888888111	111111818111111	811818181818118	118111111111811	181118818811181
181111111111181	111118181811111	811881181188118	181888888888181	111818111818111	188111818111881	118118818811811	818181111181818	888881818188888	811111181111118
111118818811111	111811818118111	888811111118888	811181111181118	811888888888118	818118111811818	188181818181881	181811181118181	181118181811181	118181181181811
811111818111118	118188888881811	818118181811818	111811111118111	188118818811881	181111181111181	888881111188888	111118111811111	181881181188181	811881818188118
818111181111818	811811111118118	181181818181181	118118111811811	888811818118888	181818181818181	111881181188111	811118818811118	111181111181111	188188888881881
118181111181811	188111181111881	181111818111181	811818818818118	818111111111818	111181181181111	181118111811181	111881818188111	811888888888118	888818181818888
181881818188181	888818111818888	111188888881111	188118181811881	181181181181181	811111111111118	811811181118118	118111818111811	818118818811818	111881111188111
811818111818118	811181181181118	188181111181881	111181818181111	118118181811811	888818818818888	181811111118181	181188888881181	111811181118111	818111818111818
111818181818111	181818818818181	118111181111811	181111818111181	811181818181118	811881111188118	818188888881818	888881181188888	188118111811881	111111111111111

*It is a **palindromic pandiagonal magic square** of order 10 with **magic sum*** $S_{10\times 10} := 4222222222222218$.

2.9 Magic Square of Order 11

2.9.1 Magic Square

Example 21. *The **pandiagonal magic square** of order 11 for the digits 1 and 8 is given by*

88888888	88111181	81811811	81181881	81118118	18818811	18181118	18111188	11881818	11818111	11188181
11818811	11181118	88881188	88111818	81818111	81188181	81118888	18811181	18181811	18111881	11888118
18118181	11888888	11811181	11181811	88881881	88118118	81818811	81181118	81111188	18811818	18188111
18818118	18188811	18111118	11881188	11811818	11188111	88888181	88118888	81811181	81181811	81111881
81188111	81118181	18818888	18181181	18111811	11881881	11818118	11188811	88881118	88111188	81811818
88111881	81818118	81188811	81111118	18811188	18181818	18118111	11888181	11818888	11181181	88881811
11181818	88888111	88118181	81818888	81181181	81111811	18811881	18188118	18118811	11881118	11811188
11881811	11811881	11188118	88888811	88111118	81811188	81181818	81118111	18818181	18188888	18111181
18181188	18111818	11888111	11818181	11188888	88881181	88111811	81811881	81188118	81118811	18811118
81111181	18811811	18181881	18118118	11888811	11811118	11181188	88881818	88118111	81818181	81188888
81811118	81181188	81111818	18818111	18188181	18118888	11881181	11811811	11181881	88888118	88118811

It is based on the 2 by 2 combinations of eleven numbers

$$\{1118, 1181, 1188, 1811, 1818, 1881, 8111, 8118, 8181, 8811, 8888\}.$$

*These eleven numbers are **upside-down** and **mirror looking** combining in pairs as* $\{1118, 8111\}$, $\{1181, 1811\}$, $\{1811, 1181\}$, $\{1818, 8181\}$, $\{1188, 8811\}$ *and* 8888*. It is **universal pandiagonal magic square**. Its **magic sum** is*

$$S_{11\times 11} := 511111106 = 10000 \times 51106 + 51106,$$

where

$$51106 = 1118 + 1181 + 1188 + 1811 + 1818 + 1881 + 8111 + 8118 + 8181 + 8811 + 8888.$$

2.9.2 Palindromic Magic Square

Example 22. *The **palindromic pandiagonal magic square** of order 11 for the digits 1 and 8 is given by*

888888888888888	881111818111188	818118111811818	811818818818118	811181181181118	188188111881881	181811181118181	181111888111181	118818181818811	118181111181811	111881818188111
118188111881811	111811181118111	888811888118888	881118181811188	818181111181818	811881818188118	811188888881118	188111818111881	181818111818181	181118818811181	118881181188811
181181818181181	118888888888811	118111818111811	111818111818111	888818818818888	881181181181188	818188111881818	811811181118118	811111888111118	188118181811881	181881111188181
188181181181881	181888111888181	181111181111181	118811888118811	118118181811811	111881111188111	888881818188888	881188888881188	818111818111818	811818111818118	811118818811118
811881111188118	811181818181118	188188888881881	181811818118181	181118111811181	118818818818811	118181181181811	111888111888111	888811181118888	881111888111188	818118181811818
881118818811188	818181181181818	811888111888118	811111181111118	188111888111881	181818181818181	181181111181181	118881818188811	118188888881811	111811818118111	888818111818888
111818181818111	888881111188888	881181818181188	818188888881818	811811818118118	811118111811118	188118818811881	181881181188181	181188111881181	118811181118811	118111888111811
118818111818811	118118818811811	111881181188111	888888111888888	881111181111188	818111888111818	811818181818118	811181111181118	188181818181881	181888888888181	181111818111181
181811888118181	181118181811181	118881111188811	118181818181811	111888888888111	888811818118888	881118111811188	818118818811818	811881181188118	811188111881118	188111181111881
811111818111118	188118111811881	181818818818181	181181181181181	118888111888811	118111181111811	111811888118111	888818181818888	881181111181188	818181818181818	811888888888118
818111181111818	811811888118118	811118181811118	188181111181881	181881818188181	181188888881181	118811818118811	118118111811811	111818818818111	888881181188888	881188111881188

*It is a **palindromic pandiagonal magic square** of order 11 with **magic sum*** $S_{11\times 11} := 511111111111106$*.*

2.10 Magic Square of Order 12

2.10.1 Blocks of Order 4

Example 23. *The **pandiagonal magic square** of order 12 for the digits 1 and 8 is given by*

18818118	81188888	11111111	88881881	18888118	81118888	11181111	88811881	18188118	81818888	11881111	88111881
11111881	88881111	18818888	81188118	11181881	88811111	18888888	81118118	11881881	88111111	18188888	81818118
88888888	11118118	81181881	18811111	88818888	11188118	81111881	18881111	88118888	11888118	81811881	18181111
81181111	18811881	88888118	11118888	81111111	18881881	88818118	11188888	81811111	18181881	88118118	11888888
18818111	81188881	11111118	88881888	18888111	81118881	11181118	88811888	18188111	81818881	11881118	88111888
11111888	88881118	18818881	81188111	11181888	88811118	18888881	81118111	11881888	88111118	18188881	81818111
88888881	11118111	81181888	18811118	88818881	11188111	81111888	18881118	88118881	11888111	81811888	18181118
81181118	18811888	88888111	11118881	81111118	18881888	88818111	11188881	81811118	18181888	88118111	11888881
18818181	81188811	11111188	88881818	18888181	81118811	11181188	88811818	18188181	81818811	11881188	88111818
11111818	88881188	18818811	81188181	11181818	88811188	18888811	81118181	11881818	88111188	18188811	81818181
88888811	11118181	81181818	18811188	88818811	11188181	81111818	18881188	88118811	11888181	81811818	18181188
81181188	18811818	88888181	11118811	81111188	18881818	88818181	11188811	81811188	18181818	88118181	11888811

It is based on the 2 by 2 combinations of twelve numbers

$$\{1118, 1181, 1188, 1811, 1818, 1888, 8111, 8181, 8188, 8811, 8818, 8881\}.$$

These 12 numbers are ***upside-down*** *and* ***mirror looking*** *combining as* $\{1118, 8111\}$, $\{1181, 1811\}$, $\{1188, 8811\}$, $\{1811, 1181\}$, $\{1818, 8181\}$, $\{1888, 8881\}$. *It is* ***universal pandiagonal magic square****. Its* ***magic sum*** *is*

$$S_{12\times 12} := 599999994 = 10000 \times 59994 + 59994,$$

where

$$59994 = 1118 + 1181 + 1188 + 1811 + 1818 + 1888 + 8111 + 8181 + 8188 + 8811 + 8818 + 8881.$$

It is ***block-wise pandigonal magic square*** *of order 12, where each block of order 4 is a* ***pandiagonal magic square*** *with equal* ***magic sums*** *given by*

$$S_{4\times 4} := \frac{599999994}{3} = 199999998.$$

2.10.2 Blocks of Order 6

Example 24. *The* ***magic square*** *of order 12 for the digits 1 and 8 is given by*

11181118	88818811	88818111	88811888	11181188	11188881	11181181	88818188	88818181	88811818	11181811	11188818
88118881	11881188	88118111	11881888	11888811	88111118	88118818	11881811	88118181	11881818	11888188	88111181
81118881	81118811	18881888	18888111	81111188	18881118	81118818	81118188	18881818	18888181	81111811	18881181
18888881	18881188	81111888	81118111	18888811	81111118	18888818	18881811	81111818	81118181	18888188	81111181
11881118	88111188	11888111	88111888	88118811	11888881	11881181	88111811	11888181	88111818	88118188	11888818
88811118	11188811	11181888	11188111	88811188	88818881	88811181	11188188	11181818	11188181	88811811	88818818
11811118	88188811	88188111	88181888	11811188	11818881	11811181	88188188	88188181	88181818	11811811	11818818
81888881	18111188	81888111	18111888	18118811	81881118	81888818	18111811	81888181	18111818	18118188	81881181
81818881	81818811	18181888	18188111	81811188	18181118	81818818	81818188	18181818	18188181	81811811	18181181
18188881	18181188	81811888	81818111	18188811	81811118	18188818	18181811	81811818	81818181	18188188	81811181
18111118	81881188	18118111	81881888	81888811	18118881	18111181	81881811	18118181	81881818	81888188	18118818
88181118	11818811	11811888	11818111	88181188	88188881	88181181	11818188	11811818	11818181	88181811	88188818

In this case the construction is based on the similar lines of Example 23. In is just a magic square with same magic sum: $S_{12\times 12} := 599999994$ *It is* ***block-wise magic square*** *of order 12, where each block of order 6 is a* ***magic square*** *with equal* ***magic sums*** *given by* $S_{6\times 6} := \dfrac{599999994}{2} = 299999997.$

2.10.3 Blocks of Order 3

Example 25. *The* ***magic square*** *of order 12 for the digits 1 and 8 is given by*

81881888	88111818	88818818	88188111	18188181	18881811	11811118	11881188	11181181	18118881	81818811	81118188
88811818	81888818	88111888	18888181	88181811	18188111	11181188	11811181	11881118	81118811	18118188	81818881
88118818	88811888	81881818	18181811	18888111	88188181	11881181	11181118	11811188	81818188	81118881	18118811
11818881	11888811	11188188	18111118	81811188	81111181	81888111	88118181	88811811	88181888	18181818	18888818
11188811	11818188	11888881	81111188	18111181	81811118	88818181	81881811	88118111	18881818	88188818	18181888
11888188	11188881	11818811	81811181	81111118	18111188	88111811	88818111	81888181	18188818	18881888	88181818
18118111	81818181	81111811	11811888	11881818	11188818	88188881	18188811	18888188	81881118	88111188	88811181
81118181	18111811	81818111	11181818	11818818	11881888	18888811	88188188	18188881	88811188	81881181	88111118
81811811	81118111	18118181	11888818	11181888	11811818	18188188	18888881	88188811	88111181	88811118	81881188
88181118	18181188	18881181	81888881	88118811	88818188	18111888	81811818	81118818	11818111	11888181	11181811
18881188	88181181	18181118	88818811	81888188	88118881	81111818	18118818	81811888	11188181	11811811	11888111
18181181	18881118	88181188	88118188	88818881	81888811	81818818	81111888	18111818	11881811	11188111	11818181

In this case the construction is based on the similar lines of Example 23. It is just a magic square with same magic sum:

$S_{12\times 12} := 599999994$ *It is* ***block-wise magic square*** *of order 12, where each block of order 3 is a* ***semi-magic square*** *with different* ***semi-magic sums****.*

2.10.4 Palindromic Magic Square

Example 26. *The* ***palindromic pandiagonal magic square*** *of order 12 for the digits 1 and 8 is given by*

188181181181881	811888888888118	111111111111111	888818818818888	188881181188881	811188888881118	111811111118111	888118818811888	181881181188181	818188888881818	118811111118811	881118818811188
111118818811111	888811111118888	188188888881881	811881181188118	111818818818111	888111111111888	188888888888881	811181181181118	118818818818811	881111111111188	181888888888181	818181181181818
888888888888888	111181181181111	811818818818118	188111111111881	888188888881888	111881181188111	811118818811118	188811111118881	881188888881188	118881181188811	818118818811818	181811111118181
811811111118118	188118818811881	888881181188888	111188888881111	811111111111118	188818818818881	888181181181888	111888888888111	818111111111818	181818818818181	881181181181188	118888888888811
188181111181881	811888818888118	111111181111111	888818888818888	188881111188881	811188818881118	111811181118111	888118888811888	181881111188181	818188818881818	118811181118811	881118888811188
111118888811111	888811181118888	188188818881881	811881111188118	111818888818111	888111181111888	188888881888881	811181111181118	118818888818811	881111181111188	181888818888181	818181111181818
888888818888888	111181111181111	811818888818118	188111181111881	888188818881888	111881111188111	811118888811118	188811181118881	881188818881188	118881111188811	818118888811818	181811181118181
811811181118118	188118888811881	888881111188888	111188818881111	811111181111118	188818888818881	888181111181888	111888818888111	818111181111818	181818888818181	881181111181188	118888818888811
188181818181881	811888111888118	111111888111111	888818181818888	188881818188881	811188111881118	111811888118111	888118181811888	181881818188181	818188111881818	118811888118811	881118181811188
111118181811111	888811888118888	188188111881881	811881818188118	111818181818111	888111888111888	188888111888881	811181818181118	118818181818811	881111888111188	181888111888181	818181818181818
888888111888888	111181818181111	811818181818118	188111888111881	888188111881888	111881818188111	811118181811118	188811888118881	881188111881188	118881818188811	818118181811818	181811888118181
811811888118118	188118181811881	888881818188888	111188111881111	811111888111118	188818181818881	888181818181888	111888111888111	818111888111818	181818181818181	881181818181188	118888111888811

It is a ***palindromic pandiagonal magic square*** *of order 12 with* ***magic sum*** $S_{13\times 13} := 599999999999994$. *It is* ***block-wise pandigonal magic square*** *of order 12, where each block of order 4 is a* ***palindromic pandiagonal magic square*** *with equal* ***magic sums*** *as* $S_{4\times 4} := \dfrac{599999999999994}{3} = 199999999999998$.

The above example of **palindromic magic square** is for the **magic square** given in Example 23. In the similar way we can write the **palindromic magic squares** for the Examples 24 and 25

2.11 Magic Square of Order 13

2.11.1 Magic Square

Example 27. *The* ***magic square*** *of order 13 for the digits 1 and 8 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

88888888	88811181	88181881	81888111	81818181	81181811	81118881	18881118	18811818	18181888	18118118	11818188	11188818
11818881	11181118	88881818	88811888	88188118	81888188	81818818	81188888	81111181	18881881	18818111	18188181	18111811
18188818	18118888	11811181	11181881	88888111	88818181	88181811	81888881	81811118	81181818	81111888	18888118	18818188
18881811	18818881	18181118	18111818	11811888	11188118	88888188	88818818	88188888	81881181	81811881	81188111	81118181
81188188	81118818	18888888	18811181	18181881	18118111	11818181	11181811	88888881	88811118	88181818	81881888	81818118
81888181	81811811	81188881	81111118	18881818	18811888	18188118	18118188	11818818	11188888	88881181	88811881	88188111
88818118	88188188	81888818	81818888	81181181	81111881	18888111	18818181	18181811	18118881	11811118	11181818	88881888
11188111	88888181	88811811	88188881	81881118	81811818	81181888	81118118	18888188	18818818	18188888	18111181	11811881
18111888	11818118	11188188	88888818	88818888	88181181	81881881	81818111	81188181	81111811	18888881	18811118	18181818
18811881	18188111	18118181	11811811	11188881	88881118	88811818	88181888	81888118	81818188	81188818	81118888	18881181
81111818	18881888	18818118	18188188	18118818	11818888	11181181	88881881	88818111	88188181	81881811	81818881	81181118
81811181	81181881	81118111	18888181	18811811	18188881	18111118	11811818	11181888	88888118	88818188	88188818	81888888
88181118	81881818	81811888	81188118	81118188	18888818	18818888	18181181	18111881	11818111	11188181	88881811	88818881

It is based on the 2 by 2 combinations of 13 numbers

$$\{1118, 1181, 1811, 1818, 1881, 1888, 8111, 8118, 8181, 8188, 8818, 8881, 8888\}.$$

*These eleven numbers are **upside-down** and **mirror looking** combining in pairs as* $\{1118, 8111\}$, $\{1181, 1811\}$, $\{1818, 8181\}$, $\{1888, 8881\}$, $\{8188, 8818\}$, $\{1881, 8118\}$ *and* 8888. *It is **universal pandiagonal magic square**. Its **magic sum** is*

$$S_{13\times 13} := 688888882 = 10000 \times 68882 + 68882,$$

where

$$68882 = 1118 + 1181 + 1811 + 1818 + 1881 + 1888 + 8111 + 8118 + 8181 + 8188 + 8818 + 8881 + 8888.$$

2.11.2 Palindromic Magic Square

Example 28. *The **palindromic pandiagonal magic square** of order 13 for the digits 1 and 8 is given by*

888888888888888	888111818111888	881818818818188	818881111188818	818181818181818	811818111818118	811188818881118	188811181118881	188118181811881	181818888818181	181181181181181	118181888181811	111888181888111
118188818881811	111811181118111	888818181818888	888118888811888	881881181188188	818881888188818	818188181881818	811888888888118	811111818111118	188818818818881	188181111181881	181881818188181	181118111811181
181888181888181	181188888881181	118111818111811	111818818818111	888881111188888	888181818181888	881818111818188	818888818888818	818111181111818	811818181818118	811118888811118	188881181188881	188181888181881
188818111818881	188188818881881	181811181118181	181118181811181	118118888811811	111881181188111	888881888188888	888188181881888	881888888888188	818811818118818	818118818811818	811881111188118	811181818181118
811881888188118	811188181881118	188888888888881	188118181811881	181818818818181	181181111181181	118181818181811	111818111818111	888888818888888	888111181111888	881818181818188	818818888818818	818181181181818
818881818188818	818118111811818	811888818888118	811111181111118	188818181818881	188118888811881	181881181188181	181181888181181	118188181881811	111888888888111	888811818118888	888118818811888	881881111188188
888181181181888	881881888188188	818888181888818	818188888881818	811811818118118	811118818811118	188881111188881	188181818181881	181818111818181	181188818881181	118111181111811	111818181818111	888818888818888
111881111188111	888881818188888	888118111811888	881888818888188	818811181118818	818118181811818	811818888818118	811181181181118	188881888188881	188188181881881	181888888888181	181111818111181	118118818811811
181118888811181	118181181181811	111881888188111	888888181888888	888188888881888	881811818118188	818818818818818	818181111181818	811881818188118	811181181181118	188888818888881	188111181111881	181818181818181
188118818811881	181881111188181	181181818181181	118118111811811	111888818888111	888811181118888	888118181811888	881818888818188	818881181188818	818181888181818	811888181888118	811188888881118	188811818118881
811118181811118	188818888818881	188181181181881	181881888188181	181188181881181	118188888881811	111811818118111	888818818818888	888181111181888	881881818188188	818818111818818	818188818881818	811811181118118
818111818111818	811818818818118	811181111181118	188881818188881	188118111811881	181888818888181	181111181111181	118118181811811	111818888818111	888881181188888	888181888181888	881888181888188	818888888888818
881811181118188	818818181818818	818118888811818	811881181188118	811181888181118	188888181888881	188188888881881	181811818118181	181118818811181	118181111181811	111881818188111	888818111818888	888188818881888

*It is a **palindromic pandiagonal magic square** of order 13 with **magic sum** $S_{13\times 13} := 6888888888888882$.*

2.12 Magic Square of Order 14

2.12.1 Magic Square

Example 29. *The **magic square** of order 14 for the digits 1 and 8 is given by*

88818881	81881181	18111818	81811118	18881811	88188118	18818111	88111881	81111888	11188811	11888188	11818818	81181188	18188181
11188188	88188818	81811181	18118881	81181118	88118111	18181888	81881818	18881881	11888181	11818811	81111188	88818118	18811811
11888118	11188181	88118811	81181181	18118818	81881888	88811881	81818881	18811818	11818188	18881188	88188111	18181811	81111118
11818181	11888111	11188118	81888188	81111181	81811881	88181818	81188818	18188881	18811188	88111888	88811811	18881118	18118811
18181188	11818118	11881888	11188111	81818181	81181818	88118881	81118811	88818818	81881881	88181811	18811118	18118188	18881181
81111881	18881818	18818881	18188818	88818811	18111811	11811118	11181188	11881181	81181888	81818111	81888118	88118181	88188188
18818818	18188811	88818188	88188181	88118118	11181181	11881188	18111118	11811811	18888881	81111818	81181881	81811888	81888111
81188811	81118188	18888181	18818118	18188111	11881118	11181811	11811181	18111188	81818818	81888881	88111818	88181881	88811888
18888111	18811888	18181881	88811818	88188881	11811188	18111181	11881811	11181118	81118118	81188181	81818188	81888811	88118818
88111181	18111881	81881118	81111811	18818188	88818181	18888118	88181888	81188111	18181818	11188818	11888811	11818881	81811188
18111888	88111118	81181811	18888811	81881188	18188188	81118181	88818111	81818118	88181181	18811881	11188881	11888818	11811818
88181118	81811811	81118818	88111188	11811881	18818811	81188188	18188118	81888181	18118111	88811181	18881888	11181818	11888881
81881811	81188881	88181188	11811888	11881818	18888818	81818811	18818181	88118188	88811118	18118118	18181181	81118111	11181881
81811818	88811188	11818111	11881881	11181888	81118881	81888818	18888188	88188811	88111811	18181118	18118181	18811181	81188118

It is based on the 2 by 2 combinations of 14 numbers

$$\{1118, 1181, 1188, 1811, 1818, 1881, 1888, 8111, 8118, 8181, 8188, 8811, 8818, 8881\}.$$

*These fourteen numbers are **upside-down** and **mirror looking** combining in pairs as* $\{1118, 8111\}$, $\{1181, 1811\}$, $\{1188, 8811\}$, $\{1811, 1181\}$, $\{1818, 8181\}$, $\{1881, 8118\}$ *and* $\{1888, 8881\}$. *It is **universal magic square**. Its **magic sum** is*

$$S_{14\times 14} := 699999993 = 10000 \times 69993 + 69993,$$

where

$$69993 = 1118 + 1181 + 1188 + 1811 + 1818 + 1881 + 1888 + 8111 + 8118 + 8181 + 8188 + 8811 + 8818 + 8881.$$

*The middle block of order 4 is a magic square with **magic sum*** $S_{4\times 4} := 52985298$

2.12.2 Palindromic Magic Square

Example 30. *The **palindromic magic square** of order 14 for the digits 1 and 8 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

888188818881888	818811818118818	181118181811181	818111181111818	188818111818881	881881181188188	188181111181881	881118818811188	811118888811118	111888111888111	118881888188811	118188181881811	811811888118118	181881818188181
111881888188111	881888181888188	818111818111818	181188818881181	811811181118118	881181111181188	181818888818181	818818181818818	188818818818881	118881818188811	118188111881811	811111888111118	888181181181888	188118111811881
118881181188811	111881818188111	881188111881188	811811818118118	181188181881181	818818888818818	888118818811888	818188818881818	188118181811881	118181888181811	188811888118881	881881111188188	181818111818181	811111181111118
118181818181811	118881111188811	111881181188111	818881888188818	811111818111118	818118818811818	881818181818188	811888181888118	181888818888181	188111888111881	881118888811188	888118111811888	188811818118881	181188111881181
181811888118181	118181181181811	118818888818811	111881111188111	818181818181818	811818181818118	881188818881188	811188111881118	888188181881888	818818818818818	881818111818188	188111181111881	181181888181181	188811818118881
811118818811118	188818181818881	188188818881881	181888181888181	888188111881888	181118111811181	118111181111811	111811888118111	118811818118811	811818888818118	818181111181818	818881181188818	881181818181188	881881888188188
188188181881881	181888111888181	888181888181888	881881818188188	881181181181188	111811818118111	118811888118811	181111181111181	118118111811811	188888818888881	811118181811118	811818818818118	818118888811818	818881111188818
811888111888118	811181888181118	188881818188881	188181181181881	181881111188181	118811181118811	111818111818111	118111818111811	181111888111181	818188181881818	818888818888818	881118181811188	881818818818188	888118888811888
188881111188881	188118888811881	181818818818181	888118181811888	881888818888188	118111888111811	181111818111181	118818111818811	111811181118111	811181181181118	811881818188118	818181888181818	818888111888818	881188181881188
881111818111188	181118818811181	818811181118818	811118111811118	188181888181881	888181818181888	188881181188881	881818888818188	811881111188118	181818181818181	111888181888111	118888111888811	118188818881811	818111888111818
181118888811181	881111181111188	811818111818118	188888111888881	818811888118818	181881888188181	811181818181118	888181111181888	818181181181818	881811818118188	188118818811881	111888818888111	118888181888811	118118181811811
881811181118188	818118111811818	811188181881118	881111888111188	118118818811811	188188111881881	811881888188118	181881181188181	818881818188818	181181111181181	888111818111888	188818888818881	111818181818111	118888818888811
818818111818818	811888818888118	881811888118188	118118888811811	118818181818811	188888181888881	818188111881818	188181818181881	881181888181188	888111181111888	181181181181181	181811818118181	811181111181118	111818818818111
818118181811818	888111888111888	118181111181811	118818818818811	111818888818111	811188818881118	818888181888818	188881888188881	881888111888188	881181181181188	181811181118181	181181818181181	188111818111881	811881181188118

*It is a **palindromic magic square** of order 14 with **magic sum*** $S_{14\times 14} := 69999999999999993$. *The middle block of order 4 is a magic square with **magic sum*** $S_{4\times 4} := 529852999159914$.

2.13 Magic Square of Order 15

2.13.1 Magic Square

Example 31. *The **magic square** of order 15 for the digits 1 and 8 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

88818881	81811188	11188118	81181181	88111811	18818818	18881111	18181881	11818188	81888111	81118811	11111818	11881118	88181888	18118181
11888181	88188818	81181811	11818111	81111188	81881818	88818811	18818881	81811111	11118188	11181881	18111181	88111888	18188118	18881118
81181118	18118118	88118811	81111818	11881111	11111811	81811881	88818818	11188181	11818881	18181188	81881888	18818111	18881181	88188188
11818118	81111181	18188111	81888188	11111881	18111118	11181818	88188811	11888818	18811811	81811888	88811111	18881188	88118181	81188881
18118811	11888111	11111188	18811111	81818181	11188881	18181181	88118188	88811818	81181888	88181118	18881811	81888118	81118818	11811881
88181881	18188188	18111111	11181811	88811118	81188118	11818818	81888181	81111888	88111181	18881818	81818111	11118811	11888881	18811188
11111888	88118881	18818181	18181118	11811818	88181181	81118111	81818118	81881188	18881881	81181111	11188188	18118818	88811811	11888811
18811818	88811881	88188881	88118818	81888811	81818188	81188181	18881888	18181811	18111188	11881181	11811118	11181111	11118111	81118118
81881181	11118181	81811118	88181188	18188881	18888111	11888188	18111818	18811881	11188811	88118118	81188818	81111811	11811111	88811888
81118188	81881111	88811181	18111881	18888118	11818811	18811888	11881811	88111118	18181818	11118818	88188181	81818881	81181188	11188111
88118111	18811118	11881818	18888181	11188818	18181888	11118118	11811188	81188811	88181111	18111811	81118881	88818188	81881881	81811181
18181111	11811811	18888188	11118881	18111888	81118181	81881118	11118181	88188118	81818818	88818111	11881188	81181881	18818811	88111818
11181188	18888811	81111881	11881888	81188188	88111111	88181811	11111118	18118111	88818181	81888881	18818118	11811181	81811818	18188818
18888818	81181818	11811888	81818811	88188111	88811188	18118881	81111111	11111181	11888118	18818188	88111881	18188181	11181118	81881811
81811811	11181888	81888818	88818118	18811181	11881881	88111188	81188111	18888881	81111118	11818181	18188811	88181818	18118188	11111111

It is based on the 2 by 2 combinations of 15 numbers

$$\{1111, 1118, 1181, 1188, 1811, 1818, 1881, 1888, 8111, 8118, 8181, 8188, 8811, 8818, 8881\}.$$

These fifteen numbers are ***upside-down*** *and* ***mirror looking*** *combining in pairs as* $\{1118, 8111\}$, $\{1181, 1811\}$, $\{1188, 8811\}$, $\{1811, 1181\}$, $\{1818, 8181\}$, $\{1881, 8118\}$, $\{1888, 8881\}$ *and 1111. It is* ***universal magic square****. Its* ***magic sum*** *is*

$$S_{13\times 13} := 711111104 = 10000 \times 71104 + 71104,$$

where

$$71104 = 1111 + 1118 + 1181 + 1188 + 1811 + 1818 + 1881 + 1888 + 8111 + 8118 + 8181 + 8188 + 8811 + 8818 + 8881.$$

2.13.2 Semi-Magic Squares

Below is an example of **semi-magic square** of order 15, where each block of order 5 is a **pandiagonal magic squares** with different magic sums.

Example 32. *The* ***semi-magic square*** *of order 15 for the digits 1 and 8 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

11111118	18188181	18811811	81888818	88111888	11811181	11888118	81111188	81188881	88818111	11181111	18118188	18881818	81818811	88181881
81881811	88118818	11111888	18181118	18818181	81181188	88818881	11818111	11881181	81118118	81811818	88188811	11181881	18111111	18888188
18181888	18811118	81888181	88111811	11118818	11888111	81111181	81188118	88811188	11818881	18111881	18881111	81818188	88181818	11188811
88118181	11111811	18188818	18811888	81881118	88818118	11811188	11888881	81118111	81181181	88188188	11181818	18118811	18881881	81811111
18818818	81881888	88111118	11118181	18181811	81118881	81188111	88811181	11818118	11881188	18888811	81811881	88181111	11188188	18111818
11811111	11888188	81111818	81188811	88811881	11181118	18118181	18881811	81818818	88181888	11111181	18188118	18811188	81888881	88118111
81181818	88818811	11811881	11881111	81118188	81811811	88188818	11181888	18111118	18888181	81881188	88118881	11118111	18181181	18818118
11881881	81111111	81188188	88811818	11818811	18111888	18881118	81818181	88181811	11188818	18188111	18811181	81888118	88111188	11118881
88818188	11811818	11888811	81111881	81181111	88188181	11181811	18118818	18881888	81811118	88118118	11111188	18188881	18818111	81881181
81118811	81181881	88811111	11818188	11881818	18888818	81811888	88181118	11188181	18111811	18818881	81888111	88111181	11118118	18181188
11181181	18118118	18881188	81818881	88188111	11111111	18188188	18811818	81888811	88111881	11811118	11888181	81111811	81188818	88811888
81811188	88188881	11188111	18111181	18888118	81881818	88118811	11111881	18181111	18818188	81181811	88818818	11811888	11881118	81118181
18118111	18881181	81818118	88181188	11188881	18181881	18811111	81888188	88111818	11118811	11881888	81111118	81188181	88811811	11818818
88188118	11181188	18118881	18888111	81811181	88118188	11111818	18188811	18811881	81881111	88818181	11811811	11888818	81111888	81181118
18888881	81818111	88181181	11188118	18111188	18818811	81881881	88111111	11118188	18181818	81118818	81181888	88811118	11818181	11881811

It is ***semi-magic square*** *as sum of the rows and columns are the same as of above magic square, but the principal diagonals sum is different.*

The above magic square is of type 5×3. Below is another example of **semi-magic square** of order 15, where each block of order 3 is also a **semi-magic square** with different **semi-magic sums**

Example 33. *The* ***semi-magic square*** *of order 15 for the digits 1 and 8 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

88111818	18111811	18188811	11811881	88188818	18811181	11111888	88818881	18881111	81818111	11881188	81118181	81888118	11181118	81188188
18181811	88118811	18111818	18818818	11811181	88181881	18888881	11111111	88811888	81111188	81818181	11888111	81181118	81888188	11188118
18118811	18181818	88111811	88181181	18811881	11818818	88811111	18881888	11118881	11888181	81118111	81811188	11188188	81188118	81881118
81811888	11888881	81111111	81888111	11181188	81188181	88118118	18111118	18188188	11811818	88181811	18818811	11111881	88818818	18881181
81118881	81811111	11881888	81181188	81888181	11188111	18181118	88118188	18118118	18811811	11818811	88181818	18888818	11111181	88811881
11881111	81111888	81818881	11188181	81188111	81881188	18118188	18188118	88111118	88188811	18811818	11811811	88811181	18881881	11118818
11818118	88181118	18818188	11111818	88811811	18888811	81811881	11888818	81111181	81881888	11188881	81181111	88118111	18111188	18188181
18811118	11818188	88188118	18881811	11118811	88811818	81118818	81811181	11881881	81188881	81881111	11181888	18181188	88118181	18118111
88188188	18818118	11811118	88818811	18881818	11111811	11881181	81111881	81818818	11181111	81181888	81888881	18118181	18188111	88111188
81881881	11188818	81181181	88111888	18118881	18181111	11818111	88181188	18818181	11118118	88811118	18888188	81811818	11881811	81118811
81188818	81881181	11181881	18188881	88111111	18111888	18811188	11818181	88188111	18881118	11118188	88818118	81111811	81818811	11881818
11181181	81181881	81888818	18111111	18181888	88118881	88188181	18818111	11811188	88818188	18888118	11111118	11888811	81111818	81811811
11118111	88811188	18888181	81818118	11881118	81118188	81881818	11181811	81188811	88111881	18118818	18181181	11811888	88188881	18811111
18881188	11118181	88818111	81111118	81818188	11888118	81181811	81888811	11181818	18188818	88111181	18111881	18818881	11811111	88181888
88818181	18888111	11111188	11888188	81118118	81811118	11188811	81181818	81881811	18111181	18181881	88118818	88181111	18811888	11818881

*It is **semi-magic square** as sum of the rows and columns are the same as of above magic square, but the principal diagonals sum is different.*

2.13.3 Palindromic Magic Square

Example 34. *The **palindromic magic square** of order 15 for the digits 1 and 8 is given by*

888188818881888	818111888111818	111881181188111	811811818118118	881118111811188	188188181881881	188811111118881	181818818818181	118181888181811	818881111188818	811188111881118	111118181811111	118811181118811	881818888818188	181181818181181
118881818188811	881888181888188	811818111818118	118181111181811	811111888111118	818818181818818	888188111881888	188188818881881	818111111111818	111181888181111	111818818818111	181111818111181	881118888811188	181881181188181	188811181118881
811811181118118	181181181181181	881188111881188	811118181811118	118811111118811	111118111811111	818118818811818	888188181881888	111881818188111	118188818881811	181811888118181	818818888818818	188181111181881	188811818118881	881881888188188
118181181181811	811111818111118	181881111188181	818881888188818	111118818811111	181111181111181	111818181818111	881888111888188	118888181888811	188118111811881	818118888811818	888111111111888	188811888118881	881181818181188	811888818888118
181188111881181	118881111188811	111111888111111	188111111111881	818181818181818	111888818888111	181811818118181	881181888181188	888118181811888	811818888818118	881811181118188	188818111818881	818881181188818	811188181881118	118118818811811
881818818818188	181881888188181	181111111111181	111818111818111	888111181111888	811881181188118	118188181881811	818881818188818	811118888811118	881111818111188	188818181818881	818181111181818	111188111881111	118888818888811	188111888111881
111118888811111	881188818881188	188181818181881	181811181118181	118118181811811	881811818118188	811181111181118	818181181181818	818811888118818	188818818818881	811811111118118	111881888188111	181188181881181	888118111811888	118888111888811
188118181811881	888118818811888	881888818888188	881188181881188	818888111888818	818181888181818	811881818188118	188818888818881	181818111818181	181111888111181	118811818118811	118111181111811	111811111118111	111181111181111	811181181181118
818811818118818	111181818181111	818111181111818	881811888118188	181888818888181	188881111188881	118881888188811	181118181811181	188118818811881	111888111888111	881181181181188	811888181888118	811181181181118	118111111111811	888118888811888
811181888181118	818811111118818	888111818111888	181118818811181	188881181188881	118188111881811	188118888811881	118818111818811	881111181111188	181818181818181	111188181881111	881881818188188	818188818881818	811811888118118	111881111188111
881181111181188	188111181111881	118818181818811	188881818188881	111888181888111	181818888818181	111181181181111	118111888111811	811888111888118	881811111118188	181181181181181	811188818881118	888181888181888	818818818818818	818111818111818
181811111118181	118118111811811	188881888188881	111188818881111	181118888811181	811181818181118	818811181118818	111811818118111	881881181188188	818188181881818	888181111181888	118811888118811	811818818818118	188188111881881	881118181811188
111811888118111	188888111888881	811118818811118	118818888818811	811881888188118	881111111111188	881818111818188	111111181111111	181181111181181	888181818181888	818888818888818	188181181181881	118111818111811	818118181811818	181888181888181
188888181888881	811818181818118	118118888811811	818188111881818	881881111188188	888111888111888	181188818881181	811111111111118	111111818111111	118881181188811	188181888181881	881118818811188	181881818188181	111811181118111	818818111818818
818118111811818	111818888818111	818888181888818	888181181181888	188111818111881	118818818818811	881111888111188	811881111188118	188888818888881	811111181111118	118181818181811	181888111888181	881818181818188	181181888181181	111111111111111

*It is a **palindromic magic square** of order 15 with **magic sum** $S_{15\times 15} := 7111111111111104$.*

2.14 Magic Square of Order 16

We observe that the 4-digits combinations considering 2 by 2 maximum goes up to 256 possibilities resulting in magic squares of orders 9 and 16. The following magic squares of order 16 are with all possible 256 combinations.

2.14.1 Magic Square

Example 35. *The **pandiagonal magic square** of order 16 for the digits 1 and 8 is given by*

81181818	18811181	88818818	11188181	81811811	18181188	88188811	11818188	81111888	18881111	88888888	11118111	81881881	18111118	88118881	11888118
88818181	11188818	81181181	18811818	88188188	11818811	81811188	18181811	88888111	11118888	81111111	18881888	88118118	11888881	81881118	18111881
11181181	88811818	18818181	81188818	11811188	88181811	18188188	81818811	11111111	88881888	18888111	81118888	11881118	88111881	18118118	81888881
18818818	81188181	11181818	88811181	18188811	81818188	11811811	88181188	18888888	81118111	11111888	88881111	18118881	81888118	11881881	88111118
81111881	18881118	88888881	11118118	81881888	18111111	88118888	11888111	81181811	18811188	88818811	11188188	81811818	18181181	88188818	11818181
88888118	11118881	81111118	18881881	88118111	11888888	81881111	18111888	88818188	11188811	81181188	18811811	88188181	11818818	81811181	18181818
11111118	88881881	18888118	81118881	11881111	88111888	18118111	81888888	11181188	88811811	18818188	81188811	11811181	88181818	18188181	81818818
18888881	81118118	11111881	88881118	18118888	81888111	11881888	88111111	18818811	81188188	11181811	88811188	18188818	81818181	11811818	88181181
81881811	18111188	88118811	11888188	81111818	18881181	88888818	11118181	81811881	18181118	88188881	11818118	81181888	18811111	88818888	11188111
88118188	11888811	81881188	18111811	88888181	11118818	81111181	18881818	88188118	11818881	81811118	18181881	88818111	11188888	81181111	18811888
11881188	88111811	18118188	81888811	11111181	88881818	18888181	81118818	11811118	88181881	18188118	81818881	11181111	88811888	18818111	81188888
18118811	81888188	11881811	88111188	18888818	81118181	11111818	88881181	18188881	81818118	11811881	88181118	18818888	81188111	11181888	88811111
81811888	18181111	88188888	11818111	81181881	18811118	88818881	11188118	81881818	18111181	88118818	11888181	81111811	18881188	88888811	11118188
88188111	11818888	81811111	18181888	88818118	11188881	81181118	18811881	88118181	11888818	81881181	18111818	88888188	11118811	81111188	18881811
11811111	88181888	18188111	81818888	11181118	88811881	18818118	81188881	11881181	88111818	18118181	81888818	11111188	88881811	18888188	81118811
18188888	81818111	11811888	88181111	18818881	81188118	11181881	88811118	18118818	81888181	11881818	88111181	18888811	81118188	11111811	88881188

It is based on the 2 by 2 combinations of 16 numbers

$$\{1111, 1118, 1181, 1188, 1811, 1818, 1881, 1888, 8111, 8118, 8181, 8188, 8811, 8818, 8881, 8888\}.$$

These 16 numbers are ***upside-down*** *and* ***mirror looking*** *combining in pairs as* $\{1118, 8111\}$, $\{1181, 1811\}$, $\{1188, 8811\}$, $\{1811, 1181\}$, $\{1818, 8181\}$, $\{1881, 8118\}$, $\{1888, 8881\}$, 1111 *and* 8888. *It is* ***pandiagonal universal magic square****. Its* ***magic sum*** *is*

$$S_{16\times 16} := 799999992 = 10000 \times 79992 + 79992,$$

where

$$79992 = 1111 + 1118 + 1181 + 1188 + 1811 + 1818 + 1881 + 1888 + 8111 + 8118 + 8181 + 8188 + 8811 + 8818 + 8881 + 8888.$$

It is ***block-wise pandigonal magic square*** *of order 16, where each block of order 4 is a* ***pandiagonal magic square*** *with equal* ***magic sums*** *given by*

$$S_{4\times 4} := \frac{799999992}{4} = 199999998.$$

2.14.2 Palindromic Magic Square

Example 36. *The* ***palindromic pandiagonal magic square*** *of order 16 for the digits 1 and 8 is given by*

188181818181881	811888181888118	111811818118111	888118181811888	181881888188181	818188111881818	118111888111811	881818111818188	188881111188881	811188888881118	111111111111111	888818888818888	181181181181181	818888818888818	118811181118811	881118818811188
111818181818111	888111818111888	188188181881881	811881818188118	118118111811811	881811888118188	181888111888181	818181888181818	111118888811111	888811111118888	188888888888881	811181111181118	118818818818811	881111181111188	181188818881181	818881181188818
888188181881888	111881818188111	811818181818118	188111818111881	881888111888188	118181888181811	818118111811818	181811888118181	888888888888888	111181111181111	811118888811118	188811111118881	881188818881188	118881181188811	818818818818818	181111181111181
811811818118118	188118181811881	888181818181888	111888181888111	818111888111818	181818111818181	881881888188188	118188111881811	811111111111118	188818888818881	888881111188888	111188888881111	818811181118818	181118818811181	881181181181188	118888818888811
188881181188881	811188818881118	111111181111111	888818818818888	181181111181181	818888888888818	118811111118811	881118888811188	188181888181881	811888111888118	111811888118111	888118111811888	181881818188181	818188181881818	118111818111811	881818181818188
111118818811111	888811181118888	188888818888881	811181181181118	118818888818811	881111111111188	181188888881181	818881111188818	111818111818111	888111888111888	188188111881881	811881888188118	118118181811811	881811818118188	181888181888181	818181818181818
888888818888888	111181181181111	811118818811118	188811181118881	881188888881188	118881111188811	818818888818818	181111111111181	888188111881888	111881888188111	811818111818118	188111888111881	881888181888188	118181818181811	818118181811818	181811818118181
811111181111118	188818818818881	888881181188888	111188818881111	818811111118818	181118888811181	881181111181188	118888888888811	811811888118118	188118111811881	888181888181888	111888111888111	818118181811818	181818181818181	881881818188188	118188181881811
181181888181181	818888111888818	118811888118811	881118111811188	188881818188881	811188181881118	111111818111111	888818181818888	181881181188181	818188818881818	118111181111811	881818818818188	188181111181881	811888888888118	111811111118111	888118888811888
118818111818811	881111888111188	181188111881181	818881888188818	111118181811111	888811818118888	188888181888881	811181818181118	118118818811811	881811181118188	181888818888181	818181181181818	111818888818111	888111111111888	188188888881881	811881111188118
881188111881188	118881888188811	818818111818818	181111888111181	888888181888888	111181818181111	811118181811118	188811818118881	881888818888188	118181181181811	818118818811818	181811181118181	888188888881888	111881111188111	811818888818118	188111111111881
818811888118818	181118111811181	881181888181188	118888111888811	811111818111118	188818181818881	888881818188888	111188181881111	818111181111818	181818818818181	881881181188188	118188818881811	811811111118118	188118888811881	888181111181888	111888888888111
181881111188181	818188888881818	118111111111811	881818888818188	188181181181881	811888818888118	111811181118111	888118818811888	181181818181181	818888181888818	118811818118811	881118181811188	188881888188881	811188111881118	111111888111111	888818111818888
118118888811811	881811111118188	181888888888181	818181111181818	111818818818111	888111181111888	188188818881881	811881181188118	118818181818811	881111818111188	181188181881181	818881818188818	111118111811111	888811888118888	188888111888881	811181888181118
881888888888188	118181111181811	818118888811818	181811111118181	888188818881888	111881181188111	811818818818118	188111181111881	881188181881188	118881818188811	818818181818818	181111818111181	888888111888888	111181888181111	811118111811118	188811888118881
818111111111818	181818888818181	881881111188188	118188888881811	811811181118118	188118818811881	888181181181888	111888818888111	818811818118818	181118181811181	881181818181188	118888181888811	811111888111118	188818111818881	888881888188888	111188111881111

It is a ***block-wise palindromic pandigonal magic square*** *of order 16, where each block of order 4 is a* ***palindromic pandiagonal magic square*** *with equal* ***magic sums****. The magic sums are given by*

$$S_{16\times 16} := 7999999999999992 \qquad and \qquad S_{4\times 4} := \frac{7999999999999992}{4} = 1999999999999998.$$

2.14.3 Bimagic Square

Example 37. *The* ***bimagic square*** *of order 16 for the digits 1 and 8 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

88888888	18811881	11181118	81118111	88818118	18881111	11111888	81188881	88181811	18118818	11888181	81811188	88111181	18188188	11818811	81881818
11188111	81111118	88881881	18818888	11118881	81181888	88811111	18888118	11881188	81818181	88188818	18111811	11811818	81888811	88118188	18181181
81111881	11188888	18818111	88881118	81181111	11118118	18888881	88811888	81818818	11881811	18111188	88188181	81888188	11811181	18181818	88118811
18811118	88888111	81118888	11181881	18881888	88818881	81188118	11111111	18118181	88181188	81811811	11888818	18188811	88111818	81881181	11818188
88181181	18118188	11888811	81811818	88111811	18188818	11818181	81881188	88888118	18811111	11181888	81118881	88818888	18881881	11111118	81188111
11881818	81818811	88188188	18111181	11811188	81888181	88118818	18181811	11188881	81111888	88881111	18818118	11118111	81181118	88811881	18888888
81818188	11881181	18111818	88188811	81888818	11811811	18181188	88118181	81111111	11188118	18818881	88881888	81181881	11118888	18888111	88811118
18118811	88181818	81811181	11888188	18188181	88111188	81881811	11818818	18811888	88888881	81118118	11181111	18881118	88818111	81188888	11111881
88118118	18181111	11811888	81888881	88188888	18111881	11881118	81818111	88811181	18888188	11118811	81181818	88881811	18818818	11188181	81111188
11818881	81881888	88111111	18188118	11888111	81811118	88181881	18118888	11111818	81188811	88818188	18881181	11181188	81118181	88888818	18811811
81881111	11818118	18188881	88111888	81811881	11888888	18118111	88181118	81188188	11111181	18881818	88818811	81118818	11181811	18811188	88888181
18181888	88118881	81888118	11811111	18111118	88188111	81818888	11881881	18888811	88811818	81181181	11118188	18818181	88881188	81111811	11188818
88811811	18888818	11118181	81181188	88881181	18818188	11188811	81111818	88118888	18181881	11811118	81888111	88188118	18111111	11881888	81818881
11111188	81188181	88818818	18881811	11181818	81118811	88888188	18811181	11818111	81881118	88111881	18188888	11888881	81811888	88181111	18118118
81188818	11111811	18881188	88818181	81118188	11181181	18811818	88888811	81881881	11818888	18188111	88111118	81811111	11888118	18118881	88181888
18888181	88811188	81181811	11118818	18818811	88881818	81111181	11188188	18181118	88118111	81888888	11811881	18111888	88188881	81818118	11881111

*It is constructed with same 16 numbers used in previous example. According to previous example, it is **universal bimagic square** with magic and bimagic sums given by*

$$S_{16\times 16} := 799999992 \quad and \quad Sb_{16\times 16} := 59797978997979800.$$

*It is **block-wise magic square** of order 16, where each block of order 4 is a **magic square** with equal **magic sums** given by*

$$S_{4\times 4} := \frac{799999992}{4} = 199999998.$$

2.14.4 Palindromic Bimagic Square

Example 38. *The **palindromic bimagic square** of order 16 for the digits 1 and 8 is given by*

111111111111111	811881181188118	888188818881888	188818888818881	111818818818111	811188888881118	888881111188888	188111181111881	118181888181811	818811818118818	881118181811188	181888111888181	118888181888811	818118111811818	881811888118188	181181818181181
888118888811888	188888818888881	111181181181111	811811111118118	888811181118888	188181111181881	111888888888111	811118818811118	881188111881188	181818181818181	118111818111811	818881888188818	881881818188188	181111888111181	118818111818811	818188181881818
188881181188881	888111111111888	811818888818118	111188818881111	188188888881881	888818818818888	811111181111118	111881111188111	181811818118181	881181888181188	818888111888818	118118181811811	181118111811181	881888181888188	818181818181818	118811888118811
811888818888118	111118888811111	188811111118881	888181181181888	811181111181118	111811181118111	188118818811881	888888888888888	818818181818818	118188111881811	181881888188181	881111818111188	818111888111818	118881818188811	181188181881181	881818111818188
118188181881811	818818111818818	881111888111188	181881818188181	118881888188811	818111818111818	881818181818188	181188111881181	111118818811111	811888888888118	888181111181888	188811181118881	111811111118111	811181181181118	888888818888888	188118888811881
881181818181188	181811888118181	118118111811811	818888181888818	881888111888188	181118181811181	118811818118811	818181888181818	888111181111888	188881111188881	111188888881111	811818818818118	888818888818888	188188818881881	111881181188111	811111111111118
181818111818181	881188181881188	818881818188818	118111888111811	181111818111181	881881888188188	818188111881818	118818181818811	188888888888881	888118818811888	811811181118118	111181111181111	188181181181881	888811111118888	811118888811118	111888818888111
818811888118818	118181818181811	181888181888181	881118111811188	818118181811818	118888111888811	181181888181181	881811818118188	811881111188118	111111181111111	188818818818881	888188888881888	811188818881118	111818888818111	188111111111881	888881181188888
118818818818811	818188888881818	881881111188188	181111181111181	118111111111811	818881181188818	881188818881188	181818888818181	111888181888111	811118111811118	888811888118888	188181818181881	111181888181111	811811818118118	888118181811888	188888111888881
881811181118188	181181111181181	118888888888811	818118818811818	881118888811188	181888818888181	118181181181811	818811111118818	888881818188888	188111888111881	111818111818111	811188181881118	888188111881888	188818181818881	111111818111111	811881888188118
181188888881181	881818818818188	818111181111818	118881111188811	181881181188181	881111111111188	818818888818818	118188818881811	188118111811881	888888181888888	811181818181118	111811888118111	188811818118881	888181888181888	811888111888118	111118181811111
818181111181818	118811181118811	181118818811181	881888888888188	818888818888818	118118888811811	181811111118181	881181181181188	811111888111118	111881818188111	188188181881881	888818111818888	811818181818118	111188111881111	188881888188881	888111818111888
111881888188111	811111818111118	888818181818888	188188111881881	111188181881111	811818111818118	888111888111888	188881818188881	118811111118811	818181181181818	881888818888188	181118888811181	118118818811811	818888888888818	881181111181188	181811181118181
888888111888888	188118181811881	111811818118111	811181888181118	888181818181888	188811888118881	111118111811111	811888181888118	881818888818188	181188818881181	118881181188811	818111111111818	881111181111188	181881111188181	118188888881811	818818818818818
188111818111881	888881888188888	811188111881118	111818181818111	188818111818881	888188181881888	811881818188118	111111888111111	181181181181181	881811111118188	818118888811818	118888818888811	181888888888181	881118818811188	818811181118818	118181111181811
811118181811118	111888111888111	188181888181881	888811818118888	811811888118118	111181818181111	188888181888881	888118111811888	818188818881818	118818888818811	181111111111181	881881181188188	818881111188818	118111181111811	181818818818181	881188888881188

It is a ***block-wise palindromic bi magic square*** *of order 16, where each block of order 4 is a* ***palindromic magic square*** *with equal* ***magic sums****. The magic sums are given by*

$$S_{16\times 16} := 7999999999999992 \qquad \text{and} \qquad S_{4\times 4} := \frac{7999999999999992}{4} = 1999999999999998.$$

3 Universal Magic Squares With Digits 2 and 5

3.1 Semi-Magic Square of Order 3

3.1.1 Semi-Magic Square

Example 39. *The* ***semi-magic square*** *of order 3 for the digits 2 and 5 is given by*

2522	5252	2225
2252	2525	5222
5225	2222	2552

*In this case it is **semi-magic square**. It has same sum in lines and rows, while principal diagonals sums are different. It is based on the 2 by 2 combinations of three numbers $\{22, 25, 52\}$. These three numbers are **upside-down** and **mirror looking** in pair $\{25, 52\}$, while for 22 is upside down but not **mirror looking**. In the mirror it becomes as 55. It is **universal semi-magic square** with different **semi-magic sums**. These magic sums are:*

$$S_{3\times 3} := 9999 = 100 \times 99 + 99, \quad \textit{and} \quad S_{3\times 3} := 13332 = 100 \times 132 + 132$$

where

$$99 = 22 + 25 + 52 \quad \textit{and} \quad 132 = 55 + 25 + 52$$

*The **magic sum** $S_{3\times 3} := 13332$ refers to **mirror looking** magic square.*

3.1.2 Palindromic Semi-Magic Square

Example 40. *The **palindromic semi-magic square** of order 3 for the digits 2 and 5 is given by*

2522252	5555555	2225222
2255522	2525252	5522255
5525255	2222222	2555552

*It is a **palindromic semi-magic square** of order 3 with **semi-magic sum** $S_{3\times 3} := 10303029$.*

3.2 Magic Square of Order 4

We observe that the 2-digits combinations considering 2 by 2 maximum goes up to 16 possibilities resulting in magic squares of orders 3 and 4. The following magic square of order 4 is with all 16 possible combinations.

3.2.1 Magic Square

Example 41. *The **pandiagonal magic square** of order 4 for the digits 2 and 5 is given by*

2552	5255	2222	5525
2225	5522	2555	5252
5555	2252	5225	2522
5222	2525	5552	2255

It is based on the 2 by 2 combinations of four numbers $\{22, 25, 52, 55\}$. *These four numbers are* ***upside-down*** *and* ***mirror looking*** *in pairs* $\{25, 52\}$ *and* $\{22, 55\}$. *It is* ***universal pandiagonal magic square***. *Its* ***magic sum*** *is*

$$S_{4\times 4} := 15554 = 100 \times 154 + 154,$$

where

$$154 = 22 + 25 + 52 + 55.$$

3.2.2 Palindromic Magic Square

Example 42. *The* ***palindromic pandiagonal magic square*** *of order 4 for the digits 2 and 5 is given by*

2552552	5255525	2222222	5525255
2225222	5522255	2555552	5252525
5555555	2252522	5225225	2522252
5222225	2525252	5552555	2255522

It is a ***palindromic pandiagonal magic square*** *of order 4 with* ***magic sum*** $S_{4\times 4} := 15555554$.

3.3 Magic Square of Order 5

We observe that the two digits combinations for two numbers maximum goes up to 16. This lead us to magic squares of orders 3 and 4. From now onwards, we shall work with three digits combinations for two numbers. It will go up to 8^{th} order magic squares.

3.3.1 Magic Square

Example 43. *The* ***pandiagonal magic square*** *of order 5 for the digits 2 and 5 is given by*

222222	225522	252225	522525	525252
522225	525525	222252	225222	252522
225252	252222	522522	525225	222525
525522	222225	225525	252252	522222
252525	522252	525222	222522	225225

It is based on the 2 by 2 combinations of five numbers $\{222, 225, 252, 522, 525\}$. *These five numbers are* ***upside-down*** *and* ***mirror looking****, but in the mirror 2 becomes 5 and 5 as 2. It is* ***universal pandiagonal square****, but with different magic sums. These magic sums are:*

$$S_{5\times5} := 1747746 = 1000 \times 1746 + 1746, \quad \text{and} \quad S_{5\times5} := 2141139 = 1000 \times 2139 + 2139,$$

where

$$1746 = 222 + 225 + 252 + 522 + 525 \quad \text{and} \quad 2139 = 555 + 552 + 525 + 255 + 252.$$

The ***magic sum*** $S_{5\times5} := 2141139$ *refers to* ***mirror looking*** *magic square.*

3.3.2 Palindromic Magic Square

Example 44. *The* ***palindromic pandiagonal magic square*** *of order 5 for the digits 2 and 5 is given by*

2225555222	2252552522	2522522252	2552525552	5252525252
2552252552	5255252552	2222525222	2255555522	2522552252
2252525252	2525555252	2552552552	5252252525	2225252222
5252552525	2222522222	2255252552	2522525252	2555555552
2525252525	2552525552	5255555525	2222552222	2252252522

It is a ***palindromic pandiagonal magic square*** *of order 5 with* ***magic sum*** $S_{5\times5} := 148081408073$.

3.4 Magic Square of Order 6

3.4.1 Magic Square

Example 45. *The* ***magic square*** *of order 6 for the digits 2 and 5 is given by*

555555	222525	222552	222225	555252	555222
252222	525252	252552	525225	525525	252555
255222	255525	522225	522552	255252	522555
522222	522252	255225	255552	522525	255555
525555	252252	525552	252225	252525	525222
222555	555525	555225	555552	222252	222222

Let's consider $T := 1000 \times A + B$, *where* $A := \{222, 252, 255, 522, 525, 555\}$ *and* $B := \{222, 225, 252, 525, 552, 555\}$. *This gives total 36 entries for the magic square of order 6. Since sum of members of A and B are 2331, this gives* ***magic square*** *of order 6 with* ***magic sum*** $S_{6\times6} := 2333331 = 1000 \times 2331 + 2331$. *Below are the values in all the three cases:*

Magic Square:	$A := \{222, 252, 255, 522, 525, 555\}$	*and*	$B := \{222, 225, 252, 525, 552, 555\}$
Upside-down:	$A := \{222, 252, 552, 225, 525, 555\}$	*and*	$B := \{222, 522, 252, 525, 255, 555\}$
Mirror Looking:	$A := \{555, 525, 225, 552, 252, 222\}$	*and*	$B := \{555, 255, 525, 252, 522, 222\}$

In all the three cases the total sums os 10 numbers appearing in A *and* B *are always same. i.e.,* 2331. *Thus, the* ***magic square*** *is* ***universal****, i.e.,* ***upside-down*** *and* ***mirror looking*** *with* ***magic sum*** $S_{6\times6} := 2333331$.

3.4.2 Palindromic Magic Square

Example 46. *The* ***palindromic magic square*** *of order 6 for the digits 2 and 5 is given by*

55555555555	22252525222	22255255222	22222522222	55525252555	55522222555
25222222252	52525252525	25255255252	52522522525	52552525525	25255555252
25522222552	25552525552	52222522225	52255255225	25525252552	52255555225
52222222225	52225252225	25522522552	25555255552	52252525225	25555555552
52555555525	25225252252	52555255525	25222522252	25252525252	52522222525
22255555222	55552525555	55522522555	55555255555	22225252222	22222222222

It is a ***palindromic magic square*** *of order 6 with* ***magic sum*** $S_{6\times6} := 233333333331$.

3.5 Magic Square of Order 7

3.5.1 Magic Square

Example 47. *The **magic square** of order 7 for the digits 2 and 5 is given by*

222222	225225	252252	255255	522522	525525	552552
525522	552525	222552	225222	252225	255252	522255
255225	522252	525255	552522	222525	225552	252222
225525	252552	255222	522225	525252	552255	222522
552252	222255	225522	252525	255552	522222	525225
522552	525222	552225	222252	225255	252522	255525
252255	255522	522525	525552	552222	222225	225252

It is based on the 2 by 2 combinations of seven numbers $\{222, 225, 252, 255, 522, 525, 552\}$. *These seven numbers are **upside-down** and **mirror looking**, but in the mirror 2 becomes 5 and 5 as 2. It is **universal pandiagonal square** with different magic sums. These magic sums are:*

$$S_{7\times 7} := 2555553 = 1000 \times 2553 + 2553, \quad \textit{and} \quad S_{7\times 7} := 2888886 = 1000 \times 2886 + 2886$$

where

$$2553 = 222 + 225 + 252 + 522 + 525 \quad \textit{and} \quad 2886 = 225 + 252 + 255 + 522 + 525 + 552 + 555$$

*The **magic sum*** $S_{7\times 7} := 2888886$ *refers to **mirror looking** magic square.*

3.5.2 Palindromic Magic Square

Example 48. *The **palindromic pandiagonal magic square** of order 7 for the digits 2 and 5 is given by*

22255555222	52525252525	25522522552	22525552522	55252225255	52255255225	25252525252
52252225225	25255255252	22252525222	52555555525	25525252552	22522522522	55225552255
22525252522	55222522255	52225552225	25252225252	22255255222	52555252525	25555555552
52555255525	25552525552	22555555522	55225252255	52222522225	25225552252	22252225222
25222522252	22225552222	52552225525	25555255552	22552525522	55255555255	52225252225
55252525255	52255555225	25225252252	22222522222	52525552525	25552225552	22555255522
25525552552	22552225522	55255255255	52252525225	25255555252	22225252222	52522522525

*It is a **palindromic pandiagonal magic square** of order 7 with **magic sum** $S_{7\times 7} := 255588888553$.*

3.6 Magic Square of Order 8

We observe that the 3-digits combinations considering 2 by 2 maximum goes up to 64 possibilities resulting in magic squares of orders 5 to 8. The following magic squares of order 8 are with all 64 possible combinations.

3.6.1 Magic Square

Example 49. *The **pandiagonal magic square** of order 8 for the digits 2 and 5 with equal sum magic squares of order 4 is given by*

222222	555522	255255	522555	225222	552522	252255	525555
255555	522255	222522	555222	252555	525255	225522	552222
522522	255222	555555	222255	525522	252222	552555	225255
555255	222555	522222	255522	552255	225555	525222	252522
222225	555525	255252	522552	225225	552525	252252	525552
255552	522252	222525	555225	252552	525252	225525	552225
522525	255225	555552	222252	525525	252225	552552	225252
555252	222552	522225	255525	552252	225552	525225	252525

*It is based on the 2 by 2 combinations of eight numbers $\{222, 225, 252, 255, 522, 525, 552, 555\}$. These eight numbers are **upside-down** and **mirror looking**. It is **universal pandiagonal magic square**. Its **magic sum** is*

$$S_{8\times 8} := 3111108 = 1000 \times 3108 + 3108,$$

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

where

$$3108 = 222 + 225 + 252 + 255 + 522 + 525 + 552 + 555.$$

*It is **block-wise pandigonal magic square** of order 8, where each block of order 4 is a **pandiagonal magic square** with equal **magic sums** given by*

$$S_{4\times 4} := \frac{3111108}{2} = 1555554.$$

3.6.2 Palindromic Magic Square

Example 50. *The **palindromic pandiagonal magic square** of order 8 for the digits 2 and 5 is given by*

2222222222	5555222555	2552555252	5225555225	2252222522	5525222525	2522555225	5255555525
2555555552	5222555222	2225222522	5552222555	2525555252	5252555525	2255222522	5522222255
5225222525	2552222552	5555555555	2222555222	5255222555	2522222252	5525555255	2252555522
5552555255	2225555222	5222222225	2555222552	5522555255	2255555522	5252222525	2525222525
2222522222	5555252555	2552525252	5225525525	2252252522	5525252525	2522525225	5255525525
2555525552	5222525222	2225252522	5552252555	2525525252	5252525525	2255252522	5522252255
5225252525	2552252552	5555255555	2222525222	5255252555	2522522252	5525525255	2252525522
5552525255	2225525222	5222522225	2555252552	5522525255	2255525522	5252252525	2525252525

*It is a **block-wise palindromic pandigonal magic square** of order 8, where each block of order 4 is a **palindromic pandiagonal magic square** with equal **magic sums**. The magic sums are given by*

$$S_{8\times 8} := 311111111108 \qquad \text{and} \qquad S_{4\times 4} := \frac{311111111108}{2} = 155555555554.$$

3.6.3 Bimagic Square

Example 51. *The **pandiagonal bimagic magic square** of order 8 for the digits 2 and 5 is given by*

225555	525222	522255	222522	255252	555525	552552	252225
255225	555552	552525	252252	225522	525255	522222	222555
222222	522555	525522	225255	252525	552252	555225	255552
252552	552225	555252	255525	222255	522522	525555	225222
522525	222252	225225	525552	552222	252555	255522	555255
552255	252522	255555	555222	522552	222225	225252	525525
525252	225525	222552	522225	555555	255222	252255	552522
555522	255255	252222	552555	525225	225552	222525	522252

It is also based on the 2 by 2 combinations of eight numbers $\{222, 225, 252, 255, 522, 525, 552, 555\}$. *These eight numbers are* ***upside-down*** *and* ***mirror looking****. It is* ***universal pandiagonal bimagic magic square****. Its* ***magic sum*** *is*

$$S_{8\times 8} := 3111108 = 1000 \times 3108 + 3108,$$

where

$$3108 = 222 + 225 + 252 + 255 + 522 + 525 + 552 + 555.$$

3.6.4 Palindromic Bimagic Square

It is also ***bimagic square*** *of order 8 with* ***bimagic sum*** *given by*

$$Sb_{8\times 8} := 1391692305276.$$

Moreover, as specified in the figure, the sum of 2×4 *entries are of same sum as of magic square, i.e.,* 3111108.

Example 52. *The* ***palindromic bimagic square*** *of order 8 for the digits 2 and 5 is given by*

2255555522	5252222525	5222555222 5	2225222522 2	2552525255 2	5555252555 5	5525525525 5	2522252225 2
2552252255 2	5555525555 5	5525252525 5	2522525225 2	2255222552 2	5252555252 5	5222222222 5	2225555522 2
2222222222 2	5225555522 5	5255222552 5	2252555252 2	2525252525 2	5522525222 55	5552252255 5	2555525555 2
2525525525 2	5522252222 55	5552525255 5	2555252555 2	2222555222 2	5225222522 5	5255555552 5	2252222252 2
5225252522 5	2222525222 2	2252252252 2	5255525552 5	5522222222 55	2525555525 2	2555222555 2	5552555255 5
5522555222 55	2525222525 2	2555555555 2	5552222255 5	5225525522 5	2222252222 2	2252525252 2	5255252552 5
5252525252 5	2255252552 2	2225525522 2	5222252222 5	5555555555 5	2552222255 2	2522555222 52	5525222525 5
5555222555 5	2552555255 2	2522222222 52	5525555552 55	5252252252 5	2255525552 2	2225252522 2	5222525222 5

*It is a **palindromic bimagic square** of order 8 with **magic sums:***

$$S_{8\times 8} := 311111111108 \qquad and \qquad Sb_{8\times 8} := 13916947251838608305276.$$

Moreover, as specified in the figure, the sum of 2×4 *entries are of same sum as of magic square, i.e.,* 311111111108.

3.7 Magic Square of Order 9

We observe that the three digits combinations considering 2 by 2 maximum goes up to 64 possibilities resulting in magic squares of orders 8. From now onwards, we shall work with four digits combinations for two numbers. It will go up to 16^{th} order magic squares.

3.7.1 Magic Square

Example 53. *The **pandiagonal magic square** of order 9 for the digits 2 and 5 is given by*

52522225	25222552	22255222	22522252	52222522	25255225	25522222	22222525	52255252
25255252	22522222	52222525	52255222	25522225	22222552	22255225	52522252	25222522
22222522	52255225	25522252	25222525	22255252	52522222	52222552	25255222	22522225
52252525	25525252	22222222	22252552	52525222	25222225	25252522	22525225	52222252
25222252	22252522	52525225	52222222	25252525	22525252	22222225	52252552	25525222
22525222	52222225	25252552	25525225	22222252	52252522	52525252	25222222	22252525
52225225	25252252	22522522	22225252	52252222	25522525	25225222	22252225	52522552
25522552	22225222	52252225	52522522	25225225	22252252	22522525	52225252	25252222
22252222	52522525	25225252	25252225	22522552	52225222	52252252	25522522	22225225

It is based on the 2 by 2 combinations of nine numbers $\{2222, 2225, 2252, 2522, 2525, 2552, 5222, 5225, 5252\}$. *These nine numbers are* ***upside-down*** *and* ***mirror looking*** *combining in pairs as* $\{2225, 5222\}$, $\{2252, 2522\}$, $\{2525, 5252\}$, $\{2552, 5225\}$, *and* 2222. *In the mirror 2 becomes 5 and 5 as 2. It is* ***universal pandiagonal square*** *with different magic sums. These magic sums are:*

$$S_{9\times 9} := 299999997 = 10000 \times 29997 + 29997, \quad \textit{and} \quad S_{9\times 9} := 399999996 = 10000 \times 39996 + 39996,$$

where

$$29997 = 2222 + 2225 + 2252 + 2522 + 2525 + 2552 + 5222 + 5225 + 5252$$

and

$$39996 = 5555 + 5552 + 5525 + 5255 + 5252 + 5225 + 2555 + 2552 + 2525$$

The ***magic sum*** $S_{9\times 9} := 399999996$ *refers to* ***mirror looking*** *magic square. Moreover, Each block of order* 3×3 *as specified in the figure is a* ***semi-magic square*** *of order 3 with* ***semi-magic sum****:* $S_{3\times 3} := 99999999$. *In the mirror looking case, this sum is* $S_{3\times 3} := 133333332$.

3.7.2 Palindromic Magic Square

Example 54. *The* ***palindromic pandiagonal magic square*** *of order 9 for the digits 2 and 5 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

222255525552222	255252252252552	522525555525225	522255252552225	225252555252522	252525525525252	252255555552252	525252525252525	222525252525222
252525252525252	522255555552225	225252525252522	222525555525222	252255525552252	525252252252525	522525525525225	222255252552222	255252555252552
525252555252525	222525525525222	252255252552252	255252525252552	522525252525225	222255555552222	225252252252522	252525555525252	522255525552225
222552525255222	252225252522252	525255555552525	522552252255225	222225555522222	255255525552552	252552555255252	522225525522225	225255252552522
255255252552552	522552555255225	222225525522222	225255555552522	252552525255252	522225252522225	525255525552525	222552252255222	252225555522252
522225555522225	225255525552522	252552252255252	252225525522252	525255252552525	222552555255222	222225252522222	255255555552552	522552525255225
225225525522522	252555252555252	522252555252225	525225252522525	222555555555222	252252525252252	255225555522552	522555525555225	222252252252222
252252252252252	525225555522525	222555525555222	222252555252222	255225525522552	522555252555225	522252525252225	225225252522522	252555555555252
522555555555225	222252525252222	255225252522552	252555525555252	522252252252225	225225555522522	222555252555222	252252555252252	525225525522525

*It is a **block-wise palindromic pandigonal magic square** of order 9, where each block of order 3 is a **palindromic semi-magic square** with equal **magic sums**. The magic sums are given by*

$$S_{9\times 9} := 3000099999989997 \qquad \text{and} \qquad S_{3\times 3} := \frac{3000099999989997}{3} = 1000033333329999 \quad \textit{(semi-magic sum).}$$

3.7.3 Bimagic Square

Example 55. *The **bimagic square** of order 9 for the digits 2 and 5 is given by*

52525252	52252222	52222525	25522225	25252552	25225222	22522522	22255225	22222252
25522522	25255225	25222252	22525252	22252222	22222525	52522225	52252552	52225222
22522225	22252552	22225222	52522522	52255225	52222252	25525252	25252222	25222525
52222222	52522525	52255252	25222552	25525222	25252225	22225225	22522252	22252522
25225225	25522252	25252522	22222222	22522525	22255252	52222552	52525222	52252225
22222552	22525222	22252225	52225225	52522252	52252522	25222222	25522525	25255252
52252525	52225252	52522222	25255222	25222225	25522552	22252252	22222522	22525225
25252252	25222522	25525225	22252525	22225252	22522222	52255222	52222225	52522552
22255222	22222225	22522552	52252252	52222522	52525225	25252525	25225252	25522222

It is constructed with same nine digits used in previous example. According to previous example explanations, it is ***universal bimagic square*** *with* ***magic*** *and* ***bimagic sums*** *are given by*

$$S_{9\times 9} := 299999997 \quad \text{and} \quad Sb_{9\times 9} := 11638163616381639.$$

In the ***mirror looking*** *case the* ***magic*** *and* ***bimagic sums*** *are given by*

$$S_{9\times 9} := 399999996 \quad \text{and} \quad Sb_{9\times 9} := 19415941238603862.$$

The sum of 9 members of each block of order 3 (specified in the figure) is the same as of magic square, i.e., 299999997. In the ***mirror looking*** *case this sum is 399999996.*

3.7.4 Palindromic Magic Square

Example 56. *The* ***palindromic bimagic square*** *of order 9 for the digits 2 and 5 is given by*

222225252522222	222555555555222	225252525252522	252255525552252	252552252255252	255225555522552	522252555252225	522525525525225	525255252552525
252252555252252	252525525525252	255255252552552	522225252522225	522555555555225	525252525252525	222255525552222	222552252255222	225225555522522
522255525552225	522552252255225	525225555522525	222252555252222	222525525525222	225255252552522	252225252522252	252555555555252	255252525252552
225255555552522	222252525252222	222525252525222	255252252252552	252225555522252	252555525555252	525225525522525	522255252552225	522552555255225
255225525522552	252255252552252	252552555255252	525255555552525	522252525252225	522525252525225	225252252252522	222225555522222	222555525555222
525252252252525	522225555522225	522555525555225	225225525522522	222552525255222	222552555255222	255255555552552	252252525252252	252525252525252
222552525255222	225225252522522	222255555552222	252525555525252	255255525552552	252252252252252	522555252555225	525252555252525	522225525522225
252555252555252	255252555252552	252225525522252	522552525255225	525225252522525	522255555552225	222525555525222	225255525552522	222252252252222
522525555525225	525255525552525	522252252252225	222555252555222	225252555252522	222225525522222	252552525255252	255225252522552	252255555552252

It is a ***palindromic bimagic square*** *of order 9 with* ***magic sums*** *given by*

$$S_{9\times 9} := 3000099999989997 \qquad \text{and} \qquad Sb_{9\times 9} := 1163883049409243186986049721639.$$

3.8 Magic Square of Order 10

3.8.1 Magic Square

Example 57. *The **magic square** of order 10 for the digits 2 and 5 is given by*

22222222	25525525	22255222	55552252	55525522	22552522	55222555	25225255	52255555	52552255
25225222	22252252	52555522	55522255	22555555	55555525	25522222	52252522	22222555	55225255
52552252	52255522	22552255	25525555	55555222	22225255	25225525	55522222	55222522	22252555
22255555	55525255	52252555	25222522	55222222	25522255	55555522	22222252	52555222	22555525
55552522	22552222	22225525	52255255	25522555	55222252	55525222	52555555	22252255	25225522
55522555	52552522	25222222	55225525	22255255	52255222	22225555	55552255	22555522	25522252
25525522	22222255	55225555	22555222	52252252	25222555	52555255	22255525	55552222	55522522
22555255	55552555	25522522	22252222	52555525	55525555	52252255	55225522	25222252	22225222
55222255	25225555	55555255	52552555	22222522	22255522	22552252	25525222	55525525	52252222
52255525	55225222	55522252	22225522	25222255	52552222	22252522	22552555	25525255	55555555

Let's consider $T := 1000 \times A + B$, *where* $A := \{2222, 2225, 2255, 2522, 2552, 5225, 5255, 5522, 5552, 5555\}$ *and* $B := \{2222, 2252, 2255, 2522, 2555, 5222, 5255, 5522, 5525, 5555\}$. *This gives total 100 entries for the magic square of order 10. Since sum of members of A and B are 38885, this gives **magic square** of order 10 with **magic sum*** $S_{10\times 10} := 388888885 = 10000 \times 38885 + 38885$. *Below are the values in all the three cases:*

Magic Square:	$A := \{2222, 2225, 2255, 2522, 2552, 5225, 5255, 5522, 5552, 5555\}$	*and*	$B := \{2222, 2252, 2255, 2522, 2555, 5222, 5255, 5522, 5525, 5555\}$
Upside-down:	$A := \{2222, 5222, 5522, 2252, 2552, 5225, 5525, 2255, 2555, 5555\}$	*and*	$B := \{2222, 2522, 5522, 2252, 5552, 2225, 5525, 2255, 5255, 5555\}$
Mirror Looking:	$A := \{5555, 2555, 2255, 5525, 5225, 2552, 2252, 5522, 5222, 2222\}$	*and*	$B := \{5555, 5255, 2255, 5525, 2225, 5552, 2252, 5522, 2522, 2222\}$

In all the three cases the total sums os 10 numbers appearing in A *and* B *are always same. i.e.,* 38885. *Thus, the **magic square** is **universal**, i.e., **upside-down** and **mirror looking** with **magic sum*** $S_{10\times 10} := 388888885$.

3.8.2 Palindromic Magic Square

Example 58. *The **palindromic magic square** of order 10 for the digits 2 and 5 is given by*

222222222222222	522525252525225	525525525525525	222522525225222	555552555255555	525222252222525	252525555525252	255255555552552	225252525252522	255552252255552
255222525222552	222522252225222	252552252255252	222252555252222	255525252525552	525522222225525	555552525255555	225225555522522	525255555552525	522525525525225
522555555555225	555525555525555	225222525222522	255522222225552	525552252255525	255252525252552	525225525522525	222525252525222	222222252222222	252552555255252
555525525525555	525552525255525	222255555552222	252525252525252	225222222222522	222552252255222	255222252222552	255552555255552	522525555525225	525222525222525
252552525255252	525222222222525	222525555525222	525555555555525	255225525522552	522552555255225	222225252522222	555552252255555	255522525225552	225222252222522
222552555255222	225255555552522	522522252225225	525252252252525	222252525252222	255525555525552	525522525225525	252525525525252	555525252525555	255222222222552
525225252522525	255252555252552	255552525255552	225225525522522	222555555555222	555522525225555	522552252255225	525522252225525	252522222225252	222225555522222
255522252225552	222252252252222	555522222225555	255225555522552	522522525225225	252555555555252	225252555252522	525252525252525	222525525525222	525525252525525
225252252252522	252522525225252	255225252522552	555522252225555	525225555522525	222225525522222	255555555555552	522522222225225	525552555255525	222552525255222
525525555525525	255525525525552	525252555252525	522552525255225	252522252225252	225225252522522	222522222225222	222222525222222	255252252252552	555555555555555

It is a ***palindromic pandiagonal magic square*** *of order 10 with* ***magic sum*** $S_{10\times 10} := 3562156221562152$.

3.9 Magic Square of Order 11

3.9.1 Magic Square

Example 59. *The* ***pandiagonal magic square*** *of order 11 for the digits 1 and 8 is given by*

55555555	55222252	52522522	52252552	52225225	25525522	25252225	25222255	22552525	22525222	22255252
22525522	22252225	55552255	55222525	52525222	52255252	52225555	25522252	25252522	25222552	22555225
25225252	22555555	22522252	22252522	55552552	55225225	52525522	52252225	52222255	25522525	25255222
25525225	25255522	25222225	22552255	22522525	22255222	55555252	55225555	52522252	52252522	52222552
52255222	52225252	25525555	25252252	25222522	22552552	22525225	22255522	55552225	55222255	52522525
55222552	52525225	52255522	52222225	25522255	25252525	25225222	22555252	22525555	22252252	55552522
22252525	55555222	55225252	52525555	52252252	52222522	25522552	25255225	25225522	22552225	22522255
22552522	22522552	22255225	55555522	55222225	52522255	52252525	52225222	25525252	25255555	25222252
25252255	25222525	22555222	22525252	22255555	55552252	55222522	52522552	52255225	52225522	25522225
52222252	25522522	25252552	25225225	22555522	22522225	22252255	55552525	55225222	52525252	52255555
52522225	52252255	52222525	25525222	25255252	25225555	22552252	22522522	22252552	55555225	55225522

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

It is based on the 2 by 2 combinations of eleven numbers

$$\{2225, 2252, 2255, 2522, 2525, 2552, 5222, 5225, 5252, 5522, 5555\}.$$

*These eleven numbers are **upside-down** and **mirror looking** combining in pairs as* $\{2225, 5222\}$, $\{2252, 2522\}$, $\{2522, 2252\}$, $\{2525, 5252\}$, $\{2255, 5522\}$ *and* 5555. *In the mirror 2 becomes 5 and 5 as 2. It is **universal pandiagonal magic square**, but with different magic sums. These magic sums are:*

$$S_{11\times11} := 411111107 = 10000 \times 41107 + 41107 \quad \textit{and} \quad S_{11\times11} := 444444440 = 10000 \times 44440 + 44440$$

where

$$41107 := 2225 + 2252 + 2255 + 2522 + 2525 + 2552 + 5222 + 5225 + 5252 + 5522 + 5555$$

and

$$44440 := 5552 + 5525 + 5522 + 5255 + 5252 + 5225 + 2555 + 2552 + 2525 + 2255 + 2222$$

*The **magic sum*** $S_{11\times11} := 444444440$ *refers to **mirror looking** magic square.*

3.9.2 Palindromic Magic Square

Example 60. *The **palindromic pandiagonal magic square** of order 11 for the digits 2 and 5 is given by*

222222222222222	222522252225222	252225222522252	252525252525252	255225525522552	255525555525552	522252222252225	522552252255225	552555252555255	555255525552555	555555555555555
555255252552555	555555525555555	222255555552222	222522222225222	252222252222252	252525222525252	255225252522552	255525525525552	522225555522225	522552222255225	552552252255255
522525555525225	552552222255255	555252252252555	555555252555555	222255525552222	222555555555222	252222222222252	252522252225252	255225222522552	255525252525552	522225525522225
255525222525552	522225252522225	522525525525225	552525555525255	555252222252555	555552252255555	222255252552222	222555525555222	252225555522252	252522222225252	255222252222552
252555555555252	255222222222552	255522252225552	522225222522225	522525252525225	552525525525255	555225555522555	555552222255555	222252252252222	222555252555222	252255525552252
222552252255222	252255252552252	252555525555252	255255555552552	255522222225552	522222252222225	522525222525225	552525252525255	555225525522555	555525555525555	222252222252222
555525525525555	222225555522222	222552222255222	252252252252252	252555252555252	255255525552552	255555555555552	522222222222225	522522252225225	552525222525255	555225252522555
552522252225255	555225222522555	555525252525555	222225525522222	222525555525222	252252222252252	252552252255252	255255252552552	255555525555552	522255555552225	522522222225225
522255525552225	522555555555225	552522222225255	555222252222555	555525222525555	222225252522222	222525525525222	252225555522252	252552222255252	255252252252552	255555252555552
255252222252552	255552252255552	522255252552225	522555525555225	552555555555255	555222222222555	555522252225555	222225222522222	222525252525222	252225525522252	252525555525252
252225252522252	252525525525252	255225555522552	255552222255552	522252252252225	522555252555225	552555525555255	555255555552555	555522222225555	222222252222222	222525222525222

*It is a **palindromic pandiagonal magic square** of order 11 with **magic sum*** $S_{11\times11} := 4168416838713867$.

3.10 Magic Square of Order 12

3.10.1 Blocks of Order 4

Example 61. *The **pandiagonal magic square** of order 12 for the digits 2 and 5 is given by*

25525225	52255555	22222222	55552552	25555225	52225555	22252222	55522552	25255225	52525555	22552222	55222552
22222552	55552222	25525555	52255225	22252552	55522222	25555555	52225225	22552552	55222222	25255555	52525225
55555555	22225225	52252552	25522222	55525555	22255225	52222552	25552222	55225555	22555225	52522552	25252222
52252222	25522552	55555225	22225555	52222222	25552552	55525225	22255555	52522222	25252552	55225225	22555555
25525222	52255552	22222225	55552555	25555222	52225552	22252225	55522555	25255222	52525552	22552225	55222555
22222555	55552225	25525552	52255222	22252555	55522225	25555552	52225222	22552555	55222225	25255552	52525222
55555552	22225222	52252555	25522225	55525552	22255222	52222555	25552225	55225552	22555222	52522555	25252225
52252225	25522555	55555222	22225552	52222225	25552555	55525222	22255552	52522225	25252555	55225222	22555552
25525252	52255522	22222255	55552525	25555252	52225522	22252255	55522525	25255252	52525522	22552255	55222525
22222525	55552255	25525522	52255252	22252525	55522255	25555522	52225252	22552525	55222255	25255522	52525252
55555522	22225252	52252525	25522255	55525522	22255252	52222525	25552255	55225522	22555252	52522525	25252255
52252255	25522525	55555252	22225522	52222255	25552525	55525252	22255522	52522255	25252525	55225252	22555522

Let's consider the set of 12 numbers:

$$A := \{2225, 2252, 2522, 2525, 2552, 2555, 5222, 5225, 5252, 5255, 5525, 5552, 5555\}.$$

*These 12 numbers are **upside-down** and **mirror looking**. See below:*

Magic Square: $A := \{2222, 2225, 2252, 2255, 2522, 2525, 2552, 5225, 5252, 5255, 5522, 5525, 5552, 5555\}$
Upside-down: $A := \{2222, 5222, 2522, 5522, 2252, 5252, 2552, 5225, 2525, 5525, 2255, 5255, 2555, 5555\}$
Mirror Looking: $A := \{5555, 2555, 5255, 2255, 5525, 2525, 5225, 2552, 5252, 2252, 5522, 2522, 5222, 2222\}$.

*The sum of 12 numbers in each case is always same, i.e, 46662. The magic square constructed above is **universal pandiagonal magic square**. Its **magic sum** is given by*

$$S_{12\times 12} := 466666662 = 10000 \times 46662 + 46662,$$

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

where

$$46662 := 2225 + 2252 + 2255 + 2522 + 2525 + 2555 + 5222 + 5252 + 5255 + 5522 + 5525 + 5552.$$

*Moreover, it is **block-wise pandigonal magic square** of order 12, where each block of order 4 is a **pandiagonal magic square** with equal **magic sums** given by*

$$S_{4\times 4} := \frac{466666662}{3} = 155555554.$$

3.10.2 Blocks of Order 6

Example 62. *The **magic square** of order 12 for the digits 2 and 5 is given by*

22252225	55525522	55525222	55522555	22252255	22255552	22252252	55525255	55525252	55522525	22252522	22255525
55225552	22552255	55225222	22552555	22555522	55222225	55225525	22552522	55225252	22552525	22555255	55222252
52225552	52225522	25552555	25555222	52222255	25552225	52225525	52225255	25552525	25555252	52222522	25552252
25555552	25552255	52222555	52225222	25555522	52222225	25555525	25552522	52222525	52225252	25555255	52222252
22552225	55222255	22555222	55222555	55225522	22555552	22552252	55222522	22555252	55222525	55225255	22555525
55522225	22255522	22252555	22255222	55522255	55525552	55522252	22255255	22252525	22255252	55522522	55525525
22522225	55255522	55255222	55252555	22522255	22525552	22522252	55255255	55255252	55252525	22522522	22525525
52555552	25222255	52555222	25222555	25225522	52552225	52555525	25222522	52555252	25222525	25225255	52552252
52525552	52525522	25252555	25255222	52522255	25252225	52525525	52525255	25252525	25255252	52522522	25252252
25255552	25252255	52522555	52525222	25255522	52522225	25255525	25252522	52522525	52525252	25255255	52522252
25222225	52552255	25225222	52552555	52555522	25225552	25222252	52552522	25225252	52552525	52555255	25225525
55252225	22525522	22522555	22525222	55252255	55255552	55252252	22525255	22522525	22525252	55252522	55255525

In this case the construction is based on the similar lines of Example 23. In is just a magic square with same magic sum: $S_{12\times 12} := 466666662$ *It is **block-wise magic square** of order 12, where each block of order 6 is a **magic square** with equal **magic sums** given by* $S_{6\times 6} := \dfrac{466666662}{2} = 233333331.$

3.10.3 Blocks of Order 3

Example 63. *The **magic square** of order 12 for the digits 2 and 5 is given by*

81881888	88111818	88818818	88188111	18188181	18881811	11811118	11881188	11181181	18118881	81818811	81118188
88811818	81888818	88111888	18888181	88181811	18188111	11181188	11811181	11881118	81118811	18118188	81818881
88118818	88811888	81881818	18181811	18888111	88188181	11881181	11181118	11811188	81818188	81118881	18118811
11818881	11888811	11188188	18111118	81811188	81111181	81888111	88118181	88811811	88181888	18181818	18888818
11188811	11818188	11888881	81111188	18111181	81811118	88818181	81881811	88118111	18881818	88188818	18181888
11888188	11188881	11818811	81811181	81111118	18111188	88111811	88818111	81888181	18188818	18881888	88181818
18118111	81818181	81111811	11811888	11881818	11188818	88188881	18188811	18888188	81881118	88111188	88811181
81118181	18111811	81818111	11181818	11818818	11881888	18888811	88188188	18188881	88811188	81881181	88111118
81811811	81118111	18118181	11888818	11181888	11811818	18188188	18888881	88188811	88111181	88811118	81881188
88181118	18181188	18881181	81888881	88118811	88818188	18111888	81811818	81118818	11818111	11888181	11181811
18881188	88181181	18181118	88818811	81888188	88118881	81111818	18118818	81811888	11188181	11811811	11888111
18181181	18881118	88181188	88118188	88818881	81888811	81818818	81111888	18111818	11881811	11188111	11818181

In this case the construction is based on the similar lines of Example 61. It is just a magic square with same magic sum: $S_{12\times 12} := 466666662$ *It is* ***block-wise magic square*** *of order 12, where each block of order 3 is a* ***semi-magic square*** *with different* ***semi-magic sums****.*

3.10.4 Palindromic Magic Square

Example 64. *The* ***palindromic pandiagonal magic square*** *of order 12 for the digits 2 and 5 is given by*

255252252252552	522555555555225	222222222222222	555525525525555	255552252255552	522255555552225	222522222225222	555225525522555	252552252255252	525255555552525	225522222225522	552225525522255
222225525522222	555522222225555	255255555552552	522552252255225	222525525525222	555222222222555	255555555555552	522252252252225	225525525525522	552222222222255	252555555555252	525252252252525
555555555555555	222252252252222	522525525525225	255222222222552	555255555552555	222552252255222	522225525522225	255522222225552	552255555552255	225552252255522	525225525522525	252522222225252
522522222225225	255225525522552	555552252255555	222255555552222	522222222222225	255525525525552	555252252252555	222555555555222	525222222222525	252525525525252	552252252252255	225555555555522
255252222252552	522555525555225	222222252222222	555525555525555	255552222255552	522255525552225	222522252225222	555225555522555	252552222255252	525255525552525	225522252225522	552225555522255
222225555522222	555522252225555	255255525552552	522552222255225	222525555525222	555222252222555	255555525555552	522252222252225	225525555525522	552222252222255	252555525555252	525252222252525
555555525555555	222252222252222	522525555525225	255222252222552	555255525552555	222552222255222	522225555522225	255522252225552	552255525552255	225552222255522	525225555522525	252522252225252
522522252225225	255225555522552	555552222255555	222255525552222	522222252222225	255525555525552	555252222252555	222555525555222	525222252222525	252525555525252	552252222252255	225555525555522
255252525252552	522555222555225	222222555222222	555525252525555	255552525255552	522255222552225	222522555225222	555225252522555	252552525255252	525255222552525	225522555225522	552225252522255
222225252522222	555522555225555	255255222552552	522552525255225	222525252525222	555222555222555	255555222555552	522252525252225	225525252525522	552222555222255	252555222555252	525252525252525
555555222555555	222252525252222	522525252525225	255222555222552	555255222552555	222552525255222	522225252522225	255522555225552	552255222552255	225552525255522	525225252522525	252522555225252
522522555225225	255225252522552	555552525255555	222255222552222	522222555222225	255525252525552	555252525252555	222555222555222	525222555222525	252525252525252	552252525252255	225555222555522

*It is a **palindromic pandiagonal magic square** of order 12 with **magic sum** $S_{13\times 13} := 4666666666666662$. It is **block-wise pandigonal magic square** of order 12, where each block of order 4 is a **palindromic pandiagonal magic square** with equal **magic sums** as* $S_{4\times 4} := \dfrac{4666666666666662}{3} = 1555555555555554$.

The above example of **palindromic magic square** is for the **magic square** given in Example 61. In the similar way we can write the **palindromic magic squares** for the Examples 62 and 63.

3.11 Magic Square of Order 13

3.11.1 Magic Square

Example 65. *The **pandiagonal magic square** of order 13 for the digits 2 and 5 is given by*

55555555	55522252	55252552	52555222	52525252	52252522	52225552	25552225	25522525	25252555	25225225	22525255	22255525
22525552	22252225	55552525	55522555	55255225	52555255	52525525	52255555	52222252	25552552	25525222	25255252	25222522
25255525	25225555	22522252	22252552	55555222	55525252	55252522	52555552	52522225	52252525	52222555	25555225	25525255
25552522	25525552	25252225	25222525	22522555	22255225	55555255	55525525	55255555	52552252	52522552	52255222	52225252
52255255	52225525	25555555	25522252	25252552	25225222	22525252	22252522	55555552	55522225	55252525	52552555	52525225
52555252	52522522	52255552	52222225	25552525	25522555	25255225	25225255	22525525	22255555	55552252	55522552	55255222
55525225	55255255	52555525	52525555	52252252	52222552	25555222	25525252	25252522	25225552	22522225	22252525	55552555
22255222	55555252	55522522	55255552	52552225	52522525	52252555	52225225	25555255	25525525	25255555	25222252	22522552
25222555	22525225	22255255	55555525	55525555	55252252	52555252	52525222	52255252	52222522	25555552	25522225	25252525
25522552	25255222	25225252	22522522	22255552	55552225	55522525	55252555	52555225	52525255	52255525	52225555	25552252
52222525	25552555	25525225	25255255	25225525	22525555	22252252	55552552	55525222	55255252	52552522	52525552	52252225
52522252	52252552	52225222	25555252	25522522	25255552	25222225	22522525	22252555	55555225	55525255	55255525	52555555
55252225	52552525	52522555	52255225	52225255	25555525	25525555	25252252	25222552	22525222	22255252	55552522	55525552

It is based on the 2 by 2 combinations of eleven numbers

$$\{2225, 2252, 2522, 2525, 2552, 2555, 5222, 5225, 5252, 5255, 5525, 5552, 5555\}.$$

*These eleven numbers are **upside-down** and **mirror looking** combining in pairs as* $\{2225, 5222\}$, $\{2252, 2522\}$, $\{2525, 5252\}$, $\{2555, 5552\}$, $\{5255, 5525\}$, $\{2552, 5225\}$ *and* 5555. *In the mirror 2 becomes 5 and 5 as 2. It is **universal pandiagonal magic square**, but with different magic sums. These magic sums are:*

$$S_{13\times 13} := 522222217 = 10000 \times 52217 + 52217 \quad \textit{and} \quad S_{13\times 13} := 488888884 = 10000 \times 48884 + 48884$$

where

$$52217 := 2225 + 2252 + 2522 + 2525 + 2552 + 2555 + 5222 + 5225 + 5252 + 5255 + 5525 + 5552 + 5555$$

and

$$48884 = 5552 + 5525 + 5255 + 5252 + 5225 + 5222 + 2555 + 2552 + 2525 + 2522 + 2252 + 2225 + 2222.$$

*The **magic sum** $S_{13\times 13} := 488888884$ refers to **mirror looking** magic square.*

3.11.2 Palindromic Magic Square

Example 66. *The **palindromic pandiagonal magic square** of order 13 for the digits 2 and 5 is given by*

225522555225522	555222525222555	552525525525255	525552222255525	525252525252525	522525222525225	522255525552225	255522252225552	255225252522552	252525555525252	252252252252252	225252555252522	222555252555222
225255525552522	222522252225222	225525252525522	555225555522555	552552252255255	525552555255525	525255252552525	522522555225225	522225252522225	255525525525552	255252222252552	252552525255252	252225222522252
252555252555252	252222555222252	225222525222522	222525525525222	225552222255522	555252525252555	552525222525255	525555525555525	525222252222525	522525252525225	522225555522225	255552252255552	255252555252552
255525222525552	255255525552552	252522252225252	252225252522252	225225555522522	222552252255222	225552555255522	555255252552555	552522555225255	525522525225525	525225525522525	522552222255225	522252525252225
522552555255225	522255252552225	255522555225552	255222525222552	252525525525252	252252222252252	225252525252522	222525222525222	225555525555522	555222525222555	552525252525255	525525555525525	525252252252525
525552525255525	525225222522525	522555525555225	522222252222225	255525252525552	255225555522552	252552252255252	252225555522252	225255525552522	222522555225222	225522525225522	555225525522555	552552222255255
555252252252555	552552555255255	525555252555525	525222555222525	522522525225225	522225525522225	255552222255552	255252525252552	252525222525252	252255525552252	225222252222522	222525252525222	225525555525522
222552222255222	225552525255522	555225222522555	552555525555255	525522252225525	525225252522525	522525555525225	522252252252225	255552555255552	255255252552552	252522555225252	252222525222252	225225525522522
252225555522252	225252252252522	222552555255222	225555252555522	555222555222555	552522525225255	525552555255525	525252222252525	522552525255225	522225222522225	255555525555552	255222525222552	252525252525252
255225525522552	252552222255252	252252525252252	225225222522522	222555525555222	225522252225522	555225252522555	552525555525255	525552252255525	525252555252525	522555252555225	522222555222225	255522525225552
522225252522225	255525555525552	255252252252552	252555255525252	252255252552252	225222555222522	222522525225222	225552555255522	555252222252555	552552525255255	525552252255525	525255525552525	522522252225225
525222525222525	522525525525225	522252222252225	255552525255552	255225222522552	252555525555252	252222252222252	225225252522522	222525555525222	225552252255522	555252555252555	552555252555255	525522555225525
552522252225255	525552525255525	525225555522525	522552252255225	522252555252225	255555252555552	255222555222552	252522525225252	252225525522252	225252222252522	222552525255222	225525222525522	555255525552555

*It is a **palindromic pandiagonal magic square** of order 13 with **magic sum** $S_{13\times 13} := 4892189221892184$.*

3.12 Magic Square of Order 14

3.12.1 Magic Square

Example 67. *The **magic square** of order 14 for the digits 2 and 5 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

22222222	25525255	55555225	52255555	52555522	25255222	25222225	22522522	22252255	55252525	55222552	55525252	52525552	22552555
55222555	22252225	52255522	55525222	55552552	52552222	25522255	52522525	22222522	22555255	25225555	25255552	22525252	55255225
52525555	55255222	22522255	52555255	55522522	25222555	25255252	22222552	22552525	22255522	25525552	52252222	55225225	55552225
55555222	52255552	55225252	22552522	55252222	52522255	22222525	25252555	22522552	55522225	22255225	25525522	52555555	25225255
55255252	25255555	22555552	22225522	52525225	25522225	22522222	52252255	52552555	25222552	55522525	22255255	55552522	55225222
52252552	22225252	25522525	22525225	22252555	55255522	55525255	55225555	55555552	25252522	22552225	52555222	25222255	52522222
22252525	22522555	52552552	25222222	22222255	55225552	55555555	55255255	55525522	25525222	52525252	22555225	25252225	52252522
22552255	25225225	25252222	25522552	22525222	55555255	55225522	55525552	55255555	52522555	52552522	22222225	52252525	22255252
22522225	22552222	25222522	25252255	25525252	55525555	55255552	55555522	55225255	52255225	22222555	52522552	22255222	52552525
25225522	55222522	55252225	22255552	25252525	22552552	52252555	25525225	52525222	52555252	55552222	22525555	22225255	55522255
25255255	55552525	55522555	52522225	25225552	22252522	52555225	22555252	25522222	22225555	52255222	55222255	55252552	22525522
52555552	55522552	22225222	55252555	55222225	52255252	52522522	22252222	25255225	55552255	22525255	25222525	22555522	25525555
55525225	52552255	52525255	55555252	22555555	22522525	22252552	25225222	52252225	55222222	25255522	55252522	25522555	22225552
25522522	52525522	22255555	55222525	52255255	22225225	22555222	52552225	25225252	22525552	55252255	55552555	55522222	25252552

Let's consider

$$T := 1000 \times A + B,$$

where

$$A := \{2222, 2225, 2252, 2255, 2522, 2525, 2552, 5225, 5252, 5255, 5522, 5525, 5552, 5555\}$$

and

$$B := \{2222, 2225, 2255, 2522, 2525, 2552, 2555, 5222, 5225, 5252, 5255, 5522, 5552, 5555\}.$$

This gives total 100 entries for the magic square of order 10. Since sum of members of A and B are 54439, this gives ***magic square*** *of order 10 with* ***magic sum*** $S_{10\times 10} := 544444439 = 10000 \times 54439 + 54439$*. Below are the values in all the three cases:*

Magic Square: $A := \{2222, 2225, 2252, 2255, 2522, 2525, 2552, 5225, 5252, 5255, 5522, 5525, 5552, 5555\}$ *and* $B := \{2222, 2225, 2255, 2522, 2525, 2552, 2555, 5222, 5225, 5252, 5255, 5522, 5552, 5555\}$

Upside-down: $A := \{2222, 5222, 2522, 5522, 2252, 5252, 2552, 5225, 2525, 5525, 2255, 5255, 2555, 5555\}$ *and* $B := \{2222, 5222, 5522, 2252, 5252, 2552, 5552, 2225, 5225, 2525, 5525, 2255, 2555, 5555\}$

Mirror Looking: $A := \{5555, 2555, 5255, 2255, 5525, 2525, 5225, 2552, 5252, 2252, 5522, 2522, 5222, 2222\}$ *and* $B := \{5555, 2555, 2255, 5525, 2525, 5225, 2225, 5552, 2552, 5252, 2252, 5522, 5222, 2222\}$

In all the three cases the total sums os 14 numbers appearing in A *and* B *are always same. i.e.,* 54439*. Thus, the* ***magic square*** *is* ***universal****, i.e.,* ***upside-down*** *and* ***mirror looking*** *with* ***magic sum*** $S_{14\times 24} := 544444439$*.*

3.12.2 Palindromic Magic Square

Example 68. *The* ***palindromic magic square*** *of order 14 for the digits 2 and 5 is given by*

555255525552555	525522525225525	252225252522252	525222252222525	255525222525552	552552252255255	255252222252552	552225525522255	522225555522225	222555222555222	225552555255522	225255252552522	522522555225225	252552525255252
222552555255222	552555252555255	525222525222525	252255525552252	522522252225225	552522222225255	252525555525252	525525252525525	255525525525552	225552525255522	225255222552522	522222555222225	555252252252555	255225222522552
225552252255522	222552525255222	552255222552255	522522525225225	252255252552252	525525555525525	555225525522555	525255525552525	255225252522552	225252555252522	255522555225552	552552222255255	252525222525252	522222252222225
225252525252522	225552222255522	222552252255222	525552555255525	522222525222225	525225525522525	552525252525255	522555252555225	252555525555252	255225555522552	552225555522255	555225222522555	255522252225552	252255222552252
252522555225252	225252252252522	225525555525522	222552222255222	525252525252525	522525252525225	552255525552255	522255222552225	555255252552555	525525525525525	552525222525255	255222252222552	252252555252252	255522525225552
522225525522225	255525252525552	255255525552552	252555252555252	555255222552555	252225222522252	225222252222522	222522555225222	225522525225522	522525555525225	525252222252525	525552252255525	552252525252255	552552555255255
255255252552552	252555222555252	555252555252555	552552525255255	552252252252255	222522525225222	225522555225522	252222252222252	225225222522522	255555525555552	522225252522225	522525525525225	525225555522525	525552222255525
522555222555225	522252555252225	255552525255552	255252252252552	252552222255252	225522252225522	222525222525222	225222525222522	252222555222252	525255252552525	525555525555525	552225252522255	552525525525255	555225555522555
255552222255552	255225555522552	252525525525252	555225252522555	552555525555255	225222555222522	252222525222252	225525222525522	222522252225222	522252252252225	522552525255225	525252555252525	525555222555525	552255252552255
552222525222255	252225525522252	525522252225525	522225222522225	255252555252552	555252525252555	255552252255552	552525555525255	522552222255225	252525252525252	222555252555222	225555222555522	225255525552522	525222555222525
252225555522252	552222252222255	522525222525225	255555222555552	525522555225525	252552555255252	522252525252225	555252222252555	525252252252525	552522525225255	255225525522552	222555525555222	225555252555522	225225252522522
552522252225255	525225222522525	522255252552225	552222555222255	225225525522522	255255222552552	522552555255225	252552252255252	525552525255525	252252222252252	555222525222555	255525555525552	222525252525222	225555525555522
525525222525525	522555525555225	552522555225255	225225555522522	225525252525522	255555252555552	525255222552525	255252525252552	552252555252255	555222525222555	252252252252252	252522525225252	522252222252225	222525555525222
525225252522525	555225525522555	225252222252522	225525525525522	222525555525222	522255525552225	525555252555525	255552555255552	552555222555255	552225222522255	252522252225252	252252525252252	255222525222552	522552252255225

It is a ***palindromic magic square*** *of order 14 with* ***magic sum*** $S_{14\times 14} := 5444444444444439$*. The middle block of order 4 is a magic square with* ***magic sum*** $S_{4\times 4} := 925492555195518$*.*

3.13 Magic Square of Order 15

3.13.1 Magic Squares

Example 69. *The* ***magic square*** *of order 15 for the digits 2 and 5 is given by*

55525552	52522255	22255225	52252252	55222522	25525525	25552222	25252552	22525255	52555222	52225522	22222525	22552225	55252555	25225252
22555252	55255525	52252522	22525222	52222255	52552525	55525522	25525552	52522222	22225255	22252552	25222252	55222555	25255225	25552225
52252225	25225225	55225522	52222525	22552222	22222522	52522552	55525525	22255252	22525552	25252255	52552555	25525222	25552252	55255255
22525225	52222252	25255222	52555255	22222552	25222225	22252525	55255522	22555525	25522522	52522555	55522222	25552255	55225252	52255552
25225522	22555222	22222255	25522222	52525252	22255552	25252252	55225255	55522525	52252555	55252225	25552522	52555225	52225525	22522552
55252552	25255255	25222222	22252522	55522225	52255225	22525525	52555252	52222555	55222252	25552525	52525222	22225522	22555552	25522255
22222555	55225552	25525252	25252225	22522525	55252252	52225222	52525225	52552255	25552552	52252222	22255255	25225525	55522522	22555522
25522525	55522552	55255552	55225525	52555522	52525255	52255252	25552555	25252522	25222255	22552252	22522225	22252222	22225222	52225225
52552252	22225252	52522225	55252255	25255552	25555222	22555255	25222525	25522552	22255522	55225225	52255525	52222522	22522222	55522555
52225255	52552222	55522252	25222552	25555225	22525522	25522555	22552522	55222225	25252525	22225525	55255252	52525552	52252255	22255222
55225222	25522225	22552525	25555252	22255525	25252555	22225225	22522255	52255522	55252222	25222522	52225552	55525255	52552552	52522252
25252222	22522522	25555255	22225552	25222555	52225252	52552225	22252252	55255225	52525525	55525222	22552255	52252552	25525522	55222525
22252255	25555522	52222552	22552555	52255255	55222222	55252522	22222225	25225222	55525252	52555552	25525225	22522252	52522525	25255525
25555525	52252525	22522555	52525522	55255222	55522255	25225552	52222222	22222252	22555225	25525255	55222552	25255252	22252225	52552522
52522522	22252555	52555525	55525225	25522252	22552552	55222255	52255222	25555552	52222225	22525252	25255522	55252525	25225255	22222222

It is based on the 2 by 2 combinations of 15 numbers

$$\{2222, 2225, 2252, 2255, 2522, 2525, 2552, 2555, 5222, 5225, 5252, 5255, 5522, 5525, 5552\}.$$

These fifteen numbers are ***upside-down*** *and* ***mirror looking*** *combining in pairs as* $\{2225, 5222\}$, $\{2252, 2522\}$, $\{2255, 5522\}$, $\{2522, 2252\}$, $\{2525, 5252\}$, $\{2552, 5225\}$, $\{2555, 5552\}$ *and 2222. In the mirror 2 becomes 5 and 5 as 2. It is* ***universal pandiagonal square****. These magic sums are:*

$$S_{15\times 15} := 566666661 = 10000 \times 56661 + 56661 \quad \textit{and} \quad S_{15\times 15} := 599999994 = 10000 \times 59994 + 59994$$

where

$$56661 := 2222 + 2225 + 2252 + 2255 + 2522 + 2525 + 2552 + 2555 + 5222 + 5225 + 5252 + 5255 + 5522 + 5525 + 5552$$

and

$$59994 = 5555 + 5552 + 5525 + 5522 + 5255 + 5252 + 5225 + 5222 + 2555 + 2552 + 2525 + 2522 + 2255 + 2252 + 2225.$$

The ***magic sum*** $S_{15\times 15} := 599999994$ *refers to* ***mirror looking*** *magic square.*

3.13.2 Semi-Magic Squares

Below is an another example of **semi-magic square** of order 15, where each block of order 5 is a **pandiagonal magic squares** with different magic sums.

Example 70. *The **semi-magic square** of order 15 for the digits 2 and 5 is given by*

22222225	25255252	25522522	52555525	55222555	22522252	22555225	52222255	52255552	55525222	22252222	25225255	25552525	52525522	55252552
52552522	55225525	22222555	25252225	25525252	52252255	55525552	22525222	22552252	52225225	52522525	55255522	22252552	25222222	25555255
25252555	25522225	52555252	55222522	22225525	22555222	52222252	52255225	55522255	22525552	25222552	25552222	52525255	55252525	22255522
55225252	22222522	25255525	25522555	52552225	55525225	22522255	22555552	52225222	52252252	55255255	22252525	25225522	25552552	52522222
25525525	52552555	55222225	22225252	25252522	52225552	52255222	55522252	22525225	22552255	25555522	52522552	55252222	22255255	25222525
22522222	22555255	52222525	52255522	55522552	22252225	25225252	25552522	52525525	55252555	22222252	25255225	25522255	52555552	55225222
52252525	55525522	22522552	22552222	52225255	52522522	55255525	22252555	25222225	25555252	52552255	55225552	22225222	25252252	25525225
22552552	52222222	52255255	55522525	22525522	25222555	25552225	52525252	55252522	22255525	25255222	25522252	52555225	55222255	22225552
55525255	22522525	22555522	52222552	52252222	55255252	22252522	25225525	25552555	52522225	55225225	22222255	25255552	25525222	52552252
52225522	52252552	55522222	22525255	22552525	25555525	52522555	55252225	22255252	25222522	25525552	52555222	55222252	22225225	25252255
22252252	25225225	25552255	52525552	55255222	22222222	25255255	25522525	52555522	55222552	22522225	22555252	52222522	52255525	55522555
52522255	55255552	22255222	25222252	25555225	52552525	55225522	22222552	25252222	25525255	52252522	55525525	22522555	22552225	52225252
25225222	25552252	52525225	55252255	22255552	25252552	25522222	52555255	55222525	22225522	22552555	52222225	52255252	55522522	22525525
55255225	22252255	25225552	25555222	52522252	55225255	22222525	25255522	25522552	52552222	55525252	22522522	22555525	52222555	52252225
25555552	52525222	55252252	22255225	25222255	25525522	52552552	55222222	22225255	25252525	52225525	52252555	55522225	22525252	22552522

*It is **semi-magic square** as sum of rows and columns are the same as of above magic square, but the principal diagonals sum is different.*

The above magic square is of type 5×3. Below is another example of **semi-magic square** of order 15, where each block of order 3 is also a **semi-magic square** with different **semi-magic sums**

Example 71. *The **semi-magic square** of order 15 for the digits 2 and 5 is given by*

55222525	25222522	25255522	22522552	55255525	25522252	22222555	55525552	25552222	52525222	22552255	52225252	52555225	22252225	52255255
25252522	55225522	25222525	25525525	22522252	55252552	25555552	22222222	55522555	52222255	52525252	22555222	52252225	52555255	22255225
25225522	25252525	55222522	55252252	25522552	22525525	55522222	25552555	22225552	22555252	52225222	52522255	22255255	52255225	52552225
52522555	22555552	52222222	52555222	22252255	52255252	55225225	25222225	25255255	22522525	55252522	25525522	22222552	55525525	25552252
52225552	52522222	22552555	52252255	52555252	22255222	25252225	55225255	25225225	25522522	22525522	55252525	25555525	22222252	55522552
22552222	52222555	52525552	22255252	52255222	52552255	25225255	25255225	55222225	55255522	25522525	22522522	55522252	25552552	22225525
22525225	55252225	25525255	22222525	55522522	25555522	52522552	22555525	52222252	52552555	22255552	52252222	55225222	25222255	25255252
25522225	22525255	55255225	25552522	22225522	55522525	52225525	52522252	22552552	52255552	52552222	22252555	25252255	55225252	25225222
55255255	25525225	22522225	55525522	25552525	22222522	22552252	52222552	52525525	22252222	52252555	52555552	25225252	25255222	55222255
52552552	22255525	52252252	55222555	25225552	25252222	22525222	55252255	25525252	22225225	55522225	25555255	52522525	22552522	52225522
52255525	52552252	22252552	25255552	55222222	25222555	25522255	22525252	55255222	25552225	22225255	55525225	52222522	52525522	22552525
22252252	52252552	52555525	25222222	25252555	55225552	55255252	25525222	22522255	55525255	25555225	22222225	22555522	52222525	52522522
22225222	55522255	25555252	52525225	22552225	52225255	52552525	22252522	52255522	55222552	25225525	25252252	22522555	55255552	25522222
25552255	22225252	55525222	52222225	52525255	22555225	52252522	52555522	22252525	25255525	55222252	25222552	25525552	22522222	55252555
55525252	25555222	22222255	22555255	52225225	52522225	22255522	52252525	52552522	25222252	25252552	55225525	55252222	25522555	22525552

*It is **semi-magic square** as sum of the rows and columns are the same as of above magic square, but the principal diagonals sum is different.*

3.13.3 Palindromic Magic Square

Example 72. *The **palindromic magic square** of order 15 for the digits 2 and 5 is given by*

555255525552555	555522555225555	222552252255222	522522525225225	552225222522255	255255252552552	255555555555552	252525525525252	225252555252522	525552222255525	522255222552225	222225252522222	225522252225522	552525555525255	252222222222252
225522222225522	552555252555255	522525222525225	225252222252522	522222555222225	525525252525525	555255222552555	255255525552552	555555555555555	222252555252222	222525552525222	252222525222252	552255555552255	252552252255252	255522252225552
522522252225225	252252252252252	552255222552255	522225252522225	225555555555522	222225222522222	555525252525555	555255252552555	222522222225222	225255525552522	252522555225252	525525555525525	255252222252552	255522525225552	552552555255255
225252252252522	522225252522225	252552222255252	525552555255525	222225525522222	252222252222252	222525252525222	552555222555255	225555252555522	255225222522552	555525555525555	555255555552555	255522555225552	552222222222255	522555525555225
252255222552252	225552222255522	222222555222222	255255555552552	555522222225555	222555525555222	252522525225252	552252555252255	555225252522555	522525555525225	552522252225255	255525222525552	525552252255525	522255252552225	225225525522522
552525525525255	252552555255252	252255555552252	222522252225222	555222252222555	522552252255225	225255252552522	525522222225525	522225555522225	552222525222255	255525252525552	555552222255555	222255222552222	225555525555522	255222555222552
222225555522222	552255525552255	255222222222552	252522252225252	225225252522522	552522525225255	522252222252225	555552252255555	525522555225525	255525525525552	522555555555225	222552555255222	252255252552252	555225222522555	225555222555522
255225252522552	555225525522555	552555525555255	552255252552255	525555222555525	555552555255555	522522222225225	255525555525552	252525222525252	252222555222252	225522525225522	225222525222522	222555555555222	222252222252222	522252252252225
525522525225525	222222222222222	555522252225555	552522555225255	252555525555252	255552222255552	225552555255522	252225252522252	255225525522552	222555222555222	552252525252255	522555252555225	522225222522225	225255555552522	555225555522555
522252555252225	525555555555525	555222525222555	252225525522252	255552252255552	225255222552522	255225555522552	225525222525522	552222252222255	252525252525252	222255252552222	552522222225255	555555525555555	522522555225225	222552222255222
552252222252255	255222252222552	225525252525522	255522222225552	222555252555222	252525555525252	222252252252222	225222555222522	522555222555225	552555555555255	252225222522252	522255525552225	555252555252555	525525252525525	555225252522555
252555555555252	225225222522522	255552555255552	222255525552222	252225555522252	522222222222225	525522252225525	222525252525222	552552252255255	555555252555555	555252222252555	225522555225522	522525525525225	255255222552552	552225252522255
222522555225222	255552222255552	522225525522225	225525555525522	522552555255225	552255555552255	552522252225255	222222252222222	252252222252252	555222222222555	525555525555525	255252252252552	225225252522522	555525252525555	252555252555252
255555252555552	522525252525225	225225555522522	555555222555555	552552222255255	555225555522555	252255525552252	522255555552225	222222525222222	225552252255522	255252555252552	552255252552255	252522222225252	222522252225222	525552222255525
555525222525555	222525555525222	525555252555525	555252252252555	255222525222552	225525525525522	552225555522255	522552222255225	255555525555552	522222252222225	225222222222522	252555222555252	552525252525255	252225525522252	222255555552222

*It is a **palindromic magic square** of order 15 with **magic sum** $S_{15\times 15} := 5696969696969691$.*

3.14 Magic Square of Order 16

We observe that the 4-digits combinations considering 2 by 2 maximum goes up to 256 possibilities resulting in magic squares of orders 9 to 16. The following magic squares of order 16 are with all possible 256 combinations.

3.14.1 Magic Square

Example 73. *The **pandiagonal magic square** of order 16 for the digits 2 and 5 is given by*

52252525	25522252	55525525	22255252	52522522	25252255	55255522	22525255	52222555	25552222	55555555	22225222	52552552	25222225	55225552	22555225
55525252	22255525	52252252	25522525	55255255	22525522	52522255	25252522	55555222	22225555	52222222	25552555	55225225	22555552	52552225	25222552
22252252	55522525	25525252	52255525	22522255	55252522	25255255	52525522	22222222	55552555	25555222	52225555	22552225	55222552	25225225	52555552
25525525	52255252	22252525	55522252	25255522	52525255	22522522	55252255	25555555	52225222	22222555	55552222	25225552	52555225	22552552	55222225
52222552	25552225	55555552	22225225	52552555	25222222	55225555	22555222	52252522	25522255	55525522	22255255	52522525	25252252	55255525	22525252
55555225	22225552	52222225	25552552	55225222	22555555	52552222	25222555	55525255	22255522	52252255	25522522	55255252	22525525	52522252	25252525
22222225	55552552	25555225	52225552	22552222	55222555	25225222	52555555	22252255	55522522	25525255	52255522	22522252	55252525	25255252	52525525
25555552	52225225	22222552	55552225	25225555	52555222	22552555	55222222	25525522	52255255	22252522	55522255	25255525	52525252	22522525	55252252
52552522	25222255	55225522	22555255	52222525	25552252	55555525	22225252	52522552	25252225	55255552	22525225	52252555	25522222	55525555	22255222
55225255	22555522	52552255	25222522	55555252	22225525	52222252	25552525	55255225	22525552	52522225	25252552	55525222	22255555	52252222	25522555
22552255	55222522	25225255	52555522	22222252	55552525	25555252	52225525	22522225	55252552	25255225	52525552	22252222	55522555	25525222	52255555
25225522	52555255	22552522	55222255	25555525	52225252	22222525	55552252	25255552	52525225	22522552	55252225	25525555	52255222	22252555	55522222
52522555	25252222	55255555	22525222	52252552	25522225	55525552	22255225	52552525	25222252	55225525	22555252	52222522	25552255	55555522	22225255
55255222	22525555	52522222	25252555	55525225	22255552	52252225	25522552	55225252	22555525	52552252	25222525	55555255	22225522	52222255	25552522
22522222	55252555	25255222	52525555	22252225	55522552	25525225	52255552	22552252	55222525	25225252	52555525	22222255	55552522	25555255	52225522
25255555	52525222	22522555	55252222	25525552	52255225	22252552	55522225	25225525	52555252	22552525	55222252	25555522	52225255	22222522	55552255

It is based on the 2 by 2 combinations of 16 numbers

$$\{2222, 2225, 2252, 2255, 2522, 2525, 2552, 2555, 5222, 5225, 5252, 5255, 5522, 5525, 5552, 5555\}.$$

*These 16 numbers are **upside-down** and **mirror looking** combining in pairs as* $\{2225, 5222\}$, $\{2252, 2522\}$, $\{2255, 5522\}$, $\{2522, 2252\}$, $\{2525, 5252\}$, $\{2552, 5225\}$, $\{2555, 5552\}$, 2222 *and* 5555. *It is **pandiagonal universal magic square**. Its **magic sum** is*

$$S_{16\times 16} := 622222216 = 10000 \times 62216 + 62216,$$

where

$$62216 = 2222 + 2225 + 2252 + 2255 + 2522 + 2525 + 2552 + 2555 + 5222 + 5225 + 5252 + 5255 + 5522 + 5525 + 5552 + 5555.$$

*It is **block-wise pandigonal magic square** of order 16, where each block of order 4 is a **pandiagonal magic square** with equal **magic sums** given by*

$$S_{4\times 4} := \frac{622222216}{4} = 155555554.$$

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

3.14.2 Palindromic Magic Square

Example 74. *The* ***palindromic pandiagonal magic square*** *of order 16 for the digits 2 and 5 is given by*

255252525252552	522555252555225	222522525225222	555225252522555	252552555255252	525255222552525	225222555222522	552525222525255	255552222255552	522255555552225	222222222222222	555525555525555	252252252252252	525555525555525	225522252225522	552225525522255
222525252525222	555222525222555	255255252552552	522552525255225	225225222522522	552522555225255	252555222555252	525252555252525	222225555522222	555522222225555	255555555555552	522252222252225	225525525525522	552222252222255	252255525552252	525552252255525
555255252552555	222552525255222	522525252525225	255222525222552	552555222555255	225252555252522	525225222522525	252522555225252	555555555555555	222252222252222	522225555522225	255522222225552	552255525552255	225552252255522	525525525525525	252222252222252
522522525225225	255225252522552	555252525252555	222555252555222	525222555222525	252525222525252	552552555255255	225255222552522	522222222222225	255525555525552	555552222255555	222255555552222	525522252225525	252225525522252	552252252252255	225555525555522
255552252255552	522255525552225	222222252222222	555525525525555	252252222252252	525555555555525	225522222225522	552225555522255	255252555252552	522555222555225	222522555225222	555225222522555	252552525255252	525255252552525	225222525222522	552525252525255
222225525522222	555522252225555	255555525555552	522252252252225	225525555525522	552222222222255	252255555552252	525552222255525	222525222525222	555222555222555	255255222552552	522552555255225	225225252522522	552522525225255	252555252555252	525252525252525
555555525555555	222252252252222	522225525522225	255522252225552	552255555552255	225552222255522	525525555525525	252222222222252	555255222552555	222552555255222	522525222525225	255222555222552	552555252555255	225252525252522	525225252522525	252522525225252
522222252222225	255525525525552	555552252255555	222255525552222	525522222225525	252225555522252	552252222252255	225555555555522	522522555225225	255225222522552	555252555252555	222555222555222	525222525222525	252525252525252	552552525255255	225255252552522
252252555252252	525555222555525	225522555225522	552225222522255	255552525255552	522255252552225	222222525222222	555525252525555	252552252255252	525255525552525	225222252222522	552525525525255	255252222252552	522555555555225	222522222225222	555225555522555
225525222525522	552222555222255	252255222552252	525552555255525	222225252522222	555522525225555	255555252555552	522252525252225	225225525522522	552522252225255	252555525555252	525252252252525	222525555525222	555222222222555	255255555552552	522552222255225
552255222552255	225552555255522	525525222525525	252222555222252	555555252555555	222252525252222	522225252522225	255522525225552	552555525555255	225252252252522	525225525522525	252522252225252	555255555552555	222552222255222	522525555525225	255222222222552
525522555225525	252225222522252	552252555252255	225555222555522	522222525222225	255525252525552	555552525255555	222255252552222	525222252222525	252525525525252	552552252255255	225255525552522	522522222225225	255225555522552	555252222252555	222555555555222
252552222255252	525255555552525	225222222222522	552525555525255	255252252252552	522555525555225	222522252225222	555225525522555	252252525252252	525555252555525	225522525225522	552225252522255	255552555255552	522255222552225	222222555222222	555525222525555
225225555522522	552522222225255	252555555555252	525252222252525	222525525525222	555222252222555	255255525552552	522552252255225	225525252525522	552222525222255	252255252552252	525552525255525	222225222522222	555522555225555	255555222555552	522252555252225
552555555555255	225252222252522	525225555522525	252522222225252	555255525552555	222552252255222	522525525525225	255222252222552	552255252552255	225552525255522	525525252525525	252222525222252	555555222555555	222252555252222	522225222522225	255522555225552
525222222222525	252525555525252	552552222255255	225255555552522	522522252225225	255225525522552	555252252252555	222555525555222	525522525225525	252225252522252	552252525252255	225555252555522	522222555222225	255525222525552	555552555255555	222255222552222

It is a ***block-wise palindromic pandigonal magic square*** *of order 16, where each block of order 4 is a* ***palindromic pandiagonal magic square*** *with equal* ***magic sums****. The magic sums are given by*

$$S_{16\times16} := 6222222222222216 \qquad \textit{and} \qquad S_{4\times4} := \frac{6222222222222216}{4} = 1555555555555554.$$

3.14.3 Bimagic Square

Example 75. *The* ***bimagic square*** *of order 16 for the digits 2 and 5 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

55555555	25522552	22252225	52225222	55525225	25552222	22222555	52255552	55252522	25225525	22555252	52522255	55222252	25255255	22525522	52552525
22255222	52222225	55552552	25525555	22225552	52252555	55522222	25555225	22552255	52525252	55255525	25222522	22522525	52555522	55225255	25252252
52222552	22255555	25525222	55552225	52252222	22225225	25555552	55522555	52525525	22552522	25222255	55255252	52555255	22522252	25252525	55225522
25522225	55555222	52225555	22252552	25552555	55525552	52255225	22222222	25225252	55252255	52522522	22555525	25255522	55222525	52552252	22525255
55252252	25225255	22555522	52522525	55222522	25255525	22525252	52552255	55555225	25522222	22252555	52225552	55525555	25552552	22222225	52255222
22552525	52525522	55255255	25222252	22522255	52555252	55225525	25252522	22255552	52222555	55552222	25525225	22225222	52252225	55522552	25555555
52525255	22552252	25222525	55255522	52555525	22522522	25252255	55225252	52222222	22255225	25525552	55552555	52252552	22225555	25555222	55522225
25225522	55252525	52522252	22555255	25255252	55222255	52552522	22525525	25522555	55555552	52225225	22252222	25552225	55525222	52255555	22222552
55225225	25252222	22522555	52555552	55255555	25222552	22552225	52525222	55522252	25555255	22225522	52252525	55552522	25525525	22255252	52222255
22525552	52552555	55222222	25255225	22555222	52522225	55252552	25225555	22222525	52255522	55525255	25552252	22252255	52225252	55555525	25522522
52552222	22525225	25255552	55222555	52522552	22555555	25225222	55252225	52255255	22222252	25552525	55525522	52225525	22252522	25522255	55555252
25252555	55225552	52555225	22522222	25222225	55255222	52525555	22552552	25555522	55522525	52252252	22225255	25525252	55552255	52222522	22255525
55522522	25555525	22225252	52252255	55552252	25525255	22255522	52222525	55225555	25252552	22522225	52555222	55255225	25222222	22552555	52525552
22222255	52255252	55525525	25552522	22252525	52225522	55555255	25522252	22525222	52552225	55222552	25255555	22555552	52522555	55252222	25225225
52255525	22222522	25552255	55525252	52225255	22252252	25522525	55555522	52552552	22525555	25255222	55222225	52522222	22555225	25225552	55252555
25555252	55522255	52252522	22225525	25525522	55552525	52222252	22255255	25252225	55225222	52555555	22522552	25222555	55255552	52525225	22552222

*It is constructed with same 16 numbers used in previous example. According to previous example, it is **universal bimagic square** with magic and bimagic sums given by*

$$S_{16\times 16} := 622222216 \quad and \quad Sb_{16\times 16} := 27833894016610552.$$

3.14.4 Palindromic Bimagic Square

Example 76. *The **palindromic bimagic square** of order 16 for the digits 2 and 5 is given by*

222222222222222	522552252255225	555255525552555	255525555525552	222525525525222	522255555552225	555552222255555	255222525222552	225252555252522	525522525225525	552225252522255	252555222555252	225555252555522	525225222522525	552522555225255	252252525252252
555225555522555	255555525555552	222252252252222	522522222225225	555222525222555	255252222252552	222555555555222	522225525522225	552255222552255	252525252525252	225222525222522	525552555255525	552552525255255	252222555222252	225525222525522	525255252552525
255552252255552	555222222222555	522525555525225	222255252552222	255255555552552	555525525525555	522222252222225	222552222255222	252522525225252	552252555252255	525552252255525	225225252522522	252225222522252	552555252555255	525252525252525	225522555225522
522555525555225	222255555552222	255522222225552	555252252252555	522252222252225	222522252225222	255225525522552	555555555555555	525525252525525	225255222552522	252552555255252	552222525222255	525222555222525	225552525255522	252255252552252	552522252225255
225255252552522	525525222525525	552225525522255	252552525255252	225552555255522	525222525222525	552525252525255	252255222552252	222225525522222	522555555555225	555252222252555	255522252225552	222522222225222	522252252252225	555555525555555	255225555522552
552252525252255	252522555225252	225225222522522	525555252555525	552555222555255	252225252522252	225522525225522	525252555252525	555222252222555	255552222255552	222255555552222	522525525525225	555255555552555	255255525552552	222552252255222	522222222222225
252522252225252	552255252552255	525552525255525	225222555222522	252222525222252	552552555255255	525255222552525	225525252525522	255555555555552	555225525522555	522522252225225	222252222252222	255252252252552	555522222225555	522225555522225	222555525555222
525522555225525	225252525252522	252555252555252	552225222522255	525225252522525	225555222555522	252252555252252	552522525225255	522552222255225	222222252222222	255525525525552	555255555552555	522255525552225	222525555525222	255222222222552	555552252255555
225525525525522	525255555552525	552552222255255	252222252222252	225222222222522	525552252255525	552255525552255	252525555525252	222555252555222	522225222522225	555525525525555	255252525252552	222252555252222	522525252525225	555225252522555	255555222555552
552522252225255	252252222252252	225555555555522	525225525522525	552225555522255	252555525555252	225252252252522	525522222225525	555552525255555	255225555522552	222525222525222	522255252552225	555255222552555	255525252525552	222222525222222	522552555255225
252255555552252	552525525525255	525222525222525	225552222255522	525555525555525	225225555522522	252522222225252	552252252252255	255225222522552	555552525255555	522252525252225	222522555225222	255522525225552	555252555252555	522555222555225	222225252522222
525252222252525	225522252225522	252225525522252	552555555555255	222552525255222	522525222525225	555222555222555	255552525255552	555255252552555	522225525522225	222552525255222	255255252552552	522525252525225	222255222552222	255552555255552	555222525222555
222552555255222	522225252522225	555525252525555	255255222552552	555252525252555	255522555225552	222225222522222	522555252555225	225522222225522	525252252252525	552555525555255	252255555552252	225225525522522	525555555555525	552252222252255	252522252225252
555552252255555	255225252522552	222522525225222	522252555252225	255525222525552	555255252552555	522552525255225	222222555222222	552525555525255	252255525552252	225552252255522	525222222222525	552222252222255	252552222255252	225255555552522	525525525525525
255222525222552	555552555255555	522255222552225	222525252525222	522522555225225	222252525252222	255555252555552	555225222522555	252252252252252	552522222225255	525225555522525	225555525555522	252555555555252	552225525522255	525522252225525	225252222252522
522225252522225	222552222255222	255252555252552	555225252522555	222225222522222	522552525255225	555222525222555	255222222222552	525255525552525	225525555525522	252222222222252	552552252255255	525552222255525	225222252222522	252525525525252	552255555552255

It is a ***block-wise palindromic bi magic square*** *of order 16, where each block of order 4 is a* ***palindromic magic square*** *with equal* ***magic sums****. The magic sums are given by*

$$S_{16\times 16} := 6222222222222216 \qquad \textit{and} \qquad S_{4\times 4} := \frac{6222222222222216}{4} = 1555555555555554.$$

The bimagic sum is given by

$$S_{16\times 16} := 27833894500561622832772166105552.$$

4 Upside-Down Magic Squares With Digits 6 and 9

4.1 Semi-Magic Square of Order 3

4.1.1 Semi-Magic Square

Example 77. *The* ***semi-magic square*** *of order 3 for the digits 6 and 9 is given by*

6966	9999	6669
6699	6969	9966
9969	6666	6999

*In this case it is **semi-magic square**. It has same sum in lines and rows, while principal diagonals sums are different. It is based on the 2 by 2 combinations of three numbers $\{66, 69, 99\}$. These three numbers are **upside-down**in pair $\{66, 99\}$, while 69 is already upside down, as it remains the same. Thus, it is **upside-down semi-magic square**. In this case the **semi-magic sum** is given by*

$$S_{3\times 3} := 23634 = 100 \times 234 + 234,$$

where

$$234 = 66 + 69 + 99$$

4.1.2 Palindromic Semi-Magic Square

Example 78. *The **palindromic semi-magic square** of order 3 for the digits 6 and 9 is given by*

6966696	9999999	6669666
6699966	6969696	9966699
9969699	6666666	6999996

*It is a **palindromic semi-magic square** of order 3 with **semi-magic sum** $S_{3\times 3} := 23636361$.*

4.2 Magic Square of Order 4

We observe that the 2-digits combinations considering 2 by 2 maximum goes up to 16 possibilities resulting in magic squares of orders 3 and 4. The following magic square of order 4 is with all possible 16 combinations.

4.2.1 Magic Square

Example 79. *The **pandiagonal magic square** of order 4 for the digits 6 and 9 is given by*

6996	9699	6666	9969
6669	9966	6999	9696
9999	6696	9669	6966
9666	6969	9996	6699

It is based on the 2 by 2 combinations of four numbers $\{66, 69, 96, 99\}$. *These four numbers are **upside-down** in pairs* $\{66, 99\}$ *and* $\{69, 96\}$. *Actually in the last both the members are remains the same in the **upside-down** case. It is **pandiagonal upside-down magic square**. Its **magic sum** is*

$$S_{4\times4} := 33330 = 100 \times 330 + 330,$$

where

$$330 = 6 + 69 + 96 + 99.$$

4.2.2 Palindromic Magic Square

Example 80. *The **palindromic pandiagonal magic square** of order 4 for the digits 6 and 9 is given by*

6996996	9699969	6666666	9969699
6669666	9966699	6999996	9696969
9999999	6696966	9669669	6966696
9666669	6969696	9996999	6699966

*It is a **palindromic pandiagonal magic square** of order 4 with **magic sum*** $S_{4\times4} := 33333330$.

4.3 Magic Square of Order 5

We observe that the two digits combinations for two numbers maximum goes up to 16. This lead us to magic squares of orders 3 and 4. From now onwards, we shall work with three digits combinations for two numbers. It will go up to 8^{th} order magic squares.

4.3.1 Magic Square

Example 81. *The **pandiagonal magic square** of order 5 for the digits 6 and 9 is given by*

666999	669699	696669	699969	969696
699669	969969	666696	669999	696699
669696	696999	699699	969669	666969
969699	666669	669969	696696	699999
696969	699696	969999	666699	669669

In this case, the construction is different. The entries are considered as $T := 1000 \times A + B$*, where* $A := \{666, 669, 696, 699, 969\}$ *and* $B := \{999, 669, 696, 699, 969\}$*. Thus, there are total, 25 entries given in above magic square. Eventhough, the sum of members of* A *and* B *are different, still, we get a **pandiagonal magic square** of order 5 with **magic sum***

$$S_{5\times 5} := 3703032 = 1000 \times 3699 + 4032,$$

where

$$3699 := 666 + 669 + 696 + 699 + 969$$

and

$$4032 := 999 + 669 + 696 + 699 + 969.$$

*Thus, we have a **upside-down pandigonal magic square** of order 5 with magic sum* $S_{5\times 5} := 3703032$.

4.3.2 Palindromic Magic Square

Example 82. *The **palindromic pandiagonal magic square** of order 5 for the digits 6 and 9 is given by*

66699999666	66969996966	69666966696	69996969996	96969696969
69966966996	96996969969	66669696666	66999999966	69669996696
66969696966	69699999696	69969996996	96966966969	66696969666
96969996969	66666966666	66996969966	69669696696	69999999996
69696969696	69969696996	96999999969	66669996666	66966966966

*It is a **palindromic pandiagonal magic square** of order 5 with **magic sum*** $S_{5\times 5} := 370303630293$.

4.4 Magic Square of Order 6

4.4.1 Magic Square

Example 83. *The **magic square** of order 6 for the digits 6 and 9 is given by*

999999	666969	666996	666669	999696	999666
696666	969696	696996	969669	969969	696999
699666	699969	966669	966996	699696	966999
966666	966696	699669	699996	966969	699999
969999	696696	969996	696669	696969	969666
666999	999969	999669	999996	666696	666666

It is also constructed similar to above two examples. It is considered as $T := 1000 \times A + B$, *where* $A := \{666, 696, 699, 966, 969, 999\}$ *and* $B := \{666, 669, 696, 969, 996, 999\}$. *Thus, there are total, 36 entries given in above magic square of order 6. Since the sum of members of A and B are 4995, this gives **magic sum** of order 6 as* $S_{6\times 6} := 4999995 = 1000 \times 4995 + 4995$. *It is **upside-down** magic square. Thus, we have an **upside-down** of order 6 with **magic sum*** $S_{6\times 6} := 4999995$.

4.4.2 Palindromic Magic Square

Example 84. *The **palindromic magic square** of order 6 for the digits 6 and 9 is given by*

99999999999	66696969666	66699699666	66666966666	99969696999	99966666999
69666666696	96969696969	69699699696	96966966969	96996969969	69699999696
69966666996	69996969996	96666966669	96699699669	69969696996	96699999669
96666666669	96669696669	69966966996	69999699996	96696969669	69999999996
96999999969	69669696696	96999699969	69666966696	69696969696	96966666969
66699999666	99996969999	99966966999	99999699999	66669696666	66666666666

*It is a **palindromic magic square** of order 6 with **magic sum*** $S_{6\times 6} := 499999999995$.

4.5 Magic Square of Order 7

4.5.1 Magic Square

Example 85. *The **magic square** of order 7 for the digits 6 and 9 is given by*

666999	969696	699669	669699	996966	966996	696969
966966	696996	666969	969999	699696	669669	996699
669696	996669	966699	696966	666996	969969	699999
969996	699969	669999	996696	966669	696699	666966
696669	666699	969966	699996	669969	996999	966696
996969	966999	696696	666669	969699	699966	669996
699699	669966	996996	966969	696999	666696	969669

In this case, the construction is little different. It is considered as $T := 1000 \times A + B$, *where* $A := \{666, 969, 699, 669, 996, 966, 696\}$ *and* $B := \{999, 696, 669, 699, 966, 996, 969\}$. *Thus, there are total, 49 entries given in above magic square of order 7. Even though, the sum of members of* A *and* B *are different, still, we get a **pandiagonal magic square** of order 7 with **magic sum***

$$S_{7\times 7} := 5666994 = 1000 \times 5661 + 5994,$$

where

$$5661 := 666 + 969 + 699 + 669 + 996 + 966 + 696$$

and

$$5994 := 999 + 696 + 669 + 699 + 966 + 996 + 969.$$

*Thus, we have a **upside-down pandigonal magic square** of order 7 with **magic sum*** $S_{7\times 7} := 5666994$.

4.5.2 Palindromic Magic Square

Example 86. *The **palindromic pandiagonal magic square** of order 7 for the digits 6 and 9 is given by*

66699999666	96969696969	69966966996	66969996966	99696669699	96699699669	69696969696
96696669669	69699699696	66696969666	96999999969	69969696996	66966966966	99669996699
66969696966	99666966699	96669996669	69696669696	66699699666	96996969969	69999999996
96999699969	69996969996	66999999966	99669696699	96666966669	69669996696	66696669666
69666966696	66669996666	96996669969	69999699996	66996969966	99699999699	96669696669
99696969699	96699999669	69669696696	66666966666	96969996969	69996669996	66999699966
69969996996	66996669966	99699699699	96696969669	69699999696	66669696666	96966966969

*It is a **palindromic pandiagonal magic square** of order 7 with **magic sum** $S_{7\times7} := 566699999661$.*

4.6 Magic Square of Order 8

We observe that the 3-digits combinations considering 2 by 2 maximum goes up to 64 possibilities resulting in magic squares of orders 5 to 8. The following magic squares of order 8 are with all 64 possible combinations.

4.6.1 Magic Square

Example 87. *The **pandiagonal magic square** of order 8 for the digits 6 and 9 with equal sum magic squares of order 4 is given by*

666666	999966	699699	966999	669666	996966	696699	969999
699999	966699	666966	999666	696999	969699	669966	996666
966966	699666	999999	666699	969966	696666	996999	669699
999699	666999	966666	699966	996699	669999	969666	696966
666669	999969	699696	966996	669669	996969	696696	969996
699996	966696	666969	999669	696996	969696	669969	996669
966969	699669	999996	666696	969969	696669	996996	669696
999696	666996	966669	699969	996696	669996	969669	696969

*It is based on the 2 by 2 combinations of eight numbers $\{666, 669, 696, 699, 966, 969, 996, 999\}$. These eight numbers are **upside-down**. Its **magic sum** is*

$$S_{8\times8} := 6666660 = 1000 \times 6660 + 6660,$$

where

$$6660 = 666 + 669 + 696 + 699 + 966 + 969 + 996 + 999.$$

*It is **block-wise pandigonal magic square** of order 8, where each block of order 4 is a **pandiagonal magic square** with equal **magic sums** given by*

$$S_{4\times 4} := \frac{6666660}{2} = 3333330.$$

4.6.2 Palindromic Magic Square

Example 88. *The **palindromic pandiagonal magic square** of order 8 for the digits 6 and 9 is given by*

6666666666	9999666999	6996999696	9669999669	6696666966	9969666969	6966999696	9699999969
6999999996	9666999666	6669666966	9996666999	6969999696	9696999696	6699666966	9966666699
9669666966	6996666996	9999999999	6666999666	9699666969	6966666696	9969999699	6696999666
9996999699	6669999666	9666666669	6999666996	9966999699	6699999966	9696666969	6969666969
6666696666	9999696999	6996969696	9669969669	6696696966	9969696969	6966969696	9699969969
6999969996	9666969666	6669696966	9996696999	6969969696	9696969696	6699696966	9966696699
9669696966	6996696996	9999969999	6666969666	9699696969	6966696696	9969969699	6696969666
9996969699	6669969666	9666696669	6999696996	9966969699	6699969966	9696696969	6969696969

*It is a **block-wise palindromic pandigonal magic square** of order 8, where each block of order 4 is a **palindromic pandiagonal magic square** with equal **magic sums**. The magic sums are given by*

$$S_{8\times 8} := 666666666660 \qquad \textit{and} \qquad S_{4\times 4} := \frac{666666666660}{2} = 333333333330.$$

4.6.3 Bimagic Square

Example 89. *The **pandiagonal bimagic magic square** of order 8 for the digits 6 and 9 is given by*

669999	969666	966699	666966	699696	999969	996996	696669
699669	999996	996969	696696	669966	969699	966666	666999
666666	966999	969966	669699	696969	996696	999669	699996
696996	996669	999696	699969	666699	966966	969999	669666
966969	666696	669669	969996	996666	696999	699966	999699
996699	696966	699999	999666	966996	666669	669696	969969
969696	669969	666996	966669	999999	699666	696699	996966
999966	699699	696666	996999	969669	669996	666969	966696

It is also based on the 2 by 2 combinations of eight numbers $\{666, 669, 696, 699, 966, 969, 996, 999\}$. *These eight numbers are* ***upside-down****. It is* ***pandiagonal upside-down bimagic magic square****. Its* ***magic sum*** *is*

$$S_{8\times 8} := 6666660 = 1000 \times 6660 + 6660,$$

where

$$6660 = 666 + 669 + 696 + 699 + 966 + 969 + 996 + 999.$$

It is also ***bimagic square*** *of order 8 with* ***bimagic sum*** *given by*

$$Sb_{8\times 8} := 5737362626268.$$

Moreover, as specified in the figure, the sum of 2×4 *entries are of same sum as of magic square, i.e.,* 6666660.

4.6.4 Palindromic Bimagic Square

Example 90. *The* ***palindromic bimagic square*** *of order 8 for the digits 6 and 9 is given by*

66999999966	96966666969	96669996669	66696669666	69969696996	99996969999	99699699699	69666966696
69966966996	99999699999	99696969699	69669696696	66996669966	96969996969	96666666669	66699999666
66666666666	96699999669	96996669969	66969996966	69696969696	99669696699	99966966999	69999699996
69699699696	99666966699	99969696999	69996969996	66669996666	96696669669	96999999969	66966666966
96696969669	66669696666	66966966966	96999699969	99666666699	69699999696	69996669996	99969996999
99669996699	69696669696	69999999996	99966666999	96699699669	66666966666	66969696966	96996969969
96969696969	66996969966	66699699666	96666966669	99999999999	69966666996	69669996696	99696669699
99996669999	69969996996	69666666696	99699999699	96966966969	66999699966	66696969666	96669696669

*It is a **palindromic bimagic square** of order 8 with **magic sums:***

$$S_{8\times 8} := 666666666660 \qquad \textit{and} \qquad Sb_{8\times 8} := 57373737374426262626268.$$

Moreover, as specified in the figure, the sum of 2×4 *entries are of same sum as of magic square, i.e.,* 666666666660.

4.7 Magic Square of Order 9

We observe that the three digits combinations considering 2 by 2 maximum goes up to 64 possibilities resulting in magic squares of orders 8. From now onwards, we shall work with four digits combinations for two numbers. It will go up to 16^{th} order magic squares.

4.7.1 Magic Square

Example 91. *The **magic square** of order 9 for the digits 6 and 9 is given by*

96966669	69666996	66699666	66966696	96666966	69699669	69966666	66666969	96699696
69699696	66966666	96666969	96699666	69966669	66666996	66699669	96966696	69666966
66666966	96699669	69966696	69666969	66699696	96966666	96666996	69699666	66966669
96696969	69969696	66666666	66696996	96969666	69666669	69696966	66969669	96666696
69666696	66696966	96969669	96666666	69696969	66969696	66666669	96696996	69969666
66969666	96666669	69696996	69969669	66666696	96696966	96969696	69666666	66696969
96669669	69696696	66966966	66669696	96696666	69966969	69669666	66696669	96966996
69966996	66669666	96696669	96966966	69669669	66696696	66966969	96669696	69696666
66696666	96966969	69669696	69696669	66966996	96669666	96696696	69966966	66669669

In this case, the construction is little different. It is considered as

$$T := 10000 \times A + B,$$

where

$$A := \{6666, 6669, 6696, 6966, 6969, 6996, 9666, 9669, 9696\}$$

and

$$B := \{6969, 6996, 6999, 9669, 9696, 9699, 9969, 9996, 9999\}.$$

*Thus, there are total 81 entries given in above magic square of order 9. Even though the sum of members of A and B are different, Still, we get **pandiagonal magic square** of order 9 with **magic sum***

$$S_{9\times 9} := 700009992 = 1000 \times 69993 + 79992,$$

where

$$69993 := 6666 + 6669 + 6696 + 6966 + 6969 + 6996 + 9666 + 9669 + 9696$$

and

$$79992 := 6969 + 6996 + 6999 + 9669 + 9696 + 9699 + 9969 + 9996 + 9999.$$

*Thus, we have a **upside-down pandigonal magic square** of order 9 with **magic sum** $S_{9\times 9} := 700009992$. Moreover, Each block of order 3×3 as specified in the figure is a **semi-magic square** of order 3 with **semi-magic sum**: $S_{3\times 3} := 233336664$.*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

4.7.2 Palindromic Magic Square

Example 92. *The* ***palindromic pandiagonal magic square*** *of order 9 for the digits 6 and 9 is given by*

666699969996666	699696696696996	966969999969669	966699696996669	669696999696966	696969969969696	696699999996696	969696969696969	666969696969666
696969696969696	966699999996669	669696969696966	666969999969666	696699969996696	969696696696969	966969969969669	666699696996666	699696999696996
969696999696969	666969969969666	696699696996696	699696969696996	966969696969669	666699999996666	669696696696966	696969999969696	966699969996669
666996969699666	696669696966696	969699999996969	966996696699669	666669999966666	699699969996996	696996999699696	966669969966669	669699696996966
699699696996996	966996999699669	666669969966666	669699999996966	696996969699696	966669696966669	969699969996969	666996696699666	696669999966696
966669999966669	669699969996966	696996696699696	696669969966696	969699696996969	666996999699666	666669696966666	699699999996996	966996969699669
669669969966966	696999696999696	966696999696669	969669696966969	666999999999666	696696969696696	699669999966996	966999696999669	666696696696666
696696696696696	969669999966969	666999969999666	666696999696666	699669969966996	966999696999669	966696969696669	669669696966966	696999999999696
966999999999669	666696969696666	699669696966996	696999969999696	966696696696669	669669999966966	666999696999666	696696999696696	969669969966969

It is a ***block-wise palindromic pandigonal magic square*** *of order 9, where each block of order 3 is a* ***palindromic semi-magic square*** *with equal* ***magic sums****. The magic sums are given by*

$$S_{9\times 9} := 7000099999989993 \qquad \textit{and} \qquad S_{3\times 3} := \frac{7000099999989993}{3} = 2333366666663331 \quad \textit{(semi-magic sum).}$$

4.7.3 Bimagic Square

Example 93. *The* ***bimagic magic square*** *of order 9 for the digits 6 and 9 is given by*

66666969	96966996	69696999	96699669	69669696	66969699	69969969	66699996	96669999
69969669	66699696	96669699	66669969	96969996	69699999	96696969	69666996	66966999
96699969	69669996	66969999	69966969	66696996	96666999	66669669	96969696	69699699
96966999	69696969	66666996	69669699	66969669	96699696	66699999	96669969	69969996
66699699	96669669	69969696	96969999	69699969	66669996	69666999	66966969	96696996
69669999	66969969	96699996	66696999	96666969	69966996	96969699	69699669	66669696
69696996	66666999	96966969	66969696	96699699	69669669	96669996	69969999	66699969
96669696	69969699	66699669	69699996	66669999	96969969	66966996	96696999	69666969
66969996	96699999	69669969	96666996	69966999	66696969	69699696	66669699	96969669

Here also the 81 combinations are done according previous example, but with different distribution resulting in a ***upside-down bimagic magic square****. Its* ***magic sums*** *are* $S_{9\times 9} := 700009992$ *and* $Sb_{9\times 9} := 56084162583030534$.

The sum of 9 members of each block of order 3 (specified in the figure) is the same as of magic square, i.e., 700009992.

4.7.4 Palindromic Bimagic Square

Example 94. *The* ***palindromic bimagic square*** *of order 9 for the digits 6 and 9 is given by*

666669696966666	666999999999666	669696969696966	696699969996696	696996696699696	699669999966996	966696999696669	966969969969669	969699696996969
696696999696696	696969969969696	699699696996996	966669696966669	966999999999669	969696969696969	666699969996666	666996696699666	669669999966966
966699969996669	966996696699669	969669999966969	666696999696666	666969969969666	669699696996966	696669696966696	696999999999696	699696969696996
669699999996966	666696969696666	666969696969666	699696696696996	696669999966696	696999969999696	969669969966969	966699696996669	966996999699669
699669969966996	696699696996696	696996999699696	969699999996969	966696969696669	966969696969669	669696696696966	666669999966666	666999969999666
969696696696969	966669999966669	966999969999669	669669969966966	666699696996666	666996999699666	699699999996996	696696969696696	696969696969696
666996969699666	669669696966966	666699999996666	696969999969696	699699969996996	696696696696696	966999696999669	969696999696969	966669969966669
696999696999696	699696999696996	696669969966696	966996969699669	969669696966969	966699999996669	666969999969666	669699969996966	666696696696666
966969999969669	969699969996969	966696696696669	666999696999666	669696999696966	666669969966666	696996969699696	699669696966996	696699999996696

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

*It is a **palindromic bimagic square** of order 9 with **magic sums** given by*

$$S_{9\times 9} := 7000099999989993 \qquad \text{and} \qquad Sb_{9\times 9} := 5608416382733678742452716397199.$$

4.8 Magic Square of Order 10

4.8.1 Magic Square

Example 95. *The **magic square** of order 10 for the digits 6 and 9 is given by*

66666666	96696969	96996996	66696696	99999699	96966669	69696999	69969999	66969696	69999669
69966696	66696669	69699669	66669699	69996969	96996666	99999696	66966999	96969999	96696996
96699999	99996999	66966696	69996666	96999669	69969696	96966996	66696969	66666669	69699699
99996996	96999696	66669999	69696969	66966666	66699669	69966669	69999699	96696999	96966696
69699696	96966666	66696999	96999999	69966996	96699699	66666969	99999669	69996696	66966669
66699699	66969999	96696669	96969669	66669696	69996999	96996696	69696996	99996969	69966666
96966969	69969699	69999696	66966996	66699999	99996696	96699669	96996669	69696666	66666999
69996669	66669669	99996666	69966999	96696696	69699999	66969699	96969696	66696996	96996969
66969669	69696696	69966969	99996669	96966999	66666996	69999999	96696666	96999699	66699696
96996999	69996996	96969699	96699696	69696669	66966969	66696666	66666696	69969669	99999999

It is based on the 2 by 2 combinations of ten numbers

$$\{6666, 6669, 6696, 6969, 6996, 6999, 9669, 9696, 9699, 9999\}.$$

*These ten numbers are **upside-down** combining in pairs as* $\{6666, 9999\}$, $\{6669, 6999\}$, $\{6696, 9699\}$, $\{6969, 9696\}$ *and* $\{6996, 9669\}$. *It is **upside-down magic square**. Its **magic sum** is*

$$S_{10\times 10} := 800660058 = 10000 \times 80058 + 80058,$$

where

$$79992 = 6666 + 6669 + 6696 + 6969 + 6996 + 6999 + 9669 + 9696 + 9699 + 9999.$$

4.8.2 Palindromic Magic Square

Example 96. *The **palindromic magic square** of order 10 for the digits 6 and 9 is given by*

666666666666666	966969696969669	969969969969969	666966969669666	999996999699999	969666696666969	696969999969696	699699999996996	669696969696966	699996696699996
699666969666996	666966696669666	696996696699696	666696999696666	699969696969996	969966666669969	999996969699999	669669999966966	969699999996969	966969969969669
966999999999669	999969999969999	669666969666966	699966666669996	969996696699969	699696969696996	969669969966969	666969696969666	666666696666666	696996999699696
999969969969999	969996969699969	666699999996666	696969696969696	669666666666966	666996696699666	699666696666996	699996999699996	966969999969669	969666969666969
696996969699696	969666666666969	666969999969666	969999999999969	699669969966996	966996999699669	666669696966666	999996696699999	699966969669996	669666696666966
666996999699666	669699999996966	966966696669669	969696696696969	666696969696666	699969999969996	969966969669969	696969969969696	999969696969999	699666666666996
969669696966969	699696999696996	699996969699996	669669969966966	666999999999666	999966969669999	966996696699669	969966696669969	696966666669696	666669999966666
699966696669996	666696696696666	999966666669999	699669999966996	966966969669669	696999999999696	669696999696966	969696969696969	666969969969666	969969696969969
669696696696966	696966969669696	699669696966996	999966696669999	969669999966969	666669969966666	699999999999996	966966666669669	969996999699969	666996969699666
969969999969969	699969969969996	969696999696969	966996969699669	696966696669696	669669696966966	666966666669666	666666969666666	699696696696996	999999999999999

*It is a **palindromic pandiagonal magic square** of order 10 with **magic sum*** $S_{10\times 10} := 8006600666006592$.

4.9 Magic Square of Order 11

4.9.1 Magic Square

Example 97. *The **pandiagonal magic square** of order 11 for the digits 6 and 9 is given by*

66666666	66696669	69666966	69696969	69966996	69996999	96669666	96699669	99699969	99969996	99999999
99969969	99999996	66669999	66696666	69666669	69696966	69966969	69996996	96666999	96699666	99699669
96696999	99699666	99969669	99999969	66669996	66699999	69666666	69696669	69966966	69996969	96666996
69996966	96666969	96696996	99696999	99969666	99999669	66669969	66699996	69669999	69696666	69966669
69699999	69966666	69996669	96666966	96696969	99696996	99966999	99999666	66669669	66699969	69669996
66699669	69669969	69699996	69969999	69996666	96666669	96696966	99696969	99966996	99996999	66669666
99996996	66666999	66699666	69669669	69699969	69969996	69999999	96666666	96696669	99696966	99966969
99696669	99966966	99996969	66666996	66696999	69669666	69699669	69969969	69999996	96669999	96696666
96669996	96699999	99696666	99966669	99996966	66666969	66696996	69666999	69699666	69969669	69999969
69969666	69999669	96669969	96699996	99699999	99966666	99996669	66666966	66696969	69666996	69696999
69666969	69696996	69966999	69999666	96669669	96699969	99699996	99969999	99996666	66666669	66696966

It is based on the 2 by 2 combinations of 11 numbers

$$\{6666, 6669, 6966, 6969, 6996, 6999, 9666, 9669, 9969, 9996, 9999\}.$$

*These 11 numbers are **upside-down** combining in pairs as* $\{6666, 9999\}$, $\{6669, 6999\}$, $\{6966, 9969\}$, $\{6969, 9696\}$, $\{6996, 9669\}$ *and* 6969 *. It is **upside-down pandiagonal magic square**. Its **magic sum** is*

$$S_{11\times 11} := 905730564 = 10000 \times 90564 + 90564,$$

where

$$90564 := 6666 + 6669 + 6966 + 6969 + 6996 + 6999 + 9666 + 9669 + 9969 + 9996 + 9999.$$

4.9.2 Palindromic Magic Square

Example 98. *The **palindromic pandiagonal magic square** of order 11 for the digits 6 and 9 is given by*

666666666666666	666966696669666	696669666966696	696969696969696	699669969966996	699969999969996	966696666696669	966996696699669	996999696999699	999699969996999	999999999999999
999699696996999	999999969999999	666699999996666	666966666669666	696666696666696	696969666969696	699669696966996	699969969969996	966669999966669	966996666699669	996996696699699
966969999969669	996996666699699	999696696696999	999999696999999	666699969996666	666999999999666	696666666666696	696966696669696	699669666966996	699969696969996	966669969966669
699969666969996	966669696966669	966969969969669	996969999969699	999696666696999	999996696699999	666699696996666	666999969999666	696699999996696	696966666669696	699666696666996
696999999999696	699666666666996	699966696669996	966669666966669	966969696969669	996969969969699	999669999966999	999996666699999	666696696696666	666999696999666	696699969996696
666996696699666	696699696996696	696999969999696	699699999996996	699966666669996	966666696666669	966969666969669	996969696969699	999669969966999	999969999969999	666696666696666
999969969969999	666669999966666	666996666699666	696696696696696	696999696999696	699699969996996	699999999999996	966666666666669	966966696669669	996969666969699	999669696966999
996966696669699	999669666966999	999969696969999	666669969966666	666969999969666	696696666696696	696996696699696	699699696996996	699999969999996	966699999996669	966966666669669
966699969996669	966999999999669	996966666669699	999666696666999	999969666969999	666669696966666	666969969969666	696669999966696	696996666699696	699696696696996	699999696999996
699696666696996	699996696699996	966699696996669	966999969999669	996999999999699	999666666666999	999966696669999	666669666966666	666969696969666	696669969966696	696969999969696
696669696966696	696969969969696	699669999966996	699996666699996	966696696696669	966999696999669	996999969999699	999699999996999	999966666669999	666666696666666	666969666969666

*It is a **palindromic pandiagonal magic square** of order 11 with **magic sum** $S_{11\times 11} := 9057305727602751$.*

4.10 Magic Square of Order 12

4.10.1 Blocks of Order 4

Example 99. *The **pandiagonal magic square** of order 12 for the digits 6 and 9 is given by*

69969669	96699999	66666666	99996996	69999669	96669999	66696666	99966996	69699669	96969999	66996666	99666996
66666996	99996666	69969999	96699669	66696996	99966666	69999999	96669669	66996996	99666666	69699999	96969669
99999999	66669669	96696996	69966666	99969999	66699669	96666996	69996666	99669999	66999669	96966996	69696666
96696666	69966996	99999669	66669999	96666666	69996996	99969669	66699999	96966666	69696996	99669669	66999999
69969666	96699996	66666669	99996999	69999666	96669996	66696669	99966999	69699666	96969996	66996669	99666999
66666999	99996669	69969996	96699666	66696999	99966669	69999996	96669666	66996999	99666669	69699996	96969666
99999996	66669666	96696999	69966669	99969996	66699666	96666999	69996669	99669996	66999666	96966999	69696669
96696669	69966999	99999666	66669996	96666669	69996999	99969666	66699996	96966669	69696999	99669666	66999996
69969696	96699966	66666699	99996969	69999696	96669966	66696699	99966969	69699696	96969966	66996699	99666969
66666969	99996699	69969966	96699696	66696969	99966699	69999966	96669696	66996969	99666699	69699966	96969696
99999966	66669696	96696969	69966699	99969966	66699696	96666969	69996699	99669966	66999696	96966969	69696699
96696699	69966969	99999696	66669966	96666699	69996969	99969696	66699966	96966699	69696969	99669696	66999966

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

It is based on the 2 by 2 combinations of twelve numbers

$$\{6669, 6696, 6699, 6966, 6969, 6999, 9666, 9696, 9699, 9966, 9969, 9996\}.$$

*These 12 numbers are **upside-down** and **mirror looking** combining as* $\{6669, 6999\}$, $\{6696, 9699\}$, $\{6699, 9966\}$, $\{6966, 9969\}$, $\{6969, 9696\}$, $\{9666, 9996\}$. *It is **upside-down pandiagonal magic square**. Its **magic sum** is*

$$S_{12\times12} := 999999990 = 10000 \times 99990 + 99990,$$

where

$$99990 = 6669 + 6696 + 6699 + 6966 + 6969 + 6999 + 9666 + 9696 + 9699 + 9966 + 9969 + 9996.$$

*It is **block-wise upside-down pandigonal magic square** of order 12, where each block of order 4 is a **pandiagonal magic square** with equal **magic sums** given by*

$$S_{4\times4} := \frac{999999990}{3} = 333333330.$$

4.10.2 Blocks of Order 6

Example 100. *The **magic square** of order 12 for the digits 6 and 9 is given by*

66696669	99969966	99969666	99966999	66696699	66699996	66696696	99969699	99969696	99966969	66696966	66699969
99669996	66996699	99669666	66996999	66999966	99666669	99669969	66996966	99669696	66996969	66999699	99666696
96669996	96669966	69996999	69999666	96666699	69996669	96669969	96669699	69996969	69999696	96666966	69996696
69999996	69996699	96666999	96669666	69999966	96666669	69999969	69996966	96666969	96669696	69999699	96666696
66996669	99666699	66999666	99666999	99669966	66999996	66996696	99666966	66999696	99666969	99669699	66999969
99966669	66699966	66696999	66699666	99966699	99969996	99966696	66699699	66696969	66699696	99966966	99969969
66966669	99699966	99699666	99696999	66966699	66969996	66966696	99699699	99699696	99696969	66966966	66969969
96999996	69666699	96999666	69666999	69669966	96996669	96999969	69666966	96999696	69666969	69669699	96996696
96969996	96969966	69696999	69699666	96966699	69696669	96969969	96969699	69696969	69699696	96966966	69696696
69699996	69696699	96966999	96969666	69699966	96966669	69699969	69696966	96966969	96969696	69699699	96966696
69666669	96996699	69669666	96996999	96999966	69669996	69666696	96996966	69669696	96996969	96999699	69669969
99696669	66969966	66966999	66969666	99696699	99699996	99696696	66969699	66966969	66969696	99696966	99699969

 2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

In this case the construction is based on the similar lines of Example **??**. *In is just a magic square with same magic sum:* $S_{12\times 12} := 999999990$ *It is* ***block-wise magic square*** *of order 12, where each block of order 6 is a* ***magic square*** *with equal* ***magic sums*** *given by* $S_{6\times 6} := \dfrac{999999990}{2} = 499999995.$

4.10.3 Blocks of Order 3

Example 101. *The* ***magic square*** *of order 12 for the digits 6 and 9 is given by*

96996999	99666969	99969969	99699666	69699696	69996966	66966669	66996699	66696696	69669996	96969966	96669699
99966969	96999969	99666999	69999696	99696966	69699666	66696699	66966696	66996669	96669966	69669699	96969996
99669969	99966999	96996969	69696966	69999666	99699696	66996696	66696669	66966699	96969699	96669996	69669966
66969996	66999966	66699699	69666669	96966699	96666696	96999666	99669696	99966966	99696999	69696969	69999969
66699966	66969699	66999996	96666699	69666696	96966669	99969696	96996966	99669666	69996969	99699969	69696999
66999699	66699996	66969966	96966696	96666669	69666699	99666966	99969666	96999696	69699969	69996999	99696969
69669666	96969696	96666966	66966999	66996969	66699969	99699996	69699966	69999699	96996669	99666699	99966696
96669696	69666966	96969666	66696969	66969969	66996999	69999966	99699699	69699996	99966699	96996696	99666669
96966966	96669666	69669696	66999969	66696999	66966969	69699699	69999996	99699966	99666696	99966669	96996699
99696669	69696699	69996696	96999996	99669966	99969699	69666999	96966969	96669969	66969666	66999696	66696966
69996699	99696696	69696669	99969966	96999699	99669996	96666969	69669969	96966999	66699696	66966966	66999666
69696696	69996669	99696699	99669699	99969996	96999966	96969969	96666999	69666969	66996966	66699666	66969696

In this case the construction is based on the similar lines of Example 99. It is just a magic square with same magic sum: $S_{12\times 12} := 999999990$ *It is* ***block-wise magic square*** *of order 12, where each block of order 3 is a* ***semi-magic square*** *with different* ***semi-magic sums****.*

4.10.4 Palindromic Magic Square

Example 102. *The* ***palindromic pandiagonal magic square*** *of order 12 for the digits 6 and 9 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

699696696696996	966999999999669	666666666666666	999969969969999	699996696699996	966699999996669	666966666669666	999669969966999	696996696699696	969699999996969	669966666669966	996669969966699
666669969966666	999966666669999	699699999996996	966996696699669	666969969969666	999666666666999	699999999999996	966696696696669	669969969969966	996666666666699	696999999999696	969696696696969
999999999999999	666696696696666	966969969969669	699666666666996	999699999996999	666996696699666	966669969966669	699966666669996	996699999996699	669996696699966	969669969966969	696966666669696
966966666669669	699669969966996	999996696699999	666699999996666	966666666666669	699969969969996	999696696696999	666999999999666	969666666666969	696969969969696	996696696696699	669999999999966
699696666696996	966999969999669	666666696666666	999969999969999	699996666699996	966699969996669	666966696669666	999669999966999	696996666699696	969699969996969	669966696669966	996669999966699
666669999966666	999966696669999	699699969996996	966996666699669	666969999969666	999666696666999	699999969999996	966696666696669	669969999969966	996666696666699	696999969999696	969696666696969
999999969999999	666696666696666	966969999969669	699666696666996	999699969996999	666996666699666	966669999966669	699966696669996	996699969996699	669996666699966	969669999966969	696966696669696
966966696669669	699669999966996	999996666699999	666699969996666	966666696666669	699969999969996	999696666696999	666999969999666	969666696666969	696969999969696	996696666696699	669999969999966
699696969696996	966999666999669	666666999666666	999969696969999	699996969699996	966699666996669	666966999669666	999669696966999	696996969699696	969699666996969	669966999669966	996669696966699
666669696966666	999966999669999	699699666996996	966996969699669	666969696969666	999666999666999	699999666999996	966696969696669	669969696969966	996666999666699	696999666999696	969696969696969
999999666999999	666696969696666	966969696969669	699666999666996	999699666996999	666996969699666	966669696966669	699966999669996	996699666996699	669996969699966	969669696966969	696966999669696
966966999669669	699669696966996	999996969699999	666699666996666	966666999666669	699969696969996	999696969696999	666999666999666	969666999666969	696969696969696	996696969696699	669999666999966

It is a ***palindromic pandiagonal magic square*** *of order 12 with* ***magic sum*** $S_{13\times 13} := 9999999999999990$. *It is* ***block-wise pandigonal magic square*** *of order 12, where each block of order 4 is a* ***palindromic pandiagonal magic square*** *with equal* ***magic sums*** *as* $S_{4\times 4} := \dfrac{9999999999999990}{3} = 3333333333333330$.

The above example of **palindromic magic square** is for the **magic square** given in Example 99. In the similar way we can write the **palindromic magic squares** for the Examples 100 and 101

4.11 Magic Square of Order 13

4.11.1 Magic Square

Example 103. *The* ***pandiagonal magic square*** *of order 13 for the digits 6 and 9 is given by*

66996699	99966696	99696996	96999666	96969696	96696966	96669996	69996669	69966969	69696999	69669669	66969699	66699969
66969996	66696669	66996969	99966999	99699669	96999699	96969969	96696699	96666696	69996996	69969666	69699696	69666966
69699969	69666699	66966696	66696996	66999666	99969696	99696966	96999996	96966669	96696969	96666999	69999669	69969699
69996966	69969996	69696669	69666969	66966999	66699669	66999699	99969969	99696699	96996696	96966996	96699666	96669696
96699699	96669969	69996699	69966696	69696996	69669666	66969696	66696966	66999996	99966669	99696969	96996999	96969669
96999696	96966966	96699996	96666669	69996969	69966999	69699669	69669699	66969969	66696699	66996696	99966996	99699666
99969669	99699699	96999969	96966699	96696696	96666996	69999666	69969696	69696966	69669996	66966669	66696969	66996999
66699666	66999696	99966966	99699996	96996669	96966969	96696999	96669669	69999699	69969969	69696699	69666696	66966996
69666999	66969669	66699699	66999969	99966699	99696696	96996996	96969666	96699696	96666966	69999996	69966669	69696969
69966996	69699666	69669696	66966966	66699996	66996669	99966969	99696999	96999669	96969699	96699969	96666699	69996696
96666969	69996999	69969669	69699699	69669969	66966699	66696696	66996996	99969666	99699696	96996966	96969996	96696669
96966696	96696996	96669666	69999696	69966966	69699996	69666669	66966969	66696999	66999669	99969699	99699969	96996699
99696669	96996969	96966999	96699669	96669699	69999969	69966699	69696696	69666996	66969666	66699696	66996966	99969996

It is based on the 2 by 2 combinations of 13 numbers

$$\{6669, 6696, 6966, 6969, 6996, 6999, 9666, 9669, 9696, 9699, 9969, 9996, 6699\}.$$

These 11 numbers are ***upside-down*** *combining in pairs as* $\{6669, 6999\}$, $\{6696, 9699\}$, $\{6966, 9969\}$, $\{6969, 9696\}$, $\{6996, 9669\}$, $\{9666, 9996\}$ *and* 6699 *. It is* ***upside-down pandiagonal magic square****. Its* ***magic sum*** *is*

$$S_{13\times 13} := 1066996689 = 10000 \times 106689 + 106689,$$

where

$$106689 := 6669 + 6696 + 6966 + 6969 + 6996 + 6999 + 9666 + 9669 + 9696 + 9699 + 9969 + 9996 + 6699.$$

4.11.2 Palindromic Magic Square

Example 104. *The* ***palindromic pandiagonal magic square*** *of order 13 for the digits 6 and 9 is given by*

669966999669966	999666969666999	996969969969699	969996666699969	969696969696969	966969666969669	966699969996669	699966696669996	699669696966996	696969999969696	696696696696696	669696999696966	666999696999666
669699969996966	666966696669666	669969696969966	999669999966999	996996696699699	969996999699969	969699696996969	966966999669669	966666969666669	699969969969996	699696666696996	696996969699696	696669666966696
696999696999696	696666999666696	669666969666966	666969969969666	669996666699966	999696969696999	996969666969699	969999969999969	969666696666969	966969696969669	966669999966669	699996696699996	699696999696996
699969666969996	696699969996696	696966696669696	696669696966696	669669999966966	666996696699666	669996999699966	999699696996999	996966999669699	969966969669969	969669969966969	966996666699669	966696969696669
966996999699669	966699696996669	699966999669996	699666969666996	696969969969696	696696666696696	669696969696966	666969666969666	669999969999966	999666696666999	996969696969699	969969999969969	969696696696969
969996969699969	969669666966969	966999969999669	966666696666669	699969696969996	699669999966996	696996696699696	696696999696696	669699696996966	666966999669666	669966969669966	999669969966999	996996666699699
999696696696999	996996999699699	969999696999969	969666999666969	966696969669669	966669969966669	699996666699996	696969696969696	696969666969696	696699969996696	669666696666966	666969696969666	669969999969966
666996666699666	669996969699966	999669666966999	996999969999699	969966696669969	969669696966969	966969999969669	966696696696669	699996999699996	699699696996996	696966999669696	696666969666696	669669969966966
696669999966696	669696696696966	666996999699666	669999696999966	999666999666999	996966969669699	969969969969969	969696666696969	966996969699669	966669666966669	699999969999996	699666696666996	696969696969696
699669969966996	696996666699696	696696969696696	669669666966966	666999969999666	669966696669966	999669696966999	996969999969699	969996696699969	969696999696969	966999696999669	966666999666669	699966969669996
966669696966669	699969999969996	699696696696996	696996999699696	696699696996696	669666999666966	666966969669666	669969969969966	999696666696999	996996969699699	969969666969969	969699969996969	966966696669669
969666969666969	966969969969669	966696666696669	699996969699996	699669666966996	696999969999696	696666696666696	669669696966966	666969999969666	669996696699966	999696999696999	996999696999699	969966999669969
996966696669699	969969696969969	969669999966969	966996696699669	966696999696669	699999696999996	699666999666996	696966969669696	696669969966696	669696666696966	666996969699666	669969666969966	999699969996999

*It is a **palindromic pandiagonal magic square** of order 13 with **magic sum*** $S_{13\times 13} := 10669966999669956$.

4.12 Magic Square of Order 14

4.12.1 Magic Square

Example 105. *The **magic square** of order 14 for the digits 6 and 9 is given by*

99969996	96996696	69666969	96966669	69996966	99699669	69969666	99666996	96666999	66699966	66999699	66969969	96696699	69699696
66699699	99699969	96966696	69669996	96696669	99669666	69696999	96996969	69996996	66999696	66969966	96666699	99969669	69966966
66999669	66699696	99669966	96696696	69669969	96996999	99966996	96969996	69966969	66969699	69996699	99699666	69696966	96666669
66969696	66999666	66699669	96999699	96666696	96966996	99696969	96699969	69699996	69966699	99666999	99966966	69996669	69669966
69696699	66969669	66996999	66699666	96969696	96696969	99669996	96669966	99969969	96996996	99696966	69966669	69669699	69996696
96666996	69996969	69969996	69699969	99969966	69666966	66966669	66696699	66996696	96696999	96969666	96999669	99669696	99699699
69969969	69699966	99969699	99699696	99669669	66696696	66996699	69666669	66966966	69999996	96666969	96696996	96966999	96999666
96699966	96669699	69999696	69969669	69699666	66996669	66696966	66966696	69666699	96969969	96999996	99666969	99696996	99966999
69999666	69966999	69696996	99966969	99699996	66966699	69666696	66996966	66696669	96669669	96699696	96969699	96999966	99669969
99666696	69666996	96996669	96666966	69969699	99969696	69999669	99696999	96699666	69696969	66699969	66999966	66969996	96966699
69666999	99666669	96696966	69999966	96996699	69699699	96669696	99969666	96969669	99696696	69966996	66699996	66999969	66966969
99696669	96966966	96669969	99666699	66966996	69969966	96699699	69699669	96999696	69669666	99966696	69996999	66696969	66999996
96996966	96699996	99696699	66966999	66996969	69999969	96969966	69969696	99669699	99966669	69669669	69696696	96669666	66696996
96966969	99966699	66969666	66996996	66696999	96669996	96999969	69999699	99699966	99666966	69696669	69669696	69966696	96699669

It is based on the 2 by 2 combinations of ten numbers

$$\{6669, 6696, 6699, 6966, 6969, 6996, 6999, 9666, 9669, 9696, 9699, 9966, 9969, 9996\}.$$

*These fourteen numbers are **upside-down** in pairs as* $\{6669, 6999\}$, $\{6696, 9699\}$, $\{6699, 6699\}$, $\{6966, 9969\}$, $\{6969, 9696\}$, $\{6996, 96699\}$ *and* $\{9666, 9996\}$. *It is **upside-down magic square**. Its **magic sum** is*

$$S_{14\times 14} := 1166666655 = 10000 \times 116655 + 116655,$$

where

$$116655 = 6669 + 6696 + 6699 + 6966 + 6969 + 6996 + 6999 + 9666 + 9669 + 9696 + 9699 + 9966 + 9969 + 9996.$$

4.12.2 Palindromic Magic Square

Example 106. *The **palindromic magic square** of order 14 for the digits 6 and 9 is given by*

99969996996999	96996969669969	69666969666696	96966696666969	69996966696996	99699669669699	69969666696996	99666996966699	96666999966669	66699666999666	66999699699966	66969969996966	96696699669669	69699696699696
66699699699666	99699969699699	96966696966969	69669969996696	96696669669669	99669666696699	69696999969696	96996969696969	69996969969996	66999696999966	66969966996966	96666699666669	99969669696999	69966966966996
66999669669966	66699696969666	99669966699669	96696696969669	69669969996696	96996999969969	99966996966999	96969969996969	69966969696996	66969699696966	69996699669996	99699666699699	69696966969696	96666669666669
66969696969666	66999666699966	66699669699666	96999699699969	96666696966669	96966996966969	99696969669699	96699696999669	69699969999696	69966699666996	99666999966699	99966966966999	69996669669996	69669966696696
69696699669696	66969669696966	66996999969966	66699666669666	96969696969969	96696969969669	99669996996699	96669666996669	99969969996999	96996969969969	99696966969699	69966669666996	69669699696696	69996696969996
96666996966669	69996969696996	69699969996996	69699696999696	99699666996999	69666966966696	66966669666966	66696699669666	66996696669966	96696999969669	96969666696969	96999669669969	99669696966699	99699699699699
69969696996996	69699666999696	99969699696999	99699696969699	99669669696699	66966969669666	66996699669966	69666669666696	66966966696966	69999969999996	96666969666669	96696996969669	96966999966969	96999666699969
96699666999669	96669699696669	69999696699996	69969669696996	69699666699696	66996669669966	66696966969666	66966969666966	69666699666696	96969696996969	96999969999969	99666969666699	99696969969699	99966999966999
69999666699996	69966999966996	69696969969696	99966969666999	99699969999699	66966699666966	69666696666696	66996966969966	66696669669666	96669669696669	96699696969669	96969699696969	96999666999969	99669696996699
99666696966699	69666996966696	96996696669969	96666966966669	69969699696996	99969696969699	69999696699996	99696999969699	96699666699669	69696969696696	66699696999666	66999666999966	66969996996966	96966699666969
69666999966696	99666669666699	96696966969669	69999666999996	96996699669969	69699699699696	96669696696669	99969666696999	96969669696969	99696696969699	69966969966996	66699969999666	66999696999966	66966969696966
99696669669699	96966966696969	96669696996669	99666699666699	66966969966966	69969666996996	96699699699669	69699669699696	96999696699969	69669666696696	99966969666999	69996999969996	66696969696666	66999969999966
96996966969969	96699969999669	99696699669699	66966999966966	66996969696966	69999696999996	96969966996969	69969696696996	99669699696699	99666696666999	69669669696696	69696696669696	96669666696669	66696996969666
96966969696969	99966699666999	66969666696966	66996969969966	66696999969666	96669969996669	96999696999969	69999699699996	99699666999699	99669666966699	69696669669696	69669696969696	69966696966996	96699669699669

*It is a **palindromic magic square** of order 14 with **magic sum*** $S_{14\times 14} := 1166666666666655$. *The middle block of order 4 is a magic square with **magic sum*** $S_{4\times 4} := 2703270332973294$.

4.13 Magic Square of Order 15

4.13.1 Magic Square

Example 107. *The **pandiagonal magic square** of order 15 for the digits 6 and 9 is given by*

99969996	99996699	66699669	96696696	99666966	69969969	69999999	69696996	66969699	96999666	96669966	66666969	66996669	99696999	69666666
66996666	99699969	96696966	66969666	96666699	96996969	99969966	69969996	99999999	66669699	66696996	69666696	99666999	69699669	69996669
96696669	69669669	99669966	96666969	66999999	66666966	99996996	99969969	66696666	66969996	69696699	96996999	69969666	69996696	99699699
66969669	96666696	69699666	96999699	66666996	69666669	66696969	99699966	66999969	69966966	99996999	99969999	69996699	99666666	96699996
69669966	66999666	66666699	69969999	99996666	66699996	69696696	99669699	99966969	96696999	99696669	69996966	96999669	96669969	66966996
99696996	69699699	69669999	66696966	99966669	96699669	66969969	96996666	96666999	99666696	69996969	99999666	66669966	66999996	69966699
66666999	99669996	69966666	69696669	66966969	99696696	96669666	99999669	96996699	69996996	96699999	66699699	69669969	99966966	66999966
69966969	99966996	99699996	99669969	96999966	99999699	96696666	69996999	69696966	69666699	66996696	66966669	66699999	66669666	96669669
96996696	66666666	99996669	99696699	69699996	69999666	66999699	69666969	69966996	66699966	99669669	96699969	96666966	66969999	99966999
96669699	96999999	99966696	69666996	69999669	66969966	69966999	66996966	99666669	69696969	66669969	99696666	99999996	96696699	66699666
99669666	69966669	66996969	69996666	66699969	69696999	66669669	66966699	96699966	99699999	69666966	96669996	99969699	96996996	99996696
69699999	66966966	69999699	66669996	69666999	96666666	96996669	66696696	99699669	99999969	99969666	66996699	96696996	69969966	99666969
66696699	69999966	96666996	66996999	96699699	99669999	99696966	66666669	69669666	99966666	96999996	69969669	66966696	99996969	69699969
69999969	96696969	66966999	99999966	99699666	99966699	69669996	96669999	66666696	66999669	69969699	99666996	69696666	66696669	96996966
99996966	66696999	96999969	99969669	69966696	66996996	99666699	96699666	69999996	96666669	66966666	69699966	99696969	69669699	66669999

In this case, the construction is little different. It is considered as

$$T := 10000 \times A + B,$$

where

$$A := \{6666, 6669, 6696, 6699, 6966, 6969, 6996, 6999, 9666, 9669, 9699, 9966, 9969, 9996, 9696\}$$

and

$$B := \{9999, 6669, 6696, 6699, 6966, 6969, 6996, 6999, 9666, 9669, 9699, 9966, 9969, 9996, 9696\}.$$

Thus, there are total 225 entries given in above magic square of order 15. Even though the sum of members of A *and* B *are different, Still, we get **magic square** of order 15 with **magic sum***

$$S_{9\times 9} := 1233336654 = 10000 \times 123321 + 126654$$

 2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16, **Zenodo**, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

where

$$123321 := 6666 + 6669 + 6696 + 6699 + 6966 + 6969 + 6996 + 6999 + 9666 + 9669 + 9699 + 9966 + 9969 + 9996 + 9696$$

and

$$126654 := 9999 + 6669 + 6696 + 6699 + 6966 + 6969 + 6996 + 6999 + 9666 + 9669 + 9699 + 9966 + 9969 + 9996 + 9696.$$

Thus, we have a ***upside-down magic square*** *of order 15 with* ***magic sum*** $S_{15\times15} := 1233336654$.

4.13.2 Semi-Magic Squares

Below is an another example of **semi-magic square** of order 15, where each block of order 5 is a **pandiagonal magic squares** with different magic sums.

Example 108. *The* ***semi-magic square*** *of order 15 for the digits 6 and 9 is given by*

66666669	69699696	69966966	96999969	99666999	66966696	66999669	96666699	96699996	99969666	66696969	69669966	69996996	96969999	99699699
96996966	99669969	66666999	69696669	69969696	96696699	99969996	66969666	66996696	96669669	96966996	99699999	66699699	69666969	69999966
69696999	69966669	96999696	99666966	66669969	66999666	96666696	96699669	99966699	66969996	69669699	69996969	96969966	99696996	66699999
99669696	66666966	69699969	69966999	96996669	99969669	66966699	66999996	96669666	96696696	99699966	66696996	69669999	69999699	96966969
69969969	96996999	99666669	66669696	69696966	96669996	96699666	99966696	66969669	66996699	69999999	96969699	99696969	66699966	69666996
66966969	66999966	96666996	96699999	99969699	66696669	69669696	69996966	96969969	99696999	66666696	69699669	69966699	96999996	99669666
96696996	99969999	66969699	66996969	96669966	96966966	99699969	66696999	69666669	69999696	96996699	99669996	66669666	69696696	69969669
66999699	96666969	96699966	99966996	66969999	69666999	69996669	96969696	99696966	66699969	69699666	69966696	96999669	99666699	66669996
99969966	66966996	66999999	96669699	96696969	99699696	66696966	69669969	69996999	96966669	99669669	66666699	69699996	69969666	96996696
96669999	96699699	99966969	66969966	66996996	69999969	96966999	99696669	66699696	69666966	69969996	96999666	99666696	66669669	69696699
66696696	69669669	69996699	96969996	99699666	66666969	69699966	69966996	96999999	99669699	66966669	66999696	96666966	96699969	99966999
96966699	99699996	66699666	69666696	69999669	96996996	99669999	66669699	69696969	69969966	96696966	99969969	66966999	66996669	96669696
69669666	69996696	96969669	99696699	66699996	69699699	69966969	96999966	99666996	66669999	66996999	96666669	96699696	99966966	66969969
99699669	66696699	69669996	69999666	96966696	99669966	66666996	69699999	69969699	96996969	99969696	66966966	66999969	96666999	96696669
69999996	96969666	99696696	66699669	69666699	69969999	96999699	99666969	66669966	69696996	96669969	96696999	99966669	66969696	66996966

It is ***semi-magic square*** *as sum of rows and columns are the as of above magic square, but the principal diagonals sum is different.*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

The above magic square is of type 5×3. Below is another example of **semi-magic square** of order 15, where each block of order 3 is also a **semi-magic square** with different **semi-magic sums**

Example 109. *The **semi-magic square** of order 15 for the digits 6 and 9 is given by*

99666996	69666969	69699969	96999669	66696966	96699699	66669696	99966696	69999966	96966999	66999996	96666699	66969666	99699999	69966669
69696969	99669969	69666996	96696966	96999699	66699669	69996696	66669966	99969696	96669996	96966699	66996999	69969999	66966669	99699666
69669969	69696996	99666969	66699699	96699669	96996966	99969966	69999696	66666696	66996699	96666999	96969996	99696669	69969666	66969999
96969696	66996696	96669966	66966999	99699996	69966699	99669666	69669999	69696669	96996996	66696969	96699969	66669669	99966966	69999699
96666696	96969966	66999696	69969996	66966699	99696999	69699999	99666669	69669666	96696969	96999969	66696996	69996966	66669699	99969669
66999966	96669696	96966696	99696699	69966999	66969996	69666669	69699666	99669999	66699969	96696996	96996969	99969699	69999669	66666966
96999666	66699999	96696669	66666996	99966969	69999969	96969669	66996966	96669699	66969696	99696696	69969966	99666999	69669996	69696699
96699999	96996669	66699666	69996969	66669969	99966996	96666966	96969699	66999669	69966696	66969966	99699696	69699996	99666699	69666999
66696669	96699666	96999999	99969969	69996996	66666969	66999699	96669669	96966966	99699966	69969696	66966696	69666699	69696999	99669996
66969669	99696966	69969699	99669696	69666696	69699966	96996999	66699996	96696699	66669666	99969999	69996669	96966996	66996969	96669969
69966966	66969699	99699669	69696696	99669966	69669696	96699996	96996699	66696999	69999999	66666669	99969666	96666969	96969969	66996996
99699699	69969669	66966966	69669966	69699696	99666696	66696699	96696999	96999996	99966669	69999666	66669999	66999969	96666996	96966969
66666999	99969996	69996699	96969666	66999999	96666669	66966996	99696969	69969969	99669669	69666966	69699699	96999696	66696696	96699966
69999996	66666699	99966999	96669999	96966669	66999666	69966969	66969969	99696996	69696966	99669699	69669669	96696696	96999966	66699696
99966699	69996999	66669996	66996669	96669666	96969999	99699969	69966996	66966969	69669699	69699669	99666966	66699966	96699696	96996696

*It is **semi-magic square** as sum of the rows and columns are the same as of above magic square, but the principal diagonals sum is different.*

4.13.3 Palindromic Magic Square

Example 110. *The **palindromic magic square** of order 15 for the digits 6 and 9 is given by*

66669666696666	66696999969666	66966699966966	66996966696966	69666969966696	69696669669696	69969969696996	99669999996699	96669699696669	96696969696969669	96966696966969	96999969999969	69999669669996	99699696969699	99969966699699
69696696969696	69969996996996	69999999999996	96669696969669	96699966999669	96969666696969	96996999969969	66969669696966	99696966969699	99966969966999	66666669666666	66699969699666	99666699666699	66999699699966	69666969696696
96966696666969	96999969699969	99669669696699	99699699699699	99966969966999	66666696966666	66699969999666	69996699669996	66999696969966	69669666996696	69699666699696	69966999966996	66969999996966	96666966966669	96696996969669
66696969696666	66966696966966	66996999969966	69669999996696	66666966966666	69966969966996	69996696669996	96669696996669	99966699666999	69699699699696	96999666999969	99669666696699	99699969999699	96699669669669	96969696969696969
69969666996996	69999666699996	96669996996669	99969669696999	69699696969696	96996969696969	99666696966699	99696999969699	96699999999669	96966966966969	66696969969666	66966669666966	66999969699966	69666699666696	66669699696666
96996996969969	99666669666699	99699969699699	96696699669669	96969699696969	66699666699666	66969666696966	66999969999966	69669669696696	66669696969666	69966969966996	69996696966996	96666999966669	99969999996999	69696966969696
66966966696966	96666969666669	69666969666696	66669969696666	69969999996996	99669699696699	66996996969966	99966696666999	69696999969696	66696699669666	69999696969996	99699666999699	96699666669669	96969996996969	96999669669969
99669696966699	99696996969699	99969666696999	96966999966969	66699669669666	69996966696996	96669966696669	96696696969669	66669996996666	96999999999969	66969699696966	66996969696966	69666669666696	69699969699696	69966699666996
69999699699969	66999966999966	96696669669669	69699996999696	96996699669969	66969696969666	99696969696699	69669666696696	96969969969669	69969669696996	99666966966699	96666996966669	99666969666999	66666999966666	66699999999666
99699999999699	69666966966696	66666996966666	69966696966996	99669969696699	96666699666669	99969699696999	69696969696969696	66696669669666	69996999969996	66999669669966	96699696969669	96969966696969	96999666699969	66969996996966
96669669696669	99969696969999	69699666999696	66699666699666	69999969999996	66999999999966	96696966696669	96966996966969	96996696969669	66969696996966	99696699669699	69669699696696	66666969696666	69966696666996	99666999966699
66996699669966	96699699699669	96966969696969	96996669666969	69666999966966	99699669669699	69669696969696	66669966696666	69969666696996	99669969996699	96669999996669	99669666966999	69696969969696	66696696696666	69999696999996
96696999969669	66669999996666	69966966966996	66966969966966	66996669669966	99969969696999	96966699666969	66699696969966	99666969666699	96666969666669	69669969996696	69699669669696	96999699699969	69999666999996	99699666699699
99969996996999	96969669696969	66699699699666	99669966996699	96669666696669	69666999966696	69699999999696	96996966696969	69996969969996	99696696669699	96699696999669	66666699666666	69969696969996	66966969696966	66996696969966
69669969696696	69696699669696	96999696999969	69996969696996	99696696969699	96699969999669	66669669696666	69969699696996	66969666996966	66999666699966	99669999966999	96969999996969	66696966969666	99666969966699	96666696666669

*It is a **palindromic magic square** of order 15 with **magic sum** $S_{15\times 15} := 12333366666663321$.*

4.14 Magic Square of Order 16

We observe that the 4-digits combinations considering 2 by 2 maximum goes up to 256 possibilities resulting in magic squares of orders 9 to 16. The following magic squares of order 16 are with all possible 256 combinations.

4.14.1 Magic Square

Example 111. *The **pandiagonal magic square** of order 16 for the digits 6 and 9 is given by*

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

96696969	69966696	99969969	66699696	96966966	69696699	99699966	66969699	96666999	69996666	99999999	66669666	96996996	69666669	99669996	66999669
99969696	66699969	96696696	69966969	99699699	66969966	96966699	69696966	99999666	66669999	96666666	69996999	99669669	66999996	96996669	69666996
66696696	99966969	69969696	96699969	66966699	99696966	69699699	96969966	66666666	99996999	69999666	96669999	66996669	99666996	69669669	96999996
69969969	96699696	66696969	99966696	69699966	96969699	66966966	99696699	69999999	96669666	66666999	99996666	69669996	96999669	66996996	99666669
96666996	69996669	99999996	66669669	96996999	69666666	99669999	66999666	96696966	69966699	99969966	66699699	96966969	69696696	99699969	66969696
99999669	66669996	96666669	69996996	99669666	66999999	96996666	69666999	99969699	66699966	96696699	69966966	99699696	66969969	96966696	69696969
66666669	99996996	69999669	96669996	66996666	99666999	69669666	96999999	66696699	99966966	69969699	96699966	66966696	99696969	69699696	96969969
69999996	96669669	66666996	99996669	69669999	96999666	66996999	99666666	69969966	96699699	66696966	99966699	69699969	96969696	66966969	99696696
96996966	69666699	99669966	66999699	96666969	69996696	99999969	66669696	96966996	69696669	99699996	66969669	96696999	69966666	99969999	66699666
99669699	66999966	96996699	69666966	99999696	66669969	96666696	69996969	99699669	66969996	96966669	69696996	99969666	66699999	96696666	69966999
66996699	99666966	69669699	96999966	66666696	99996969	69999696	96669969	66966669	99696996	69699669	96969996	66696666	99966999	69969666	96699999
69669966	96999699	66996966	99666699	69999969	96669696	66666969	99996696	69699996	96969669	66966996	99696669	69969999	96699666	66696999	99966666
96966999	69696666	99699999	66969666	96696996	69966669	99969996	66699669	96996969	69666696	99669969	66999696	96666966	69996699	99999966	66669699
99699666	66969999	96966666	69696999	99969669	66699996	96696669	69966996	99669696	66999969	96996696	69666969	99999699	66669966	96666699	69996966
66966666	99696999	69699666	96969999	66696669	99966996	69969669	96699996	66996696	99666969	69669696	96999969	66666699	99996966	69999699	96669966
69699999	96969666	66966999	99696666	69969996	96699669	66696996	99966669	69669969	96999696	66996969	99666696	69999966	96669699	66666966	99996699

It is based on the 2 by 2 combinations of 16 numbers

$$\{6666, 6669, 6696, 6699, 6966, 6969, 6996, 6999, 9666, 9669, 9696, 9699, 9966, 9969, 9996, 9999\}.$$

These eleven numbers are ***upside-down*** *combining in pairs as* $\{6666, 9999\}$, $\{6669, 6999\}$, $\{6696, 9699\}$, $\{6699, 9966\}$, $\{6966, 9969\}$, $\{6969, 9696\}$, $\{6996, 9669\}$ *and* $\{9666, 9996\}$. *It is a* ***pandiagonal upside-down magic square****. Its* ***magic sum*** *is*

$$S_{16\times16} := 1333333320 = 10000 \times 133320 + 133320,$$

where

$$133320 = 6666 + 6669 + 6696 + 6699 + 6966 + 6969 + 6996 + 6999 + 9666 + 9669 + 9696 + 9699 + 9966 + 9969 + 9996 + 9999.$$

It is ***block-wise pandigonal magic square*** *of order 66, where each block of order 4 is a* ***pandiagonal magic square*** *with equal* ***magic sums*** *given by*

$$S_{4\times4} := \frac{1333333320}{4} = 333333330.$$

2-Digits Universal and Upside-Down Palindromic Magic and Bimagic Squares: Orders 3 to 16,
Zenodo, April 07, 2020, pp. 1-103, http://doi.org/10.5281/zenodo.3743362

4.14.2 Palindromic Magic Square

Example 112. *The* ***palindromic pandiagonal magic square*** *of order 16 for the digits 6 and 9 is given by*

69969696969996	96699696999669	66696696969666	99669696966999	69699699699696	96969666996969	66966699666966	99696966969699	69999666699996	96669999996669	66666666666666	99996999969999	69669669669696	96999969999969	66996696669966	99669969966699
66696969696666	99966969666999	69969696996996	96699696969669	66966966696966	99696699669699	69699666699696	96969699696969	66669999966666	99996666669999	69999999999996	96669666696669	66996996996966	99666696666699	69669996996696	96999696699969
99969969696999	66699696699666	96696969696669	69966969666996	99699666999699	66969699696966	96966966966969	69696699669696	99999999999999	66669666696666	96666999966669	69996666669996	99669969996699	66999669699966	96996996996969	69666696666696
96696696969669	69966969696996	99969696969999	66699696999666	96966699666969	69696966969696	99699699699699	66969966996966	96666666666669	69996999969996	99999666699999	66669999996666	96996696669969	69666996966696	99669669696699	66999969999966
69999669699996	96669969996669	66666669666666	99996969969999	69669666696696	96999999999969	66996666669966	99666999966699	69969699696996	96699666999669	66696699669666	99669666966999	69699696699696	96969969996969	66966696966966	99696969696699
66669969966666	99996696669999	69999969999996	96669669696669	66996999969966	99666666666699	69669999996696	96999666699969	66696966696966	99966699666999	69699666996996	96699699699669	66966969696966	99696696969699	69699696999696	96969696969969
99999969999999	66669669696666	96666969966669	69996669669996	99669999996699	66999666699966	96996999969969	69666666666696	99969966696999	66699699699666	96696966969669	69966699666996	99699696999699	66969696969966	96966969666969	69696696969696
96666669666669	69996996969996	99999669669999	66669969996666	96996666669969	69666999966696	99669666696699	66999999999966	96696699669669	69966966966996	99969699696999	66699666999666	96966696666969	69696969696696	99699696699699	66969969696966
69669699696696	96999666999969	66996699669966	99666966966699	69999696969996	96669969696669	66666969666666	99996969699999	69699669699696	96969969996969	66966669666966	99696969969699	69969666696996	96699999999669	66696666669666	99966999966999
66996966696966	99666699666699	69669666996696	96999699699969	66666969666666	99996696969999	69999696999996	96669696969669	66966996996966	99696669669699	69699969999696	96969669696969	66696999969666	99966666666999	69969999996996	96699666699669
99669966996699	66999699699966	96996966969969	69666699666696	99999696999999	66669696969666	96666969696669	69996696969996	99699969999699	66969669696966	96966996966969	69696669669696	99969999996999	66699666699666	96696999969669	69966666666996
96996699669969	69666966966696	99669699696699	66999666999966	96666969666669	69996969696996	99996969699999	66669969696666	96966696666969	69696996969696	99699669669699	66969996996966	96696666669669	69966999966996	99969666696999	66699999999666
69699666699696	96969999996969	66966666666966	99696999969699	69969669696996	96699969999669	66696669669666	99966969966999	69669696969696	96999969699969	66996696969966	99666969696699	69999699699996	96669966696669	66666699666666	99996966696999
66966999966966	99696666669699	69699999999696	96969666696969	66696996969666	99966696666999	69969969996996	96699696699669	66996969696966	99666969666699	69669969996696	96999696969969	66666966966666	99996699669999	69999666699996	96669699696669
99699999999699	66969666696966	96966999966969	69696666669696	99969969996999	66699669669666	96696996969669	69966696666996	99669696996699	66999696969966	96996969696969	69666696966696	99999966699999	66669699696666	96666966966669	69996699669996
96966666666969	69696999969696	99699666699699	66969999996966	96696696669669	69966969966996	99969669696999	66699969999666	96996696669969	69666969666696	99669696969699	66999696999966	96666699666669	69996966696996	99999699699999	66669666996666

It is a ***block-wise palindromic pandigonal magic square*** *of order 16, where each block of order 4 is a* ***palindromic pandiagonal magic square*** *with equal* ***magic sums****. The magic sums are given by*

$$S_{16\times 16} := 133333333333333320 \qquad \text{and} \qquad S_{4\times 4} := \frac{133333333333333320}{4} = 33333333333333330.$$

4.14.3 Bimagic Square

Example 113. *The* ***bimagic square*** *of order 16 for the digits 6 and 9 is given by*

99999999	69966996	66696669	96669666	99969669	69996666	66666999	96699996	99696966	69669969	66999696	96966699	99666696	69699699	66969966	96996969
66699666	96666669	99996996	69969999	66669996	96696999	99966666	69999669	66996699	96969696	99699969	69666966	66966969	96999966	99669699	69696696
96666996	66699999	69969666	99996669	96696666	66669669	69999996	99966999	96969969	66996966	69666699	99699696	96999699	66966696	69696969	99669966
69966669	99999666	96669999	66696996	69996999	99969996	96699669	66666666	69669696	99696699	96966966	66999969	69699966	99666969	96996696	66969699
99696696	69669699	66999966	96966969	99666966	69699969	66969696	96996699	99999669	69966666	66696999	96669996	99969999	69996996	66666669	96699666
66996969	96969966	99699699	69666696	66966699	96999696	99669969	69696966	66699996	96666999	99996666	69969669	66669666	96696669	99966996	69999999
96969699	66996696	69666969	99699966	96999969	66966966	69696699	99669696	96666666	66699669	69969996	99996999	96696996	66669999	69999666	99966669
69669966	99696969	96966696	66999699	69699696	99666699	96996966	66969969	69966999	99999996	96669669	66696666	69996669	99969666	96699999	66666996
99669669	69696666	66966999	96999996	99699999	69666996	66996669	96969666	99966696	69999699	66669966	96696969	99996966	69969969	66699696	96666699
66969996	96996999	99666666	69699669	66999666	96966669	99696996	69669999	66666969	96699966	99969699	69996696	66696699	96669696	99999969	69966966
96996666	66969669	69699996	99666999	96966996	66999999	69669666	99696669	96699699	66666696	69996969	99969966	96669969	66696966	69966699	99999696
69696999	99669996	96999669	66966666	69666669	99699666	96969999	66996996	69999966	99966969	96696696	66669699	69969696	99996699	96666966	66699969
99966966	69999969	66669696	96696699	99996696	69969699	66699966	96666969	99669999	69696996	66966669	96999666	99699669	69666666	66996999	96969996
66666699	96699696	99969969	69996966	66696969	96669966	99999699	69966696	66969666	96996669	99666996	69699999	66999996	96966999	99696666	69669669
96699969	66666966	69996699	99969696	96669699	66696696	69966969	99999966	96996996	66969999	69699666	99666669	96966666	66999669	69669996	99696999
69999696	99966699	96696966	66669969	69969966	99996969	96666696	66699699	69696669	99669666	96999999	66966996	69666999	99699996	96969669	66996666

It is constructed with same 16 numbers used in previous example. According to previous example, it is ***universal bimagic square*** *with magic and bimagic sums given by*

$$S_{16\times 16} := 1333333320 \quad \textit{and} \quad Sb_{16\times 16} := 114747472525252536.$$

Moreover, each block of order 4 is a magic square with ***magic sum*** $S_{4\times 4} := 333333330.$

4.14.4 Palindromic Magic Square

Example 114. *The* ***palindromic bimagic square*** *of order 16 for the digits 6 and 9 is given by*

66666666666666	96699696699669	99969996996999	69996999969996	66696996969666	96669999996669	99999666699999	69966696666996	66969699696966	96996696669969	99669696966699	69699666999696	66999696999966	96966966966969	99696699669699	69669696696696
99966999966999	69999969999996	66696696696666	96696666669669	99996696669999	69969666696996	66699999999666	96666996966669	99669966996699	69696969669696	66966696666966	96999699699969	99699696699699	69666699666696	66996966669966	96969969696969
69999696699996	99966666666999	96696999969669	66699969996666	69969999996996	99996969969999	96666696666669	66699666699666	69696696669696	99669699696699	96999666999969	66966969696966	69666966696696	99699696999699	96969696696969	66996699669966
96699969999669	66666999966666	69996666669996	99969669696999	96669666696669	66696669669666	69966969966996	99999999999999	96996969696969	66969966996966	69699699699696	99666969666699	96966699666969	66999696699966	69669969696696	99696966969699
66969969696966	96996966696969	99666699666699	69699696969696	66999699699966	96966696966969	99696969696699	69669666996696	66669969966666	96699999999669	99969666696999	69996696669996	66696666669666	96669669696669	99999969999999	69966999966996
99669696969699	69696699669696	66966966696966	96999696999969	99699666999699	69666969696696	66996696669966	96969699696969	99966696666999	69999666699996	66669999996666	96696996969669	99996999969999	69699969996996	66699669699666	96666666666669
69696966969696	99669696996699	96999696699969	66966699666966	69666969666696	99699699699699	96969666996969	66996969696966	69999999999996	99669969966999	96696669669669	66669666696666	69969669696996	99966666669999	96669999966669	66699969999666
96996699669969	66969696969666	69699696999696	99669666966699	96966969696969	66999666999966	69669699696696	99696696669699	96699666699669	66666696666666	69996969969996	99969999996999	96669969996669	66696999969666	69966666666996	99999696699999
66996969969966	96969999996969	99699666699699	69666696666696	66966666666966	96999696699969	99669969996699	69696999969696	66699696999666	96666966966669	99996699669999	69969696969996	66669699696666	96696696669669	99669696966999	69999666999996
99696696669699	69669666696696	66999999999966	96966969966969	99666999966699	69699969999696	66969669696966	96996666669969	99996969699999	69966699666996	66696966696666	96669969696669	99969666996999	69996969969996	66666969666666	96696999699669
69669999996696	99696996969699	96966696666969	66999666699966	69699696699696	99666666666699	96996999969969	66969996996966	69966966966996	99999696999999	96669696696669	66966999669666	69996696669996	99969699696999	96699666999669	66669696966666
96969666696969	66996696669966	69666969966696	99699999999699	96999969999969	66966999966966	69696666669696	99669696696699	96666699666669	66699696969666	69969969696996	99969666969999	96696969696669	66699666996666	69999699699996	99966969666999
66699699699666	96666969666669	99996969699999	69699666996996	66669696996666	96696966969669	99666999666999	69999696699996	66996666669966	96969669696969	99699969999699	69666999966696	66966969966966	96999999999969	99669666696699	69696669669696
99999666999999	69966969696996	66966969669666	96669699696669	99969696969999	69996699669996	66666966966666	96699696999669	99696999969699	69669969996696	66999669699966	96966666666969	99666696666699	69699666699696	66969999996966	96996969969969
69966969666996	99999699699999	96669966696669	66696969696666	69996966969996	99699696996999	96699696699669	66666699666666	69669669696696	99696666669699	96966999966969	66999969999966	69699999999696	99666969966699	96996696669969	66969666696966
96669696966669	66699666999666	69696999696996	99996696969999	96696699669669	66669696966666	69999696999996	99966966966999	96969996996969	66996999969966	69666666666696	99699696699699	96999666699969	66966669666966	69696969969696	99669999996699

*It is a **block-wise palindromic bi magic square** of order 16, where each block of order 4 is a **palindromic magic square** with equal **magic sums**. The magic sums are given by*

$$S_{16\times 16} := 1333333333333320 \qquad \textit{and} \qquad S_{4\times 4} := \frac{1333333333333320}{4} = 333333333333330.$$

The bimagic sum is given by

$$S_{16\times 16} := 114747546754675029253245324525 36.$$

5 Author's Contributions to Magic Squares

The **item-wise** author's work on magic squares is as follows:

i. ***Digital Numbers** Magic Squares - [5, 6, 7, 8, 9, 10, **?**];*

ii. ***Block-Wise Construction** of **Bimagic Squares** - [11];*

iii. *Connections with **Genetic Tables** and **Shannon's entropy** - [12];*

iv. ***Selfie** and **Palindromic-type** Magic Squares - [13, 28, **?**];*

v. ***Intervally Distributed*** *and* ***Block-Wise*** *Magic Squares - [14, 15, 16, 28];*

vi. ***Multi-digits*** *and* ***Number Patterns*** *Magic Squares - [17, 27];*

vii. ***Perfect Square Sum*** *Magic Squares with* ***Uniformity***, ***Minimum Sum*** *and* ***Pythagorean Triples*** *- [18, 19];*

viii. ***Block-Wise*** *Constructions of Magic and Bimagic Squares - [20, 21, 22, 23, 26, 29, 31];*

ix. ***Magic Crosses:*** *Repeated and Non Repeated Entries - [24];*

x. *Representations of* ***Letters*** *and* ***Numbers*** *With Equal Sums Magic Squares of Orders 4 and 6 - [25].*

xi. ***Bordered*** *Magic Squares - [32, 33, 34, 35, 37].*

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