Mathematical and numerical modelling of ice sheets and glaciers

Tutorial held at the Workshop "Mathematical Approach to Climate Change Impacts",
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ILTS

Contents


1. Introduction

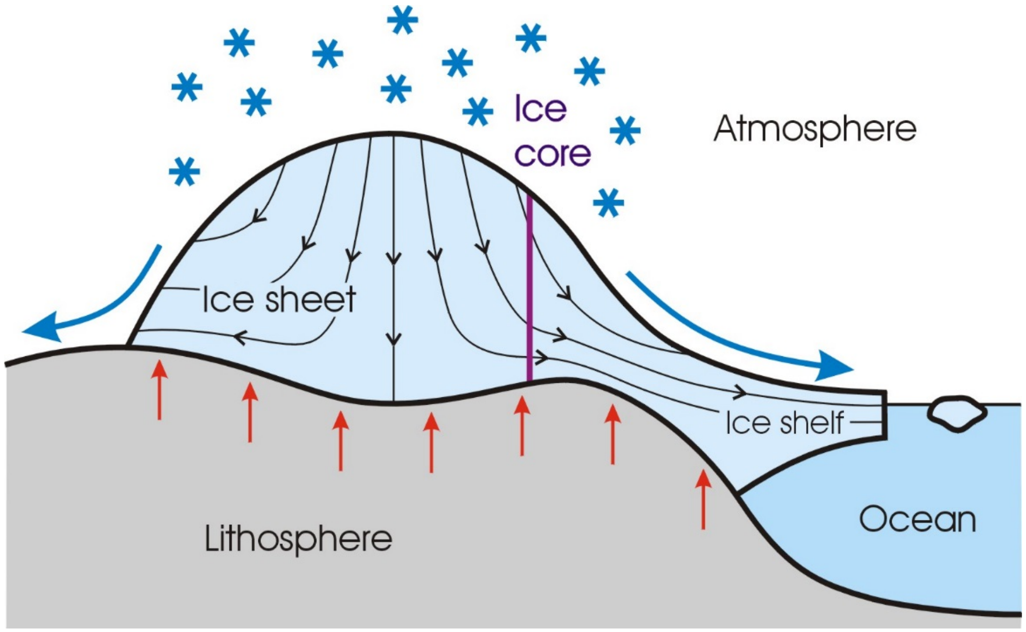
Terminology

Ice sheets

→ grounded ice masses of continental size, area > 50,000 km^2 (Antarctica, Greenland).

Ice shelves

→ floating ice masses, connected to an ice sheet (Antarctica).



Vertical exaggeration factor ~200…500

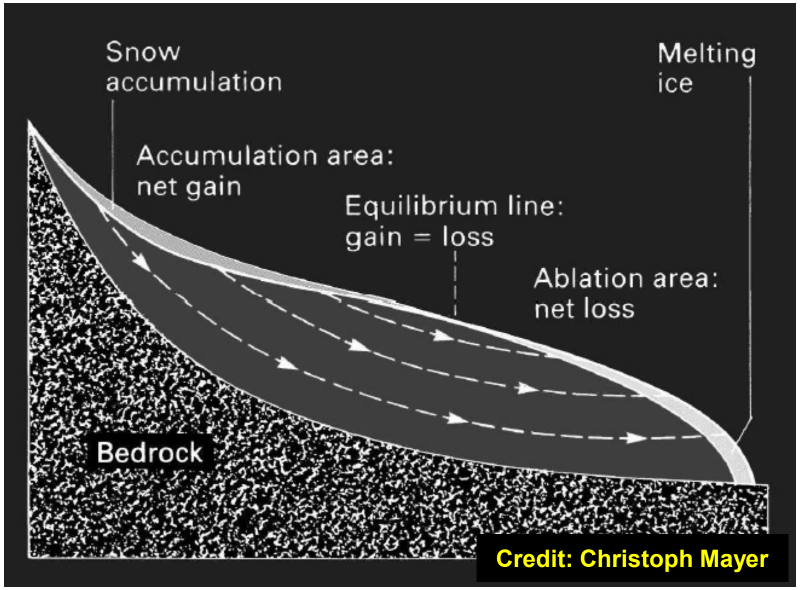
Terminology

Ice caps

→ extended grounded ice masses, area < 50,000 km^2 (Austfonna, Vatnajökull, North/South Patagonian Icefields...).

Glaciers

→ small grounded ice masses in mountainous regions, constrained by topographical features.

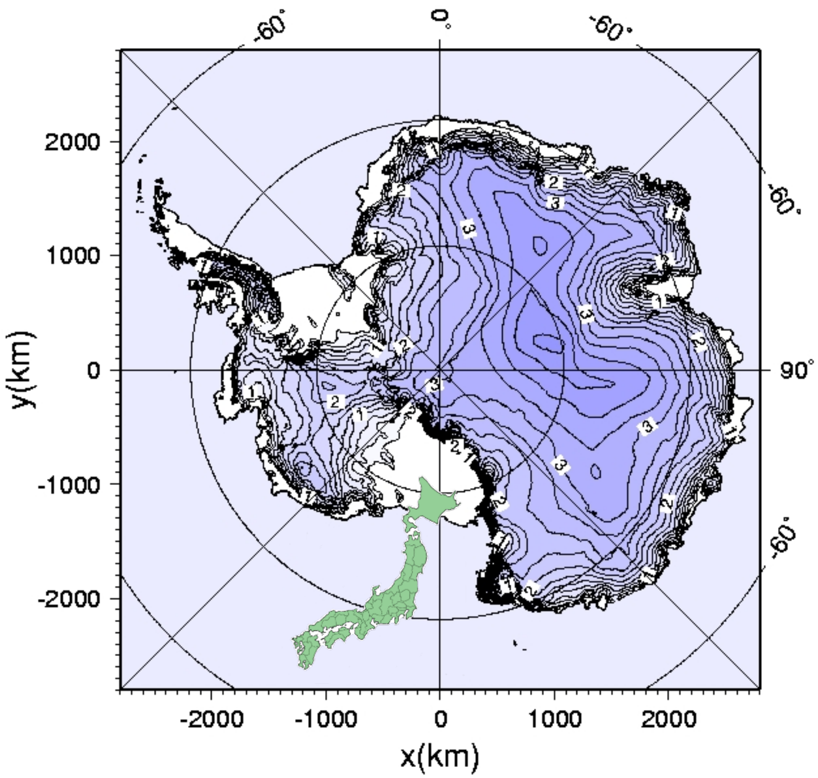


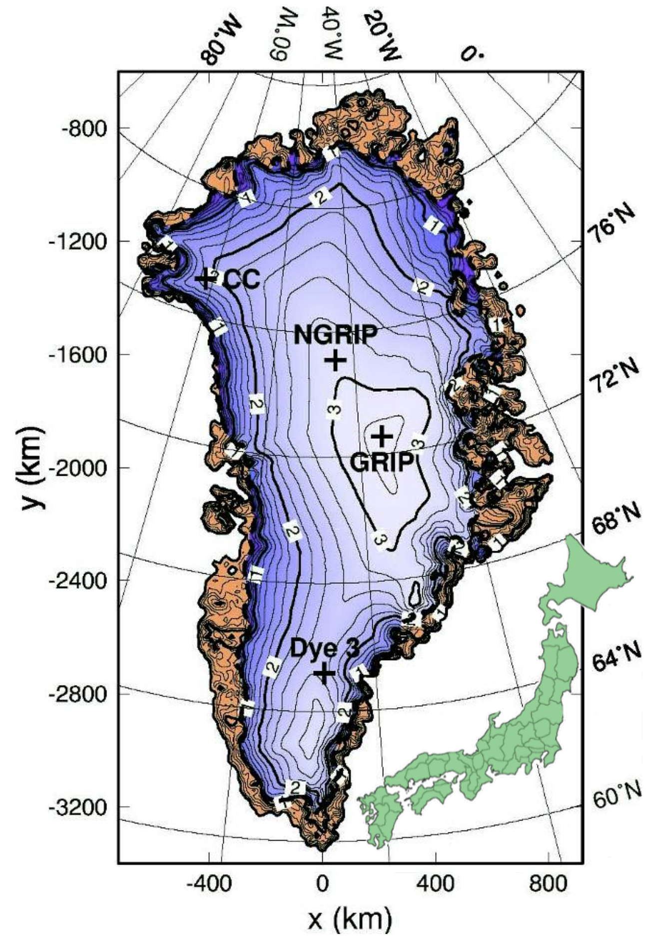
Remark: "Glacier" is sometimes also used as an umbrella term for all grounded ice bodies (ice sheets, ice caps and glaciers as defined above).

Ice sheets

Greenland ice sheet

Antarctic ice sheet
(with ice shelves)





Glaciers and ice caps

Can be found on every continent (polar/sub-polar areas, mountains).

Number: ~ 200,000 (~ 70 ice caps).

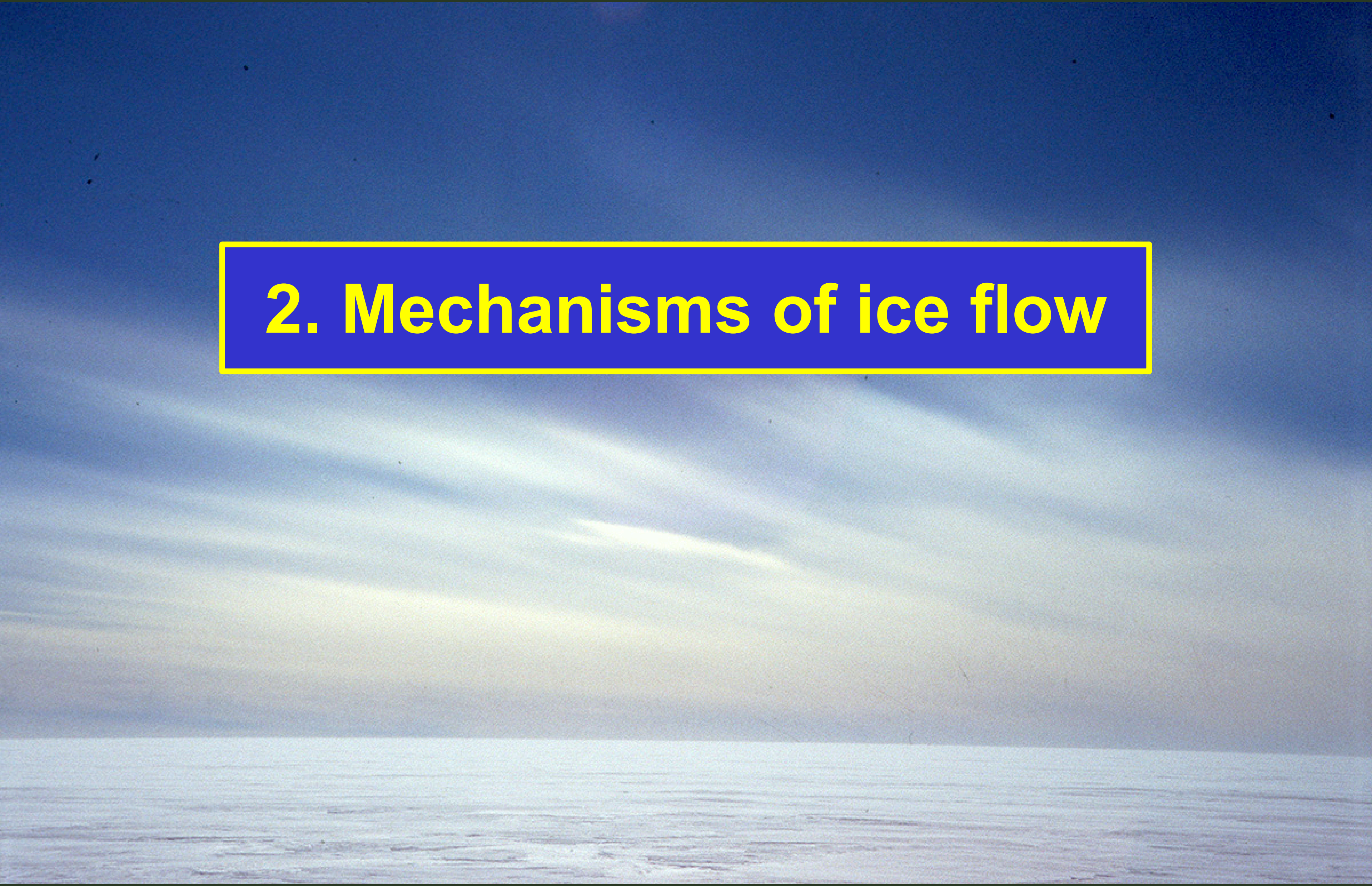
Many different types:
Valley glaciers, cirque glaciers, hanging glaciers, tidewater glaciers, rock glaciers…

Photo credit: www.glaciers-online.net

Inventory

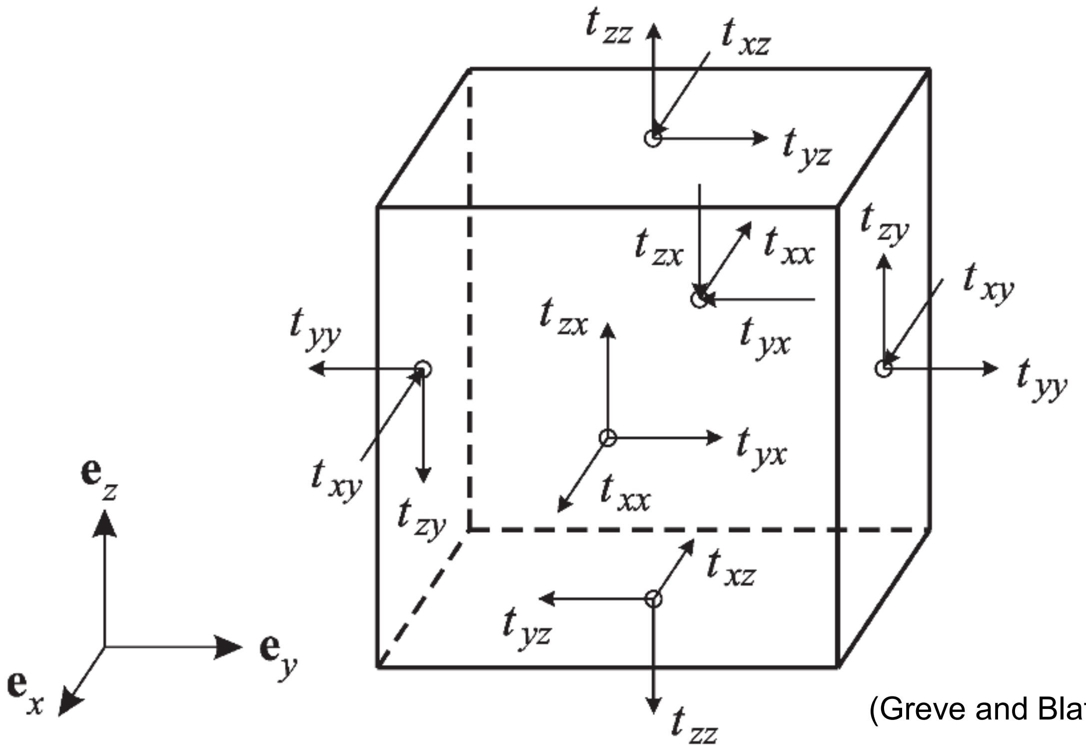
	Glaciers and ice caps	Greenland ice sheet	Antarctic ice sheet
Area (10^6 km^2)	0.73*	1.80	12.3
Volume (metres of sea level equivalent)	0.41*	7.36	58.3
Turnover time (vol/accum, years)	~ 50 – 1000**	~ 5000	~ 12000

Main source: Vaughan et al. (2013) [IPCC AR5 Ch. 4].
(*) Sum for all glaciers and ice caps. (**) Range of values for individual glaciers and ice caps.


2. Mechanisms of ice flow

Stress and strain

Cauchy stress tensor T



(Greve and Blatter 2009)

Normal stresses (t_{ii}) and shear stresses (t_{ij})
acting on the surface of a cube aligned with x, y, z.

Stress and strain

Stress deviator T^D

For incompressible materials like glacier ice:

$$\mathrm{T} = -p\,\mathrm{I} + \mathrm{T}^\mathrm{D} \qquad [\; t_{ij} = -p\,\delta_{ij} + t^\mathrm{D}_{ij} \;]$$

Pressure p: free field.

Traceless stress deviator T^D:
to be described by a material equation (flow law).

Conservation of angular momentum
→ both T and T^D are symmetric.

Stress and strain

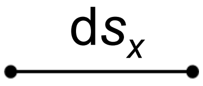
Strain-rate (stretching) tensor D

Symmetric part of the velocity gradient:

$$\mathrm{D} = \mathrm{sym\ grad}\ \mathbf{v} \qquad [\ D_{ij} = \tfrac{1}{2}\,(\ v_{i,j} + v_{j,i}\ )\ ]$$

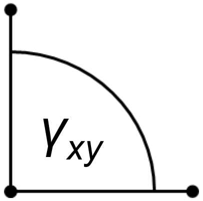
Diagonal elements D_{ii}: dilatation rates, e.g.

$$D_{xx} = (\mathrm{d}s_x)^{\bullet} / \mathrm{d}s_x$$



Off-diagonal elements D_{ij}: ½ × shear rates, e.g.

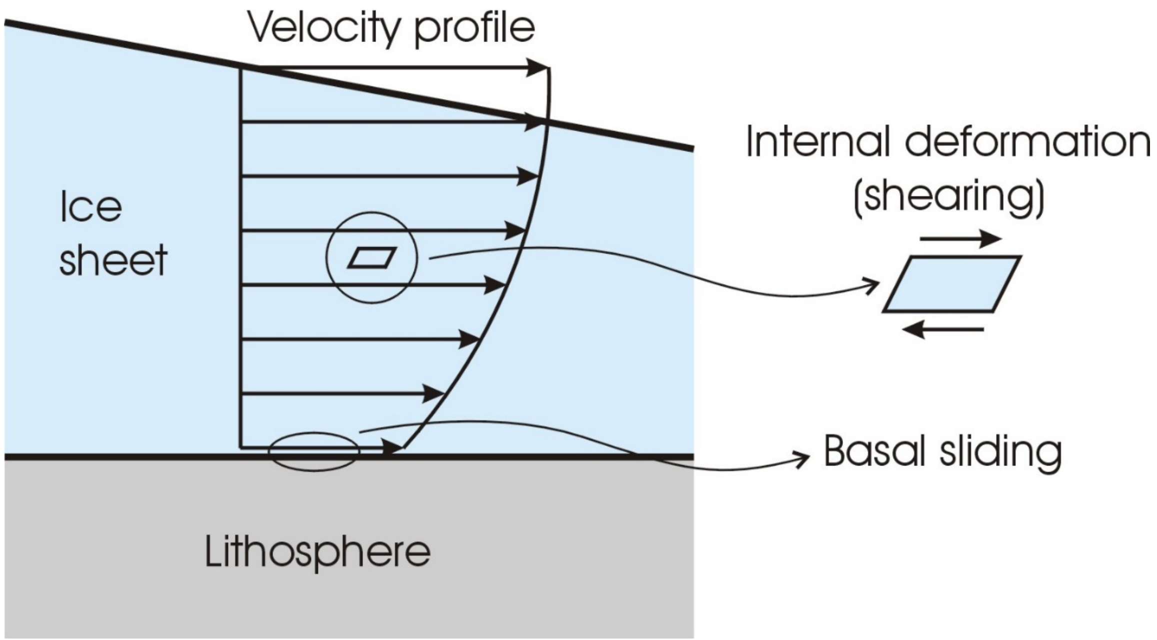
$$D_{xy} = (\gamma_{xy})^{\bullet} / 2$$



Why does ice flow?

Two mechanisms

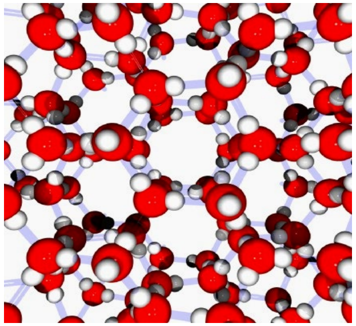
- Internal deformation (ice = viscous fluid).
- Basal sliding.



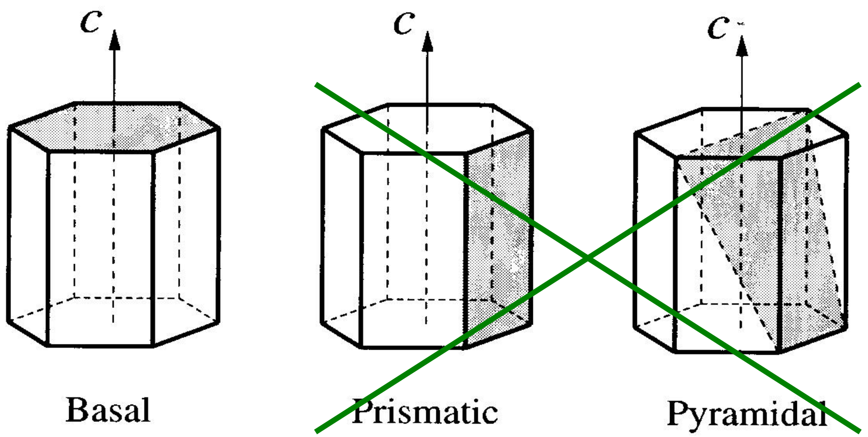
Internal deformation

Ice Ih: hexagonal crystal structure.

Loose-packed lattice, packing factor only 34% (close packing of spheres 74%).



Deformation along crystallographic planes (mainly basal, to a much lesser extent prismatic and pyramidal)
→ strong anisotropy.



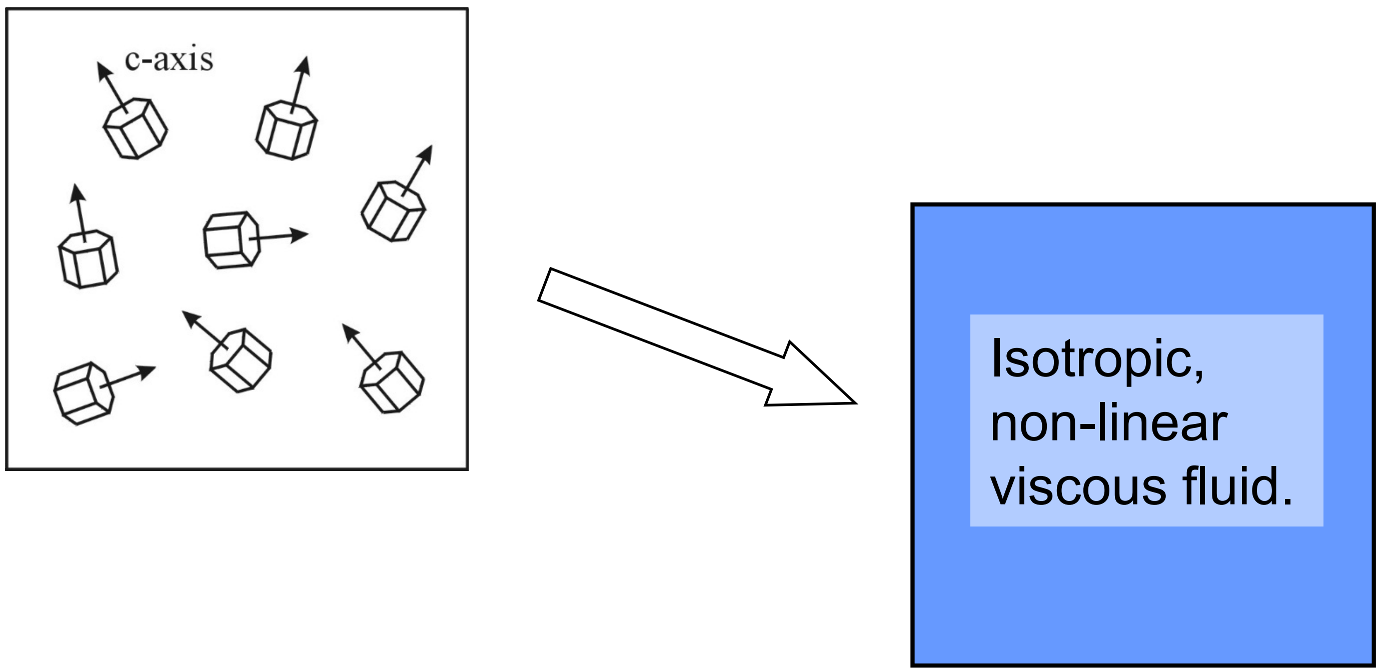
(Faria et al. 2014)

Deck-of-cards model

Internal deformation

Macroscopic description:

Polycrystalline ice $\rightarrow$ control volume contains an ensemble of randomly oriented ice crystallites (a.k.a. grains).



Isotropic, non-linear viscous fluid: Glen's flow law

$$\mathsf{D} = \underbrace{E A(T') f(\sigma)}_{= 2 \times \text{fluidity} = 1/(2 \times \text{viscosity})} \mathsf{T}^{\mathrm{D}}$$

Fluidity factors:

- Creep function: Power law $f(\sigma) = \sigma^{n-1}$, stress exponent $n = 3$.
- Rate factor: Arrhenius law $A(T') = A_0\, e^{-Q/RT'}$.
- Enhancement factor E (equal to 1 for pure isotropic ice).

Basal sliding

Two different processes:
sliding on hard rock vs. sliding on deformable sediment.

Difficult to measure, not well understood!

Often "Weertman-type" parameterization is used:

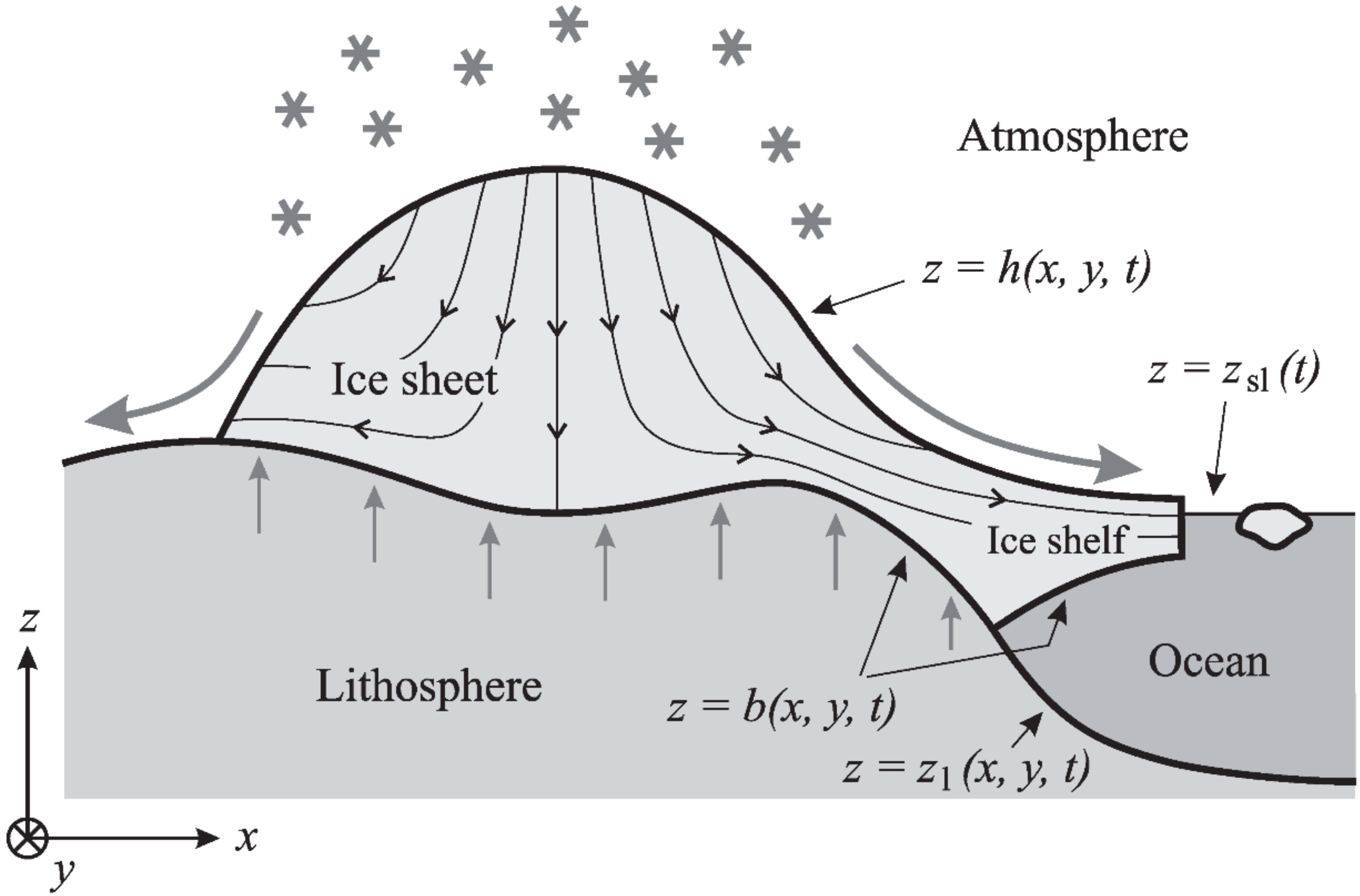
$$v_\mathrm{b} \propto \frac{\tau_\mathrm{b}^p}{P_\mathrm{b}^q}$$

v_b – basal sliding velocity
τ_b – basal shear stress
P_b – basal pressure

$$(p,q) = \begin{cases} (3,0),\ (3,1) \text{ or } (3,2) & \text{for hard rock sliding} \\ (1,0) & \text{for sediment sliding} \end{cases}$$


3. Dynamics

Geometry

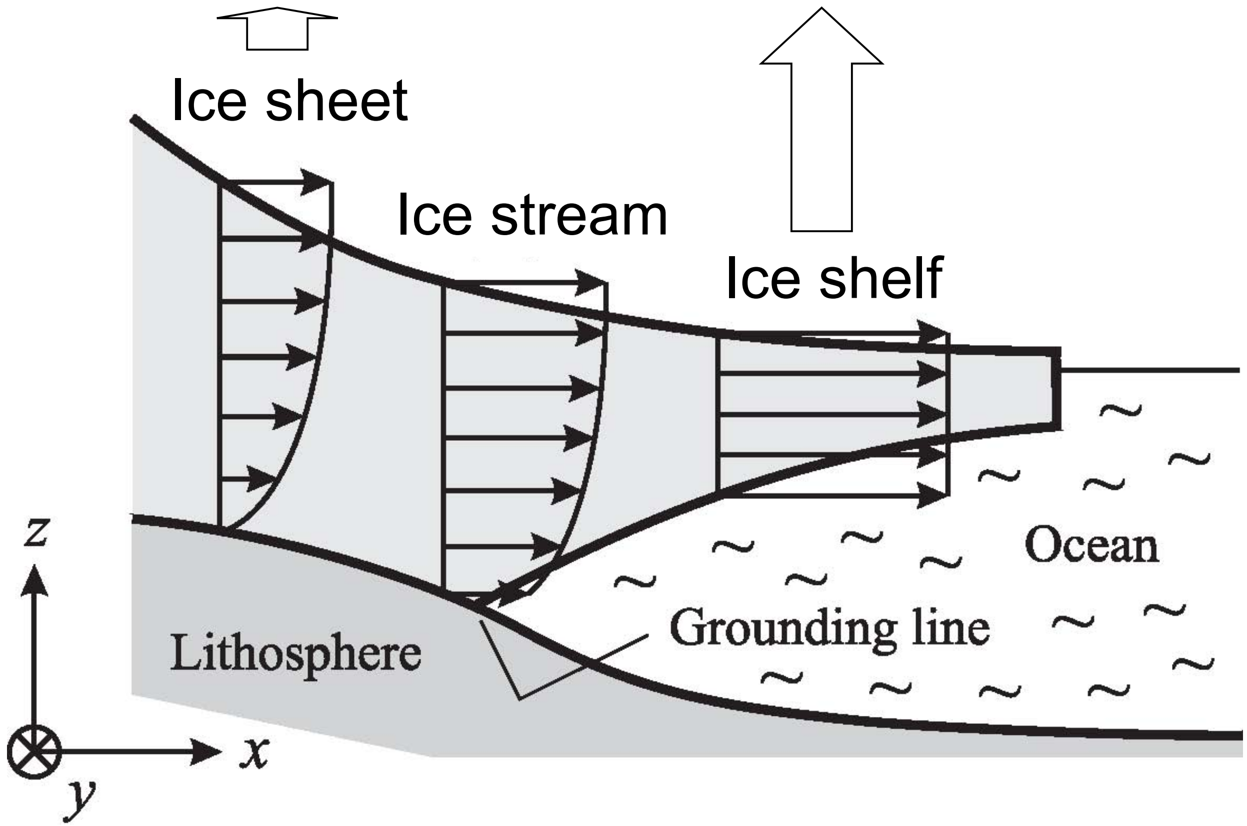


Atmosphere
$z = h(x, y, t)$
Ice sheet
$z = z_{sl}(t)$
Ice shelf
Ocean
Lithosphere
$z = b(x, y, t)$
$z = z_l(x, y, t)$
z
x
y

Grounded vs. floating ice

Shear flow, dominated by p, t_{xz}, t_{yz}

Plug flow, dominated by p, t^{D}_{xx}, t^{D}_{yy}, t^{D}_{zz}, t_{xy}



Full Stokes (FS) flow problem

3-d momentum balance on a flat Earth

→ *Fr* = [*U*]^2 / (*g*[*H*]) ~ 10^{-15}

→ *Fr/Ro* = 2Ω[*U*][*L*] / (*g*[*H*]) ~ 5 x 10^{-8} >>> FS

$$\frac{\partial t_{xx}}{\partial x} + \frac{\partial t_{xy}}{\partial y} + \frac{\partial t_{xz}}{\partial z} \quad \cancel{- 2\rho(\Omega_y v_z - \Omega_z v_y)} \quad = \quad \rho \cancel{\frac{\mathrm{d}v_x}{\mathrm{d}t}}\,,$$

$$\frac{\partial t_{xy}}{\partial x} + \frac{\partial t_{yy}}{\partial y} + \frac{\partial t_{yz}}{\partial z} \quad \cancel{- 2\rho(\Omega_z v_x - \Omega_x v_z)} \quad = \quad \rho \cancel{\frac{\mathrm{d}v_y}{\mathrm{d}t}}\,,$$

$$\frac{\partial t_{xz}}{\partial x} + \frac{\partial t_{yz}}{\partial y} + \frac{\partial t_{zz}}{\partial z} - \rho g \cancel{- 2\rho(\Omega_x v_y - \Omega_y v_x)} \quad = \quad \rho \cancel{\frac{\mathrm{d}v_z}{\mathrm{d}t}}\,.$$

Grounded ice: Hydrostatic and shallow ice approximations

Full Stokes → Hydrostatic approximation → SIA

$$
\begin{aligned}
\frac{\partial \overset{(-p)}{\cancel{t_{xx}}}}{\partial x} + \cancel{\frac{\partial t_{xy}}{\partial y}} + \frac{\partial t_{xz}}{\partial z} &= 0\,, \\
\cancel{\frac{\partial t_{xy}}{\partial x}} + \frac{\partial \overset{(-p)}{\cancel{t_{yy}}}}{\partial y} + \frac{\partial t_{yz}}{\partial z} &= 0\,, \\
\cancel{\frac{\partial t_{xz}}{\partial x}} + \cancel{\frac{\partial t_{yz}}{\partial y}} + \frac{\partial \overset{(-p)}{\cancel{t_{zz}}}}{\partial z} &= \rho g\,.
\end{aligned}
$$

SIA force balance

Hydrostatic pressure:

$$p = \rho g (h - z)$$

Vertical shear stresses:

$$
\begin{aligned}
t_{xz} &= -\rho g (h - z) \frac{\partial h}{\partial x} \\
t_{yz} &= -\rho g (h - z) \frac{\partial h}{\partial y}
\end{aligned}
$$

At the ice base ($z = b$):

$$\underbrace{\qquad}_{-\boldsymbol{\tau}_{\mathrm{drag}}} \qquad\qquad \underbrace{\qquad\qquad\qquad\qquad}_{\boldsymbol{\tau}_{\mathrm{driving}}}$$

$$\Rightarrow \quad \boldsymbol{\tau}_{\mathrm{driving}} + \boldsymbol{\tau}_{\mathrm{drag}} = \mathbf{0}$$

Floating ice: Hydrostatic and shallow shelf approximations

Full Stokes → Hydrostatic approximation → SSA

$$
\begin{aligned}
\frac{\partial t_{xx}}{\partial x} + \frac{\partial t_{xy}}{\partial y} + \cancel{\frac{\partial t_{xz}}{\partial z}} &= 0\,,\\
\frac{\partial t_{xy}}{\partial x} + \frac{\partial t_{yy}}{\partial y} + \cancel{\frac{\partial t_{yz}}{\partial z}} &= 0\,,\\
\cancel{\frac{\partial t_{xz}}{\partial x}} + \cancel{\frac{\partial t_{yz}}{\partial y}} + \frac{\partial t_{zz}}{\partial z} &= \rho g\,.
\end{aligned}
$$

SSA force balance

Hydrostatic vertical normal stress:

$$t_{zz} = -\rho g(h - z)$$

Vertically integrated horizontal force balance:

$$\underbrace{\begin{aligned} 2\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{yy}}{\partial x} + \frac{\partial N_{xy}}{\partial y} \\ 2\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xx}}{\partial y} + \frac{\partial N_{xy}}{\partial x} \end{aligned}}_{\boldsymbol{\tau}_{\text{membrane}}} \begin{aligned} &= \\ &= \end{aligned} \underbrace{\begin{aligned} \rho g H \frac{\partial h}{\partial x} \\ \rho g H \frac{\partial h}{\partial y} \end{aligned}}_{-\boldsymbol{\tau}_{\text{driving}}}$$

$$\Rightarrow \quad \boldsymbol{\tau}_{\text{driving}} + \boldsymbol{\tau}_{\text{membrane}} = \mathbf{0}$$

Shelfy stream approximation (SStA): SSA with additional basal drag

$$\boldsymbol{\tau}_{\text{driving}} + \boldsymbol{\tau}_{\text{membrane}} \quad = \mathbf{0}$$

$$+\boldsymbol{\tau}_{\text{drag}}$$

$$\Rightarrow \quad \boldsymbol{\tau}_{\text{driving}} + \boldsymbol{\tau}_{\text{membrane}} + \boldsymbol{\tau}_{\text{drag}} = \mathbf{0}$$

Plug flow as for ice shelves (v_x, v_y independent of z), but with some resistance due to the basal drag!

Computation of the velocity field

Choose appropriate force balance,

insert Glen's flow law

+ mass balance div $\boldsymbol{v} = 0$

$\rightarrow$ system of PDEs for the 3-d velocity field…

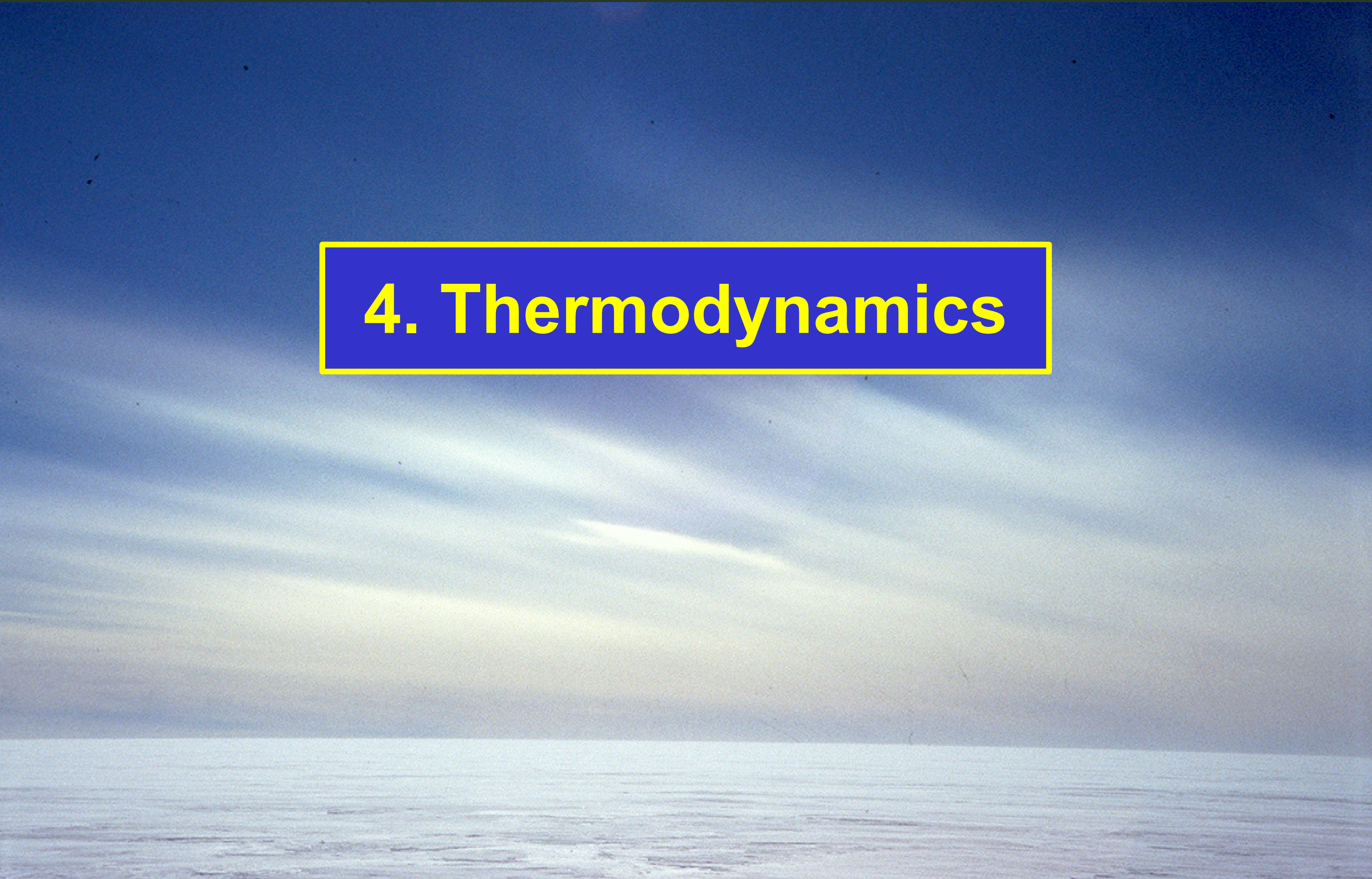
(SIA: easier, just integrals over depth)

No evolution equation!

Boundary conditions: Stress-free condition at the surface,
basal sliding parameterization,
pressure (air/ocean) at the sides.

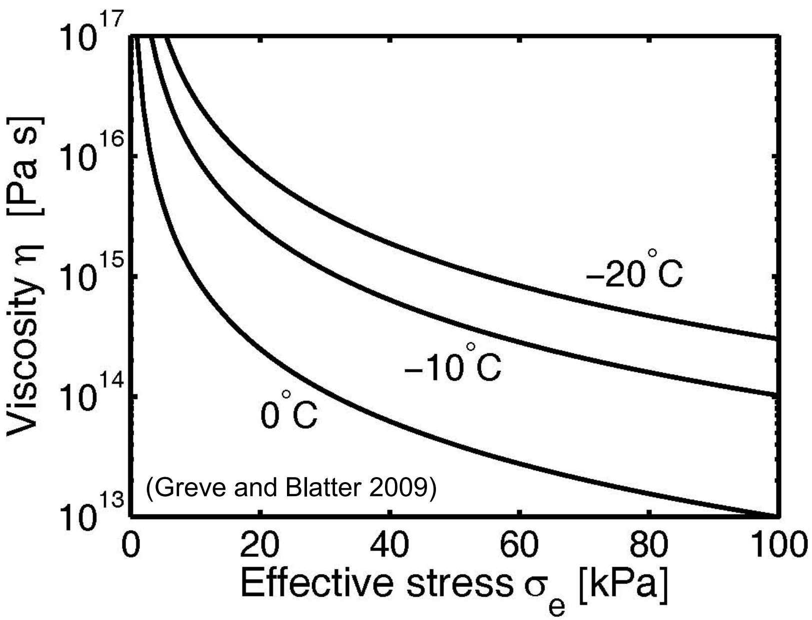
For the full sets of equations, see
Greve, R. and H. Blatter, 2009, *Dynamics of Ice Sheets and Glaciers*, Springer.

4. Thermodynamics

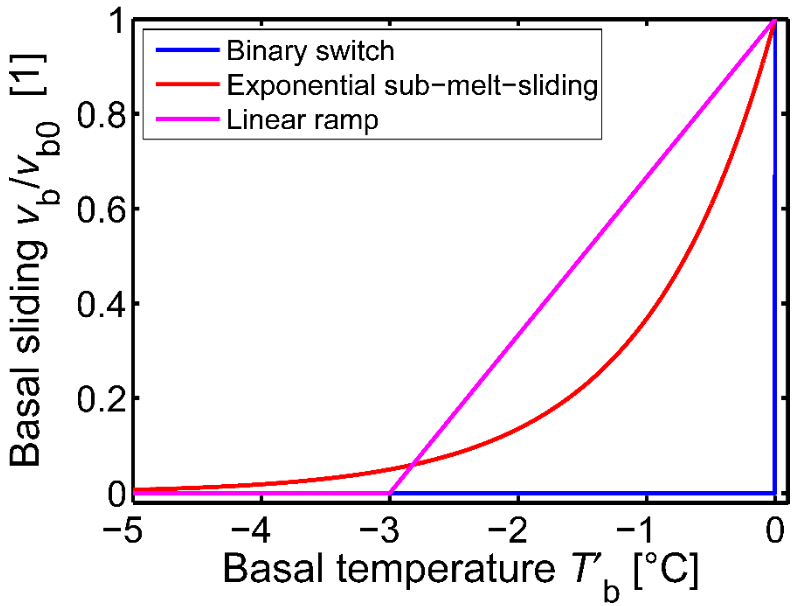


Ice flow depends strongly on temperature

Viscosity of polycrystalline ice



Basal sliding



Thermodynamic material equations

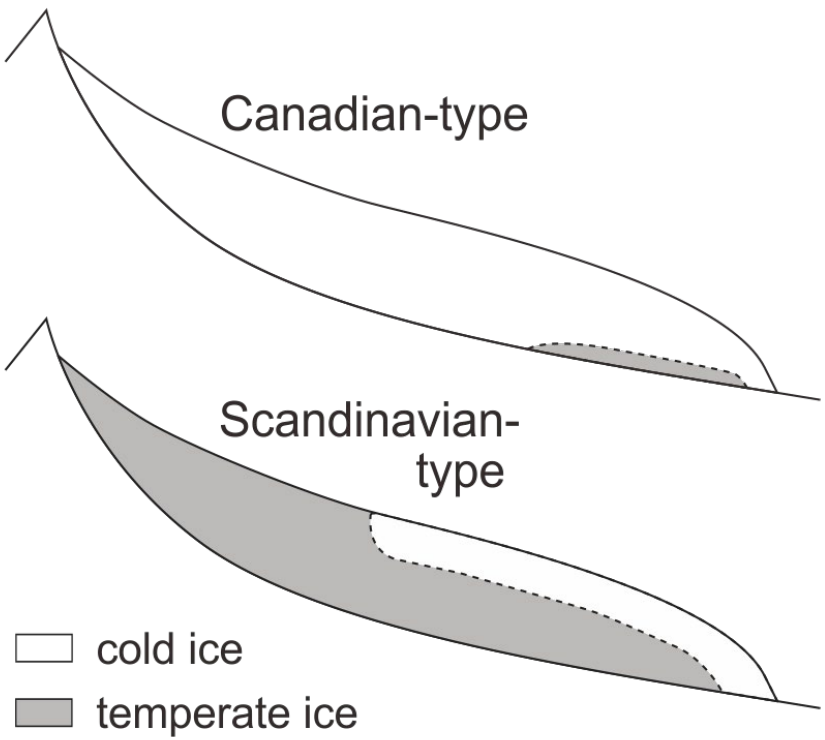
Fourier's law of heat conduction:

$$\mathbf{q} = -\kappa(T)\,\mathrm{grad}\,T$$

Caloric equation of state:

$$u(T) = \int_{T_0}^{T} c(T')\,\mathrm{d}T'$$

Coexistence of cold and temperate ice ("polythermal")



(Aschwanden et al. 2012)

Temperate ice can contain small amounts of water
→ reduces ice viscosity.

Cold-temperate transition surface CTS
→ (1) melting conditions
(2) freezing conditions

Cold-ice method

Temperature equation:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{\rho c}\,\mathrm{div}\,(\kappa\,\mathrm{grad}\,T) + \frac{\Phi}{\rho c}$$

Secondary condition:

$$T \le T_\mathrm{m} \quad (\,\text{where } T_\mathrm{m} = T_0 - \beta\,p\,)$$

Water content:

$$W = 0$$

Polythermal method

Temperature equation as before, but only solved in cold ice.

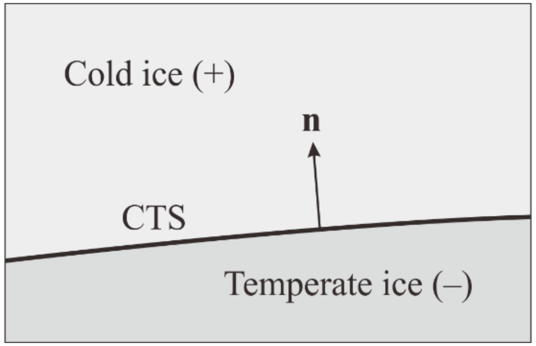
Water-content equation in temperate ice:

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{1}{\rho}\,\mathrm{div}\,(\nu\,\mathrm{grad}\,W) + \frac{\Phi}{\rho L}$$

Transition conditions at the CTS:

(1) melting conditions ($a_\mathrm{m}^\perp > 0$)

$$\frac{\partial T^+}{\partial \mathbf{n}} = \frac{\partial T^-}{\partial \mathbf{n}}\,, \qquad W^+ = W^- = 0$$



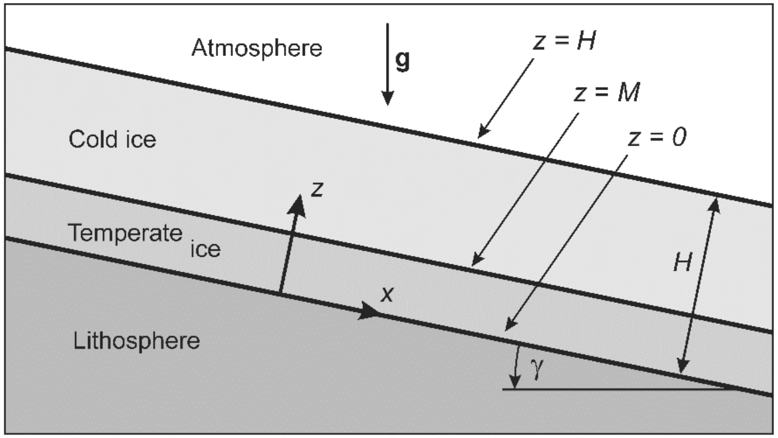
(2) freezing conditions ($a_\mathrm{m}^\perp < 0$)

$$\kappa\left(\frac{\partial T^+}{\partial \mathbf{n}} - \frac{\partial T^-}{\partial \mathbf{n}}\right) = L\rho W^- a_\mathrm{m}^\perp\,, \qquad W^+ = 0$$

Polythermal method

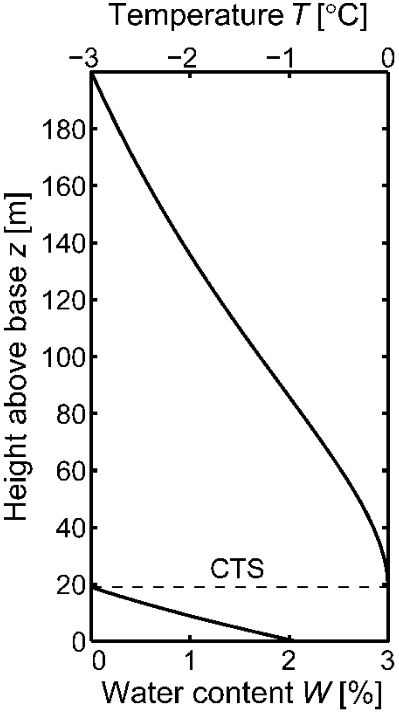
Analytical solution for the parallel-sided slab

Geometry

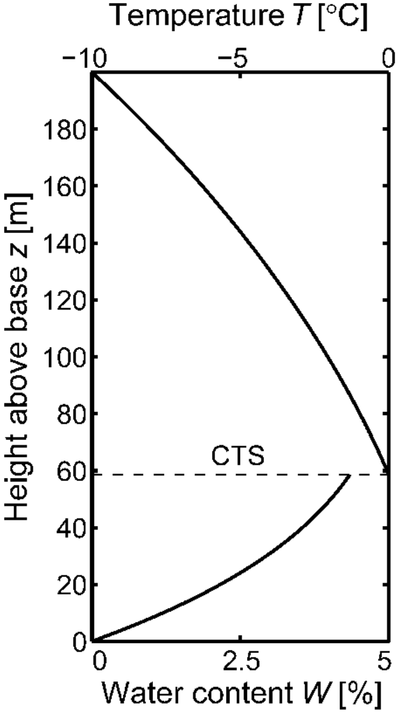


$$H = 200\,\mathrm{m}$$
$$\gamma = 4^\circ$$
$$\kappa = \mathrm{const}$$
$$c = \mathrm{const}$$
$$\beta = 0$$

Melting conditions, $a_m^{\perp} = +0.2$ m/a



Freezing conditions, $a_m^{\perp} = -0.2$ m/a



Enthalpy method

One common thermodynamic field

Enthalpy h = fct(temperature T, water content W)

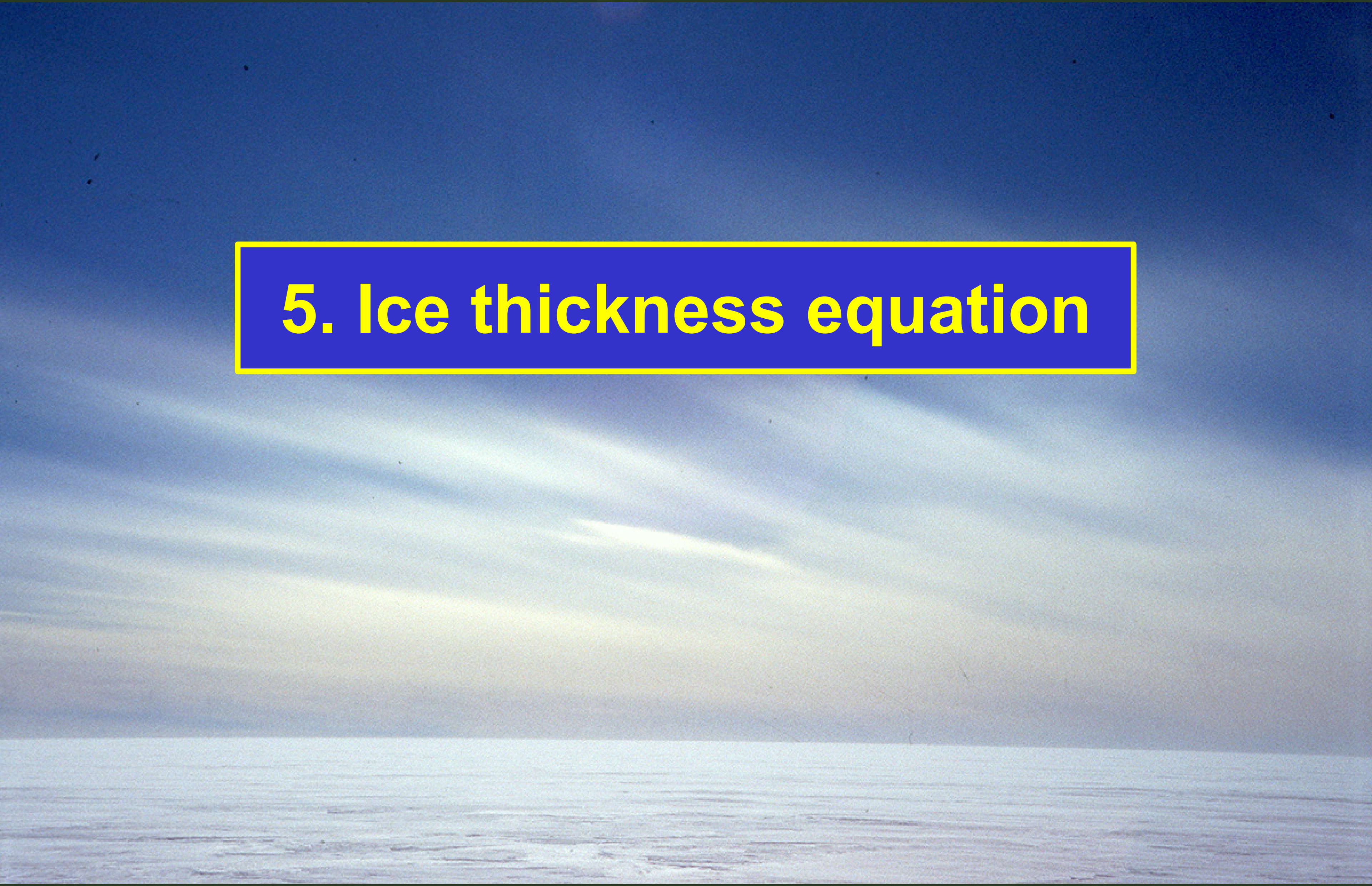
for cold and temperate ice: (Aschwanden et al. 2012)

$$h(T,W) = \int_{T_0}^{T} c(T')\,\mathrm{d}T' + LW$$

Enthalpy equation for cold and temperate ice:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \mathrm{div}\,(k\,\mathrm{grad}\,h) + \frac{\Phi}{\rho}$$

$$\text{with } k = \begin{cases} \dfrac{\kappa}{\rho c} & \text{for cold ice} \\[2ex] \dfrac{\nu}{\rho} & \text{for temperate ice} \end{cases}$$


5. Ice thickness equation

Ice thickness equation

Geometry, processes:

H : ice thickness

a_s : surface mass balance

a_b : basal melting rate

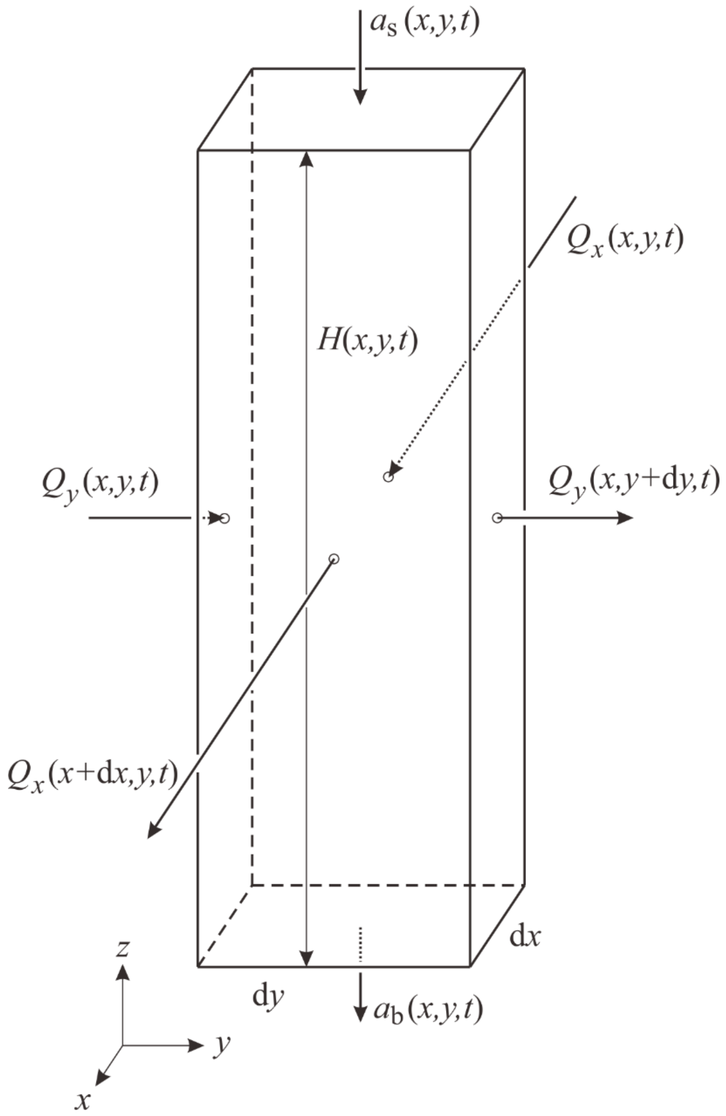
$$\mathbf{Q} = \int_b^h \mathbf{v}_\mathrm{h}\,\mathrm{d}z \quad : \text{ volume flux}$$

$\mathbf{v}_\mathrm{h}$: horizontal velocity

h : ice surface

b : ice base

$$H(x,y,t) = h(x,y,t) - b(x,y,t)$$



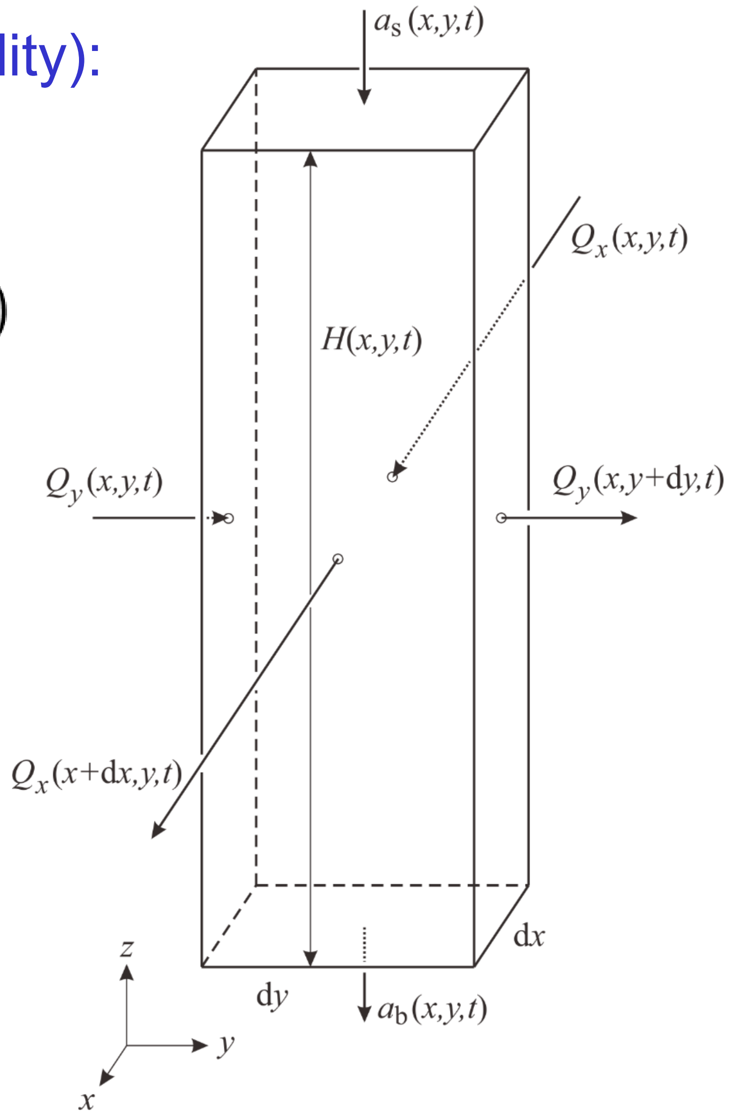
Ice thickness equation

Volume balance (due to incompressibility):

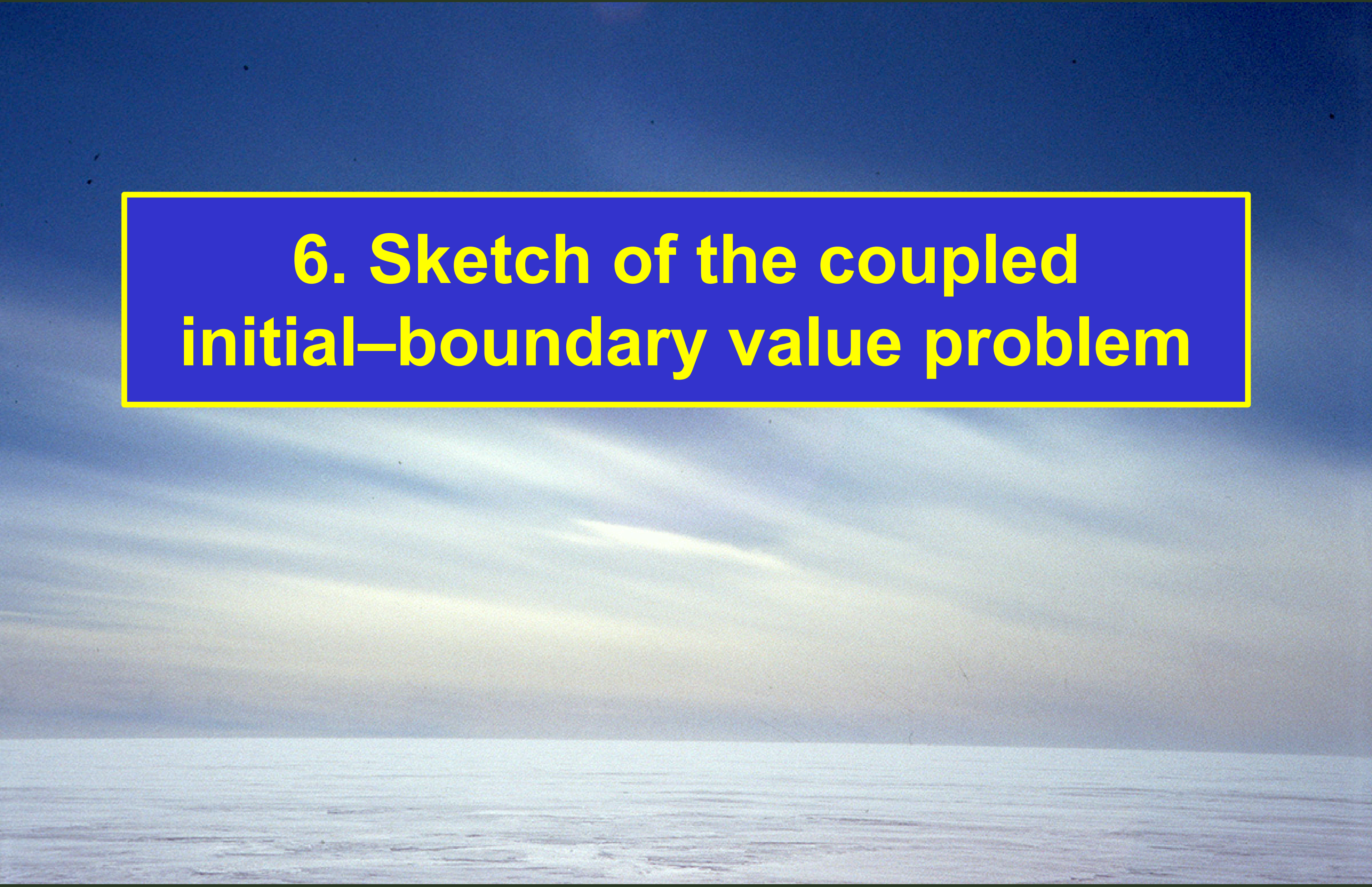
$$\frac{\mathrm{d}}{\mathrm{d}t}(\text{Volume of ice column}) = -\text{Volume fluxes (outflow positive)} + \text{Volume supplies}$$

$$\text{Volume of ice column} = H\,\mathrm{d}x\,\mathrm{d}y$$

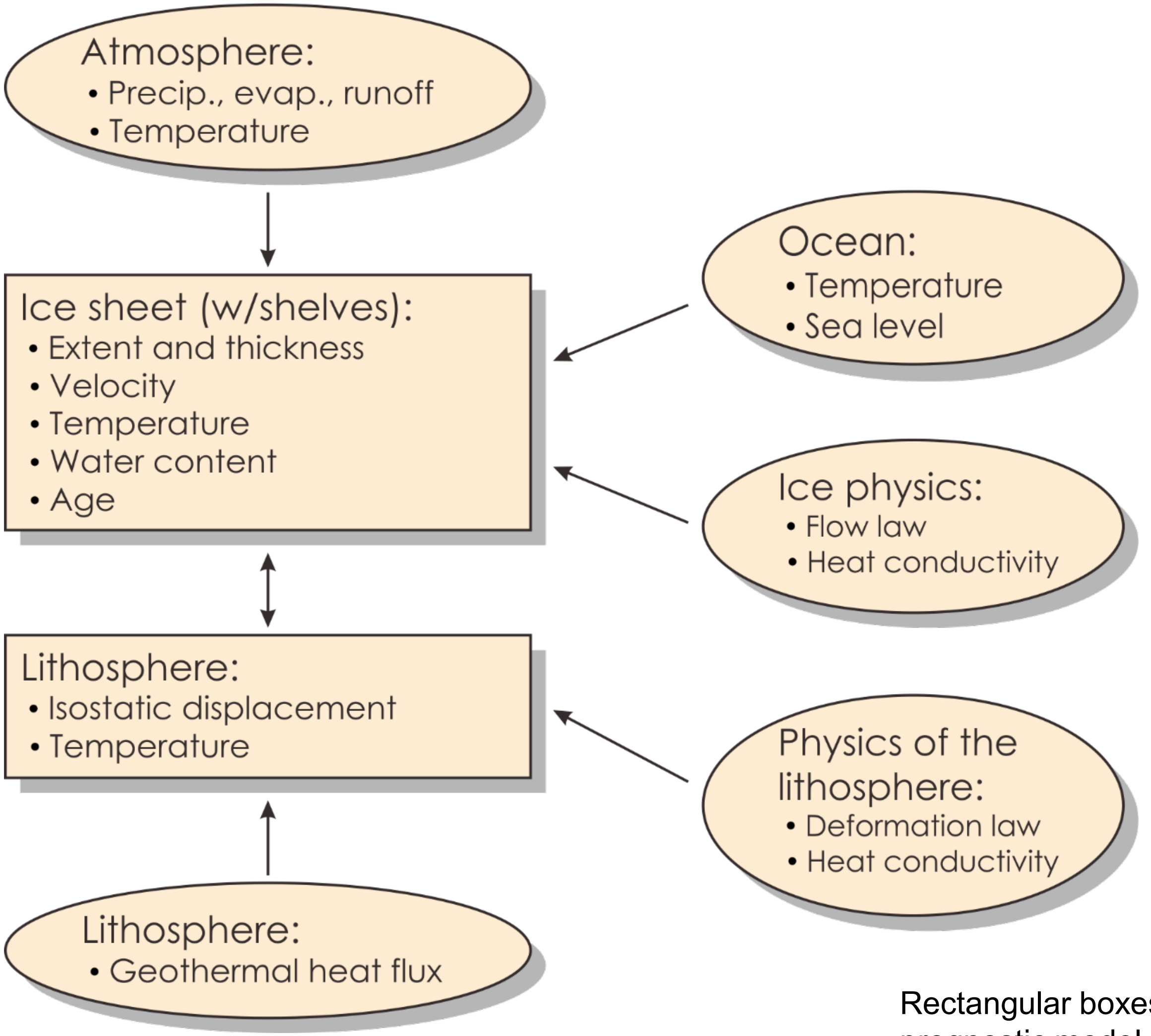
$$\frac{\partial H}{\partial t} = -\mathrm{div}\,\mathbf{Q} + a_\mathrm{s} - a_\mathrm{b}$$



6. Sketch of the coupled initial–boundary value problem



Sketch of the coupled initial–boundary value problem



Rectangular boxes: prognostic model components. Ovals: model input.

7. Analytical solutions

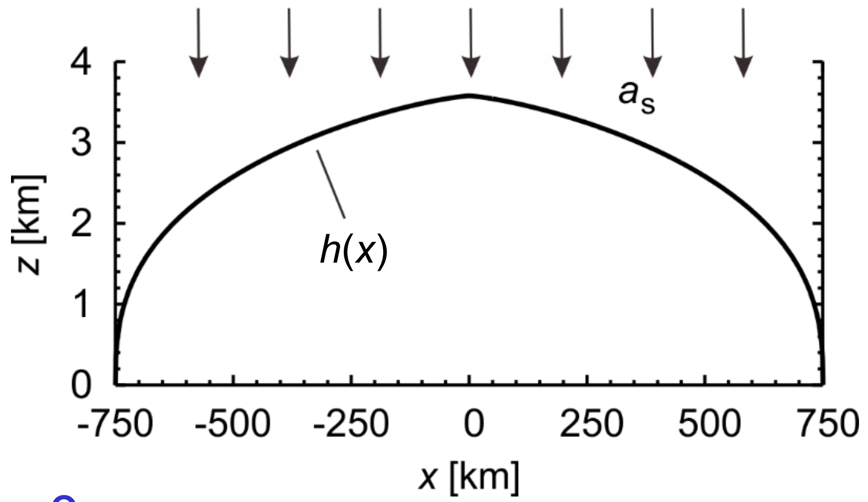
Vialov profile

Only for highly simplified problems, e.g., the **Vialov profile:**

2-d ice sheet (only x–z, no y), SIA.

Flat, rigid bed: $b = 0$, $\partial b/\partial t = 0$.

Extent between
$x = -L$ and $x = +L$
($L = 750$ km),
zero thickness at the margins.



Surface mass balance $a_s = \text{const} > 0$.

No basal melting: $a_b = 0$, no basal sliding.

Constant rate factor: $A = \text{const} \rightarrow$ no dependence on T.

↪ Steady-state surface (= thickness) profile $h(x)$.

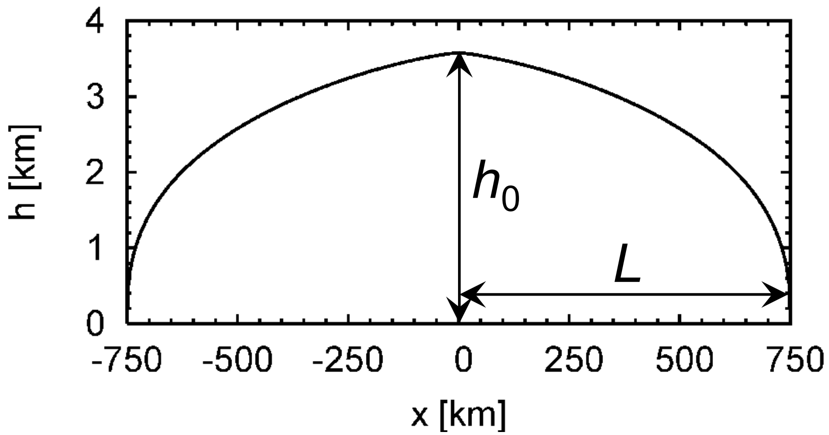
Vialov profile

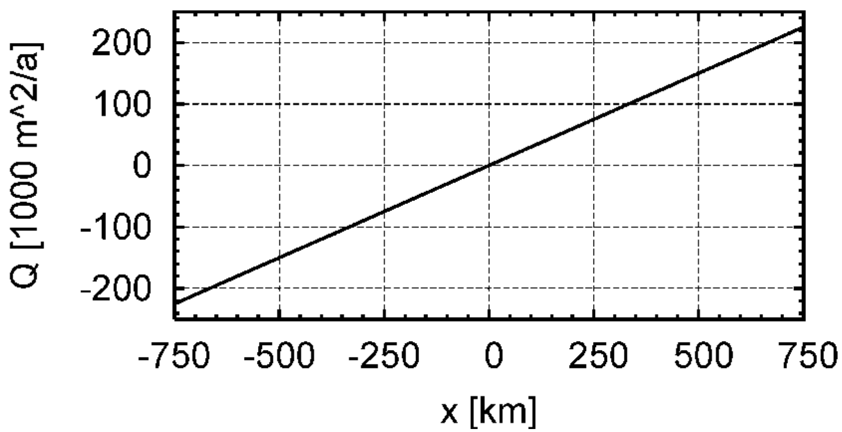
Profile *h*(*x*)

$$h = h_0 \left[1 - \left(\frac{|x|}{L} \right)^{(n+1)/n} \right]^{n/(2n+2)}$$

(Vialov 1958)

$$\text{with} \quad h_0 = 2^{n/(2n+2)} \left(\frac{a_s}{A_0} \right)^{1/(2n+2)} L^{1/2}\,, \qquad A_0 = \frac{2A(\rho g)^n}{n+2}$$





L = 750 km, n = 3, a_s = 0.3 m a^{-1}, A = 10^{-16} a^{-1} Pa^{-3}
(~ Greenland west-east transect)

Vialov profile

Aspect ratio (shallowness parameter)

$$\varepsilon = \frac{h_0}{L} \sim \frac{1}{L^{1/2}}$$

Large ice bodies are shallower than small ones!

Sensitivity to surface mass balance (snowfall rate)

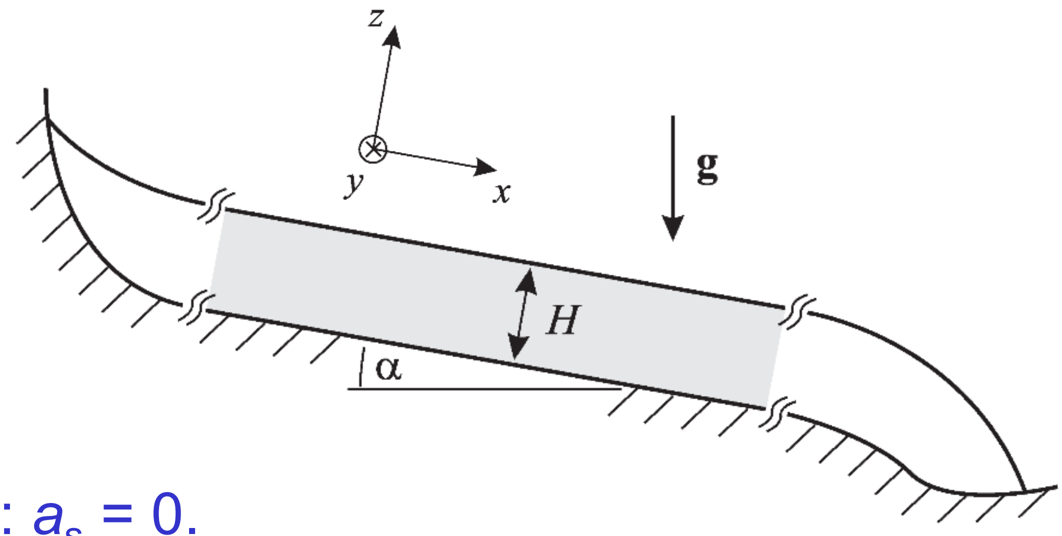
$$h_0 \sim a_\mathrm{s}^{1/8} \quad (\text{for } n = 3)$$

Very weak sensitivity!

Parallel-sided slab

2-d glacier (only x–z, no y), FS.

Flat, rigid bed:
$b = 0$, $\partial b/\partial t = 0$.



Constant thickness H
and inclination angle α.

Uniformity in x-direction.

No surface mass balance: $a_\mathrm{s} = 0$.

No basal melting: $a_\mathrm{b} = 0$, no basal sliding.

Constant rate factor: $A = \mathrm{const} \rightarrow$ no dependence on T.

Constant heat conductivity: $\kappa = \mathrm{const}$.

→ Steady-state velocity $v_\mathrm{x}(z)$ and temperature $T(z)$.

Parallel-sided slab

Velocity profile $v_x(z)$

$$v_x = \frac{2A(\rho g \sin\alpha)^n}{n+1} \left[H^{n+1} - (H-z)^{n+1} \right]$$

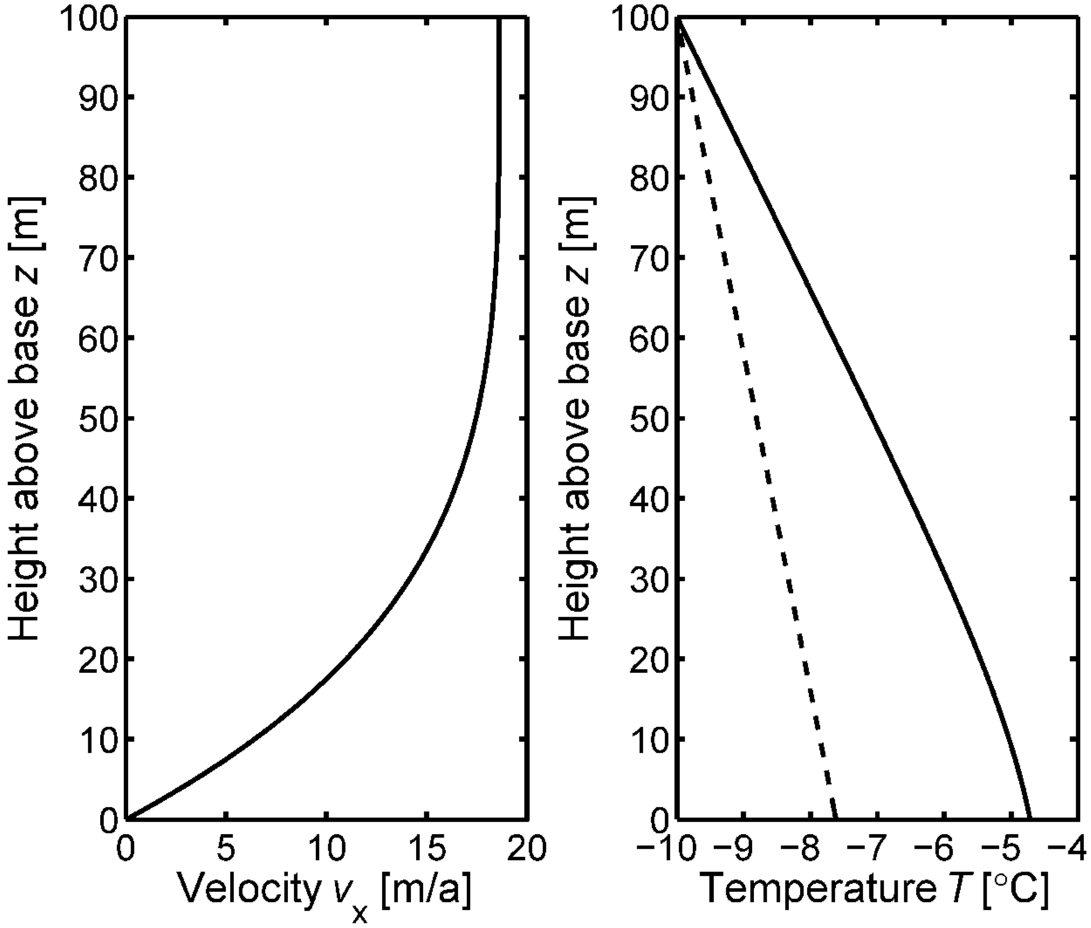
Quartic function of z for $n = 3$.

Temperature profile $T(z)$ (cold glacier assumed)

$$\begin{aligned} T &= T_\mathrm{s} + \frac{q_\mathrm{geo}}{\kappa}(H-z) \\ &\quad + \frac{2AH^{n+3}(\rho g \sin\alpha)^{n+1}}{\kappa(n+2)} \left[1 - \frac{z}{H} - \frac{1}{n+3}\left(\frac{H-z}{H}\right)^{n+3} \right] \end{aligned}$$

Linear contribution due to heat conduction,
nonlinear contribution due to viscous dissipation (strain heating).

Parallel-sided slab



H = 100 m, α = 10°, T_s = –10°C, q_{geo} = 50 mW m^{-2},
n = 3, A = 10^{-16} a^{-1} Pa^{-3}

8. Numerical solutions and models

Numerical solutions

Non-linear,
thermo-mechanically coupled,
free-surface flow problem

$\rightarrow$ in general, numerical solution techniques are required:

- Finite difference methods (FDM).
- Finite elements methods (FEM).
- Finite volume methods (FVM).
- Others...

Model SICOPOLIS

```
...
kr=KRMAX
kc=0
lgs_a0(kr) = ccb2
lgs_a1(kr) = -(ccb1+ccb2)
lgs_a2(kr) = ccb1
lgs_b(kr)  = ccb3+ccb4

do kc=1, KCMAX-1

   lgs_a0(KRMAX+kc) = -0.5d0*(ct1(kc)-ct2(kc)-ct3(kc)-ct4(kc)) &
                      -ct5(kc)*ct6(kc-1)
   lgs_a1(KRMAX+kc) = 1.0d0+ct5(kc)*(ct6(kc)+ct6(kc-1))
   lgs_a2(KRMAX+kc) = 0.5d0*(ct1(kc)-ct2(kc)-ct3(kc)-ct4(kc)) &
                      -ct5(kc)*ct6(kc)

#if ADV_HOR==1

   lgs_b(KRMAX+kc) = temp_c(kc,j,i) + ct7(kc) &
      -dtt_2dxi* &
         ( (vx_c(kc,j,i)-abs(vx_c(kc,j,i))) &
           *(temp_c(kc,j,i+1)-temp_c(kc,j,i)) &
           *insq_g11_sgx(j,i) &
          +(vx_c(kc,j,i-1)+abs(vx_c(kc,j,i-1))) &
           *(temp_c(kc,j,i)-temp_c(kc,j,i-1)) &
           *insq_g11_sgx(j,i-1) ) &
      -dtt_2deta* &
         ( (vy_c(kc,j,i)-abs(vy_c(kc,j,i))) &
           *(temp_c(kc,j+1,i)-temp_c(kc,j,i)) &
           *insq_g22_sgy(j,i) &
          +(vy_c(kc,j-1,i)+abs(vy_c(kc,j-1,i))) &
           *(temp_c(kc,j,i)-temp_c(kc,j-1,i)) &
           *insq_g22_sgy(j-1,i) )

#elif ADV_HOR==2

   lgs_b(KRMAX+kc) = temp_c(kc,j,i) + ct7(kc) &
                     -dtt_dxi *(ftx_c_r(kc)-ftx_c_l(kc)) &
                     -dtt_deta*(fty_c_r(kc)-fty_c_l(kc))

#endif

end do

kc=KCMAX
lgs_a0(KRMAX+kc) = 0.0d0
lgs_a1(KRMAX+kc) = 1.0d0
lgs_b(KRMAX+kc)  = temp_s(j,i)
...
```

"SImulation
COde
for
POLythermal
Ice
Sheets"

- Open-source model, mainly delevoped at ILTS (www.sicopolis.net).
- Coded in Fortran.
- Shallow ice + shallow shelf approximations.
- Finite difference method.

SICOPOLIS – Sigma transformation

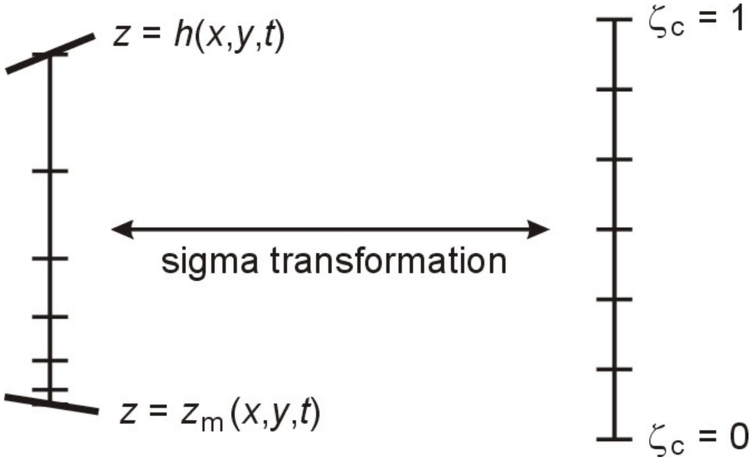
Vertical ice columns mapped on [0,1] intervals.

Separate mappings for cold-ice layer,
temperate-ice layer [polythermal method only],
lithosphere (rock) layer

→ vertical coordinates ζ_c, ζ_t, ζ_r.

Cold-ice layer:
Densification of grid points
close to the base

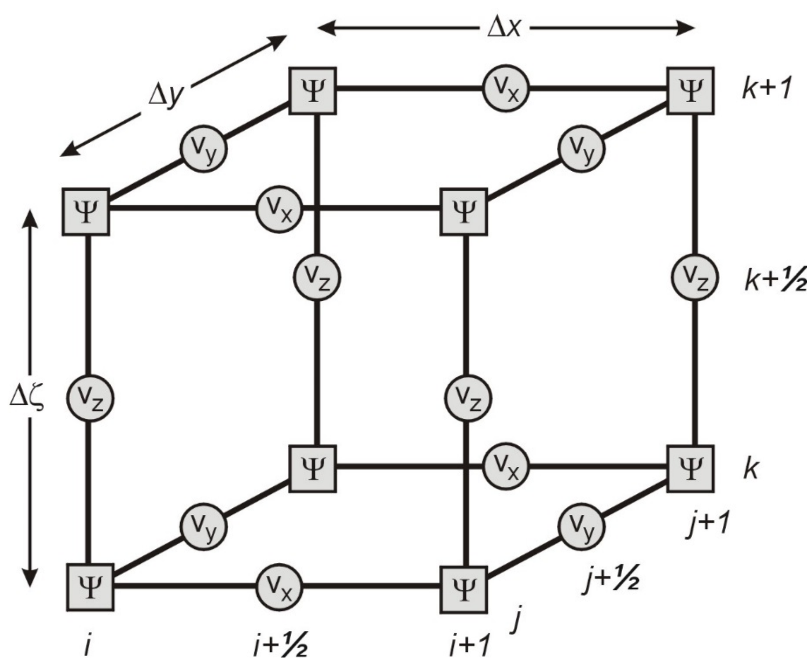
→ parameter *a*.



SICOPOLIS – Numerical solution technique

Finite difference method.

Staggered grid (Arakawa-C grid):



- Velocities (v_x, v_y, v_z) and volume fluxes (Q_x, Q_y) are defined in between grid points.
- Other field quantities (Ψ) are defined on grid points.

SICOPOLIS – Numerical solution technique

2nd-order central differences for diffusive terms.

1st-order upstreaming for advective terms.

Time-stepping (ice thickness equation):

- Time-step Δt (same for velocity and isostasy).
- **Over-implicit** in the linear part, explicit in the non-linear part.

Time-stepping (temperature, water content and age):

- Time-step $\widetilde{\Delta t}$ (integer multiple of Δt).
- Implicit in the vertical, explicit in the horizontal derivatives.

Model Elmer/Ice

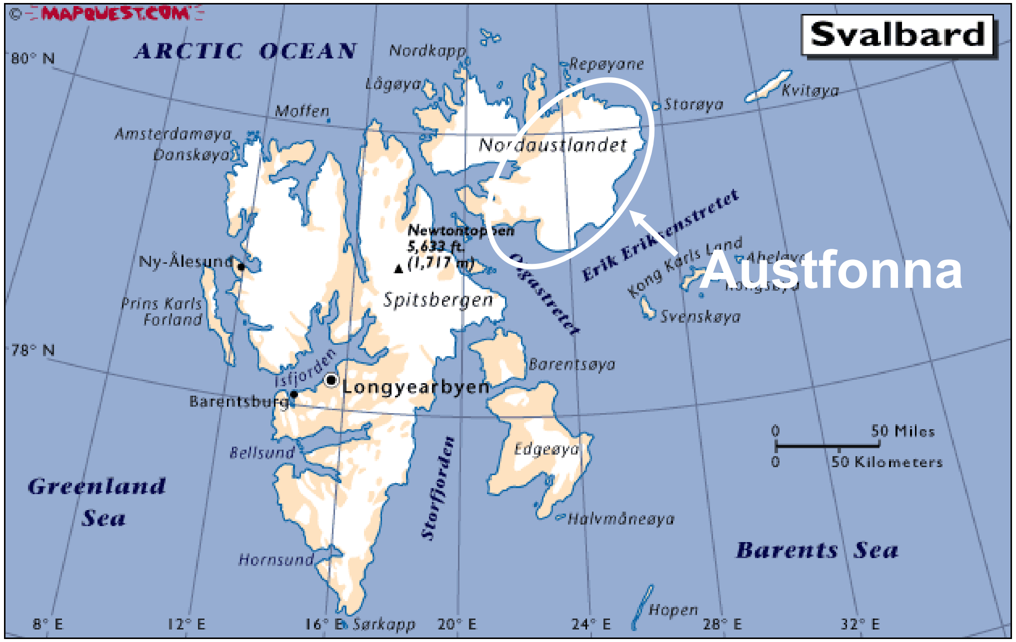
elmerice.elmerfem.org

- Add-on package to Elmer (multi-physics FEM suite mainly developed by CSC – IT Center for Science, Espoo, Finland).
- Open-source model.
- Solves the full Stokes (FS) equations.
- Applicable to ice sheets, ice shelves, ice caps and glaciers.

9. Selected applications

Application of SICOPOLIS to the Austfonna ice cap, Svalbard

(Dunse et al. 2011)



Objective:

To reproduce the observed surge-recovery cycles of several drainage basins of Austfonna.

Simulated surface velocity field over 1000 years of present-day climate conditions

Animation

→ supplementary material

SeaRISE

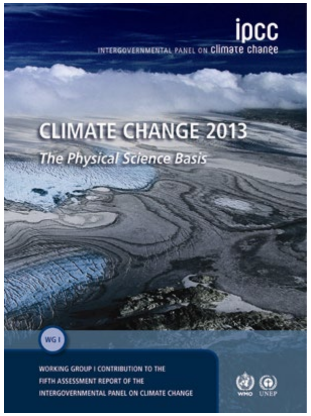
= **Sea**-level **R**esponse to **I**ce **S**heet **E**volution

International multi-ice-sheet model community effort.

Objective:

To predict the likely range of contributions of the Greenland and Antarctic ice sheets to sea level rise over the next 100's of years under global warming conditions.

Input for the Fifth Assessment Report (AR5) of the Intergovernmental Panel on Climate Change (IPCC 2013)



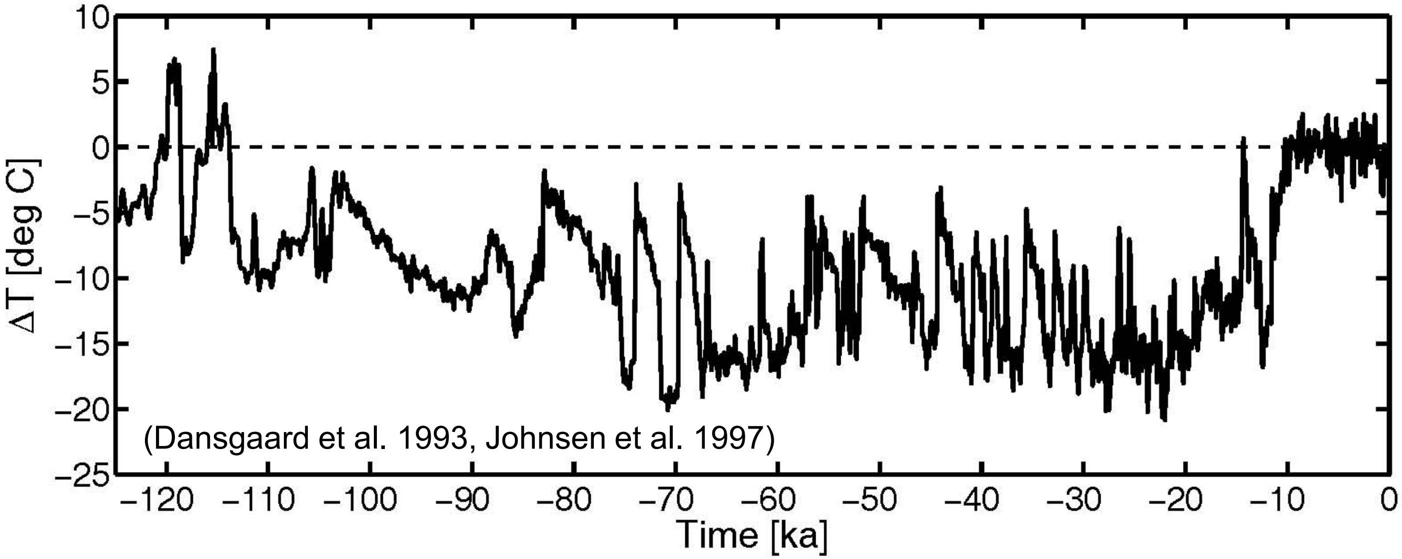
Paleoclimatic spin-up for Greenland (with SICOPOLIS)

Grid spacing: Δx = 5, 10, 20 km.

Model time: t = –125 ka ... 0 ka (one glacial cycle).

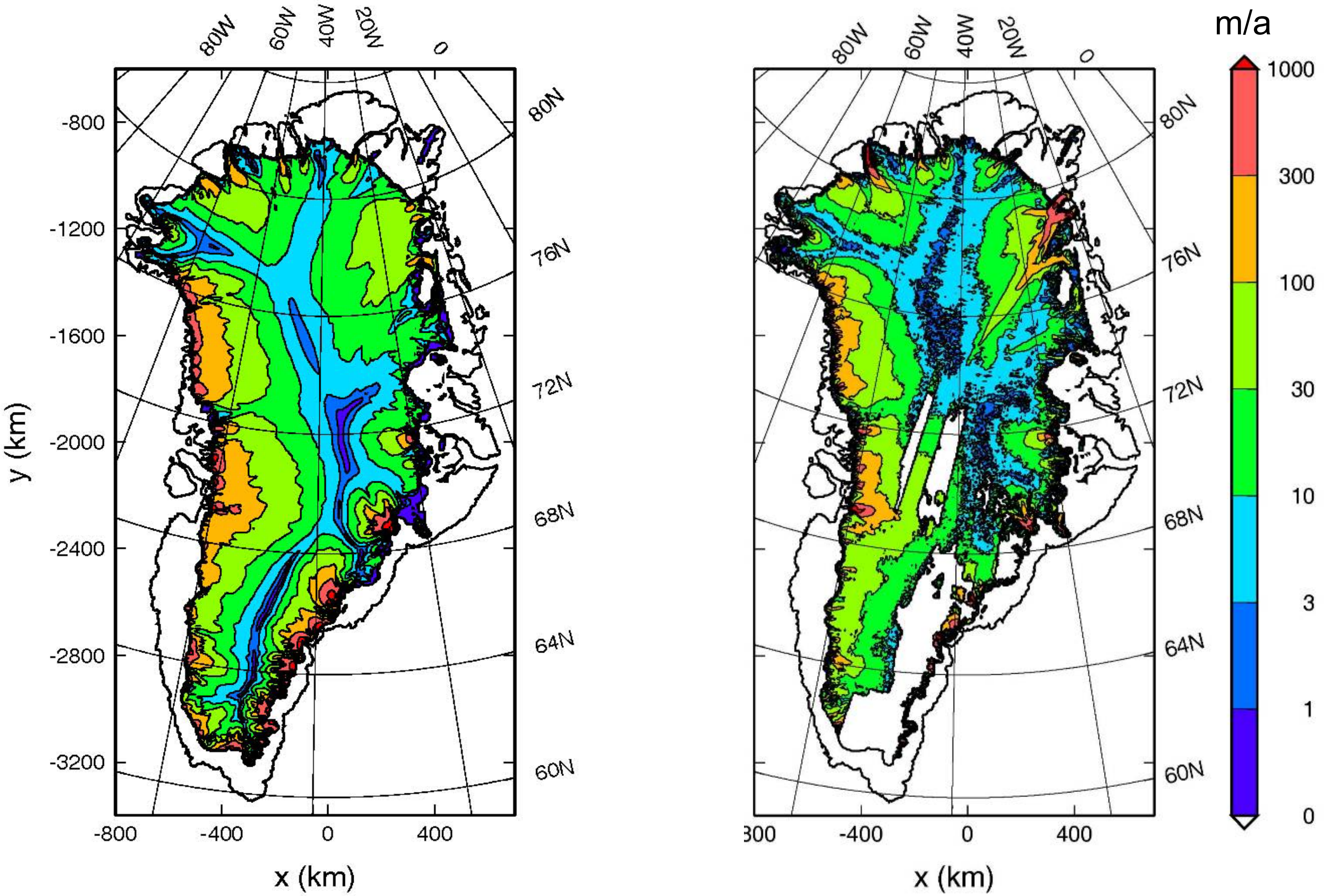
Fixed topography (except last 100 a).

Surface temperature anomaly from GRIP $\delta^{18}O$ record:



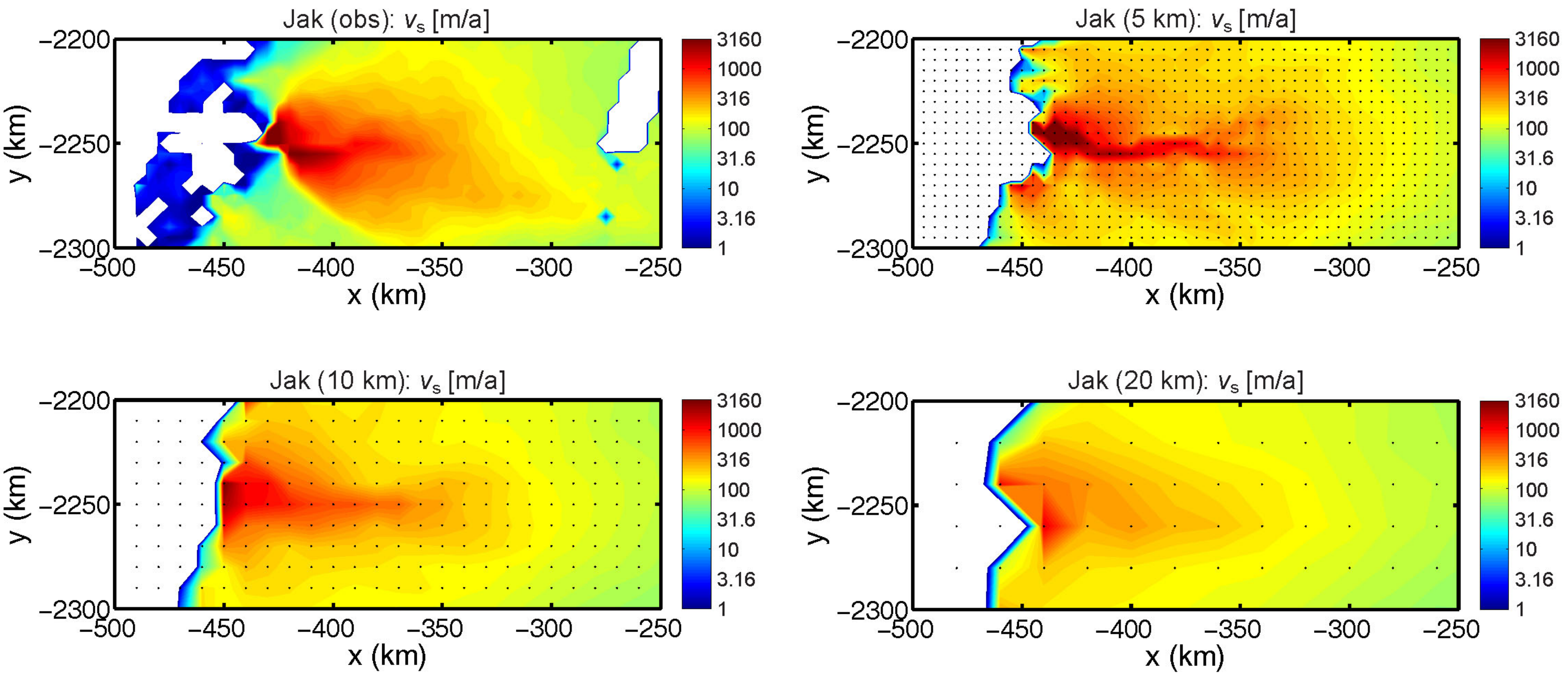
Paleoclimatic spin-up, results for Δx = 5 km

Surface velocity | Obs. (Joughin et al. 2010)



Ice stream patterns depend strongly on resolution

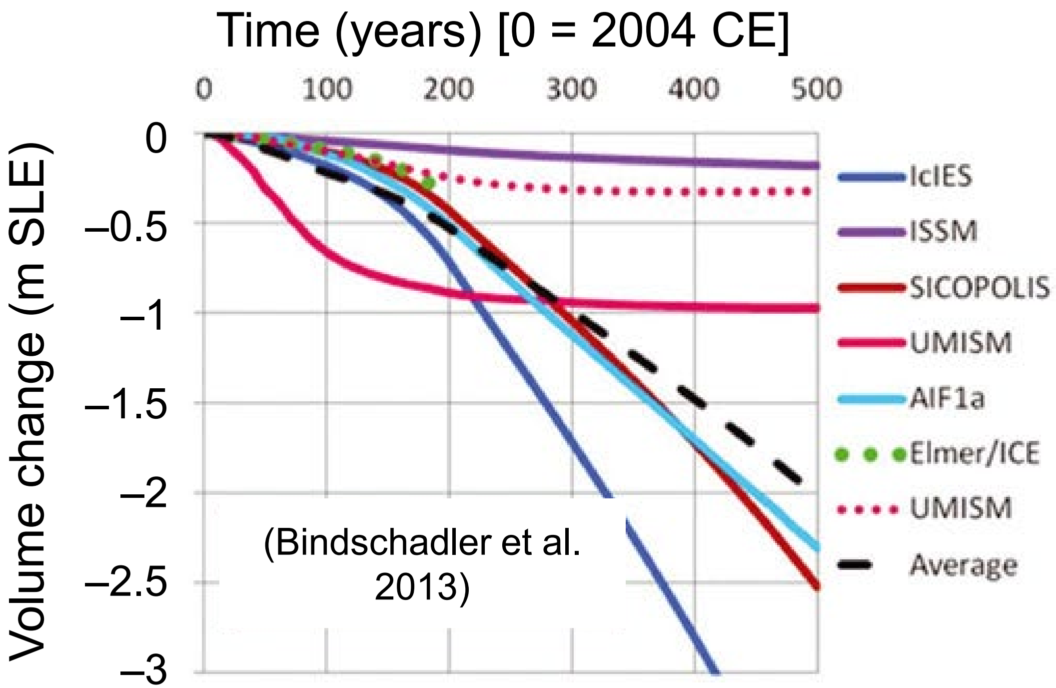
Jakobshavn ice stream



(Greve and Herzfeld 2013)

SeaRISE experiment R8, mimics IPCC's RCP8.5 "business as usual" scenario

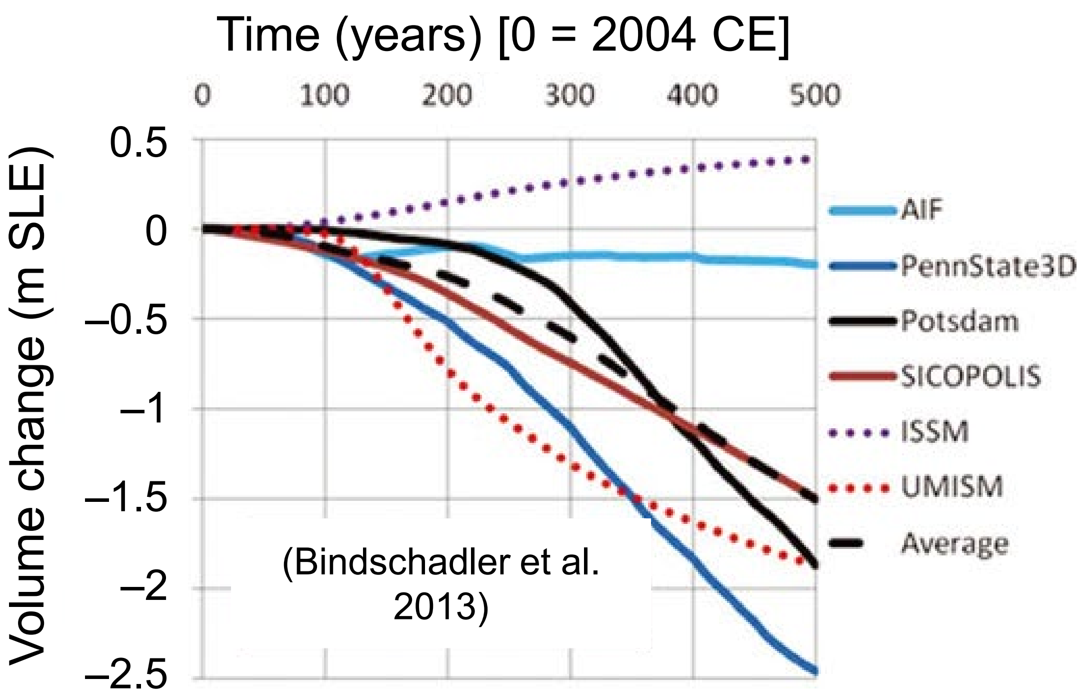
Simulated volume change of the GrIS (all models):



Average loss after 100 a: 0.22 m SLE
200 a: 0.53 m SLE
500 a: 2.02 m SLE

SeaRISE experiment R8 for Antarctica

Simulated volume change of the AIS (all models):



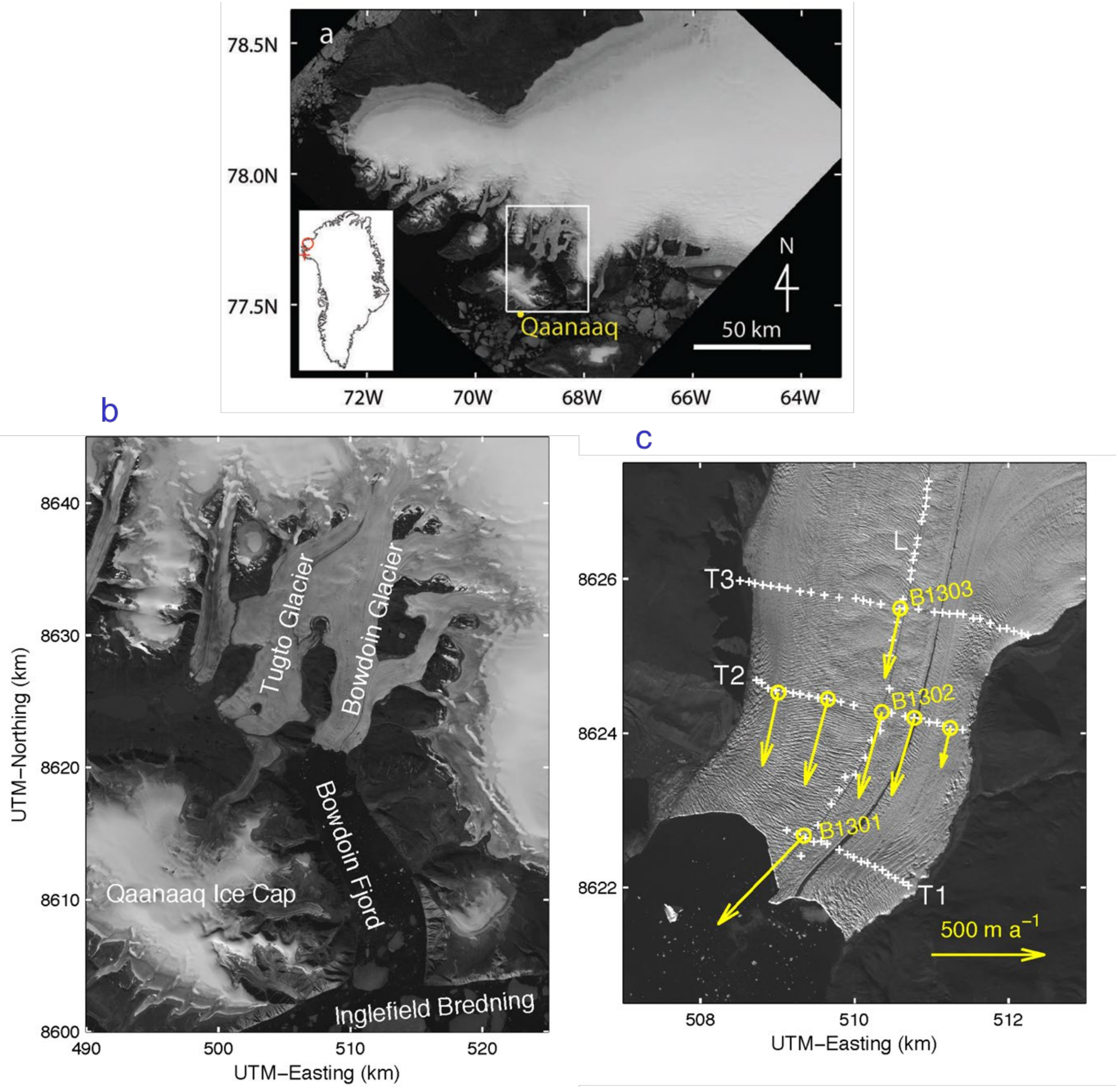
Average loss after 100 a: 0.08 m SLE
200 a: 0.27 m SLE
500 a: 1.51 m SLE

Application of Elmer/Ice to Bowdoin Glacier, Greenland

Bowdoin Glacier:

Marine-terminating outlet glacier located in NW Greenland.

Field surveys (2013–2016), satellite data analysis.



(Sugiyama et al. 2015)

Control run – Set-up

Diagnostic simulation,
resolution ~ 70 m.

Temperature field:
Steady state w/o basal sliding.

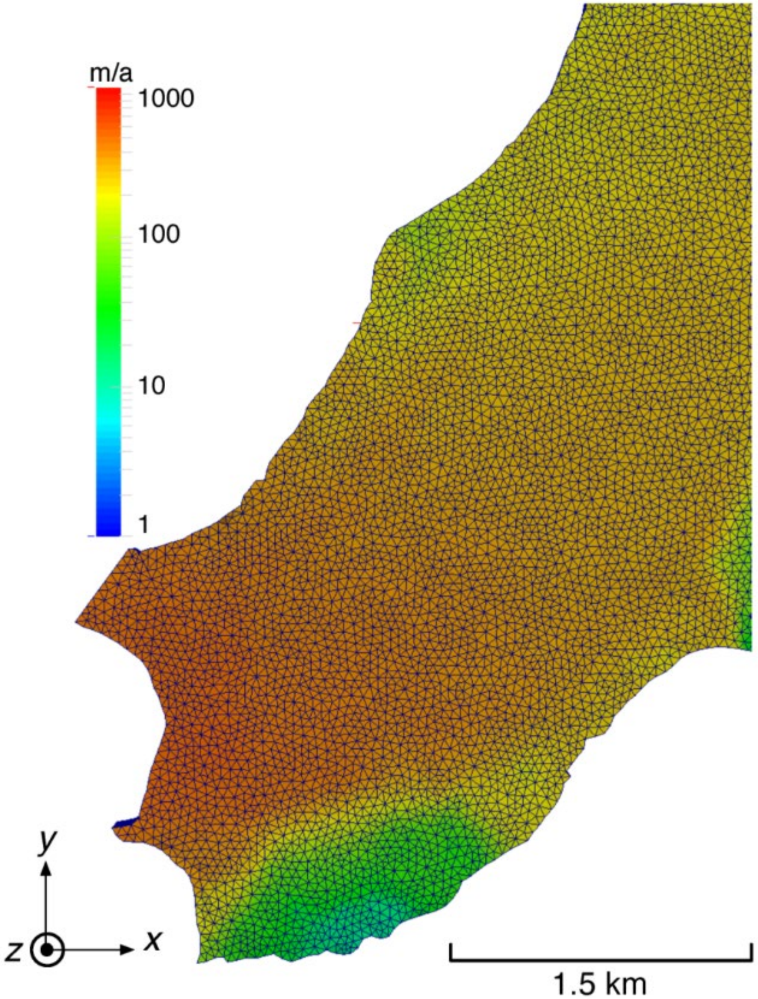
Control inverse method:
Minimize cost function

$$J_{\mathrm{tot}} = J_0 + \lambda J_{\mathrm{reg}}$$

(J_0: misfit between modelled and observed surface velocities,
J_{reg}: regularization)

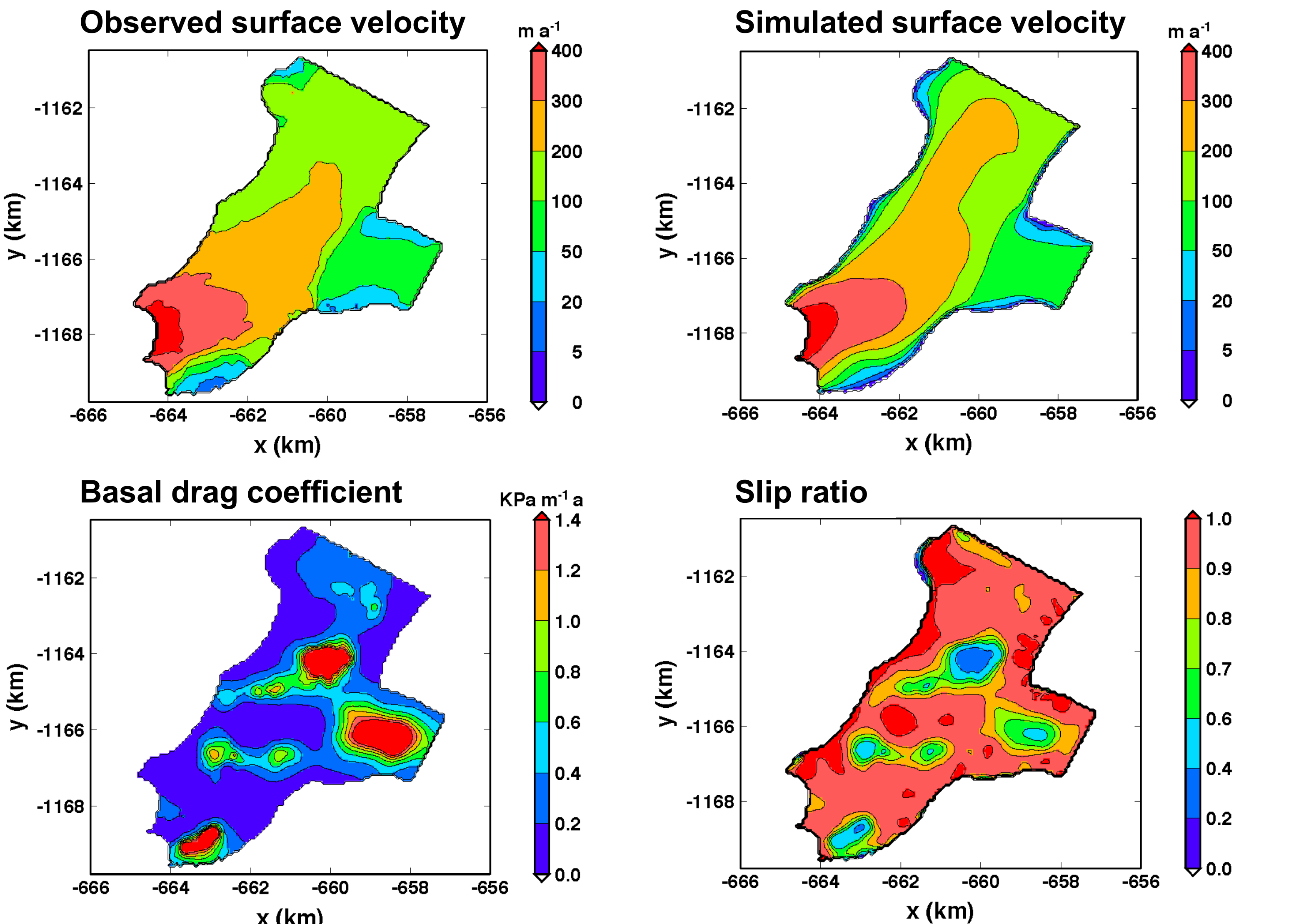
→ distribution of the basal drag β

(simplified sliding law $\tau_b = \beta v_b$)



Observed surface velocities:
Sugiyama et al. (2015)

Control run – Results



Seddik et al. (in preparation)

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  (development of the theory,
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Thank you