INTERFERENCE-RESISTANT LPTV-MMSE EQUALIZATION

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ABSTRACT

The problem of recovering a digital communication signal distorted by a linear time-invariant channel and contaminated by severe co-channel or adjacent-channel digital interference is addressed in this paper. The proposed linear periodically time-varying (LPTV) receiver jointly performs channel equalization and interference suppression, without requiring explicit knowledge or estimation of the interfering channel. Simulation results confirm the effectiveness of the proposed technique, whose performance exhibit a remarkable robustness with respect to varying interference power level.

1 INTRODUCTION

The problem of channel equalization arises mainly in high-speed digital communications, both in wired systems (in order to compensate for amplitude and phase distortion introduced by non-ideal transmission media) and in wireless ones (in order to mitigate multipath propagation effects). The aim of equalization is mainly to reduce the harmful effects of intersymbol interference (ISI) and noise. Since channel ISI is typically modeled as the result of linear time-invariant (LTI) filtering (in wired systems) or quasi-LTI filtering (in wireless systems) performed on the transmitted symbols, and thermal noise is assumed to be wide-sense stationary (WSS), the equalizer is constrained to have an LTI structure, and is designed under a zero-forcing or minimum mean-square error (MMSE) criterion. Such LTI equalizers have been receiving a great deal of attention in the open literature in the last decades, both in their blind and non-blind (i.e., trained) versions, and are widely implemented in modern high-speed communication systems.

However, in both wired and wireless systems, the increasing demand for high-speed transmission is rapidly saturating the scarce available bandwidth resources. Therefore, in highly-congested settings, one might be faced with the more challenging problem of counteracting not only the ubiquitous presence of ISI, but also the deleterious effects of *co-channel interference* (CCI) and *adjacent-channel interference* (ACI), which arise because of multiple users sharing the same communication resources, in dual-rate or multi-rate transmission schemes, as the effect of imperfect separation between different communication systems (due to crosstalk and nonlinearities), and in overlay systems. In many cases, the desired and the interfering signals might exhibit different symbol rates: this happens, for example, in wired systems, due to crosstalk between adjacent pairs transmitting at different rates; or in wireless multi-rate systems, where the desired transmission in a given cell might be disturbed by a cochannel transmission with different rate carried out in a neighboring cell.

In all these cases, due to the presence of the interfering signal, optimal zero-forcing and MMSE linear structures turn out to be *time-varying* rather than LTI. More precisely, since all digitally modulated signals are cyclostationary [1], i.e., they exhibit periodicities in their second- and/or higher-order statistics, such optimal filters turn out to be *periodically* time-varying (LPTV) or *almost-periodically* time varying, whose theory and applications have received a great deal of interest in recent years [2, 3, 4]. In this paper, the concepts of LPTV filtering are applied to derive new equalization structures, optimized under the MMSE criterion, which can cope with the presence of high-level CCI and/or ACI, without requiring explicit knowledge or estimation of the interfering channel.

2 THE MATHEMATICAL MODEL

Let us consider the complex envelope of the received signal in a digital communication system, which is given by

$$r_a(t) = u_a(t) + i_a(t) + w_a(t) \,, \tag{1}$$

where $u_a(t)$, $i_a(t)$, and $w_a(t)$ denote the desired signal, the CCI or ACI, and thermal noise, respectively. We will assume that both $u_a(t)$ and $i_a(t)$ are digital communication signals employing linear modulation formats, with different signaling rates T_U and T_I, respectively, i.e.,

$$u_a(t) = \sum_{k=-\infty}^{\infty} s(k)\, c_a(t - kT_U) \,, \tag{2}$$

$$i_a(t) = \sum_{k=-\infty}^{\infty} s_I(k)\, c_{I,a}(t - kT_I)\, e^{j2\pi f_I t}\, e^{j\phi_I} \,, \tag{3}$$

where $s(k)$ and $s_I(k)$ are the (complex) symbol sequences (assumed zero-mean and iid), $c_a(t)$ and $c_{I,a}(t)$ denote the overall impulse responses (possibly including transmitting filters, channel, and receiving filters) of the signal and interference channel, and, finally, f_I and ϕ_I are the frequency offset and the carrier phase of the interference. In the following, we will assume customarily that $u_a(t)$, $i_a(t)$, and $w_a(t)$ are statistically independent, and that $w_a(t)$ can be modeled as a WSS zero-mean random process.

Our aim is to extract the desired symbol stream $s(k)$ by counteracting the effects of both ISI, interference $i_a(t)$, and noise. To this end, the continuous-time received signal (1) is sampled at rate N/T_U, with $N \geq 1$ denoting the *oversampling* factor, obtaining the discrete-time signal

$$r(n) = u(n) + i(n) + w(n) \,, \tag{4}$$

where

$$u(n) \triangleq u_a(nT_U/N) = \sum_{k=-\infty}^{\infty} s(k)\, c(n-kN) \,, \tag{5}$$

and $c(n) \triangleq c_a(nT_U/N)$, $i(n) \triangleq i_a(nT_U/N)$, and, finally, $w(n) \triangleq w_a(nT_U/N)$. For $N = 1$ we obtain a model appropriate for *baud-spaced equalization*, whereas $N > 1$ is appropriate for *fractionally-spaced equalization*.

Observe that the continuous-time signal $u_a(t)$ [see (2)] is second-order cyclostationary with period T_U and, therefore, the discrete-time signal $u(n)$ in (5) is second-order cyclostationary with period N. Then, it is customary to represent $u(n)$ in terms of its *polyphase components*:

$$u^{(i)}(n) \triangleq u(nN+i) = \sum_{k=-\infty}^{\infty} s(k) c^{(i)}(n-k) \,, \tag{6}$$

for $i = 0, 1, \ldots, N-1$, where $c^{(i)}(n) \triangleq c(nN+i)$ is the polyphase decomposition of $c(n)$. The advantage of this representation is well-known: the polyphase components given by (6) turn out to be jointly WSS rather than cyclostationary. Accordingly, let us consider the polyphase decomposition of $r(n)$, that is, $r^{(i)}(n) \triangleq r(nN+i)$, $i = 0, 1, \ldots, N-1$, and collect the N phases in the N-column vector $\boldsymbol{r}(n) \triangleq [r^{(0)}(n),\, r^{(1)}(n), \ldots, r^{(N-1)}(n)]^T$, with T denoting transpose, obtaining therefore the compact vector model

$$\boldsymbol{r}(n) = \sum_{k=0}^{L_c-1} \boldsymbol{c}(k)\, s(n-k) + \boldsymbol{i}(n) + \boldsymbol{w}(n) \,, \tag{7}$$

where $\boldsymbol{c}(n)$, $\boldsymbol{i}(n)$, and $\boldsymbol{w}(n)$ are built similarly to $\boldsymbol{r}(n)$, and it is assumed, although not strictly required in the following, that $c_a(t) = 0$ for $t \notin [0, L_c T_U[$.

3 LPTV-MMSE EQUALIZATION

The aim of the linear equalizer is to extract the nth desired symbol $s(n)$ by simultaneously minimizing the effect of ISI, interference, and noise. The equalizer (see Fig. 1) is composed by a bank of N time-varying finite impulse-response (FIR) filters having impulse responses $f_n^{(i)}(m)$ of length L_e, each operating on a different phase of $r(n)$, whose outputs are summed together to yield an estimate $\hat{s}(n)$ of the nth transmitted symbol $s(n)$:

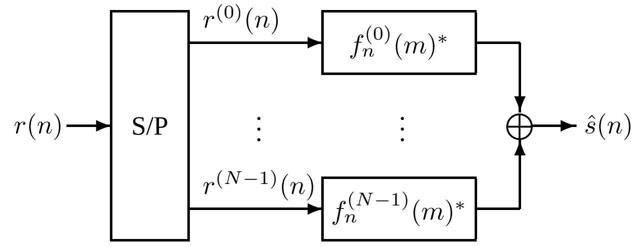

Figure 1: The proposed equalizer scheme.

$$\hat{s}(n) = \sum_{i=0}^{N-1} \sum_{m=0}^{L_e-1} [f_n^{(i)}(m)]^*\, r^{(i)}(n-m) \,, \tag{8}$$

where $*$ denotes conjugation. Equation (8) can be expressed in a more compact form as:

$$\hat{s}(n) = \boldsymbol{b}^H(n)\, \boldsymbol{z}(n) \,, \tag{9}$$

where H denotes conjugate transpose, and $\boldsymbol{z}(n)$ and $\boldsymbol{b}(n)$ are (NL_e)-column vectors:

$$\boldsymbol{z}(n) \triangleq [\boldsymbol{r}^T(n), \boldsymbol{r}^T(n-1), \ldots, \boldsymbol{r}^T(n-L_e+1)]^T , \tag{10}$$

$$\boldsymbol{b}(n) \triangleq [\boldsymbol{f}_n^T(0), \boldsymbol{f}_n^T(1), \ldots, \boldsymbol{f}_n^T(L_e-1)]^T \,, \tag{11}$$

with $\boldsymbol{f}_n(\cdot) \triangleq [f_n^{(0)}(\cdot), f_n^{(1)}(\cdot), \ldots, f_n^{(N-1)}(\cdot)]^T$. The optimal weights $\boldsymbol{b}(n)$ are singled out by minimizing the (time-varying) mean-square error $\text{MSE}(n) \triangleq E[|\hat{s}(n) - s(n)|^2]$, with $E[\cdot]$ denoting statistical averaging, which yields:

$$\boldsymbol{b}(n) = \sigma_s^2\, \boldsymbol{R}_{\boldsymbol{zz}}^{-1}(n)\, \widetilde{\boldsymbol{c}} \,, \tag{12}$$

wherein $\boldsymbol{R}_{\boldsymbol{zz}}(n) \triangleq E[\boldsymbol{z}(n)\boldsymbol{z}^H(n)]$, $\sigma_s^2 \triangleq E[|s(n)|^2]$, and $\widetilde{\boldsymbol{c}} \triangleq [\boldsymbol{c}^T(0), 0, 0, \ldots, 0]^T$ is an (NL_e)-column vector. From (12), it results that the optimal weight vector depends on n, since $\boldsymbol{R}_{\boldsymbol{zz}}(n)$ is time-varying due to the presence of the interference. By appropriately exploiting these time-varying properties, the LPTV-MMSE equalizer defined by (9) and (12) is able to outperform its LTI counterpart, which is based on *time-averaged measurements* of $\boldsymbol{R}_{\boldsymbol{zz}}(n)$.

To gain further insight about this aspect, set $M = L_c + L_e$ and observe that $\boldsymbol{z}(n)$ can be explicitly written as:

$$\boldsymbol{z}(n) = \boldsymbol{C}\boldsymbol{s}(n) + \boldsymbol{j}(n) + \boldsymbol{v}(n) \,, \tag{13}$$

where $\boldsymbol{C}$ is the $NL_e \times (M-1)$ block-Toeplitz channel matrix, given by:

$$\boldsymbol{C} = \begin{pmatrix} \boldsymbol{c}(0) & \ldots & \boldsymbol{c}(L_c-1) & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{c}(0) & \ldots & \boldsymbol{c}(L_c-1) & \boldsymbol{0} \\ \vdots & & & & \vdots \\ \vdots & & & & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{c}(0) & \ldots & \boldsymbol{c}(L_c-1) \end{pmatrix} , \tag{14}$$

and

$$s(n) \stackrel{\triangle}{=} [s(n), s(n-1), \ldots, s(n-M+1)]^T, \quad (15)$$

$$j(n) \stackrel{\triangle}{=} [i^T(n), i^T(n-1), \ldots, i^T(n-L_e+1)]^T, \quad (16)$$

$$v(n) \stackrel{\triangle}{=} [w^T(n), w^T(n-1), \ldots, w^T(n-L_e+1)]^T. \quad (17)$$

Accounting for (13), $R_{zz}(n)$ can be decomposed as follows:

$$R_{zz}(n) = \sigma_s^2 C C^H + R_{jj}(n) + R_{vv}\,, \quad (18)$$

where $R_{jj}(n)$ and R_{vv} are the correlation matrices of $j(n)$ and $v(n)$, respectively. It is worth noting that the only time-varying quantity in (18) is $R_{jj}(n)$, which accounts for the presence of the interfering signal. Moreover, since the elements of $j(n)$ are the polyphase components of $i(n)$, its time-varying features can be determined by studying the cross-correlation functions of the polyphase components of $i(n)$, that is,

$$R_{ij}(n,m) \stackrel{\triangle}{=} E[i^{(i)}(n)\, i^{(j)}(n-m)^*] \quad (19)$$

for $i, j = 0, 1, \ldots, N-1$. By straightforward calculations, it can be shown that, for any fixed value of m, $R_{ij}(n,m)$ can be written as a *generalized Fourier series* [5] in n:

$$R_{ij}(n,m) = \sum_{\ell=-\infty}^{\infty} R_{ij}^{(\ell)}(m)\, e^{j2\pi \ell n \frac{T_U}{T_I}}\,. \quad (20)$$

Therefore, time-varying properties of $R_{ij}(n,m)$ [and, hence, of $R_{jj}(n)$ and $R_{zz}(n)$] are governed by the ratio T_U/T_I: when T_U/T_I is integer, $R_{zz}(n)$ is time-invariant; when $T_U/T_I = p/q$ (rational), $R_{zz}(n)$ is simply periodic in n with period P such that $p\,P$ is the smallest multiple of q; when T_U/T_I is not a rational number, $R_{zz}(n)$ is *almost* periodic [5] in n.

On the basis of periodic or almost periodic properties of $R_{zz}(n)$, one can easily design exact and approximate, non-adaptive as well as adaptive time-varying equalizers, which are expected to outperform their LTI counterparts in the presence of severe CCI or ACI. We provide here details on a batch-type (non-adaptive) implementation, under the assumption that $R_{zz}(n)$ is periodic with period P, i.e., the equalizer turns out to be LPTV with period P. Let $z(n)$, $n = 0, 1, \ldots, KP-1$ represent the available data, the matrices $R_{zz}(\ell)$ for $\ell = 0, 1, \ldots, P-1$ can be estimated by means of synchronized averaging [1]:

$$\widehat{R}_{zz}(\ell) = \frac{1}{K} \sum_{n=0}^{K-1} z(nP+\ell)\, z^H(nP+\ell) \quad (21)$$

and the equalizer weights obtained as:

$$\widehat{b}(\ell) = \sigma_s^2\, \widehat{R}_{zz}^{-1}(\ell)\widetilde{c}\,, \quad (22)$$

for $\ell = 0, 1, \ldots, P-1$, under the assumption that the desired channel (and therefore $\widetilde{c}$) is known. Since the optimum filter (12) is periodic with period P, the LPTV equalizer is implemented by choosing cyclically at time n the weight $\widehat{b}(\ell)$ such that $\ell = (n \mod P)$.

Finally, note that a major assumption of the proposed equalization technique is that [see (12) and (22)] the channel of the desired user is known or estimated, whereas knowledge of the channel of the interfering signal is not required. In practice, channel identification can be achieved by resorting to suitable training sequences, whereas a blind implementation of this same LPTV equalizer is proposed in [6].

4 NUMERICAL RESULTS

To assess the benefits of the proposed LPTV equalizer, its performance has been evaluated via computer simulations and compared with those of the conventional LTI equalizer.

In all the experiments, the following common simulation setting is assumed. The ratio T_U/T_I is 5/3, i.e., the proposed equalizer is LPTV with period $P = 3$. The channels $c_a(t)$ and $c_{I,a}(t)$ are modeled as truncated approximations of a two-ray multipath channel, whose impulse response is, for $t \in (0, T_0)$,

$$p(t) = a_1\, e^{j2\pi\xi_1}\, g(t-\tau_1) + a_2\, e^{j2\pi\xi_2}\, g(t-\tau_2)\,, \quad (23)$$

where $g(t)$ is a Nyquist-shaped pulse with 35 % excess bandwidth and $a_1 = 1\,[1]$, $a_2 = 0.8\,[0.75]$, $\xi_1 = 0.15\,[0.25]$, $\xi_2 = 0.6\,[0.9]$, $\tau_1 = 0.25T_U\,[0]$, $\tau_2 = T_U\,[T_I]$, and $T_0 = 4T_U\,[5T_I]$ for the desired signal [for the interference, respectively]. It results that $L_c = 4$ for the desired-user channel, and the equalizer length is set to $L_e = 2$, whereas the oversampling factor is $N = 5$. The thermal noise is modeled as a complex circular Gaussian process, and the signal-to-noise ratio (SNR) is set to 30 dB. The sample-size is $K = 5000$: note that the LPTV equalizer is based [see (22)] on estimation of P different correlation matrices, each performed on K samples, whereas the LTI equalizer requires estimation of a single correlation matrix, performed using all KP samples. As performance measure, we chose the signal-to-interference-plus-noise ratio (SINR) at the receiver output, averaged over 100 independent trials, which is monotonically related to MSE and, moreover, is insensitive to constellation scaling and/or rotation.

In the first experiment, we tested the performance of the proposed equalizer in an environment where the desired signal is BPSK, while the interference is a QPSK CCI, with $f_I = 0$ and $\phi_I = 0$. Figure 1 reports the received eye diagram, together with those obtained by means of LTI and LPTV equalization, respectively, for a signal-to-interference ratio (SIR) of 0 dB. Results show that the LPTV equalizer provides good performance, whereas the LTI equalizer is practically useless. For further investigation, we report in Fig. 2 the values of the output SINR (in dB) obtained with the two equalizers, as a function of SIR ranging from -10 to 40 dB. The LPTV equalizer exhibits a very robust performance with respect to SIR, whereas the LTI equalizer is slightly superior only when the SIR is very high. This can be expected, since when the SIR increases, the time-varying

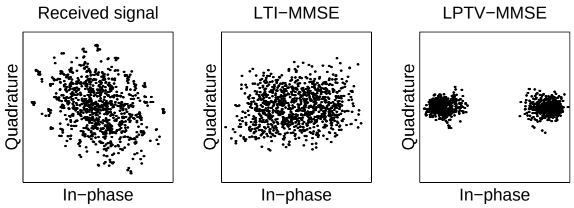

Figure 2: Eye-diagrams (first experiment, SIR = 0 dB).

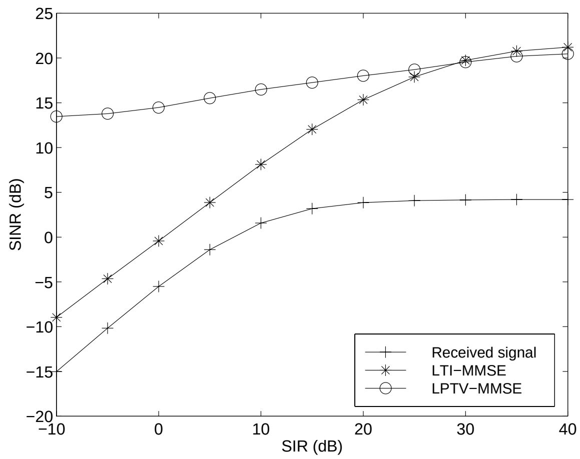

Figure 3: SINR versus SIR (first experiment).

features of the available data vanish: in this case, the LTI equalizer benefits from a higher level of statistical accuracy, since its weights are estimated on the basis of KP samples, rather than K. However, the performance of the LTI equalizer exhibits a marked performance degradation as long as the SIR decreases, in which case the time-varying features of the data become preponderant.

In the second experiment, we exchanged the roles of the desired signal and interference. More specifically, the desired signal is now QPSK, whereas the interference is a BPSK CCI, with $f_I = 0$ and $\phi_I = 0$. Figure 3 reports the received eye diagram, together with those obtained by means of LTI and LPTV equalization, respectively, for a SIR of 0 dB. Moreover, Figure 4 reports the SINR in dB as a function of SIR, ranging from -10 to 40 dB. The results are very similar to those of the first experiment: also in this case, the LPTV equalizer exhibits a very robust performance with respect to SIR, whereas the LTI is slightly superior only when the SIR is very high.

5 CONCLUSIONS

In this paper, we proposed a new equalization structure, which is able not only to compensate for channel distortion (i.e., ISI) but also to mitigate the effect of strong interfering signals. In order to perform such a task, the structure of the equalizer needs to be LPTV rather than LTI. Simulation results show that the LPTV equalizer exhibits satisfactory performances for values of SIR well below 0 dB, which makes it a good candidate to separate different information

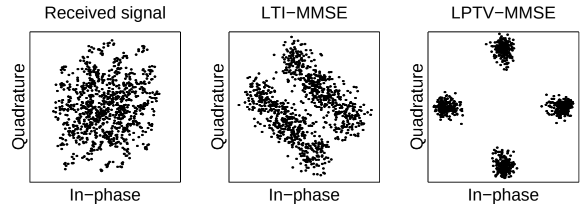

Figure 4: Eye-diagrams (second experiment, SIR = 0 dB).

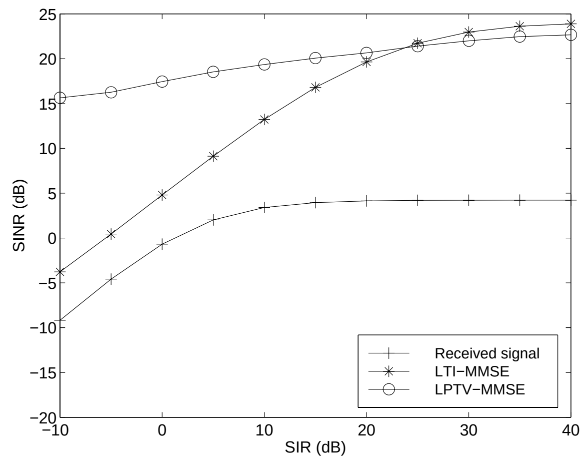

Figure 5: SINR versus SIR (second experiment).

streams (with different symbol rates) in a multi-rate system. It is worthwhile to note that similar conclusions hold [6] also when the LPTV equalizer operates blindly.

6 References

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