An MCDM Method under Neutrosophic Cubic Fuzzy Sets with Geometric Bonferroni Mean Operator

D. Ajay [1,*], Said Broumi [2], J. Aldring [3]

[1] Assistant Professor, Department of Mathematics, Sacred Heart College, Tamilnadu, India ; dajaypravin@gmail.com

[2] Laboratory of Information Processing, Faculty of Science Ben MSik, University Hassan II, Casablanca, Morocco ; broumisaid78@gmail.com

[3] Research Scholar, Department of Mathematics, Sacred Heart College, Tamilnadu, India ; jaldring24@gmail.com

[*]Correspondence: dajaypravin@gmail.com

Abstract. Neutrosophic cubic fuzzy sets ($NCFSs$) involve interval valued and single valued neutrosophic sets, and are used to describe uncertainty or fuzziness in a more efficient way. Aggregation of neutrosopic cubic fuzzy information is crucial and necessary in a decision making theory. In order to get a better solution to decision making problems under neutrosophic cubic fuzzy environment, this paper introduces an aggregating operator to neutrosophic cubic fuzzy sets with the help of Bonferroni mean and geometric mean, and proposes neutrosophic cubic fuzzy geometric Bonferroni mean operator ($NCFGBM^{u,v}$) with its properties. Then, an efficient decision making technique is introduced based on weighted operator $WNCFGBM_w^{u,v}$. An application of the established method is also examined for a real life problem.

Keywords: Neutrosophic Sets; Cubic Fuzzy Sets; Bonferroni Geometric Mean; Aggregation Operators; MCDM

---

1. Introduction

Fuzzy set [1] deals with fuzziness in terms of degree of truthness or membership within the range of interval $[0, 1]$. The traditional fuzzy sets are not efficient when the decision makers face more complex problems and it is difficult to quantify their truth values. Y.B.Jun et al. [2] introduced the notion of cubic sets which represents the degree of belongingness or certainty by interval valued fuzzy sets and single valued fuzzy sets simultaneously. Therefore, cubic sets are made up of two parts, where the first one is the interval valued fuzzy sets which represents belongingness in a particular range of interval, and the second one is exact belongingness or fuzzy sets.

Smarandache [3] introduced the philosophical idea of neutrosophic sets (NS) which is formulated from the general concept of fuzzy sets and many real life applications are avaliable under NS. Ajay, D., et al. used neutrosophic theory in fuzzy SAW method [4] and Abdel-Basset.M et al. utilized neutrosophic sets to asseses the uncertainty of linear time-cost tradeoffs [5] and also they applied to resource levelling problem in construction projects [6]. Further, biploar neutrosophic sets have been used in medical diagnosis [7] and decision making suituations [8]. Moreover, Y.B.Jun et al. [9] and M.Ali et al. [10] effectively utilized cubic fuzzy sets to the neutrosophic sets and introduced the concept of neutrosophic cubic fuzzy sets (NCFSs) with some basic operations. Therefore the hybrid form of neutrosophic cubic fuzzy set may be more adequate to address problems of more complexity using interval valued and exact valued neutrosophic information and it has been broadly used in the fields of MCDM [12–19]. Neutrosophic cubic fuzzy sets contain more information than general form of NS and therefore NCFSs provide better and efficient solution in MCDM.

Aggregating the fuzzy information plays an important role in decision theory and in particular decision making in real life problems. Variety of aggregating operators exist, but very few aggregating operators are available under neutrosophic cubic fuzzy numbers such as Heronian mean operators [21], Einstein Hybrid Geometric Aggregation Operators [22, 23], Dombi Aggregation Operators [24], weighted arithmetic averaging (NCNWAA) operator and weighted geometric averaging (NCNWGA) operator [25]. Still the Bonferroni geometric mean aggregating operator has not been studied in NCF environment. So the main purposes of this study are: (1) to establish a neutrosophic cubic fuzzy Bonferroni weighted geometric mean operator $WNCFBWGM_w^{u,v}$.(2) to develop an MCDM method using $WNCFBWGM_w^{u,v}$ operator to rank the alternatives under NCFS environment.

The content of the paper is organized as follows. Section 2 and 3 briefly introduce the basic concepts and operations of neutrosophic cubic fuzzy sets. The concepts of Bonferroni mean and geometric Bonferroni mean are explained in section 4. The neutrosophic cubic fuzzy geometric Bonferroni mean $NCFGBM^{u,v}$ and weighted neutrosophic cubic fuzzy geometric Bonferroni mean $WNCFGBM_w^{u,v}$ operators are established and examined with their properties in section 5. An MCDM method based on $WNCFGBM_w^{u,v}$ is presented in section 6. Finally conclusions and scope for future research are given in section 7.

2. **Neutrosophic Cubic Fuzzy Set**

Definition 2.1. [9] Let X be a non empty universal set or universe of discourse. A neutrosophic cubic fuzzy set $\tilde{S}$ in X is constructed in the following form:

$$\tilde{S} = \{x, \langle T(x), I(x), F(x)\rangle\,; \langle T_\lambda(x), I_\lambda(x), F_\lambda(x)\rangle\,|x \in X\}$$

where $T(x), I(x), F(x)$ are interval valued neutrosophic sets; $T(x) = [T^-(x), T^+(x)] \subseteq [0, 1]$ is the degree of truth interval values; $I(x) = [I^-(x), I^+(x)] \subseteq [0, 1]$ is the degree of indeterminacy interval values; $F(x) = [F^-(x), F^+(x)] \subseteq [0, 1]$ is the degree of falsity interval values; and $\langle T_\lambda(x), I_\lambda(x), F_\lambda(x)\rangle \in [0, 1]$ are truth, indeterminacy, and falsity degrees of membership values respectively. For convenience, a neutrosophic cubic fuzzy element in a neutrosophic cubic fuzzy set (NCFSs) $\tilde{S}$ is simply denoted by $\tilde{S} = \{\langle T, I, F\rangle ; \langle T_\lambda, I_\lambda, F_\lambda\rangle\}$, where $\langle T, I, F\rangle \subseteq [0, 1]$ and $\langle T_\lambda, I_\lambda, F_\lambda\rangle \in [0, 1]$, satisfying the conditions that $0 \leq \langle T^+, I^+, F^+\rangle \leq 3$ and $0 \leq \langle T_\lambda, I_\lambda, F_\lambda\rangle \leq 3$.

Definition 2.2. [10] Let $\tilde{S}$ be a neutrosophic cubic fuzzy set in X given by

$$\tilde{S} = \left\{ \left[T^-(x), T^+(x)\right], \left[I^-(x), I^+(x)\right], \left[F^-(x), F^+(x)\right]; \langle T_\lambda(x), I_\lambda(x), F_\lambda(x)\rangle \,|x \in X \right\}$$

$\tilde{S}$ is said to be internal NCFSs if $T^-(x) \leq T_\lambda(x) \leq T^+(x), I^-(x) \leq I_\lambda(x) \leq I^+(x), F^-(x) \leq F_\lambda(x) \leq F^+(x) \forall x$; $\tilde{S}$ is said to be external NCFSs if $T_\lambda(x) \notin [T^-(x), T^+(x)], I_\lambda(x) \notin [I^-(x), I^+(x)], F_\lambda(x) \notin [F^-(x), F^+(x)] \forall x$.

Definition 2.3. Let $\tilde{S}$ be a neutrosophic cubic fuzzy set in X. Then the support of neutrosophic cubic fuzzy set $\tilde{S}^*$ is defined by

$$\begin{aligned}\tilde{S}^* = \{ &\left[T^-(x), T^+(x)\right] \supset [0,0], \left[I^-(x), I^+(x)\right] \supset [0,0], \left[F^-(x), F^+(x)\right] \subset [1,1]; \\ &\langle T_\lambda(x) > 0, I_\lambda(x) > 0, F_\lambda(x) < 1\rangle \,|x \in X\}\end{aligned}$$

Definition 2.4. [25] Let $\tilde{S}$ be a non empty neutrosophic cubic fuzzy number given by

$$\begin{aligned}\tilde{S} &= \{x, \langle T(x), I(x), F(x)\rangle ; \langle T_\lambda(x), I_\lambda(x), F_\lambda(x)\rangle \,|x \in X\} \\ &= \left\{ \left[T^-(x), T^+(x)\right], \left[I^-(x), I^+(x)\right], \left[F^-(x), F^+(x)\right]; \langle T_\lambda(x), I_\lambda(x), F_\lambda(x)\rangle \,|x \in X \right\},\end{aligned}$$

then its score, accuracy and certainty functions can be defined respectively, as follows:

$$s(\tilde{S}) = \frac{\frac{\left[4+T^-(x)-I^-(x)-F^-(x)+T^+(x)-I^+(x)-F^+(x)\right]}{6} + \frac{\left[2+T_\lambda(x)-I_\lambda(x)-F_\lambda(x)\right]}{3}}{2}, \tag{1}$$

$$a(\tilde{S}) = \frac{\left[\left(T^-(x) - F^-(x) + T^+(x) - F^+(x)\right)/2 + T_\lambda(x) - F_\lambda(x)\right]}{2}, \tag{2}$$

$$c(\tilde{S}) = \frac{\left[\left(T^-(x) + T^+(x)\right)/2 + T_\lambda(x)\right]}{2}; \qquad s(\tilde{S}), a(\tilde{S}), c(\tilde{S}) \in [0, 1] \tag{3}$$

3. Operations on NCFNs

Let $A_i(x) = \left\{ \left[T_i^-, T_i^+\right], \left[I_i^-, I_i^+\right], \left[F_i^-, F_i^+\right]; \langle T_{\lambda i}, I_{\lambda i}, F_{\lambda i}\rangle \,|x \in X \right\} (i = 1, 2, 3, \cdots n)$ and $A_j(y) = \left\{ \left[T_j^-, T_j^+\right], \left[I_j^-, I_j^+\right], \left[F_j^-, F_j^+\right]; \langle T_{\lambda j}, I_{\lambda j}, F_{\lambda j}\rangle \,|y \in Y \right\} (j = 1, 2, 3, \cdots n)$ be two collections of NCFNs. Then the following operations are defined [25]:

(1) **Union**

$$A_i(x) \cup A_j(y) = \Big\{ \Big[min(T_i^-, T_j^-), max(T_i^+, T_j^+) \Big], \Big[max(I_i^-, I_j^-), min(I_i^+, I_j^+) \Big],$$
$$\Big[max(F_i^-, F_j^-), min(F_i^+, F_j^+) \Big]; \langle max(T_{\lambda i}, T_{\lambda j}), min(I_{\lambda i}, I_{\lambda j}), min(F_{\lambda i}, F_{\lambda j}) \rangle \Big\}$$

(2) **Intersection**

$$A_i(x) \cap A_j(y) = \Big\{ \Big[max(T_i^-, T_j^-), min(T_i^+, T_j^+) \Big], \Big[min(I_i^-, I_j^-), max(I_i^+, I_j^+) \Big],$$
$$\Big[min(F_i^-, F_j^-), max(F_i^+, F_j^+) \Big]; \langle min(T_{\lambda i}, T_{\lambda j}), max(I_{\lambda i}, I_{\lambda j}), max(F_{\lambda i}, F_{\lambda j}) \rangle \Big\}$$

(3) **Complement**

$$A_i^c(x) = \left\{ \left[F_i^-, F_i^+ \right], \left[1 - I_i^-, 1 - I_i^+ \right], \left[T_i^-, T_i^+ \right]; \langle F_{\lambda i}, 1 - I_{\lambda i}, T_{\lambda i} \rangle | x \in X \right\}$$

(4) $A_i(x) \subseteq A_j(y)$ if and only if $\left[T_i^-, T_i^+ \right] \subseteq \Big[T_j^-, T_j^+ \Big], \left[I_i^-, I_i^+ \right] \supseteq \Big[I_j^-, I_j^+ \Big], \left[F_i^-, F_i^+ \right] \supseteq \Big[F_j^-, F_j^+ \Big]$ and $T_{\lambda i} \leq T_{\lambda i}, I_{\lambda i} \geq I_{\lambda j}, F_{\lambda i} \geq F_{\lambda j} \forall x \in X, y \in Y$.

(5) $A_i(x) = A_j(y)$ if and only if $A_i(x) \subseteq A_j(y)$ and $A_i(x) \supseteq A_j(y)$ i.e. $\left[T_i^-, T_i^+ \right] = \Big[T_j^-, T_j^+ \Big], \left[I_i^-, I_i^+ \right] = \Big[I_j^-, I_j^+ \Big], \left[F_i^-, F_i^+ \right] = \Big[F_j^-, F_j^+ \Big]; \langle T_{\lambda i}, I_{\lambda i}, F_{\lambda i} \rangle = \langle T_{\lambda j}, I_{\lambda j}, F_{\lambda j} \rangle$

(6) For $\omega > 0$

$$\omega A_i = \left\{ \left[1 - \left(1 - T_i^- \right)^\omega, 1 - \left(1 - T_i^+ \right)^\omega \right], \left[\left(I_i^- \right)^\omega, \left(I_i^+ \right)^\omega \right], \left[\left(F_i^- \right)^\omega, \left(F_i^+ \right)^\omega \right]; \right.$$
$$\left. \langle 1 - (1 - T_{\lambda i})^\omega, (I_{\lambda i})^\omega, (F_{\lambda i})^\omega \rangle \right\}$$

(7) For $\omega > 0$

$$(A_i)^\omega = \left\{ \left[\left(T_i^- \right)^\omega, \left(T_i^+ \right)^\omega \right], \left[1 - \left(1 - I_i^- \right)^\omega, 1 - \left(1 - I_i^+ \right)^\omega \right], \right.$$
$$\left. \left[1 - \left(1 - F_i^- \right)^\omega, 1 - \left(1 - F_i^+ \right)^\omega \right]; \langle (T_{\lambda i})^\omega, 1 - (1 - I_{\lambda i})^\omega, 1 - (1 - F_{\lambda i})^\omega \rangle \right\}$$

(8) **Algebraic Sum**

$$A_i(x) \oplus A_j(y) = \Big\{ \Big[T_i^- + T_j^- - T_i^- T_j^-, T_i^+ + T_j^+ - T_i^+ T_j^+ \Big], \Big[I_i^- I_j^-, I_i^+ I_j^+ \Big],$$
$$\Big[F_i^- F_j^-, F_i^+ F_j^+ \Big]; \langle T_{\lambda i} + T_{\lambda j} - T_{\lambda i} T_{\lambda j}, I_{\lambda i} I_{\lambda j}, F_{\lambda i} F_{\lambda j} \rangle \Big\}$$

(9) **Algebraic Product**

$$A_i(x) \otimes A_j(y) = \Big\{ \Big[T_i^- T_j^-, T_i^+ T_j^+ \Big], \Big[I_i^- + I_j^- - I_i^- I_j^-, I_i^+ + I_j^+ - I_i^+ I_j^+ \Big],$$
$$\Big[F_i^- + F_j^- - F_i^- F_j^-, F_i^+ + F_j^+ - F_i^+ F_j^+ \Big]; \langle T_{\lambda i} T_{\lambda j}, I_{\lambda i} + I_{\lambda j} - I_{\lambda i} I_{\lambda j}, F_{\lambda i} + F_{\lambda j} - F_{\lambda i} F_{\lambda j} \rangle \Big\}$$

4. **Geometric Bonferroni Mean**

Bonferroni proposed the concept of Bonferroni mean (BM) which is defined as follows:

Definition 4.1. [11] Let $s_i (i = 1, 2, \ldots, n)$ be n number of positive crisp data. For any $u, v \geq 0$,

$$B^{u,v}(s_1, s_2, \ldots, s_n) = \left(\frac{1}{n(n-1)} \sum_{\substack{i,j=1, \\ i \neq j}}^{n} \left(s_i^u s_j^v \right) \right)^{\frac{1}{u+v}} \tag{4}$$

We call Eq.(4) as the Bonferroni mean (BM) operator. Especially, if v=0, Eq.(4) reduces to the generalized mean operator given by

$$\begin{aligned} B^{u,0}(s_1, s_2, \ldots, s_n) &= \left(\frac{1}{n} \sum_{i=1}^{n} s_i^u \left(\frac{1}{(n-1)} \sum_{\substack{j=1, \\ i \neq j}}^{n} \left(s_j^0 \right) \right) \right)^{\frac{1}{u+0}} \\ &= \left. \frac{1}{n} \sum_{i=1}^{n} s_i^u \right)^{\frac{1}{u}} \end{aligned} \tag{5}$$

If $u = 1$ and $v = 0$, the above equation produces the very known arithmetic mean (AM):

$$B^{1,0}(s_1, s_2, \ldots, s_n) = \frac{1}{n} \sum_{i=1}^{n} s_i^u \tag{6}$$

With the usual notion of geometric mean and the BM, the geometric Bonferroni mean operator is formulated.

Definition 4.2. Let $u, v > 0$, and $s_i (i = 1, 2, \ldots, n)$ be a collection of non negative crisp numbers. If

$$GB^{u,v}(s_1, s_2, \ldots, s_n) = \frac{1}{(u+v)} \prod_{\substack{i,j=1, \\ i \neq j}}^{n} (us_i + vs_j)^{\frac{1}{n(n-1)}} \tag{7}$$

then $GB^{u,v}$ is called the geometric Bonferroni mean (GBM).

Obviously, the GBM statisfies the following properties:

(1) $GB^{u,v}(0, 0, \ldots, 0) = 0$

(2) $GB^{u,v}(s_1, s_2, \ldots, s_n) = s$ if $s_i = s$, for all $i = 1, 2, \ldots, n$.

(3) $GB^{u,v}(s_1, s_2, \ldots, s_n) \geq GB^{u,v}(t_1, t_2, \ldots, t_n)$ if $s_i \geq t_i$ $\forall i$ that is, $GB^{u,v}$ is monotonic.

(4) $\text{Min}(s_i) \leq GB^{u,v} \leq \text{Max}(s_i)$.

Furthermore, if $v = 0$, $Eq.(7)$ generates the geometric mean:

$$GB^{u,0}(s_1, s_2, \ldots, s_n) = \frac{1}{u} \prod_{\substack{i,j=1, \\ i \neq j}}^{n} (us_i)^{\frac{1}{n(n-1)}} = \prod_{i=1}^{n} (s_i)^{\frac{1}{n}} \tag{8}$$

5. Neutrosophic Cubic Fuzzy Geometric Bonferroni Mean

Definition 5.1. Let $\tilde{S}_i = \left\{\left[T_i^-, T_i^+\right], \left[I_i^-, I_i^+\right], \left[F_i^-, F_i^+\right]; \langle T_{\lambda i}, I_{\lambda i}, F_{\lambda i}\rangle\right\}$ be a collection of neutrosophic cubic fuzzy numbers $(NCFN)$. For any $u, v > 0$,

$$NCFGBM^{u,v}\left(\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_n\right) = \frac{1}{u+v}\left(\bigotimes_{\substack{i,j=1,\\ i\neq j}}^{n}\left((u\tilde{S}_i \oplus v\tilde{S}_j)^{\frac{1}{n(n-1)}}\right)\right)$$

is called the neutrosophic cubic fuzzy geometric bonferroni mean operator.

Theorem 5.2. *Let $u, v > 0$ and $\tilde{S}_i = \left\{\left[T_i^-, T_i^+\right], \left[I_i^-, I_i^+\right], \left[F_i^-, F_i^+\right]; \langle T_{\lambda i}, I_{\lambda i}, F_{\lambda i}\rangle\right\}$ be a collection of neutrosophic cubic fuzzy numbers $(NCFN)$. Then the aggregated value is calculated using the operator $NCFGBM^{u,v}$*

$$NCFGBM^{u,v}\left(\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_n\right) = \frac{1}{u+v}\left(\bigotimes_{\substack{i,j=1,\\ i\neq j}}^{n}\left((u\tilde{S}_i \oplus v\tilde{S}_j)^{\frac{1}{n(n-1)}}\right)\right)$$

$$= \left\{\left[1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-T_i^-)^u(1-T_j^-)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}},\right.\right.$$

$$\left. 1-\left(1-\prod_{\substack{i,j=1,\\ i\neq j}}^{n}\left(1-(1-T_i^+)^u(1-T_j^+)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right],$$

$$\left[\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(I_i^-)^u(I_j^-)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}, \left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(I_i^+)^u(I_j^+)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right], \quad (9)$$

$$\left[\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(F_i^-)^u(F_j^-)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}, \left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(F_i^+)^u(F_j^+)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right];$$

$$\left\langle 1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}(1-(1-T_{\lambda i})^u(1-T_{\lambda j})^v)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}},\right.$$

$$\left.\left.\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}(1-(I_{\lambda i})^u(I_{\lambda j})^v)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}, \left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}(1-(F_{\lambda i})^u(F_{\lambda j})^v)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right\rangle\right\}.$$

Proof. . Using the operational laws on $NCFN$ described in section (3), we have

$$u\tilde{S}_i = \{ [1-(1-T_i^-)^u, 1-(1-T_i^+)^u], [(I_i^-)^u, (I_i^+)^u], [(F_i^-)^u, (F_i^+)^u]; \langle 1-(1-T_{\lambda i})^u, (I_{\lambda i})^u, (F_{\lambda i})^u \rangle \}$$

$$v\tilde{S}_j = \left\{ \left[1-\left(1-T_j^-\right)^v, 1-\left(1-T_j^+\right)^v\right], \left[\left(I_j^-\right)^v, \left(I_j^+\right)^v\right], \left[\left(F_j^-\right)^v, \left(F_j^+\right)^v\right]; \langle 1-(1-T_{\lambda j})^v, (I_{\lambda j})^v, (F_{\lambda j})^v \rangle \right\}$$

$$u\tilde{S}_i \oplus v\tilde{S}_j = \left\{ \left[1-(1-T_i^-)^u(1-T_j^-)^v, 1-(1-T_i^+)^u(1-T_j^+)^v\right], \left[(I_i^-)^u(I_j^-)^v, (I_i^+)^u(I_j^+)^v\right], \left[(F_i^-)^u(F_j^-)^v, (F_i^+)^u(F_j^+)^v\right]; \langle 1-(1-T_{\lambda i})^u(1-T_{\lambda j})^v, (I_{\lambda i})^u(I_{\lambda j})^v, (F_{\lambda i})^u(F_{\lambda j})^v \rangle \right\}.$$

Next, we have the following equation which has been derived by Xu and Yager [28].

$$\begin{aligned} &\bigotimes_{\substack{i,j=1,\\ i\neq j}}^{n} \left(u\tilde{S}_i \oplus v\tilde{S}_j\right)^{\frac{1}{n(n-1)}} \\ &= \Bigg\{ \left[\prod_{\substack{i,j=1,\\ i\neq j}}^{n} \left(1-(1-T_i^-)^u(1-T_j^-)^v\right)^{\frac{1}{n(n-1)}}, \prod_{\substack{i,j=1,\\ i\neq j}}^{n} \left(1-(1-T_j^+)^u(1-T_j^+)^v\right)^{\frac{1}{n(n-1)}} \right], \\ &\left[1-\prod_{\substack{i,j=1,\\ i\neq j}}^{n} \left(1-(I_i^-)^u(I_j^-)^v\right)^{\frac{1}{n(n-1)}}, 1-\prod_{\substack{i,j=1,\\ i\neq j}}^{n} \left(1-(I_i^+)^u(I_j^+)^v\right)^{\frac{1}{n(n-1)}} \right], \\ &\left[1-\prod_{\substack{i,j=1,\\ i\neq j}}^{n} \left(1-(F_i^-)^u(F_j^-)^v\right)^{\frac{1}{n(n-1)}}, 1-\prod_{\substack{i,j=1,\\ i\neq j}}^{n} \left(1-(F_i^+)^u(F_j^+)^v\right)^{\frac{1}{n(n-1)}} \right]; \\ &\Bigg\langle \prod_{\substack{i,j=1,\\ i\neq j}}^{n} (1-(1-T_{\lambda i})^u(1-T_{\lambda j})^v)^{\frac{1}{n(n-1)}}, 1-\prod_{\substack{i,j=1,\\ i\neq j}}^{n} \left(1-(I_{\lambda i}^u)(I_{\lambda j}^v)\right)^{\frac{1}{n(n-1)}}, \\ &1-\prod_{\substack{i,j=1,\\ i\neq j}}^{n} \left(1-(F_{\lambda i}^u)(F_{\lambda j}^v)\right)^{\frac{1}{n(n-1)}} \Bigg\rangle \Bigg\}. \end{aligned} \tag{10}$$

Using NCF operational laws, Eq.(10) yields neutrosophic cubic fuzzy geometric bonferroni mean operator $NCFGBM^{u,v}(\tilde{S}_1, \tilde{S}_2, \cdots, \tilde{S}_n)$ given by $Eq.(9)$. In addition, it satisfies the following conditions

$$\left[1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-T_i^-)^u(1-T_j^-)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}},\right.$$

$$\left.1-\left(1-\prod_{\substack{i,j=1,\\ i\neq j}}^{n}\left(1-(1-T_i^+)^u(1-T_j^+)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right]\subseteq[0,,1],$$

$$\left[\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(I_i^-)^u(I_j^-)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}},\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(I_i^+)^u(I_j^+)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right]\subseteq[0,1],$$

$$\left[\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(F_i^-)^u(F_j^-)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}},\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(F_i^+)^u(F_j^+)^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right]\subseteq[0,1];$$

$$0\leq 1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}(1-(1-T_{\lambda i})^u(1-T_{\lambda j})^v)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\leq 1,$$

$$0\leq\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}(1-(I_{\lambda i})^u(I_{\lambda j})^v)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\leq 1,$$

$$0\leq\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}(1-(F_{\lambda i})^u(F_{\lambda j})^v)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\leq 1$$

which completes the proof of the theorem. □

We discuss some of the important properties of the $NCFGBM^{u,v}$:

(1) **Idempotency**: Suppose the colletive data of neutrosophic cubic fuzzy numbers $\tilde{S}_i=\left\{\left[T_i^-,T_i^+\right],\left[I_i^-,I_i^+\right],\left[F_i^-,F_i^+\right];\langle T_{\lambda i},I_{\lambda i},F_{\lambda i}\rangle\right\}(i=1,2,3,\cdots n)$ are equal, for

any $u, v > 0$, the aggregate operator be

$$\begin{aligned} NCFGBM^{u,v}(\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_n) &= NCFGBM^{u,v}(\tilde{S}, \tilde{S}, ..., \tilde{S}) \\ &= \frac{1}{u+v}\left(\bigotimes_{\substack{i,j=1,\\ i\neq j}}^{n}\left(\left(u\tilde{S}\oplus v\tilde{S}\right)^{\frac{1}{n(n-1)}}\right)\right) \\ &= \frac{1}{u+v}\left(\bigotimes_{\substack{i,j=1,\\ i\neq j}}^{n}\left((u+v)\,\tilde{S}\right)^{\frac{1}{n(n-1)}}\right) \\ &= \frac{1}{u+v}\left((u+v)\,\tilde{S}\right)^{\frac{n(n-1)}{n(n-1)}} = \tilde{S} \end{aligned} \tag{11}$$

(2) **Commuatativity**: Let $\tilde{S}_i(i = 1, 2, 3, \cdots n)$ be a collection of neutrosophic cubic numbers. For any $u, v > 0$,

$$NCFGBM^{u,v}\left(\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_n\right) = NCFGBM^{u,v}\left(\dot{\tilde{S}}_1, \dot{\tilde{S}}_2, \ldots, \dot{\tilde{S}}_n\right) \tag{12}$$

Let $\left(\dot{\tilde{S}}_1, \dot{\tilde{S}}_2, \ldots, \dot{\tilde{S}}_n\right)$ be any permuation of $\left(\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_n\right)$. Then

$$\begin{aligned} NCFGBM^{u,v}\left(\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_n\right) &= \frac{1}{u+v}\left(\bigotimes_{\substack{i,j=1,\\ i\neq j}}^{n}\left((u\tilde{S}_i\oplus v\tilde{S}_j)^{\frac{1}{n(n-1)}}\right)\right) \\ &= \frac{1}{u+v}\left(\bigotimes_{\substack{i,j=1,\\ i\neq j}}^{n}\left((u\dot{\tilde{S}}_i\oplus v\dot{\tilde{S}}_j)^{\frac{1}{n(n-1)}}\right)\right) \\ &= NCFGBM^{u,v}\left(\dot{\tilde{S}}_1, \dot{\tilde{S}}_2, \ldots, \dot{\tilde{S}}_n\right) \end{aligned}$$

(3) **Monotonicity**: Let $\tilde{S}_i$ $(i = 1, 2, 3, \cdots n)$ and $\tilde{S}_j$ $(j = 1, 2, 3, \cdots n)$ be two collections of neutrosophic cubic numbers. For any $u, v > 0$, if $[T_i^-, T_i^+], \subseteq [T_j^-, T_j^+], [I_i^-, I_i^+] \supseteq [I_j^-, I_j^+], [F_i^-, F_i^+] \supseteq [F_j^-, F_j^+]; T_{\lambda i} \leq T_{\lambda j}, I_{\lambda i} \geq I_{\lambda j}, F_{\lambda i} \geq F_{\lambda j} \quad (\forall i, j = 1, 2, 3, \ldots n)$, Then

$$NCFGBM^{u,v}\left(\tilde{S}_i\right) \leq NCFGBM^{u,v}\left(\tilde{S}_j\right) \tag{13}$$

(4) **Boundedness**: Let $\tilde{S}_i = \left\{\left[T_i^-, T_i^+\right], \left[I_i^-, I_i^+\right], \left[F_i^-, F_i^+\right]; \langle T_{\lambda i}, I_{\lambda i}, F_{\lambda i}\rangle\right\} (i = 1, 2, 3, \cdots n)$ be a collection of neutrosophic cubic fuzzy numbers, and let

$$\tilde{S}_i^- = \left\{inf\left(\left[T_i^-, T_i^+\right]\right), sup\left(\left[I_i^-, I_i^+\right]\right), sup\left(\left[F_i^-, F_i^+\right]\right); min\left(T_{\lambda i}\right), max\left(I_{\lambda i}\right), max\left(F_{\lambda i}\right)\right\},$$

$$\tilde{S}_i^+ = \left\{sup\left(\left[T_i^-, T_i^+\right]\right), inf\left(\left[I_i^-, I_i^+\right]\right), inf\left(\left[F_i^-, F_i^+\right]\right); max\left(T_{\lambda i}\right), min\left(I_{\lambda i}\right), min\left(F_{\lambda i}\right)\right\}.$$

For any $u, v > 0$,

$$\tilde{S}_i^- \leq NCFGBM^{u,v}\left(\tilde{S}_i\right)(i = 1, 2, 3, \ldots n) \leq \tilde{S}_i^+ \tag{14}$$

Thus the boundedness is easily obtained.

If parameters u and v are modified in $NCFGBM^{u,v}$, then a special case can be obtained as follows:

If $v \to 0$, then by equation (9), we have

$$NCFGBM^{u,v}\left(\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_n\right) = \frac{1}{u+v}\left(\bigotimes_{\substack{i,j=1,\\ i\neq j}}^{n}\left(\left(u\tilde{S}_i \oplus v\tilde{S}_j\right)^{\frac{1}{n(n-1)}}\right)\right) = \frac{1}{u}\left(\bigotimes_{i=1}^{n}\left((u\tilde{S}_i)^{\frac{1}{n}}\right)\right)$$

$$= \left\{\left[1-\left(1-\prod_{i=1}^{n}\left(1-(1-T_i^-)^u\right)^{\frac{1}{n}}\right)^{\frac{1}{u}}, 1- \quad 1-\prod_{i=1}^{n}\left(1-(1-T_i^+)^u\right)^{\frac{1}{n}}\right)^{\frac{1}{u}}\right],\right.$$

$$\left[\ 1-\prod_{i=1}^{n}\left(1-(I_i^-)^u\right)^{\frac{1}{n}}\right)^{\frac{1}{u}},\ 1-\prod_{i=1}^{n}\left(1-(I_i^+)^u\right)^{\frac{1}{n}}\right)^{\frac{1}{u}}\right],$$

$$\left[\ 1-\prod_{i=1}^{n}\left(1-(F_i^-)^u\right)^{\frac{1}{n}}\right)^{\frac{1}{u}},\ 1-\prod_{i=1}^{n}\left(1-(F_i^+)^u\right)^{\frac{1}{n}}\right)^{\frac{1}{u}}\right];$$

$$\left.\left\langle 1- \quad 1-\prod_{i=1}^{n}\left(1-(1-T_{\lambda i})^u\right)^{\frac{1}{n}}\right)^{\frac{1}{u}},\ 1-\prod_{i=1}^{n}\left(1-(I_{\lambda i})^u\right)^{\frac{1}{n}}\right)^{\frac{1}{u}},\ 1-\prod_{i=1}^{n}\left(1-(F_{\lambda i})^u\right)^{\frac{1}{n}}\right)^{\frac{1}{u}}\right\rangle\right\}.$$

which we call the generalized neutrosophic cubic fuzzy geometric mean ($NCFBGM^{u,v}$).

5.1. *Weighted Neutrosophic Cubic Fuzzy Bonferroni Geometric Mean*

Generally weighted aggregating operator plays a significant role in decision-making processes to aggregate information. Therefore we propose a weighted aggregate operator based on neutrosophic cubic fuzzy bonferroni geometric mean ($WNCFGBM_w^{u,v}$).

Definition 5.3. Let $\tilde{S}_i = \left\{\left[T_i^-, T_i^+\right], \left[I_i^-, I_i^+\right], \left[F_i^-, F_i^+\right]; \langle T_{\lambda i}, I_{\lambda i}, F_{\lambda i}\rangle\right\}$ be a collection of neutrosophic cubic numbers (NCN), and $w = (W_1, W_2, \ldots, W_n)^T$ the wieght vector of $\tilde{S}_i = \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n$, where w_i indicates the importance degree of $\tilde{S}_i$ such that $w_i > 0$ and $\sum_{i=1}^{n} w_i = 1$ $(i = 1, 2, 3, \ldots, n)$. For any $u, v > 0$,

$$WNCFGBM_w^{u,v}\left(\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_n\right) = \frac{1}{u+v}\left(\bigotimes_{\substack{i,j=1,\\ i\neq j}}^{n}\left(\left(u(\tilde{S}_i)^{w_i} \oplus v(\tilde{S}_j)^{w_j}\right)^{\frac{1}{n(n-1)}}\right)\right) \tag{15}$$

is called the weighted neutrosophic cubic fuzzy geometric bonferroni mean operator.

Theorem 5.4. *Let $u, v > 0$ and $\tilde{S}_i$ $(i = 1, 2, 3, \ldots, n)$ be a collection of neutrosophic cubic fuzzy numbers (NCFN), whose weight vector is $w_i = (W_1, W_2, \ldots, W_n)^T$, which satisfies that*

$w_i > 0$, *and* $\sum_{i=1}^{n} w_i = 1$ $(i = 1, 2, 3, \ldots, n)$. *Then the aggregated value using the operator is*

$$WNCFGBM_w^{u,v}(\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_n) = \frac{1}{u+v}\left(\bigotimes_{\substack{i,j=1,\\ i\neq j}}^{n}\left(\left(u(\tilde{S}_i)^{w_i} \oplus v(\tilde{S}_j)^{w_j}\right)^{\frac{1}{n(n-1)}}\right)\right)$$

$$= \left\{\left[1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-(T_i^-)^{w_i})^u(1-(T_j^-)^{w_j})^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}},\right.\right.$$

$$\left. 1-\left(1-\prod_{\substack{i,j=1,\\ i\neq j}}^{n}\left(1-(1-(T_i^+)^{w_i})^u(1-(T_j^+)^{w_j})^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right],$$

$$\left[\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-(1-I_i^-)^{w_i})^u(1-(1-I_j^-)^{w_j})^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}},\right.$$

$$\left.\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-(1-I_i^+)^{w_i})^u(1-(1-I_j^+)^{w_j})^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right], \tag{16}$$

$$\left[\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-(1-F_i^-)^{w_i})^u(1-(1-F_j^-)^{w_j})^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}},\right.$$

$$\left.\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}\left(1-(1-(1-F_i^+)^{w_i})^u(1-(1-F_j^+)^{w_j})^v\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right];$$

$$\left\langle 1-\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}(1-(1-(T_{\lambda i})^{w_i})^u(1-(T_{\lambda j})^{w_j})^v)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}},\right.$$

$$\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}(1-(1-(1-I_{\lambda i})^{w_i})^u(1-(1-I_{\lambda j})^{w_j})^v)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}},$$

$$\left.\left.\left(1-\prod_{\substack{i,j=1\\ i\neq j}}^{n}(1-(1-(1-F_{\lambda i})^{w_i})^u(1-(1-F_{\lambda j})^{w_j})^v)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{u+v}}\right\rangle\right\}.$$

Proof. The proof is identical with the proof of theorem (5.2) and therefore is omitted. □

6. An application of weighted neutrosophic cubic fuzzy geometric bonferroni mean operator to MCDM problems

In this section, we propose an algorithm for MCDM method based on neutrosophic cubic fuzzy geometric Bonferroni mean operators and illustrate it with a numerical example.

Algorithm. Let $\tilde{A}_i = \{\tilde{\gamma}_1, \tilde{\gamma}_2, \ldots, \tilde{\gamma}_n\}$ and $\tilde{C}_j = \{\tilde{\eta}_1, \tilde{\eta}_2, \ldots, \tilde{\eta}_m\}$ be collections of n alternatives and m attributes respectively. According to the appropriate weight of attributes $(\widehat{\omega}_j)^T = \{\tilde{\omega}_1, \tilde{\omega}_2, \ldots \tilde{\omega}_m\}$ is determined, which satisfies the condition that $\tilde{\omega}_j > 0$ and $\sum \widehat{\omega}_j = 1$. Then the following steps are used in process of MCDM method.

Step 1. Construct neutrosophic cubic fuzzy decision matrix $D = [N_{ij}]_{n\times m}$.

Step 2. The decision matrix is aggregated using $NCFGBM^{u,v}$ or $WNCFGBM_w^{u,v}$ to m attributes.

Step 3. Utilize the score formula (Eq.1) to calculate the values of $s(\tilde{A}_i)$

Step 4. The n alternatives are ranked according to their score values

6.1. *Numerical Example and Investigation*

An illustrative example on the selection problem of investment alternatives is adapted (Ref. [25, 26]) to validate the proposed MCDM method with NCF data. A company wants a sum of money to be invested in an industry. Then the committee suggests the following four feasible alternatives: (a) $\tilde{\gamma}_1$ is a textile company; (b) $\tilde{\gamma}_2$ is an automobile company; (c) $\tilde{\gamma}_3$ is a computer company; (d) $\tilde{\gamma}_4$ is a software company. Suppose that three attributes namely, (1) $\tilde{\eta}_1$ is the risk; (2) $\tilde{\eta}_2$ is the growth; (3) $\tilde{\eta}_3$ is the environmental impact; are taken into the evaluation requirements of the alternatives. The weight vectors of the three attributes $\tilde{\eta}_j (j = 1, 2, 3)$ are $(\widehat{\omega}_j)^T = (0.32, 0.38, 0.3)$ respectively. Then the experts or decision makers are asked to evaluate each alternative on attributes by the form of NCFNs. Thus, the assessment data can be represented by neutrosophic cubic decision matrix $D = [S_{ij}]_{m\times n}$.

step 1. Neutrosophic cubic fuzzy decision matrix $D = [S_{ij}]_{4\times 3}$

$$D = \begin{bmatrix} \begin{pmatrix} [0.5,0.6], & [0.1,0.3], \\ [0.2,0.4]; & \langle 0.6,0.2,0.3\rangle \end{pmatrix}, & \begin{pmatrix} [0.5,0.6], & [0.1,0.3], \\ [0.2,0.4]; & \langle 0.6,0.2,0.3\rangle \end{pmatrix}, & \begin{pmatrix} [0.2,0.4], & [0.7,0.8], \\ [0.8,0.9]; & \langle 0.3,0.8,0.9\rangle \end{pmatrix} \\ \begin{pmatrix} [0.6,0.8], & [0.1,0.2], \\ [0.2,0.3]; & \langle 0.7,0.1,0.2\rangle \end{pmatrix}, & \begin{pmatrix} [0.6,0.7], & [0.1,0.2], \\ [0.2,0.3]; & \langle 0.6,0.1,0.2\rangle \end{pmatrix}, & \begin{pmatrix} [0.3,0.4], & [0.6,0.7], \\ [0.8,0.9]; & \langle 0.3,0.6,0.9\rangle \end{pmatrix} \\ \begin{pmatrix} [0.4,0.6], & [0.2,0.3], \\ [0.1,0.3]; & \langle 0.6,0.2,0.2\rangle \end{pmatrix}, & \begin{pmatrix} [0.5,0.6], & [0.2,0.3], \\ [0.3,0.4]; & \langle 0.6,0.3,0.4\rangle \end{pmatrix}, & \begin{pmatrix} [0.3,0.5], & [0.7,0.8], \\ [0.6,0.7]; & \langle 0.4,0.8,0.7\rangle \end{pmatrix} \\ \begin{pmatrix} [0.7,0.8], & [0.1,0.2], \\ [0.1,0.2]; & \langle 0.8,0.1,0.2\rangle \end{pmatrix}, & \begin{pmatrix} [0.6,0.7], & [0.1,0.2], \\ [0.1,0.3]; & \langle 0.7,0.1,0.2\rangle \end{pmatrix}, & \begin{pmatrix} [0.3,0.4], & [0.6,0.7], \\ [0.7,0.8]; & \langle 0.3,0.7,0.8\rangle \end{pmatrix} \end{bmatrix}$$

step 2. The decision matrix is aggregated by $WNCFGBM_w^{u,v}(\tilde{S}_{i1}, \tilde{S}_{i2}, \tilde{S}_{i3})(i = 1, 2, \ldots n)$ operators (Using Eq.16) to the three $(\tilde{\eta}_j, j = 1, 2, 3)$ attributes.

If we take the parameter values $u = v = 1$, then using $\tilde{A}_i = WNCFGBM_w^{(1,1)}$, we get the following values

$$\tilde{A}_1 = \{[0.7345, 0.8126], [0.0881, 0.1861], [0.1453, 0.2523]; \langle 0.7951, 0.1453, 0.2093 \rangle\},$$
$$\tilde{A}_2 = \{[0.7951, 0.8635], [0.0790, 0.1307], [0.1453, 0.2093]; \langle 0.8124, 0.0790, 0.1642 \rangle\},$$
$$\tilde{A}_3 = \{[0.7378, 0.8287], [0.1307, 0.1861], [0.1195, 0.1876]; \langle 0.8126, 0.1674, 0.1703 \rangle\},$$
$$\tilde{A}_4 = \{[0.8124, 0.8635], [0.0790, 0.1307], [0.0881, 0.1674]; \langle 0.8491, 0.0881, 0.1453 \rangle\}.$$

step 3. Utilizing Eq.(1), the score values $s(\tilde{A}_i)$ are found

$$s(\tilde{A}_1) = 0.8130, s(\tilde{A}_2) = 0.8527, s(\tilde{A}_3) = 0.8244, s(\tilde{A}_4) = 0.8702.$$

step 4. Since the values $s(\tilde{A}_4) > s(\tilde{A}_2) > s(\tilde{A}_3) > s(\tilde{A}_1)$, the rank of alternatives are in the order of $\tilde{\gamma}_4 > \tilde{\gamma}_2 > \tilde{\gamma}_3 > \tilde{\gamma}_1$.

From the results, we could see that the ranking order and the best choice of alternatives are the same as the results in [25, 26].

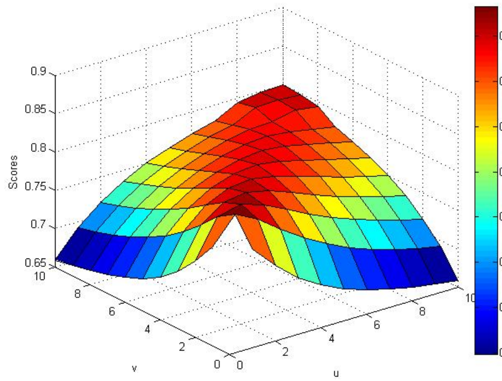
If the parameters $u = v = 2$, then using $\tilde{A}_i = WNCFGBM_w^{(2,2)}$, we get the following aggregate values

$$\tilde{A}_1 = \{[0.7306, 0.8111], [0.0950, 0.1940], [0.1542, 0.2619]; \langle 0.7916, 0.1542, 0.2204 \rangle\},$$
$$\tilde{A}_2 = \{[0.7916, 0.8563], [0.0847, 0.1376], [0.1542, 0.2204]; \langle 0.8055, 0.0847, 0.1757 \rangle\},$$
$$\tilde{A}_3 = \{[0.7371, 0.8283], [0.1376, 0.1940], [0.1354, 0.1945]; \langle 0.8111, 0.1797, 0.1841 \rangle\},$$
$$\tilde{A}_4 = \{[0.8055, 0.8563], [0.0847, 0.1376], [0.0950, 0.1797]; \langle 0.8395, 0.0950, 0.1542 \rangle\}.$$

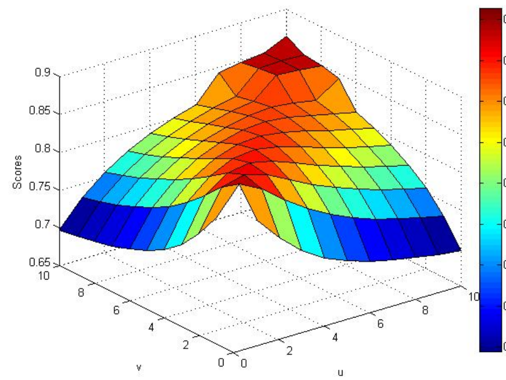
Then we calculate the score of the alternatives $s(\tilde{A}_1) = 0.8059$, $s(\tilde{A}_2) = 0.8451$, $s(\tilde{A}_3) = 0.8165$, $s(\tilde{A}_4) = 0.8621$.

Since $s(\tilde{A}_4) > s(\tilde{A}_2) > s(\tilde{A}_3) > s(\tilde{A}_1)$, the order of the rank is $\tilde{\gamma}_4 > \tilde{\gamma}_2 > \tilde{\gamma}_3 > \tilde{\gamma}_1$.

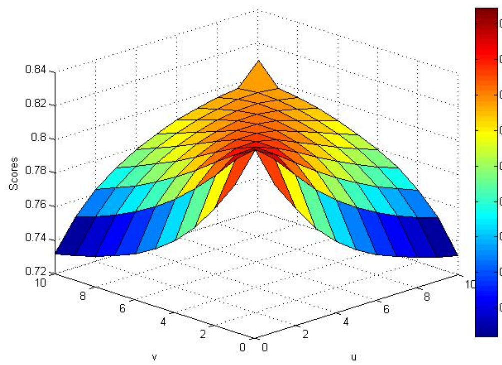
As the values of parameters u and v change according to the subjective preference of the decision maker, we can find that the ranking order of the alternatives are the same, which indicates that the proposed method can obtain the most optimistic results than the existing MCDM methods based on GBM [29]. For a detailed comparision, we represent the scores of each alternatives in Fig.1 by changing the values of parameters u, v between 0 and 10.



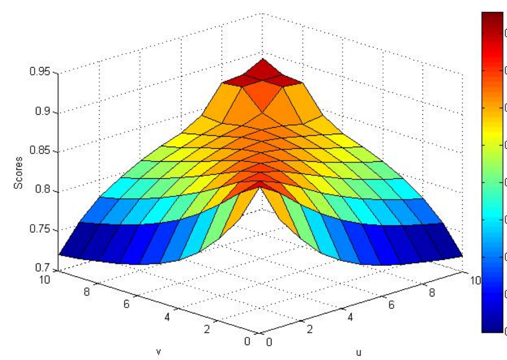
(A) Scores of alternative $\tilde{\gamma}_1$



(B) Scores of alternative $\tilde{\gamma}_2$



(C) Scores of alternative $\tilde{\gamma}_3$



(D) Scores of alternative $\tilde{\gamma}_4$

FIGURE 1. Scores of alternative $\tilde{\gamma}_i$ obtained by $WNCFGBM_w^{u,v}$

7. Conclusions

In this paper, we have applied geometric Bonferroni mean to neutrosophic cubic fuzzy sets. A new aggregating operator $NCFGBM^{u,v}$ has been established and its properties are discussed. The MCDM method is developed based on the weighted operator $WNCFGBM_w^{u,v}$ and is verified with a numerical example where four alternatives are ranked under three criteria. The graphical representation of the results depicted above shows that the ranking of the alternatives remains unaffected when the parameters are changed due to subjective preferences. This proves that the method is objective and moreover the result obtained, when compared with the results of existing techniques, shows that the proposed method is more effective in dealing with neutrosophic fuzzy information. In future, $NCFGBM^{u,v}$ operator could be applied to various other MCDM methods.

References

[1] Zadeh, L.A. (1965). Fuzzy sets. Inf Control, 8, 338-353

[2] Jun, Y.B., Kim, C.S., Yang, K.O. (2012). Cubic sets. Annals of Fuzzy Mathematics and Informatics, 4, 83-98

[3] Smarandache, F. (1999). A Unifying Field in Logics, Neutrosophic Logic, Neutrosophy, Neutrosophic Set and Neutrosophic Probabilty. 4th (eds) American Research Press, Rehoboth, DE, USA

[4] Ajay, D., Manivel, M., Aldring, J. (2019). Neutrosophic Fuzzy SAW Method and Its Application. The International journal of analytical and experimental modal analysis, Vol. XI, Issue VIII, 881-887

[5] Abdel-Basset, M., Mumtaz, A., Asmaa, A. (2020). Uncertainty assessments of linear time-cost tradeoffs using neutrosophic set. Computers and Industrial Engineering. https://doi.org/10.1016/j.cie. 2020.106286

[6] Abdel-Basset, M., Mumtaz, A., Asma, A. (2019) Resource levelling problem in construction projects under neutrosophic environment. The Journal of Supercomputing. https://doi.org/10.1007/s11227-019-03055-6

[7] Abdel-Basset, M., Mohamed, M., Elhoseny, M., Chiclana, F., Zaied, A. E. N. H. (2019). Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases. Artificial Intelligence in Medicine, 101, 101735

[8] Ajay, D., Aldring, J. (2019). A Decision Making Technique Based on Similarity Measure and Entropy of Bipolar Neutrosophic Sets. The International journal of analytical and experimental modal analysis, Vol. XI, Issue IX, 520-529

[9] Jun, Y.B., Smarandache, F., Kim, C.S. (2017). Neutrosophic Cubic Sets. New Mathematics and Natural Computation, 13, 41-54

[10] Alia, M., Delib, I., Smarandache, F. (2016). The theory of neutrosophic cubic sets and their applications in pattern recognition. Journal of Intelligent and Fuzzy Systems, 30, 1-7

[11] Bonferroni, C. (1950). Sulle Medie multiple di potenze. Bolletino Matematica Italiana, 5, 267-270

[12] Banerjee, D., Giri, B.C., Pramanik, S. (2017). GRA for multi attribute decision making in neutrosophic cubic set environment. Neutrosophic Sets and Systems, 15, 60-69

[13] Pramanik, S., Dalapati, S., Alam, S. (2017). Neutrosophic cubic MCGDM method based on similarity measure. Neutrosophic Sets and Systems, 16, 44-56

[14] Luand, Z., Ye, J. (2017). Cosine measures of neutrosophic cubic sets for multiple attribute decision-making. Symmetry, 9, 1-10

[15] Paramanik, S., Dey, P.P., Giri, B.C., Smarandache, F. (2017). An extended TOPSIS for multi-attribute decision making problems with neutrosophic cubic information. Neutrosophic Sets and Systems, 17, 20-28

[16] Zhan, J., Khan, M., Gulistan, M., Ali, A. (2017). Applications of neutrosophic cubic sets in multi-criteria decision-making. International Journal for Uncertainty Quantication, 7, 377-394

[17] Ye, J. (2017). Linguistic neutrosophic cubic numbers and their multiple attribute decision-making method. Information, 8

[18] Cui, W.H., Ye, J. (2019). Logarithmic similarity measure of dynamic neutrosophic cubic sets and its application in medical diagonsis. computers in industry, 111, 198-206

[19] Xue, H., Yu, M., Chen, C. (2019). Research on Novel Correlation Coefficient of Neutrosophic Cubic Sets and Its Applications. Mathematical Problems in Engineering. https://doi.org/10.1155/2019/7453025

[20] Pramanik, S., Dalapati, S., Alam, S., Roy, T.K. (2018). NC-VIKOR Based MAGDM Strategy under Neutrosophic Cubic Set Environment. Neutrosophic Sets and Systems, 19

[21] Gulistan, M., Mohammad, M., Karaaslan, F., Kadry, S., Khan, S., Wahab, H.A. (2019). Neutrosophic cubic Heronian mean operators with applications in multiple attribute group decision-making using cosine similarity functions. International Journal of Distributed Sensor Networks. https://doi.org/10.1177/1550147719877613

[22] Gulistan, M., Khan, M., Kadry, S., Alhazaymeh, K. (2019). Neutrosophic Cubic Einstein Hybrid Geometric Aggregation Operators with Application in Prioritization Using Multiple Attribute Decision-Making Method. Mathematics. https://doi.org/10.3390/math7040346

[23] Khan, M., Gulistan, M., Yaqoob, N., Khan, M., Smarandache, F. (2019). Neutrosophic Cubic Einstein Geometric Aggregation Operators with Application to Multi-Criteria Decision Making Method. Symmetry. https://doi.org/10.3390/sym11020247

[24] Shi, L., Ye, J. (2019). Dombi Aggregation Operators of Neutrosophic Cubic Sets for Multiple Attribute Decision-Making. Algorithms. https://doi.org/10.3390/a11030029

[25] Ye, J. (2018). Operations and aggregation method of neutrosophic cubic numbers for multiple attribute decision-making. Soft Computing. https://doi.org/10.1007/s00500-018-3194-x

[26] Lu, Z., Ye, J. (2017). Cosine Measures of Neutrosophic Cubic Sets for Multiple Attribute Decision-Making. Symmetry. https://doi.org/10.3390/sym9070121

[27] Jun, Y.B., Smarandache, F., Ki, C.S. (2017). P-union and P-intersection of neutrosophic cubic sets. An. St. Univ. Ovidius Constanta. https://doi.org/10.1515/auom-2017-0009

[28] Xu, Z.S., yager, R.R. (2006). Some geometric aggregtion operators based on intuitionistic fuzzy sets. International journal of general systems, 35, 417-433

[29] Xia, M., Xu, Z., Zhu, B.(2013). Geometric Bonferroni means with their application in multi-criteria decision making. Knowledge-Based Systems, 40, 88-100

Received: Oct 08, 2019. Accepted: Mar 15, 2020