Mathematical derivation of

a refined four-stream radiative transfer model for row planted crops

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ABSTRACT:

In the main text, the equations of the results are shown, namely the DRF-related equations. In order to further illustrate our research work, this material provides detailed mathematical physical ideas about the derivation and solution for the modified the four-stream (MFS) radiative transfer equations. The material includes:

A) Nomenclature table;

B) Derivation of horizontal radiative transfer equation for row crops;

C) Area fractions of each component in row crops;

D) Solving of the DRFs on the boundary of the canopy closure;

E) Solving of the DRF of between-row based on integral raditive transfer equation.

A. Nomenclature table

Nomenclature table is the symbol of the physical quantities involved in the row modeling of canopy reflectance. Most of the physical quantities follow the original four-stream radiative transfer equations, and only the physical quantities required for the horizontal radiative transfer equations and the row modeling of canopy reflectance are added.

(Blod: vector and matrix**, Non-boldface:** scalar**)**

A-1 Radiance and flux density

Unit: W m^{-2} nm^{-1}, **General symbol:** E

a) Radiance

Unit: W m^{-2} sr^{-1} nm^{-1}, **General symbol:** L

L_i The radiance in the scattering direction

L_o The radiance in the viewing direction

L_b The horizontal radiance of lateral "wall" A

L_d The horizontal radiance of lateral "wall" B

b) Flux density

Unit: W m^{-2} nm^{-1}, **General symbol:** E

E_s Downward specular irradiance (collimated flux density) on a horizontal plane

E_- Downward Hemispherical diffuse flux density

E_+ Upward Hemispherical diffuse flux density

$E_o(\theta_o)$ Flux-equivalent radiance in the viewing direction

$E_o(\theta_i)$ The Lebesgue integral form of the horizontal radiance (referring to L_b and L_d), and it denotes E_b or E_d with the same radiation energy as $E_o(\theta_o)$

E_b Diffuse horizontal hemispheric flux density through the lateral “wall” A

E_d Diffuse horizontal hemispheric flux density through the lateral “wall” B

A-2 The coefficients and optical functions

Unit: m^{-1}

a) The coefficients of the continuous crops

1-1 The coefficients of the specular flux

k Extinction coefficient for the specular flux

K Extinction coefficient in the viewing direction

s' Forward scatter coefficient for specular flux

s Backscatter coefficient for specular flux

w Bidirectional scattering coefficient

1-2 The coefficients of the uniform diffuse flux

κ Extinction coefficient for diffuse flux, $\kappa = 1$

σ' Forward scattering coefficient for diffuse flux

σ Backscatter coefficient for diffuse flux

a' Absorption coefficient for diffuse flux

a Attenuation coefficient for diffuse flux, $a = a' + \sigma'$

v Directional backscatter coefficient for diffuse incidence

v' Directional forward scatter coefficient for diffuse incidence

b) The coefficients of the canopy closure of row crops

1-1 The coefficients of the specular flux

m' Bidirectional scattering coefficient for specular flux to horizontal diffuse flux

o_- Attenuation coefficient of flux from $E_s(0)$ to $E_{\|}$

o_+ Enhancement coefficient of flux from $E_s(-1)$ to $E_{\|}$

1-2 The coefficients of the uniform diffuse flux

n' Attenuation coefficient for the horizontal diffuse flux

g Radiative converted coefficient describing the proportion of downward diffuse flux converting to horizontal diffuse flux of the lateral "wall"

g' Radiative converted coefficient describing the proportion of upward diffuse flux converting to horizontal diffuse flux of the lateral "wall"

o_1 Attenuation coefficient of flux from $E_-(0)$ to E'_-

o_2 Enhancement coefficient of flux from $E_+(-1)$ to E'_+

o_3 Radiative converted coefficient from $E'_\pm$ to $E_{\|}$

1-3 The coefficients of soil particles in the between-row

a_s Attenuation coefficient of soil particle

ω^s Single albedo of soil particle

w^s Bi-directional scattering coefficient of soil particle

b and c Adjustment parameters of soil scattering phase function in the between-row

c) optical function

Unit: dimensionless

G The projection of a unit leaf area onto the surface normal to the direction θ (J. Ross's G-function)

$p(\delta)$ Scattering phase function of soil particle

$\mathbf{K}$ Transfer probability of collision

f_0 Source function of the medium

$\mathbf{k}$ Transfer probability (matrix)

A-3 Reflectance, transmittance and radiative transfer ratio

Unit: dimensionless **General symbol:** r ($\mathbf{R}$), τ ($\mathbf{T}$), ρ ($\mathbf{H}$) or g ($\mathbf{G}$)

a) Directional reflectance factors on the surface

$R_{\perp}$ Directional reflectance factor (DRF) in the vertical direction

$R_{\|}$ DRF in the horizontal direction

R_b DRF of lateral "wall" A

R_d DRF of lateral "wall" B

R_c DRF at the top of canopy closure

R_{br} DRF at the top of between-row

R_{c_1} The single-scattering of the canopy closure

R_{c_m} The multiple-scattering of the canopy closure

R_{br_1} The single-scattering of the between-row

R_{br_m} The multiple-scattering of the between-row

b) Reflectance factors and transmittance factors on the surface

r_{so}^{*} Bidirectional reflectance on the surface

r_{do}^{*} Hemispheric-directional reflectance on the surface

r_{sd}^{*} Directional-hemispherical reflectance on the surface

r_{dd}^{*} Bi-hemisphere reflectance on the surface

c) Reflectance factor in the layer

r_{so} Bidirectional reflectance in the layer

r_{sd} Directional-hemispherical reflectance in the layer

r_{do} Hemispherical-directional reflectance in the layer

r_{dd} Bi-hemisphere reflectance in the layer

d) Transmittance factor in the layer

τ_{ss} Transmittance in the direction of solar beam in the layer

τ_{sd} Directional-hemispherical transmittance in the layer

τ_{dd} Bi-hemisphere transmittance in the layer

τ_{do} Hemispherical-directional transmittance in the layer

τ_{oo} Transmittance in the direction of observation in the layer

e) Radiative transfer ratio in the layer

$\rho_{d\bar{d}}$ The radiative transfer ratio from downward diffuse to the lateral "wall" in the layer

$\rho'_{d\bar{d}}$ The radiative transfer ratio from upward diffuse to the lateral "wall" in the layer

$\rho_{s\bar{d}}$ The radiative transfer ratio of directional horizontal hemispherical direction in the layer

$\rho_{\bar{d}\bar{d}}$ The radiative transfer ratio of horizontal bi-hemispherical direction in the layer

f) Single-scattering and multiple-scattering

$r_{so\_v}^{1}$ Single-scattering of specular flux in the canopy closure

$r_{so\_v}^{m}$ Multiple-scattering of specular flux in the canopy closure

$r_{so\_s}^{1}$ Single-scattering of specular flux from the soil in the canopy closure

$r_{so\_s}^{m}$ Multiple-scattering of specular flux between soil and vegetation in the canopy closure

r_{do}^{1} Single-scattering of diffuse flux in the canopy closure

r_{do}^{m} Multiple-scattering of diffuse flux in the canopy closure

A-4 Angle parameters

Unit: rad, ° **General symbol:** θ (φ)

θ zenith angle

φ Azimuth angle

φ_{so} Relative azimuth angle ($|\varphi_s - \varphi_o|$)

φ_r Row azimuth angle (general symbol of $\varphi_{sr} = |\varphi_s - \varphi_r|$ or $\varphi_{or} = |\varphi_o - \varphi_r|$)

α Inclined angle projected in the perpendicular plane of row canopy

β Azimuth of the inclined angle

Unit: sr **General symbol:** Ω

Ω Solid angle

A-5 Vegetation physical parameters

a) Structural parameter

Unit: m

A_1 Row width

A_2 Distance of between-rows

h The height of the canopy

l The path length of vegetation

N_u Number of row cycle

$f(\theta_l)$ The leaf inclination distribution function (LADF)

$P_o(\Omega, x, z)$ Gap probabilities in the viewing direction

$P_{so}(\Omega_s, \Omega_o, x, z)$ Bi-directional gap probabilities at each point

Ω_E Clumping index

b) Medium-density

Unit: m^{-1}

L' Differential leaf area index (also named as leaf area density) in the vertical direction of the continuous crops

L'_{row} Differential leaf area index for canopy closure in the vertical direction

U Differential leaf area index for the canopy closure in the horizontal direction

Unit: m m^{-1}

L Leaf area index

L_{row} Leaf area index for canopy closure

L_E Effective leaf area index

c) Area fraction **Unit:** dimensionless

$S_{closure\_s}(z)$ Fraction of observed canopy illuminated by the specular flux in the canopy closure

$S_{closure\_s}(h)$ Fraction of observed soil illuminated by the specular flux in the canopy closure

$S_{closure\_d}$ Fraction of canopy closure illuminated by the diffuse flux

$S_{ill\_between\_row\_s}$ Fraction of observed soil background in the between-row area illuminated by the specular flux

$S_{between\_row\_d}$ Fraction of between-row background illuminated by the diffuse flux

B. Derivation of horizontal radiative transfer equation for row crops

Eq. (4) in the main text is $\frac{dE_o(\theta_o)}{L'dz} = wE_s + vE_- + v'E_+ - KE_o(\theta_o)$. It is an approximation of the one-dimensional radiative transfer equation for continuous vegetation [1], and was derived from

$$\frac{dE_s}{L'dz} = -kE_s \tag{B-1}$$

$$\frac{d\pi L_o}{L'dz} = w(\mu_s, \varphi_s, \mu_o, \varphi_o)E_s + \int_{4\pi} w(\mu_i, \varphi_i, \mu_o, \varphi_o)L_i \mu_i d\Omega_i - K\pi L_o \tag{B-2}$$

Similar to Eq. (B-2), the horizontal radiative transfer equation of canopy closure in row planted crops is

$$\frac{d\pi L_{||}}{Udx} = m'(\mu_s, \varphi_s, \mu_{i||}, \varphi_{i||})E_s + \int_{4\pi} w(\mu_i, \varphi_i, \mu_{i||}, \varphi_{i||})L_i \mu_i d\Omega_i - n'\pi L_{||} \tag{B-3}$$

where the horizontal scattering direction is denoted by $i\,||$, $L_{||}$ is the horizontal radiance in the lateral “walls” with angles varying between $[-\pi,0)\cup[0,\pi)$ (Fig. B-1(b-c)). m' is the bidirectional scattering coefficient for specular flux to horizontal diffuse flux (its expression reference B-1 in this section), n' is the attenuation coefficient for horizontal diffuse flux. Compared to the attenuation coefficient for vertical diffuse flux (a) [2], n' is computed by using leaf inclined angle rather than normal leaf angle, and $n'=1-\frac{\rho+\tau}{2}+\frac{\rho-\tau}{2}\sin^2\theta_l$, here r and τ are the leaf directional-hemisphere reflectance and transmittance, respectively, θ_l . U is horizontal differential leaf area index, and there is $\int_0^h L'_{row}dz \approx L'_{row}h=\int_{-\frac{A_1}{2}}^{\frac{A_1}{2}} Udx \approx UA_1$, and L'_{row} the differential leaf area index (leaf area density) for canopy closure in the vertical direction, and $L'_{row}=(A_1+\mathrm{A}_2)Lf(\theta_l)d\theta_l/A_1h$, then there is $U=\frac{(A_1+A_2)Lf(\theta_i)d\theta_i}{A_1^2}$. Eq. (B-3) is divided into two equations, i.e., the equation describing the horizontal radiative transfer in the lateral “wall” A

$$\frac{d\pi L_b}{Udx}=m'E_s+\int_{4\pi} w\left(\mu_i,\varphi_i,\mu_{i\|},\varphi_{i\|}\right)L_i\mu_i d\Omega_i-n'\pi L_b \tag{B-4}$$

and the equation describing the horizontal radiative transfer in the lateral “wall” B

$$\frac{d\pi L_d}{Udx}=\int_{4\pi} w\left(\mu_i,\varphi_i,\mu_{i\|},\varphi_{i\|}\right)L_i\mu_i d\Omega_i-n'\pi L_d \tag{B-5}$$

where L_b is the radiance of the lateral “wall” A with angles varying within $[-\pi,0)$ (Fig. A-1(b)). L_d is the radiance of the lateral “wall” B with angles varying within $[0,\pi)$ (Fig. B-1(c)).

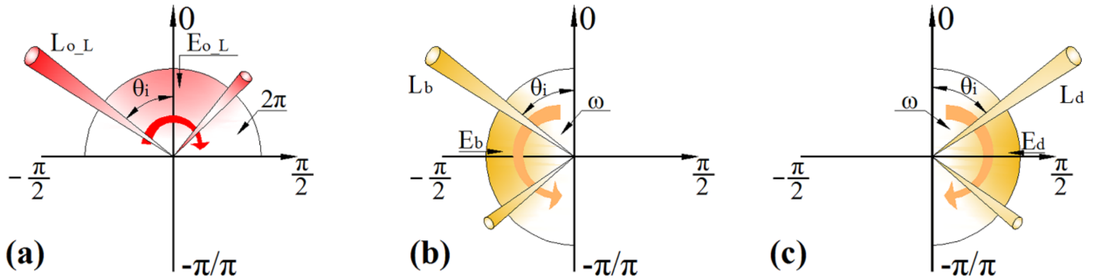

Fig. B-1 Sketch of the one-dimensional coordinate system of angle. (a) Zenith angle for the vertical radiation, (b) angle for the radiation of the lateral "wall" A, and (c) angle for the radiation of the lateral "wall" B. The color-coding is explained as: orange+arrow = Riemann integral of the radiance in the horizontal direction, red+arrow = Lebesgue integral of the radiance in the horizontal direction.

Eqs. (B, 4-5) are changed into

$$\frac{d\pi L_b}{Udx} = m'E_s + \int_{4\pi} w\left(\mu_i, \varphi_i, \mu_{i\|}, \varphi_{i\|}\right) L_i \mu_i d\Omega_i - n' \int_0^{\pi} L_b d\theta_i \tag{B-6}$$

$$\frac{d\pi L_d}{Udx} = \int_{4\pi} w\left(\mu_i, \varphi_i, \mu_{i\|}, \varphi_{i\|}\right) L_i \mu_i d\Omega_i - n' \int_0^{\pi} L_d d\theta_i \tag{B-7}$$

where θ_i is the scattering angle (Fig. A-1(b-c)). $\int_0^{\pi} L_b d\theta_i$ and $\int_0^{\pi} L_d d\theta_i$ are the Riemann integral (integral commonly used in calculus, and its illustration see yellow arrow with rotation in Fig. B-1(b-c)). To simplify Eqs. (B, 6-7), $\int_0^{\pi} L_b d\theta_i$ and $\int_0^{\pi} L_d d\theta_i$ needs to be converted in mathematical form. We give a mathematical **Definition** and **Theorem** in [3].

Definition: Let $f(x)$ be a bounded function,V is nondegenerate interval, and is recorded as $M_f(V)=sup\{f(x)|x \in V\cap[a,b]\}$, $m_f(V)=inf\{f(x)|x \in V\cap[a,b]\}$,$w_f= M(V)- m(V)$. here $w_f(x)= inf\{w(x)|V$ is Open interval, and $x \in V\}$, $w_f(V)$ is the amplitude of f on $x \in V\cap[a,b]$, and $w_f(x)$ is the amplitude of f on point x. When the function is determined,

$w_f(x)$ and $w_f(V)$ are abbreviated as $w_f(x)$ and $w_f(V)$, respectively.

Theorem: If the bounded function f is Riemann integrable in $[a,b]$, then a Lebesgue integrable function also exists in $[a,b]$, and their values after integration are equal, i.e., $\int_a^b f(x)dx = \int_{[a,b]} f(x)dx$.

According to the **Theorem**, Riemann integrals exist an equal Lebesgue integral (Lebesgue integral is an extension of the Riemann integral on the additive measure of set in real analysis in mathematics, if Riemann integral is understood to divide the integration interval vertically, and Lebesgue integral can divide the value range horizontally. Its illustration see red arrow with rotation in Fig. B-1(a)). Therefore, there are $\int_0^\pi L_b d\theta_i = \int_\omega L_b d\chi$ and $\int_0^\pi L_d d\theta_i = \int_\omega L_d d\chi$, where χ is the Lebesgue measure of the two-dimensional space consisting of the X direction and Z direction, ω is set in additive measure space (i.e., measure space is a mathematical concept), and it varies within $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \wedge \left\{\left[-\frac{\pi}{2}, -\pi\right] \vee \left[\frac{\pi}{2}, \pi\right]\right\}$. Then, Eqs. (B, 6-7) become

$$\frac{d\pi L_b}{Udz} = m'E_s + \int_{4\pi} w\left(\mu_i, \varphi_i, \mu_{i\|}, \varphi_{i\|}\right) L_i \mu_i d\Omega_i - n' \int_\omega L_b d\chi \tag{B-8}$$

$$\frac{d\pi L_d}{Udz} = \int_{4\pi} w\left(\mu_i, \varphi_i, \mu_{i\|}, \varphi_{i\|}\right) L_i \mu_i d\Omega_i - n' \int_\omega L_d d\chi \tag{B-9}$$

The general form of $\int_\omega L_b d\chi$ and $\int_\omega L_d d\chi$ in the Eq. (B, 8-9) is $n'\pi L_{\|}$ in Eq. (A-3), and it is obtained by analogy to $K\pi L_o$ in the approximate radiative transfer equation derived by verhoef (i.e., Eq. (B-2)), which represents the extinction of L_o within the layer (i.e., inner canopy closure) (detailed derivation on pages 21-27 in [1]). We assume that the horizontal radiative transfer of horizontal diffuse flux inner (E_b and

E_d) canopy closure does not change with height, then vertical radiance in the viewing direction (L_o), radiance of the lateral "wall" A (L_b), and radiance of the lateral "wall" B (L_d) are equal in the canopy closure. Therefore, $-n'\int_{\omega} L_b d\chi = -n'\int_{\omega} L_o d\chi = -n'\int_{0}^{\pi} L_o d\theta_i$ and $-n'\int_{\omega} L_d d\chi = -n'\int_{\omega} L_o d\chi = -n'\int_{0}^{\pi} L_o d\theta_i$. Then, Eqs. (B, 8-9) can be further simplified as

$$\frac{d\pi L_b}{Udz} = m'E_s + \int_{4\pi} w\left(\mu_i, \varphi_i, \mu_{i\|}, \varphi_{i\|}\right) L_i \mu_i d\Omega_i - n'E_o\left(\theta_i\right) \tag{B-10}$$

$$\frac{d\pi L_d}{Udz} = \int_{4\pi} w\left(\mu_i, \varphi_i, \mu_{i\|}, \varphi_{i\|}\right) L_i \mu_i d\Omega_i - n'E_o\left(\theta_i\right) \tag{B-11}$$

Here $E_o\left(\theta_i\right)$ is $E_o\left(\theta_o\right)$ in the Eq. (1-d), and just have different mathematical forms, and $E_o\left(\theta_i\right)$ is the Lebesgue integral form of the horizontal radiance (L_b and L_d), and it denotes E_b or E_d having the same radiation energy with $E_o\left(\theta_o\right)$. According to the analysis of B-1 in the section, then, Eqs. (B, 10-11) are rewritten as

$$\frac{dE_b}{Udz} = m'E_s + gE_- + g'E_+ - n'E_o\left(\theta_i\right) \tag{B-12}$$

$$\frac{dE_d}{Udz} = gE_- + g'E_+ - n'E_o\left(\theta_i\right) \tag{B-13}$$

In Eqs. (B, 10-11), details of the two issues, including the $\int_{4\pi} w\left(\mu_i, \varphi_i, \mu_{i\|}, \varphi_{i\|}\right) L_i \mu_i d\Omega_i$ simplified by the radiative converted coefficient in horizontal diffuse flux, the mathematical form of the bidirectional scattering coefficient for specular flux and horizontal diffuse flux (m'), are clarified as follows.

B-1 Radiative converted coefficient in horizontal diffuse flux

According to Beer's law and mathematical set theory, the diffuse upward flux through the diagonal area of canopy closure ($E'_{+}(*)$) (When the canopy closure is assumed in a two-dimensional space, it is a rectangle or square, and the diagonal of rectangular or square at this time is diagonal area of the canopy closure, i.e., the area shows in Fig. B-2), the diffuse downward flux through the diagonal area of canopy closure ($E'_{-}(*)$) and the diffuse internal flux on the surface of the lateral “wall” ($E_{\parallel}(B)$) are modeled as following formula

$$\begin{aligned} E'_{-}(*) &= E_{-}(0)e^{0} - E_{-}(0)e^{-\kappa L'_{row}\Delta z} - E_{-}(0)e^{-2\kappa L'_{row}\Delta z} \cdots - E_{-}(0)e^{-3\kappa L'_{row}\Delta z} \\ &- E_{-}(0)e^{n\kappa L'_{row}\Delta z} \approx E_{-}(0)\frac{1}{1+e^{-L'_{row}\Delta z}} = E_{-}(0)o_{1} \end{aligned} \tag{B-14}$$

$$\begin{aligned} E'_{+}(*) &= E_{+}(-1)e^{0} - E_{+}(-1)e^{-\kappa L'_{row}\Delta z} - E_{+}(-1)e^{-2\kappa L'_{row}\Delta z} \cdots - E_{+}(-1)e^{-3\kappa L'_{row}\Delta z} \\ &- E_{+}(-1)e^{n\kappa L'_{row}\Delta z} \approx E_{+}(-1)\frac{1}{1+e^{-L'_{row}\Delta z}} = E_{-}(0)o_{2} \end{aligned} \tag{B-15}$$

$$E_{\parallel}(B) = n'E'_{\pm}(*) = \mathrm{o}_{3}E'_{\pm}(*) \tag{B-16}$$

where $E'_{\pm}(*)$ denotes $E'_{+}(*)$ and $E'_{-}(*)$. κ is the extinction coefficient for diffuse flux, and $\kappa = 1$ [2]. The cross-correlation function of the leaves and the normalized method with 20 sub-layers in the SAIL are used in the step length [1, 4], hence $\Delta z = -\ln\left[1-0.05\times\left(1-e^{-\frac{d_{so}}{l_L^*}}\right)\right]\frac{l_L^*}{d_{so}}$ (it is an expression derived from the code in the SAIL program) , and $d_{so} = \sqrt{\tan^2\theta_s + \tan^2\theta_o - 2\tan\theta_s\tan\theta_o\cos\varphi}$ [1]. Then, the attenuation coefficient of flux from $E_{-}(0)$ to E'_{-} is

$$o_{1} = \frac{1}{1+e^{-L'_{row}\Delta z}} \tag{B-17}$$

The enhancement coefficient of flux from $E_{+}(-1)$ to E'_{+} is

$$o_{2} = \frac{1}{1+e^{-L'_{row}\Delta z}} \tag{B-18}$$

The radiative converted coefficient from $E'_{\pm}$ to $E_{\parallel}$ is

$$o_3 = n' \tag{B-19}$$

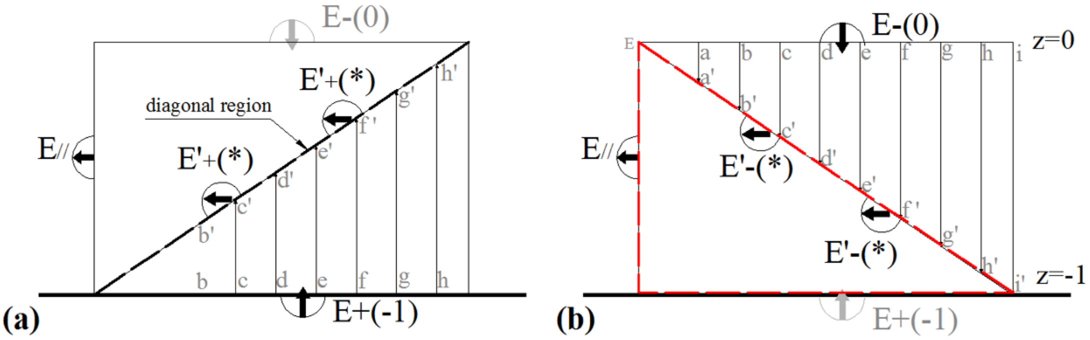

Fig. B-2 Sketch of the radiative transfer process of the diffuse flux in the horizontal direction. (a) Radiative transfer process of diffuse flux from the bottom boundary surface ($E_+(-1)$) to the lateral "wall" ($E_{\parallel}(B)$). (b) Radiative transfer process of diffuse flux from the top boundary surface ($E_-(0)$) to the lateral "wall"($E_{\parallel}(B)$). Here, $E'_+(*)$ and $E'_-(*)$ are upward and downward diffuse flux through the diagonal area, respectively.

Combining Eqs. (B, 17-19), the radiative converted coefficient that describes the proportion of downward diffuse flux converting to horizontal diffuse flux of the lateral "wall" is

$$g = o_3 o_1 = \frac{n'}{1+e^{-L'_{row}\Delta z}} = \frac{2-\left(\sin^2\theta_l - 1\right)\rho - \left(\sin^2\theta_l + 1\right)\tau}{2\left(1+e^{-L'_{row}\Delta z}\right)} \tag{B-20}$$

and the radiative converted coefficient that describes the proportion of upward diffuse flux converting to horizontal diffuse flux of the lateral "wall" is

$$g' = o_3 o_2 = \frac{n'}{1+e^{-L'_{row}\Delta z}} = \frac{2-\left(\sin^2\theta_l - 1\right)\rho - \left(\sin^2\theta_l + 1\right)\tau}{2\left(1+e^{-L'_{row}\Delta z}\right)} \tag{B-21}$$

B-2 Bidirectional scattering coefficient for specular flux and horizontal diffuse flux

Coefficients in the four-stream radiative transfer equations are calculated based on the SAIL model [2]. Thereafter, the coordinate system of the SAIL model is introduced, and ***X*** and ***Y*** for row crops are considered. Finally, the coordinate system of the leaf in row crops is established (Fig. B-3). For one-dimensional radiation transfer issue, ***Y*** is assumed to be an isotropic direction for row crops, hence it is ignored. Accordingly, the vectors in the generalized coordinates are

$$\mathbf{l} = \left(\sin\theta_l \cos\varphi_l;\quad \sin\theta_l \sin\varphi_l;\quad \cos\theta_l\right)$$

$$\mathbf{s} = \left(\sin\theta_s;\quad 0;\quad \cos\theta_s\right)$$

$$\mathbf{o} = \left(\sin\theta_i \cos\varphi_i;\quad \sin\theta_i \sin\varphi_i;\quad \cos\theta_i\right)$$

$$\mathbf{Z} = \left(0;\quad 0;\quad 1\right)$$

$$\mathbf{X} = \left(\pm B;\quad 0;\quad 0\right) \tag{B-22}$$

The horizontal path length of lateral "wall" in the ***X***-axies is $\frac{A_1}{2h}\left|\sin\varphi_{or}\right|$, here φ_{or} is the angle between viewing azimuth (φ_o) angle and row azimuth angle (φ_r). Therefore, the boundary condition of the canopy closure in the ***X***-axies is $\pm B = \pm\frac{A_1}{2h}\left|\sin\left|\varphi_r - \varphi_o\right|\right|$. Since the direction of the path length is always ignored, the sign of B is removed here, and $B = \frac{A_1}{2h}\left|\sin\left|\varphi_r - \varphi_o\right|\right|$.

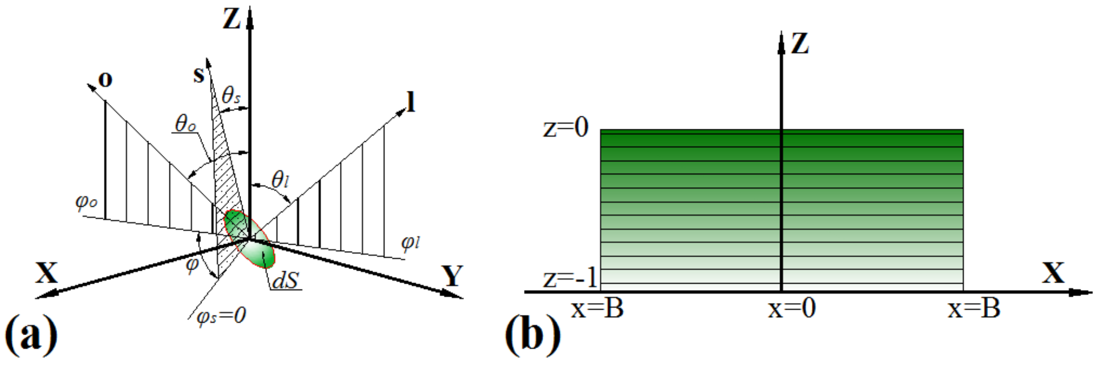

Fig. B-3 Sketch of the orientations of unit vectors **l**, **s**, **o**, **X**, **Y**, and **Z**, relative to the leaf and canopy closure. (a) The coordinate vector of leaf and (b) the coordinate vector of canopy closure.

The vertical conversion factor in the SAIL model is introduced into the horizontal direction here [2], then the horizontal conversion factor of the solar direction is

$$f_{s\|} = (\mathbf{s}\cdot\mathbf{l})/(\mathbf{s}\cdot\mathbf{X}) = B\cot\theta_s\left[\cos\theta_l\left(1+\tan\theta_s\tan\theta_l\cos\varphi_l\right)\right] = B\cot\theta_s f_s \qquad \text{(B-23)}$$

Here, f_s is the vertical conversion factors in the solar direction [2]. Using the transition angle in the viewing direction ($\beta_s = \arccos\left(-1/\tan\theta_o\tan\theta_l\right)$) [2], $f_{s\|}$ can be divided into two parts

$$\begin{aligned} f_{s\|} &= f_{st\|} + f_{sb\|} = 2\int_0^{\beta_s} f_{s\|}d\varphi_l + 2\int_0^{\beta_s} -f_{s\|}d\varphi_l = 2B\cot\theta_s\cos\theta_l\left(\beta_s+\tan\theta_s\tan\theta_l\sin\beta_s\right) \\ &+2B\cot\theta_s\cos\theta_l\left(\beta_s-\pi+\tan\theta_s\tan\theta_l\sin\beta_s\right) = B\cot\theta_s k \end{aligned} \qquad \text{(B-24)}$$

Eqs. (B, 17-18) are analogized for direct solar radiation, the attenuation coefficient of flux from $E_s(0)$ to $E_{\|\_diffuse}$ is

$$o_- = \frac{n'}{1+e^{-kL'_{row}\Delta z}} \qquad \text{(B-25)}$$

and the enhancement coefficient of flux from $E_s(-1)$ to $E_{\|\_diffuse}$ is

$$o_+ = \frac{n'}{1+e^{-kL'_{row}\Delta z}} \qquad \text{(B-26)}$$

Scattering efficiency factors ($Q_{sc}\left(E_1,E_2\right)$) similar to the one in SAIL method are used [2], and the bidirectional scattering coefficient for specular flux and horizontal diffuse flux is

$$
\begin{aligned}
m' &= \frac{1}{2\pi} f_{st\|}\left(ro_+ + \tau o_-\right) + f_{sb\|}\left(ro_+ + \tau o_-\right) = \frac{1}{2\pi}\left(f_{st\|} + f_{sb\|}\right)\left(ro_+ + \tau o_-\right) = B\cot\theta_s k\left(ro_+ + \tau o_-\right) \\
&= \frac{A_1\left|\sin\left|\varphi_r - \varphi_o\right|\right|\cot\theta_s k\left(r+\tau\right)\left[2-\left(\sin^2\theta_l - 1\right)\rho - \left(\sin^2\theta_l + 1\right)\tau\right]}{4h\left(1+e^{-kL'_{row}\Delta z}\right)}
\end{aligned}
\tag{B-27}
$$

Here, r and τ are the leaf directional-hemisphere reflectance and transmittance, respectively.

C. Area fractions of each component in row crops

The parameters of S with different subscripts in Eq. (30), Eqs. (32-33), and Eq. (36-37) in the main text are area fractions of each component (Table C-1), and they are the integral of the gap probability considering clumping index.

Table C-1 Area fractions of each component in the scene

Flux type	Canopy closure	Between-row
Specular flux	$S_{closure\_s}(z) = \frac{1}{A_1}\int_0^h\int_0^{A_1} P_{so}\left(\Omega_s,\Omega_o,x,z\right)dxdz$ $S_{closure\_s}(h) = \begin{cases} \frac{1}{A_1+A_2}\int_0^{A_1+A_2} P_{so}\left(\Omega_s,\Omega_o,x,h\right)dx & L<1 \\ \frac{1}{A_1}\int_0^{A_1} P_{so}\left(\Omega_s,\Omega_o,x,h\right)dx & L\geq 1 \end{cases}$	$S_{between\_row\_s} = \begin{cases} \frac{1}{A_1+A_2}\int_0^{A_1+A_2} P_{so}\left(\Omega_s,\Omega_o,x,h\right)dx & L<1 \\ \frac{1}{A_2}\int_{A_1}^{A_1+A_2} P_{so}\left(\Omega_s,\Omega_o,x,h\right)dx & L\geq 1 \end{cases}$
Diffuse flux	$S_{closure\_d} = \frac{1}{A_1}\left[\left(A_1+A_2\right) - \int_{A_2}^{A_1+A_2} P_o\left(\Omega_o,x,h\right)dx - S_{sc}\int_0^{A_1+A_2} P_o\left(\Omega_o,x,h\right)dx\right]$	$S_{between\_row\_d} = \frac{1}{A_2}\int_{A_1}^{A_1+A_2} P_o\left(\Omega_o,x,h\right)dx$

Here $S_{sc} = e^{kL_{row}\Omega_E}$

Integral in Table C-1 uses the numerical integration method. For the calculation of the numerical integral, we use Simpson method, which can reduce the cumulative error to the fourth derivative ($P^{(4)}(\upsilon)$ in Table C-2).

Table C-2 Numerical integration of Simpson method for Area fractions of each component

Method	Iterative equation	Error of x-axis	Error of z-axis
Simpson method	$\sum_{z=0}^{80}\sum_{x=0}^{40}\frac{d-c}{6}\begin{Bmatrix} \frac{b-a}{6}\left[P(a,c)+4P\left(\frac{b-a}{2},z\right)+P(b,c)\right]+ \\ 4\left\{\frac{b-a}{6}\left[P\left(a,\frac{d-c}{2}\right)+4P\left(\frac{b-a}{2},\frac{d-c}{2}\right)+P\left(b,\frac{d-c}{2}\right)\right]\right\} \\ +\left\{\frac{b-a}{6}\left[P(a,d)+4P\left(\frac{b-a}{2},d\right)+P(b,d)\right]\right\} \end{Bmatrix}$	$-\frac{1}{2880}(b-a)^5 P^{(4)}(\upsilon),\upsilon\in[a,b]$	$-\frac{1}{2880}(d-c)^5 P(\upsilon)^{(4)},\upsilon\in[d,c]$

Here a and b are the starting point and ending point for the integral step in the *X*-axis, c and d are the starting point and ending point for the integral step in the *Z*-axis.

For the gap probability in the viewing direction ($P_o(\Omega,x,z)$) and the gap probability for both sun and viewing directions ($P_{so}(\Omega_s,\Omega_o,x,z)$), [5] gives the equations without considering the clumping effect of leaves. However, for the leaves of most crops in the real world, they are not random distribution, but have clumping effect. According to research in [6, 7], we use clumping index (Ω_E) to modified equation in [5], and the gap probability considering clumping index in the viewing direction is

$$P_o(\Omega,x,z)=e^{-G(\theta_o)\cdot L'_{row}\cdot l(\Omega_o,x,z)\cdot\Omega_E} \tag{C-1}$$

Here $l(\Omega,x,z)$ is the path length of vegetation, and their specific calculation equation can refer to [8], $G(\theta)$ is the projection of a unit leaf area onto the surface normal to the direction θ, and it is $k\cos\theta_s$ (or $K\cos\theta_o$, depending on whether it is the solar or the viewing direction). $\Omega_E=\frac{L_E}{L}$, here L_E is the effective leaf area index. The the gap probability considering clumping index for both sun and viewing directions is

$$
\begin{aligned}
&P_{so}\left(\Omega_s,\Omega_o,x,z\right)=P_s\left(\Omega_s,x,z\right)P_o\left(\Omega_s,x,z\right)C_{hopspot}\\
&=\exp\left\{L'_{row}\left[\begin{array}{l}-G\left(\theta_s\right)\cdot l_s\cdot\Omega_E-G\left(\theta_o\right)\cdot l_o\cdot\Omega_E\\ +\Omega_E\sqrt{G\left(\theta_s\right)\cdot l_s\cdot G\left(\theta_o\right)\cdot l_o}\,\dfrac{l_L^*}{l_{so}}\left(1-e^{-\frac{l_{so}}{l_L^*}}\right)\end{array}\right]\right\}
\end{aligned}
\tag{C-2}
$$

here $\cos\xi=\cos\theta_s\cos\theta_o+\sin\theta_s\sin\theta_o\cos\left|\varphi_s-\varphi_o\right|$, l_L^* is the canopy dimension parameter. l_s and l_o are the path length of vegetation in the sun direction and the path length of vegetation in the viewing direction, respectively. $l_{so}=\sqrt{l_s^2+l_o^2-2l_sl_o\cos\xi}$. In Eqs. (C, 1-2), two key parameters need to be discussed, i.e., the path length of vegetation ($l\left(\Omega,x,z\right)$) and canopy dimension parameter (l_L^*).

C-1 The path length of vegetation

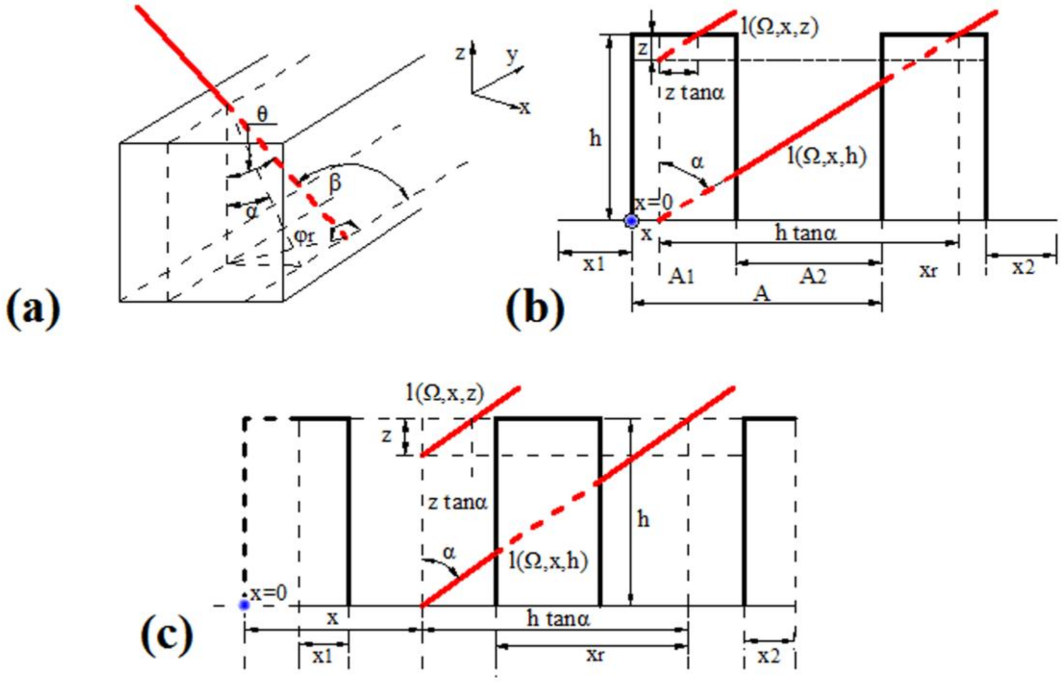

Fig. C-1 Sketch of the path length of vegetation and area fractions in row crops. (a) Geometric relationship of the path length of vegetation; (b) calculation of the path length of vegetation in the canopy closure; (c) calculation of the path length of vegetation in the between-row. Here, x_1 is the length of the incomplete area in x-axis at the direction of the incident hemisphere, x_2 is the length of the incomplete area in x-axis at the opposite direction of the incident hemisphere.

In the calculation of path length of vegetation, the inclined angle projected in the perpendicular plane of row canopy (α) and the azimuth of inclined angle (β) are defined (Fig. C-1(a)):

$$\alpha = \arctan\left(\tan|\theta|\sin\varphi_r\right) \tag{C-3}$$

$$\beta = \arcsin\left(\frac{\sin\varphi_r \sin|\theta|}{\sin\alpha}\right) \tag{C-4}$$

Where α , β , θ and φ_r are the general symbol, which refers to the solar or the viewing direction, α and β have the sign of positive and negative in the hemisphere space. In the [8], the method to calculate the path length of vegetation in the canopy closure is introduced (Fig. C-1(b)). To calculate the DRFs distribution in row planted crops, the method of calculating path length of vegetation is extended to the between-row background (Fig. C-1(c)) and along row plane (AR). The coordinate origin of *X*-axes is the vertical bisector of the canopy closure (Fig. B-3(b)), to facilitate the calculation, the coordinate origin of *X*-axes moved the length of $\frac{A_1}{2}$ toward the negative half axis, Therefore, A, A_1, x, x_r have the sign of positive and negative in the hemisphere space. Then, the path length of vegetation is

$$l(\Omega,x,z)=\begin{cases}\dfrac{N_uA_1-x-x_r}{\sin\alpha\sin\beta} & (x_r\le A_1)\wedge(x\le A_1)\wedge(\theta\ne 0)\wedge\left[(\varphi_r\ne 0)\vee(\varphi_r\ne 180)\right]\\ \dfrac{(N_u+1)A_1-x}{\sin\alpha\sin\beta} & (x_r> A_1)\wedge(x\le A_1)\wedge(\theta\ne 0)\wedge\left[(\varphi_r\ne 0)\vee(\varphi_r\ne 180)\right]\\ \dfrac{\left[(N_u-1)A_1+x_r\right]}{\sin\alpha\sin\beta} & (x_r\le A_1)\wedge(x> A_1)\wedge(\theta\ne 0)\wedge\left[(\varphi_r\ne 0)\vee(\varphi_r\ne 180)\right]\\ \dfrac{N_uA_1}{\sin\alpha\sin\beta} & (x_r> A_1)\wedge(x> A_1)\wedge(\theta\ne 0)\wedge\left[(\varphi_r\ne 0)\vee(\varphi_r\ne 180)\right]\\ z & (\theta=0)\wedge(x\le A_1)\\ 0 & \left[(\theta=0)\wedge(x>A_1)\right]\vee(\varphi_r=0)\vee(\varphi_r=180)\\ z/\cos\theta & (\varphi_r=0)\vee(\varphi_r=180)\end{cases} \quad \text{(C-5)}$$

Here $\wedge$ and $\vee$ are the mathematical logic symbol for "and" and "or", respectively. Eq. (C-5) is an example in the positive *X*-axis. For *x* on the negative axis, $\ge$, $>$ and $\le$, $<$ need to be interchanged in the limited conditions, while positive sign and negative sign need to be interchanged in the variables. N_u is the number of row cycle, and $N_u=\dfrac{z\tan\alpha+x-x_r}{A}$. x_r is the remainder of row cycles, and $x_r=\mathrm{mod}\left(\dfrac{z\tan\alpha+x}{A}\right)$. x and z are *X*- and *Z*- axes in the position of space, respectively.

C-2 The canopy dimension parameter

According to [9], [10] and [5], there are

$$l_L^*=\frac{l_L}{h}=\frac{f_L\sqrt{w_*l_*}}{h} \quad \text{(C-6)}$$

$$l_L^*=\frac{l_L}{h}=\frac{c_L\sqrt{w_*l_*}}{h} \quad \text{(C-7)}$$

Eqs. (C, 6-7) can apply to the calculation for five-leaf shape (triangle, square, rectangle, ellipse, and circle).

$$l_L^* = \frac{l_L}{h} = l^* \sqrt{\frac{w_* \pi}{l_*}} \Big/ 2h^2 \qquad \text{(C-8)}$$

Eq. (C-8) is the calculation for the square leaf.

$$l_L^* = \frac{l_L}{h} \frac{\sqrt{S_\triangle \pi}}{h} \qquad \text{(C-9)}$$

Eq. (C-9) is the calculation for triangular leaf. Eq. (C-9) come from [5], which cannot be derived from the literature provided by the paper of DRM (i.e., Eq. (C, 6-8) in this section). Here l_L^* is the canopy dimension parameter, l_L is an average length of the chord of leaves. w_* is the average width of the leaf, l_* is the average length of the leaf. f_L is a correction factor for leaf shape and orientation, c_L is a general expression for a leaf with an arbitrary shape. $S_\triangle$ is the area of triangle leaf.

According to [9], the original derivation equation of Eqs. (C, 6-8) is $l_L = \sqrt{A_L}$, A_L is leaf area. The spatial plane and its chord length do not seem to have the above mathematical relationship, Eqs. (C, 6-9) are used to calculate the gap probability, and the physical dimension will have problems. Therefore, the length of an average chord of leaves is re-derived.

a) Elliptic or circular leaves

According to the chord length formula of the ellipse in the polar coordinate, the mean chord length of horizontal leaf for the vertical viewing direction is

$$l_{L\_hor} = \frac{2ep}{1-\left(e\cos\varphi_o\right)^2} \qquad \text{(C-10)}$$

Where e is eccentricity, and $e = \frac{\sqrt{\left(0.5\times l^*\right)^2 - \left(0.5\times w^*\right)^2}}{\left(0.5\times l^*\right)}$, p is the distance from the ellipse focus to the directrix, and $p=\frac{\left(0.5\times w^{*}\right)^{2}}{\sqrt{\left(0.5\times l^{*}\right)^{2}-\left(0.5\times w^{*}\right)^{2}}}+\sqrt{\left(0.5\times l^{*}\right)^{2}-\left(0.5\times w^{*}\right)^{2}}$. The average chord length of horizontal leaf for the viewing direction will change with the leaf inclination angle from l_{L_hor} to w_{*} or l_{*}, respectively. This transformation is a synthesis of affine transformation and orthogonal transformation, and the change coefficient for affine transformation is θ_{l} (Fig. C-2(b)). Therefore, two equations in two orthogonal directions are derived as

$$l_{L\_hor1}=\cos\theta_{l} l_{L\_hor}+\sin\theta_{l} l_{*} \tag{C-11}$$

$$l_{L\_hor2}=\cos\theta_{l} l_{L\_hor}+\sin\theta_{l} w_{*} \tag{C-12}$$

The plant planting orientation (row azimuth angle for row crops) and spatial distribution of leaves are considered, the average chord length for the alternate or opposite leaves (botanical definition) is

$$l_{L}=\left|\cos\varphi_{or}\right| l_{L\_hor2}+\left|\sin\varphi_{or}\right| l_{L\_hor1} \tag{C-13}$$

This type of distribution includes corn, wheat, etc. The average chord length for the clustered or whorled leaves (botanical definition) is

$$l_{L}=0.5\times\left(l_{L\_hor1}+l_{L\_hor2}\right) \tag{C-14}$$

This type of distribution includes beets, potatoes, etc.

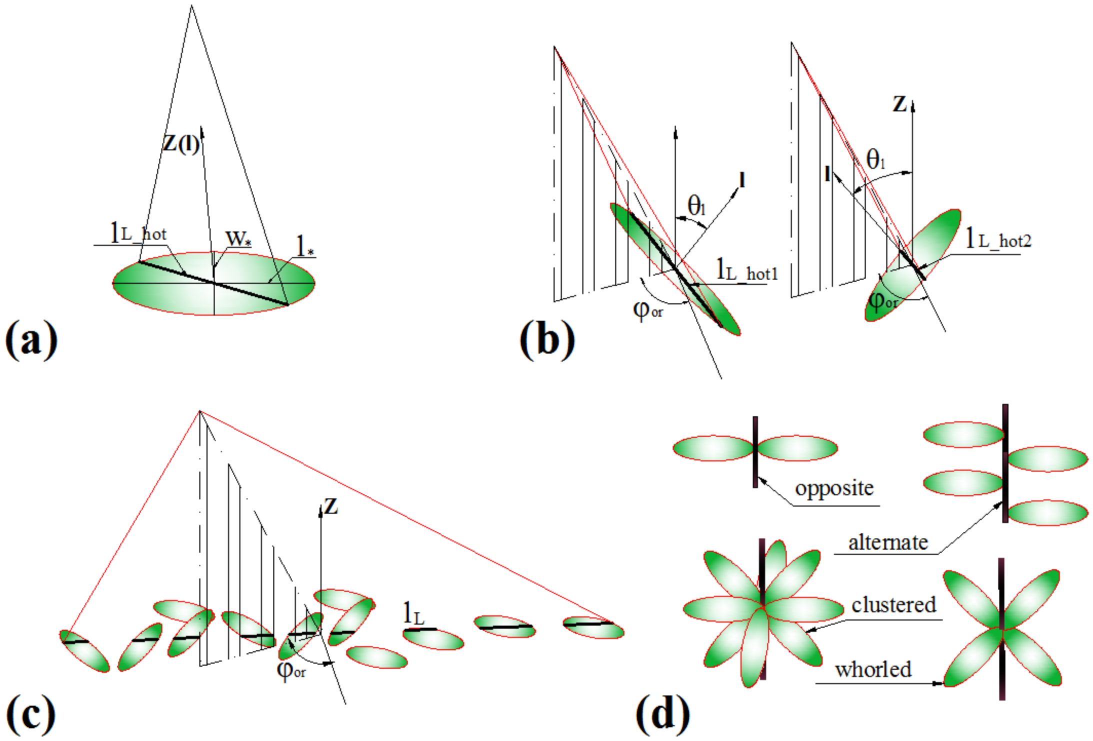

Fig. C-2. The sketch of structure and distribution of ellipse (or circle) leaves. (a) Geometric relationship of ellipse structure; (b) chord length of leaves under varying leaf inclined angle; (c) Geometric relationship of average chord length and (d) distribution pattern of leaves on shoots.

The canopy dimension parameter is

$$l_L^* = \frac{l_L}{h} \tag{C-15}$$

Using Eq. (C-15) to calculate gap probability will cause dimensional problems. This phenomenon is also a problem that is not noticed in the [9], [10] and [5]. According to [1], the relative optical height ($\frac{z}{h}$) is used to modify this problem. Then

$$l_L^* = \frac{l_L h}{z} \tag{C-16}$$

Using the transformation $\frac{z}{h} \rightarrow z$, and the clumping effect of leaves is considered, therefore, Eq. (C-16) is modified to

$$l_L^* = \Omega_E l_L h \qquad \text{(C-17)}$$

Combining Eqs. (C, 10-15), the canopy dimension parameter for corn in the paper is

$$l_L^* = \begin{cases} \Omega_E h \left\{ \dfrac{l_*^3 \cos\theta_l \left(|\sin\varphi_{or}| + |\cos\varphi_{or}|\right)}{w_*^2 \cos^2\varphi_o} + \sin\theta_l \left(|\sin\varphi_{or}|\, l_* + |\cos\varphi_{or}|\, w_*\right) \right\} & (a) \\ \Omega_E h \left\{ \dfrac{l_*^2 \cos\theta_l}{w_*^2 \cos^2\varphi_o} + 0.5 \sin\theta_l \left(l_* + w_*\right) \right\} & (b) \end{cases} \qquad \text{(C-18)}$$

Here the function (a) in Eq. (C-18) is the alternate or opposite leaves, and function (b) in Eq. (C-18) is the clustered or whorled leaves. The canopy dimension parameter is a function of the average width of leaf, the average length of leaf, leaf inclined angle, plant planting orientation (row azimuth angle for row crops), the height of the canopy, the spatial distribution of leaves and leaf shape. The average width of the leaf (w_*) and the average length of the leaf (l_*) in Eq. (C-18) are very easy to measure.

b) Triangular leaves

Considering the comparison of RGM (the mian text), the triangular leaves are used. The triangle has no chord length, and the side length is used for derivation. $l_{*\triangle}$ and $w_{*\triangle}$ are defined as the short sides of the horizontal triangle leaf, which are very easy to acquire in the computer scene. The three sides of a triangular leaf under varying leaf inclination angle are

$$s_1 = \cos\theta_l \left(0.5 \times l_{*\triangle}\right) + \sin\theta_l \times l_{*\triangle} \qquad \text{(C-19)}$$

$$s_2 = \cos\theta_l \left(0.5 \times w_{*\triangle}\right) + \sin\theta_l \times w_{*\triangle} \qquad \text{(C-20)}$$

$$s_3 = \cos\theta_l \left(0.5 \times \sqrt{w_{*\triangle}^2 + l_{*\triangle}^2}\right) + \sin\theta_l \times \sqrt{w_{*\triangle}^2 + l_{*\triangle}^2} \qquad \text{(C-21)}$$

the canopy dimension parameter (leaf curvature is not considered) is

$$l_L^* = l_{L\triangle} h = \frac{\sum_{i=1}^{n_\triangle} h\left(\frac{\vartheta_1}{\vartheta_1+\vartheta_2+\vartheta_3} s_1 + \frac{\vartheta_2}{\vartheta_1+\vartheta_2+\vartheta_3} s_2 + \frac{\vartheta_3}{\vartheta_1+\vartheta_2+\vartheta_3} s_3\right)}{n_\triangle}$$
$$= \frac{\sum_{i=1}^{n_\triangle} \frac{h\left(0.5\cos\theta_l + \sin\theta_l\right)\left(\vartheta_1 l_{*\triangle} + \vartheta_2 w_{*\triangle} + \vartheta_3 \sqrt{w_{*\triangle}^2 + l_{*\triangle}^2}\right)}{\left(\vartheta_1+\vartheta_2+\vartheta_3\right)}}{n_\triangle} \tag{C-22}$$

Here $l_{L\triangle}$ is the length of the visible line in the triangular leaf, ϑ_1, ϑ_2 and ϑ_3 are the random number from 0 to1, $n_\triangle$ is the number of triangular leaves, and is easy to count in computer scene. $\frac{\vartheta_1}{\vartheta_1+\vartheta_2+\vartheta_3}$, $\frac{\vartheta_2}{\vartheta_1+\vartheta_2+\vartheta_3}$ and $\frac{\vartheta_3}{\vartheta_1+\vartheta_2+\vartheta_3}$ are the random probability of triangular edges.

D. Solving of the DRFs on the boundary of the canopy closure

D-1 Construction of the layer scattering matrix

The boundary conditions (i.e., z=0, z=-1 and x=B in Fig. B-3(b)) are considered, Eqs. (6-11) in the main text become scattering matrix in the canopy closure, and it is

$$\begin{bmatrix} E_s(-1) \\ E_-(-1) \\ E_+(0) \\ E_o(0) \\ E_b(B) \\ E_d(B) \end{bmatrix} = \begin{bmatrix} \tau_{ss} & 0 & 0 & 0 & 0 & 0 \\ \tau_{sd} & \tau_{dd} & r_{dd} & 0 & 0 & 0 \\ r_{sd} & r_{dd} & \tau_{dd} & 0 & 0 & 0 \\ r_{so} & r_{do} & \tau_{do} & \tau_{oo} & 0 & 0 \\ \rho_{s\bar{d}} & \rho_{d\bar{d}} & \rho'_{d\bar{d}} & \rho_{\bar{d}\bar{d}} & 0 & 0 \\ 0 & \rho_{d\bar{d}} & \rho'_{d\bar{d}} & \rho_{\bar{d}\bar{d}} & 0 & 0 \end{bmatrix} \begin{bmatrix} E_s(0) \\ E_-(0) \\ E_+(-1) \\ E_o(-1) \\ E_b(0) \\ E_d(0) \end{bmatrix} \tag{D-1}$$

r in Eq. (D-1) is reflectance factors in the homogeneous scattering layer, τ in Eq. (D-1) is transmittance factors in the homogeneous scattering layer, and they are derived from the four-stream radiation transfer theory (pers. comm. W. Verhoef, 2018).

ρ is radiative transfer ratio in the homogeneous scattering layer (the specific derivation is detailed in D-2 in this section). Their subscripts represent the properties of incident and outgoing radiation, and can be summarized by the followas: *s* represents specular flux in direction of direct solar radiation; *o* represents specular flux in direct viewing direction; *d* represents diffuse flux of the vertical hemisphere; $\bar{d}$ represents diffuse flux of the horizontal hemisphere. These parameters describe the theory for bidirectional reflectance distribution Function (BRDF) inside the canopy closure. Eq. (D-1) is changed into the notation of matrix-vector, there is $\mathbf{\Phi}_{out} = \mathbf{S\Phi}_{in}$, in which $\mathbf{S}$ is the layer scattering matrix for the specular and diffuse fluxes. The relationship of the sources and sinks in the radiative transfer of specular and diffuse flux within the canopy closure is illustrated in Fig. D-1.

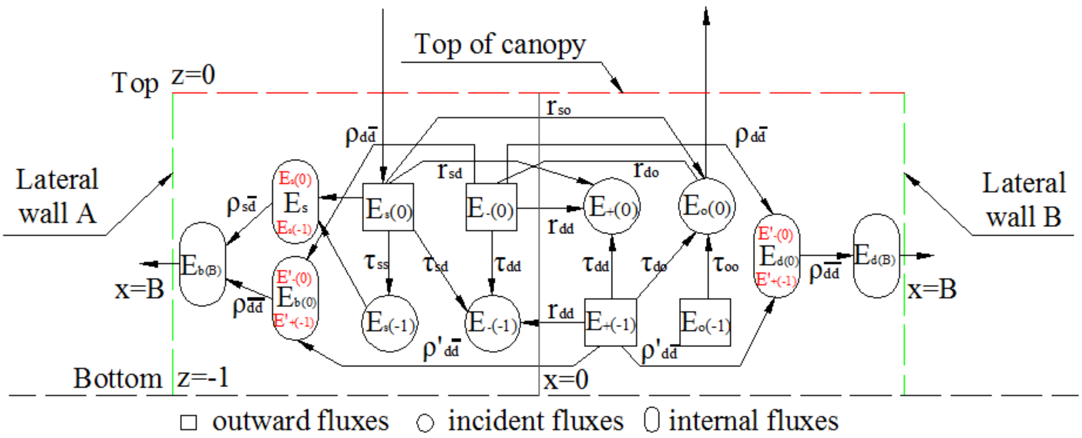

Fig. D-1 Interactions of fluxes for an isolated homogeneous scattering layer in canopy closure of row planted crops.

D-2 Derivation of the radiative transfer ratio

For the diffuse flux vectors in the vertical direction (i.e., E_- and E_+), the system of the differential equation is

$$\frac{d}{L'dz}\begin{bmatrix} E_- \\ E_+ \end{bmatrix} = \begin{bmatrix} a & -\sigma \\ \sigma & -a \end{bmatrix}\begin{bmatrix} E_- \\ E_+ \end{bmatrix} \tag{D-2}$$

This equation is diagonalized, and its eigenvalues and eigenvectors are

$$\mathbf{\Lambda}_{dd} = \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix}, \quad \mathbf{P}_{dd} = \begin{bmatrix} 1 & \dfrac{a-m}{\sigma} \\ \dfrac{\sigma}{a+m} & 1 \end{bmatrix} \tag{D-3}$$

For the horizontal diffuse flux, the system of the differential equation is

$$\frac{d}{Udz}\begin{bmatrix} E_b \\ E_d \end{bmatrix} = \begin{bmatrix} g & g' \\ g & g' \end{bmatrix}\begin{bmatrix} E_- \\ E_+ \end{bmatrix} \tag{D-4}$$

Eq. (D-4) is diagonalized, and its eigenvalues and eigenvectors are

$$\mathbf{\Lambda}_{hdd} = \begin{bmatrix} 0 & 0 \\ 0 & g+g' \end{bmatrix}, \quad \mathbf{P}_{hdd} = \begin{bmatrix} -g' & 1 \\ g & 1 \end{bmatrix} \tag{D-5}$$

In matrix analysis and geometry, the eigenvector is the basis of the matrix, which determines the direction of the matrix. Therefore, the eigenvector is used in the calculation. For Eqs. (D, 3 and 5), there are the following relationships

$$\begin{aligned}\begin{bmatrix} \rho_{d\bar{d}}+\delta_1 & \rho'_{d\bar{d}}+\delta_2 \\ \rho_{d\bar{d}}+\delta_1 & \rho'_{d\bar{d}}+\delta_2 \end{bmatrix} &= \begin{bmatrix} 1 & \dfrac{a-m}{\sigma} \\ \dfrac{\sigma}{a+m} & 1 \end{bmatrix}^{-1}\begin{bmatrix} -g' & 1 \\ g & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & r_\infty \\ r_\infty & 1 \end{bmatrix}^{-1}\begin{bmatrix} -g' & 1 \\ g & 1 \end{bmatrix} = \begin{bmatrix} \dfrac{g'+gr_\infty}{1-r_\infty^2} & \dfrac{g'r_\infty+g}{1-r_\infty^2} \\ \dfrac{r_\infty-1}{1-r_\infty^2} & \dfrac{1-r_\infty}{1-r_\infty^2} \end{bmatrix}\end{aligned} \tag{D-6}$$

the infinite reflectance is defined as $r_\infty = \dfrac{a-m}{\sigma} = \dfrac{\sigma}{a+m}$ [11]. $\rho_{d\bar{d}}$ is the radiative transfer ratio from downward diffuse to the lateral "wall". $\rho'_{d\bar{d}}$ is the radiative transfer ratio from upward diffuse to the lateral "wall". δ_1 and δ_2 are cross-radiation coefficient for $\rho_{d\bar{d}}$ and $\rho'_{d\bar{d}}$, respectively. There are two pairs of solutions for Eq. (D-6). $\frac{r_\infty - 1}{1 - r_\infty^2}$ is overflowing, and the downside-solutions of the matrix is omitted. Therefore, only upside-solutions of the matrix are used

$$\rho_{d\bar{d}} + \delta_1 = \frac{g' + g r_\infty}{1 - r_\infty^2} \tag{D-7}$$

$$\rho'_{d\bar{d}} + \delta_2 = \frac{g' r_\infty + g}{1 - r_\infty^2} \tag{D-8}$$

then

$$\rho_{d\bar{d}} + \delta_1 = \left\{ \frac{g'}{1 - r_\infty^2} + \frac{g r_\infty}{1 - r_\infty^2} \right\} \approx g r_\infty \left[1 + r_\infty + r_\infty^2 \cdots \right] + g' \left[1 + r_\infty + r_\infty^2 \cdots \right] \tag{D-9}$$

$$\rho'_{d\bar{d}} + \delta_2 = \left[\frac{g' r_\infty}{1 - r_\infty^2} + \frac{g}{1 - r_\infty^2} \right] \approx g' r_\infty \left[1 + r_\infty + r_\infty^2 \cdots \right] + g \left[1 + r_\infty + r_\infty^2 \cdots \right] \tag{D-10}$$

Here

$$\delta_1 = g' \left[1 + r_\infty + r_\infty^2 \cdots \right] \tag{D-11}$$

$$\delta_2 = g \left[1 + r_\infty + r_\infty^2 \cdots \right] \tag{D-12}$$

Therefore, there are

$$\rho_{d\bar{d}} = g r_\infty \left[1 + r_\infty + r_\infty^2 \cdots \right] \approx \frac{g r_\infty}{1 - r_\infty^2} \tag{D-13}$$

$$\rho'_{d\bar{d}} = g' r_\infty \left[1 + r_\infty + r_\infty^2 \cdots \right] \approx \frac{g' r_\infty}{1 - r_\infty^2} \tag{D-14}$$

In Eqs. (D, 13-14), the conversion factor is multiplied by the infinite reflectance from one interaction to *n* interactions, which more satisfies the physical meaning. Superposition principle (mathematical physics) [12] is used to decompose the radiation field, and there is a physical relationship for the diffuse horizontal hemispheric flux density through the lateral “wall” A

$$E_b\left(B\right)=\int_{-1}^{0}E_b\left(B,z\right)dz=\int_{-1}^{0}e^{Um'z}E_s\left(0,z\right)dz=e^{Um'z}E_s\left(0,z\right)\Big|_{-1}^{0}=E_s\left(0\right)\left(1-e^{-Um'}\right) \quad \text{(D-15)}$$

Here $\dfrac{dE_b\left(B,z\right)}{Udz}\approx m'E_b\left(0,z\right)\Rightarrow E_b\left(B,z\right)=e^{Um'z}E_b\left(0,z\right)$. The radiative transfer ratio of directional horizontal hemispherical direction is

$$\rho_{\overline{dd}}=1-e^{-Un'} \quad \text{(D-16)}$$

similarly, the radiative transfer ratio of horizontal bi-hemispherical direction is

$$\rho_{\overline{dd}}=1-e^{-Un'} \quad \text{(D-17)}$$

D-3 Solving of DRFs on the boundary of the canopy closure

The block matrices are used to calculate the DRF. They are

$$\mathbf{E}^d=\begin{bmatrix}E_s\\E_-\end{bmatrix},\ \mathbf{E}^u=\begin{bmatrix}E_+\\E_o\end{bmatrix},\ \mathbf{E}^e=\begin{bmatrix}E_b\\E_d\end{bmatrix},$$

$$\mathbf{T}_d=\begin{bmatrix}\tau_{ss}&0\\\tau_{sd}&\tau_{dd}\end{bmatrix},\ \mathbf{R}_b=\begin{bmatrix}0&0\\r_{dd}&0\end{bmatrix},\ \mathbf{R}_t=\begin{bmatrix}r_{sd}&r_{dd}\\r_{so}&r_{do}\end{bmatrix},$$

$$\mathbf{T}_u=\begin{bmatrix}\tau_{dd}&0\\\tau_{do}&\tau_{oo}\end{bmatrix},\ \mathbf{H}_{bl}=\begin{bmatrix}\rho_{s\bar{d}}&\rho_{d\bar{d}}\\0&\rho_{d\bar{d}}\end{bmatrix},\ \mathbf{H}_{da}=\begin{bmatrix}\rho'_{d\bar{d}}&\rho_{\overline{dd}}\\\rho'_{d\bar{d}}&\rho_{\overline{dd}}\end{bmatrix} \quad \text{(D-18)}$$

Then, Eq. (D-1) is simplified as

$$\begin{bmatrix}\mathbf{E}^d\left(b\right)\\\mathbf{E}^u\left(t\right)\\\mathbf{E}^e\left(s\right)\end{bmatrix}=\begin{bmatrix}\mathbf{T}_d&\mathbf{R}_b&0\\\mathbf{R}_t&\mathbf{T}_u&0\\\mathbf{H}_{bl}&\mathbf{H}_{da}&0\end{bmatrix}\begin{bmatrix}\mathbf{E}^d\left(t\right)\\\mathbf{E}^u\left(b\right)\\\mathbf{E}^e\left(i\right)\end{bmatrix} \quad \text{(D-19)}$$

in which the indices refer to the bottom of the canopy in the vertical direction (b), top of the canopy in the vertical direction (t), inner part of the canopy in the horizontal direction (i), surface of the canopy in the horizontal direction (s), downward direction (d), upward direction (u), horizontal direction in lateral "wall" A (bl), and horizontal direction in lateral "wall" B (da).

For a non-Lambert surface, there is

$$\mathbf{E}^u\left(b\right)=\mathbf{R}_s\mathbf{E}^d\left(b\right) \tag{D-20}$$

Defining $\mathbf{R}_s$ as the matrix of the non-Lambert reflectance factor of soil, there is $\mathbf{R}_s=\begin{bmatrix} r_{sd}{}^s & r_{dd}{}^s \\ r_{so}{}^s & r_{do}{}^s \end{bmatrix}$. Then, the relationship between $\mathbf{E}^u\left(t\right)$ and $\mathbf{E}^d\left(t\right)$ can be expressed as

$$\mathbf{E}^u\left(t\right)=\mathbf{R}_\perp\mathbf{E}^d\left(t\right) \tag{D-21}$$

where $\mathbf{R}_\perp$ is the matrix of the reflectance factors at the top surface of the canopy, and

$$\mathbf{R}_\perp=\mathbf{R}_t+\mathbf{T}_u\left(\mathbf{I}-\mathbf{R}_s\mathbf{R}_b\right)^{-1}\mathbf{R}_s\mathbf{T}_d \tag{D-22}$$

Here each element in $\mathbf{R}_\perp$ is $\mathbf{R}_\perp=\begin{bmatrix} r_{sd}{}^* & r_{dd}{}^* \\ r_{so}{}^* & r_{do}{}^* \end{bmatrix}$.

a) The DRF at the top of canopy closure

According to the DRF of the top of canopy derived from the original four-stream radiative transfer equations, i.e., $R=\dfrac{r_{so}^*E_s\left(0\right)+r_{do}^*E_-\left(0\right)}{E_s\left(0\right)+E_-\left(0\right)}$ in [11], here $E_s\left(0\right)$ is the specular flux on top of canopy and $E_-\left(0\right)$ is the diffuse flux on top of canopy. From this equation, the bi-directional reflectance factor (r_{so}^*) and the hemispherical-directional reflectance factor (r_{do}^*) need to be calculated. [1] gives the result of a derivation of Eq. (D-22):

$$r_{so}^*=r_{so}+\tau_{ss}\tau_{oo}r_s+\left\{\left[\left(\tau_{ss}+\tau_{sd}\right)\tau_{do}+\left(\tau_{sd}+\tau_{ss}r_sr_{dd}\right)\tau_{oo}\right]\frac{1}{1-r_sr_{dd}}\right\}r_s \tag{D-23}$$

$$r_{do}^*=r_{do}+\left[(\tau_{do}+\tau_{oo})\tau_{dd}\frac{1}{1-r_sr_{dd}}\right]r_s \tag{D-24}$$

According to (pers. comm. W. Verhoef, 2018), r_{so} in Eq. (D-23) is bidirectional reflectance in the layer, and it consists of the single-scattering of specular flux in the layer (r_{so}^{1}) and the multiple-scattering of specular flux in the layer (r_{so}^{m}). The single-scattering of specular flux in the canopy, and its is

$$r_{so}^{1} = wL'S \tag{D-25}$$

we consider the row structure effect to modify the row structure on the differential leaf area index (leaf area density) for canopy in the vertical direction of continuous crops (L') and area fractions of canopy (S), Eq. (D-25) is modified, then the single-scattering of specular flux in the canopy closure ($r_{so\_v}^{1}$) is

$$r_{so\_v}^{1} = wL'_{row}S_{closure\_s}(z) \tag{D-26}$$

Here L'_{row} and $S_{closure\_s}(z)$ are parameters after considering the influence of the row structure on the canopy closure (Table C-1). L'_{row} is the differential leaf area index (leaf area density) for canopy closure in the vertical direction, and $L'_{row}=\left(A_1+\mathrm{A}_2\right)Lf\left(\theta_l\right)d\theta_l \big/ A_1 h$. Similarly, the multiple-scattering of specular flux in the canopy closure is

$$r_{so\_v}^{m} = S_{closure\_d}\left[\begin{array}{l} \dfrac{\left(v+v'r_\infty\right)T_1+\left(r_\infty v+v'\right)T_2}{1-r_\infty^2} \\ -\left(Q_v \quad P_v\right)\begin{bmatrix} 1 & r_\infty e^{-mL_{row}\Omega_E} \\ r_\infty e^{-mL_{row}\Omega_E} & 1 \end{bmatrix}^{-1}\begin{pmatrix} Q_s \\ P_s \end{pmatrix} r_\infty \Big/ 1-r_\infty^2 \end{array}\right] \tag{D-27}$$

Here T_1, T_1, Q_v, Q_s, P_v and P_s are functions derived from four-stream radiative transfer theory (pers. comm. W. Verhoef, 2018). According to [1], $\tau_{ss}\tau_{oo}r_s$ in Eq. (D-23) is the single-scattering of specular flux from the soil in the canopy, and we consider the row structure with reference to Eq.(D-26), the single-scattering of specular flux from the soil in the canopy closure ($r_{so\_s}^{1}$) is

$$r_{so\_s}^{1} = S_{closure\_s}\left(h\right) r_s \tag{D-28}$$

According to [1], $\left\{\left[\left(\tau_{ss}+\tau_{sd}\right)\tau_{do}+\left(\tau_{sd}+\tau_{ss}r_s r_{dd}\right)\tau_{oo}\right]\frac{1}{1-r_s r_{dd}}\right\} r_s$ in Eq. (D-23) is multiple-scattering between soil and vegetation in the canopy for specular flux. Similar to the modification of row structure in Eq. (D-26), the multiple-scattering between soil and vegetation in the canopy closure for specular flux is

$$r_{so\_s}^{m} = \left(S_{closure\_d} r_s\right)\left\{\left[\left(\tau_{ss}+\tau_{sd}\right)\tau_{do}+\left(\tau_{sd}+\tau_{ss}r_s r_{dd}\right)\tau_{oo}\right]\frac{1}{1-r_s r_{dd}}\right\} \tag{D-29}$$

According to [1], r_{do} in Eq. (D-24) is the single-scattering of diffuse flux in the canopy. Similar to the previous modification of row structure, the single-scattering of diffuse flux in the canopy closure is

$$r_{do}^{1} = r_{do} S_{closure\_d} \tag{D-30}$$

According to [1], $\left[(\tau_{do}+\tau_{oo})\tau_{dd}\frac{1}{1-r_s r_{dd}}\right] r_s$ in Eq. (D-24) is the multiple-scattering of diffuse flux in the canopy. Similar to the modification of row structure in Eq. (D-26), the multiple-scattering of diffuse flux in the canopy closure is

$$r_{do}^{m} = S_{closure\_d}\left[(\tau_{do}+\tau_{oo})\tau_{dd}\frac{1}{1-r_s r_{dd}}\right] r_s \tag{D-31}$$

According to Eqs. (D, 26 and 28), the bi-directional reflectance factor for single-scattering of specular flux ($r_{so\_c\_1}^{*}$) is

$$\begin{aligned} r_{so\_1}^{*} &= r_{so\_v}^{1} + r_{so\_s}^{1} \\ &= wL'_{row} S_{closure\_s}(z) + S_{closure\_s}(h) r_s \end{aligned} \tag{D-32}$$

According to Eqs. (D, 27 and 29), the bi-directional reflectance factor for multiple-scattering of specular flux ($r_{so\_c\_m}^{*}$) is

$$r_{so\_m}^{*}=r_{so\_v}^{m}+r_{so\_s}^{m}$$
$$=S_{closure\_d}\left[\begin{array}{l}\dfrac{\left(v+v'r_\infty\right)T_1+\left(r_\infty v+v'\right)T_2}{1-r_\infty^2}\\ -\left(Q_v \quad P_v\right)\begin{bmatrix}1 & r_\infty e^{-mL'_{row}\Omega_E}\\ r_\infty e^{-mL'_{row}\Omega_E} & 1\end{bmatrix}^{-1}\begin{pmatrix}Q_s\\ P_s\end{pmatrix} r_\infty \Big/ 1-r_\infty^2\end{array}\right]$$
$$+\left(S_{closure\_d}r_s\right)\left\{\left[\left(\tau_{ss}+\tau_{sd}\right)\tau_{do}+\left(\tau_{sd}+\tau_{ss}r_s r_{dd}\right)\tau_{oo}\right]\frac{1}{1-r_s r_{dd}}\right\} \quad \text{(D-33)}$$

According to Eqs. (D, 30 and 31), the hemispherical-directional reflectance factor (this physical quantity describes diffuse flux) ($r_{do\_c}^{*}$)is

$$r_{do}^{*}=r_{do}^{1}+r_{do}^{m}$$
$$=S_{closure\_d}\left\{r_{do}+\left[(\tau_{do}+\tau_{oo})\tau_{dd}\frac{1}{1-r_s r_{dd}}\right]r_s\right\} \quad \text{(D-34)}$$

According to the DRF at the top of canopy derived from the original four-stream radiative transfer equations, i.e., $R=\dfrac{r_{so}^{*}E_s\left(0\right)+r_{do}^{*}E_{-}\left(0\right)}{E_s\left(0\right)+E_{-}\left(0\right)}$ in [11], the nature of the incident radiant flux is considered, i.e., specular flux and diffuse flux. The single-scattering of the canopy closure (R_{c_1}) is

$$R_{c\_1}=\frac{r_{so\_1}^{*}E_s\left(0\right)}{E_s\left(0\right)+E_{-}\left(0\right)} \quad \text{(D-35)}$$

and the multiple-scattering of the canopy closure (R_{c_m}) is

$$R_{c\_m}=\frac{r_{so\_m}^{*}E_s\left(0\right)+r_{do}^{*}E_{-}\left(0\right)}{E_s\left(0\right)+E_{-}\left(0\right)} \quad \text{(D-36)}$$

Note: $r_{so\_1}^{*}$, $r_{so\_m}^{*}$, and r_{do}^{*} are $r_{so\_c\_1}^{*}$, $r_{so\_c\_m}^{*}$, and $r_{do\_c}^{*}$ in the text. To distinguish them from reflectance factor of between-row, hence, a *c* is added to the subscript in the mian text .

b) The DRF of lateral “wall” A and the DRF of lateral “wall” B

Combining Eqs. (D, 4-6), there are

$$\mathbf{E}^e(s)=\left[\mathbf{H}_{bl}\mathbf{R}_\perp^{-1}+\mathbf{H}_{da}\mathbf{T}_u^{-1}\left(\mathbf{I}-\mathbf{R}_t\mathbf{R}_\perp^{-1}\right)\right]\mathbf{E}^u(t) \tag{D-37}$$

$$\mathbf{E}^e(s)=\left[\mathbf{H}_{bl}\mathbf{R}_s^{-1}\mathbf{T}_d^{-1}\left(\mathbf{I}-\mathbf{R}_b\mathbf{R}_s\right)+\mathbf{H}_{da}\right]\mathbf{E}^u(b) \tag{D-38}$$

We define $\mathbf{G}_1$ as the ratio matrix of the radiative transfer from the top surface to the lateral “wall” of the canopy closure, and $\mathbf{G}_2$ as the ratio matrix of the radiative transfer from the bottom surface to the lateral “wall” of the canopy closure, namely

$$\mathbf{G}_1=\begin{bmatrix} g_{s\bar d} & g_{d\bar d} \\ g_{so} & g_{\bar d o}\end{bmatrix}=\mathbf{H}_{bl}\mathbf{R}_\perp^{-1}+\mathbf{H}_{da}\mathbf{T}_u^{-1}\left(\mathbf{I}-\mathbf{R}_t\mathbf{R}_\perp^{-1}\right) \tag{D-39}$$

$$\mathbf{G}_2=\begin{bmatrix} g'_{s\bar d} & g'_{d\bar d} \\ g'_{so} & g'_{\bar d o}\end{bmatrix}=\mathbf{H}_{bl}\mathbf{R}_s^{-1}\mathbf{T}_d^{-1}\left(\mathbf{I}-\mathbf{R}_b\mathbf{R}_s\right)+\mathbf{H}_{da} \tag{D-40}$$

Combining with Eqs. (D, 4-6), there are

$$\mathbf{E}^e(s)=\mathbf{G}_1\mathbf{E}^u(t) \tag{D-41}$$

$$\mathbf{E}^e(s)=\mathbf{G}_2\mathbf{E}^u(b) \tag{D-42}$$

Then, multiplying Eq. (D-41) by $\mathbf{E}^d(t)^{-1}$ produces

$$\mathbf{E}^e(s)\times\mathbf{E}^d(t)^{-1}=\mathbf{G}_1\mathbf{E}^u(t)\times\mathbf{E}^d(t)^{-1}=\mathbf{G}_1\mathbf{R}_\perp \tag{D-43}$$

where $\times$ denotes the vector product.

Combining Eqs. (B, 17-18), Eq. (D-42), and Eq. (D-43), Eqs. (D, 41-42) can be written as

$$\frac{1}{o_1}\mathbf{E}^e(s)\times\mathbf{E}'_-(*)^{-1}=\mathbf{G}_1\mathbf{R}_\perp \tag{D-44}$$

$$\frac{1}{o_2}\mathbf{E}^e(s)\times\mathbf{E}'_+(*)^{-1}=\mathbf{G}_2 \tag{D-45}$$

where $\mathbf{E}'_-(*)$ and $\mathbf{E}'_+(*)$ are the vectors of the downward diffuse flux and upward diffuse flux in the diagonal area of the canopy closure, respectively, as shown in Fig. B-2. The symbol $*$ represents the diagonal area. The sum of Eq. (D-44) and Eq. (D-45) is

$$\mathbf{E}^{e}\left(s\right)^{-1}\times\mathbf{E}'_{-}\left(*\right)+\mathbf{E}^{e}\left(s\right)^{-1}\times\mathbf{E}'_{+}\left(*\right)=\left(\mathbf{G}_1\mathbf{R}_{\perp}o_1\right)^{-1}+\left(\mathbf{G}_2o_2\right)^{-1} \tag{D-46}$$

Eq. (B-45) is resolved as

$$\left[\mathbf{E}'_{-}\left(*\right)+\mathbf{E}'_{+}\left(*\right)\right]\times\mathbf{E}^{e}\left(s\right)^{-1}=\left(\mathbf{G}_1\mathbf{R}_{\perp}o_1\right)^{-1}+\left(\mathbf{G}_2o_2\right)^{-1} \tag{D-47}$$

Combined with Eq. (B-19), there is $\mathbf{E}^{e}(i)=\left[\mathbf{E}'_{+}\left(*\right)+\mathbf{E}'_{-}\left(*\right)\right]o_3$. Then, Eq. (D-47) becomes

$$\begin{aligned}\mathbf{R}_{\|}&=\mathbf{E}^{e}\left(s\right)\times\mathbf{E}^{e}\left(i\right)^{-1}=\left[\left(\mathbf{G}_1\mathbf{R}_{\perp}o\right)^{-1}+\left(\mathbf{G}_2o'\right)^{-1}\right]^{-1}o_3^{-1}\\&=\mathbf{G}_1\mathbf{R}_{\perp}o_1\mathbf{G}_2o_2\left[\mathbf{G}_1\mathbf{R}_{\perp}o_1+\mathbf{G}_2o_2\right]^{-1}o_3^{-1}=\begin{bmatrix}r_{\|s\bar{d}} & r_{\|d\bar{d}}\\ r_{\|so} & r_{\|\bar{d}o}\end{bmatrix}\end{aligned} \tag{D-48}$$

where $\mathbf{R}_{\|}$ is the matrix of horizontal transmittance factor in the lateral "wall". Here, the soil is assumed to be Lambertian, namely $r_{sd}{}^{s}=r_{dd}{}^{s}=r_{so}{}^{s}=r_{do}{}^{s}=r_s$, and the elements of $\mathbf{R}_{\|}$ are

$$r_{\|s\bar{d}}=\frac{o_1o_2\rho'_{d\bar{d}}\left(D_1+\tau_{dd}\rho_{s\bar{d}}\tau_{oo}\right)}{o_3\left(M_1+o_1\tau_{dd}\rho_{s\bar{d}}\tau_{oo}\right)} \tag{D-49}$$

$$r_{\|d\bar{d}}=\frac{o_1o_2\rho_{\bar{d}\bar{d}}\left(D_1+\tau_{dd}\rho_{s\bar{d}}\tau_{oo}\right)}{o_3M_2} \tag{D-50}$$

$$r_{\|so}=\frac{o_1o_2\rho'_{d\bar{d}}D_1}{o_3M_1} \tag{D-51}$$

$$r_{\|\bar{d}o}=\frac{o_1o_2\rho_{\bar{d}\bar{d}}D_1}{o_3M_2} \tag{D-52}$$

in which

$$\begin{aligned}D_1=\;&\rho_{\bar{d}\bar{d}}r_{dd}\tau_{do}-\rho_{\bar{d}\bar{d}}r_{do}\tau_{dd}-\rho_{\bar{d}\bar{d}}\overset{*}{r}_{dd}\tau_{do}+\rho_{\bar{d}\bar{d}}\overset{*}{r}_{do}\tau_{dd}-\rho'_{d\bar{d}}r_{dd}\tau_{oo}+\rho_{\bar{d}\bar{d}}r_{sd}\tau_{do}\\&-\rho_{\bar{d}\bar{d}}r_{so}\tau_{dd}-\rho_{\bar{d}\bar{d}}\overset{*}{r}_{sd}\tau_{do}+\rho_{\bar{d}\bar{d}}r_{so}\tau_{dd}-\rho'_{d\bar{d}}r_{sd}\tau_{oo}+\rho'_{d\bar{d}}\overset{*}{r}_{sd}\tau_{oo}+\rho_{d\bar{d}}\tau_{dd}\tau_{oo}\end{aligned} \tag{D-53}$$

$$M_1 = o_1\rho_{\overline{dd}} r_{sd}\tau_{do} - o_1\rho_{\overline{dd}} r_{so}\tau_{dd} - o_1\rho_{\overline{dd}} r_{sd}^{*}\tau_{do} + o_1\rho_{\overline{dd}} r_{so}^{*}\tau_{dd}$$
$$-o_1\rho'_{d\bar{d}} r_{sd}\tau_{oo} + o_1\rho'_{d\bar{d}} r_{sd}^{*}\tau_{oo} + o_2\rho'_{d\bar{d}}\tau_{dd}\tau_{oo} \quad \text{(D-54)}$$

$$M_2 = o_1\rho_{\overline{dd}} r_{dd}\tau_{do} - o_1\rho_{\overline{dd}} r_{do}\tau_{dd} - o_1\rho_{\overline{dd}} r_{dd}^{*}\tau_{do} + o_1\rho_{\overline{dd}} r_{so}^{*}\tau_{dd} - o_1\rho'_{d\bar{d}} r_{dd}\tau_{oo}$$
$$+o_1\rho'_{d\bar{d}} r_{dd}^{*}\tau_{oo} + o_2\rho_{\overline{dd}}\tau_{dd}\tau_{oo} + o_1\rho_{d\bar{d}}\tau_{dd}\tau_{oo} \quad \text{(D-55)}$$

According to DRF derived from the original four-stream radiative transfer equations, i.e., $R = \dfrac{r_{so}^{*}E_s(0) + r_{do}^{*}E_-(0)}{E_s(0) + E_-(0)}$ in [11], this equation shows that the DRF is the ratio of the reflected flux ($r_{so}^{*}E_s(0) + r_{do}^{*}E_-(0)$) to the incident flux ($E_s(0) + E_-(0)$) at the top of the canopy. Similarly, we need to calculate incident flux in the lateral "wall". Incident flux in the lateral "wall" includes specular flux and diffuse flux. The specular flux in surface of lateral "wall" is

$$E_{\|s}(B) = \rho_{s\bar{d}} E_s(0) \quad \text{(D-56)}$$

and the diffuse flux in surface of lateral "wall" is

$$E_{\|\pm}(B) = \rho_{d\bar{d}}E_-(0) + \rho'_{d\bar{d}}E_+(-1) = \rho_{d\bar{d}}E_-(0) + \rho'_{d\bar{d}}\left\{\left[E_-(-1) + E_s(-1)\right]r_s\right\}$$
$$= \rho_{d\bar{d}}E_-(0) + \rho'_{d\bar{d}}\left\{\left[e^{-L_{row}}E_-(0) + e^{-kL_{row}}E_s(0)\right]r_s\right\} \quad \text{(D-57)}$$

Finally, DRF of the lateral "wall" A is

$$R_b = \frac{r_{\|s\bar{d}}E_{\|s}(B) + r_{\|\bar{d}o}E_{\|\pm}(B)}{E_{\|s}(B) + E_{\|\pm}(B)} \quad \text{(D-58)}$$

and the DRF of the lateral "wall" B is

$$R_d = \frac{r_{\|\bar{d}o}E_{\|s}(B) + r_{\|d\bar{d}}E_{\|\pm}(B)}{E_{\|s}(B) + E_{\|\pm}(B)} \quad \text{(D-59)}$$

E. Solving of the DRF of between-row based on integral raditive transfer equation

The between-row area consists of two lateral "walls" (A' and B'), between-row background (C') and escaping surface ($abcd$ in Fig. E-1). Radiation transfer in this area is influenced by two mediums, i.e., vegetation leaf and soil particle. The differential-integral form of the radiative transfer equation is transformed into an integral form to describe the radiative transfer among these four components [13], there is

$$f\left(z,\Omega\right)=\int_{4\pi}\mathbf{K}\left(x,z,\Omega'\rightarrow\Omega\right)f\left(z,\Omega'\right)d\Omega+f_0\left(z,\Omega\right) \tag{E-1}$$

Here $f_0\left(z,\Omega\right)$ is the source function of a medium. $\mathbf{K}\left(x,z,\Omega'\rightarrow\Omega\right)$ is the transfer probability of collision, and $\mathbf{K}\left(x,z,\Omega'\rightarrow\Omega\right)=\mathbf{k}\left(x,z,\Omega'\rightarrow\Omega\right)*\mathbf{a}\left(x,z,\Omega'\rightarrow\Omega\right)$, here $*$ is symbol of hadamard product. $\mathbf{k}\left(x,z,\Omega'\rightarrow\Omega\right)$ is the matrix of transfer probabilities between lateral "walls", between-row background and escaping surface, which is composed of the probabilities of four components, and its expression reference E-1 in this section. $\mathbf{a}\left(x,z,\Omega'\rightarrow\Omega\right)$ is the matrix of light attenuation coefficients, and its expression reference E-2 in this section. Finally, the equation is solved in E-3 in this section.

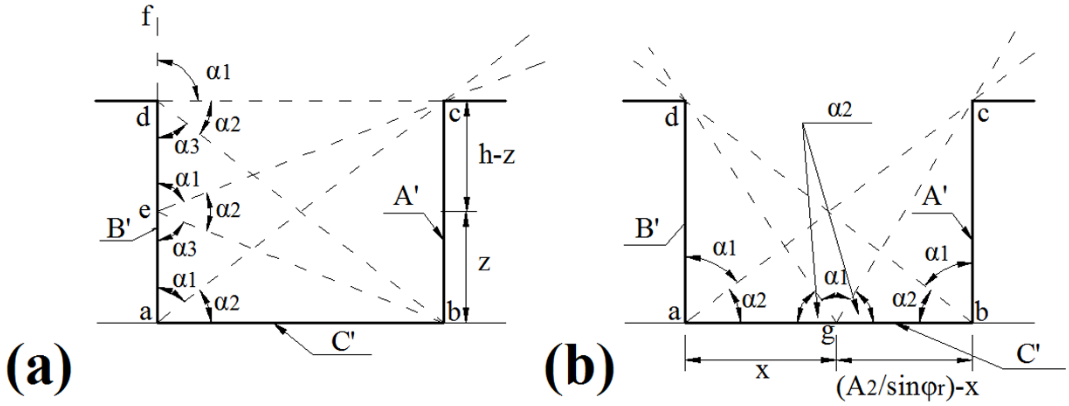

Fig. E-1 Sketch of transfer probability matrix in the between-row area. (a) Transfer probability matrix in the *Z*-axis; (b) transfer probability matrix in the *X*-axis.

E-1 Transfer probability

According to Fig. E-1, when the value of the solar azimuth angle is $0^\circ \le \varphi_s < 180^\circ$, B' is the lateral "wall" A. Similarly, when the value of the solar azimuth angle is $180^\circ \le \varphi_s < 360^\circ$, A' is the lateral "wall" A. The related angles are shown in Fig. E-1, α_1 is the angle of radiation escaping in the between-row, α_2 is the angle of radiative transfer between both lateral "walls" in the between-row, α_3 is the angle of radiative transfer between lateral "wall" and soil background in the between-row. The transfer probability in the between-row is the ratio of these angles to the sum angle. Therefore, the average probability of radiation escaping from the lateral "wall" ($k_{B(A)\to}$) is $\frac{\angle fdc}{\pi}$=0.5, $z=h$, $\frac{\angle fec}{\pi}$, $z \in (0,h)$, and $\frac{2\angle fac}{\pi}$, $z=0$. Then

$$k_{B(A)\to A(B)}=\frac{1}{h}\int_0^h P_{B(A)\to A(B)}dz,$$
$$P_{B(A)\to A(B)}=\begin{cases}\frac{1}{\pi}\arctan\left(\frac{h\sin\varphi_{or}}{A_2}\right) & z=h\\ \frac{1}{\pi}\left\{\pi-\arctan\left[\frac{A_2}{\sin\varphi_{or}\left(h-z\right)}\right]-\arctan\left(\frac{A_2}{z\sin\varphi_{or}}\right)\right\} & z\in\left(0,h\right)\\ \frac{2}{\pi}\arctan\left(\frac{h\sin\varphi_{or}}{A_2}\right) & z=0\end{cases} \tag{E-2}$$

Eq. (E-2) is a linear decreasing function, in which $k_{B(A)\to}$ decreases with the increase of the depth of canopy closure. Similarly, the average probability of radiation transferring between two lateral "walls" is

$$k_{B(A)\to A(B)}=\frac{1}{h}\int_0^h P_{B(A)\to A(B)}dz,$$
$$P_{B(A)\to A(B)}=\begin{cases}\frac{1}{\pi}\arctan\left(\frac{h\sin\varphi_{or}}{A_2}\right) & z=h\\ \frac{1}{\pi}\left\{\pi-\arctan\left[\frac{A_2}{\sin\varphi_{or}\left(h-z\right)}\right]-\arctan\left(\frac{A_2}{z\sin\varphi_{or}}\right)\right\} & z\in\left(0,h\right)\\ \frac{2}{\pi}\arctan\left(\frac{h\sin\varphi_{or}}{A_2}\right) & z=0\end{cases} \tag{E-3}$$

Eq. (E-3) is a hyperbolic function, in which $k_{B(A)\to A(B)}$ decreases first, then increases with the increase of depth of canopy closure. The average probability of radiation transferring from the lateral "wall" to between-row background is

$$k_{B(A)\to C}=\frac{1}{h}\int_0^h P_{B(A)\to C}dz,\quad P_{B(A)\to C}=\begin{cases}\frac{1}{\pi}\arctan\left(\frac{A_2 h}{z\sin\varphi_{or}}\right) & z=h\\ \frac{1}{\pi}\arctan\left(\frac{A_2}{z\sin\varphi_{or}}\right) & z\in\left(0,h\right)\\ 0 & z=0\end{cases} \tag{E-4}$$

Eq. (E-4) is an incremental function with the increase of depth. Using the same mathematical principles, the average probability of the radiation escaping from the between-row background is

$$k_{C\to} = \frac{1}{A_2}\int_0^{A_2} P_{c\to}\,dx,$$

$$P_{c\to} = \begin{cases} \dfrac{2}{\pi}\arctan\left(\dfrac{A_2 h}{\sin\varphi_{or}}\right) & (x = A_2)\wedge(x=0) \\ \dfrac{1}{\pi}\left[\pi - \arctan\left(\dfrac{h\sin\varphi_r}{A_2 - x\sin\varphi_{or}}\right) - \arctan\left(\dfrac{h}{x}\right)\right] & x\in(0, A_2) \end{cases} \tag{E-5}$$

and the average probability that radiation transferring from between-row background to lateral "wall" is

$$k_{C\to A(B)} = \frac{1}{A_2}\int_0^{A_2} P_{C\to A(B)}\,dx,$$

$$P_{C\to A(B)} = \begin{cases} \dfrac{2}{\pi}\arctan\left(\dfrac{h\sin\varphi_{or}}{A_2}\right) & (x = A_2)\wedge(x=0) \\ \dfrac{\arctan\left(\dfrac{h}{x}\right) + \arctan\left(\dfrac{h\sin\varphi_{or}}{A_2 - x\sin\varphi_{or}}\right)}{\pi} & x\in(0, A_2) \end{cases} \tag{E-6}$$

Eqs. (E , 2-6) are elements of matrix of transfer probability between lateral "wall", between-row background and escape surface. Therefore, the matrix of transfer probability is

$$\mathbf{k}\left(x,z,\Omega'\to\Omega\right) = \begin{cases} \begin{bmatrix} k_{B\to B} & k_{B\to A} & k_{B\to C} & k_{B\to escape} \\ k_{A\to B} & k_{A\to A} & k_{A\to C} & k_{A\to escape} \\ k_{C\to B} & k_{C\to A} & k_{C\to C} & k_{C\to escape} \\ k_{escape\to B} & k_{escape\to A} & k_{escape\to C} & k_{escape\to escape} \end{bmatrix} & 0^\circ \le \varphi_s < 180^\circ \\ \begin{bmatrix} k_{A\to A} & k_{A\to B} & k_{A\to C} & k_{A\to escape} \\ k_{B\to A} & k_{B\to B} & k_{B\to C} & k_{B\to escape} \\ k_{C\to A} & k_{C\to B} & k_{C\to C} & k_{C\to escape} \\ k_{escape\to A} & k_{escape\to B} & k_{escape\to C} & k_{escape\to escape} \end{bmatrix} & 180^\circ \le \varphi_s < 360^\circ \end{cases} \tag{E-7}$$

$$= \begin{bmatrix} 0 & k_{B(A)\to A(B)} & k_{B(A)\to C} & k_{B(A)\to} \\ k_{B(A)\to A(B)} & 0 & k_{B(A)\to C} & k_{B(A)\to} \\ k_{C\to A(B)} & k_{C\to A(B)} & 0 & k_{C\to} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

E-2 Light attenuation coefficient

Radiation transfer in this area is influenced by two mediums, i.e., vegetation in two lateral "walls" (A' and B') and soil of between-row (C'). For attenuation coefficient in two lateral "walls", we continue to use the attenuation coefficient for horizontal diffuse flux (n'). For attenuation coefficient for soil of between-row (a_s), we derived backward by using the modified Hapke model [14]. According to (pers. comm. W. Verhoef, 2018), there is

$$a_s = 2 - \frac{4r_s\left(\cos\theta_s + \cos\theta_o\right)}{p\left(\delta\right)\cos\theta_s^2\cos\theta_o^2}\left(1 - \frac{b}{4}\right) \tag{E-8}$$

Here $p\left(\delta\right)$ is the scattering phase function of soil particle, which represents the second-order Legendre polynomial (an approximation of spherical function) [15], and $p\left(\delta\right) = 1 + b\cos\delta + c\frac{3\cos^2\delta - 1}{2}$, and $\cos\delta = \cos\theta_s\cos\theta_o + \sin\theta_s\sin\theta_o\cos\varphi_{so}$. Here b and c are the adjustment parameters for the second-order Legendre polynomial in the scattering phase function of soil particle, and they can be determined by [16]. According to Eq. (E-7), and then combined with the calculation rules of hadamard product (i.e., $\mathbf{K}\left(x,z,\Omega'\to\Omega\right) = \mathbf{k}\left(x,z,\Omega'\to\Omega\right) * \mathbf{a}\left(x,z,\Omega'\to\Omega\right)$), $\mathbf{a}\left(x,z,\Omega'\to\Omega\right)$ is the matrix of light attenuation coefficient

$$\mathbf{a}\left(x,z,\Omega'\to\Omega\right) = \begin{bmatrix} 0 & n' & a_s & 0 \\ n' & 0 & a_s & 0 \\ n' & n' & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{E-9}$$

E-3 Solving equations

Converting Eq. (E-1) into an operator notation, there is

$$f = \mathbf{K}f + f_0 \tag{E-10}$$

in which

$$\begin{aligned}&\mathbf{K}\left(x,z,\Omega'\rightarrow\Omega\right)=\mathbf{k}\left(x,z,\Omega'\rightarrow\Omega\right)*\mathbf{a}\left(x,z,\Omega'\rightarrow\Omega\right)\\&=\begin{bmatrix}0 & n'k_{B(A)\rightarrow A(B)} & a_s k_{B(A)\rightarrow C} & k_{B(A)\rightarrow}\\ n'k_{B(A)\rightarrow A(B)} & 0 & a_s k_{B(A)\rightarrow C} & k_{B(A)\rightarrow}\\ n'k_{C\rightarrow A(B)} & n'k_{C\rightarrow A(B)} & 0 & k_{C\rightarrow}\\ 0 & 0 & 0 & 0\end{bmatrix}\end{aligned} \tag{E-11}$$

According to the Riemann series principle, for $|\mathbf{K}|<1$, and we let $f_0 = \mathbf{J}$. Eq. (E-11) becomes

$$f = \mathbf{J}+\mathbf{KJ}+\mathbf{K}^2\mathbf{J}+\cdots\approx\mathbf{J}+\mathbf{JK}\left(\mathbf{I}-\mathbf{K}\right)^{-1} \tag{E-12}$$

Eq. (E-12) represents the collision of radiation in the between-row, which is composed of single scattering and multiple scattering (i.e., up to n-scattering). The between-row area includes two lateral “walls” (A' and B'), between-row background (C') and escaping surface ($abcd$ in Fig. D-1). The DRF on the A', B', C' and escaping surface should be the DRF of the soil of between-row (R_s) and the DRF of the two lateral walls (R_b *and* R_d) and 0, respectively.

$$R_s = \frac{r_s E_s(0)}{E_s(0)+E_-(0)} \tag{E-13}$$

Then, we let single scattering of between-row for A', B', C' and escaping surface be

$$\mathbf{R}_{b\_r\_1}=\mathbf{J}=\begin{cases}\begin{bmatrix}R_b & R_d & R_s & 0\end{bmatrix} & 0^\circ\leq\varphi_s<180^\circ\\ \begin{bmatrix}R_d & R_b & R_s & 0\end{bmatrix} & 180^\circ\leq\varphi_s<360^\circ\end{cases} \tag{E-14}$$

and substitute these initial values into Eq. (C-12), there is

$$\mathbf{R}_{b\_r} = \mathbf{R}_{b\_r\_1} + \mathbf{R}_{b\_r\_m} = \mathbf{J} + \mathbf{JK}\left(\mathbf{I} - \mathbf{K}\right)^{-1} \tag{E-15}$$

Here $\mathbf{R}_{b\_r\_m}$ is multiple scattering matrices of A', B', C' and escaping surface. The final calculation result of $\mathbf{R}_{b\_r\_m}$ is $\begin{bmatrix} R_{b\_m} & R_{d\_m} & R_{s\_m} & 0 \end{bmatrix}^T$. In the between-row, we focus on between-row background, i.e., multiple scattering of soil in the between row, and both initial values are calculated to be the same value, there is

$$R_{s\_m} = \frac{a_s k_{B(A)\to C}\left(R_b + R_d\right) + 2a_s n' k_{B(A)\to C} k_{C\to A(B)} R_s}{1 - k_{B(A)\to A(B)} - 2a_s n' k_{B(A)\to C} k_{C\to A(B)}} \tag{E-16}$$

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