USE OF LINEAR AND POWER TRANSFORMATIONS FOR ESTIMATING THE POPULATION MEAN IN TWO-OCCASION SUCCESSIVE SAMPLING

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ABSTRACT
The paper advocates the problem of estimating population mean on the current (second) occasion using auxiliary information in successive sampling over two-occasions. A class of estimators based on transformed auxiliary variable is derived. The bias and mean square error of the proposed estimator have been obtained. The suggested estimator has been compared with simple mean estimator when there is no matching and the optimum estimator, which is a combination of the means of the matched and un-matched portion of the sample at the second occasion. Optimum replacement policy and the efficiency of the suggested estimator have been discussed. Numerical Illustration is given in support of the present study.

KEYWORDS: Auxiliary variable, study variable, successive sampling, mean square error.

MSC: 62D05

RESUMEN
Este paper trata el problema de estimar la media de la población una clase de estimadores basada en la variable transformada. Son obtenidos el sesgo y el error cuadrático medio del estimador propuesto. El estimador ha sido comparado con el común estimador de la media cuando no hay sobrelapamiento con el estimador óptimo, el que es una combinación de las medias de la porción en la actual, (segunda) ocasión, usando información auxiliar en el muestreo sucesivo en dos-ocasiones soprelapada y no soprelapada de la muestra en la segunda ocasión. Se sugiere una política de reemplazo optimal y se discute sobre la eficiencia del estimador sugerido. Una ilustración numérica es dada para soportar el presente estudio

PALABRAS CLAVE: variable auxiliar, variable de estudio, muestreo sucesivo, error cuadrático medio.

1. INTRODUCTION

In successive (rotation) sampling, it is not uncommon in practice to use the information collected on the preceding occasions to improve the precision of the estimates on the current occasion. The problem of sampling on two successive occasions with a partial replacement of sampling units was first introduced by Jessen (1942) in the analysis of a survey that collected from data. After Jessen (1942) several authors including Patterson (1950), Eckler (1955), Rao and Graham (1964), Singh et al. (1992) and among others have developed the theory of successive sampling. Feng and Zou (1997) and Biradar and Singh (2001) used the auxiliary information on both the occasions for estimating the current population mean in the successive sampling. Singh (2005), Singh and Vishwakarma (2007a, 2007b, 2009), Singh and Pal (2014), Singh and Pal(2015a, 2015b, 2015c), Singh and Pal (2016a, 2016b, 2016c) Singh and Pal (2017a, 2017b, 2017c, 2017d, 2017e), Pal and Singh (2017a, 2017b) have used the auxiliary information on both the occasion and envisaged several estimators for the estimating the population mean on current occasion in two- occasion successive (rotation) sampling.

The procedure discussed in the above studies have used information only on the population mean $\bar{Z}$ of the auxiliary variable *z*, while in various survey situations information on other parameters of the auxiliary variable *z* such as coefficient of variation C_z, population standard deviation S_z, population coefficients of skewness $\beta_1(z)$ and kurtosis $\beta_2(z)$; and the correlation coefficients between study variable *y* and *z* (ρ_{yz}); and the auxiliary variable *x* and *z* (ρ_{xz}) are known, for instance see Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh and Tailor (2003), Kadilar and Cingi (2004), Chandra and Singh (2005), Koyuncu and Kadilar (2009), Sousa et al.(2010) and Singh and Solanki (2013a, 2013b).

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Keeping this in view we have suggested an effective and efficient estimation procedure , which provides a cost effective estimate of the population mean on the current (second) occasion in two-occasion successive (rotation) sampling.

Let $U= (U_1, U_2,..., U_N)$ be the finite population of size N units, which has been sampled over two occasions. Let $x\,(y)$ be the variable under study on the first (second) occasion respectively. It is assumed that information on an auxiliary variable z (stable over occasion) is readily available for the both the occasions. It is assumed that the population under investigation is large, and the sample size is constant on each occasion. A simple random sample of n units is drawn without replacement (WOR) on the first occasion. A random sub sample of m (= $n\lambda$) units is retained (soprelapada) from the sample drawn on the first occasion for its use on the current (second) occasion, while a fresh sample of size $u= (n\text{-}m) = n\,\mu$ units is drawn on the current (second) occasion, from the entire population by simple random sampling without replacement (SRSWOR) procedure so that the sample size on the current (second) is also n. The fractions of the soprelapada and fresh samples are respectively designated by λ and μ such that $\lambda+\mu=1$.

In what follows we shall use the following notations throughout this paper.

$\bar{X}\,,\bar{Y}\,,\ \bar{Z}$: The population means of the variables x, y and z respectively.

$\bar{x}_m,\ \bar{x}_n\,,\bar{y}_u,\ \bar{y}_m\,,\bar{z}_u\,,\ \bar{z}_n$: The sample means of the respective variables based on the sample sizes indicated in suffices.

$C_x,\ C_y,\ C_z$: The coefficients of variation of the variables x, y and z respectively,

$\rho_{yx},\ \rho_{yz},\ \rho_{xz}$: The correlation coefficients between the variables shown in suffices .

$S_x^2=(N-1)^{-1}\sum_{i=1}^{N}(x_i-\bar{X})^2$, $S_y^2=(N-1)^{-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2$, $S_z^2=(N-1)^{-1}\sum_{i=1}^{N}(z_i-\bar{Z})^2$ are the population mean squares of x, y and z respectively,

$f=n/N$: Sampling fraction. *fda*: fist degree of approximation.

The rest part of the paper is prepared as follows: In Section 2, the estimators of population mean have been suggested and the expressions of their bias and the mean square error are obtained. Section 3 addresses the problem of optimal replacement policy while Section 4 has focused on efficiency comparisons and empirical study of proposed estimators. Section 5 dealt with the optimum estimator along with its properties. Concluding remarks are given in Section 6.

2. DEVELOPMENT OF THE ESTIMATOR

To develop the estimator of the population mean $\bar{Y}$ on the current (second) occasion in two-occasion successive sampling, motivated by Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh and Agnihotri (2008) and Srivastava (1967) we have given two estimators using linear as well as power transformation simultaneously over auxiliary variable z. One is based on a sample of size $u = n\,\mu$ drawn, afresh on the second occasion and is defined by

$$P_u=\bar{y}_u\left(\frac{a\bar{Z}+b}{a\bar{z}_u+b}\right)^{\alpha}, \tag{2.1}$$

where α being a constant which takes real values +ive (-ive) for generating ratio-type (product-type) estimators and '0 (zero)' for the usual unbiased estimator which does not utilize information on auxiliary variable z ; and (a,b) are suitably chosen scalars . The scalars (a,b) may assume real values as well as parametric values C_z (coefficient of variation of the auxiliary variable z), ρ_{xz} (correlation coefficient between x and z), $\beta_1(z)$ (coefficient of skewness of z), $\beta_2(z)$ (coefficient of kurtosis of z), $\bar{Z}$ (population mean of z) and S_z (standard deviation of z) etc.

Motivated by Singh et al. (2004), we consider the second estimator P_m based on the sample of size m common with both the occasions, as

$$P_m=\bar{y}_m\left(\frac{\bar{x}_n}{\bar{x}_m}\right)^{\alpha_1}\left(\frac{a\bar{Z}+b}{a\bar{z}_n+b}\right)^{\alpha_2}\left(\frac{a\bar{Z}+b}{a\bar{z}_m+b}\right)^{\alpha_3} \tag{2.2}$$

where (a,b) are same as defined earlier and α_i's (i = 1, 2, 3) are unknown scalar to be suitably determined (or α_i's (i = 1, 2, 3) may take real values +ive (-ive) and '0').

Now combining the two estimators P_u and P_m, we propose the final estimator of the population mean $\bar{Y}$ on the current occasion, as

$$P = \omega P_u + (1-\omega)P_m, \tag{2.3}$$

where ω is an unknown constant to be determined under certain criterion.

It is interesting to mention that for $\alpha = 1$ in (2.1) and ($\alpha_1, \alpha_2, \alpha_3$) = (1, 1, 0) in (2.2) the estimators P_u and P_m respectively reduce to the Singh and Majhi (2013) estimators and hence combined estimator 'P' is more general than that of the Singh and Majhi (2013) estimator.

3. BIAS AND MEAN SQUARED ERROR (*MSE*) OF THE PROPOSED ESTIMATOR

We note that the estimators P_u and P_m are rectified versions of ratio and chain ratio- type estimators respectively, they are biased estimators of the population mean $\bar{Y}$. Thus the combined estimator P in (2.3) would be a biased estimator of the population mean $\bar{Y}$. The bias and mean squared error (*MSE*) of the estimator 'P' are obtained up to the first order approximation. To obtain the bias and mean square error of suggested class of estimators 'P' we define following quantities

$$\bar{y}_u = \bar{Y}(1+e_{0u}),\ \bar{y}_m = \bar{Y}(1+e_{0m}),\ \bar{x}_n = \bar{X}(1+e_{1n}),$$

$$\bar{x}_m = \bar{X}(1+e_{1m}),\ \bar{z}_u = \bar{Z}(1+e_{2u}),\ \bar{z}_m = \bar{Z}(1+e_{2m}) \text{ and } \bar{z}_n = \bar{Z}(1+e_{2n}).$$

such that

$$E(e_{0u}) = E(e_{0m}) = E(e_{1n}) = E(e_{1m}) = E(e_{2u}) = E(e_{2n}) = 0$$

and

$$E(e_{0u}^2) = \hbar C_y^2,\ E(e_{0m}^2) = \lambda C_y^2,\ E(e_{1m}^2) = \lambda C_x^2,\ E(e_{1n}^2) = \ell C_x^2,\ E(e_{2u}^2) = \hbar C_z^2,$$

$$E(e_{2n}^2) = \ell C_z^2,\ E(e_{2m}^2) = \lambda C_z^2,\ E(e_{0u}e_{0m}) = -\wp C_y^2,\ E(e_{0u}e_{1m}) = -\wp \rho_{yx} C_y C_x,$$

$$E(e_{0u}e_{1n}) = -\wp \rho_{yx} C_y C_x,\ E(e_{0u}e_{2u}) = \hbar \rho_{yz} C_y C_z,\ E(e_{0u}e_{2m}) = -\wp \rho_{yz} C_y C_z,$$

$$E(e_{0u}e_{2n}) = -\wp \rho_{yz} C_y C_z,\ E(e_{0m}e_{1m}) = \lambda \rho_{yx} C_y C_x,\ E(e_{0m}e_{1n}) = \ell \rho_{yx} C_y C_x,$$

$$E(e_{0m}e_{2u}) = -\wp \rho_{yz} C_y C_z,\ E(e_{0m}e_{2m}) = \lambda \rho_{yz} C_y C_z,\ E(e_{0m}e_{2n}) = \ell \rho_{yz} C_y C_z,$$

$$E(e_{1m}e_{1n}) = \ell \rho_{yx} C_y C_x,\ E(e_{1m}e_{2u}) = -\wp \rho_{xz} C_x C_z,\ E(e_{1m}e_{2n}) = \ell \rho_{xz} C_x C_z,$$

$$E(e_{1m}e_{2m}) = \lambda \rho_{xz} C_x C_z,\ E(e_{1n}e_{2u}) = -\wp \rho_{xz} C_x C_z,\ E(e_{1n}e_{2n}) = \ell \rho_{xz} C_x C_z,$$

$$E(e_{1n}e_{2m}) = \ell \rho_{xz} C_x C_z,\ E(e_{2u}e_{2n}) = -\wp C_z^2,\ E(e_{2u}e_{2m}) = -\wp C_z^2,\ E(e_{2m}e_{2n}) = \lambda C_z^2.$$

Where $\ell = \left(n^{-1} - N^{-1}\right), \hbar = \left(u^{-1} - N^{-1}\right), \lambda = \left(m^{-1} - N^{-1}\right), \wp = N^{-1}$.

3.1 The Bias and *MSE* of the Estimator 'P_u'

We now express P_u at (2.1) in terms of e's as

$$P_u = \bar{Y}\frac{(1+e_{0u})}{(1+\phi e_{2u})^{\alpha}}$$

$$= \bar{Y}(1+e_{0u})(1+\phi e_{2u})^{-\alpha}, \tag{3.1}$$

where $\phi = a\bar{Z}/(a\bar{Z}+b)$.

We assume that $|\phi e_{2u}| < 1$, so that $(1+\phi e_{2u})^{-\alpha}$ is expandable. Now expanding the right hand side of (3.1), multiplying out and neglecting terms of e's having power greater than two we have

$$P_u \cong \bar{Y}\left[1 + e_{0u} - \alpha\phi e_{2u} - \alpha\phi e_{0u}e_{2u} + \frac{\alpha(\alpha+1)}{2}\phi^2 e_{2u}^2\right],$$

or

$$(P_u - \bar{Y}) \cong \bar{Y}\left[e_{0u} - \alpha\phi e_{2u} - \alpha\phi e_{0u} e_{2u} + \frac{\alpha(\alpha+1)}{2}\phi^2 e_{2u}^2\right]. \quad (3.2)$$

Taking expectation on both sides of (3.2), we get the bias of the proposed class of estimators P_u up to first degree of approximation as

$$B(P_u) = \left(\frac{\bar{Y}\alpha\phi}{2}\right)\hbar C_z^2\{\phi(\alpha+1) - 2k_{yz}\}.$$

where $k_{yz} = \rho_{yz} C_y / C_z$.

Squaring both sides of (3.2) and neglecting terms of e's having power greater than two, we have

$$(P_u - \bar{Y})^2 \cong \bar{Y}^2[e_{0u} - \alpha\phi e_{2u}]^2$$
$$\cong \bar{Y}^2[e_{0u}^2 + \alpha^2\phi^2 e_{2u}^2 - 2\alpha\phi e_{0u} e_{2u}]. \quad (3.2)$$

Taking expectation on both sides of (2.7), we get the *MSE* of the proposed class of estimators P_u up to first degree of approximation as

$$MSE(P_u) = \bar{Y}^2\hbar[C_y^2 + \alpha\phi C_z^2(\alpha\phi - 2k_{yz})]$$
$$= \bar{Y}^2\hbar\eta_0,$$

where $\eta_0 = [C_y^2 + \alpha\phi C_z^2(\alpha\phi - 2k_{yz})]$.

Thus we state the following theorem.

Theorem 3.1 To the first degree of approximation the bias and *MSE* of the estimator P_u are respectively, given by

$$B(P_u) = \left(\frac{\bar{Y}\alpha\phi}{2}\right)\hbar C_z^2\{\phi(\alpha+1) - 2k_{yz}\}. \quad (3.4)$$

$$MSE(P_u) = \bar{Y}^2\hbar[C_y^2 + \alpha\phi C_z^2(\alpha\phi - 2k_{yz})]$$
$$= \bar{Y}^2\hbar\eta_0, \quad (3.5)$$

where $\eta_0 = [C_y^2 + \alpha\phi C_z^2(\alpha\phi - 2k_{yz})]$.

3.2 The Bias and *MSE* of the Estimator 'P_m'

To obtain the bias and *MSE* of the estimator P_m at (2.2), we express P_m in terms of e's:

$$P_m = \bar{Y}(1+e_{0m})(1+e_{1n})^{\alpha_1}(1+e_{1m})^{-\alpha_1}(1+\phi e_{2n})^{-\alpha_2}(1+\phi e_{2m})^{-\alpha_3}, \quad (3.6)$$

We suppose that $|e_{1n}| < 1$, $|e_{1m}| < 1$, $|\phi e_{2n}| < 1$ and $|\phi e_{2m}| < 1$, so that $(1+e_{1n})^{\alpha_1}$, $(1+e_{1m})^{-\alpha_1}$ $(1+\phi e_{2n})^{-\alpha_2}$ and $(1+\phi e_{2m})^{-\alpha_3}$ are expandable. Expanding the right hand side of (2.9), multiplying out and neglecting terms of e's having power greater than two, we have

$$P_m \cong \bar{Y}\Big[1 + e_{0m} + \alpha_1(e_{1n} - e_{1m}) - \phi(\alpha_2 e_{2n} + \alpha_3 e_{2m}) + \alpha_1(e_{0m}e_{1n} - e_{0m}e_{1m}) - \phi(\alpha_2 e_{0m} e_{2n}$$
$$+ \alpha_3 e_{0m} e_{2m}) + \alpha_1\alpha_2\phi(e_{1m}e_{2n} - e_{1n}e_{2n}) + \alpha_1\alpha_3\phi(e_{1m}e_{2m} - e_{1n}e_{2m}) + \phi^2\alpha_2\alpha_3 e_{2m}e_{2n}$$
$$+ \alpha_1\left\{\frac{(\alpha_1-1)}{2}e_{1n}^2 + \frac{(\alpha_1+1)}{2}e_{1m}^2 - \alpha_1 e_{1m}e_{1n}\right\} + \frac{\phi^2}{2}\left\{\alpha_2(\alpha_2+1)e_{2n}^2 + \alpha_3(\alpha_3+1)e_{2m}^2\right\}\Big]$$

or

$$(P_m - \bar{Y}) \cong \bar{Y}\Big[e_{0m} + \alpha_1(e_{1n} - e_{1m}) - \phi(\alpha_2 e_{2n} + \alpha_3 e_{2m}) + \alpha_1(e_{0m}e_{1n} - e_{0m}e_{1m}) - \phi(\alpha_2 e_{0m} e_{2n}$$
$$+ \alpha_3 e_{0m} e_{2m}) + \alpha_1\alpha_2\phi(e_{1m}e_{2n} - e_{1n}e_{2n}) + \alpha_1\alpha_3\phi(e_{1m}e_{2m} - e_{1n}e_{2m}) + \phi^2\alpha_2\alpha_3 e_{2m}e_{2n}$$
$$+ \alpha_1\left\{\frac{(\alpha_1-1)}{2}e_{1n}^2 + \frac{(\alpha_1+1)}{2}e_{1m}^2 - \alpha_1 e_{1m}e_{1n}\right\} + \frac{\phi^2}{2}\left\{\alpha_2(\alpha_2+1)e_{2n}^2 + \alpha_3(\alpha_3+1)e_{2m}^2\right\}\Big] \quad (3.7)$$

Taking expectation on both sides of (3.7), we get the bias of the proposed class of estimators P_m up to first degree of approximation as

$$B(P_m) = (\bar{Y}/2)[\Gamma\alpha_1 C_x^2(\alpha_1 - 2k_{yx} + 1) + \ell C_z^2\{\alpha_2\phi(\phi(\alpha_2 + 1) - 2k_{yz})$$
$$+ \alpha_3\phi(\alpha_2\phi - 2k_{xz}\alpha_1)\} + \lambda\alpha_3\phi C_z^2\{\phi(\alpha_3 + 1) + 2\alpha_1 k_{xz} - 2k_{yz}\}]$$

Squaring both sides of (3.7) and neglecting terms of e's having power greater than two, we have

$$\begin{aligned}(P_m - \bar{Y})^2 &\cong \bar{Y}^2[e_{0m} + \alpha_1(e_{1n} - e_{1m}) - \phi(\alpha_2 e_{2n} + \alpha_3 e_{2m})]^2 \\ &\cong \bar{Y}^2[e_{0m}^2 + \alpha_1^2(e_{1n} - e_{1m})^2 + \phi^2(\alpha_2 e_{2n} + \alpha_3 e_{2m})^2 \\ &+2\alpha_1(e_{0m}e_{1n} - e_{0m}e_{1m}) - 2\phi(\alpha_2 e_{0m}e_{2n} + \alpha_3 e_{0m}e_{2m}) \\ &-2\alpha_1\phi\{\alpha_2(e_{1n}e_{2n} - e_{1m}e_{2n}) + \alpha_3(e_{1n}e_{2m} - e_{1m}e_{2m})\}].\end{aligned} \tag{3.8}$$

Taking expectation on both sides of (3.8), we get the *MSE* of the proposed class of estimators P_m up to first degree of approximation as

$$MSE(P_m) = \bar{Y}^2(m^{-1}\eta_1 + n^{-1}\eta_2 - N^{-1}\eta_3),$$

where $\eta_1 = [C_y^2 + \alpha_1 C_x^2(\alpha_1 - 2k_{yx}) + \alpha_3\phi C_z^2(\alpha_3\phi - 2k_{yz} + 2\alpha_1 k_{xz})]$,

$$\eta_2 = [\alpha_1 C_x^2(2k_{yx} - \alpha_1) + \phi C_z^2\{\alpha_2(\alpha_2\phi - 2k_{yz}) + 2\alpha_3(\alpha_2\phi - \alpha_1 k_{xz})\}],$$

and

$$\eta_3 = (\eta_1 + \eta_2) = [C_y^2 + \phi C_z^2(\alpha_2 + \alpha_3)\{\phi(\alpha_2 + \alpha_3) - 2k_{yz}].$$

Thus we state the following theorem.

Theorem 3.2 To the first degree of approximation the bias and *MSE* of the estimator P_m are respectively, given by

$$B(P_m) = \left(\frac{\bar{Y}}{2}\right)[\Gamma\alpha_1 C_x^2(\alpha_1 - 2k_{yx} + 1) + \ell C_z^2\{\alpha_2\phi(\phi(\alpha_2 + 1) - 2k_{yz}) + \alpha_3\phi(\alpha_2\phi - 2k_{xz}\alpha_1)\}$$
$$+ \lambda\alpha_3\phi C_z^2\{\phi(\alpha_3 + 1) + 2\alpha_1 k_{xz} - 2k_{yz}\}] \tag{3.9}$$

$$MSE(P_m) = \bar{Y}^2(m^{-1}\eta_1 + n^{-1}\eta_2 - N^{-1}\eta_3) \tag{3.10}$$

where η_i 's (i = 1, 2, 3) are same as defined earlier.

3.3 The Covariance between P_u and P_m

The covariance between P_u and P_m to the first degree of approximation is obtained as follows:

$$\begin{aligned}Cov(P_u, P_m) &= E[(P_u - \bar{Y})(P_m - \bar{Y})] \\ &\cong \bar{Y}^2 E[(e_{ou} - \alpha\phi e_{2u})\{e_{0m} - \alpha_1(e_{1m} - e_{1n}) - \phi(\alpha_2 e_{2n} + \alpha_3 e_{2m})\}] \\ &\cong \bar{Y}^2 E[e_{ou}e_{0m} - \alpha_1(e_{ou}e_{1m} - e_{ou}e_{1n}) - \phi(\alpha_2 e_{ou}e_{2n} + \alpha_3 e_{ou}e_{2m}) \\ &- \alpha\phi e_{ou}e_{0m} + \phi\alpha\alpha_1(e_{1m}e_{2u} - e_{1n}e_{n2u}) + \phi^2\alpha(\alpha_2 e_{2u}e_{2n} + \alpha_3 e_{2u}e_{2m})] \\ &= -\left(\frac{\bar{Y}^2}{N}\right)[C_y^2 + \phi C_z^2\{\alpha\phi(\alpha_2 + \alpha_3) - k_{yz}(\alpha + \alpha_2 + \alpha_3)\}]\end{aligned} \tag{3.11}$$

Thus we state the following theorem.

Theorem 3.2 To the first degree of approximation the covariance between of the estimators P_u and P_m is given by

$$Cov(P_u, P_m) = -\left(\frac{\bar{Y}^2}{N}\right)\eta_4 \tag{3.12}$$

where $\eta_4 = [C_y^2 + \phi C_z^2\{\alpha\phi(\alpha_2 + \alpha_3) - k_{yz}(\alpha + \alpha_2 + \alpha_3)\}]$.

The *MSE* of the combined estimator 'P' given by

$$MSE(P) = MSE(P_m) + \omega^2[MSE(P_u) + MSE(P_m) - 2Cov(P_u, P_m)]$$

$$-2\omega[MSE(P_m) - Cov(P_u, P_m)] \tag{3.13}$$

Substituting the values of $MSE(P_u)$, $MSE(P_m)$ and $Cov(P_u, P_m)$ from (3.5), (3.10) and (3.12), we get the *MSE* of P to the first degree of approximation, as

$$MSE(P) = \bar{Y}^2 \left[\left(m^{-1}\eta_1 + n^{-1}\eta_2 - N^{-1}\eta_3 \right) + \omega^2 \left\{ u^{-1}\eta_0 + m^{-1}\eta_1 + n^{-1}\eta_2 - \left(\frac{\phi^2 C_z^2}{N} \right)(\alpha_2 + \alpha_3 - \alpha) \right\} \right.$$

$$\left. - 2\omega \left\{ m^{-1}\eta_1 + n^{-1}\eta_2 - \left(\frac{\phi C_z^2}{N} \right)(\alpha_2 + \alpha_3 - \alpha)\left(\phi(\alpha_2 + \alpha_3) - k_{yz} \right) \right\} \right] \tag{3.14}$$

Now we make the following assumption:

Assumption 3.1

(i) Population size *N* is large enough (i.e. $N \to \infty$) so that the finite population correction (fpc) terms are ignored.

(ii) Since *x* and *y* denoted the same study variable over two occasions and *z* is the stable auxiliary variable correlated to *x* and *y* , therefore we assume that the coefficients of variation of *x*, *y*, *z* are approximately equal (i.e. $C_y \cong C_x \cong C_z$) for instance see Murthy (1967), Reddy (1978), Cochran(1977) and Fen and Zou (1977).

(iii) " $\rho_{xz} = \rho_{yz}$ ". This is an intuitive assumption, which has been considered by Cochran (1977), Fen and Zou (1977) and Singh and Priyanka (2008).

Under the assumption 3.1, the *MSE* of P at (3.14) reduce to:

$$MSE(P) = [m^{-1}\eta_1^* + n^{-1}\eta_2^* + \omega^2(u^{-1}\eta_0^* + m^{-1}\eta_1^* + n^{-1}\eta_2^*) - 2\omega(m^{-1}\eta_1^* + n^{-1}\eta_2^*)],$$

where $\eta_0^* = [1 + \alpha\phi(\alpha\phi - 2\rho_{yz})]$, $\eta_1^* = [1 + \alpha_1(\alpha_1 - 2\rho_{yx}) + \alpha_3\phi(\alpha_3\phi - 2\rho_{yz} + 2\alpha_1\rho_{yz})]$

and $\eta_2^* = [\alpha_1(2\rho_{yx} - \alpha_1) + \phi\{\alpha_2(\alpha_2\phi - 2\rho_{yz}) + 2\alpha_3(\alpha_2\phi - \alpha_1\rho_{yz})\}]$.

We express the $MSE(P)$ at (3.15) in terms $\mu(= u/n)$ as

$$MSE(P) = \frac{S_y^2}{n\mu(1-\mu)}[\mu(\eta_1^* + \eta_2^*) - \mu^2\eta_2^* + \omega^2\{\eta_0^* + \mu(\eta_1^* + \eta_2^* - \eta_0^*) - \mu^2\eta_2^*\}$$

$$-2\omega\mu\{(\eta_1^* + \eta_2^*) - \mu\eta_2^*\}] \tag{3.16}$$

This is minimized when

$$\omega_{opt} = \frac{\mu(\eta_1^* + \eta_2^* - \mu\eta_2^*)}{\{\eta_0^* + \mu(\eta_1^* + \eta_2^* - \eta_0^*) - \mu^2\eta_2^*\}}$$

Thus the resulting minimum *MSE* of *d* is given by

$$\min.MSE(P) = \frac{S_y^2\eta_0^*(\eta_1^* + \eta_2^* - \mu\eta_2^*)}{n\{\eta_0^* + \mu(\eta_1^* + \eta_2^* - \eta_0^*) - \mu^2\eta_2^*\}}$$

Thus we state the following theorem.

Theorem 3.4 The optimum value of ω [which minimizes the $MSE(P)$ in (3.16)] and minimum *MSE* of P are respectively given by

$$\omega_{opt} = \frac{\mu(\eta_1^* + \eta_2^* - \mu\eta_2^*)}{\{\eta_0^* + \mu(\eta_1^* + \eta_2^* - \eta_0^*) - \mu^2\eta_2^*\}}, \tag{3.17}$$

and

$$\min.MSE(P) = \frac{S_y^2\eta_0^*(\eta_1^* + \eta_2^* - \mu\eta_2^*)}{n\{\eta_0^* + \mu(\eta_1^* + \eta_2^* - \eta_0^*) - \mu^2\eta_2^*\}}. \tag{3.18}$$

where η_i's (*i* = 1, 2, 3) are same as defined earlier.

Corollary 3.1 If there is complete matching i.e. $\mu = 0$, then

$$\min.MSE(P) = \frac{S_y^2\eta_0^*}{n}. \tag{3.19}$$

Corollary 3.2 If there is no matching i.e. $\mu = 1$, then

$$\min.MSE(P) = \frac{S_y^2 \eta_0^*}{n}. \tag{3.20}$$

In both the cases, $\min.MSE(P)$ has the same value. This provides an implication that there must be an optimum choice of μ , other than extreme values so that $\min.MSE(P)$ will be fewer than the quantity given in (3.19) or (3.20). Thus, for obtaining current estimate (neither the case of "complete matching" nor the case of "no matching") more precise, it is always advisable to replace the sample partially.

4. OPTIMUM REPLACEMENT POLICY

To determine the value of μ so that the population mean $\bar{Y}$ on current (second) occasion may be estimated with maximum efficiency, we minimize $\min.MSE(P)$ in equation (3.18) with respect to μ and hence we get

$$\hat{\mu} = \frac{(\eta_1^* + \eta_2^*) \pm \sqrt{\eta_0^* \eta_1^*}}{\eta_2^*} = \mu_0(say). \tag{4.1}$$

The real values of $\hat{\mu}$ exists iff the quantity under square root is greater than are equal to zero i.e. $\eta_0^* \eta_1^* \geq 0$. For any combination of (α , α_1 , α_2, α_3, ϕ, ρ_{yx} and ρ_{yz}) which satisfies the condition $\eta_0^* \eta_1^* \geq 0$, two real values of $\hat{\mu}$ are possible , hence to select a value of $\hat{\mu}$, it should be noted that $0 \leq \hat{\mu} \leq 1$, all other values of $\hat{\mu}$ are inadmissible. Putting the admissible value of $\hat{\mu}$ say μ_0 from equation (4.1) into equation (3.18), we get the optimum value of $\min.MSE(P)$, which is shown in equation (4.2):

$$\min.MSE(P)_{opt} = \frac{S_y^2 \eta_0^* (\eta_1^* + \eta_2^* - \mu_0 \eta_2^*)}{n\{\eta_0^* + \mu_0(\eta_1^* + \eta_2^* - \eta_0^*) - \mu_0^2 \eta_2^*\}}. \tag{4.2}$$

5. EFFICIENCY COMPARISON

The percent relative efficiencies of the proposed estimator 'P' with respect to (i) $\bar{y}_n$,when there is no matching, and (ii) $\hat{\hat{Y}} = \psi \bar{y}_u + (1-\psi)\bar{y}_d$, when no auxiliary information is used at any occasion, where $\bar{y}_d = \bar{y}_m + \beta_{yx}(\bar{x}_n - \bar{x}_m)$, have been obtained for different values of (α , α_1 , α_2, α_3, ρ_{yx} , ρ_{yz}) and β_{yx} being the population regression coefficients of y on x . Since $\bar{y}_n$ and $\hat{\hat{Y}}$ are unbiased estimators of the population $\bar{Y}$, the variance of $\bar{y}_n$ and optimum variance of $\hat{\hat{Y}}$ for large N (i.e. $N \to \infty$) are, respectively given

$$V(\bar{y}_n) = \frac{S_y^2}{n}, \tag{5.1}$$

and

$$V(\hat{\hat{Y}}) = [1 + \sqrt{(1-\rho_{yx}^2)}]\frac{S_y^2}{n}, \tag{5.2}$$

The percent relative efficiencies E_1 and E_2 of P (under optimal condition) with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ are respectively given by

$$E_1 = \frac{V(\bar{y}_n)}{\min.MSE(d)_{opt}} \times 100 = \frac{[\eta_0^* + \mu_0(\eta_1^* + \eta_2^* - \eta_0^*) - \mu_0^2 \eta_2^*]}{\eta_0^*(\eta_1^* + \eta_2^* - \mu_0 \eta_2^*)} \times 100 \tag{5.3}$$

and

$$E_2 = \frac{V(\hat{\bar{Y}})_{opt}}{\min.MSE(d)_{opt}} \times 100 = \frac{[1+\sqrt{(1-\rho_{yx}^2)}][\eta_0^* + \mu_0(\eta_1^* + \eta_2^* - \eta_0^*) - \mu_0^2\eta_2^*]}{2\eta_0^*(\eta_1^* + \eta_2^* - \mu_0\eta_2^*)} \times 100 \tag{5.4}$$

To illustrate our results we have computed the percent relative efficiencies of E_1 and E_2 for (i) $\alpha = \alpha_1 = 0.50, \alpha_2 = \alpha_3 = 0.25$, (ii) $\alpha = 0.75, \alpha_1 = \alpha_2 = 0.50, \alpha_3 = 0.25$, (iii) $\alpha = \alpha_1 = 0.75, \alpha_2 = 0.50, \alpha_3 = 0.25$, and (iv) $\alpha = 1.00, \alpha_1 = 0.75, \alpha_2 = 0.50, \alpha_3 = 0.25$ and different values of ρ_{yx} and ρ_{yz}; and findings are shown in Tables 5.1 and 5.2.

Table 5.1: Optimum values μ_0 and percent relative efficiencies of the estimator P

ρ_{yx}	ρ_{yz}	ϕ	$\alpha = \alpha_1 = 0.50, \alpha_2 = \alpha_3 = 0.25$			$\alpha = 0.75, \alpha_1 = \alpha_2 = 0.50, \alpha_3 = 0.25$		
			μ_0	E_1	E_2	μ_0	E_1	E_2
0.5	0.5	0.2	0.53	116.03	108.26	0.52	119.82	111.79
		0.3	0.52	120.19	112.14	0.52	125.34	116.95
		0.4	0.52	124.05	115.74	0.51	129.96	121.25
		0.5	0.52	127.52	118.98	0.51	133.39	124.46
		0.6	0.52	130.52	121.78	0.51	135.41	126.34
		0.7	0.51	132.96	124.05	0.51	135.88	126.77
	0.6	0.2	0.53	118.21	110.29	0.52	123.25	115.00
		0.3	0.52	123.74	115.45	0.51	131.11	122.33
		0.4	0.52	129.17	120.52	0.51	138.42	129.15
		0.5	0.51	134.39	125.39	*	-	-
		0.6	0.51	139.29	129.96	0.50	149.82	139.78
		0.7	0.50	143.74	134.11	0.49	153.13	142.87
	0.7	0.2	0.52	120.48	112.41	0.52	126.90	118.40
		0.3	0.52	127.52	118.98	0.51	137.47	128.27
		0.4	0.51	134.76	125.73	0.50	148.16	138.23
		0.5	0.51	142.09	132.57	0.49	158.51	147.89
		0.6	0.50	149.39	139.39	0.48	167.97	156.72
		0.7	0.50	156.53	146.05	0.48	175.88	164.10
0.6	0.5	0.2	0.55	120.07	108.07	0.54	124.04	111.63
		0.3	0.54	124.46	112.02	0.54	129.87	116.88
		0.4	0.54	128.55	115.69	0.53	134.75	121.28
		0.5	0.54	132.23	119.01	0.53	138.39	124.55
		0.6	0.53	135.41	121.87	0.53	140.54	126.49
		0.7	0.53	137.99	124.19	0.53	141.06	126.95
	0.6	0.2	0.54	122.37	110.14	0.54	127.66	114.89
		0.3	0.54	128.23	115.40	0.53	135.96	122.36
		0.4	0.54	133.97	120.58	0.52	143.70	129.33
		0.5	0.53	139.51	125.56	0.52	150.46	135.41
		0.6	0.53	144.71	130.24	0.52	155.80	140.22
		0.7	0.52	149.43	134.49	0.51	159.33	143.40
	0.7	0.2	0.54	124.77	112.29	0.53	131.51	118.36
		0.3	0.54	132.23	119.01	0.52	142.68	128.41
		0.4	0.53	139.90	125.91	0.52	154.00	138.60

		0.5	0.53	147.68	132.91	0.51	164.99	148.49
		0.6	0.52	155.45	139.90	0.50	175.06	157.55
		0.7	0.52	163.05	146.74	0.50	183.50	165.15
0.7	0.5	0.2	0.57	124.78	106.94	0.56	128.96	110.52
		0.3	0.56	129.46	110.96	0.56	135.17	115.85
		0.4	0.56	133.82	114.70	0.55	140.38	120.31
		0.5	0.56	137.75	118.06	0.55	144.27	123.65
		0.6	0.56	141.15	120.98	0.55	146.59	125.64
		0.7	0.56	143.92	123.35	0.55	147.16	126.13
	0.6	0.2	0.57	127.23	109.05	0.56	132.80	113.82
		0.3	0.56	133.48	114.40	0.55	141.65	121.40
		0.4	0.56	139.62	119.66	0.55	149.91	128.48
		0.5	0.55	145.55	124.74	0.54	157.14	134.68
		0.6	0.55	151.12	129.52	0.54	162.88	139.60
		0.7	0.55	156.18	133.86	0.54	166.70	142.87
	0.7	0.2	0.56	129.79	111.24	0.56	136.90	117.33
		0.3	0.56	137.75	118.06	0.55	148.81	127.54
		0.4	0.55	145.96	125.10	0.54	160.90	137.91
		0.5	0.55	154.30	132.25	0.53	172.69	148.01
		0.6	0.54	162.65	139.40	0.53	183.51	157.28
		0.7	0.54	170.82	146.41	0.52	192.62	165.09

Note: '*' indicates μ_0 does not exist.

Table 5.2: Optimum values μ_0 and percent relative efficiencies of the estimator P

ρ_{yx}	ρ_{yz}	ϕ	$\alpha=\alpha_1=0.75, \alpha_2=0.50, \alpha_3=0.25$			$\alpha=1.00, \alpha_1=0.75, \alpha_2=0.50, \alpha_3=0.25$		
			μ_0	E_1	E_2	μ_0	E_1	E_2
0.5	0.5	0.2	0.51	117.01	109.17	0.74	119.78	111.75
		0.3	0.50	122.10	113.92	*	-	-
		0.4	0.50	126.28	117.82	*	-	-
		0.5	0.49	129.31	120.64	**0.24**	**130.30**	**121.57**
		0.6	0.49	130.96	122.19	0.58	130.26	121.53
		0.7	0.49	131.13	122.34	0.90	126.51	118.04
	0.6	0.2	0.51	120.23	112.18	1.00	125.00	116.63
		0.3	0.50	127.49	118.95	*	-	-
		0.4	0.49	134.17	125.19	**0.07**	**136.93**	**127.76**
		0.5	0.48	139.90	130.53	0.26	143.26	133.66
		0.6	0.48	144.31	134.64	0.38	146.58	136.76
		0.7	0.47	147.05	137.20	0.49	146.55	136.73
	0.7	0.2	0.50	123.65	115.37	*	-	-
		0.3	0.49	133.44	124.50	*	-	-
		0.4	0.48	143.23	133.63	**0.15**	**148.61**	**138.66**
		0.5	0.47	152.61	142.39	0.24	159.47	148.78
		0.6	0.46	161.05	150.27	0.31	167.83	156.59

		0.7	0.45	167.96	156.71	0.38	172.56	161.00
0.6	0.5	0.2	0.54	122.92	110.63	0.60	125.45	112.90
		0.3	0.53	128.38	115.54	0.62	131.53	118.37
		0.4	0.52	132.87	119.58	0.62	135.74	122.16
		0.5	0.52	136.11	122.50	0.58	137.61	123.85
		0.6	0.52	137.88	124.09	0.49	137.22	123.50
		0.7	0.52	138.05	124.24	0.34	135.19	121.67
	0.6	0.2	0.53	126.38	113.74	0.63	130.05	117.05
		0.3	0.52	134.18	120.76	0.70	139.61	125.65
		0.4	0.52	141.36	127.22	0.79	147.95	133.16
		0.5	0.51	147.52	132.76	0.91	153.95	138.56
		0.6	0.50	152.25	137.02	1.00	156.25	140.63
		0.7	0.50	155.19	139.67	**0.07**	**154.89**	**139.40**
	0.7	0.2	0.53	130.05	117.05	0.67	135.17	121.65
		0.3	0.52	140.57	126.51	0.85	149.73	134.76
		0.4	0.51	151.11	136.00	*	-	-
		0.5	0.50	161.22	145.10	*	-	-
		0.6	0.49	170.32	153.29	**0.02**	**174.67**	**157.20**
		0.7	0.48	177.76	159.99	0.30	182.06	163.86
0.7	0.5	0.2	0.57	130.32	111.70	0.60	132.92	113.92
		0.3	0.56	136.27	116.79	0.61	139.46	119.52
		0.4	0.56	141.15	120.98	0.60	144.04	123.45
		0.5	0.55	144.68	124.00	0.58	146.21	125.32
		0.6	0.55	146.59	125.64	0.54	145.87	125.02
		0.7	0.55	146.75	125.77	0.49	143.30	122.82
	0.6	0.2	0.56	134.09	114.92	0.61	137.81	118.11
		0.3	0.56	142.59	122.21	0.63	147.86	126.72
		0.4	0.55	150.42	128.92	0.64	156.40	134.05
		0.5	0.54	157.14	134.68	0.63	162.45	139.23
		0.6	0.54	162.30	139.11	0.59	165.31	141.68
		0.7	0.53	165.50	141.85	0.52	164.95	141.38
	0.7	0.2	0.56	138.09	118.36	0.63	143.15	122.69
		0.3	0.55	149.57	128.19	0.66	157.66	135.13
		0.4	0.54	161.09	138.07	0.71	172.00	147.41
		0.5	0.53	172.16	147.56	0.76	184.70	158.31
		0.6	0.52	182.14	156.10	0.80	193.81	166.11
		0.7	0.51	190.29	163.09	0.76	197.41	169.19

Note: '*' indicates μ_0 does not exist.

It is observed from Tables 5.1 and 5.2 that the values of percent relative efficiencies E_1 and E_2 are larger than 100 for the parametric values considered here. Thus the proposed estimator 'P' is more efficient than the usual unbiased estimator $\bar{y}_n$ and $\hat{\hat{Y}}$. It is interesting to observe from Table 5.2 that the minimum value μ_0 is 0.02, which shows that the fraction of fresh sample to be replaced at the current occasion is as low as about 2 percent of the total sample size, which highly price is saving. Also for this value of μ_0 (i.e. μ_0 =0.02) the proposed estimator ' P ' is more efficient than $\bar{y}_n$ and $\hat{\bar{Y}}$ with substantial gain in efficiency.

On the whole we conclude that there is enough scope of selecting the values of scalars (α , α_1, α_2, α_3, a, b) with different choices of correlations ρ_{yx} and ρ_{yz}; for obtaining better estimators than $\bar{y}_n$ and $\hat{\bar{Y}}$ with considerable gain in efficiency.

6. CONCLUSIONS

In the present paper we have made the use of linear and power transformations simultaneously over the auxiliary variable .We have suggested a class of estimators ' P ' for estimating the population mean on current (second) occasion in two-occasion successive sampling. We have studied the properties of suggested the class of estimator ' P ' under the large sample approximation. Optimum replacement policy relevant to the suggested estimation procedures has been discussed. The proposed estimator is compared with usual estimators numerically.

It is evident from Tables 5.1 and 5.2 the use of information on a transformed auxiliary variable is highly rewarding in improving the precision of the suggested estimator. The principal interesting thing in this study is that the proposed linear and power transformations are quite effective in reducing the cost of the survey [for instance see Table 5.2, the values of μ_0 are like 0.02, 0.07, 0.15, 0.24 etc]. Thus the suggested estimators may be recommended to the survey statisticians / practitioners for their practical applications.

RECEIVED: NOVEMBER , 2018
REVISED: JULY, 2019

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